The manuscript

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THE THEORETICAL DESCRIPTION OF NON-EQUILIBRIUM CRITICAL BEHAVIOR OF DISORDERED TWO-DIMENSIONAL XY-MODEL BY MONTE-CARLO METHODS

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The abstract
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Abstract sent «__» April 2016 г.

General description of work

Actuality of the theme

The features of the critical behavior of systems near the critical point of second order phase transition are related to the occurrence of abnormally in critical condition large, long-lived and strongly interacting fluctuations of basic thermodynamic quantities. Dimension reduction of systems leads to increase the effects of the fluctuation.

A special place among the low-dimensional systems occupy a two-dimensional systems with continuous symmetry. It is known that in these systems, long-range order is destroyed for all finite temperatures. However, the case of two-dimensional XY-model is characterized by the implementation of this system the Berezinskii-Kosterlitz-Thouless topological phase transition with temperature of transition T_{BKT}. The phase transition is associated with the dissociation of associated vortex-antivortex pairs at the transition point. A feature of this the system is abnormally strong spatial and temporal correlation of states of the system in all of the low-temperature phase $T < T_{\mathrm{BKT}}$, characterized by a power law decay, while the thermodynamic phase transitions effects of strong correlation performed only near the critical point. This makes it possible to observe the slow dynamics of two-dimensional XY-model not only near the critical point and in all low temperature range $T \leq T_{\mathrm{BKT}}$.

A two-dimensional XY-model is one of the basic models of the fundamental studies of phase transitions and critical phenomena and is used to describe a broad range of real physical systems [1, 2], in particular ultrathin magnetic films, planar magnetics «light plane» type, superfluid films, two-dimensional crystals and two-dimensional turbulence.

Non-equilibrium relaxation and the effect of structural disorder on the critical properties are of considerable scientific interest, although the equilibrium properties of two-dimensional XY-model far enough investigated. It is connected with the phenomena of slow relaxation of the initial non-equilibrium state, in particular the effects of aging, memory and violations of the fluctuation-dissipation theorem, as well as the influence on them a strong vortex of disequilibrium, a quasi-order clustering areas and vortex pinning on defects.

The aims

– study of the influence of structural disorder on the non-equilibrium effects of aging and violation of the fluctuation-dissipation theorem in the system.

– determine the effect of the non-equilibrium dynamics of the vortex and spin-wave excitations in the elementary process of slow relaxation of the system critical initial vortex vortex-free and non-equilibrium state.

– study of non-equilibrium critical clustering system with defects of various

initial non-equilibrium states, the study of the mutual influence of local ordering process in a quasi-order and non-equilibrium processes of vortex dynamics.

– numerical study of lateral stiffness structurally disordered system in a low-temperature phase, determine the applicability of the spin-wave approximation and compared with the results for the homogeneous system.

– study of dynamic critical properties of two-dimensional XY-model as part of dissipative model critical relaxation and models conserved order parameter.

Scientific novelty of the results

1. For the first time carried out a numerical simulation of non-equilibrium critical relaxation structurally disordered two-dimensional XY-model across the low-temperature phase $T \leq T_{\mathrm{BKT}}$ at a wide range of concentrations of spin p from two different non-equilibrium initial states – low-temperature $T_0 = 0$ and high-temperature $T_0 \gg T_{\mathrm{BKT}}$. The effects of aging and disorders fluctuation-dissipation theorem in a wide range of waiting times t_w are investigated are are determined by the asymptotic behavior transition $t_w \to \infty$ on the dynamic scaling modes. The temperature dependence of the asymptotic values of the fluctuation-dissipation ratio X^{∞}: power dependence for the initial high-temperature initial state and universal linear dependence – for low-temperature initial state.

2. For the first time subaging effects were found for relaxation pure and disordered system from the initial low-temperature condition. It was investigated scaling laws and appropriate determined by the temperature and concentration dependence of the subaging exponent $\mu(T,p)$. Shown, that $\mu(T,p) = \eta(T,p)$, where $\eta(T,p)$ – exponent of the anomalous dimensionality of the system, determining power law decay of the spatial and temporal correlation of states of the system in the low-temperature phase.

3. For the first time it is shown that the spin-wave approximation is fundamentally not possible to get the correct temperature dependence of the transverse rigidity to the system defects, and to adequately describe the properties of disordered systems even at low temperatures requires consideration of the interaction of the vortex component to the field of structural defects.

4. For the first time carried out direct numerical study clustering in pure and disordered two-dimensional XY-model. The effect of the anomalous deceleration cluster coarsening process in a disordered system.

5. For the first time it developed and implemented a method of searching for and identifying the elementary vortex excitations in structurally disordered system. The dynamic process depending non-equilibrium vortex pinning excitations. It is shown that the dynamics of cluster coarsening determined by the dynamics of free and pinned vortex excitations and anomalous slowing down of clustering due to the slowdown in the dynamics of the vortex due to the pinning of vortex excitations

into defects.

6. It was shown that the non-equilibrium critical behavior of two-dimensional XY-model in the whole low-temperature phase $T \leq T_{\mathrm{BKT}}$ described in terms of the relaxation model A with the order parameter is not conserved, and the use of model B to save the parameters of the order is possible only at low temperatures $T \ll T_{\mathrm{BKT}}$.

The scientific and practical significance of the work

The scientific significance of the work is due to the importance of identifying the nature of non-equilibrium processes critical relaxation in structurally disordered systems and the need to develop and test methods for numerical study of non-equilibrium critical phenomena. The special significance of the work of becoming when interpreting the results of field experiments on the slow dynamics of the system, in particular in violation of communication between the correlation functions and response functions.

The practical significance of the work is due to the use of two-dimensional XY-model to describe the behavior and properties of a wide variety of physical systems, so identified in the thesis especially non-equilibrium behavior of the model should be considered in their description. In particular, the behavior of the magnetic structures Ni/Cu(100), Co/Cu(100), Co-Ni/Cu(100) on the basis of ultra thin magnetic films Ni, Co и Co-Ni with thicknesses $N < 4 - 6$ monolayers on metal substrate Cu(100) it describes a two-dimensional XY-model. Therefore, in the artificial magnetic structures, created on their basis, and with the effect of giant magnetoresistance, is necessary in the processes of switching on and off of an external magnetic field to take into account non-equilibrium effects due to the slow dynamics of these systems.

The results obtained in the thesis the results make a significant contribution to the physics of phase transitions and critical behavior of the field of study of low-dimensional systems with continuous symmetry, characterized by an abnormally slow dynamics.

The main provisions for the defense

1. Methods of numerical study of non-equilibrium critical properties of pure and disordered two-dimensional XY-model with the relaxation of various initial non-equilibrium states and method of determining the temperature and concentration dependencies of asymptotic values of the fluctuation-dissipation ratio $X^{\infty}(T,p)$ using scaling dependencies and a set of wide range of waiting times t_w.

2. Non-equilibrium relaxation of critical systems low-temperature initial state demonstrates subaging effects with aging exponent $\mu(T,p) = \eta(T,p)$. The temperature dependence of the asymptotic value of the fluctuation-dissipation ratio $X^{\infty}(T,p)$ characterized by a universal dependence from $T/T_{\mathrm{BKT}}(p)$ for different

spin concentrations p.

3. At the relaxation of the system from the initial high-temperature state the temperature dependence of the asymptotic value of the fluctuation-dissipation ratio $X^{\infty}(T,p)$ it has a power form. Introduction to the structure of the system results in defects to a decrease in the critical value of the fluctuation-dissipative ratio T_{BKT} due process of vortex pinning.

4. Obtained numerically and analytically transverse rigidity values in a uniform and structurally disordered systems indicate the existence of nonlinear anharmonic spin-wave effects and contributions from intervortex cooperation in clean system. The spin-wave approximation is not applicable to a disordered system due to the pinning of vortex excitations into defects.

5. Methods of study non-equilibrium process of clustering and non-equilibrium dynamics of a vortex in a homogeneous and structurally disordered two-dimensional XY-model with the relaxation of a high-temperature and low-temperature initial state. The anomalous deceleration coarsening process in structurally disordered system due to the «fringing» effect of the vortex subsystem.

6. Relaxation of the two-dimensional XY-model Metropolis dynamics described correctly in the low-temperature phase is not conserved order parameter, while continuing with Kawasaki dynamics parameter is applicable only to describe the low-temperature properties in $T \ll T_{\mathrm{BKT}}$.

Testing of work

The main results of scientific work were reported and discussed at the All-Russian scientific seminar «Computational Physics: Algorithms, Methods and Results» (Tarusa, 2011), at regional scientific and practical conferences «Young people of the third millennium» (Omsk, 2011, 2012, 2013, 2014), at the VIII International Scientific and Technical Conference «Dynamics of systems, mechanisms and machines» (Omsk, 2012), on the scientific and practical seminars «Computational physics and supercomputing technology» (Omsk, 2012, 2013, 2014), on «XXV IUPAP Conference on Computational Physics» (Moscow, 2013), on «Moscow International Symposium on Magnetism (MISM)» (Moscow, 2014), on «Twenty-first Scientific Conference of students-physicists and young scientists» (Omsk, 2015), on the International Conference «Spin physics, spin chemistry and spin technology» (St. Petersburg, 2015), on «International Conference on Computer Simulation in Physics and beyond» (Moscow, 2015), on the seminar «Methods of supercomputer simulations» (Tarusa, 2015), as well as scientific seminars Theoretical Physics Department of Omsk State University.

Publications

List of publications of the author on the topic of the thesis includes 25 works, published in Russian and international journals, collections of papers and conference

proceedings, of which 8 papers in journals of citation database Scopus and Web of Science, and the list of VAK, monograph and an article in the collection, as well as 4 of the certificate of state registration of computer programs.

The structure and scope of the thesis

The thesis consists of an introduction, four chapters and a conclusion. The volume of the thesis – 162 pages of typewritten text, including 35 figures, 7 tables and a list of references of 274 titles.

Summary of work

In the introduction grounded topicality of the chosen theme of the thesis and the main aims of research.

In the first chapter, having the nature of a survey, briefly set out a number of concepts and methods in the description of the critical behavior. The features of low-dimensional systems with continuous symmetry, relaxation processes in systems with slow dynamics and clustering processes in the coarsening systems. A review of current developments in this scientific field.

A two-dimensional XY-model is a lattice model with continuous symmetry, and the Hamiltonian of system is

$$H = -J \sum_{<i,j>} p_i p_j \mathbf{S}_i \mathbf{S}_j, \tag{1}$$

where $J > 0$ – exchange integral, $\mathbf{S}_i$ – classic flat spin, associated with i-node two-dimensional lattice, p_i – the number of filling: $p_i = 1$, if in i-node of lattice it is a spin, and $p_i = 0$, if a node is defective. Defects were distributed on a two-dimensional square lattice linear size L uniformly and uncorrelated with concentration $c_{\text{imp}} = 1 - p$, where p – spin concentration.

In the second chapter carried out numerical study non-equilibrium processes clustering (см. рис. 1) and vortex dynamics in pure and disordered system at the start of the different initial states (см. рис. 2).

To investigate clustering XY-dimensional model was developed a special algorithm for determining the geometric parameters of fields a quasi-order. The continuous phase ϕ_i i-spin $\mathbf{S}_i = (\cos\phi_i, \sin\phi_i)$ sampled on a discrete set of values $\{\phi_i\}_k$, with which the produced classic search clusters on the lattice.

Vortex dynamics studied using the developed search algorithm and identification of vortex excitations in a uniform and structurally disordered system. To search for and identification of antivortices and vortices in disordered system before the process determined minimum modeling all disjoint contours around the individual defects and their clusters. Along these contours in the simulation determines the set of spin phase, for which and issued a conclusion about the presence of a vortex.

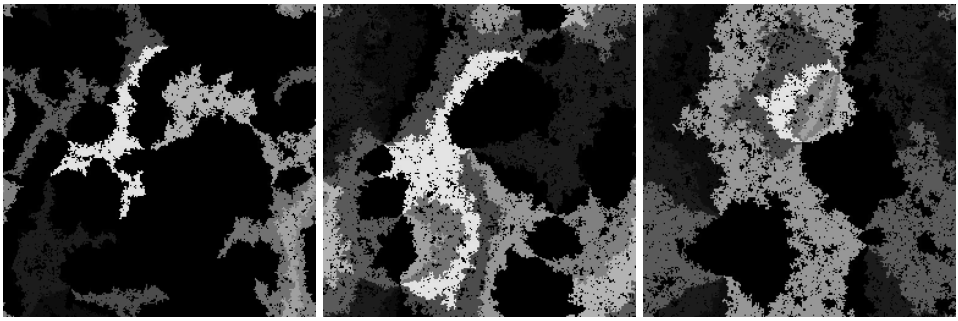

Рис. 1: Visualization of the non-equilibrium process of cluster coarsening during relaxation of the initial high-temperature state. The growth regions of a quasi-order (Fig. shows shades) at times 2000, 70000 and 200000 MCS/s.

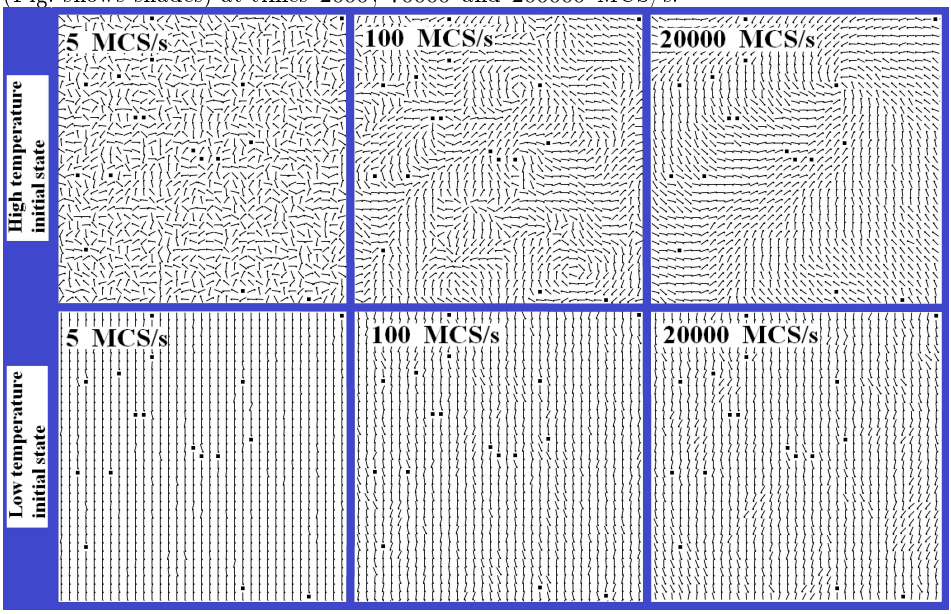

Рис. 2: Visualization non-equilibrium critical relaxation structurally disordered two-dimensional XY-model from an initial high temperature (above) and low temperature (below) state. Arrows – classic film back. Squares – structural defects. Observation time: 5, 100 and 20000 MCS/s, from left to right.

As a result, depending on the dynamic simulation obtained the size of large clusters of medium size and the number of clusters clusters, the concentration of free and pinned vortex system. Modeling carried out for all the low-temperature phase $T \leq T_{\mathrm{BKT}}$ for a wide range of variation of the impurity concentration for various initial non-equilibrium states.

The relaxation of the system from the initial high-temperature state is accom-

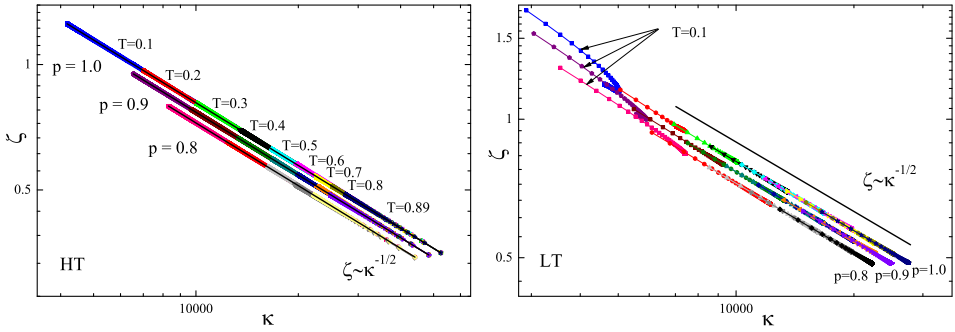

Рис. 3: The parametric dependence of the average cluster size ζ at their number κ and approximation $\zeta \sim \kappa^{-1/2}$ relaxation of the system from high-temperature (left) and low-temperature (right) initial states for different temperatures T and spin concentrations p.

panied by a process of critical cluster coarsening. With the relaxation of the system from the initial low-temperature condition the process of cluster fragmentation – one large cluster It is fragmented into smaller. It has been found that the introduction of the system structural defects leads to an anomalous slowdown of clustering. Inertial effects are revealed in the process of clustering system – in the process of coarsening the size of large clusters first rises above the equilibrium, and then thereto is reduced, and in the process of fragmentation of the opposite – large cluster size is reduced below the equilibrium value and then increases slowly. Note that inertial effects are manifested on a roughening long times, while at the fragmentation process – for small times. Inertia effects are observed in dynamic dependencies for sizes larger clusters and the total number of clusters, but It does not appear in the dependencies for the average cluster size. Dynamic dependence is characterized by high functional complexity. The robbery observed logarithmic and exponential depending on various time intervals. During fragmentation, especially at low temperatures, there are sharp bends and changes of dynamic modes.

The simulation result showed that the average size of the parametric dependence on their number of clusters has the form $\zeta \sim \kappa^{-1/2}$ (см. рис. 3). This led to the conclusion that the clustering in the non-equilibrium relaxation of the critical two-dimensional XY-model is a dense filling of a plane clustered curve without self-intersections. Thus clustering is reduced to the plane clustering and curve to it a dense packing on plane.

The time dependence of the concentration of the vortex of elementary excitations in the system at the initial relaxation of a high-temperature condition characterized by multi-mode. At low temperatures found the initial, intermediate and final power modes, access to the equilibrium and crossover area between them. With increasing temperature, the intermediate and final degree regimes merged

into a single power mode. Observed concentrations of intersection of the dynamic curves for different temperatures. This is due to the high initial concentration of vortex excitations and nonlinearity intervortex interaction. Introduction to structural disorder system slows down the relaxation of the vortex subsystem, which is associated with the non-equilibrium vortex pinning on defects.

Study of graphic visualization and parametric dependencies performance clustering and vortex dynamics (see Fig. 4) It showed that the formation of a quasi-order fields in the system is interconnected with the vortex subsystem through the «fringing» effect of vortices. Vortex excitations being inhomogeneous areas, prevent the formation and growth of clusters. The dynamics of vortex component are significantly slowed as a result of non-equilibrium pinning on defects in disordered system. This leads to abnormally slowdown clustering system. Thus, through the «fringing» effect and non-equilibrium vortex pinning field of structural defects substantially affect the clustering process, causing anomalous slowing down.

In the third chapter was performed a numerical study of non-equilibrium effects of aging in a pure and disordered system in the whole of the low-temperature phase of the different initial states (see Fig. 2), as well as to calculate the temperature dependence of the asymptotic values of the fluctuation-dissipation relations $X^{\infty}(T,p)$ and the value of the effective temperature $T^{\text{eff}}(T,p)$.

The effects of aging are manifested primarily in two-time dependence of the autocorrelation function and system response functions. The expressions for the autocorrelation function $C(t,t_w)$ and generalized dynamic susceptibility $\chi(t,t_w)$ for the system are of the form

$$C(t,t_w) = \overline{\left[\left\langle \frac{1}{pL^2}\sum_i p_i \vec{S}_i(t)\vec{S}_i(t_w)\right\rangle\right]}; \tag{2}$$

$$\chi(t,t_w) = \overline{\left[\left\langle \frac{1}{pL^2h^2}\sum_i p_i \vec{h}_i\vec{S}_i(t)\right\rangle\right]}, \tag{3}$$

where the angle brackets $\langle\ldots\rangle$ correspond to statistical averaging procedure, and square brackets $[\ldots]$ – averaging procedure on different impurity configurations for structurally disordered systems. Time variable t_w It characterizes the age of the sample and is called waiting time.

Fluctuation-dissipation theorem relates the equilibrium response function $R(t,t_w) = R_{\text{eq}}(t-t_w)$ and the autocorrelation function $C(t,t_w) = C_{\text{eq}}(t-t_w)$ for time $t > t_w \gg t_{\text{rel}}$ with using relation $TR_{\text{eq}}(t) = -dC_{\text{eq}}(t)/dt$.

For systems with slow dynamics introduced a generalization of the fluctuation-dissipation theorem to describe non-equilibrium processes by specifying the flu-

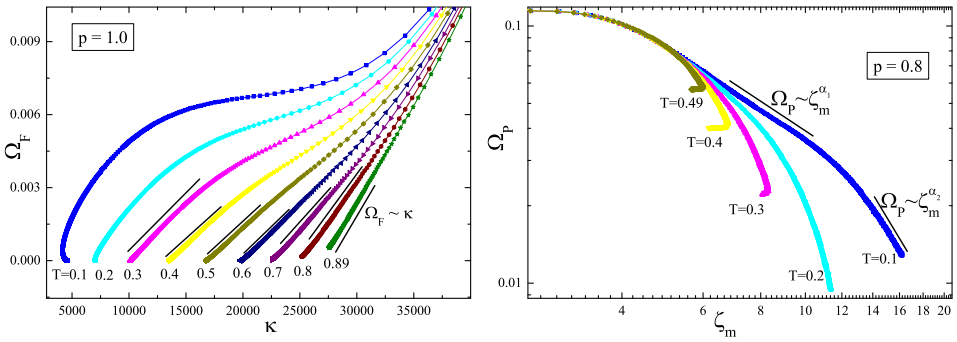

Рис. 4: The parametric dependence of the concentration of free vortex excitations Ω_F on the number of clusters in the system κ for a homogeneous system and the concentration dependence of the pinned vortex Ω_P from the large cluster size ζ_m for disordered system with spin concentration $p = 0.8$.

ctuation-dissipation ratio:

$$X(t, t_w) = \frac{TR(t, t_w)}{\partial_{t_w} C(t, t_w)}. \tag{4}$$

In equilibrium state $X(t > t_w \gg t_{rel}) = 1$.

In a non-equilibrium state at the manifestation of the effects of aging $X(t, t_w) \neq 1$. The entered limit value of the fluctuation-dissipation ratio $X^\infty = \lim_{t_w \to \infty} \lim_{t \to \infty} X(t, t_w)$ are becomes the new universal characteristic of the non-equilibrium behavior of various systems [5].

Designation X^∞ allows us to introduce the concept of effective temperature $T_{eff} = T/X^\infty$, as the characteristics, defining the direction of heat flow in establishing equilibrium in system. Value T_{eff} can be measured directly with systems «glass» phase during their non-equilibrium evolution. The importance of determination T_{eff} for structurally disordered systems due to the fact, than T_{eff} characterized by fluctuations of the local critical temperature under non-equilibrium processes, arising due to the strong interaction of the spin-density fluctuations through the field of structural defects.

Study the effects of aging was carried out from two essential various initial non-equilibrium states – high-temperature with $T_0 \gg T_{BKT}(p)$ and low-temperature with $T_0 = 0$. In a system with $T \gg T_{BKT}(p)$ in equilibrium there is a high concentration of free vortex excitations that allows the use of such a state as the primary vortex state for modeling non-equilibrium critical relaxation in the low-temperature phase. In a system with $T = 0$ the system is in the ground state, and all the spins have the same direction, so that when the temperature rises, remaining in the low-temperature phase, the main role is played by the spin-wave excitation. This choice of initial states allowed to investigate in detail the impact

of vortex excitations and spin-wave effects on the non-equilibrium relaxation of critical systems.

To investigate violations of the fluctuation-dissipation theorem and determining the values of the fluctuation-dissipation ratio X^{∞} Simulation was carried out for a large set of waiting times t_w from a wide range of values. Universal dynamic values were defined as the asymptotic behavior $t_w \to \infty$ values, the values of which were selected in the areas of dynamic scaling system. All this has allowed to monitor and examine the output of the asymptotic behavior of the system in order to identify universal characteristics.

The relaxation of the system from the initial high-temperature state accompanied by a violation of the classic dynamic scaling. This was taken into account in the analysis of time dependencies through the use of logarithmic dynamic correction depending on the correlation length of the system $\xi(t)$. It was using 16 waiting times t_w with values from 10 to 10^4 Monte-Carlo steps per spin (MCS/s). The two-time dependence of the autocorrelation function of the observed two dynamic power relaxation mode: initial $t-t_w < t_w$ and long-time $t-t_w \gg t_w$, with dynamic crossover between them. Calculated exponent dependencies modes for different temperatures and spin concentrations. Analysis of the results showed that the increase in the concentration of defects leads to increased aging effects in the system, with the long-term relaxation mode is shifted to higher values $t-t_w$. The asymptotic values fluctuation-dissipation ratio $X(t_w)$ determined only on scaling areas. The final temperature dependencies $X^{\infty}(T,p)$ (см. рис. 5 (a)) was approximated by power functions of temperature with an indicator $\lambda(p)$, decreases with increasing proportion of structural disorder in the system. Result $X^{\infty}(T = T_{\mathrm{BKT}}, p = 1.0)$ is in satisfactory agreement with the results of the spin-wave calculation, however, taking into account the error explained in modeling the vortex component in the system and taking full account of the spin-wave effects and spin diffusion. Study the effects of aging with the relaxation of the system from the initial low-temperature condition showed no expressed asymptotics $t_w \to \infty$ variables on the scaling plots for reasonable waiting times. With this it may also be related to the absence of specific literature of finite values $X^{\infty}(T,p)$ for this type of relaxation despite the considerable number of publications on this topic. Simulation was carried out for a comprehensive study of the system from the initial low-temperature state of relaxation critical for large values of waiting times $t_w \sim 50000$ MCS/s. As a result, new dynamic critical scaling were found $C(t,t_w) = t_w^{\eta/2}\Phi(\xi(t-t_w)/\xi(t_w)^{\mu})$ and effects on the system subaging. calculation of the scaling exponent have been implemented subaging $\mu(T,p)$, whose value is within statistical error coincided with the values of the anomalous dimension $\eta(T,p)$ system for all considered temperatures T and spin concentrations p. Using

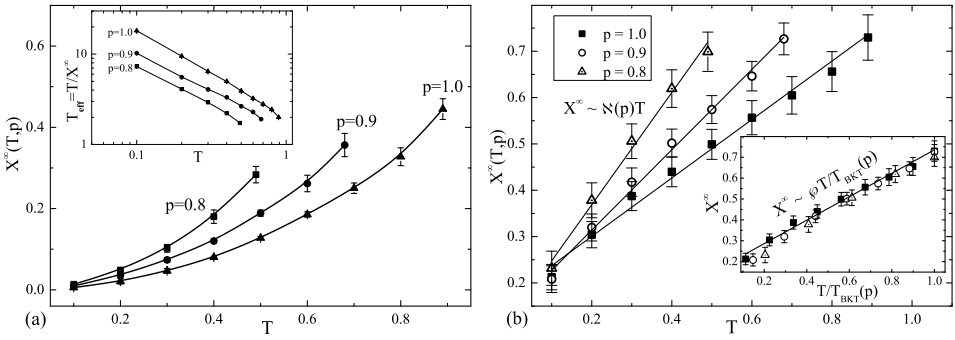

Рис. 5: The temperature dependence of the asymptotic value of the fluctuation-dissipation ratio $X^\infty(T,p)$ for the relaxation of the system from the initial high temperature (a) and low-temperature (b) states for different spin concentration p. On inset in (a) – the temperature dependence of the effective temperature T_{eff}. On inset in (b) – the linear approximation X^∞ as function of $T/T_{BKT}(p)$.

the asymptotic behavior $t_w \to \infty$ values $X(t_w)$ at temporary sites dynamic scaling subaging were obtained temperature dependencies $X^\infty(T,p)$ для различных p (см. рис. 5 (b)). These dependencies were approximated by linear functions of the form $X^\infty \sim \aleph(p)T$, where $\aleph(p)$ increases with increasing impurity concentration. Temperature dependence for all p We were approximated by a linear dependence $X^\infty \sim \wp T/T_{BKT}(p)$ with $\wp = 0.561 \pm 0.023$. This led to the conclusion that the relaxation of the system from the initial low-temperature condition throughout the low-temperature phase are the predominant contribution of the spin-wave excitation.

In the fourth chapter it carried out a numerical study of equilibrium and dynamic critical properties of homogeneous and structurally disordered two-dimensional XY-model.

The temperature dependence of exponents Δ of decay of autocorrelation function $C(t-t') \sim (t-t')^\Delta$ for two different models of the dynamics of critical phenomena: single-spin relaxation Metropolis dynamics with the order parameter is not conserved and two-spin exchange Kawasaki dynamics with conserved order parameter. Comparison of the relationships between themselves and with the existing analytical results showed that the Metropolis dynamics adequately implements the dynamic critical behavior of two-dimensional XY-model across the low-temperature phase $T \leq T_{\text{BKT}}$, and Kawasaki dynamics – only at low temperatures $T \ll T_{\text{BKT}}$.

Using the method of correlation functions relations different spatial scales were calculated temperature $T_{\text{BKT}}(p)$ for different spin concentrations $p = 1.0$, 0.9 и 0.8. The result for the homogeneous system agrees well with results of other studies.

Calculate the temperature dependence of the transverse stiffness $\rho(T,p)$ for the system in all of the low-temperature phase. For pure system ($p = 1.0$) in low-temperature area $T \ll T_{\mathrm{BKT}}$ there is good agreement with the results of the spin-wave approximation. However, with the increase in temperature is observed mismatch associated with vortex processes and anharmonic spin-wave contributions. In structurally ordered system ($p < 1.0$) value $\rho(T = 0, p < 1.0) \neq 1.0$, which leads to failure of the spin-wave approximation even in the area $T \ll T_{\mathrm{BKT}}(p)$. This is due to the pinning of vortex excitations into structural defects, significantly alters the critical properties of the system. From the condition that the free energy of vortices, expressed through transverse stiffness of the system, the transition temperature was determined $T_{\mathrm{BKT}}(p)$ for different spin concentration p. They are in good agreement with the values obtained through the relations of the correlation functions.

Implemented calculation of the temperature dependence of the anomalous dimension system (the Fisher critical exponent) $\eta(T,p)$ for all the low-temperature phase and $p = 1.0$, 0.9 and 0.8. The results for the pure system are in good agreement with the results of other studies, and for disordered system – in accordance with the Harris criterion.

In conclusion It states the main results and conclusions of the thesis.

The main results and conclusions

1. Developed a numerical technique to study the effects of aging in non-equilibrium critical behavior of homogeneous and structurally disordered two-dimensional XY-model across the low-temperature phase $T \leq T_{\mathrm{BKT}}$ for a wide spin concentration range p at a relaxation of the various initial non-equilibrium states and method of determining the temperature and concentration dependencies asymptotic values of the fluctuation-dissipation relations X^{∞} using dynamic scaling of dependencies and a wide range of waiting times t_w.

2. The temperature dependence of the asymptotic values of the fluctuation-dissipation relations $X^{\infty}(T,p)$ in all low-temperature phase $T \leq T_{\mathrm{BKT}}(p)$ for different spin concentration p for different initial non-equilibrium states.It is shown that the effects of structural defects leads to a decrease of the critical values $X^{\infty}(T = T_{\mathrm{BKT}}(p), p)$ for vortex initial state and do not affect on this value – at the spin-wave relaxation of the vortex-free state. The first obtained approximation exponential dependence $X^{\infty}(T,p) \sim T^{\lambda(p)}$ relaxation from the initial high-temperature state. The first obtained approximation $X^{\infty} = f[T/T_{BKT}(p)] \sim \wp T/T_{BKT}(p)$ c $\wp = 0.561 \pm 0.023$ for all p relaxation from the initial low-temperature condition has shown that the main contribution in this case, the dynamics of the relaxation given spin waves.

3. First subaging effects were found in the non-equilibrium system relaxation of

initial low-temperature state. The temperature dependence of the index subaging $\mu(T,p)$. From a comparison with the temperature dependence of the anomalous dimension $\eta(T,p)$ it was shown that $\eta(T,p) = \mu(T,p)$.

4. For the first time it is shown that the relaxation dynamics of the order parameter is not conserved with well reproduces the dynamic properties of two-dimensional XY-model across the low-temperature phase $T \leq T_{\mathrm{BKT}}$, while conservative dynamics of the continuing order parameter – only at low temperatures $T \ll T_{\mathrm{BKT}}$.

5. Implemented calculation of the temperature dependence of the transverse stiffness system $\rho(T,p)$ for structurally disordered systems. Shown, that $\rho(T = 0, p < 1.0) \neq 1.0$. Shown the not applicability of the spin-wave approximation for a system with defects. Using $\rho(T,p)$ determining the phase transition temperature Berezinskii-Kosterlitz-Thouless $T_{\mathrm{BKT}}(p)$, which are in accordance with values obtained by other methods.

6. It was developed the methods of the study of non-equilibrium processes clustering areas a quasi-order and study the non-equilibrium dynamics of a vortex throughout the low-temperature phase $T \leq T_{\mathrm{BKT}}$ for a wide spin concentration range p at a relaxation of the various initial non-equilibrium states.

7. First discovered the anomalous effect of slowing down the clustering process in structurally disordered system. It is shown that this effect is associated with «fringing» effect of vortices and pinning process of vortex excitations into defects. It was developed the processes of the cluster and the cluster coarsening of fragmentation in the system. Detected multi-mode dynamic depending on different characteristic values clustering and vortex dynamics.

The main results of the thesis were published in

Publications in the editions of the VAK list:

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