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Domain Decomposition and Uzawa-Type Iterative Method for Elliptic Variational Inequality

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Abstract—Finite element approximation of an elliptic variational inequality with quaisilinear operator and constraints to the solution and its gradient is constructed. Corresponding discrete problem is splitted into subproblems by non-overlapping domain decomposition technique and constrained saddle point problem is constructed for the splitted problem. Block relaxation-Uzawa iterative solution method is applied to this resulting saddle point problem. Existence of a solution to saddle point problem and convergence of the iterative method are proved.

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INTRODUCTION

Domain decomposition methods for the variational inequalitie s and free boundary problems have been investigated in the numerous publications [1–22] (the list is not complete). The most attention has been paid to Schwarz-type (or, overlapping subdomains) methods for the variational inequalities with pointwise constraints to solution, such as obstacle problems, contact problems, Stefan problems on a fixed time etc. The corresponding results can be found in [1–15]. Non-overlapping domain decomposition method has been applied to variational inequalities with pointwise constraints to a solution in the supposition that the location of free boundary [16–19].

Convergence of Uzawa-type iterative methods is thoroughly investigated for a long time. The main results on this subject can be find in the monographs [23–25].

Our current research is based on the results of the papers [20–22] for the finite dimensional constrained saddle point problems. In [20] a sufficient convergence condition in terms of matrices inequalities for Uzawa iterative method has been proved. In [21, 22] a generalization of this result for wider class of saddle point problems and for so-called block relaxation-Uzawa iterative solution method have been done.

In this paper we construct and investigate an iterative method for a finite element scheme approximating an elliptic variational inequality with quaisilinear operator and constraints to the solution and its gradient. This method is based on the domain decomposition technique applied to the constructed mesh approximation of the variational inequality in the whole domain. The decomposition of the initial mesh problem is not connected with any information on the location of free boundaries. For the resulting "decomposed" discrete problem corresponding saddle point problem is constructed. This saddle point problem is solved by a preconditioned Uzawa-type iterative method combined with block relaxation method. The implementation of every step of the iterative method consists of the solution of a non-coupled systems of subproblems and updating of Lagrange multipliers on the artificial boundaries, thus, the main part of the algorithm can be parallelized.

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