structure and to symmetric wormholes. Black hole horizons in space-times with zero scalar curvature are shown to be either simple or double. The same is generically true for horizons inside a matter distribution, but in special cases there can be horizons of any order. A few simple examples are discussed. A natural application of the above results is the brane world concept, in which the trace of the 4D gravity equations is the only unambiguous equation for the 4D metric, and its solutions can be continued into the 5D bulk according to the embedding theorems.

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A DEGENERATE CONICS CONSTRUCTION FROM CONTROL POINTS OF A PLANAR CUBIC BÉZIER CURVE

Conic sections play an important role in computational geometry, computer graphics and other fields related with CAGD and their useful geometric properties have been and are still being discovered (1).

The aim of this paper is to construct a family of conics from five points of a planar cubic Bézier curve, four of them are the control points of a cubic Bézier curve and the fifth one is any point on this curve and research the degenerate cases.

The Bézier curve of degree n can be generalized as follows.

$$\mathbf{b}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{b}_i = (1-t)^n \mathbf{b}_0 + \\ + \binom{n}{1} (1-t)^{n-1} t \mathbf{b}_1 + \cdots + \binom{n}{n-1} (1-t) t^{n-1} \mathbf{b}_{n-1} + \\ + t^n \mathbf{b}_n, \quad t \in [0,1] \quad (2)$$

where given points $\mathbf{b}_0, \mathbf{b}_1, \ldots, \mathbf{b}_n$ are the control points of a Bézier curve.

The conic can be constructed by the Braikenridge – Maclaurin construction by applying the Braikenridge – Maclaurin theorem, which is the converse of Pascal's theorem. Pascal's theorem states that given six points on a conic (a hexagon), the lines defined by opposite sides intersect in three collinear points. This can be reversed to construct the possible locations for a sixth point, given five existing ones.

So to construct a conic we will use four control points of a cubic Bézier curve $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ and the fifth point given by (2), i.e. any point on the curve.

The implicit equation of conic is given by

$$\begin{vmatrix} x^2 & xy & y^2 & x & y & 1 \\ x_0^2 & x_0y_0 & y_0^2 & x_0 & y_0 & 1 \\ x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x^2(t_*) & x(t_*)y(t_*) & y^2(t_*) & x(t_*) & y(t_*) & 1 \end{vmatrix} = 0, \qquad (1)$$

where $x(t_*)$ and $y(t_*)$ are coordinates of the point $\mathbf{b}(t)$ on a curve with parameter $t \in [0,1]$.

If the three of five points are collinear then the conic always becomes a pair of intersecting straight lines, i.e. degenerate conic. There might be only $C_3^5 = 10$ cases when three of five points are collinear and this cases have been fully analysed in this work. Let's consider a simple example.

Construct a conics family from five points of cubic Bézier curve with given control points $\mathbf{b}_0 = (1,3)$, $\mathbf{b}_1 = (3,7), \mathbf{b}_2 =$ $= (8,4)$, $\mathbf{b}_3 = (9,2)$. Find all values of parameter t when conic becomes degenerate. The following figures show only degenerate conic sections.

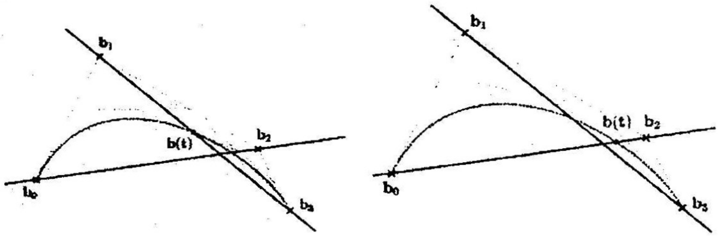

Fig. The same degenerate conic sections at $t_1 = \frac{34+\sqrt{714}}{13} \simeq 0.559$ and $t_2 = \frac{26-\sqrt{130}}{21} \simeq 0.695$.

REFERENCES

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