

# Application of harmonic analysis for the quantitative description of Earth surface topography

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## Abstract

The paper is focused at perspectives of Discrete Fourier Transform (DFT) as a tool for morphometric studies. Characterized are the mathematical basics and MathCAD algorithm of one- and two-dimensional DFT when applied to topographical profiles and digital elevation models (DEMs). One-dimensional DFT allows to replace an empirical spatial series with a periodic function as a sum of waves with different frequencies and wavelengths. Amplitude of each wave describes the contribution of the corresponding frequency to the total elevation variations along the studied profile, which allows to compare different harmonics (waves of certain frequency) in terms of their importance in shaping the Earth surface. Two-dimensional DFT allows also to determine the direction of wave distribution. In an ideal fractal erosional system in which different-order thalwegs have equal ranking both by width and by depth of incision, strict decrease of harmonic amplitude with increasing spatial frequency would occur. However this feature is only rarely occurring in the real Earth surface. Consequently, the DFT method can be used for spatial classification and regionalization according to topographic patterns formally described by harmonic components of the Earth surface elevation.

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## Keywords

Amplitude spectra, Earth surface morphometry, Fourier transform of topographic data, Harmonic analysis, Periodicity

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