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IRREDUCIBLE, SINGULAR, AND CONTIGUOUS DEGREES

I. I. Batyrshin

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We study structures of degrees of stronger algorithmic reducibilities inside the degrees of weaker algorithmic ones. Results in this area are reviewed for algorithmic reducibilities m-, 1-, tt-, wtt-, T-, e-, s-, Q-, and we formulate questions that are still not settled for these. A computably enumerable Q-degree which consists of one computably enumerable m-degree is constructed.

INTRODUCTION

Algorithmic reducibilities which make it possible to reduce decidability of some problems to decidability of others are the key tool for studying complexity of mathematical problems in computability theory. If a set A R-reduces to a set B (written $A \leq_R B$), where R is one of the algorithmic reducibilities, then the set B is meant to be more complex than the set A, and the reducibility R produces an algorithm for computing A using B as an oracle.

The basic algorithmic reducibilities used for these purposes are transitive and reflexive and thereby generate a degree structure on 2^ω: for every set $A \subseteq \omega$, the reducibility R specifies an R-degree $a_R = \{B \subseteq \omega \mid A \equiv_R B\}$ of this set.

We say that a reducibility r is *stronger* than a reducibility R if $A \leq_r B \Rightarrow A \leq_R B$ for all sets A and B. It is clear that in this case, for any set A, its r-degree lies inside its R-degree (written $a_r \subseteq a_R$). In computability theory, therefore, one of the objects of investigation is the structure of degrees of a stronger reducibility inside the degrees of a weaker one. In particular, attention is focused on the following natural questions: whether there exists an R-degree consisting of exactly n

Kazan (Volga Region) Federal University, ul. Kremlevskaya 18, Kazan, 420008 Russia; batyrshin@gmail.com.
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