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SECONDARY SCHOOL MATHEMATICS TEACHERS' DISPOSITION TOWARD MISTAKES: CROSS-CULTURAL MIXED METHODS STUDY

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Abstract. The reported mixed methods study focused on exploration of mathematics teachers' dispositions toward errors in two countries – Mexico and USA. More specifically, this study addressed borderland secondary teachers' dispositions toward mistakes in teaching and learning mathematics. An explanatory sequential method design was used that involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative analysis. The instrument - Error Orientation Questionnaire (EOQ) - was used to collect quantitative data on teachers' disposition toward errors in the context of their own learning and their students learning. Following up on EOQ, the semi-structured interview was used to collect qualitative data. Two research questions guided the study. First, we were interested in what dispositions toward mathematical mistakes do secondary teachers in the US and Mexico have. The second question addressed ways the interview aimed at teachers' dispositions toward errors helped to explain the quantitative results.

Keywords: teacher disposition, orientation toward errors, secondary school mathematics, cross-cultural study.

Introduction

There are just a few things that can be stated categorically and one of them is the fact that we all make mistakes; starting from this premise a question immediately arises about why if everybody commits errors, these are usually frowned upon. As the saying goes, we learn from our mistakes. Latin proverb says, by ignorance we mistake, and by mistakes we learn. Or seen from another angle, as the Chinese proverb says, success in the end erases all the mistakes along the way, and, the mistakes of others are good teachers, says the Russian Proverb. All these proverbs and sayings seem to provide a clear understanding of mistake benefit, though doubt still exists as to whether those could be taken with sincerity or are just a way of comfort.

Mistakes are part of life, as such, they are immersed in all human practices. Even though, in some cultures, mistakes are much related to blame, some others cultures use those to improve human activity in all domains. Education is no exception, errors are perceived in different ways, even a same mistake would have a different treatment depending on cultural issues. For example, U.S. and Chinese students produce similar types and quantity of mathematical errors, but teachers' attitudes toward errors are very different.

Learning from mistakes is an essential part of how students learn mathematics (NCTM, 2000; de Estudio, 2011). However, teachers do not always support their students to engage with mistakes productively (Gojak, 2013; Ingram, Pitt, & Baldry, 2015). The way that teachers respond to productive errors can encourage or discourage student thinking and learning (Gojak, 2013). In this sense, research studies have shown that student reactions toward errors are closely related to how teachers handle mistakes during their teaching (Ingram, Pitt, & Baldry, 2015).

Besides, talking about mistakes and how we perceive them makes think about dispositions. The dispositions construct has been recently adopted by education researchers. Hence, disposition as a construct has been modified. At the beginning, dispositions were conceived as tendencies (Splitter, 2003), now a wide range of elements have been incorporated to understand the complexity of this construct. In this regard, Beyers (2011) conceptual framework for students' dispositions with respect to mathematics will serve as a model for defining and categorizing teacher dispositions toward errors. In this sense, mathematics teachers' dispositions toward errors are studied from three different mental functions: cognitive, affective, and conative.

The purpose of this cross-cultural study is to explore mathematics teachers' dispositions toward errors in Mexico and USA using the Error Orientation Questionnaire (EOQ). Thus, this study addresses borderland secondary teachers' dispositions toward mistakes in teaching and learning mathematics. An explanatory sequential mixed methods design was used, since it involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative data. The research question that guides this mixed methods study are: 1) What dispositions toward mathematical mistakes do secondary teachers in the US and Mexico have? 2) In what ways do the interview data reporting teachers' dispositions toward errors on the context of their own learning and their students learning help to explain the quantitative results about dispositions toward errors reported on the EOQ?

Research on teacher beliefs about errors in mathematics

Mathematics education research has studied errors in general from many different perspectives. On the one hand, some approaches focused on errors and misconceptions with the aim of understanding their sources or causes to prevent or correct them (Tariq, 2008; Tsitsipis, Stamovlasis, & Papageorgiou, 2012). Others addressed errors based on mathematics mistakes/errors diagnosis pattern analysis (e.g., Ayres, 2001; Livy & Vale, 2011; Riccomini, 2005). All these studies are some examples of how quantitative research paradigm has been used to address mathematical errors. Addressing errors using teachers' beliefs as an overarching perspective to understand errors is an emerging topic in mathematics education research. Using a mix methods approach, Tulis (2013) found that teachers' beliefs about errors will be impact their error management in the classroom, which in turn is highly likely to influence students' attitudes towards learning from mistakes. Using a qualitative approach, Borasi (1987) found that mathematics teachers perceive their students' errors as valuable tools and resources for remediation teaching strategies. However, these types of teachers' beliefs and interpretations about errors do not enable students to participate in a process to detect their own error, and even more important to participate also in explaining, analyzing, correcting, and discussing them. Borasi (1994) proposed a group of strategies to help teachers to capitalize on errors to engage their students in their mathematical learning processes. Similarly, Santagata (2005) examines teachers' beliefs about mistakes and their error handling practices and how cultural factors impact both. She states that teachers' mistakes handling activities are influenced by their beliefs and cultural practices. Teacher positive beliefs about errors and appropriate ways of error- handling practices will impact and change their students' attitudes about errors (Bray, 2011; Tulis, 2013). Teachers who perceive and manage errors positively encourage students to move forward to analyze conceptual mistakes instead of mistakes related to accuracy. From this perspective, students and teachers can change their attitudes about errors and improve their analysis skills, stimulate critical reasoning, and increase their enthusiasm by analyzing their erroneous answers (Tsamir, Rasslan, and Dreyfus, 2006).

Teacher Disposition

Since, this study explores teachers' dispositions, it is essential to understand this term. According to Katz (1993) "a disposition is a tendency to exhibit frequently, consciously, and voluntarily a pattern of behavior that is directed to a broad goal." In this regard, some authors state that experiences and environment conditions supports the manifestation of dispositions (Rogoff, Gauvain, and Ellis, 1990). Spliter (2010) makes a disposition definition compilation especially in the education field and we can notice that a common denominator in those definitions are words like "tendencies", "attitudes", "beliefs", "values", "actions", "patterns", and "behaviors"; he focuses his discussion in how some researchers have focused their research efforts on defining dispositions origins, nature, characteristics, and scopes. Also, others focused on discussion of specific types of dispositions. Katz (1993) states that "dispositions are less likely to be acquired through didactic processes than to be modeled by young children as they are around people who exhibit them." Thus, it is important that teacher model productive dispositions to their students. Dewey states from an epistemological perspective that dispositions are not state of possession, but state of performance (as is cited by Dottin, 2008).

Mathematical Disposition

There is a body of literature addressing students' and teachers' dispositions toward mathematics exist in research literature. These studies began to arise when an evaluation standard named as "mathematical disposition was proposed by the National Council of Teachers of Mathematics (NCTM). This concept was

released on The Curriculum and Evaluation Standards for School Mathematics (1989). In this document, NCTM stated that "disposition refers not simply to attitudes but to a tendency to think and to act in positive ways," adding that "this kind of information is best collected through informal observation" (NCTM 1989, p. 233). In this regard, disposition goes beyond the idea of including attitudes but it includes habits of mind as, interest, curiosity, perseverance, confidence in using mathematics, interest on the role that mathematics play on the society and culture. The National Research Council (in Kilpatrick, Swafford, and Findell [2001]) defines productive disposition as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131).

Beyers (2011) argues that there are three different types of dispositions toward mathematics, since they are mental processes. Beyers proposes the following dispositional function types: cognitive, affective, and conative (2011). In this way, Beyers (2011) argues that organizing dispositions toward mathematics by considering those three modes of mental functioning allows us to analyzed students' (teachers') dispositions in a systematic way. This framework not only offers a clear definition of dispositions in regards to mathematics, but it offers a research possibility to understand how dispositional cognitive, affective, and conative functioning can influence their learning in mathematics in regard to different elements, including errors.

To conclude, some of the above studies have analyzed the evolution of research of mathematics education of teachers' beliefs about errors and their strategies to handle them by using quantitative methods. Some others have focused on cultural aspects of educational practices related to mathematics teachers' beliefs about mistakes, while others have studied errors-handling strategies from a student learning perspective. Only a few studies have explored teachers' beliefs about errors and the relationships between beliefs and practices by using qualitative methods. The idea of using the construct of disposition toward mathematics is an emerging topic, which has not been addressed with a specific focus on mistakes. Thus, this research will provide relevant information of teachers' cognitive, affective, and conative dispositions toward errors and the relationships between their beliefs and their instructional practices to analyze the pertinence of using mistakes as part of their instructional approaches.

Methods

Methods and Data Sources

A mixed methods sequential explanatory design (Creswell, Plano Clark, Gutmann, & Hanson, 2003) was used to answer the study research questions. It involved two phases: 1) quantitative and 2) qualitative phase. The methodology and data gathering process for each phase is described below.

In this study, quantitative data contributed to identify the types of teachers' dispositions toward errors, and in this way, the result guide the design of the quantitative data collection instruments. Then, a qualitative multiple case (four cases) was used to explain teachers' dispositions toward errors in their teaching and learning processes. Therefore, the quantitative data provided a general picture about teachers dispositions toward errors, while the qualitative part will explain these dispositions in teachers and their students learning processes. The priority in this study will be given to the qualitative phase, since this study focuses on the in-depth explanation of the results obtain by the EOQ.

Phase I – Quantitative

The purpose of the first phase was to determine secondary mathematics teachers' error orientation in Juarez-El Paso border region, based on teachers' beliefs, as determined by the EOQ. The survey instrument was developed by Rywobiak et al. (1999), and it consists of eight domains on attitudes in dealing with errors at work. All constructs were validated by the authors. The domains comprise "error competence", "learning from errors", "error risk taking", "error strain", "error anticipation", "covering up errors". In a second study, items were added to the domain and two additional domains, "error communication" and "thinking about errors", were included. The EOQ was adapted to a teacher environment.

Data Collection and Analysis.

The pilot study was conducted by using a non-probabilistic convenience sampling procedure for selecting the participants. The EOQ was administrated face-by-face by the researcher to N=164 teachers in US (n=78) and Mexico (n=86). The EOQ was administered in conjunction with demographic questions and

open-ended responses in different sites. For the US teachers, the site was the University only. On the contrary, the site in Mexico included four middle schools and three high schools.

Univariate and multivariate statistical procedures were conducted to analyzed data. Demographic information was analyzing using cross tabulations and frequency counts. A table of correlations was constructed with the aim of selecting the qualitative participants.

Phase II – Qualitative

A multiple case study design (n=4) was used. The unit of analysis was disposition toward errors by two teachers from Mexico and two from US. The interview protocol was grounded in the quantitative results that were obtained in the first phase. A total of sixteen open-ended questions were designed.

Data collection and Analysis.

The interviews were conducted with each of the four participants. Each interview was audio recorded and transcribed by using express scribe transcription software. The data analysis was made by using the NVivo-11 software. This software was used on the coding line by line process. Then, those codes were used with the aim of determining the emerging themes that provide the resources for understanding teachers' dispositions.

Preliminary Findings

Results of the Pearson correlation indicated that there was a significant negative association between years of experience and the teacher error orientation scores, (r (76) = -0.58322, $p < .001$). Whereas, for the case of Mexico, the correlation between teachers' years of experience and the error orientation scores was found to be non-statistically significant, (r (82) = .0522, $p < .001$). Some other results show that the correlation between cognitive and affective error orientation domains was found to be statistically significant, r (162) =.173, p <.05. Whereas, the correlation between cognitive and conative was found not statistically significant, r (162) = 0.155, p <.05. As well as, the correlation between affective and conative was found not statistically significant, r (112)= -.0849, p<.05. Also, we found a difference between males and females in regards to error strain domain, more specifically to feel embarrassed when they make a mistake, since we find an average of 71% women versus 48% men.

Conclusion

The main results of the study suggest that there are multiple emerging themes in teacher disposition toward errors such as error competence, learning from errors, error strain, error communication, to name a few. The study also found that there is a gender difference between teachers' affective disposition toward mistakes: female teachers expressed embarrassment toward their own mistakes, whereas male teachers did not. With regard to students' mistakes, however, female teachers demonstrated productive dispositions, considering mistakes learning opportunities. In this same regard, male teachers expressed that when they make a mistake in front of the class they try to joke about it. They feel that making mistakes in teaching mathematics is a way of connecting to student learning.

Regarding students' mistakes, both male teachers stated that they attempt to connect student mistakes to real life situations. Findings also showed that there is commonality between Mexican and the U.S. teachers' cognitive dispositions toward errors. Participating teachers reported using students' mistakes as a pedagogical resource that could support deeper understanding through asking student to debate different ways of thinking. Overall, teachers expressed productive dispositions toward students' mistakes since they thought that errors could help students to make connections in learning mathematics.

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