Degrees of Categoricity of Rigid Structures

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Abstract. We prove that there exists a properly 2-c.e. Turing degree $\mathbf{d}$ which cannot be a degree of categoricity of a rigid structure.

Keywords: Categoricity spectrum · Strong degree of categoricity · Rigid structure · 2-c.e. Turing degrees

1 Introduction

The study of effective categoricity for computable structures goes back to the works of Fröhlich and Shepherdson [1], and Mal'tsev [2,3]. In recent years, the focus of the research in the area is on computable categoricity relative to Turing degrees.

Definition 1. *Let* $\mathbf{d}$ *be a Turing degree. A computable structure* $\mathcal{A}$ *is* $\mathbf{d}$-computably categorical *if for every computable copy* $\mathcal{B}$ *of* $\mathcal{A}$*, there is a* $\mathbf{d}$*-computable isomorphism from* $\mathcal{A}$ *onto* $\mathcal{B}$*. The* categoricity spectrum *of* $\mathcal{A}$ *is the set*

$$\mathrm{CatSpec}(\mathcal{A}) = \{\mathbf{d} : \mathcal{A} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

A Turing degree $\mathbf{d}$ *is the* degree of categoricity *of* $\mathcal{A}$ *if* $\mathbf{d}$ *is the least degree in the spectrum* $\mathrm{CatSpec}(\mathcal{A})$*.*

Categoricity spectra and degrees of categoricity were introduced in [4]. Suppose that n is a natural number and α is an infinite computable ordinal. Fokina, Kalimullin, and Miller [4] proved that each Turing degree $\mathbf{d}$ that is 2-c.e. in and above $\mathbf{0}^{(n)}$ is the degree of categoricity for a computable structure. Csima, Franklin, and Shore [5] extended this result to hyperarithmetical degrees. They

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