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Optimal by the Order Methods of Solving Integral Equations in a Special Case

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Abstract—We study an integral equation of the third kind with fixed singularities of the kernel. We suggest and substantiate special generalized versions of spline methods for approximate solving these equations in the space of distributions. We show that the constructed methods are optimal by the order.

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We investigate the linear third kind integral equation with fixed singularities of the kernel (TKIEFS)

$$Ax \equiv x(t)\prod_{j=1}^{l}(t-t_j)^{m_j} + \int_{-1}^{1} K(t,s)[(s+1)^{p_1}(1-s)^{p_2}]^{-1}x(s)ds = y(t) \qquad (1)$$

where $t \in I \equiv [-1,1]$, $t_j \in (-1,1)$, $m_j \in \mathbb{N}$ $(j=\overline{1,l})$; $p_1, p_2 \in \mathbb{R}^+$, K and y are known continuous functions satisfying some smooth properties of pointwise type, $x(t)$ is the desired function; here the integral is understood in the sense of Hadamard finite part ([1], P. 144–150). Equations of the type (1) find increasingly wide use in the theory and applications. Investigation of some important problems of the elasticity theory, electron transfer, particle scattering (see, e.g., [2, 3] and the bibliography in [2]), and study of mixed type equations [4] lead to such equations. Natural classes of solutions to the third kind integral equation (TKIE) are, as a rule, some special spaces of distributions (SD) of type D or V. We mean by D (V) the SD constructed on the base of the functional which is the "Dirac δ-function" (correspondingly, the "Hadamard finite part integral"). The considered equations can be solved exactly only in few particular cases, therefore, creation of theoretically proved effective methods of their approximate solving in SD is an important and actively developing direction of mathematical analysis and computing mathematics. In [5–9], some results in the field were obtained; there were suggested and substantiated special methods of solving (1) in spaces of type D. The first results on approximate solution of TKIEFS in SD of the type V were obtained in [10] where it was developed and substantiated a "polynomial" method of solving equations of the type (1) in a space X of distributions.

In the paper we suggest new variants of spline-methods specially suitable for approximate solving (1) in SD of the type V. We give their theoretic substantiation in the sense of [11] (Chap. 1), on some class F generated by H_ω^r and show that the developed methods are optimal by the order among all direct projective methods of solving the investigated equations in the space $X \equiv V^{\{p\}}\{m;0\}$.

1. Basic spaces. Let $C \equiv C(I)$ be the space of continuous on I functions with the usual max-norm and $m \in \mathbb{N}$. Following [12], we say that a function $f \in C$ belongs to $C\{m;0\} \equiv C_0^{\{m\}}(I)$, if at the point

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