**КАЗАНСКИЙ (ПРИВОЛЖСКИЙ) ФЕДЕРАЛЬНЫЙ УНИВЕРСИТЕТ**
Институт языка
Кафедра английского языка для естественнонаучных дисциплин

# English For Students Of Mathematics

Учебное пособие для студентов
Института Математики и Механики им. Н.И. Лобачевского

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Данное учебное пособие представляет собой практические задания по ESP (английский язык для специальных целей) для студентов уровня Pre-Intermediate Института математики и механики I и II курсов. Пособие состоит из 3 разделов, приложения и словаря.

Цель данного пособия развить у студентов математической специальности навыки работы со специализированными текстами, включая навыки просмотрового и поискового чтения, навыки монологической речи и навыки ведения дискуссии по актуальным математическим проблемам, расширить словарный запас за счет специальной лексики, а также развить навыки технического перевода с английского на русский и с русского на английский языки.

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## Предисловие

Предлагаемое вниманию учебное пособие предназначено для студентов I и II курсов **уровня Pre-Intermediate и Intermediate** института математики и механики.

**Цель** данного пособия состоит в том, чтобы научить студентов работать со специальными математическими текстами среднего уровня трудности и расширить их запас специальной лексики, научить вести дискуссию по наиболее актуальным математическим проблемам и привить навыки технического перевода.

**Пособие состоит из 3 разделов, приложения и русско-английского словаря.**

**Первый раздел** включает в себя 5 блоков, каждый из которых содержит лексику определенной математической тематики, тексты и упражнения к текстам.

Работа с каждым из блоков состоит из нескольких этапов. Первый этап – текстовый. На этом этапе происходит знакомство с новой лексикой, осуществляется работа с текстом. Цель данного этапа заключается в адекватном восприятии текстов, их наиболее полном понимании и осознании. Второй этап – послетекстовый, практический. Он связан с выполнением лексико-грамматичеких упражнений, нацеленных на закрепление новой специализированной лексики и грамматических конструкций, на развитие навыков монологической и диалогической речи, а также навыков перевода с английского на русский и с русского на английский языки. В основу последовательности расположения предлагаемых упражнений положен принцип усложнения: от более простых упражнений к более сложным. Для более успешного усвоения специальной математической лексики в начале каждого Unit приводится список терминов, которые отрабатываются в данном блоке, что несомненно облегчает работу как студентам, так и преподавателям.

**Второй раздел** представляет собой **с**писок математических знаков и символов с объяснением их прочтения на английском языке.

**Третий раздел** рассчитан на самостоятельную работу студентов и состоит из 14 текстов для дополнительного чтения из аутентичных и отечественных монографий различной математической тематики.

**Приложение** включает в себя образец **GMAT** (Graduate Management Admission Test), что дает возможность студентам познакомиться с форматом данного экзамена, сдача которого необходима для участия в программах PhD по математическим и экономическим специальностям.

В конце учебного пособия представлен **русско-английский словарь** математических терминов, встречающихся в данном пособии. Необходимость включения именно русско-английского словаря объясняется отсутствием такого рода словарей и острой необходимостью последнего при выполнении перевода с русского языка на английский язык.

При составлении методического пособия были использованы следующие **источники**:

1. Леонтьев В.В., Булатов В.В. Английский язык для математиков: Учебное пособие. - Волгоград: Издательство Волгоградского государственного университета, 2001. – 156 с.
2. Шаншиева С.А. Английский язык для математиков (интенсивный курс для начинающих): Учебник. – 2-е изд., доп. и перераб. – М.: Изд-во МГУ, 1991 – 400с.
3. Eugene D. Jaffe, M.B.A., Ph.D., Stephen Hilbert, Ph. D. Barron’s GMAT (how to prepare for the graduate management admission test) 12th edition.
4. The NEW MATRICULATION ALGEBRA. Being the Tutorial Algebra, Elementary Course by R. Deakin, M.A. Lond. and Oxon. With a Section on Graphs, 3s. 6d.

## CONTENTS


| | |
|---|---|
| Предисловие | 3 |
| **PART 1** | 6 |
| UNIT 1 "Numbers" | 6 |
| UNIT 2 "Fundamental arithmetical operations" | 17 |
| UNIT 3 "Advanced operations" | 27 |
| UNIT 4 "Higher mathematics" | 36 |
| Checking vocabulary in Advanced Operations and Highest Mathematics | 47 |
| UNIT 5 "Geometry" | 49 |
| Checking vocabulary in Geometry | 77 |
| **PART 2** | 83 |
| MATHEMATICAL SYMBOLS AND EXPRESSIONS | 83 |
| READING OF MATHEMATICAL EXPRESSIONS | 86 |
| **PART 3 ADDITIONAL READING** | 88 |
| Text 1 "Matriculation algebra" | 88 |
| a) Definitions | 90 |
| b) Addition and subtraction | 94 |
| c) Multiplication | 96 |
| Text 2 "Base two numerals" | 99 |
| Text 3 "Closure property" | 100 |
| Text 4 "Something about mathematical sentences" | 101 |
| Text 5 "Rational numbers" | 103 |
| Text 6 "Decimal numbers" | 103 |
| Text 7 "The differential calculus" | 105 |
| Text 8 "Rays, angels, simple closed figures" | 106 |
| Text 9 "Something about Euclidean and Non-Euclidean geometry" | 109 |
| Text 10 "Circles" | 110 |
| Text 11 "The Solids" | 111 |
| Text 12 "Polyhedron" | 112 |
| Text 13 "The Pythagorean property" | 113 |
| Text 14 "Square root" | 115 |
| **APPENDIX:** SAMPLE TEST FROM **GMAT** | 117 |
| **ACTIVE VOCABULARY** | 129 |

# UNIT I
# NUMBERS

| BASIC TERMINOLOGY | | |
|---|---|---|
| **9658** | **ABSTRUCT NUMBER**<br><br>**A FOUR FIGURE NUMBER** | отвлеченное число<br><br>4-х значное число |
| **9** | **thousands** | тысячи |
| **6** | **hundreds** | сотни |
| **5** | **tens** | десятки |
| **8** | **units** | единицы |
| **5 КG.** | **CONCRETE NUMBER** | именованное число |
| **2** | **CARDINAL NUMBER** | количественное число |
| **$2^{nd}$** | **ORDINAL NUMBER** | порядковое число |
| **+ 5** | **P0SITIVE NUMBER** | положительное число |
| **- 5** | **NEGATIVE NUMBER** | отрицательное число |
| **a, b, c......** | **ALGEBRAIC SYMBOLS** | алгебраические символы |
| **3 1/3** | **MIXED NUMBER** | смешанное число |
| **3** | **WHOLE NUMBER (INTEGER)** | целое число |
| **1/3** | **FRACTION** | дробь |
| **2, 4, 6, 8** | **EVEN NUMBERS** | четные числа |
| **1,3,5,7** | **ODD NUMBERS** | нечетные числа |
| **2, 3, 5, 7** | **PRIME NUMBERS** | простые числа |
| **3+2-1** | **COMPLEX NUMBER** | комплексное число |
| **3** | **REAL PART** | действительное число |
| **2-1** | **IMAGINARY PART** | мнимая часть |
| **2/3** | **PROPER FRACTION** | правильная дробь |
| **2** | **NUMERATOR** | числитель |
| **3** | **DENOMINATOR** | знаменатель |
| **3/2** | **IMPRORER FRACTION** | неправильная дробь |

## TEXT I. INTRODUCTION TO REAL - NUMBER SYSTEM

**Mathematical analysis** studies **concepts** related in some way to **real numbers,** so we begin our study of analysis with the real number system. Several methods are used to introduce real numbers. One method starts with the positive **integers 1, 2, 3 …..** as **undefined** concepts and uses them to build a larger system, the positive rational numbers (quotients of positive integers), their negatives, and zero. **The rational numbers,** in turn, are then used to construct **the irrational numbers,** real numbers like $\sqrt{2}$ and $\pi$ which are not rational. The rational and irrational numbers together constitute the real number system.

Although these matters are an important part of the foundations of mathematics, they will not be described in detail here. **As a matter of fact,** in most phases of analysis it is only the **properties** of real numbers that concerns us, rather than the methods used to construct them.

For **convenience,** we use some elementary set **notation** and **terminology.** Let **S** denote a set (a collection of objects). The notation $\mathbf{x \in S}$ means that the object **x** is in the set, and we write $\mathbf{x \notin S}$ to indicate that **x** is not in **S**.

A set S is said to be **a subset** of **T**, and we write $\mathbf{S \subseteq T}$, if every object in **S** is also in **T**. A set is called nonempty if it **contains** at least one object.

We **assume** there exists a nonempty set **R** of objects, called real numbers, which **satisfy** ten axioms. The axioms **fall** in a natural way into three groups which we refer as the **field axioms, order axioms, completeness axioms** (also called the upper-bound axioms or the axioms of continuity

**I. Translate the definitions of the following mathematical terms.**

1. mathematics - the group of sciences (including arithmetic, geometry, algebra, calculus, etc.) dealing with quantities, magnitudes, and forms, and their relationships, attributes, etc., by the use of numbers and symbols;
2. negative - designating a quantity less than zero or one to be subtracted;
3. positive - designating a quantity greater than zero or one to be added;
4. irrational - designating a real number not expressible as an integer or as a quotient of two integers;
5. rational - designating a number or a quantity expressible as a quotient of two integers, one of which may be unity;

6. integer - any positive or negative number or zero: distinguished from fraction;

7. quotient - the result obtained when one number is divided by another number;

8. subset - a mathematical set containing some or all of the elements of a given set;

9. field - a set of numbers or other algebraic elements for which arithmetic operations (except for division by zero) are defined in a consistent manner to yield another element of a set.

10. order - a) an established sequence of numbers, letters, events, units,

b) a whole number describing the degree or stage of complexity of an algebraic expression;

c) the number of elements in a given group.

(From Webster's New World Dictionary).

**II. Match the terms from the left column and the definitions from the right column:**

| | |
|---|---|
| 1. negative | a) designating a number or a quantity expressible as a quotient of two integers, one of which may be unity |
| 2. positive | b) a set of numbers or other algebraic elements for which arithmetic operations (except for division by zero) are defined in a consistent manner to yield another element of a set |
| 3. rational | c) designating a quantity greater than zero or one to be added |
| 4. irrational | d) the number of elements in a given group |
| 5. order | e) designating a real number not expressible as an integer or as a quotient of two integers |
| 6. quotient | f) a mathematical set containing some or all of the elements of a given set |

| | |
|---|---|
| 7. subset | g) a quantity less than zero or one to be subtracted |
| 8. field | h) any positive or negative number or zero: distinguished from fraction |
| 9. order | i) the result obtained when one number is divided by another number |

**III. Read and decide which of the statements are true and which are false. Change the sentences so they are true.**

1. A real number $x$ is called positive if $x > 0$, and it is called negative if $x < 0$.
2. A real number $x$ is called nonnegative if $x=0$.
3. The existence of a relation > satisfies the only axiom: If $x < y$, then for every $z$ we have $x + z < y + z$.
4. The symbol $\geq$ is used similarly as the symbol $\leq$.

**IV. Translate the following sentences into English.**

1. В этой системе используются положительные и отрицательные числа.
2. Положительные и отрицательные числа представлены (to represent) отношениями целых положительных чисел.
3. Рациональные (rational) числа, в свою очередь, используются для создания иррациональных (irrational) чисел.
4. В совокупности рациональные и иррациональные числа составляют систему действительных чисел.
5. Математический анализ - это раздел математики, изучающий функции и пределы.
6. Множество $X$ является подмножеством другого множества $У$ в том случае, если все элементы множества $X$ одновременно являются элементами множества $У$.
7. Аксиомы, удовлетворяющие множеству действительных чисел, можно условно разделить на три категории.

## TEXT II. RATIONAL AND IRRATIONAL NUMBERS

Quotients of **integers a/b** (where $b \neq 0$) are called rational numbers. For example, **1/2, -7/5,** and **6** are rational numbers. The set of rational numbers, which we **denote** by **Q,** contains **Z** as a subset. The students of mathematics should **note** that all the field axioms and the order axioms are satisfied by **Q.**

We assume that every student of mathematical department of universities is **familiar with** certain **elementary** properties of rational numbers. For example, if *a* and *b* are rational numbers, their **average** $(a+b)/2$ is also rational and **lies between** *a* and *b.* Therefore between any two rational numbers there are **infinitely** many rational numbers, which **implies** that if we are given a certain rational number we cannot speak of the "next largest" rational number.

Real numbers that are not rational are called irrational. For example, $\boldsymbol{e}$, $\boldsymbol{\pi}$, $\boldsymbol{e^{\pi}}$ are irrational.

**Ordinarily** it is not too easy **to prove** that some particular number is irrational. There is no simple proof, for example, of **irrationality** of $\boldsymbol{e^{\pi}}$. However, the irrationality of certain numbers such as $\sqrt{2}$ is not too difficult to establish and, in fact, we can easily prove the following theorem:

*If* ***n*** *is a positive integer which is not a* **perfect square,** *then* $\sqrt{n}$ is *irrational.*

*Proof.* **Suppose** first that ***n*** contains no square **factor** > 1. We assume that $\sqrt{\mathbf{n}}$ is rational and **obtain** a **contradiction.** Let $\sqrt{n} = a/b$, where *a* and *b* are integers having no factor **in common.** Then $nb^2 = a^2$ and, since the left **side** of this equation is a **multiple** of *n,* so too is $a^2$. However, *if* $a^2$ is a multiple of *n, a* itself must be a multiple of *n,* since *n* has no square factors > 1. (This is easily seen by examining **factorization** of *a* into its prime factors). This means that $a = cn$, where *c* is some integer. Then the **equation** $nb^2 = a^2$ becomes $nb^2 = c^2n^2$, *or* $b^2 = nc^2$. The same argument shows that *b* must be also a multiple of *n.* Thus *a* and *b* are both multiples of *n,* which **contradicts** the fact that they have no factors in common. This completes the **proof** if *n* has no **square** factor > 1.

If *n* has a square factor, we can write $n = m^2k$, where $k > 1$ and *k* has no square factor > 1. Then $\sqrt{n} = m\sqrt{k}$; and if $\sqrt{n}$ were rational, the number $\sqrt{k}$ would also be rational, contradicting that was just proved.

## I. Match the terms from the left column and the definitions from the right column:

| | |
|---|---|
| 1. perfect square | a) any of two or more quantities which form a product when multiplied together |
| 2. factor | b) the numerical result obtained by dividing the sum of two or more quantities by the number of quantities |
| 3. multiple | c) the process of finding the factors |
| 4. average | d) a number which is a product of some specified number and another number |
| 5. factorization | e) a quantity which is the exact square of another quantity |

## II. Translate into Russian.

An irrational number is a number that can't be written as an integer or as quotient of two integers. Thee irrational numbers are infinite, non-repeating decimals. There're two types of irrational numbers. Algebraic irrational numbers are irrational numbers that are roots of polynomial equations with rational coefficients. Transcendental numbers are irrational numbers that are not roots of polynomial equations with rational coefficients; $\pi$ and *e* are transcendental numbers.

## III. Give the English equivalents of the following Russian words and word combinations:

отношения целых, множитель, абсолютный квадрат, аксиома порядка, разложение на множители, уравнение, частное, рациональное число, элементарные свойства, определенное рациональное число, квадратный, противоречие, доказательство, среднее (значение).

## IV. Translate the following sentences into English and answer to the questions in pairs.

1. Какие числа называются рациональными?
2. Какие аксиомы используются для множества рациональных чисел?
3. Сколько рациональных чисел может находиться между двумя любыми рациональными числами?
4. Действительные числа, не являющиеся рациональными, относятся к категории иррациональных чисел, не так ли?

## V. Translate the text from Russian into English.

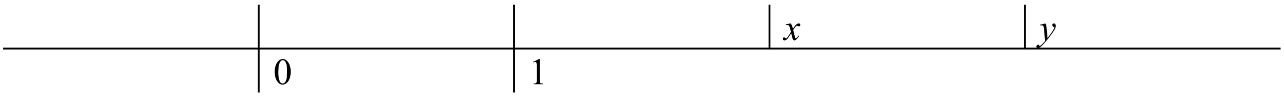
Обычно нелегко доказать, что определенное число является иррациональным. Не существует, например, простого доказательства иррациональности числа $e^{\pi}$ . Однако, нетрудно установить иррациональность
определенных чисел, таких как $\sqrt{2}$ , и, фактически, можно легко доказать следующую теорему: если $n$ является положительным целым числом, которое не относится к абсолютным квадратам, то $\sqrt{n}$ является иррациональным.

## TEXT III. GEOMETRIC REPRESENTATION OF REAL NUNBERS AND COMPLEX NUMBERS

The real numbers are often represented geometrically as **points** on **a line** (called **the real line** or **the real axis**). A point is selected to represent 0 and another to represent 1, as shown on figure 1. This choice determines **the scale**.

Under an appropriate set of **axioms** for **Euclidean geometry**, each point on the real line corresponds to one and only one real number and, conversely, each real number is represented by one and only one point on the line. It is customary to refer to **the point x** rather than the point representing the real number *x*.

**Figure 1**




**The order relation** has a simple **geometric interpretation**. If $x < y$, the point *x* lies to the left of the point *y*, as shown in Figure 1. Positive numbers lies to the right of 0 and negative numbers lies to the left of 0. If $\boldsymbol{a} < \boldsymbol{b}$, a point *x* satisfies $a < x < b$ if and only if *x* is between ***a*** and ***b***.

Just as real numbers are represented geometrically by points on a line, so **complex numbers** are represented by points in a plane. The complex number $x = (x^1,x^2)$ can be thought of as the "point" with coordinates $(x^1,x^2)$.

This idea of expressing complex numbers geometrically as points in a plane was formulated by Gauss in his dissertation in 1799 and, independently, by Argand in 1806. Gauss later coined the somewhat unfortunate phrase "complex number". Other geometric interpretations of complex numbers are possible. Riemann found **the sphere** particularly convenient for this **purpose**. Points of the sphere are **projected** from the North Pole onto the **tangent plane** at the South Pole and, thus there corresponds to each point of the plane a definite point of the sphere. With the exception of the North Pole itself, each point of the sphere corresponds to exactly one point of the plane. The correspondence is called **a stereographic projection**.

I. **Match the terms from the left column and the definitions from the right column:**

| | |
|---|---|
| 1. an axis | a) a prescribed collection of points, numbers or other objects satisfying the given |

| | |
|---|---|
| | condition |
| 2. a scale | b) the act or result of interpretation; explanation, meaning |
| 3. an axiom | c) a straight line through the center of a plane figure of a solid, especially one around which the parts are symmetrically arranged |
| 4. complex | d) a system of numerical notation |
| 5. a point | e) not simple, involved or complicated |
| 6. an inequality | f) a statement or proposition which needs no proof because its truth is obvious, or one hat is accepted as truee without proof |
| 7. a set | g) the relation between two unequal quantities, or the expression of this relationship |
| 8. interpretation | h) an element in geometry having definite position, but no size, shape or extension |

**II. Read and decide which of the statements are true and which are false. Change the sentences so they are true.**

1. A fraction is the indicated quotient of two expressions.

2. A fraction in its lowest terms is a fraction whose numerator and denominator have some common factors.

3. Fractions in algebra have in general the same properties as fractions in arithmetic.

4. The numerator of a fraction is the divisor and the denominator is the dividend.

5. In order to reduce a fraction to its lowest terms we must resolve the numerator and denominator by the factors common to both.

**III Match the terms from the left column and the definitions from the right column:**

| | |
|---|---|
| 1. fraction | a) the number or quantity by which the dividend is divided to produce the quotient |
| 2. expression | b) any quantity expressed in terms of a numerator and denominator |
| 3. divisor | c) a showing by a symbol, sign, figures |
| 4. dividend | d) the term above or to the left of the linee in a fraction |
| 5. common factor | e) the term below or to the right of the line in a fraction |
| 6. numerator | f) to change in denomination or form without changing in value |
| 7. denominator | g) factor common to two or more numbers |
| 8. to reduce | h) the number or quantity to be divided |

**IV. Write a short summary to the text.**

A fraction is a quotient of two numbers usually indicated by *a/b.* The dividend *a* is called the numerator and the divisor *b* is called a denominator. Fractions are classified into 5 categories: common (simple, vulgar) fractions, complex fractions, proper fractions, improper fractions and mixed fractions.

In common fractions both numerator and denominator are integers. In complex fractions the numerator and denominator are themselves fractions. In proper fractions the numerator is less than the denominator. In improper fractions the denominator is greater than the denominator. And, at last, a mixed fraction is an integer together with a proper fraction.

**V. Translate the paragraphs into Russian.**

A) Stereographic projection is a conformal projection of a sphere onto a plane. A point *P (the plane)* is taken on the sphere and the plane is perpendicular to the diameter through *P*. Points on the sphere, *A,* are mapped by straight line from *P* onto the plane to give points *A'*.

B) A tangent plane is a plane that touches a given surface at a particular point. Specifically, it is a plane in which aall the lines that pass through the point are tangents to the surface at the point. If the surface is a conical or cylindrical surface then the tangent plane will touch it along a line (the element of contact).

C) Argand's diagram or complex plane is any plane with a pair of mutually perpendicular axes which is used to represent complex numbers by identifying the complex number $a + ib$ with the point in the plane whose coordinates are (a, b). It's named after Jean Robert Argand (1768 - 1822), although the method's first exposition, in 1797, was by Casper Wessel (1745 - 1818), and the idea can be found in the work of John Wallis (1616 - 1703).

D) Complex numbers can be represented on an Argand diagram using two perpendicular axes. The real part is the x-coordinate and the imaginary part is they y - coordinate. Any complex number is then represented either by the point (a, b) or by a vector from the origin to this point. This gives an alternative method of expressing complex numbers in the form $r(\cos \Theta + i \sin \Theta)$, where $r$ is the length of the vector and $\Theta$ is the angle between the vector and the positive direction of $x$-axis.

# UNIT 2

# FUNDAMENTAL ARITHMETICAL OPERATIONS

| BASIC TERMINOLOGY | | |
|---|---|---|
| **I. ADDITION** | **сложение** | |
| 3 + 2=5 → в этом примере: | | |
| 3&2 | ADDENDS | слагаемые |
| + | PLUS SIGN | знак плюс |
| =<br>5 | EQUALS SIGN<br>THE SUM | знак равенства<br>сумма |
| **II. SUBTRACTION** | **вычитание** | |
| 3 - 2 = 1 → в этом примере: | | |
| 3 | THE MINUEND | уменьшаемое |
| - | MINUS SIGN | знак минус |
| 2 | THE SUBTRAHEND | вычитаемое |
| 1 | THE DIFFERENCE | разность |
| **III. MULTIPLICATION** | **умножение** | |
| 3 x 2 = 6 → в этом примере: | | |
| 3 | THE MULTIPLICAND | множимое |
| x | MULTIPLICATION SIGN | знак умножения |
| 2 | THE MULTIPLIER | множитель |
| 6 | THE PRODUCT | произведение |
| 3&2 | FACTORS | сомножители |
| **IV. DIVISION** | **деление** | |
| 6:2 = 3 → в этом примере: | | |
| 6 | THE DIVIDEND | делимое |
| : | DIVISION SIGN | знак деления |
| 2 | THE DIVISOR | делитель |
| 3 | THE QUOTIENT | частное |

**Note:**

**23** is read “twenty three”

**578** is read “five hundred (and) seventy eight”

**3578** is read “three thousand five hundred (and) seventy eight”

**7425629** is read “seven million four hundred twenty five thousand six hundred and twenty nine”

a (one) hundred books hundreds of books

| | | |
|---|---|---|
| 7 + 5 = 12 | is read<br>or<br>or<br>or | **seven plus five equals twelve**<br>**seven plus five is equal to twelve**<br>**seven plus five is (are) twelve**<br>**seven added to five makes twelve** |
| 7 – 5 = 2 | is read<br>or<br>or<br>or | **seven minus five equals two**<br>**seven minus five is equal two**<br>**five from seven leaves two**<br>**difference between five and seven is two** |
| 5 x 2 = 10 | is read<br>or<br>or | **five multiplied by two is equal to ten**<br>**five multiplied by two equals ten**<br>**five times two is ten** |
| 0 : 2 = 5 | is read<br>or | **ten divided by two is equal to five**<br>**ten divided by two equals five** |

**I. Read and write the numbers and symbols in full according to the way they are pronounced:**

76, 13, 89, 53, 26, 12, 11, 71, 324, 117, 292, 113, 119; 926, 929, 735, 473, 1002, 1026, 2606, 7354, 7013, 3005, 10117, 13526, 17427, 72568, 634113, 815005, 905027, 65347005, 900000001, 10725514, 13421926, 65409834, 815432789, 76509856, 1000000, 6537.

425 - 25 = 400

730 - 15=715

222 - 22 = 200

1617 + 17 = 1634

1215 + 60 = 1275

512 ÷ 8 = 64

1624 ÷ 4 = 406

456 ÷ 2 = 228

$135 \times 4 = 540$

$450 \times 3 = 1350$

$107 \times 5 = 535$

$613 \times 13 = 7969$

$1511 + 30 = 1541$

$34582 + 25814 = 60396$

$768903 - 420765 = 348138$

$1634986 - 1359251 = 275735$

$1000 \div 100 = 10$

$810 \div 5 = 162$

$100 \times 2 = 200$

## TEXT I. FOUR BASIC OPERATIONS OF ARITHMETIC

We cannot live a day without numerals. Numbers and numerals are everywhere. On this page you will see number names and numerals. The number names are: zero, one, two, three, four and so on. And here are the corresponding numerals: 0, 1, 2, 3, 4, and so on. In a numeration system numerals are used to represent numbers, and the numerals are grouped in a special way. The numbers used in our numeration system are called digits. In our Hindu-Arabic system we use only ten digits: 0, 1, 2, 3, 4. 5, 6, 7, 8, 9 to represent any number. We use the same ten digits over and over again in a place-value system whose base is ten. These digits may be used in various combinations. Thus, for example, 1, 2, and 3 are used to write 123, 213, 132 and so on.

One and the same number could be represented in various ways. For example, take 3. It can be represented as the sum of the numbers 2 and 1 or the difference between the numbers 8 and 5 and so on.

A very simple way to say that each of the numerals names the same number is to write an equation — a mathematical sentence that has an equal sign ( = ) between these numerals. For example, the sum of the numbers 3 and 4 equals the sum of the numbers 5 and 2. In this case we say: three plus four (3+4) is equal to five plus two (5+2). One more example of an equation is as follows: the difference between numbers 3 and 1 equals the difference between numbers 6 and 4. That is three minus one (3—1) equals six minus four (6—4). Another example of an equation is 3+5 = 8. In this case you have three numbers. Here you add 3 and 5 and get 8 as a result. 3 and 5 are addends (or summands) and 8 is the sum. There is also a plus (+) sign and a sign of equality ( = ). They are mathematical symbols.

Now let us turn to the basic operations of arithmetic. There are four basic operations that you all know of. They are addition, subtraction, multiplication and division. In arithmetic an operation is a way of thinking of two numbers and getting one number. We were just considering an operation of addition. An equation like 7—2 = 5 represents an operation of subtraction. Here seven is the minuend and two is the subtrahend. As a result of the operation you get five. It is the difference, as you remember from the above. We may say that subtraction is the inverse operation of addition since 5 + 2 = 7 and 7 — 2 = 5. The same might be said about division and multiplication, which are also inverse operations. In multiplication there is a number that must be multiplied. It is the multiplicand. There is also a multiplier. It is the number by which we multiply. When we are multiplying the multiplicand by the multiplier we get the product as a result. When two or more numbers are multiplied, each of them is called a factor. In the expression five multiplied by two (5×2), the 5 and the 2 will be factors. The multiplicand and the multiplier are names for factors.

In the operation of division there is a number that is divided and it is called the dividend; the number by which we divide is called the divisor. When we are dividing the dividend by the divisor we get the quotient. But suppose you are dividing 10 by 3. In this case the divisor will not be contained a whole number of times in the dividend. You will get a part of the dividend left over. This part is called the remainder. In our case the remainder will be 1. Since multiplication and division are inverse operations you may check division by using multiplication.

There are two very important facts that must be remembered about division.

*a)* The quotient is 0 (zero) whenever the dividend is 0 and the divisor is not 0. That is, $0 \div n$ is equal to 0 for all values of *n* except n = 0.

*b)* Division by 0 is meaningless. If you say that you cannot divide by 0 it really means that division by 0 is meaningless. That is, n: 0 is meaningless for all values of *n*.

**I. Translate the definitions of the following mathematical terms.**

1. To divide – to separate into equal parts by a divisor;
2. Division – the process of finding how many times (*a number*) is contained in another number (the *divisor*);
3. Divisor – the number or quantity by which the dividend is divided to produce the quotient;
4. Dividend – thee number or quantity to be divided;
5. To multiply – to find the product by multiplication;
6. Multiplication – the process of finding the number or quantity *(product)* obtained by repeated additions of a specified number or quantity;
7. Multiplier – the number by which another number (the *multiplicand*) is multiplied;
8. Multiplicand – the number that is multiplied by another (the *multiplier*);
9. Remainder – what is left undivided when one number is divided by another that is not one of its factors;
10. Product – the quantity obtained by multiplying two or more quantities together;
11. To check – to test, measure, verify or control by investigation, comparison or examination.

(From Webster's New World Dictionary).

## I. Match the terms from the left column and the definitions from the right column:

**a)**

| | |
|---|---|
| 1. algebra | a) a number or quantity be subtracted from another one |
| 2. to add | b) to take away or deduct (one number or quantity from another) |
| 3. addition | c) the result obtained by adding numbers or quantities |
| 4. addend | d) the amount by which one quantity differs from another |
| 5. to subtract | e) to join or unite (to) so as to increase the quantity, number, size, etc. or change the total effect |
| 6. subtraction | f) a number or quantity from which another is to be subtracted |
| 7. subtrahend | g) equal in quantity value, force, meaning |
| 8. minuend | h) an adding of two or moree numbers to get a number called the sum |
| 9. equivalent | i) a mathematical system using symbols, esp. letters, to generalize certain arithmetical operations and relationships |

**b)**

| | |
|---|---|
| 1. to divide | a) to test, measure, verify or control by investigation, comparison or examination |
| 2. division | b) the process of finding the number or quantity (product) obtained by repeated additions of a specified number or quantity |
| 3. dividend | c) the number by which another number is multiplied |
| 4. divisor | d) what is left undivided when one number is divided by another that is not one of its factors |
| 5. to multiply | e) to separate into equal parts by a divisor |

| | |
|---|---|
| 6. multiplication | f) the process of finding how many times a number is contained in another number |
| 7. multiplicand | g) the number or quantity to be divided |
| 8. multiplier | h) the quantity obtained by multiplying two or more quantities together |
| 9. remainder | i) the number that is multiplied by another |
| 10.product | j) the number or quantity by which the dividend is divided to produce the quotient |
| 11. to check | k) to find the product by multiplication |

**II. Read the sentences and think of a word which best fits each space.**

1. Subtraction is ... of addition.
2. Addition and subtraction are arithmetical ... .
3. Positive and negative numbers are known as ... numbers.
4. Minuend is a number from which we ... subtrahend.
5. The process of checking subtraction consists of adding subtrahend to ….
6. In arithmetic only ... numbers with no ... in front of them are used.
7. The multiplicand is a number, which must be ... by a multiplier.
8. The number *y* which we divide is ... .
9. Division and multiplication as well as addition and ... are inverse.
10. Division by ... is meaningless.
11. The multiplicand and ... the names for factors.
12. The product is get as the result of multiplying multiplicand and ... .
13. The ... is the part of the dividend left over after the division if the ... isn't contained a whole ... of times in the dividend.

**V. Complete the following definitions.**

**a)** *Pattern*: The operation, which is the inverse of addition is subtraction.

1. The operation, which is the inverse of subtraction … .
2. The quantity, which is subtracted … .
3. The result of adding two от more numbers … .

4. The result of subtracting two oт more numbers … .

5. To find the sum … .

6. To find the difference … .

7. The quantity number or from which another number (quantity) is subtracted … .

8. The terms of the sum … .

**b)** *Pattern*: A number that is divided is a dividend.

1. The process of cumulative addition … .

2. The inverse operation of multiplication … .

3. A number that must be multiplied … .

4. A number by which we multiply … .

5. A number by which we divide … .

6. A part of the dividend left oover after division … .

7. The number which is the result of the operation of multiplication … .

**V. Numbers and Fundamental Arithmetical Operations.**

Choose the correct term corresponding to the following definitions:

a) A quotient of one number by another.

square root mixed number division

integer fraction divisor

b) The inverse operation of multiplication.

addition fraction subtraction

quotient division integer

c) A whole number that is not divisible by 2.

integer prime number odd number

complex number even number negative number

d) A number that divides another number.

dividend division divisor

division sign quotient remainder

e) The number that is multiplied by another.

multiplication remainder multiplicand

multiplier product dividend

**VI. Read and translate the following sentences. Write two special questions to each of them. Then make the sentences negative.**

1. Everybody can say that division is an operation inverse of addition.
2. One can say that division and multiplication are inverse operations.
3. The number which must be multiplied is multiplicand.
4. We multiply the multiplicand by the multiplier.
5. We get the product as the resu1t of multiplication.
6. If the divisor is contained a whole number of times in the dividend, we won't get any remainder.
7. The remainder is a part of the dividend left over after the operation is over.
8. The addends are numbers added in addition.

**VII. Give the English equivalents of the following Russian words and word combinations:**

вычитаемое, величина, уменьшаемое, алгебраическое сложение, эквивалентное выражение, вычитать, разность, сложение, складывать, слагаемое, сумма, числительное, числа со знаками, относительные числа, деление, умножение, делить, остаток, частное, произведение, выражение, обратная операция, делитель, делимое, множитель, множимое, сомножители, сумма, знак умножения, знак деления.

**VIII. Translate the text into English.**

Сложением в математике именуется действие (operation), выполняемое над двумя числами, именуемыми (named) слагаемыми, для получения искомого числа, суммы. Данное действие можно определить увеличение величины одного числа на другое число.

При сложении двух дробей необходимо привести обе дроби к общему знаменателю, а затем сложить числители. Сложение чисел одновременно и коммутативно, и ассоциативно. При сложении комплексных чисел надо складывать действительную (real) и мнимую (imaginary) части раздельно (separately).

Действие, обратное (inverse to) сложению в математике называется вычитанием. Это процесс, в котором даны два числа и требуется найти третье, искомое число. При этом, при прибавлении искомого третьего числа к одному из данных, должно получиться второе из данных чисел.

## IX. Write a summary to the text.

### PRIMES

A prime is a whole number larger than 1 that is divisible only by 1 and itself. So 2, 3, 5, 7, ... , 101, ... , 1093 ... are all primes. Each prime number has the following interesting property: if it divides a product, then it must divide at least one of the factors.

No other number bigger than I have this property. Thus 6, which is not a prime, divides the product of 3 and 4 (namely 12), but does not divide either 3 or 4. Every natural number bigger than 1 is either a prime or can be written as a product of primes. For instance 18 = 2 × 3 × 3, 37 is a prime, 91 = 7 x 13.

The term can also be used **analogously** in some other situations where division is **meaningful**. For instance, in the context of aall integers, an integer **n** other than **0, +1**, is a prime integer, if its **only** integer divisors are **+1** and **+n.**

The positive prime integers are just the **ordinary** natural prime numbers 2, 3, 5 and the negative prime integers are -2, -3, -5.

# UNIT 3

# ADVANCED OPERATIONS

| BASIC TERMINOLOGY | | |
|---|---|---|
| **I. RAISING TO A POWER** | **возведение в степень** | |
| $3^2 = 9$ | | |
| **3** | **THE BASE** | основание |
| **2** | **THE EXPONENT (INDEX)** | показатель степени |
| **9** | **VALUE OF THE POWER** | значение степени |
| **II. EVOLUTION (EXTRACTING A ROOT) - извлечение корня** | | |
| $\sqrt[3]{8} = 2$ | | |
| **3** | **THE INDEX (DEGREE) OF THE ROOT** | показатель корня |
| **8** | **THE RADICAND** | подкоренное выражение |
| **2** | **VALUE OF THE ROOT** | значение корня |
| √ | **RADICAL SIGN** | знак корня |
| **III. EQUATIONS** | **уравнения** | |
| 1. **3x + 2 = 12** | **SIMPLE EQUATION** | линейное уравнение |
| **3&2** | **THE COEFFICIENTS** | коэффициенты |
| **X** | **THE UNKNOWN QUANTITY** | неизвестная величина |
| 2. **4a + 6ab - 2ac = 2a(2 + 3b - c)** | **IDENTICAL EQUATION** | тождественное уравнение |
| 3. **2:50 = 4:x** | **CONDITIONAL EQUATION** | условное уравнение |
| **x = 100** | **SOLUTION** | решение |
| **IV. CALCULATIONS LOGARITHMIC логарифмические вычисления** | | |
| $\log_{10}3 = 0.4771$ | | |
| **Log** | **LOGARITHM SIGN** | знак логарифма |
| **10** | **THE BASE** | основание |
| **0.** | **THE CHARACTERISTIC** | характеристика |
| **4771** | **THE MANTISSA** | мантисса |

## TEXT I. EQUATIONS

**An equation** is **a symbolic statement** that two expressions are equal. Thus **x + 3 = 8** is an equation, stating that **x + 3** equals **8**. There are two kinds of equations: **conditional equations,** which are generally called equations and **identical equations** which are generally called **identities.**

An identity is an equality whose two members (sides) are equal for all values of **the unknown quantity** (or quantities) contained in it.

**An equation in one unknown** is an equality which is true for only one value of the unknown.

To solve an equation in one unknown means to find values of the unknown that make the left member equal to the right member.

Any such value which satisfies the equation is called **the solution** or **the root** of the equation.

Two equations are equivalent if they have the same roots. Thus, $\mathbf{x - 2 = 0}$ and $\mathbf{3x - 6 = 0}$ are **equivalent equations,** since they both have the single root $\mathbf{x = 2}$.

In order to solve an equation it is permissible to:

a) add the same number to both members;

b) subtract the same number from both members;

c) multiply both members by the same number;

d) divide both members by the same number with the single exception
of the number zero.

These operations are permissible because they lead to equivalent equations.

Operations **a)** and **b)** are often replaced by an equivalent operation called **transposition.** It consists in changing a term from one member of the equation to the other member and changing its signs.

An equation of the form $\mathbf{ax + b = 0}$ where $\mathbf{a \neq 0}$ is an equation of **the first degree in the unknown** x. Equations of the first degree are solved by the permissible operations listed in this text. The solution is incomplete until the value of the unknown so found is substituted in the original equation and it is shown to satisfy this equation.

***Example:*** **Solve:** **x ÷ 3x = 6**

**Solution:** Divide both members by 3 ÷ x = 2

**Check:** Substitute **2** for **x** in the original equation: **3(2)** = **6, 6** = **6.**

| | | |
|---|---|---|
| **$\sqrt[3]{8} = 2$** | **is read** | **the cube root of eight is two** |
| **2 : 50 = 4 : x** | **is read** | **two is to fifty as four is to x** |
| **Log 10 3** | **is read** | **logarithm of three to the base of ten** |

## I. Read and decide which of the statements are true and which are false. Change the sentences so they are true.

1. An equation is a symbolic statement that two expressions are equal.
2. There' s only one kind of equations. It is called an identical equation.
3. An equation in one unknown is an equality which is true for various values of the unknown.
4. Two or more equations are equivalent if they have the same roots.
5. To solve an equation in one unknown means to find values of the unknown such that make the left member equal to the right member.
6. An equation of the form $ax + b = 0$, where $a \neq 0$ is an equation of the first degree in the unknown $x$.
7. In order to solve an equation it is permissible to add the same number to both members, to subtract the same number from both members, to multiply both members by the same member and divide both members be the same number with the single exception of the number one.

## II. Give the Russian equivalents of the following English words and word combinations.

1. equation
1. statement

2. conditional equation
3. identical equation
4. identity
5. unknown quantity
6. solution (root)
7. simple equation
8. permissible operation
9. transposition
10. equation in one unknown
11. equation of the first degree
12. substitution
13. equivalent equations

**III. Give the English equivalents of the following Russian words and word combinations:**

тождество, перестановка, корень, решение, неизвестная величина, основа, условное уравнение, степень, показатель степени, высказывание (формулировка), эквивалентная операция, тождественное уравнение, уравнение с одним неизвестным, уравнение первой степени, подстановка, подкоренное выражение, линейное уравнение.

**IV. Translate the following sentences into English.**

1. Математика как наука состоит из таких областей, как арифметика, алгебра, геометрия, математический анализ и т.д.

2. Математическое выражение $x + 3 = 8$ - это уравнение, показывающее что $x + 3$ и $8$ равны. Таким образом, считается, что уравнение - это символическое высказывание, показывающее равенство двух или более математических выражений.

3. Уравнение типа $x + 3 = 8$ содержит одно неизвестное.

4. Для того чтобы решить уравнение; необходимо выполнить определенные

математические операции, такие как сложение и вычитание, умножение и деление.

5. Решить уравнение означает найти значения неизвестных, которые удовлетворяют уравнению.

6. Уравнение - это выражение равенства между двумя величинами.

7. Все уравнения 2-й, 3-й и 4-й степени решаются в радикалах.

8. Линейное уравнение может быть записано в форме $3x + 2 = 12$.

## III. Summarize the major point of the text.

Girolamo Cardano was a famous Italian mathematician, physician and astronomer who lived in the 16 century. He was born in 1501 and died in 1576 at the age of 75. He was noted for the first publication of the solution to the general cubic equation in his book on algebra called "Ars magna" ("The Great Art"). The book also contained the solution of the general biquadrate equation found by Cardano's former assistant Ferrari.

Cardano was also known for his speculations on philosophical and theological matters, and, in mathematics, for his early work in the theory of probability, published posthumously in "A Book on Games of Chance".

## VI. Read and translate the text.

An equation is a statement that two mathematical expressions are equal. A conditional equation is true only for certain values of the variables. Thus,
$3x + y = 7$ is true only for certain values of $x$ and $y$. Such equations are distinguished from identities, which are true for aall values of the variables. Thus,
$(x + y) = x^2 + 2xy + y^2$ which is true for aall values of $x$ and $y$, is an identity. Sometimes the symbol ≡ is used to distinguish an identity from a conditional equation.

## VII. Translate the text into English.

### VOCUBALARY

| | |
|---|---|
| выражаться | be expressed |
| дифференциал | differential |
| искомая величина | an unknown quantity |
| независимая переменная | an independent variable |
| обращаемый | making into |
| переменная | a variable |
| порядок | an order |
| приложение | an application |
| производная | a variable |
| свойство | a property, a characteristics |
| соотношение | a correlation |
| тождество | an identity |
| уравнение в частных производных | an equation in quotient variables |
| функция | a function |

В алгебре для нахождения неизвестных величин пользуются уравнениями. На основании условий задачи составляют **соотношение**, связывающее неизвестную величину с данными, составляют уравнение и, затем, решая его, находят **искомую величину**. Аналогично этому в анализе для нахождения неизвестной функции по данным ее **свойствам** составляют уравнение, связывающее неизвестную функцию и величины, задающие ее свойства, и, поскольку эти последние **выражаются** через **производные** (или **дифференциалы**) того или иного **порядка**, приходят к соотношению, связывающему неизвестную функцию и ее производные или дифференциалы. Это уравнение называется дифференциальным уравнением. Решая его, находят искомую функцию. Из всех отделов анализа дифференциальные уравнения являются одним из самых важных по своим **приложениям**; и это не удивительно: решая дифференциальные уравнения, т.е., находя неизвестную функцию, мы устанавливаем закон, по которому происходит то или иное явление.

Не существует каких-либо общих правил для составления дифференциальных уравнений по условиям конкретной задачи. Условия задачи должны быть таковы, чтобы позволяли составить соотношение, связывающее **независимое переменное**, функцию и ее производную (или производные).

Порядком дифференциального уравнения называется наивысший из порядков входящих в него производных. Если в уравнение входят неизвестная функция нескольких переменных и ее производные (**частные производные)**, то уравнение называется **уравнением в частных производных**.

Обыкновенным **дифференциальным уравнением l-го порядка** называется *соотношение, связывающее независимое переменное, неизвестную функцию этого переменного и ее производную 1-го порядка.* Решением дифференциального уравнения мы будем называть всякую **дифференцируемую функцию,** удовлетворяющую этому уравнению, т.е. **обращаемую** его в **тождество** (по крайней мере, в некотором **промежутке** изменения *x)*;

### VIII. Match the words and the definitions:

fraction, geometry, complex number, algebra, positive number, conditional equation, mantissa, identical equation, characteristic, square root, cube root, equation

1. a whole part of a logarithm;
2. a number that when multiplied by itself gives a given number;
3. a statement that two mathematical expressions are equal; ,
4. a statement that two mathematical expressions are equal for all values of their variables;
5. the branch of mathematics that deals with the general properties of numbers;

6. a number of the type $a + ib$;

## X. Read and decide which of the statements are true and which are false. Change the sentences so they are true.

**1.** For a positive number *n*, the logarithm of *n* (written *log n*) is the power to which some number *b* must be raised to give *n.*

**2.** Common logarithms are logarithms to the base *e* (2.718 ...).

**3.** Common logarithms for computation are used in the form of an integer (the characteristic) plus a positive decimal fraction (the mantissa).

**4.** Logarithms don't obey any laws.

## XI. Match the terms from the left column and the definitions from the right column:

| | |
|---|---|
| 1. logarithm | a) to put (facts, statistics; etc.) in a table of columns |
| 2. base | b) the decimal part of a logarithm to the base 10 as distinguished from the integral part called *the characteristic* |
| 3. antilogarithm | c) a logarithm to the base *e* |
| 4. characteristic | d) any number raised to a power by an exponent |
| 5. mantissa | e) the exponent expressing the power to which a fixed number *(the base)* must be raised in order to produce a given number *(an antilogarithm)* |
| 6. natural logarithm | f) the resulting number when a base is raised to power by a logarithm |
| 7. to tabulate | g) a) the act of computing, calculation, b) a method of computing. |
| 8. computation | h) the whole number, or integral part, of a logarithm as distinguished from the *mantissa* |

## XI. Translate the text into English.

### Натуральные логарифмы

Число *e* имеет очень важное значение (to be of great importance) в высшей математике, его можно сравнить со значением *P* в геометрии. Число *e* применяется как основание натуральных, или неперовых логарифмов, имеющих широкое применение (application) в математическом анализе. Так, с их помощью многие формулы могут быть представлены в более простом виде, чем при пользовании десятичными логарифмами. Натуральный логарифм имеет символ ln.

# UNIT 4

# HIGHER MATHEMATICS

| **BASIC TERMINOLOGY** | | |
|---|---|---|
| **1. SERIES** **ряд** | | |
| **2 + 4 + 6 + 8** | **ARITHMETICAL SERIES** | арифметический ряд |
| **2 + 4 + 8+ 16** | **GEOMETRIC SERIES** | геометрический ряд |
| **2, 4, 6, 8, 16 .....** | **ELEMENTS** | элементы |
| **II. INFINITESIMAL CALCULUS исчисление бесконечно малых величин** | | |
| **dy/dx** | **DERIVATIVE** | производная |
| **dy, dx** | **THE DIFFERENTIALS** | дифференциалы |
| **d** | **DIFFERENTIAL SIGN** | знак дифференциала |
| $\int axdx = a\int xdx = ax^2/2 + c$ | **INTEGRAL** | интеграл |
| **x** | **THE VARIABLE** | переменная (величина) |
| **dx** | **THE DIFFERENTIAL** | дифференциал |
| **c** | **CONSTANT OF INTEGRATION** | постоянная интегрирования |
| $\int$ | **THE INTEGRAL SIGN** | знак интеграла |

## TEXT I. INTEGRAL AND DIFFERENTIAL CALCULUS

**Calculus** is a branch of mathematics using the idea of **a limit** and generally divided into two parts: **integral** and **differential calculus**.

Integral and differential calculus can be used for finding **areas**, **volumes**, **lengths** of **curves**, centroids and **moments of inertia** of **curved figures**. It can be traced back to **Eudoxus of Cnidus** and his method of **exhaustion**. **Archimedes** developed a way of finding the arrears of curved by considering them to be divided by many parallel line **segments**, and extended it to determine the volumes of certain solids; for this he is sometimes called “father of the integral calculus”.

In the early $17^{th}$ century interest again developed in measuring volumes by integration methods. **Kepler** used a procedure for finding the volumes of solids by taking them to be composed of an infinite set of infinitesimally small elements. These ideas were generated by **Cavalieri** in his "Geometria indivisibilibus continuorum nova" and a volume of indivisible areas; i.e., the concept used by **Archimedes** in "The Method'. Cavalieri thus developed what became known as his "method of indivisible". **John Wallis**, in "Arithmetica infinitorum" arithmetized **Cavalieri's** ideas. In this period **infinitesimal methods** were extensively used to find lengths and areas of curves.

Differential calculus is concerned with the **rates of changes** of functions with respect of changes in the independent variable. It came out of problems of finding **tangents** to curves, and an account of the method is published in Isaac Barrow's "Lectiones geometricae". **Newton** had discovered the method and suggested that **Barrow** include it in his book. In his original theory **Newton** regarded a function as a changing quality – **a fluent** – and **the derivative** or rate of change he called **a fluxion**. **The slope** of a curve at a point was found by taking a small element at the point and finding the **gradient** of a straight line through this element. **The binomial** theorem was used to find **the limiting case**, i.e., **Newton's** calculus was an application of **infinite series**. He used the notation **x'** and **y'** for fluxions and **x''** and **y''** for fluxions of fluxions. Thus, if **x=f(t)**, where **x** is the distance and **t** – the time for a moving body, then **x'** is the **instantaneous velocity** and **x''** – **the instantaneous acceleration. Leibniz** had also discovered the method by 1676 publishing it in 1684. **Newton** did not publish until 1687. A bitter dispute arose over the priority for the discovery. In fact it is now known that the two made their discoveries independently and that **Newton** made his about ten years before Leibniz, although **Leibniz** published first. The modern notation of **dy/dx** and the elongated **s** for **integration** is due to **Leibniz**.

From about this time integration came to be regarded simply as the inverse process of **differentiation**. In the 1820s **Cauchy** put the differential and integral calculus on a more secure footing by using the concept of a limit. Differentiation he defined by the **limit** of a ratio and the integration by the limit of a type of sum. The limit definition of integral was made more general by **Riemann**.

In the $20^{th}$ century the idea of an integral has been extended. Originally integration was concerned with elementary ideas of measure (i.e., lengths, areas and volumes) and with **continuous functions**. With the advent of set theory functions came to be regarded as **one-to-one mapping**, not necessarily continuous, and more general and abstract concepts of measure were introduced. **Lévesque** put forward a definition based on the **Lévesque measure** of a set. Similar extensions of the concept have been made by other mathematicians.

NOTE! **dx** is read "differential of **x**"
**dy/dx** is read "derivative of **y** with respect to **x**"

### I. Read and translate the sentences.

1. Integral and differential calculus can be used for finding areas, volumes, lengths of curves.
2. In the early 17th century interest again developed in measuring volumes by integration methods.
3. Kepler used a procedure for finding the volumes of solids by taking them to be composed of an infinite set of infinitesimally small elements.
4. Cauchy put the calculus on a more secure footing by using the concept of a limit.

### II. Match the terms from the left column and definitions from the right column:

| | |
|---|---|
| 1. calculus | a) a fixed quantity or value which a varying quantity is |

| | |
|---|---|
| | regarded as approaching indefinitely |
| 2. differential calculus | b)the rate of continuous change in variable quantities |
| 3. integral calculus | c) the point in a body, or in a system of bodies, at which, for certain purposes, the entire mass may be assumed to be concentrated |
| 4. limit | d) the branch of mathematics dealing with derivatives and their applications |
| 5. volume | e) having the three dimensions of length, breadth and thickness (prisms and other solid figures) |
| 6. centroid | f) a) a part of a figure, esp. of a circle or sphere, marked off or made separate by a line or plane, as a part of a circular area bounded by an arc and its chord, b) any of a finite sections of a line |
| 7. curvee | g) the path of a moving point, thought of as having length but not breadth, whether straight or curved |
| 8. solid | h) the combined methods of mathematical analysis of differential and integral calculus |
| 9. line | i) the limiting value of a rate of change of a function with respect to variable; the instantaneous rate of change, or slope, of a function |
| 10. segment | j) the sum of a sequence, often infinite, of terms usually separated by plus or minus signs |
| 11. derivative | k) the slope of a tangent line to a given curve at a designated point |
| 12. fluxion | l) the branch of higher mathematics that deals with integration and its use in finding volumes, areas, equations of curves, solutions of differential equations |
| 13. slope | m) a one-dimensional continuum of in a space of two or more dimensions |
| 14. series | n) any system of calculation using special symbolic notation |
| 15. infinitesimal calculus | o) the amount of space occupied in three dimensions; cubic contents or cubic magnitude |

**III. Read the sentences and think of a word which best fits each space.**

1. The branch of mathematics dealing derivatives and their applications is called … .
2. Differential calculus deals with … and their applications.
3. We must measure all three dimensions of a solid if we want to find its … .
4. The idea of a … is the central idea of differential calculus.
5. The method of … which is the combine methods of mathematical analysis of differential and integral calculus is very popular in modern mathematics.
6. There are a lot of … around us in our everyday life.

**IV. Give the Russian equivalents of the following words and word combinations:**

| | | | |
|---|---|---|---|
| 1. calculus | 2. limit | 3. integral calculus | 4. differential calculus |
| 5. area | 6. volume | 7. length | 8. curve |
| 9. centroid | 10. moment of inertia | 11. curved figure | 12. exhaustion |
| 13. line segment | 14. solid | 15. infinitesimal method | 16. rate of change |
| 17. independent variable | 18. tangent | 19. fluent | 20. derivative |
| 21. fluxion | 22. slope of a curve | 23. gradient | 24. straight line |
| 25. binomial | 26. limiting case | 27. infinite series | 28. distance |

| | | | |
|---|---|---|---|
| theorem | | | |
| 29. instantaneous velosity | 30. instantaneous acceleration | 31. integration | 32. limit of ratio |
| 33.limit of sum | 34. continuous function | 35.one-to-one mapping | 36. measure of a set |
| 37. differenitation | 38. infinitesimal calculus | | |

## V. Complete the sentences.

1. The branch of mathematics dealing with derivatives and their applications is called ... .

2. Differential calculus deals with ... and their applications.

3. We must measure aall three dimensions of a solid if we want to find its... .

4. The idea of a ... is the central idea of differential calculus.

5. There're a lot of ... around us in our everyday life.

6. The method of ...., which is the combined methods of mathematical analysis of differential and integral calculus is very popular in modem mathematics.

## VI. Read and translate the following sentences. Write 3-4 special questions to each of them:

1. The derivative of a function *f* at a point *x* is defined as the limit.

2. The derivative is denoted in the following way:

*f' (x)* =_______lim Δy= lim */(x+~) - ЛX)*___(which is read: *f'* primed of *x* is equal to the limit of delta *y* over delta *x* with delta *x* tending to zero).

3. That notation of the derivative is commonly used by aall mathematicians.

4. The notion of derivative is justly considered to be one of most important in mathematical analysis.

5. Usually when we say that a function has a derivative *f'(x)* at point *x* it is implied that derivative is finite.

6. The function *f* has a derivative at aall points of the closed interval.

7. In order to compute $f\ '(x+)$ (or $f\ '(x-)$ ) one must remember that the function *f* must be defined at the point *x* and on the right of it in a certain neighborhood.

## VII. Give the English equivalents of the following words and word combinations:

конечная производная, закрытый интервал, начальная и конечная точки, окрестность, открытый интервал, внутренняя точка, бесконечная производная, бесконечно малое приращение, интеграл, стремиться к нулю, коэффициент бесконечно малого приращения.

## VIII. Match the words and the definitions:

segment, point, open interval, positive number, equality, infinity, absolute value, end points, closed interval, derivative, increment, integration, differentiation.

1. numbers defining an interval;
2. the interval which contains the end points;
3. the interval which doesn't contain the end points;
4. a positive and negative change in a variable;
5. an element of geometry having position but no magnitude;
6. the idea of something that is unlimited;
7. the process of finding a function with a derivative that is a given number;

## IX. Translate the definitions of the following mathematical terms:

1. argument - a variable whose value can be determined freely without reference to other variables;
2. increment - the quantity, usually small, by which a variable increases or is

increased;

3. integral - a) the result of integrating a fraction, b) a solution of a differential equation;
4. interior - situated within; on the inside; inner;
5. neighborhood - the set of aall points which lie within a stated distance of a given point.

## TEXT II. A SEQUENCE AND A SERIES

**A sequence** is a succession of terms $a_1$, $a_2$, $a_3$, $a_4$, formed according to some rule or law.

*Examples are:* 1) 1, 4, 9, 16, 25 ...

2)1,-1, 1,-1, 1 ...

3) $x/1!$, $x^2/2!$, $x^3/3!$, $x^4/4!$ ...

It is not necessary for the **terms** to be **distinct.** The terms are ordered by **matching** them one by one with the positive integers, 1, 2, 3, …The n- th term is thus «n, where *n* is a positive integer. Sometimes the terms are matched with the non-negative integers, 0, 1, 2, 3, .... A **finite sequence** has a finite (i.e. limited) number of terms, as in the first example above. An **infinite sequence** has an unlimited number of terms, i.e. there is no last term, as in the second and third examples. An infinite sequence can however approach a **limiting value** as the number of terms *n* becomes very great. Such a sequence is described as a **convergent sequence** and is said to tend to a limit as *n* tends to infinity.

With some sequences, the n-th term (or **general term)** expresses directly the rule by which the terms are formed. This is the case in the three examples above, where the n-th terms are **$n^2$, $(-l)^{n+1}$, $x^n/n!$ n, 1** respectively. A sequence is then a function of *n,* the general term being given by $a_n = f(n)$ and having as its **domain** the set of positive integers (or sometimes the set of non-negative integers). A sequence with general term $a_n$ can be written $(a_n)$ or $\{ a_n \}$.

A **series** is the indicated sum of the terms of a sequence. In the case of a finite sequence $a_1, a_2, a_3, ..., a_n$. the corresponding series is

$$a_1, a_2, a_3 + \ldots + a_n = \sum_{1}^{n} a_n$$

This series has a finite or limited number of terms and is called a **finite series.** The Greek letter S is the summation sign, whose **upper** and **lower limits** indicate the values of the variable *n* over which the sum is calculated; in this case the set of positive integers 1, 2,... N.

In the case of an infinite sequence $a_1 + a_2 + a_3 + ... + a_n = \sum a_n.$

this type of series has an unlimited number of terms end is called an **infinite series.**

The n-th term, $a_n$, of a finite or infinite series is known as the general term. An infinite series can be either a **convergent series** or a **divergent series** depending on whether or not it converges to a **finite** sum. **Convergence** important characteristic of a series.

## I. Read and decide which of the statements are true and which are false.

**Change the sentences so they are true.**

1. A sequence is a succession of terms $a_1, a_2, a_3, a_4$, ... formed according to some rule or law.
2. There're three types of sequences: finite sequence, infinite sequence and infinitesimal sequence.
3. An infinite sequence has a finite (i.e. limited) number of terms.
4. An infinite sequence can however approach a limiting value.
5. A convergent sequence tends to a limit as the number **n** tends to infinity.
6. A series is the indicated sum of the terms of a sequence.

7. The Greek letter ∑ which is used for indicating series is the summation sign.
8. Only a finite series can be either a convergent series or a divergent series depending on whether or not it converges to a finite sum.

## II. Match the terms from the left column and the definitions from the right column:

| | |
|---|---|
| 1. to converge | a) an ordered set of quantities or elements an oordered set of quantities or elements |
| 2. sequence | b) a) indefinitely large; greater than any finite number however large<br>b) capable of being put into one-to-one correspondence with a part of itself |
| 3. finite | c) the sum of a sequence, often infinite, of terms usually separated by plus sign or minus sign |
| 4. infinite | d) 1) capable of being reached, completed or surpassed by counting (said of numbers or sets),<br>2). neither infinite nor infinitesimal (said of magnitudes) |
| 5. series | e) to fit (things) together; make similar or corresponding |
| 6. to match | f) to approach a definite limit, as the sum of certain infinite series of numbers |

## III. Give the Russian equivalents of the following words and word combinations:

non-negative integer, general term, upper limit, finite sequence, divergent series, lower limit, finite series, infinite sequence, domain, infinite series, summation, convergent, limiting value, convergent series.

## IV. Give the English equivalents of the following words and word combinations:

сходящийся ряд, предельное значение, конечная последовательность, область определения, суммирование, расходящийся ряд, бесконечная последовательность, конечный ряд, бесконечный ряд, сходящаяся последовательность, верхний предел, общий член, нижний предел, неотрицательное целое число.

### V. Read and translate the following sentences, write 3-4 special questions to each of them.

1. The terms are ordered by matching them one by one with the positive integers.
2. A finite sequence has a finite number of terms.
3. An infinite sequence can approach a limiting value.
4. Upper and lower limits indicate the values of the variable.
5. An infinite series can be either a convergent or a divergent series.

### VI. Translate the text into English.

**Ряды**

Выражение вида $u_1 + u_2 + u_3 + .........+ u_n + .........$ где $u_1, u_2, u_3$ - члены некоторой бесконечной последовательности, называется бесконечным рядом или просто рядом. Член $u$ называется общим членом ряда. Обозначим сумму $n$ первых членов ряда через $S_n$, т.е. $S_n = u_1 + u_2 + u_3 + ... + u_n$

Сумма $S_n$ называется частичной суммой ряда. При изменении $n$ меняется и $S_n$; при этом возможны два случая:

1). Величина $S_n$ при $n \to \infty$ имеет предел $S$, т.е. $\lim_{n \to \infty} S_n = S$

2) Величина $S_n$ при $n \to \infty$ предела не имеет или предел ее равен.

В первом случае ряд называется сходящимся, а число $S = \lim_{n \to \infty} S_n$ его суммой. Во втором случае ряд называется расходящимся. Такой ряд суммы не имеет.

# CHECKING VOCABULARY IN
# ADVANCED OPERATIONS & HIGHER MATHEMATICS

## I. Choose the appropriate answer.

1. A variable whose limit is zero:

(A) infinitesimal (D) unknown quantity
(B) derivative (E) constant
(C) absolute value (F) limit

2. A positive and negative change in a variable:

(A) increment (D) derivative
(B) argument (E) infinity
(C) function (F) series

3. The interval which doesn't contain the end points:

(A) segment (D) partly open interval
(B) closed interval (E) straight line
(C) open interval (F) curve

4. An equation which is true for aall values of the variable:

(A) conditional equation (D)simple linear equation
(B) identical equation (E) differential equation
(C) integral equation (F) quadratic equation

5. The indicated sum of the terms of a sequence:

(A) finite sequence (D) general term
(B) series (E) summation
(C) infinite sequence (F) I don't know

## II. Give the English equivalents of the following words and word combinations:

бесконечно малая величина, извлечение корня, значение степени, подкоренное выражение, возведение в степень, тело, криволинейные фигуры, касательная, бесконечность, сходящаяся последовательность.

## III. Translate the text without using a dictionary.

### *INTEGRAL EQUATIONS*

It is an equation that involves an integral of an unknown function. A general integral equation of the third kind has the form

$$u(x)g(x) = f(x) + \lambda \int^{b} K(x,y)g(y)dy \quad a$$

where the functions *u(x)*, *f(x)* and *K(x, y)* are known and *g* is the unknown function. The function *K* is the **kernel (1)** of the integral equation and is the parameter.

The limits of integration may be constants or may be functions of *x*. If *u(x)* is zero, the equation becomes an integral equation of the first kind - i.e. it can be put in the form:

$$f(x) = \lambda \int_{a}^{b} K(x,y)g(y)\,dy$$

If *u(x)=1,* the equation becomes an integral equation of the second kind:

$$g(x) = f(x) + \lambda \int_{a}^{b} K(x, y)g(y)dy$$

An equation of the second kind is said to be **homogeneous (2)** if *f(x)* is zero.

If the limits of integration, *a* and *b*, are constants then the integral equation is a Fredholm integral equation. If a is *a* constant and *b* is the variable *x*, the equation is a Volterra integral equation.

# UNIT 5
# GEOMETRY

## BASIC TERMINOLOGY

| | |
|---|---|
| POINT | точка |
| LINE | линия |
| ANGLE | угол |
| POINT OF INTERSECTION | точка пересечения |
| ANGULAR POINT | угловая точка, вершина |
| STRAIGHT LINE | прямая (линия) |
| RAY | луч |
| PENCIL OF RAYS | пучок лучей |
| CURVED LINE | кривая линия |
| RIGHT ANGLE | прямой угол |
| REFLEX ANGLE | угол в пределах 180° и 360° |
| ACUTE ANGLE | острый угол |
| OBTUSE ANGLE | тупой угол |
| CORRESPONDING ANGLE | соответственный угол |
| ADJACENT ANGLE | прилежащий угол |
| SUPPLEMENTARY ANGLE | дополнительный угол [до 180°] |
| COMPLEMENTARY ANGLE | дополнительный угол [до 90°] |
| INTERIOR ANGLE | внутренний угол |
| EXTERIOR ANGLE | внешний угол |
| PLANE TRIANGLE | плоский треугольник |
| EQUILATERAL TRIANGLE | равносторонний треугольник |
| ISOSCELES TRIANGLE | равнобедренный треугольник |
| ACUTE-ANGLED TRIANGLE | остроугольный треугольник |
| OBTUSE-ANGLED TRIANGLE | тупоугольный треугольник |
| RIGHT-ANGLED TRIANGLE | прямоугольный треугольник |
| QUADRILATERAL | четырехугольник |
| SQUARE | квадрат |
| RECTANGLE | прямоугольник |
| RHOMBUS | ромб |
| RHOMBOID | ромбоид |
| TRAPEZIUM | трапеция |
| DELTOID | дельтоид |
| IRREGULAR QUADRILATERALS | неправильный четырёхугольник |
| POLYGON | многоугольник |

| | |
|---|---|
| REGULAR POLYGON | правильный многоугольник |
| CIRCLE | окружность, круг |
| CENTER | центр |
| CIRCUMFERENCE (PERIPHERY) | окружность, периферия |
| DIAMETER | диаметр |
| SEMICIRCLE | полукруг, полуокружность |
| RADIUS | радиус |
| TANGENT | касательная |
| POINT OF CONTACT | точка касания |
| SECANT | секущая |
| CHORD | хорда |
| SEGMENT | сегмент |
| ARC | дуга |
| SECTOR | сектор |
| RING (ANNULUS) | кольцо |
| CONCENTRIC CIRCLES | концентрические окружности |
| AXIS OF COORDINATES | координатная ось |
| AXIS OF ABSCISSAE | ось абсциссы |
| AXIS OF ORDINATE | ось ординаты |
| VALUES OF ABSCISSAE AND ORDINATES | значения абсциссы ординат |
| CONIC SECTION | коническое сечение |
| PARABOLA | парабола |
| BRANCHES OF PARABOLA | ветви параболы |
| VERTEX OF PARABOLA | вёршина параболы |
| ELLIPSE | эллипс |
| (sing. FOCUS) FOCI OF THE ELLIPSE | фокусы эллипса |
| TRANSVERSE AXIS (MAJOR AXIS) | пересекающая ось (главная ось) |
| CONJUGATE AXIS (MINOR AXIS) | сопряженная ось (малая ось) |
| HYPERBOLA | гипербола |
| ASYMPTOTE | асимптота |
| SOLIDS | твердые тела |
| CUBE | куб |
| PLANE SURFACE (A PLANE) | плоская поверхность (плоскость) |
| EDGE | грань |
| PARALLELEPIPED | параллелепипед |
| TRIANGULAR PRISM | трехгранная призма |
| CYLINDER | цилиндр |
| CIRCULAR PLANE | плоскость круга |
| SPHERE | сфера |
| CONE | конус |

## TEXT I.

## THE MEANING OF GEOMETRY

1. Geometry is a very old subject. 2. It probably began in Babylonia and Egypt. 3. Men needed practical ways for measuring their land, for building pyramids, and for defining volumes. 4. The Egyptians were mostly concerned with applying geometry to their everyday problems. 5. Yet, as the knowledge of Egyptians spread to Greece the Greeks found the ideas about geometry very intriguing and mysterious. 6. The Greeks began to ask "Why? Why is that true?" 7. In 300 B. C. all the known facts about Greek geometry were put into a logical sequence by Euclid. 8. His book, called Elements, is one of the most famous books of mathematics. 9. In recent years men have improved on Euclid's work. 10. Today geometry includes not only the study of the shape and size of the earth and all things on it, but also the study of relations between geometric objects. 11. The most fundamental idea in the study of geometry is the idea of a point. 12. We will not try to define what *a* point is, but instead discuss some of its properties. 13. Think of a point as an exact location in space. 14. You cannot see a point, feel a point, or move a point, because it has no dimensions. 15. There, are points (locations) on the earth, in the earth, in the sky, on the sun, and everywhere in space. 16. When writing about points, you represent the points by dots. 17. Remember the dot is only a picture of a point and not the point itself. 18. Points are commonly referred to by using capital letters. 19. The dots below mark points and are referred to as point *A,* point *B,* and point *C.*

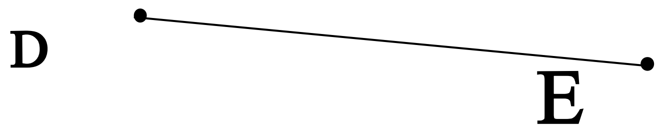
.*B*

.*A*

.*C*

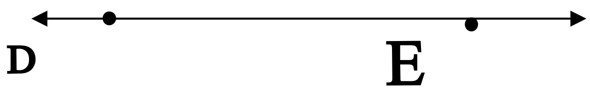
*Lines and Line Segments*

20. If you mark two points on your paper and, by using a ruler, draw a straight line between them, you will get a figure. 21. The figure below is a picture of a line segment.




22. Points *D* and *E* are referred to as endpoints of the line segment. 23. The line segment includes point *D,* point *E,* and all the points between them.

24. Imagine extending the segment indefinitely. 25. It is impossible to draw the complete picture of such an extension but it can be represented as follows.




26. Let us agree on using the word line to mean a straight line. 27. The figure above is a picture of line *DE* or line *ED.*

**I. Read and decide which of the statements are true and which are false. Change the sentences so they are true.**

1. A curve can be considered as the path of a moving point.

2. There're two types of curves: algebraic curves and transcendental curves.

3. Open curves have no end points and closed curves have a lot of end points.

4. A curve that does not lie in a plane is a skew or twisted curve.

5. A curvature is the rate of change of direction of a curve at a particular point on that curve.

6. The angle $\delta\psi$ through which the tangent to a curve moves as the point of contact moves along an arc PQ is the total curvature of this arc.

7. We define the mean curvature of any arc taking into account both the total curvature and the arc length.

8. At any point on a surface the curvature doesn't vary with direction.

## I. Match the terms from the left column and definitions from the right column:

| | |
|---|---|
| 1. curvature | a) 1) any straight line extending from the center to the periphery of a circle or sphere, 2) the length of such a line |
| 2. graph | b) the rate of deviation of a curve or curved surface from a straight line or plane surface tangent to it |
| 3. arc | c) a curve or surface showing the values of a function |
| 4. radius | d) any part of a curve, esp. of a circle |

## II. Read the sentences and think of a word which best fits each space.

**a)** 1. The Egyptians were mostly concerned with applying … to their everyday problems.

2. In 300 B.C. all the known facts about Greek geometry were put into a logical sequence by … .

3. Today geometry includes not only the study of the … and … of the earth and all things on it, but also the study of relations between geometric … .

4. The most fundamental idea in the study of geometry is the idea of a … .

**5.** You cannot see a … , feel a … , or move a … , because it has no dimensions.

**For ideas**: shape, point (4), size, geometry, Euclid, object.

**b)** 1. ... are generally studied as graphs of equations using a coordinate systems.

2. Only ... curves (or arcs) have end points.

3. A curve that does entirely in a plane is a ... curve.

4. A curve that does not lie in ... is a skew or twisted curve.

5. The rate of change of direction of a curve at a particular point on that curve is called a ...

6. The angle $\delta\psi$ through which the tangent to a curve moves as the point of contact moves along any arc is the ... of this arc.

7. The ... of any arc is defined as the total curvature divided by the arc length.

8. The circle of curvature at any point on a curve is the circle that is ... to the curve at that point.

9. There are two ... in which the radius of curvature has an absolute maximum and absolute minimum.

10. The principal curvatures at the point are the curvatures in two ... directions.

## TEXT II.

## ANGLES

An **angle is a configuration** of two lines (the sides or arms) meeting at a point (the **vertex).** Often an angle is regarded as the measure of **rotation** involved in moving from one **initial axis** to coincide with another final axis (termed a directions angle). If the amount and sense of the rotation are specified the angle is a rotation angle, and is positive if measured in an **anticlockwise** sense and negative if in a **clockwise** sense.

Angles are classified according to their measure:

-**Null** (or zero) **angle** - zero rotation (0°).

-**Right angle** - a quarter of a complete turn (90°)

-**Flat (or straight) angle** - half a complete turn (180°).

-**Round angle** (or perigon) - one complete turn (360°),

-**Acute angle** - between 0° and 90°.

-**Obtuse angle** - between 90° and 180°.

-**Reflex angle** - between 180°and 360°.

- The angle of **elevation** of a point *A* from another point *B* is the angle between the line *AB* and the horizontal plane through *B,* with *A* lying above the plane. The angle of **depression** is similarly defined with A lying below the plane. The angle at point *B* made by lines *AB* and *CB* is denoted by $\angle ABC$.

## I. Read and decide which of the statements are true and which are false. Change the sentences so they are true.

1. An angle is often regarded as the measure of rotation involved in moving from one initial axis to coincide with another final axis.
2. There're eleven types of angles in their classification according to their measure.
3. 90° - it is the measure of an acute angle.
4. An angle is positive if it is measured in a clockwise sense.
5. The measure of a reflex angle is between 180°and360°.
6. The main difference of an angle of elevation of a points and its angle of depression is the following one: in the case of the angle of elevation the point *A* lies above the plane and in the case of the angle of depression - below the plane.

## II. Match the terms from the left column and definitions from the right column:

| | |
|---|---|
| 1. an angle | a) formed by, or with reference to, a straight line or plane perpendicular to a base |

| | |
|---|---|
| 2. null | b) of less than 90 degrees |
| 3. right | c) designating an angle greater than a straight angle (180 degrees) |
| 4. obtuse | d) height above a surface, as of the earth |
| 5. flat | e) the shape made by two straight lines meeting at a common point, the vertex, or by two planes, meeting along an edge |
| 6. acute | f) a decrease in force, activity, amount, etc. - a decrease in force, activity, amount, etc. |
| 7. reflex | g) greater than 90 degrees and less than 180 degrees greater than 90 degrees and less than 180 degrees |
| 8. elevation | h) designating of, or being zero, as: a) having all zero elements *(null* matrix), b) having a limit of zero *(null* sequence), c) having no members whatsoever *(null* set) |
| 9. depression | i) absolute, positive |

**III. Give the Russian equivalents of the following words and word combinations:**

1. side (arm)
2. acute angle
3. angle of depression
4. direction angle
5. sense of rotation
6. clockwise sense
7. vertex
8. obtuse angle
9. rotation
10. reflex angle
11. rotation angle
12. angle of elevation
13. right angle

14. flat (straight) angle
15. round angle (perigon)
16. null (zero) angle

### III. Give the English equivalents of the following words and word combinations:

тупой угол, развёрнутый угол, нулевой угол, угол возвышения, угол понижения, прямой угол, полный угол, сторона, направление вращения, вершина, угол в пределах от 180° 360, вращение (поворот), острый угол, по часовой стрелке, против часовой стрелки, угол вращения, направляющий угол.

### IV. Translate the sentences into English.

1. Если две стороны и угол между ними одного треугольника равны соответственно двум сторонам и углу между ними другого треугольника, то такие треугольники равны.
2. Две прямые называются перпендикулярными, если они пересекаются под прямым углом.
3. Какой угол называется прилежащим?
4. Докажите, что вертикальные углы равны.
5. Сумма трех этих углов равна 270°.

### VI. Read the sentences and think of a word which best fits each space.

1. An angle is a ... of two lines (the sides or …) meeting at a point called the vertex.
2. Flat (or ...) angle means half a ... turn.
3. An obtuse angle is greater than an ... angle.
4. The measure of a ... angle is between 180°and 360°.
5. Angles are classified according to their....

6. Clockwise means the ... in which the hands of a clock rotate.
7. The largest angle is the ... angle being 360 degrees.

### VII. Answer the questions on the text "Angles".

1. What is an angle?
2. Can one say that an angle is regarded as the measure of rotation involved in moving from one initial axis to coincide with another final axis?
3. What are characteristics of a null angle?
4. An acute angle is an angle between 0° and 90°, isn't it?
5. What are characteristics of an obtuse angle?
6. What are characteristics of a reflex angle?
7. Is there any difference between the angle of depression and the angle of elevation?

## TEXT III.

## A POLYGON

A **polygon** is a figure formed by three or more points (**vertices**) joined by line segments (sides). The term is usually used to denote a closed **plane** figure in which no two sides **intersect**. In this case the number of sides is equal to the number of **interior** angles. If all the interior angles are less than or equal to 180°, the figure is **a convex** polygon; if it has one or more interior angles greater than 180°, it is a **concave** polygon, A polygon that has all its sides equal is an **equilateral** polygon; one with all its interior angles equal is an equiangular polygon. Note that an equilateral polygon need not be equiangular, or vice versa, except in case of an equilateral **triangle**. A polygon that is both equilateral and equiangular is said to be **regular**. The **exterior** angles of a regular polygon are each equal to 360° /**n**, where **n** is a number of sides.

The distance from the center of a regular polygon to one of its vertices is called the long radius, which is also a radius of the **circumcircle** of the polygon. The **perpendicular** distance from the center to one of the sides is called the short radius or **apothem**, which is also the radius of the **inscribed circle** of the polygon.

A regular star polygon is a figure formed by joining every m-the point, starting with a given point, of the n points that divide a circle's circumference into **n** equal parts, where **m** and **n** are relatively **prime**, and **n** is equal two or greater than 3. This star polygon is denoted by **{m/n}.** When **m = 1**, the resulting figure is a regular polygon. The star polygon **{5/2}** is the **pentagram**.

## I. Read and decide which of the statements are true and which are false.

## Change the sentences so they are true.

1. A polygon is a figure formed only by three vertices joined by line segments.
2. No sides of a polygon usually intersect.
3. All the interior angles of a convex polygon are greater than or equal to180°.
4. An equilateral polygon is a polygon whose no sides are equal.
5. An equiangular polygon is a polygon whose all interior angles are equal.
6. The regular polygon is both equilateral and equiangular.
7. The perpendicular distance from the center to one of the sides is called the long radius.
8. The long radius is also the radius of the inscribed circle of the polygon.
9. The star polygon is usually denoted by **{m/n}.**

## II. Match the terms from the left column and definitions from the right column:

| | |
|---|---|
| 1. apothem | a) to draw a figure inside another figure so that their |

| | |
|---|---|
| | boundaries touch at as many points as possible |
| 2. circle | b) a) at right angles to a given plane or line, b) exactly upright; vertical; straight up or down |
| 3. convex | c) a) any of the four angles formed on the inside of two straight lines when crossed by a transversal, b) the angle formed inside a polygon by two adjacent sides |
| 4. concave | d) any figure of five lines |
| 5. equilateral | e) a geometrical figure having three angles and three sides |
| 6. equiangular | f) a) the point of intersection of the two sides of an angle, b) a comer point of a triangle, square, cube, parallelepiped, or other geometric figure bounded by lines, planes, or lines and planes |
| 7. exterior angle | g) curving outward like the surface of a sphere |
| 8. to inscribe | h) having all angles equal |
| 9. interior angle | i) a closed plane figure, esp. one with more than four sides and angles |
| 10. pentagram | j) a) any of the four angles formed on the outside of two straight lines when crossed by a transversal, b) an angle formed by any side of a polygon and the extension of the adjacent side |
| 11. perpendicular | k) a plane figure bounded by a singly curved line, every point of which is equally distant from the point at the center of the figure |
| 12. polygon | l) hollow and curved like the inside half of a hollow ball |
| 13. triangle | m) having all sides equal |
| 14. vertex | n) the perpendicular from the center of a regular polygon to any one of its sides |

## III. Give the Russian equivalents of the following words and word combinations:

1. exterior angle

2. circumcircle
3. perpendicular
4. apothem
5. inscribed circle
6. star polygon
7. relatively prime
8. regular star polygon
9. pentagram
10. closed plane figure
11. interior angle
12. convex polygon
13. concave polygon
14. equilateral polygon
15. equiangular polygon
16. equilateral triangle
17. regular polygon
18. long radius

## IV. Give the English equivalents of the following words and word combinations:

равносторонний треугольник, правильный треугольник, замкнутая плоская фигура, апофема (радиус вписанного круга), вписанная окружность, внутренний угол, взаимно простые, радиус описанного круга, выпуклый многоугольник, описанная окружность, пентаграмма (пятиугольная звезда), равноугольный многоугольник, вогнутый многоугольник, многоугольник в виде звезды, правильный многоугольник в виде звезды, внешний угол, перпендикуляр, равносторонний многоугольник.

## V. Answer the questions on the text.

1. What figure is usually understood in geometry as a polygon?
2. No two sides in a polygon intersect, do they?
3. What is the usual measure of the interior angles of a convex polygon and of a concave polygon respectively?
4. A polygon that has all its sides equal is an equiangular polygon, isn't it?
5. A polygon in which all its interior angles are equal is an equilateral polygon, isn't it?
6. Must an equilateral polygon be equiangular?

## VI. Read and translate the sentences and the questions.

1. This figure is formed by three points.
2. This term is usually used to denote a closed plane figure.
3. This polygon has only one interior angle.
4. This polygon is said to be regular.
5. The points divide a circle's circumference into equal parts.
6. What are the main characteristics of a regular polygon?
7. The distance from the center of a regular polygon to one of its vertices is called the long radius, isn't it?
8. Is the apothem also the radius of the inscribed circle of the polygon?
9. What figure is called a regular star polygon?
10. What figure is usually understood in geometry as a polygon?

## VII. Match the words and the definitions:

interior angle, convex polygon, exterior angle, circumcircle, pentagram, concave polygon, inscribed circle, regular polygon, long radius.

1) An angle formed outside a polygon.

2) A circle circumscribed about a given polygon.

3) A polygon that has all its angles less than or equal to 180°.

4) An angle between two sides of a polygon lying within the polygon.

5) A symmetrical five-pointed star polygon.

6) A polygon that has at least one interior angle greater than 180°.

## VIII. Translate the text into English.

### ТРЕУГОЛЬНИКИ.

Выпуклый треугольник называется правильным, если все его стороны равны и равны все его углы.

**Многоугольник называется вписанным в окружность, если все его вершины лежат на некоторой окружности. Многоугольник называется описанным около окружности, если все его стороны касаются данной окружности.**

Правильный выпуклый многоугольник является одновременно вписанным в окружность и описанным около нее.

Углом выпуклого многоугольника при определенной вершине называется угол, образованный его сторонами, которые сходятся в этой вершине. Внешним углом выпуклого многоугольника при данной вершине называется угол, смежный с внутренним углом многоугольника при этой вершине.

## TEXT IV.

## THE CARTESIAN COORDINATE SYSTEM

The Cartesian coordinate system is a coordinate system in which the position of a point is determined by its distances from reference lines (axes). In two dimensions two lines are used; commonly the lines are at right angles, forming a rectangular coordinate system. The horizontal axis is x-axis and the vertical axis is the y-axis. The point of intersection O is the origin of the coordinate system. Distances along the x-axis to the right of the origin are usually taken as positive, distances to the left - as negative. Distances along the y-axis above the origin are positive; distances below are negative. The position of a point anywhere in the plane can then be specified by two numbers, the coordinates of the point, written (x,y). The x-coordinate (or abscissa) is the distance of the point from the y-axis in a direction parallel to the x-axis (i.e. horizontally). The y-coordinate (or ordinate) is the distance of the point from the x-axis in a direction parallel to the y-axis (vertically). The origin O is the point (0,0). The two axes divide the plane into four quadrants, numbered anticlockwise starting from the top right (positive) quadrant.

Cartesian coordinates were first introduced in the 17th century by Rene Descartes. Their discovery allowed the application of algebraic methods to geometry and the study of hitherto unknown curves. As a point in Cartesian coordinates is represented by an order pair of numbers, so is a line represented by an equation. Thus, $y = x$ represent a set of points for which the x-coordinate equals the y-coordinate; i.e. $y = x$ is a straight line through the origin at 45° to the axes. Equations of higher degree represent curves; for example, $x^2 + y^2 = 4$ is a circle of radius 2 with its center at the origin. A curve drawn in a Cartesian coordinate system for a particular equation or function is a graph of the equation or function.

The axes in planar Cartesian coordinate system need not necessarily be at right angles to each other. If the x- and y- axes make an angle than 90° the system is said to be an oblique coordinate system. Distances from the axes are then measured along lines parallel to the axes.

Cartesian coordinate system can also be for three dimensions by including a third axis - z-axis - through the origin perpendicular to the other two. The position of point is then given by three coordinates (x,y,z). The coordinate axes may be left-handed or right-handed, depending on the way positive directions are given to the axes. In a right-handed system if the thumb of the right hand points in the positive direction of the x-axis, the first and second fingers can be pointed in the positive direction of the y- and z-axes respectively. A left-handed system is the **mirror image** of this (i.e. determined by using the left hand).

I. **Read and decide which of the statements are true and which are false. Change the sentences so they are true.**

1. The Cartesian coordinate system is a coordinate system in which the position of a point is determined by its distances from axes.

**2.** The vertical axis on the coordinate system is ***x***-axis and the horizontal axis is the ***y***-axis.

**3.** The point of intersection **O** is the origin of the coordinate system.

**4.** Distances along the ***x*-**axis to the right of the origin are usually taken as negative and distances to the left - as positive.

**5.** The position of a point anywhere in the plane is specified by at least four numbers.

**6.** The abscissa is the distance of the point from the ***y***-axis in the vertical direction.

7. The ordinate is the distance of the point from the ***x***-axis in the horizontal direction.

8. The two axes divide the plane into four quadrants numbered clockwise starting from the top left (negative) quadrant.

9. If a point in Cartesian coordinates is represented by an order pair of numbers, but a line is represented by an equation.

10. The axes in planar Cartesian coordinate system can be at various right angles to each other (right, obtuse, etc.).

11. Cartesian coordinate system can also be for three dimensions by including a third axis - ***z***-axis - through the origin perpendicular to the other two

axes.

12. The coordinate axes may be left-handed or right-handed, depending on the way positive directions are given to the axes.

### II. Match the terms from the left column and the definitions from the right column:

| | |
|---|---|
| 1. abscissa | a) the vertical Cartesian coordinate on a plane, measured from the ***x***-axis along a line parallel with the ***y***-axis to point P |
| 2. axis (axes) | b) perpendicular, or at a right angle, to the plane of the horizon; upright, straight up or down, etc. |
| 3. Cartesian coordinates | c) a) of or pertaining to a point on a surface at which the curvature is zero, b) of or lying in the plane |
| 4. horizontal | d) any of the four parts formed by rectangular coordinate axes on a plane surface |
| 5. oblique | e) the horizontal Cartesian coordinate on a plane, measured from the ***y***-axis to point P |
| 6. ordinate | f) a) a straight line through the center of a plane figure or a solid, esp. one around which the parts are symmetrically arranged, b) a straight line for measurement or reference, as in a graph |
| 7. origin | g) with its axes not perpendicular to its base |
| 8. planar | h) parallel to the plane of the horizon, not vertical |
| 9. quadrant | i) in a system of Cartesian coordinates, the point at which the axes intersect; base point where the abscissa and the ordinate equal zero |
| 10. vertical | j) a pair of numbers that locate a point by its distances from two fixed, intersecting, usually perpendicular lines in the same plane |

### III. Read and translate the sentences.

1. Two lines are used in this system.
2. These axes divide the plane into four quadrants.
3. The Cartesian coordinate system was introduced in the 17th century.

4. This particular discovery allowed the application of algebraic methods to the geometry.

5. The coordinates can be used for three dimensions.

6. The axes may be left-handed or right-handed, depending on the way positive directions are given to the axes.

7. The system is said to be an oblique coordinate system.

## IV. Give the English equivalents of the following words and word combinations:

базисная прямая, прямоугольная координатная система, система прямоугольных (декартовых) координат, система косоугольных координат, система плоскостных декартовых координат, начало координат, квадрант, вертикальное отображение, ордината, абсцисса, левосторонняя координатная ось, вертикальная ось, горизонтальная ось.

## V. Read the sentences and think of a word which best fits each space.

1. The position of a point in the ... system is determined by its distances axes.

2. The point **O** of ... of two axes is the ... of the coordinate system.

3. ... is the distance of the point from the ***y***-axis in a direction parallel to the ***x***-axis.

4. ... is the distance of the point from the ***x***-axis in a direction parallel to the ***y***-axis.

5. The two axes divide the plane into four ....

6. A point in Cartesian coordinates is represented by an ... of numbers

7... . in Cartesian coordinates are represented by equations of higher degree.

8.The ... in planar Cartesian coordinate system can be both at right and obtuse ... to each other.

9. Cartesian coordinate system can also be for two and three .... . Three imensions include a third axis - *z*-axis – through ….to the other two axes.

10. The coordinate axes may be left-handed or right-handed, depending on the way ... are given to the axes.

## VI. Translate the paragraphs into English.

1. Прямые ***x, y, z*** называются координатными осями, точка их пересечения **0** - началом координат, а плоскости ***xy, yz, xz*** – координатными плоскостями.

2. В декартовой системе прямоугольных координат на плоскости каждой точке соответствует пара действительных чисел ***x*** и ***y***, определяющих положение данной точки на плоскости, и, наоборот, каждой паре действительных чисел ***x*** *и* ***y*** соответствует только одна точка на плоскости.

3. Координатой ***y*** точки **М** называется число, измеряющее расстояние от данной точки до прямой **Ох** и взятое со знаком «плюс», если данная точка **М** расположена выше прямой **Ох**, и со знаком «минус», если точка **М** расположена ниже прямой **Ох.** Координату ***y*** называют ординатой, а ось **О*у*** - осью ординат.

## VII. Read and translate the following sentences. Write 2-3 special and tag questions to each of them.

1. It is surface composed of plane polygonal surfaces.
2. This term is used for closed solid figures.
3. These figure played a significant part in Greek geometry.
4. That polyhedron has identical polyhedral angles.
5. Other polyhedra can be generated by truncated the other regular polyhedra.

6. He used them in his complicated model.

7. These solids were known to Plato.

8. There are some possible convex regular polyhedra in this text.

9. A plane cuts other faces.

10. The vertices lie at the centers of the edges of the original cube.

### VIII. Put the words in the correct order to make the sentences.

1. were, ancient, solids, known, Greeks, these, to.
2. meeting, points, are, the vertex, two, there, at.
3. a figure, formed, a polygon, three or more, is, by, points.
4. figure, is, three-dimensional, this, a, geometric.
5. an angle, one quarter, turn, right angle, equal to, compete, is, a, of, a.
6. the eccentricity, the conic, of, the constant, is.
7. central conics, the hyperbola, known, and, are, the ellipse, as.
8. by, a, three, is, a triangle, figure, line, formed, closed, plane, segments.

### IX. Give the English equivalents of the following words and word combinations:

многогранный угол, замкнутая пространственная фигура, тело геометрически правильной формы, прямая призма, архимедово тело, кубооктаэдр, грань (плоская поверхность), антипризма, икосододекаэдр, ребро, усеченный куб, полуправильный многогранник, платоново тело восьмигранник (октаэдр), куб, четырехгранник (тетраэдр), выпуклый многогранник, вогнутый многогранник, двадцатигранник (икосаэдр), однородный многогранник.

### X. Read and decide which of the statements are true and which are false. Change the sentences so they are true.

**1.** A polyhedron is a surface composed of plane triangular surfaces.

**2.** The sides of the polygons, joining two faces, are its edges.

**3.** There're two types of polyhedra: concave and convex ones.

**4.** The faces of a regular polyhedron are formed by identical (congruent) regular polygons.

**5.** A tetrahedron has got six square faces.

**6.** An octahedron has got eight triangular faces.

**7.** A dodecahedron has got twenty triangular faces.

**8.** The five regular solids were known to Plato and so they're often called Platonic solids.

**9.** A uniform polyhedron is a polyhedron that has identical polyhedral angles at all its vertices and has all its faces formed by regular polygons.

**10.** An icosidodecahedron, a cuboctahedron and truncated cube represent the so called "semiregular polyhedra".

### XI. Match the terms from the left column and the definitions from the right column:

| | |
|---|---|
| 1. congruent | a) a solid figure with twenty plane surfaces |
| 2. cube | b) a solid figure with eight plane surfaces |
| 3. dodecahedron | c) a plane figure with five angles and five sides |
| 4. icosahedron | d) a solid figure, esp. one with more than six plane surfaces |
| 5. identical | e) a solid figure whose ends are parallel, polygonal, and equal in size and shape, and whose sides are parallelograms |
| 6. octahedron | f) a solid figure with four triangular faces |
| 7. pentagon | g) a solid figure with twelve plane faces |

| | |
|---|---|
| 8. polyhedron | h) 1. a) cut off or replaced by a plane face (said of the angles or edges of a crystal or solid figure), b) having its angles or edges cut off or replaced in this way (said of the crystal or solid figure); 2. having a vertex cut off by a plane that is not parallel to the base(said of a cone or pyramid). |
| 9. prism | i) of figures, having identical shape and size |
| 10. tetrahedron | j) a solid with six equal, square sides |
| 11. truncated | k) 1. the very same; 2. exactly alike or equal; |

## XII. Read the definitions and decide what terms are defined.

a) A solid figure that has four triangular faces.

b) One of the plane regions bounding a polyhedron.

c) A solid figure that has six identical faces.

d) A line joining two vertices of a geometric figure.

e) A polyhedron that has eight faces.

f) A polyhedron that has twelve pentagonal faces.

## XIII. Translate the text into English.

### МНОГОГРАННИК

Многогранником называется тело, ограниченное конечным числом плоскостей. Это значит, что вся его поверхность расположена в конечном числе плоскостей. Многогранник называется выпуклым, если он лежит по одну сторону каждой из ограничивающих его плоскостей. Общая часть поверхности выпуклого многогранника и ограничивающей его плоскости называется гранью. Стороны граней называются ребрами многогранника, а вершины - вершинами многогранника.

Поясним данное определение на примере куба. Куб есть выпуклый многогранник. Его поверхность состоит из шести квадратов: ABCD, BEFC, ... Они являются его гранями. Ребрами куба являются стороны этих квадратов; AB, BC, BE, ... . Вершинами куба являются вершины квадратов A, B, C, D, E,... . У куба шесть граней, двенадцать ребер и восемь вершин.

## XIV. Read and translate the following sentences. Write 2-3 special and tag questions to each of them:

1. The given figure is formed from two congruent polygons with their corresponding sides parallel and the parallelograms formed by joining the corresponding vertices of the polygons.
2. A right prism is one in which the lateral edges are at right angles to the bases.
3. One base is displaced with respect to the other, but remains parallel to it.
4. The term "cone" is often used loosely for "conical surface".
5. The common vertex isn't coplanar with the base.
6. The pyramid which has its axis perpendicular to its base is a right pyramid.
7. The given surface is composed of plane polygonal surfaces.
8. This term is used for closed solid figures.
9. Greeks thought that these figures played a significant part in geometry.
10. That polyhedron has identical polyhedral angles.
11. Other polyhedra can be generated by truncated the other regular polyhedron.
12. Kepler used these solids in his complicated model.
13. These solids were already known to Plato.
14. The given plane cuts other faces.
15. We see that all vertices lie at the centers of the edges of the original cube.

## XV. Read the definitions and decide what terms are defined.

1) A solid figure that has four triangular faces.
2) One of the plane regions bounding a polyhedron.
3) A solid figure that has six identical faces.
4) A line joining two vertices of a geometric figure.
5) A polyhedron that has eight faces.
6) A polyhedron that has twelve pentagonal faces.

## XVI. Read and decide which of the statements are true and which are false. Change the sentences so they are true.

1. A prism is a solid figure formed from three congruent polygons with their corresponding sides perpendicular.
2. Prisms are named according to the base, thus, a triangular prism has two triangular bases.
3. There're only two types of prisms: right and regular.
4. A cone is a solid figure formed by a circle and curve on a plane and all the lines joining points of the base to a fixed point
5. The curved area of the cone forms its lateral surface.
6. A cone that has its axis perpendicular to its base is an oblique cone.
7. The altitude of a cone is the line parallel to the plane of the base.
8. A pyramid is a solid figure formed by a polygon (the base) and a number of triangles (lateral faces) with a common vertex that is coplanar with the base.
9. A pyramid that has its axis perpendicular to its base is a right pyramid.
10. The volume of any pyramid is l/3Ah, where A is the area of the base.
11. The slant height of the pyramid is the altitude of a face and the total surface area of the lateral faces is l/2**sp**, where **p** is the perimeter of the base polygon.
12.

## XVII. Translate the definitions of the following mathematical terms.

1. altitude – the perpendicular distance from the base of a figure to its highest point or to the side parallel to the base;
2. circular – in the shape of a circle; round;
3. cone – a flat-based, single-pointed solid formed by a rotating straight line that traces out a closed-curved base from a fixed vertex point that is not in the same plane as the base; esp. one formed by tracing a circle from a vertex perpendicular to the center of the base;
4. coplanar – in the same plane: said of figures, points, etc;
5. generator (generatrix) – a point, line or plane, whose motion generates a curve, plane, or figure;
6. height – the distance from the bottom to the top;
7. hexagon – a plane figure with six angles and six sides;
8. hexagonal – having a six-sided base or section: said of a solid figure;
9. lateral – of, at, or toward the side; sideways;
10. parallelogram – a plane figure with four sides, having the opposite sides parallel and equal;
11. pyramid – a solid figure having a polygonal base and four sloping, triangular sides meeting at the top;
12. quadrangle – a plane figure with four angles and four sides;
13. rectangle – any four-sided plane figure with four right angles;
14. slant – an oblique or inclined surface, line, direction, etc; slope; incline;
15. triangular – of or shaped like a triangle; three-cornered;
16. volume – the amount of space occupied in three dimensions; cubic contents or cubic magnitude.

## XVIII. Translate the sentences into English.

1. Отрезки, соединяющие вершину конуса с точками окружности основания, называются образующими конуса.
2. Объем любой треугольной пирамиды равен одной трети произведения площади основания на высоту.
3. Осью правильной пирамиды называется прямая, содержащая ее высоту.
4. Ребра призмы, соединяющие вершины оснований, называются боковыми ребрами.
5. Многогранником называется тело, ограниченное конечным числом плоскостей,
6. Боковая поверхность прямой призмы равна произведению периметра основания на высоту призмы.
7. Пирамидой именуется геометрическая фигура с многоугольным основанием и четырьмя сторонами в виде треугольников, сходящимися в вершине пирамиды.
8. Противоположные стороны параллелограмма равны и параллельны.
9. Конус – это твердое тело с одной вершиной и основанием в виде плоскости.
10. Правильной считается пирамида с осью, перпендикулярной основанию.

**11.** Конусы, призмы и пирамиды названы по типу их оснований.

**12.** Высотой конуса именуется перпендикулярное расстояние от его вершины до плоскости основания.

## XIX. Translate the text into English.

## ПРИЗМА

Призмой называется многогранник, образованный заключенными между двумя параллельными плоскостями отрезками всех параллельных прямых, которые пересекают плоский многоугольник в одной из плоскостей. Грани призмы, лежащие в этих плоскостях, называются основаниями призмы. Другие грани называются боковыми гранями. Все боковые грани - параллелограммы. Ребра призмы, соединяющие вершины оснований, называются боковыми ребрами. Все боковые ребра призмы параллельны.

Высотой призмы называется расстояние между плоскостями ее оснований. Отрезок, соединяющий две вершины, не принадлежащие одной грани, называется диагональю призмы. Призма называется прямой, если ее боковые ребра перпендикулярны основаниям. В противном случае призма называется наклонной. Прямая призма называется правильной, если ее основания являются правильными многоугольниками.

## CHECKING VOCABULARY IN GEOMETRY

### I. Choose the correct variant of the answer.

**1. An angle equal to one-half of a complete turn:**

**(A)** flat angle (D) obtuse angle

**(B)** right angle (E) reflex angle

**(C)** round angle (F) acute angle

**2. A type of conic that has an eccentricity greater than 1:**

(A) parabola (D) focus

(B) hyperbola (E) transverse axis

(C) ellipse (F) circle

**3. A plane figure formed by four intersecting lines:**

(A) angle (D) quadrilateral

(B) cube (E) star polygon

(C) triangle (F) square

**4. A surface composed of plane polygonal surface:**

(A) polyhedron (D) quadrilateral

(B) polygon (E) circle

(C) isosceles (F) dodecahedron

**5. A line either straight or continuously bending without angles:**

(A) curvature (D) curve

(B) straight line (E) height

(C) ray (F) circle

## II. Give the English equivalents of the following words and word combinations:

соответственный угол, тупоугольный треугольник, касательная дуга, хорда, кольцо, окружность, пространство, уравнение прямой в отрезках, вектор положения точки, пространственная кривая, прямолинейная координата, по часовой стрелке, против часовой стрелки, угол вращения, выпуклый многоугольник, равноугольный многоугольник.

## III. Give the Russian equivalents of the following words and word combinations:

1. edge;
2. origin of coordinates;
3. reference line;
4. mirror image;
5. translation of axes;
6. generating angle;
7. semi-regular polyhedron;
8. truncated cube;
9. oblique cone;
10. slant height.

## IV. Write special questions using the words in brackets.

1. This figure is formed from two congruent polygons. **(What, How many)**
2. The polyhedron has got identical polyhedral angles. **(What angles)**
3. Rene Descartes used these; equations in his complicated model. **(Who, Where)**

4. The vertices lie at the centers of the edges. **(What, Where)**

5. They could be formed by different section. **(What...by, What)**

6. Cavaliery had discovered this system by 1774. **(Who, What system)**

## V. Translate the text without using a dictionary.

### PARABOLA

Parabola is a type of conic that has an eccentricity equal to 1. It is an open curve symmetrical about a line (its axis). The point at which the curve cuts the axis is the vertex. In a Cartesian coordinate system the parabola has a standard equation of the form "$y^2 = 4ax$".

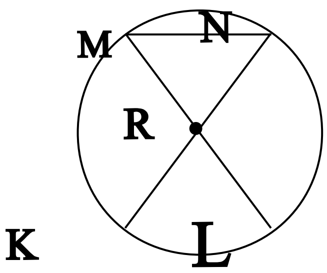
Here, the axis of the parabola is the x-axis, the directrix is the line $x = -a$, and the focus is the point (a,0). The length of the chord through the focus perpendicular to the axis is equal to 4a.

The **focal property (1)** of the parabola is that for any point P on the curve, the tangent at P (APB) makes equal angles with a line from the focus F to P and with a line parallel to the x-axis. This is also called the **reflection property** (2), since for a parabolic reflector light from a **source (3)** at the focus would be reflected in a **beam (4)** parallel to the x-axis and **sound** (5) would be similarly reflected.

### Notes:

1) **focal property** - фокальное свойство
2) **reflection property** - свойство отражения
3) **source** - источник
4) **beam -** луч
5) **sound** – звук

### VI. Use the figure for completing the following statements.




1. *RM* is called a………………. of the circle.
2. *KN is* twice as long as...............................
3. *LM* is called a………………………….of a circle.
4. *RL* has the same length as ……………………….
5. ▲ *MRN* is an............................... triangle.
6. Point *R* is called the ..................................of the circle and the ... .......................of ∟*KRL.*
7. *MN* is called ................................... of a circle.
8. *MN* is called an ..........................................
9. ∟*MRN* is an..........................................angle.
10. ∟*MRK* is a........................................ angle.
11. No matter how short an arc is, it is................... at least slightly.
12. The term circumference means...................................
13. A diameter is a chord which................................. .
14. A circle is a set of points in a plane each of which......................... .
15. We cannot find the circumference of a circle by adding.....................

### VII. Translate the following sentences.

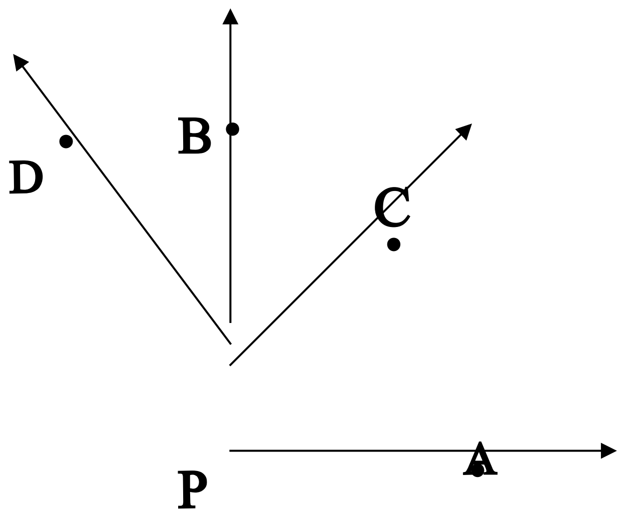
1. Сумма углов треугольника равна 180◦.
2. В треугольнике может быть только один тупой угол и два острых.
3. В равностороннем треугольнике все углы равны.
4. Углы при основании в равнобедренном треугольнике равны.

5. В прямоугольном треугольнике сумма квадрата катетов равна квадрату гипотенузе.
6. В прямоугольнике противоположные стороны равны и параллельны.
7. Параллельные линии не пересекаются.
8. При помощи циркуля можно начертить окружность.
9. Площадь круга равна $\pi R^2$.
10. Любая точка лежащая на окружности равноудалена от центра.
11. Мы всегда можем вычислить площадь криволинейной трапеции.
12. Синусоиду можно растянуть вдоль оси координат.

### VIII. Complete the sentences with the following words:

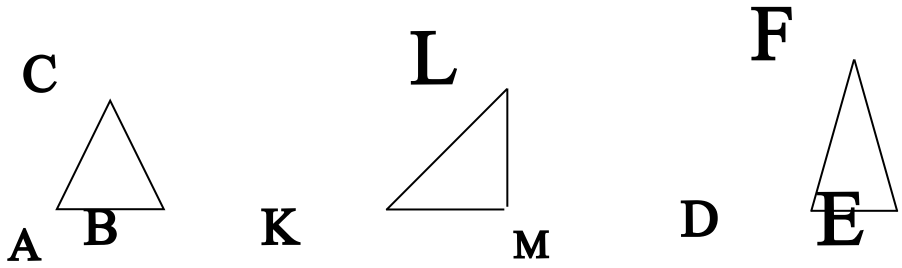
legs, a ray, polygon, a radius, a center, a hypotenuse, triangles, π, a diameter, a circle, circumference, an angle, an obtuse, a chord, an acute, an equilateral.

1. You certainly remember that by extending a line segment in one direction we obtain ... .
2. The following symbol ∟ is frequently used in place of the word ... .
3. Since PD (except for point P) lies in the exterior of ∟APB, we say that ∟APD is greater than a right angle and call it ... .




4. A simple closed. Figure formed by line segments is called... .

5. This is true of..... – geometric figures having three sides.
6. ▲DEF is called .... which means that its two sides have the same measure.
7. ▲ABC is referred as .... triangle.




8. In ▲MKL, ∟M is the right angle sides MK and ML are called the..., and side KL is called the... .
9. A .... is a set of points in a plane each of which is equidistant, that is the same distance from some given point in the plane called...
10. A line segment joining any point of the circle with the center is called…. of a circle is a line segment whose endpoints are points on the circle.
11. ….is a chord which passes through the center of the circle.
12. Instead of speaking of the perimeter of a circle, we usually use the term...
13. The number c/d or c/2*r which is the same for all circles, is designated by … .

# PART II

## MATHEMATICAL SYMBOLS AND EXPRESSIONS

| | |
|---|---|
| $+$ | addition, plus, positive – знак сложения или положительной величины |
| — | subtraction, minus, negative – знак вычитания или отрицательной величины |
| $\pm$ | plus or minus – плюс минус |
| *X* или · | multiplication sign, multiplied by – знак умножения, умноженный на … |
| ÷ или / | division, divided by – знак деления, деленный на ... |
| $a/b$ | *a* divided by *b* – *a* деленное на *b* |
| : | dividied by, ratio sign – делённое; знак отношения |
| :: | equals; as – знак пропорции |
| $<$ | less than – менее |
| $\nless$ | not less than – не менее |
| $>$ | greater than – более |
| $\ngtr$ | not greater than – не более |
| $\approx$ | approximately equal – приблизительно равно |
| ∽ | similar to – подобный |
| $=$ | equals – равно |
| $\neq$ | not equal to – не равно |
| $\doteq$ | approaches – достигает значения |
| $\sim$ | difference – разность |
| $\infty$ | infinity – бесконечность |
| $\therefore$ | therefore – следовательно |
| $\because$ | since, because – так как |
| $\sqrt{}$ | square root – квадратный корень |
| $\sqrt[3]{}$ | cube root – кубичный корень |
| $\sqrt[n]{}$ | *n*th root – корень *n-й* степенн |
| $\leq$ | equal to or less than – меньше или равно |
| $\geq$ | equal to or greater than – больше или равно |
| $a^n$ | the *n*th power of *a* – *a* в *n*-й степени |
| $a_1$ | *a* sub 1 – *a* первое |
| $a_n$ | *a* sub *n* – *a n*-е |
| ∟ | angle – угол |
| $\perp$ | perpendicular to – перпендикулярно к |
| $\parallel$ | parallel to – параллельно |
| *log* или $log_{10}$ | common logarithm, or Briggsian logarithm десятичный логарифм |
| $log_e$ или *1n* | natural logarithm, or hyperbolic logarithm, or Naperian logarithm – |

натуральный логарифм

*e base* (2.718) of natual systems of logarithms – основание натуральных логарифмов

*sin* sine – синус (sin)

*cos* cosine – косинус (cos)

*tan* tangent – тангенс (tg)

*ctn* или *cot* cotangent – котангенс (ctg

*sec* secant – секанс (sec)

*csc* cosecant – косеканс (cosec)

*vers* versine, versed sine – синус-верзус

*covers* coversine, coversed sine – косинус-верзус

$sin^{1-}$ antisine – арксинус (arcsin)

$cos^{1-}$ anticosine – арккосинус (arccos)

*sinh* hyperbolic sine – синус гиперболический (sh)

*cosh* hyperbolic cosine – косинус гиперболический (ch)

*tanh* hyperbolic tangent – тангенс гиперболический (th)

*f(x)* или *(x)* function of *x* – функция от *x*

*f'* *f* primed – производная

$\Delta x$ increment of *x* – приращение *x*

$\sum$ summation of – знак суммирования

$\int$ integral of – интеграл от

*a*

$\int$ integral between the limits *a* and *b* – интеграл в пределах от *a* до *b*

*b*

○ или ⊙ circ1e; circumference – круг; окружность

*(),[],{}* parentheses, brackets, and braces – круглые, квадратные и фигурные скобки

$\overline{AB}$ length of *1ine* from *A to B* – длина отрезка *AB*

*μ* micron = 0,001 mm – микрон ($10^{3-}$ *мм*)

*mμ*

millimicron = 0,001μ – миллимикрон ($10^{7-}$ *см*)

$^{o}$ degree – градус

′ minute – минута

′′ second – секунда

\# 1. № (номер), если знак предшествует числу; 2. англ. фунт, если знак поставлен после числа

℄ centre line – центральная линия, линия центров

*1st* first – первый

*2nd* second – второй

*3rd* third – третий

*4th* fourth – четвертый (все однозначные порядковые от 4 до 9 имеют окончание *th*)

*5′* 1. пять футов; 2. угол в 5 мин

*9″* 1. девять дюймов; 2. угол в 9 сек

*.5* (англичане и американцы иногда не пишут нуль uелых)

*1.5* (англичане и американцы отделяют знаки десятичных дробей не запятой, а точкой, ставя ее вверху, в середине или внизу строки)

*7,568 = 7568; 1,000,000=* $10^6$ (англичане и американцы в многозначных числах отделяют каждые три цифры запятой)

$.0^5103$ *= 00000103 = 0,00000103* (англичане и американцы иногда записывают, таким образом, для краткости малые дроби, впрочем, в большинстве случаев они пользуются общепринятой записью *103* x $10^{5-}$ )

*2/0, 3/0* и т. д. означают номера размеров проводов *00, 000* и т.д, согласно британскому стандартному калибру проводов (SWG)

## READING OF MATHEMATICAL EXPRESSIONS

1. $x > y$ «*x* is greater than *y*»
2. $x < y$ «*x* is less than *y*»
3. $x = 0$ «*x* is equal to *zero*»
4. $x \leq y$ «*x* is equal or less than *y*»
5. $x < y < z$ «*y* is greater than *x* but less than *z*»
6. *xv* «*x* times or *x* multiplied by *y*»
7. $a + b$ «*a* plus *b*»
8. $7 + 5 = 12$ «*seven* plus *five* equals *twelve*; *seven* plus *five* is equal to *twelve*; *seven* and *five* is (are) *twelve*; *seven* added to *five* makes *twelve*»
9. $a - b$ «*a* minus *b*»
10. $7 - 5 = 2$ «*seven* minus *five* equals *two*; *five* from *seven* leaves *two*; difference between *five* and *seven* is *two*; *seven* minus *five* is equal to *two*»
11. $a \times b$ «*a* multiplied by *b*»
12. $5 \times 2 = 10$ «*five* multiplied by *two* is equal to *ten*; *five* multiplied by *two* equals *ten*; *five* times two is *ten*»
13. $a : b$ «*a* divided by *b*»
14. $a/b$ «*a* over *b,* or *a* divided by *b*»
15. $10 : 2 = 5$ «*ten* divided by *two* is equal to *five; ten* divided by *two* equals *five*»
16. $a = b$ «*a* equals *b,* or *a* is equal to *b*»
17. $b \neq 0$ «*b* is not equal to *0*»
18. $m : ab$ «*m* divided by *a* multiplied by *b*»
19. $\sqrt{ax}$ «The square root of *ax*»
20. ½ «one second»
21. ¼ «one quarter»
22. $-7/5$ «minus seven fifth»
23. $a^4$ «*a* fourth, *a* fourth power or *a* exponent 4»
24. $a^n$ «*a* *n*th, *a* *n*th power, or *a* exponent *n*»
25. $e^{\pi}$ «*e* to the power *π* »
26. $\sqrt[n]{b}$ «The *n*th root of *b*»
27. $\sqrt[3]{8}$ «The *cube* root of *eight* is *two*»
28. $\log_{10} 3$ «Logarithm of *three* to the base of *ten*»
29. $2 : 50 = 4 : x$ «*two* is to *fifty* as *four* is to *x*»
30. $4!$ «factorial 4»
31. $(a + b)^2 = a^2 + 2ab + b^2$ «The square of the sum of two numbers is equal to the square of the first number, plus twice the product of the first and second, plus the square of the second»
32. $(a-b)^2 = a^2 - 2ab + b^2$ «The square of the difference of two numbers is equal to the square of the first number minus twice the product of the first and second, plus

the square of the second»

*33.* $\Delta x$ «Increment of *x*»

*34.* $\Delta x \rightarrow 0$ «delta *x* tends to *zero*»

*35.* $\sum$ «Summation of ...»

*36.* $dx$ «Differential of *x*»

*37.* $dy/dx$ «Derivative of *y* with respect to *x*»

*38.* $d^2y/dx^2$ «Second derivative of *y* with respect to *x*»

*39.* $d^ny/dx^n$ «*n*th derivative of *y* with respect to *x*»

*40.* $dy/dx$ «Partial derivative of *y* with respect to *x*»

*41.* $d^ny/dx^n$ «*n*th partial derivative of *y* with respect to *x*»

*42.* $\int$ «Integral of ...»

*43.* $\int_b^a$ «Integral between the limits *a* and *b*»

*44.* $\sqrt[5]{d^n}$ «The fifth root of *d* to the *n*th power»

*45.* $\sqrt{a+b/\ a-b}$ «The square root of *a* plus *b* over *a* minus *b*»

*46.* $a^3 = log_c d$ «*a* cubed is equal to the logarithm of *d* to the base *c*»

*47.* $\int_\tau^t f[S,\ \varphi(S)]\ ds$ «The integral of *f* of *S* and *φ* of *S*, with respect to *S* from *τ* to *t*»

*48.* $\dfrac{d^2y}{ds^2} + (l + b\ (S))y = 0$ «The second derivative of *y* with respect to *s*, plus *y* times the quantity *l* plus *b* of *s,* is equal to *zero*»

*49.* $X_{a-b} = e^{tl}$ «*X* sub *a* minus *b* is equal to *e* to the power *t* times *l*»

*50.* $f(z) = K_{ab}$ «*f* of *z* is equal to *K* sub *ab*»

*51.* $\dfrac{d^2u}{dt^2} = 0$ «The second partial (derivative) of *u* with respect to *t* is equal to zero»

# PART III

## ADDITIONAL READING

## I. MATRICULATION ALGEBRA
## DEFINITIONS

1. ALGEBRA is the science which deals with quantities. These quantities may be represented either by figures or by letters. Arithmetic also deals with quantities, but in Arithmetic the quantities are always represented by figures. Arithmetic therefore may be considered as a branch of Algebra.

2. In Algebra it is allowable to assign any values to the letters used; in Arithmetic the figures must have definite values. We are therefore able to state and prove theorems in Algebra as being true, universally, for all values; whereas in Arithmetic only each particular sum is or is not correct. Instances of this will frequently occur to the student of Algebra, as he advances in the subject.

3. This connection of Arithmetic and Algebra the student should recognize from the first. He may expect to find the rules of Arithmetic included in the rules of Algebra. Whenever he is in a difficulty in an algebraical question, he will find it useful to take a similar question in Arithmetic with simple figures, and the solution of this simple sum in Arithmetic will often help him to solve correctly his algebraical question.

4. All the signs of operation used in Arithmetic are used in Algebra with the same significations, and all the rules for arithmetical operations are found among the rules for elementary Algebra. Elementary Algebra, how ever, enables the student to solve readily and quickly many problems which would be either difficult or impossible in Arithmetic.

5. **Signs and abbreviations.** — The following signs and abbreviations are used in Algebra :—
+ **plus**, the sign of addition.
− **minus**, the sign of subtraction.
× **into**, or **multiplied by**, the sign of multiplication.
÷ **by**, or **divided by**, the sign of division.
~ **the sign of difference** ; thus, a~6 means the difference between *a* and *b,* whichever is the larger.
= **is**, or **are**, **equal to**.
.•. **therefore**.

6. The sign of multiplication is often expressed by a **dot** placed between the two quantities which are to be multiplied together.

Thus, 2.3 means 2×3; and *a. b* means a × *b.*

This dot should be placed low down, in order to distinguish it from the decimal point in numbers. Thus 3.4 means 3×4; but 3·4 means 3 *decimal point* 4, that is 3 + ·4.

More often between letters, or between a number and a letter, no sign of multiplication is placed.

Thus 3*a* means 3 × *a;* and *bcd* means *b* × *c* × *d.*

1. The operation of division is often expressed by writing the dividend over the divisor, and separating them by a line.

Thus $\frac{a}{b}$ means *a* ÷ *b.* For convenience in printing this line is sometimes written in a slanting direction between the terms ; thus *a/b* = $\frac{a}{b}$.

The words *sum, difference, multiplier, multiplicand, product, divisor, dividend,* and *quotient* are used in Algebra with the same meanings as in Arithmetic.

8. **Expressions and terms**. — Quantities in Algebra are represented by figures and by letters. The letters may have any values attached to them, provided the same letter always has the same value in the same question.

The letters at the beginning of the alphabet are generally used to denote *known* quantities, and the letters at the end of the alphabet are used to denote quantities whose values are *unknown.* For example, in the expression *ax* + *by* — *c,* it is generally considered that *a*, *b,* and *c* denote known values, but *x* and *y* denote unknown values.

An **algebraical expression** is a collection of one or more signs, figures, and letters, which are used to denote **one** quantity.

**Terms** are parts of an expression which are connected by the signs + or —.

A **simple expression** consists of only one term.

A **compound expression** consists of two or more terms.

Thus *a, bc,* and *3d* are simple expressions; and *x* + *3yz* — *2xy* is a compound expression denoting one quantity ; and *x, 3yz,* and *2xy* are terms of the expression.

A **binomial** expression is a compound expression consisting of only **two** terms; *e.g., a*+*b* is a binomial expression.

A **trinomial** expression is a compound expression consisting of only **three** terms; *e.g., a* — *b* + *c* is a trinomial expression.

A **multinomial expressi**on is a compound expression consisting of more than three terms.

**Positive ter**ms are terms which are preceded by the sign +.

**Negative terms** are terms which are preceded by the sign −.

When a term is preceded by no sign, the sign + is to be understood. The first term in an expression is generally positive, and therefore has no sign written before it.

Thus, in *a* + 2*b* — 3*c*, *a* and 2*b* are positive terms, and 3*c* is a negative term.

**Like terms** are those which consist of the same letter or the same combination of letters. Thus, *a, 3a,* and 5*a* are like terms; *bc,* 2*bc*, and 6*bc* are like terms ; but *ab* and *ac* are unlike terms.

9. The way in which the signs of multiplication and division are abbreviated or even omitted in Algebra will serve to remind the student of the important rule in Arithmetic that the operations of multiplication and division are to be performed before operations of addition and subtraction.
For example — $2 \times 3 + 4 \div 2 — 5 = 6 + 2 — 5 = 3$.
A similar sum in Algebra would be

$$ab + \quad — e.$$

From the way in which this is written, the student would expect that he must multiply *a* by *b,* and divide *c* by *d,* before performing the operations of addition and subtraction.

10. **Index, Power, Exponent.**—When several like terms have to be multiplied together, it is usual to write the term only once, and to indicate the number of terms that have to be multiplied together by a small figure or letter placed at the right-hand top corner of the term.
Thus:—
$a^2$ means *a.a,* or a × *a.*
$a^3$ means *a.a. a,* or *a*×*a*×*a.*
$a^4$ means *a.a.a.a,* or *a*× *a*× *a* × *a.*
$a^2$ is read ***a* square;** $a^3$ is read ***a* cube ;** $a^4$ is read ***a* to the fourth power,** or, more briefly, ***a* to the fourth;** $a^7$ is read ***a* to the seventh power, or *a* to the seventh;** and so on.

Similarly, $(3a)^4 = 3a \times 3a \times 3a \times 3a = 81a^4$; and $a^b$ means that *b a*'s are to be multiplied together.

11. Instead of having several like terms to multiply together, we may have a number of like expressions to multiply together. Thus, $(b + c)^3$ means that *b* + *c* is to be multiplied by *b* + (b + c). This will be explained more fully when the use of brackets has been *c*, and the product multiplied again by *b*+*c*; *i.e.,* $(b + c)^3 = (b + c) \times (b + c) \times$ explained.

12. The small figure or letter placed at the right-hand top corner of a quantity to indicate how many of the quantities are to be multiplied together is called an **index,** or **exponent.** This index or exponent, instead of being a number or letter, may also be a compound expression, or, in fact, any quantity; but we, at first, restricts ourselves to positive integral indices. We say, therefore, that an **index** or **exponent** is an integral quantity, usually expressed in small characters, and placed at the right-hand top corner of another quantity, to express how many of this latter quantity are to be multiplied together. A **power** is a product obtained by multiplying some quantity by itself a certain number of times.

13. Notice carefully that an index or an exponent expresses how many of a given quantity are to be multiplied together. For example, $a^5$ means that five *a*'s are to be multiplied together. In other words, the index expresses how many factors are to be used. The index, if a whole number, is always greater by one than the number of times that the given quantity has to be multiplied by itself. For example, the 5 in $a^5$ expresses the fact that five factors, each equal to *a,* are to be multiplied together; or, in other words, that *a* is to be multiplied by itself **four** times. Thus, $a^5 = a \times a \times a \times a \times a$. This fact is often overlooked by beginners.

14. **Factor, Coefficient, Co-Factor.** — A term or expression may consist of a number of symbols, either numbers or letters, which are multiplied together. For example, the term $15a^2bc$ consists of the numbers 3 and 5 and the letters *a, a, b, c* all multiplied together.

**A factor** (Lat. **facere,** to make) of an expression is a quantity which, when multiplied by another quantity, makes, or produces, the given expression. In the above example 3, 5, *a, b, c,* and also 15, *ab, ac, &c*., are all factors of $15a^2bc$. For we may consider that

$$15a^2bc = 3 \times 5\, a \times a \times b \times c;$$

or that $\quad 15a^2bc = 15 \times ab \times ac;$

or that $\quad 15a^2bc = 15 \times a^2\, bc;$

or that $\quad 15a^2\, bc = ab \times 15ac$ ; *&c*.

15. It is evident that the term $15a^2\, bc$ may be broken up into factors in several ways. Sometimes the factors of a quantity may be broken up again into simpler factors. Thus the factors 15 and $a^2\, bc$ may be broken up again into 5 and 3 and into *ab* and *ac*; and *ab* and *ac* may be broken up again into *a* and *b*, and into *a* and *c*. When a quantity has been broken up into its simplest factors, these factors are called the **simple** or **prime** factors of the quantity. In whatever way we begin to break up a given integral quantity into factors, if we continue to break each factor into simpler factors as long as this is possible, we shall always arrive at the same set of simple factors from the same integral quantity. There is therefore only one set of simple or prime factors for the same integral quantity. In the above example the simple factors of $15a^2bc$ are 3, 5, *a, a*, *b*, *c*.

16. When a quantity is broker up into only two factors, either of these factors may be called the **Coefficient or Co-Factor** of the other factor. For example, in $15a^2bc$ we may call 15 the coefficient of $a^2bc,$ or $15a^2$ the coefficient of *bc,* or *3ab* the coefficient of 5*ac*, *&c*. It is convenient, however, to use the word coefficient in the sense of numerical coefficient, and to speak of 15 as the coefficient of $a^2\, bc$ in $15a^2bc$. In this sense the *coefficient* of a quantity is the numerical factor of the quantity.

17. In Arithmetic the factors of a whole number or integer are always taken to be whole **numbers** or **integers.** The factors of a

fraction may be either integers or fractions. For example, the factors of 6/5 may be either 3 and 2/5, or 2 and 3/5, or 6 and 1/5; or, again, the factors of *3/4* may be taken as ½ and 2/3, or as 3 and 1/4. In the case of fractions, a fraction can be broken up into different sets of simple factors in an infinite number of ways.

18. The coefficient of a quantity may be either integral or fractional. Thus in $5/6a^2b$ the coefficient is 5/6. When no coefficient is expressed, the coefficient **one** is to be understood. Thus *ab* means **once** *ab,* just as in Arithmetic 23 means **once** 23.

19. **Roots.** — We have seen that $a \times a = a^2$. Here we multiply the quantity *a* by itself and so get $a^2$. Suppose we reverse this process; that is, we have a quantity given us, and we try to find some quantity which, when multiplied by itself, will produce the given quantity. For example, what quantity multiplied by itself will give $a^2$? Evidently, *a* is the required answer. Again, what number multiplied by itself will produce 16? Here 4 is the answer. In these cases we are said to find a root of $a^2$, and of 16.

A **root** of a given quantity is a quantity which, when multiplied by itself a certain number of times, will produce the given quantity.

20. The **square root** of a given quantity is that quantity which, when two of them are multiplied together, produces the given quantity. Thus, the square root of $a^2$ is *a*; because two *a*'s multiplied together produce $a^2$. Again, the square root of 16 is 4, because two fours multiplied together produce 16.

The square root of a quantity is indicated by the sign √, which was originally the first letter in the word **radix**, the Latin for **root**. Thus, $\sqrt{16} = 4$ ; $\sqrt{a^2} = a$.

21. The **cube root** of a given quantity is that quantity which, when three of the latter are multiplied together, produces the given quantity. The cube root of a quantity is indicated by the sign $\sqrt[3]{\ }$. Thus, $\sqrt[3]{64} = 4$, because $4 \times 4 \times 4 = 64$. Similarly, $\sqrt[3]{a^3} = a$, because $a \times a \times a = a^3$.

22. In like manner $\sqrt[4]{\ }$, $\sqrt[5]{\ }$, $\sqrt[6]{\ }$ &c., are used to indicate the fourth, fifth, sixth, &c., roots of a quantity. Thus, $\sqrt{64} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$. Similarly, $\sqrt[5]{a^5} = a$; $\sqrt[7]{a^7} = a$ ; $\sqrt[3]{x^3} = x$ ; $\sqrt[3]{y^3} = y$.

23. With regard to Square and Cube Boot, the student may notice that in Mensuration, if the area of a square is given, the length of each side of the square is expressed by the square root of the quantity expressing the area. For example, a square whose area is 16 square feet has each side 4 feet long. Similarly, a cube whose content is 27 cubic feet has each edge 3 feet long.

24. **Brackets**. — In Arithmetic each number, as, for example, 13, is thought of as one number. It is true that 13 is equal to the sum of certain other numbers; e.g., 6

+ 4 + 3 = 13; but we do not necessarily consider 13 as made up of these numbers, 6, 4, and 3. So also in Algebra each expression must be considered as expressing one quantity, e.g., $a + b — c$ represents the one quantity which is obtained by adding 5 to $a$ and then subtracting $c$ from the sum of $a$ and $b$.

So also each of the expressions in Exercises I ***a*** and I ***b*** represents one quantity. The answer to each of these examples is the numerical value of the example when the letters $a$, $b$, $c$, $d$, $e$, and $f$ have the numerical values mentioned.

25. When an expression is made up of terms containing the signs +, —, ×, and ÷, either expressed or understood, we know from Arithmetic that the operations of multiplication and division are considered as indicating a closer relation than the operations of addition and subtraction. The operations of multiplication and division must be performed first, before the operations of addition and subtraction. For example,

$$3 + 8 \div 2 — 2\times3 = 3 + 4—6 = 1.$$

Exactly the same rule applies in Algebra. For example, consider the expression $a + bc — d\div e + f$. Here we must first multiply $b$ by $c$, and divide $d$ by $e$. Then we add the product to $a$, then subtract the quotient, and finally add $f$ to get the result.

26. Frequently, however, it is necessary to break this rule about the order of operations, and we may wish some part of an expression to be considered as forming but one term. This is indicated by placing in brackets that part which is to be considered as one term.

For example, in Arithmetic, $(3 + 7) \times 2 = 10 \times 2 = 20$. Here we treat 3 + 7 as one term, and therefore we place it in brackets. If we leave out the brackets, $3 + 7\times2 = 3 + 14 = 17$. Exactly the same thing is done in Algebra. For example, $(a + b) \times c$ means that the sum of $a$ and $b$ is to be multiplied by $c$; whereas $a+b\times c$ means that first of all $b$ is to be multiplied by $c$, and then the product is to be added to $a$.

27. **Negative quantities.** — In Arithmetic, in questions involving subtraction, we are always asked to take a smaller quantity from a larger quantity. For example, if we have to find the difference between 5 and 7, we say 7—5 = 2. But suppose we are asked to subtract 7 from 5. Arithmetically this is impossible. In Algebra such a question is allowable. We say that 5 — 7 = 5 —5—2 = —2, and we arrive at a negative answer, namely —2. In Algebra, therefore, we may either subtract 5 from 7, or 7 from 5; and we consider it correct to write $a — 5$, whether $a$ is larger or smaller than $b$.

28. Instead of considering abstract numbers like 5 and 7, let us suppose that we have to deal with concrete quantities such as £5 and £7. Suppose a tradesman made a profit of £5 one day, and then lost £7 the next day. How should we express his total profit? We should say £5 —£7 = —£2; his profits on the two days amounted to —£2; or, in other words, he lost +£2. It appears then that the negative result of —£2 profit can be expressed as a positive result of + £2 loss.

29. Again, suppose a ship sails 5 miles towards a harbour, and then is carried back by wind and tide 7 miles away from the harbour. We might say that the ship has advanced (5 — 7) miles, or —2 miles towards the harbour; or that the ship has retired + 2 miles from the harbour.

30. In both these examples a negative answer can be expressed as a positive answer by altering the form of the answer. This can always be done with concrete quantities, and, in Arithmetic, whenever we arrive at a negative result, we transpose the form of the answer and express the result as a positive answer. In Algebra, however, it is convenient to leave a negative result and even to speak of a negative quantity without expressing any positive quantity. Thus we speak of —$a$, or of —3$b$, &c., as well as of +$a$ or of +3$b$, &c.

31. The signs, therefore, + and — are used to distinguish quantities of opposite kinds. Every term in an algebraical expression and also every factor in every term must be thought of as being preceded by either + or —. If no sign is expressed, the sign + is understood. This use of the signs + and — is so constant and so important that + and — are often spoken of as *the* signs in an expression, and to change the signs in an expression means to change all + signs to —, and all — signs to +. For example, $a + b—c$ is the same expression as $-a—b+c$ with the signs changed.

This use of + and — before each term must not be confused with the use of the same signs to mark operations of addition and subtraction.

## ADDITION AND SUBTRACTION

32. We know, from Arithmetic, that the operations of addition and subtraction are mutually opposed. If we add to and subtract from the same number some other number, we shall not alter the number with which we started. For example, suppose we start with 7. Add and subtract 3, thus: 7 + 3—3 = 7; adding and subtracting 3 has not altered the 7. This is true in Algebra.

In Arithmetic we can add together two or more abstract numbers and express them more shortly as a single number, thus : 2 + 3 + 5 = 10; but in Algebra we can only add together and express more shortly terms which are alike, thus: $2a + 3a + 5a = 10a$. Terms which are unlike cannot be added together; thus $a$ + b + $c$ cannot be expressed in a shorter form.

33. The rules for addition are as follows: —

*(1) Only like terms can be added.*

(2) *Add together all the like terms that are positive and all the like terms that are negative; subtract the smaller of these sums from the larger, and prefix the sign of the larger sum.*

Remember that when no numerical coefficient is expressed the coefficient 1 is understood.

34. **Subtraction.** — If we add together —7 and 20, we get 13. If we subtract + 7 from 20, we get 13. Therefore to subtract + 7 from 20 gives the same result as adding -7 to 20.

Conversely, since +7 added to 20 gives 27, we might infer that —7 subtracted from 20 would give 27, and this would be correct. Hence we can also infer a general rule for subtraction, viz.: — *Change the signs of all the terms in the express-ion which has to lie subtracted, and then proceed as in addition.*

*For example:* — Subtract $3a—4b$ from $8a + 2b$.

Set down as in addition, and change the signs in the lower line, thus:

$$\begin{array}{r} 8a + 2b \\ \underline{-3a + 4b} \\ 5a + 6b \end{array}$$

By adding, we get 5a + 6i as the difference required.

In working subtraction sums, the signs in the lower lines should be changed mentally. The above sum would then appear thus:

$$\begin{array}{r} 8a + 2b \\ \underline{3a - 4b} \\ 5a + 6b. \end{array}$$

The actual process of working this sum after setting it down would be as follows: — Begin with the $a$'s, minus 3 and plus 8; the plus is the larger by 5; therefore set down $5a$, omitting the plus sign because $5a$ is the first term. Again, plus 4 and plus 2 give plus 6; therefore set down plus $6b$.

35. As in addition, like terms must be arranged under like terms. Take another example. From $5a^3 + 5a^2 — 7a + 3a^4—5$ take $5a^2 — 6a + 7 — 2a^4 + 2a^3$. Arrange in order thus:

$$\begin{array}{c} 3a^4 + 5a^3 + 5a^2 - 7a - 5 \\ \underline{-2a^4 + 2a^3 + 5a^2 - 6a + 7} \\ 5a^4 + 3a^3 — a — 12. \end{array}$$

The working of this question was as follows: — Begin with $a^4$ plus 2 and plus 3 give plus $5a^4$. Then, for $a^3$, minus 2 and plus 5, the plus is the larger by 3; therefore set down + $3a^3$. Then $a^2$ minus 5 and plus 5 give 0; therefore set nothing down. Then $a$ plus 6 and minus 7, the minus is the larger by 1; therefore set down $—a$, the 1 being understood before the $a$. Lastly, minus 7 and minus 5 give —12.

36. Notice that in Algebra we do not consider which expression is the larger in a subtraction sum. The answer may be either a positive or a negative quantity; and so in Algebra we may subtract either a larger quantity from a smaller, or a smaller quantity from a larger. Also the letters used in an algebraical expression may have **any** value, so that we cannot always tell which, is the larger of two expressions. We usually, therefore, pay no attention in Algebra to the magnitudes of the quantities we use.

37. Since subtraction and addition are inverse operations, we can prove the accuracy of our work in an addition sum by subtracting one or more of the expressions added

together from the sum; and we can prove the accuracy of a subtraction sum by adding the expression subtracted to the remainder. To take an example from Arithmetic: 7+5+3=15; to prove that this is correct, we subtract 3 from 15, thus: 15— 3 = 12; then subtract 5 from 12, thus: 12 — 5 = 7; we have now come back to 7, which is the number we started with; so we infer that our addition was correct. In subtraction, 14—5 = 9 and 5 + 9 = 14. This will be evident to the student from his knowledge of Arithmetic.

## MULTIPLICATION

38. In Arithmetic we say 2×3 = 6. No notice is taken of signs; but, if this be expressed fully and correctly, we should say + 2× +3 = +6. Therefore, when two terms with **plus** signs are multiplied together the product is **plus**.
Suppose +2 × -3 or -2 × +3. Evidently the product will not be the same in either of these cases as in +2 × +3. Therefore we assume that +2 × -3 = -6 and -2 × +3= -6.
*Therefore, when one term has a plus sign and the other term has a minus sign the product is minus.*
Again, suppose -2 × -3. This is different from the last two cases, and we assume that -2 × -3 = +6. Therefore, when two terms with **minus** signs are multiplied together the product is **plus**.
From these results we can infer the rule of signs.
**Rule of signs**. — *Like signs produce plus; unlike produce minus.*

39. The application of the rule of signs is very important when we come to deal with indices or powers, and roots of quantities. For example:
$(+a)^2 = +a \times +a = +a^2 = a^2$.
$(-a)^2 = -a \times -a = +a^2 = a^2$.
$(+a)^3 = +a \times +a \times +a = +a^3 = a^3$.
$(-a)^3 = -a \times -a \times -a = -a^3$.
We see that a **plus quantity raised to any power produces a plus result; a minus quantity raised to an even power produces a plus result,** e.g., $(-a)^6 = a^6$; but **a minus quantity raised to an odd power produces a minus result**,
e.g., $(-a)^7 = -a^7$.
Again, with roots $\sqrt{(a^2)}$ = either $+a$ or $-a$, since $+a \times +a = a^2$, and $-a \times -a = a^2$ also.
So that the square root of a positive or plus quantity is either plus or minus; that is, every positive quantity which is an exact square has two roots, these roots being of opposite sign — the one plus and the other minus.
Since like signs produce plus, we cannot find the square root of any negative or minus quantity, e.g., $\sqrt{(-a^2)}$ is impossible quantity, for $-a \times -a = +a^2$, and $a \times a = +a^2$.
Again, $\sqrt[3]{(+a^3)} = +a^3$, since $+a \times +a \times +a = +a^3$; and $\sqrt[3]{(-a^3)} = -a^3$, since $-a \times -a \times -a = -a^3$.
We see, therefore, that apparently there is only one real or possible cube root of a given quantity, but this given quantity may be either plus or minus.

Similarly, for higher powers; if we are asked to find the 4th, 6th, 8th, or any even root of a, given quantity, we can only do so when the given quantity is plus, and then we can find two real roots, one of each sign. But, if we are asked to find the 5th, 7th, 9th, or any odd root of a given quantity, we may be able to do so whatever the sign of the quantity is, but we can only find **one real** root, and the sign of this root will be the same as the sign of the given quantity.

40. These conclusions must be understood to be true only in a limited sense. It is only in a few cases that any root can be obtained exactly; as, for example, the square roots of 4, 9, 16, &c., of $a^2$, $a^4$, $a^6$, &g. ; the cube roots of 8, 27, &o., and of $a^3$, $a^6$, $a^9$, &c. But we can calculate roots of numbers to some required degree of accuracy, or we can express the roots algebraically without actually calculating them, e.g., $\sqrt[5]{(a^4)}$, $\sqrt[7]{(a^2)}$, $\sqrt[8]{(a^3)}$, &g. The student also will learn afterwards to consider that every quantity has just as many roots as the power of the root, e.g., there are 5 fifth roots of any quantity, 6 sixth roots, 7 seventh roots, and so on. One or more of these roots will be real, and the rest only imaginary.

41. We have already seen that when any term is multiplied by itself the product may be expressed in a simple form by the use of an index or power. Thus
$a\times a=a^2$; $b\times b\times b=b^3$; $c\times c\times c\times c=c^4$; and so on. By reversing the process, $b^4=b\times b\times b\times b$ and $b\times b= b^2$.
Therefore $b^4\times b^2=b\times b\times b\times b\times b\times b=b^6$.
Hence we infer that different powers of the same form may be multiplied by writing the quantity with an index equal to the sum of the indices of the multipliers. In the above example, 4 + 2 = 6; therefore $b^4\times b^2=b^6$.
Similarly, $b^3\times b^5=b^8$.
Also, since $a= a^1$, $a^2\times a= a^3$, $a^4\times a= a^5$; and so on.
Also we have seen that when two different terms are multiplied together the product may be expressed by writing the two terms side by side. Thus: $a\times b = ab$; $c^2\times d^4=c^2d^4$.

42. In Arithmetic the student knows that, if several numbers have to be multiplied together, the numbers may be taken in any order. For example:
2 ×3 ×4 = 2 ×4 ×3 = 3 ×4 ×2 = 4 ×3 ×2, &c., for each product is equal to 24. So also in Algebra the terms in any product may be taken in any order. So that
$abc = acb = bca = bac = cab = cba$.
If, therefore, in Algebra we have to multiply together two or more simple factors, we may place the numerical factors all together, and we may gather together any factors which are powers of the same quantity, and apply the rule for the multiplication of indices. For example:
$3a^2b^3c^4\times 2ab^2c^3=3\times 2\times a^2\times a\times b^3\times b^2\times c^4\times c^3=6\times a^3\times b^5\times c^7=6a^3b^5c^7$.

43. In the multiplication of simple expressions like the above, the student will find it advisable to take the numbers first, then the letters in alphabetical order, and, lastly, to apply the rule of signs.

44. **Dimension and degree**. — If we take a simple expression and write down separately all the letters used as factors of the expression, and if we then count the letters, we obtain the number of the dimensions, or the degree of the term. Thus $3a^2 b^3 = 3\times a\times a\times b\times b\times b$, is of five dimensions, or of the fifth degree; or $3a^2 b^3$ is of two dimensions in $a$, and of three dimensions in $b$. In multiplication, the dimensions of the product must be equal to the sum of the dimensions of the factors. With integral indices, the dimension or degree of any term is equal to the sum of all the indices; thus $3abc^2$ is of the fourth degree, the indices being 1, 1, 2.

45. The following considerations will enable the student to test the correctness of his work. Notice that —

(1) *There are as many lines as there are terms in the multiplier.*

(2) *There are as -many terms in each line as there are terms in the multiplicand*

(3) With regard to signs, *a plus sign in the multiplier will leave all the signs the same as in the multiplicand*. Conversely, *a minus sign in the multiplier will change all the signs of the multiplicand in the corresponding line of the product.*

(4) It is advisable *to arrange both multiplicand and multiplier in descending powers of some letter*, because by so doing we shall find that the products produced in the working will be easier to arrange in columns.

46. **A compound expression in which all the terms are of the same dimension is said to be homogeneous.**

Since the dimension of every term in a product is equal to the sum of the dimensions of its factors, it follows that, if we multiply together two homogeneous expressions, we shall obtain a homogeneous product.

47. The following rules will therefore enable us to read off the product when two binomial expressions, such as $x + 7$ and $x — 8$, are multiplied together.

(1) *The first and last terms in the product are obtained by multiplying together the two first terms, and then the two last terms.*

(2) *The coefficient of the middle term in the product is obtained by adding together, algebraically, the two last terms;* e.g.,

$(a + 6)(a + 4) = a^2 + 10a+24$.

Similar rules will hold if, instead of a number, we use any other kind of term for the second term in each multiplier; e.g., $(a + 2b)(a + 3b) = a^2 +5ab+6b^2$.

## II. BASE TWO NUMERALS

During the latter part of the seventeenth century a great German philosopher and mathematician Gottfried Wilhelm von Leibnitz (1646—1716), was doing research on the simplest numeration system. He developed a numeration system using only the symbols 1 and 0. This system is called a base two or binary numeration system.

Leibnitz actually built a mechanical calculating machine which until recently was standing useless in a museum in Germany. Actually he made his calculating machine some 3 centuries before they were made by modern machine makers.

The binary numeration system introduced by Leibnitz is used only in some of the most complicated electronic computers. The numeral 0 corresponds to *off* and the numeral 1 corresponds to *on* for the electrical circuit of the computer.

Base two numerals indicate groups of ones, twos, fours, eights, and so on. The place value of each digit in $1101_{two}$ is shown by the above words *(on* or *off)* and also by powers of 2 in base ten notation as shown below.

The numeral $1101_{two}$ means $(1x2^3)$[4] + $(1x2^2)$ + (0x2) + (1x1) = (1x8) + (1x4) + (0x2) + (1x1) = 8 + 4 + + 0 + 1 = 13. Therefore $1101_{two}$ = 13

| ...$2^3$ Eights | $2^2$[4] Fours | **2** Twos | 1 Ones |
|---|---|---|---|
| 1 | 1 | 0 | 1 |

A base ten numeral can be changed to a base two numeral by dividing by powers of two.

From the above you know that the binary system of numeration is used extensively in high-speed electronic computers. The correspondence between the two digits used in the binary system and the two positions (on and off) of a mechanical switch used in an electric circuit accounts for this extensive use.

The binary system is the simplest[5] place-value, power-position system of numeration. In every such numeration system there must be symbols for the numbers zero and one. We are using 0 and 1 because we are well familiar with them.

The binary numeration system has the advantage of having only two digit symbols but it also has a disadvantage of using many more digits to name the same numeral in base two than in base ten. See for example:

476 = $111011100_{two}$

It is interesting to note that any base two numeral looks like a numeral in any other base. The sum of 10110 and 1001 appears the same in any numeration system, but the meaning is quite different. Compare these numerals:

| | | |
|---|---|---|
| 10110 two | 10110ten | 10110seven |
| + 1001 two | + 1001ten | + 1001 seven |

## III. CLOSURE PROPERTY

In this lesson we shall be concerned with the closure property.

If we add two natural numbers, the sum will also be a natural number. For example, 5 is a natural number and 3 is a natural number. The sum of these two numbers, 8, is also a natural number. Following are other examples in which two natural numbers are being added and the sum is another natural number. 19+4 = 23 and only 23; 6+6=12 and only 12; 1429+357=1786 and only 1786. In fact, if you add any two natural numbers, the sum is again a natural number. Because this is true, we say that the set of natural numbers is closed under addition.

Notice that in each of the above equations we were able to name the sum. That is, the sum of 5 and 3 exists, or there is a number which is the sum of 19 and 4. In fact, the sum of any two numbers exists. This is called the existence property.

Notice also that if you are to add 5 and 3, you will get 8 and only 8 and not some other number. Since there is one and only one sum for 19+4, we say that the sum is unique. This is called the uniqueness property.

Both uniqueness and existence are implied in the definition of closure.

Now, let us state the closure property of addition.

If $a$ and $b$ are numbers of a given set, then $a+ b$ is also a number of that same set. For example, if $a$ and $b$ are any two natural numbers, then $a + b$ exists, it is unique, and it is again a natural number.

If we use the operation of subtraction instead of the operation of addition, we shall not be able to make the statement we made above. If we are to subtract natural numbers, the result is sometimes a natural number, and sometimes not. 11—6 = 5 and 5 is a natural number, while 9—9 = 0 and 0 is not a natural number.

Consider the equation 4—7 = $n$. We shall not be able to solve it if we must have a natural number as an answer. Therefore, the set of natural numbers is not closed under subtraction.

What about the operation of multiplication? Find the product of several pairs of natural numbers. Given two natural numbers, is there always a natural number which is the product of the two numbers?

Every pair of natural numbers has a unique product which is again a natural number. Thus the set of natural numbers is closed under multiplication.

In general, the closure property may be defined as follows: if $x$ and $y$ are any elements, not necessarily the same, of set $A$ ($A$ capital) and * (asterisk) denotes an operation *, then set $A$ is closed under the operation asterisk if $(x*y)$ is an element of set $A$.

To summarize, we shall say that there are two operations, addition and multiplication, for which the set of natural numbers is closed. Given any two natural numbers $x$ and $y$, $x + y$ and $x$ x $y$ are again natural numbers. This implies that the sum and the product of two natural numbers exists. It so happens that with the set of natural numbers (but not with every mathematical system) the results of the operations of addition and multiplication are unique.

It should be pointed out that it is practically impossible to find the sum or the product of *every* possible pair of natural numbers. Hence, we have to accept the closure property without proof, that is, as an axiom.

## IV. SOMETHING ABOUT MATHEMATICAL SENTENCES

In all branches of mathematics you need to write many sentences about numbers. For example, you may be asked to write an arithmetic sentence that includes two numerals which may name the same number or even different numbers. Suppose that for your sentence you choose the numerals 8 and 11—3 which name the same number. You can denote this by writing the following arithmetic sentence, which is true: 8= 11—3.

Suppose that you choose the numerals 9+6 and 13 for your sentence. If you use the equal sign ( = ) between the numerals you will get the following sentence 9+6=13. But do 9+6 and 13 both name the same number? Is 9+6= 13 a true sentence? Why or why not?

You will remember that the symbol of equality ( = ) in an arithmetic sentence is used to mean *is equal to.* Another symbol that is the symbol of non-equality (≠) is used to mean *is not equal to.* When an equal sign ( = ) is replaced by a non-equal sign (≠), the opposite meaning is implied. Thus the following sentence (9+6≠13) is read: nine plus six is not equal to thirteen. Is it a true sentence? Why or why not?

An important feature about a sentence involving numerals is that it is either true or false, but not both.

A mathematical sentence that is either true or false, but not both is called a closed sentence. To decide whether a closed sentence containing an equal sign ( = ) is true or false, we check to see that both elements, or expressions, of the sentence name the same number. To decide whether a closed sentence containing a non-equal sign (≠) is true or false, we check to see that both elements do not name the same number.

As a matter of fact, there is nothing incorrect or wrong, about writing a false sentence; in fact, in some mathematical proofs it is essential that you write false sentences. The important thing is that you must be able to determine whether arithmetic sentences are true or false.

The following properties of equality will help you to do so.

Reflexive: $a = a$

Symmetric: If $a = b,$ then $b$ — $a.$

Transitive: If $a = b$ and $b = c,$ then $a = c.$

The relation of equality between two numbers satisfies these basic axioms for the numbers a, *b,* and *c.*

Using mathematical symbols, we are constantly building a new language. In many respects it is more concise and direct than our everyday language. But if we are going to use this mathematical language correctly we must have a very good understanding of the meaning of each symbol used.

You already know that drawing a short line across the = sign (equality sign) we change it to ≠ sign (non-equality sign). The non-equality symbol (≠) implies either of the two things, namely: is greater then or is less than. In other words, the sign of non-equality (≠) in 3+4≠6 merely tells us that the numerals 3+4 and 6 name different numbers; it does not tell us which numeral names the greater or the lesser of the two numbers.

If we are interested to know which of the two numerals is greater we use the conventional symbols meaning less than (<) or greater than (>). These are inequality symbols or ordering symbols because they indicate order of numbers. If you want to say that six is less than seven, you will write it in the following way: $6<7$. If you want to show that twenty is greater than five, you will write $20>5$.

The signs which express equality or inequality (=, ≠, >, <) are called relation symbols because they indicate how two expressions are related.

## V. RATIONAL NUMBERS

In this chapter you will deal with rational numbers. Let us begin like this.

John has read twice as many books as Bill. John has read 7 books. How many books has Bill read?

This problem is easily translated into the equation $2n = 7$, where $n$ represents the number of books that Bill has read. If we are allowed to use only integers, the equation $2n=7$ has no solution. This is an indication that the set of integers does not meet all of our needs.

If we attempt to solve the equation $2n = 7$, our work might appear as follows.

$2n=7$, $\frac{2n}{2} = \frac{7}{2}$, $\frac{2}{2} \times n = \frac{7}{2}$, $1 \times n = \frac{7}{2}$, $n = \frac{7}{2}$ .

The symbol, or fraction, 7/2 means 7 divided by 2. This is not the name of an integer but involves a pair of integers. It is the name for a rational number. *A rational number* is the quotient of two integers (divisor and zero). The rational numbers can be named by fractions. The following fractions name rational numbers:

$\frac{1}{2}$, $\frac{8}{3}$, $\frac{0}{5}$, $\frac{3}{1}$, $\frac{9}{4}$.

We might define a rational number as any number named by $\frac{a}{n}$ where $a$ and $n$ name integers and $n \neq 0$.

Let us dwell on fractions in some greater detail.

Every fraction has a numerator and a denominator. The denominator tells you the number of parts of equal size into which some quantity is to be divided. The numerator tells you how many of these parts are to be taken.

Fractions representing values less than 1, like $\frac{2}{3}$ (two thirds) for example, are called proper fractions. Fractions which name a number equal to or greater than 1, like $\frac{2}{2}$ or $\frac{3}{2}$, are called improper fractions.

There are numerals like $1\frac{1}{2}$ (one and one second), which name a whole number and a fractional number. Such numerals are called mixed fractions.

Fractions which represent the same fractional number like $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8},$ and so on, are called equivalent fractions.

We have already seen that if we multiply a whole number by I we shall leave the number unchanged. The same is true of fractions since when we multiply both integers named in a fraction by the same number we simply produce another name for the fractional number.

For example, $1 \times \frac{1}{2} = \frac{1}{2}$ We can also use the idea that I can be as expressed a fraction in various ways: $\frac{2}{2}, \frac{3}{3}, \frac{4}{4},$ and so on.

Now see what happens when you multiply $\frac{1}{2}$ by $\frac{2}{2}$. You will have $\frac{1}{2} = 1 \frac{1}{2} = \frac{2}{2} \times \frac{1}{2} = \frac{2x1}{2x2} = \frac{2}{4}$. As a matter of fact in the above operation you have changed the fraction to its higher terms.

Now look at this: $\frac{6}{8} : 1 = \frac{6}{8} : \frac{2}{2} = \frac{6 : 2}{8 : 2} = \frac{3}{4}$.

In both of the above operations the number you have chosen for 1 is $\frac{2}{2}$ In the second example you have used division to change $\frac{6}{8}$ to lower terms, that is to $\frac{3}{4}$ .The numerator and the denominator in this fraction are relatively prime and accordingly we call such a fraction the simplest fraction for the given rational number.

You may conclude that dividing both of the numbers named by the numerator and the denominator by the same number, not 0 or 1 leaves the fractional number unchanged. The process of bringing a fractional number to lower terms is called reducing a fraction.

To reduce a fraction to lowest terms, you are to determine the greatest common factor. The greatest common factor is the largest possible integer by which both numbers named in the fraction are divisible.

From the above you can draw the following conclusion[6]: mathematical concepts and principles are just as valid in the case of rational numbers (fractions) as in the case of integers (whole numbers).

## VI. DECIMAL NUMERALS

In our numeration system we use ten numerals called digits. These digits are used over and over again in various combinations. .Suppose, you have been given numerals 1, 2, 3 and have been asked to write all possible combinations of these digits. You may write 123, 132, 213 and so on. The position in which each digit is written affects its value. How many digits are in the numeral 7086? How many place value positions does it have? The diagram below may prove helpful. A comma separates each group or period. To read 529, 248, 650, 396, you must say: five hundred twenty-nine billion, two hundred forty-eight million, six hundred fifty thousand, three hundred ninety-six.

| Billions period | Millions period | Thousands period | Ones period |
|---|---|---|---|
| billions Hundred billions | millionsHundred millions | One- Hundred- thousands | OnesTens Hundreds |
| 5 2 9, | 2 4 8, | 6 5 0, | 3 9 6 |

But suppose you have been given a numeral 587.9 where 9 has been separated from 587 by a point, but not by a comma. The numeral 587 names a whole number. The sign (.) is called a decimal point.

All digits to the left of the decimal point represent whole numbers. All digits to the right of the decimal point represent fractional parts of 1.

The place-value position at the right of the ones place is called tenths. You obtain a tenth by dividing 1 by 10. Such numerals like 687.9 are called decimals.

You read .2 as two tenths. To read .0054 you skip two zeroes and say fifty four ten thousandths.

Decimals like .666..., or .242424..., are called repeating decimals. In a repeating decimal the same numeral or the same set of numerals is repeated over and over again indefinitely.

We can express rational numbers as decimal numerals. See how it may be done.

$\frac{31}{100} = 0.31. \quad \frac{4}{25} = \frac{4 \times 4}{4 \times 25} = \frac{16}{100} = 0.16$

The digits to the right of the decimal point name the numerator of the fraction, and the number of such digits indicates the power of 10 which is the denominator. For example, .217 denotes numerator 217 and a denominator of $10^3$ (ten cubed) or 1000.

In our development of rational numbers we have named them by fractional numerals. We know that rational numerals can just as well be named by decimal numerals. As you might expect, calculations with decimal numerals give the same results as calculations with the corresponding fractional numerals.

Before performing addition with fractional numerals, the fractions must have a common denominator. This is also true of decimal numerals.

When multiplying with fractions, we find the product of the numerators and the product of denominators. The same procedure is used in multiplication with decimals.

Division of numbers in decimal form is more difficult to learn because there is no such simple pattern as has been observed for muliplication.

Yet, we can introduce a procedure that reduces all decimal-division situations to one standard situation, namely the situation where the divisor is an integer. If we do so we shall see that there exists a simple algorithm that will take care of all possible division cases.

In operating with decimal numbers you will see that the arithmetic of numbers in decimal form is in full agreement with the arithmetic of numbers in fractional form.

You only have to use your knowledge of fractional numbers.

Take addition, for example. Each step of addition in fractional form has a corresponding step in decimal form.

Suppose you are to find the sum of, say, .26 and 2.18. You can change the decimal numerals, if necessary, so that they denote a common denominator. We may write .26 = .260 or 2.18 = 2.180. Then we add the numbers just as we have added integers and denote the common denominator in the sum by proper placement of the decimal point.

We only have to write the decimals so that all the decimal points lie on the same vertical line. This keeps each digit in its proper place-value position.

Since zero is the identity element of addition it is unnecessary to write .26 as . 260, or 2.18 as 2.180 if you are careful to align the decimal points, as appropriate.

## VII. THE DIFFERENTIAL CALCULUS

No elementary school child gets a chance of learning the differential calculus, and very few secondary school children do so. Yet I know from my own experience that children of twelve can learn it. As it is a mathematical tool used in most branches of science, this form a bar between the workers and many kinds of scientific knowledge. I have no intention of teaching the calculus, but it is quite easy to explain what it is about, particularly to skilled workers. For a very large number of skilled workers use it in practice without knowing that they are doing so.

The differential calculus is concerned with rates of change. In practical life we constantly come across pairs of quantities which are related, so that after both have been measured, when we know one, we know the other. Thus if we know the distance along the road from a fixed point we can find the height above sea level from a map with contour**.** If we know a time of day we can determine the air temperature on any particular day from a record of a thermometer made on that day. In such cases we often want to know the rate of change of one relative to the other.

If $x$ and $y$ are the two quantities then the rate of change of y relative to x is written $dy/dx$. For example if $x$ is the distance of a point on a railway from London, measured in feet, and $y$ the height above sea level, $dy/dx$ is the gradient of the railway. If the height increases by 1 foot while the distance $x$ increases by 172 feet, the average value of $dy/dx$ is 1/172. We say that the gradient is 1 to 172. If $x$ is the time measured in hours and fractions of an hour, and $y$ the number of miles gone, then $dy/dx$ is the speed in miles per hour. Of course, the rate of change may be zero, as on level road, and negative when the height is diminishing as the distance $x$ increases. To take two more examples, if $x$ the temperature, and y the length of a metal bar, *dy/dx*—:—*y* is the coefficient of expansion, that is to say the proportionate increase in length per degree. And if *x* is the price of commodity, and *y* the amount bought per day, then *-dyldx* is called the elasticity of demand.

For example people must buy bread, but cut down on jam, so the demand for jam is more elastic than that for bread. This notion of elasticity is very important in the academic economics taught in our universities. Professors say that Marxism is out of date because Marx did not calculate such things. This would be a serious criticism if the economic "laws" of 1900 were eternal truths. Of course Marx saw that they were nothing of the kiad and "elasticity of demand" is out of date in England today for the very good reason that most commodities are controlled or rationed.

The mathematical part of the calculus is the art of calculating *dy/dx* if *y* has some mathematical relations to *x,* for example is equal to its square or logarithm. The rules have to be learned like those for the area and volume of geometrical figures and have the same sort of value. No area is absolutely square, and no volume is absolutely cylindrical. But there are things in real life like enough to squares and cylinders to make the rules about them worth learning. So with the calculus. It is not exactly true that the speed of a falling body is proportional to the time it has been falling. But there is close enough to the truth for many purposes.

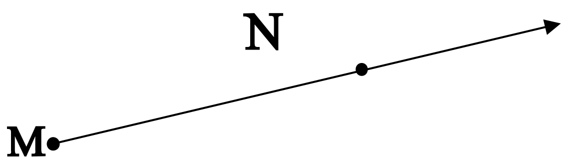
The differential calculus goes a lot further. Think of a bus going up a hill which gradually gets steeper. If *x* is the horizontal distance, and *y* the height, this means that the slope *dy/dx* is increasing. The rate of change of *dy/dx* with regard to *y* is written $d^{2y}/dx^2$. In this case it gives a measure of the curvature of the road surface. In the same way if *x* is time and distance, $d^{2y}/dx^2$ is the rate of change of speed with time, or acceleration. This is a quantity which good drivers can estimate pretty well, though they do not know they are using the basic ideas of the differential calculus.

If one quantity depends on several others, the differential calculus shows us how to measure this dependence. Thus the pressure of a gas varies with the temperature and the volume. Both temperature and volume vary during the stroke of a cylinder of a steam or petrol engine, and the calculus is needed for accurate theory of their action.

Finally, the calculus is a fascinating study for its own sake. In February 1917 I was one of a row wounded officers lying on stretchers on a steamer going down the river Tigris in Mesopotamia. I was reading a mathematical book on vectors, the man next to me was reading one on the calculus. As antidotes to pain we preferred them to novels. Some parts of mathematics are beautiful, like good verse or painting. The calculus is beautiful, but not because it is a product of "pure thought". It was invented as a tool to help men to calculate the movement of stars and cannon balls. It has the beauty of really efficient machine.

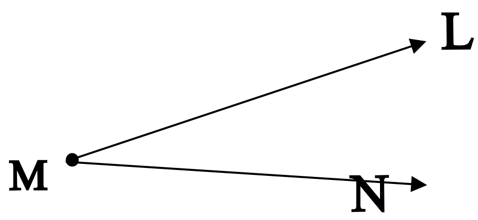
## VIII. RAYS, ANGLES, SIMPLE CLOSED FIGURES

1. You certainly remember that by extending a line segment in one direction we obtain a ray. 2. Below is a picture of such an extension.




3. The arrow indicated that you start at point *M,* go through point *N,* and on without end. 4. This results in what is called ray *MN*, which is denoted by the symbol *MN.* 5. Point *M* is the endpoint in this case. 6. Notice that the letter naming the endpoint of a ray is given when first naming the ray.

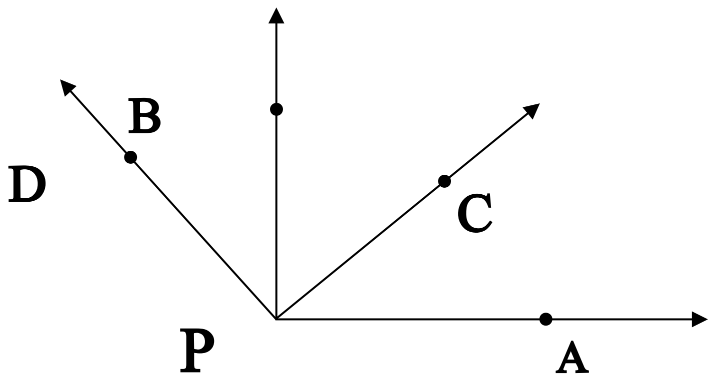
7. From what you already know you may deduce that drawing two rays originating from the same endpoint forms an angle. 8. The common point of the two rays is the vertex of the angle.




9. Angles, though open figures, separate the plane into three distinct sets of points: the interior, the exterior, and the angle. 10. The following symbol ∠ is frequently used in place of the word angle.

11. The angle pictured above could be named in either of the following ways: a) angle *LMN* (or *∠LMN);* b) angle *NML* (or ∟*NML).*

12. The letter naming the vertex of an angle occurs as the middle letter in naming each angle. 13. Look at the drawing below.

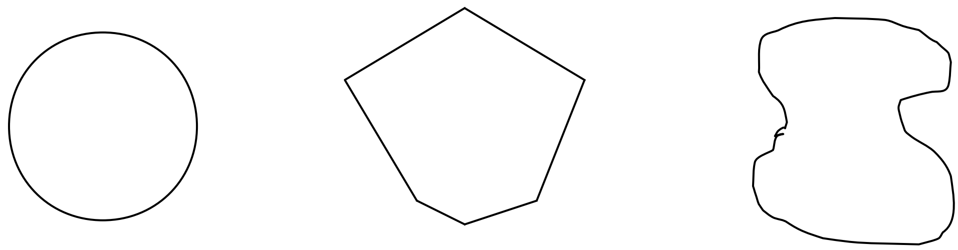



13. Ray *PA* $(\overrightarrow{PA})$ and ray *PB* $(\overrightarrow{PB})$ form a right angle, which means that the angle has a measure of 90° (degrees). 15. Since $\overrightarrow{PC}$ (except for point *P)* lies in the *interior* of *∠APB,* we speak of *∠CPA* being less than a right angle and call it an acute angle with a degree

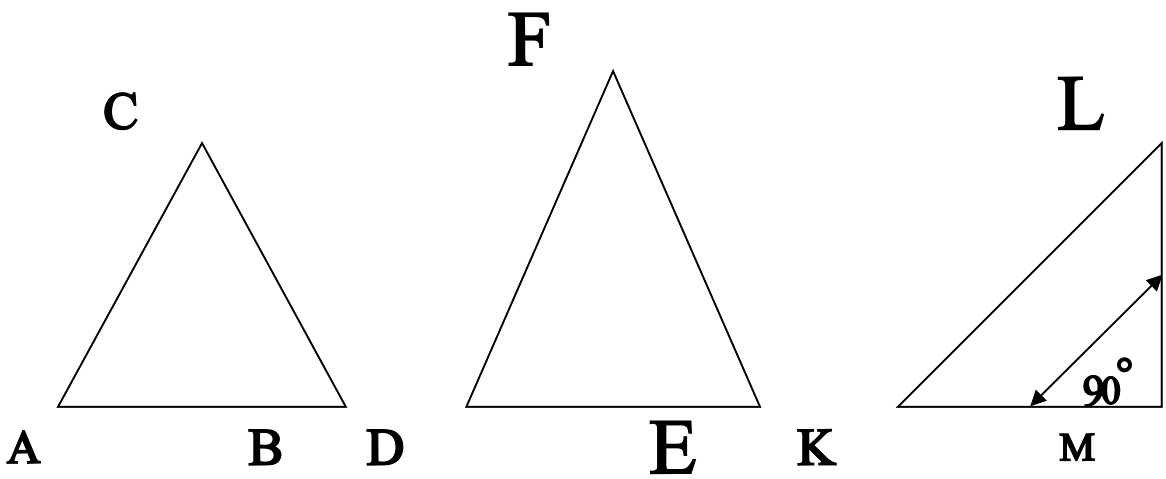
measure less than 90°. 16. Since $\overrightarrow{PD}$ (except for point *P)* lies in the *exterior* of ∠APB, we say that ∠APD is greater than a right angle and call it an obtuse angle with a degree measure greater than 90°.

*Simple Closed Figures*

17 A simple closed figure is any figure drawn in a plane in such a way that its boundary never crosses or intersects itself and encloses part of the plane. 18. The following are examples of simple closed figures. 19. Every simple closed figure separates the plane into three distinct sets of points. 20. The interior of the figure is the set of all points in the part of the plane enclosed by the figure. 21. The exterior of the figure is the set of points in the plane which are outside the figure. 22. And finally, the simple closed figure itself is still another set of points.



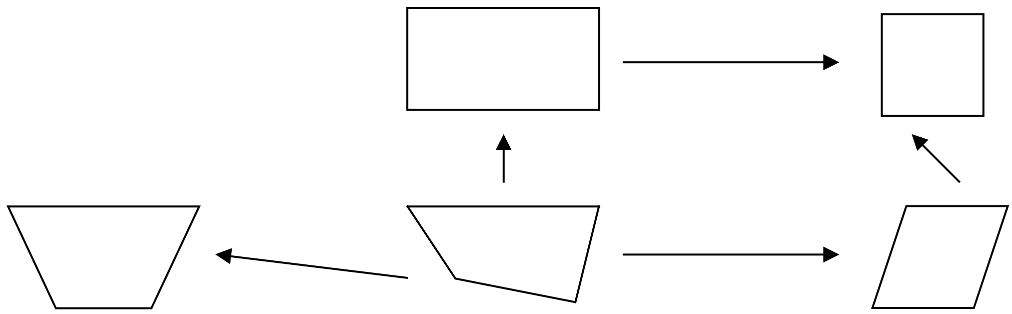
23. A simple closed figure formed by line segments is called a polygon. 24. Each of the line segments is called a side of the polygon. 25. Polygons may be classified according to the measures of the




angles or the measure of the sides. 26. This is true of triangles — geometric figures having three sides — as-well as of quadrilaterals, having four sides.

27. In the picture above you can see three triangles. Δ*ABC* is referred to as an equilateral triangle. 29. The sides of such a triangle all have the same linear

measure. 30. $\Delta DEF$ is called an isosceles triangle which means that its two sides have the same measure. 31. You can see it in the drawing above. 32. $\Delta ALMK$ being referred to as a right triangle means that it contains one right angle. 33. In $\Delta MKL$, $\angle M$ is the right angle, sides $MK$ and $ML$ are called the legs, and side $KL$ is called the hypotenuse. 34. The hypotenuse refers only to the side opposite to the right angle of a right triangle. Below you can see quadrilaterals.



35. A parallelogram is a quadrilateral whose opposite sides are parallel. 36. Then the set of all parallelograms is a subset of all quadrilaterals. Why? 37. A rectangle is a parallelogram in which all angles are right angles. 38. Therefore we can speak of the set of rectangles being a subset of the set of parallelograms. 39. A square is a rectangle having four congruent sides as well as four right angles. 40. Is every square a rectangle? Is every rectangle a square? Why or why not? 41. A rhombus is a parallelogram in which the four sides are congruent. 42. Thus, it is evident that opposite sides of a rhombus are parallel and congruent. 43. Is defining a square as a special type of rhombus possible? 44. A trapezoidal has only two parallel sides. 45. They are called the bases of a trapezoidal.

## IX. SOMETHING ABOUT EUCLIDEAN AND NON-EUCLIDEAN GEOMETRIES

1. It is interesting to note that the existence of the special quadrilaterals discussed above is based upon the so-called parallel postulate of Euclidean geometry. 2. This postulate is now usually stated as follows: Through a point not on line *L,* there is no more than one line parallel to *L.* 3. Without assuming (не допуская) that there exists at least one parallel to a given line through a point not on the given line, we could not state the definition of the special quadrilaterals which have given pairs of parallel sides. 4. Without the as sumption that there exists no more than one parallel to a given line through a point not on the given line, we could not deduce the conclusion we have stated (сформулировали) for the special quadrilaterals. 5. An important aspect of geometry (or any other area of mathematics) as a deductive

system is that the conclusions which may be drawn are consequences (следствие) of the assumptions which have been made. 6. The assumptions made for the geometry we have been considering so far are essentially those made by Euclid in Elements. 7. In the nineteenth century, the famous mathematicians Lobachevsky, Bolyai and Riemann developed non-Euclidean geometries. 8. As already stated, Euclid assumed that through a given point not on a given line there is no more than one parallel to the given line. 9. We know of Lobachevsky and Bolyai having assumed independently of (не зависимо от) one another that through a given point not on a given line there is more than one line parallel to the given line. 10. Riemann assumed that through a given point not on a given line there is no line parallel to the given line. 11. These variations of the parallel postulate have led (привели) to the creation (создание) of non-Euclidean geometries which are as internally consistent (непротиворечивы) as Euclidean geometry. 12. However, the conclusions drawn in non-Euclidean geometries are often completely inconsistent with Euclidean conclusions. 13. For example, according to Euclidean geometry parallelograms and rectangles (in the sense (смысл) of a parallelogram with four 90-degree angles) exist; according to the geometries of Lobachevsky and Bolyai parallelograms exist but rectangles do not; according to the geometry of Riemann neither parallelograms nor rectangles exist. 14. It should be borne in mind that the conclusions of non-Euclidean geometry are just as valid as those of Euclidean geometry, even though the conclusions of non-Euclidean geometry contradict (противоречат) those of Euclidean geometry. 15. This paradoxical situation becomes intuitively clear when one realizes that any deductive system begins with undefined terms. 16. Although the mathematician forms intuitive images (образы) of the concepts to which the undefined terms refer, these images are not logical necessities (необходимость). 17. That is, the reason for forming these intuitive images is only to help our reasoning (рассуждение) within a certain deductive system. 18. They are not logically a part of the deductive system. 19. Thus, the intuitive images corresponding to the undefined terms straight line and plane are not the same for Euclidean and non-Euclidean geometries. 20. For example, the plane of Euclid is a flat surface; the plane of Lobachevsky is a saddle-shaped (седлообразный) or pseudo-spherical surface; the plane of Riemann is an ellipsoidal or spherical surface.

## X. CIRCLES

1. If you hold the sharp end of a compass fixed on a sheet of paper and then turn the compass completely around you will draw a curved line enclosing parts of a plane. 2. It is a circle. 3. A circle is a set of points in a plane each of which is equidistant, that is the same distance from some given point in the plane called the center. 4. A line segment joining any point of the circle with the center is called a radius. 5. In the figure above $R$ is the center and $RC$ is the radius. 6. What other radii are shown? 7. A chord bf a circle is a line segment whose endpoints are points on the circle. 8. A diameter is a chord which passes through the center of the circle. 9. In the figure above $AB$ and $BC$ are chords and $AB$ is a diameter. 10. Any part of a circle

containing more than one point forms an arc of the circle. 11. In the above figure, the points $C$ and $A$ and all the points in the interior of $\angle ARC$ that are also points of the circle are called arc

$AC$ which is symbolized as $\overset{\frown}{AC}$. 12. $\overset{\frown}{ABC}$ is the arc containing points $A$ and $C$ and all the points of the circle which are in the exterior of $\angle ABC$. 13. Instead of speaking of the perimeter of a circle, we usually use the term circumference to mean the distance around the circle. 14. We cannot find the circumference of a circle by adding the measure of the segments, because a circle does not contain any segments. 15. No matter how short an arc is, it is curved at least slightly. 16. Fortunately mathematicians have discovered that the ratio of the circumference ($C$) to a diameter *(d)* is the same for all

circles. This ratio is expressed $\frac{C}{d}$ 17. Since $d = 2r$ (the length of a diameter is equal to twice the length of a radius of the same circle), the following denote the same ratio.

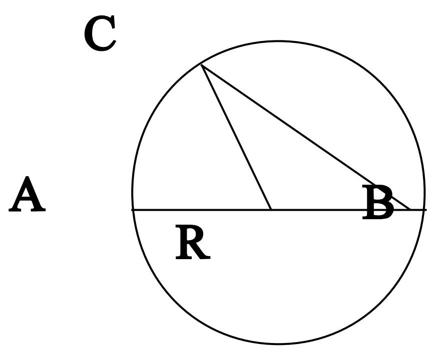
$$\frac{C}{d} = \frac{C}{2r} \quad \text{since } d=2r$$

18. The number $\frac{C}{d}$ or $\frac{C}{2r}$ which is the same for all circles, is designated by $\pi$
19. This allows us to state the following:

$$\frac{C}{d} = \pi \qquad \text{or} \qquad \frac{C}{2r} = \pi$$

20. By using the multiplication property of equation, we obtain the following:

$$C = \pi d \text{ or } C = 2\pi r.$$




## XI. THE SOLIDS

A solid is a three-dimensional figure, e.g. a prism or a cone.

A prism is a solid figure formed from two congruent polygons with their corresponding sides parallel (the bases) and the parallelogram (lateral faces) formed by joining the corresponding vertices of the polygons. The lines joining the vertices of the polygons are lateral edges. Prisms are named according to the base - for example, a triangular prism has two triangular bases (and three lateral faces); a quadrangular prism has bases that are quadrilaterals. Pentagonal, hexagonal, etc. prism have bases that are pentagons, hexagons, etc.

A right prism is one in which the lateral edges are at right angles to the bases (i.e. the lateral faces are rectangles) - otherwise the prism is an oblique prism (i.e. one base is displaced with respect to the other, but remains parallel to it). If the bases are regular polygons and the prism is also a right prism, then it is a regular prism.

A cone is a solid figure formed by a closed plane curve on a plane (the base) and all the lines joining points of the base to a fixed point (the vertex) not in the plane of the base. The closed curve is the directrix of the cone and the lines to the vertex are its **generators** (or **elements).** The curved area of the cone forms its lateral **surface.** Cones are named according to the base, e.g. a **circular** cone or an **elliptical** cone. If the base has a center of symmetry, a line from the vertex to the center is the axis of the cone. A cone that has its axis perpendicular to its base is a right cone; otherwise the cone is a oblique cone. The **altitude** of a cone (h) is the perpendicular distance from the plane of the base to the vertex. The **volume** of any cone is **l/3hA,** where A is the area of the base. A right circular cone (circular base with perpendicular axis) has **a slant height** (s), equal to the distance from the edge of the base to the vertex (the length of a generator). The term "cone" is often used loosely for "conical surface".

A **pyramid** is a solid figure (a polyhedron) formed by a polygon (the base) and a number of triangles (lateral faces) with a common vertex that is not **coplanar** with the base. Line segments from the common vertex to the vertices of the base are lateral edges of the pyramid. Pyramids are named according to the base: a triangular pyramid (which is a tetrahedron), a square pyramid, a pentagonal pyramid, etc.

If the base has a center, a line from the center to the vertex is the axis of the pyramid. A pyramid that has its axis perpendicular to its base is a right pyramid; otherwise, it is an oblique pyramid, then it is also a regular pyramid.

The altitude (h) of a pyramid is the perpendicular distance from the base to the vertex. The volume of any pyramid is **l/3Ah,** where A is the area of the base. In a regular pyramid all the lateral edges have the same length. The slant height (s) of the pyramid is the altitude of a face; the **total surface area** of the lateral faces is **l/2sp,** where **p** is the perimeter of the base polygon.

## XII. POLYHEDRON

**A polyhedron** is a surface composed of plane polygonal surfaces **(faces).** The sides of the **polygons,** joining two faces, are its **edges.** The corners, where three or more faces meet, are its vertices. Generally, the term "polyhedron" is used for **closed solid** figure. A convex polyhedron is one for which a plane containing any face does not cut other faces; otherwise the polyhedron is concave.

A **regular** polyhedron is one that has **identical (congruent)** regular polygons forming its faces and has all its **polyhedral** angles congruent. There are only five possible convex regular polyhedra:

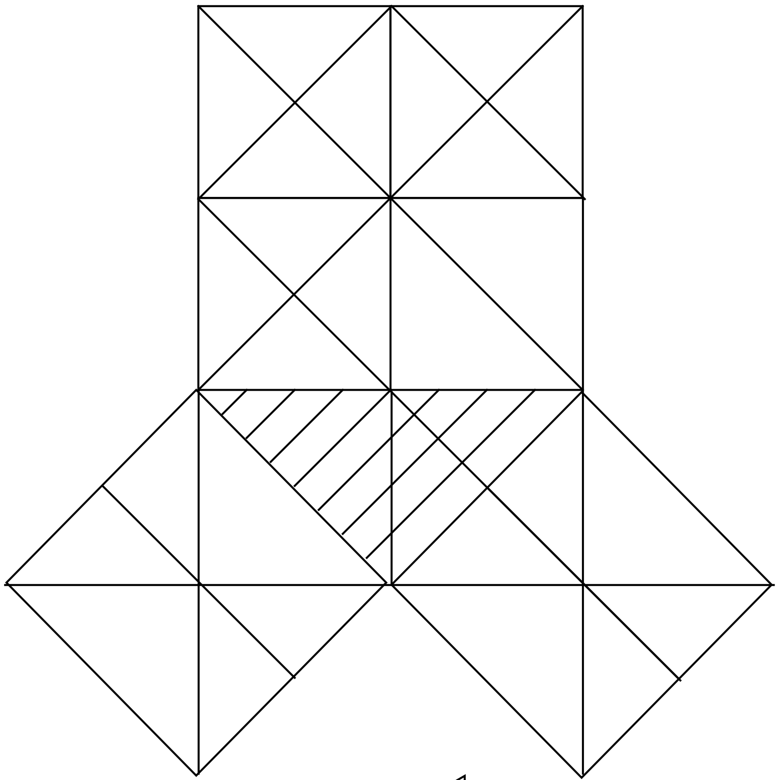
1) **tetrahedron** - four triangular faces,
2) **cube** - six square faces,
3) **octahedron** - eight triangular faces,
4) **dodecahedron** - twelve pentagonal faces,
5) **icosahedron** - twenty triangular faces.

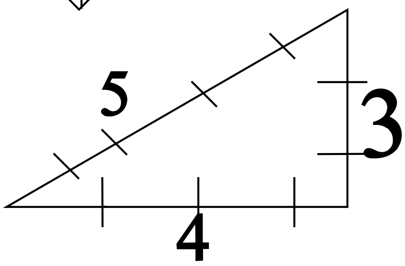
The five regular solids played a significant part in Greek geometry. They were known to Plato and are often called **Platonic** solids. Kepler used them in his complicated model of the solar system.

A **uniform** polyhedron is a polyhedron that has identical polyhedral angles at all its vertices, and has all its faces formed by regular polygons (not necessarily of the same type). The five regular polyhedra are also uniform polyhedra. Right **prisms** and antiprisms that have regular polygons as bases are also uniform. In addition, there are thriteen **semiregular** polyhedra, the so-called **Archimedian** solids. For example, the **icosidodecahedron** has 32 faces - 20 triangles and 12 pentagons. It has 60 edges and 30 vertices, each vertex beeing the meeting point of two triangles and two **pentagons.** Another example is the **truncated** cube, obtained by cutting the corners off a cube. If the corners are cut so that the new vertices lie at the centers of the edges of the original cube, a **cuboctahedron** results. **Truncating** the cuboctahedron and "distorting" the rectangular faces into squares yields another Archimedian solid. Other uniform polyhedra can be generated by truncating the four other regular polyhedra or the icosidodecahedron.

## XIII. THE PYTHAGOREAN PROPERTY

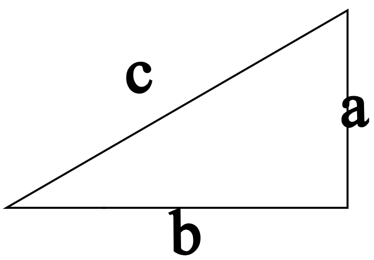
The ancient Egyptians discovered that in stretching ropes of lengths 3 units, 4 units and 5 units as shown below, the angle formed by the shorter ropes is a right angle. 2. The Greeks succeeded in finding other sets of three numbers which gave right triangles and were able to tell without drawing the triangles which ones should be right triangles, their method being as follows. 3. If you look at the illustration you will see a triangle with a dashed interior. 4. Each side of it is used as the side of a square. 5. Count the number of small triangular regions in the interior of each square. 6. How does the number of small triangular regions in the two smaller squares compare with the number of triangular regions in the largest square? 7. The Greek philosopher and mathematician Pythagoras noticed the relationship and is credited with the proof of this property known as the Pythagorean Theorem or the Pythagorean Property. 8. Each side of a right triangle being used as a side of a square, the sum of the areas of the two smaller squares is the same as the area of the largest square.

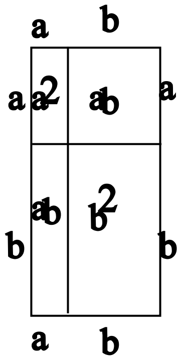


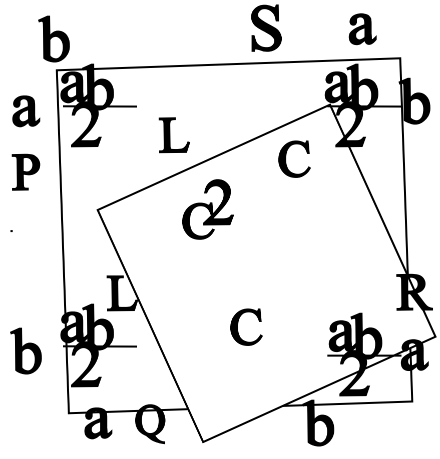



**Proof of the Pythagorean Theorem**

9. We should like to show that the Pythagorean Property is true for all right triangles, there being several proofs of this property. 10. Let us discuss one of them. 11. Before giving the proof let us state the Pythagorean Property in mathematical language. 12. In the triangle above, *c* represents the measure of the hypotenuse, and *a* and *b* represent the measures of the other two sides.






13. If we construct squares on the three sides of the triangle, the area-measure will be $a^2$, $b^2$ and $c^2$. 14. Then the Pythagorean Property could be stated as follows: $c^2 = a^2 + b^2$. 15. This proof will involve working with areas. 16. To prove that $c^2 = a^2 + b^2$ for the triangle above, construct two squares each side of which has a measure $a + b$ as shown above. 17. Separate the first of the two squares into two squares and two rectangles as shown. 18. Its total area is the sum of the areas of the two squares and the two rectangles.

$$A = a^2+2ab+b^2$$

19. In the second of the two squares construct four right triangles. 20. Are they congruent? 21. Each of the four triangles being congruent to the original triangle, the hypotenuse has a measure *c*. 22. It can be shown that *PQRS* is a square, and its area is $c^2$. 23. The total area of the second square is the sum of the areas of the four triangles and the square *PQRS*. $A = c^2+4\ (½\ ab)$. The two squares being congruent to begin with[2], their area measures are the same. 25. Hence we may conclude the following:

$$a^2+2ab+b^2 = c^2+4(½\ ab)$$

$$(a^2 + b^2) + 2ab = c^2 + 2ab$$

26. By subtracting $2ab$ from both area measures we obtain $a^2+ b^2 = c^2$ which proves the Pythagorean Property for all right triangles.

## XIV. SQUARE ROOT

1. It is not particularly useful to know the areas of the squares on the sides of a right triangle, but the Pythagorean Property is very useful if we can use it to find the length of a side of a triangle. 2. When the Pythagorean Property is expressed in the form $c^2 = a^2+ b^2$, we can replace any two of the letters with the measures of two sides of a right triangle. 3. The resulting equation can then be solved to find the measure of the third side of the triangle. 4. For example, suppose the measures of the shorter sides of a right triangle are 3 units and 4 units and we wish to find the measure of the longer side. 5. The Pythagorean Property could be used as shown below:

$$c^2 = a^2+b^2, \quad c^2 = 3^2+4^2, \quad c^2 = 9+16, \quad c^2 = 25.$$

6. You will know the number represented by $c$ if you can find $a$ number which, when used as a factor twice, gives a product of 25. 7. Of course, 5x5 = 25, so $c = 5$ and 5 is called the positive square root (корень) of 25. 8. If a number is a product of two equal factors, then either (любой) of the equal factors is called a square root of the number. 9. When we say that $y$ is the square root of $K$ we merely (всего лишь) mean that $y^2 = K$. 10. For example, 2 is a square root of 4 because $2^2 = 4$. 11. The product of two negative numbers being a positive number, —2 is_also a square root of 4 because $(—2)^2 = 4$. The following symbol √ called a radical sign is used to denote the positive square root of a number. 13. That is $\sqrt{K}$ means the positive square root of $K$. 14. Therefore $\sqrt{4} = 2$ and $\sqrt{25} = 5$. 15. But suppose you wish to find the $\sqrt{20}$. 16. There is no integer whose square is 20, which is obvious from the following computation. $4^2 = 16$ so $\sqrt{16} = 4$; $a^2 = 20$ so $4<a<5$, $5^2 = 25$, so $\sqrt{25} = 5$. 17. $\sqrt{20}$ is greater than 4 but less than 5. 18. You might try to get a closer approximation of $\sqrt{20}$ by squaring some numbers between 4 and 5. 19. Since $\sqrt{20}$ is about as near to $4^2$ as[1] to $5^2$, suppose we square 4.4 and 4.5.

$$4.4^2 = 19.36 \quad a^2 = 20 \quad 4.5^2 = 20.25$$

20. Since $19.36<20<20.25$ we know that $4.4<a<4.5$. 21. 20 being nearer to 20.25 than to 19.36, we might guess that $\sqrt{20}$ is nearer to 4.5 than to 4.4. 22. Of course, in order to make sure[2] that $\sqrt{20} = 4.5$, to the nearest tenth, you might select values between 4.4 and 4.5, square them, and check the results. 23. You could continue the process indefinitely and never get the exact value of 20. 24. As a matter of fact, $\sqrt{20}$ represents an irrational number which can only be expressed approximately as rational number. 25. Therefore we say that $\sqrt{20} = 4.5$ approximately (to the nearest tenth).

## APPENDIX
## SAMPLE TEST FROM GMAT

1. A trip takes 6 hours to complete. After traveling $\frac{1}{4}$ of an hour, $1\frac{3}{8}$ hours, and $2\frac{1}{3}$ hours, how much time does one need to complete the trip?
(A) $2\ ^{1}/_{12}$ hours
(B) 2 hours, $2\frac{1}{2}$ minutes
(C) 2 hours, 5 minutes
(D) $2\frac{1}{8}$ hours
(E) 2 hours, $7\frac{1}{2}$ minutes

2. It takes 30 days to fill laboratory dish with bacteria. If the size of the bacteria doubles each day, how long did it take for the bacteria to fill one half of the dish?
**(A)** 10 days
**(B)** 15 days
**(C)** 24 days
**(D)** 29 days
**(E)** 29.5 days

3. A car wash can wash 8 cars in 18 minutes. At this rate how many cars can the car wash wash in 3 hours?
(A) 13
(B) 40.5
(C) 80
(D) 125
(E) 405

4. If the ratio of the areas of 2 squares is 2 : 1, then the ratio of the perimeters of the squares is
(A) 1: 2
(B) 1: $\sqrt{2}$
(C) $\sqrt{2}$ : 1
(D) 2 : 1
(E) 4: 1

5. There are three types of tickets available for a concert: orchestra, which cost $12 each; balcony, which cost $9 each; and box, which cost $25 each. There were P orchestra tickets, B balcony tickets, and R box tickets sold for the concert. Which of the following expressions gives the percentage of the ticket proceeds due to the sale of orchestra tickets?

**P**

(A) $100 \times \frac{\quad}{(P+B+R)}$

(B) $100 \times \frac{12P}{(12P + 9B + 25R)}$

(C) $\frac{12P}{(12P + 9B + 25R)}$

(D) $100 \times \frac{(9B + 25R)}{(12P + 9B + 25R)}$

(E) $100 \times \frac{(12P + 9B + 25R)}{(12P)}$

6. City B is 5 miles east of City A. City C is 10 miles southeast of City B. Which of the following is the closest to the distance from City A to City C?

(A) 11 miles
(B) 12 miles
(C) 13 miles
(D) 14 miles
(E) 15 miles

7. If $3x - 2y = 8$, then $4y - 6x$ is:

(A) -16
(B) -8
(C) 8
(D) 16
(E) cannot be determined

8. It costs 10c. a kilometer to fly and 12c. a kilometer to drive. If you travel 200 kilometers, flying $x$ kilometers of the distance and driving the rest, then the cost of the trip in dollars is:

(A) 20
(B) 24
(C) $24 - 2x$
(D) $24 - 0.02x$
(E) $2.400 - 2x$

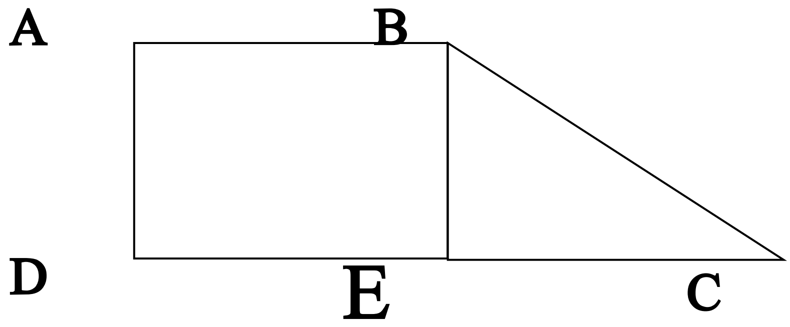
9. If the area of a square increases by 69%, then the side of the square increases by:
(A) 13%
(B) 30%
(C) 39%
(D) 69%
(E) 130%

10. There are 30 socks in a drawer. 60% of the socks are red and rest are blue. What is the minimum number of socks that must be taken from the drawer without looking in order to be certain that at least two blue socks have been chosen?
(A) 2
(B) 3
(C) 14
(D) 16
(E) 20

11. How many squares with sides ½ inch long are needed to cover a rectangle that is 4 feet long and 6 feet wide?
(A) 24
(B) 96
(C) 3,456
(D) 13,824
(E) 14,266

12. In a group of people solicited by a charity, 30% contributed \$40 each, 45 % contributed \$20 each, and the rest contributed \$12 each. What percentage of the total contributed came from people who gave \$40?
(A) 25%
(B) 30%
(C) 40%
(D) 45%
(E) 50%

13. A trapezoid ABCD is formed by adding the isosceles right triangle BCE with base 5 inches to the rectangle ABED where DE is t inches. What is the area of the trapezoid in square inches?
(A) $5t + 12.5$
(B) $5t + 25$
(C) $2.5t + 12.5$
(D) $(t + 5)^2$
(E) $t^2 + 25$




14. A manufacturer of jam wants to make a profit of $75 by selling 300 jars of jam. It costs 65c. each to make the first 100 jars of jam and 55c. each to make each jar after the first 100. What price should be charged for the 300 jars of jam?
(A) $75
(B) $175
(C) $225
(D) $240
(E) $250

15. A car traveled 75% of the way from town A to town B by traveling for T hours at an average speed of V mph. The car travels at an average speed of S mph for the remaining part of the trip. Which of the following expressions represents the time the car traveled at S mph?
(A) VT/ S
(B) VS/4T
(C) 4VT/3S
(D) 3S/VT
(E) VT/3S

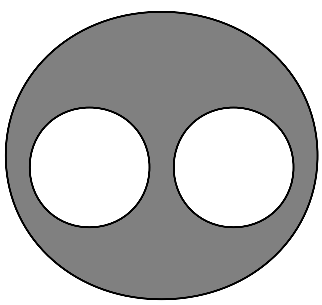
16. A company makes a profit of 7% selling goods which cost $2,000; it also makes a profit of 6% selling a machine that cost the company $5,000. How much total profit did the company make on both transactions?
(A) $300
(B) $400
(C) $420
(D) $440
(E) $490

17. The ratio of chickens to pigs to horses on a farm can be expressed as the triple ratio 20: 4: 6. If there are 120 chickens on the farm, then the number of horses on the farm is
(A) 4

(B) 6
(C) 24
(D) 36
(E) 60

18. If $x^2 - y^2 = 15$ and $x + y = 3$, then $x - y$ is
(A) – 3
(B) 0
(C) 3
(D) 5
(E) cannot be determined

19. What is the area of the shaded region? The radius of the outer is *a* and the radius of each of the circles inside the large circle is *a/3*.
(A) 0
(B) $(\frac{1}{3})\equiv a^2$
(C) $(\frac{2}{3})\equiv a^2$
(D) $(\frac{7}{9})\equiv a^2$
(E) $(\frac{8}{9})\equiv a^2$



20. If $2x - y = 4$, then $6x - 3y$ is
(A) 4
(B) 6
(C) 8
(D) 10
(E) 12

21. A warehouse has 20 packers. Each packer can load ⅛ of a box in 9 minutes. How many boxes can be loaded in 1½ hours by all 20 packers?
**(A)** 1¼
**(B)** 10¼
**(C)** 12½

**(D)** 20
**(E)** 25

22. In Motor City 90% of the population own a car, 15 % own a motorcycle, and everybody owns one or the other or both. What is the percentage of motorcycle owners who own cars?

*(A)* 5%
*(B)* 15%
*(C)* 33⅓%
*(D)* 50%
*(E)* 90%

23. Towns A and C are connected by a straight highway which is 60 miles long. The straight-line distance between town A and town B is 50 miles, and the straight-line distance from town B to town C is 50 miles. How many miles is it from town B to the point on the highway connecting towns A and C which is closest to town B?

**(A)** 30
**(B)** 40
**(C)** $30\sqrt{2}$
**(D)** 50
**(E)** 60

24. A chair originally cost $ 50.00. The chair was offered for sale at 108% of its cost. After a week the price was discounted 10% and the chair was sold. The chair was sold for

(A) $45.00
(B) $48.00
(C) $49.00
(D) $49.50
(E) $54.00

25. A worker is paid x dollars for the first 8 hours he works each day. He is paid y dollars per hour for each hour he works in excess of 8 hours. During one week he works 8 hours on Monday, 11 hours on Tuesday, 9 hours on Wednesday, 10 hours on Thursday, and 9 hours on Friday. What is his average daily wage in dollars for the five-day week?

(A) $x + (7/5)\,y$
(B) $2x + y$
(C) $(5x + 8y)/\,5$
(D) $8x + (7/5)\,y$
(E) $5x+7y$

26. A club has 8 male and 8 female members. The club is choosing a committee of 6 members. The committee must have 3 male and 3 female members. How many different committees can be chosen?
(A) 112,896
(B) 3,136
(C) 720
(D) 112
(E) 9

27. A motorcycle costs $ 2,500 when it is brand new. At the end of each year it is worth $^4/_5$ of what it was at the beginning of the year. What is the motorcycle worth when it is three years old?
**(A)** $1,000
**(B)** $1,200
**(C)** $1,280
**(D)** $1,340
**(E)** $1,430

28. If $x + 2y = 2x + y$, then $x - y$ is equal to
**(A)** 0
**(B)** 2
**(C)** 4
**(D)** 5
**(E)** cannot be determined

29. Mary, John, and Karen ate lunch together. Karen's meal cost 50% more than John's meal and Mary's meal cost $^5/_6$ as much as Karen's meal. If Mary paid $2 more than John, how much was the total that the three of them paid?
(A) $2833
(B) $30.00
(C) $35.00
(A) $37.50
(B) $40.00

30. If the angles of triangle are in the ratio 1 : 2 :2, then the triangle
(A) is isosceles
(B) is obtuse
(C) is a right triangle
(D) is equilateral
(E) has one angle greater than 80º

31. Successive discounts of 20% and 15% are equal to a single discount of

(A) 30%
(B) 32%
(C) 34%
(D) 35%
(E) 36%

32. It takes Eric 20 minutes to inspect a car. Jane only needs 18 minutes to inspect a car. If they both start inspecting cars at 8:00 a.m., what is the first time they will finish inspecting a car at the same time?
(A) 9:30 a.m.
(B) 9:42 a.m.
(C) 10:00 a.m.
(D) 11:00 a.m.
(E) 2:00 p.m.

33. If $x/y = 4$ and $y$ is not 0, what percentage (to the nearest percent) of $x$ is $2x - y$
(A) 25
(B) 57
(C) 75
(D) 175
(E) 200

34. If $x > 2$ and $y > -1$, then
(A) $xy > -2$
(B) $-x < 2y$
(C) $xy < -2$
(D) $-x > 2y$
(E) $x < 2y$

35. If $x = y = 2z$ and $x \bullet y \bullet z = 256$, then $x$ equals
(A) 2
(B) $2\sqrt[3]{2}$
(C) 4
(D) $4\sqrt[3]{2}$
(E) 8

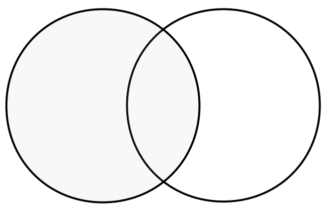
36. If the side of square increases by 40%, then the area of the square increases by
(A) 16%
(B) 40%
(C) 96%

(D) 116%
(E) 140%

37. If 28 cans of soda cost $21.00, then 7 cans of soda should cost
(A) $5.25
(B) $5.50
(C) $6.40
(D) $7.00
(E) $10.50

38. If the product of 3 consecutive integers is 210, then the sum of the two smaller integers is
(A) 5
(B) 11
(C) 12
(D) 13
(E) 18

39. Both circles have radius 4 and the area enclosed by both circles is 28≡. What is the area of the shaded region?



(A) 0
(B) 2≡
(C) 4≡
(D) $4\equiv^2$
(E) 16≡

40. If a job takes 12 workers 4 hours to complete, how long should it take 15 workers to complete the job?
(A) 2 hr 40 min
(B) 3 hr
(C) 3 hr 12 min
(D) 3 hr 24 min
(E) 3 hr 30 min

41. If a rectangle has length L and the width is one half of the length, then the area of the rectangle is

(A) L

(B) $L^2$

(C) ½ $L^2$

(D) ¼ $L^2$

(E) 2L

42. What is the next number in the arithmetic progression 2, 5, 8 .......?

(A) 7
(B) 9
(C) 10
(D) 11
(E) 12

43. The sum of the three digits *a, b*, and *c* is 12. What is the largest three-digit number that can be formed using each of the digits exactly once?

(A) 921
(B) 930
(C) 999
(D) 1,092
(E) 1,200

44. What is the farthest distance between two points on a cylinder of height 8 and the radius 8?

(A) 8 √2
(B) 8 √3
(C) 16
(D) 8 √5
(E) 8 (2≡ + 1)

45. For which values of *x* is $x^2 - 5x + 6$ negative?

(A) x < 0
(B) 0 < x < 2
(C) 2 < x < 3
(D) 3 < x < 6
(E) x > 6

46. A plane flying north at 500 mph passes over a city at 12 noon. A plane flying east at the same time altitude passes over the same city at 12:30p.m. The

plane is flying east at 400 mph. To the nearest hundred miles, how far apart are the two planes at 2 p.m.?

(A) 600 miles
(B) 1,000 miles
(C) 1,100 miles
(D) 1,200 miles
(E) 1,300 miles

47. A manufacturer of boxes wants to make a profit of $x$ dollars. When she sells 5,000 boxes it costs 5¢ a box to make the first 1,000 boxes and then it costs $y$¢ a box to make the remaining 4,000 boxes. What price in dollars should she charge for the 5,000 boxes?

(A) $5{,}000 + 1{,}000y$
(B) $5{,}000 + 1{,}000y + 100x$
(C) $50 + 10y + x$
(D) $5{,}000 + 4{,}000y + x$
(E) $50 + 40y + x$

48. An angle of $x$ degrees has the property that its complement is equal to $\frac{1}{6}$ of its supplement where $x$ is

(A) 30
(B) 45
(C) 60
(D) 63
(E) 72

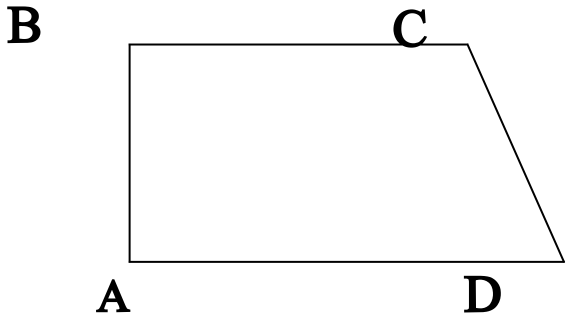
49. The angles of a triangle are in the ratio 2 : 3 : 4. The largest angle in the triangle is

(A) 30º
(B) 40º
(C) 70º
(D) 75º
(E) 80º

50. If $x < y$, $y < z$, and $z > w$, which of the following statements is always true?

(A) $x > w$
(B) $x < z$
(C) $y = w$
(D) $y > w$
(E) $x < w$

51 ABCD has area equal to 28. BC is parallel to AD. BA is perpendicular to AD. If BC is 6 and AD is 8, then what is CD?




(A) $2\sqrt{2}$
(B) $2\sqrt{3}$
(C) 4
(D) $2\sqrt{5}$
(E) 6

52. Write formulas according to descriptions:

1. *a* plus *b* over *a* minus *b* is equal to *c* plus *d* over *c* minus *d*.
2. *a* cubed is equal to the logarithm of *d* to the base *c*.
3. a) $\varphi$ of *z* is equal to *b*, square brackets, parenthesis, *z* divided by *c* sub *m* plus 2. close parenthesis, to the power *m* over *m* minus 1, minus 1, close square brackets;

   b) $\varphi$ of *z* is equal to *b* multiplied by the whole quantity: the quantity two plus *z* over *c* sub *m,* to the power *m* over *m* minus 1, minus1.
4. the absolute value of the quantity $\varphi$ sub *j* of *t* one, minus $\varphi$ sub *j* of *t* two, is less than or equal to the absolute value of the quantity *M* of $t_1$ minus $\beta$ over *j*, minus *M* of $t_2$ minus $\beta$ over *j*.
5. *R* is equal to the maximum over *j* of the sum from *i* equals one to *i* equals *n* of the modulus of *aij* of *t,* where *t* lies in the closed interval *a b* and where *j* runs from one to *n*.
6. the limit as *n* becomes infinite of the integral of *f* of *s* and $\varphi_n$ of s plus delta *n* of s, with respect to *s,* from $\tau$ to *t*, is equal to the integral of *f* of s and $\varphi$ of s, with respect to s, from $\tau$ to *t*.
7. $\psi$ sub *n* minus *r* sub *s* plus 1 of *t* is equal to *p* sub *n* minus *r* sub s plus 1, times *e* to the power *t* times $\lambda$ sub *q* plus s.
8. *L* sub *n* adjoint of *g* is equal to minus 1 to the *n,* times the *n*-th derivative of *a* sub zero conjugate times *g,* plus, minus one to the *n* minus 1, times the *n* minus first derivative of *a* sub one conjugate times *g,* plus ... plus *a* sub *n* conjugate times *g*.
9. the partial derivative of *F* of lambda sub *i* of *t* and *t*, with respect to lambda, multiplied by lambda sub *i* prime of *t,* plus the partial derivative of *F* with arguments lambda sub *i* of *t* and *t*, with respect to *t,* is equal to 0.

10. the second derivative of *y* with respect to *s*. plus *y,* times the quantity 1 plus *b* of *s,* is equal to zero.
11. *f* of z is equal to φ sub *mk* hat, plus big *0* of one over the absolute value of *z,* as absolute *z* becomes infinite, with the argument of *z* equal to gamma.
12. *D* sub *n* minus 1 prime of *x* is equal to the product from *s* equal to zero to *n* of, paranthesis, 1 minus *x* sub *s* squared, close paranthesis, to the power epsilon minus 1.
13. *K* of *t* and *x* is equal to one over two*πi*, times the integral of *K* of *t* and *z,* over *ω* minus *ω* of *x*, with respect to *ω* along curve of the modulus of *ω* minus one half, is equal to rho.
14. The second partial (derivative) of *u* with respect to *t,* plus *a* to the fourth power, times the Laplacian of the Laplacian of *u,* is equal to zero, where *a* is positive.
15. *D* sub *k* of *x* is equal to one over two *πi*, times integral from *c* minus *i* infinity to *c* plus *i* infinity of dzeta to the *k* of, *ω, x* to the *ω* divided by *ω,* with respect to *ω,* where *c* is greater than 1.

## ACTIVE VOCABULARY

Предлагаемый словарь содержит выражения, которые были представлены в данном пособии как новая лексика. **Слова и выражения расположены в алфавитном порядке.**

### -А-

| | |
|---|---|
| абсолютная величина, абсолютное значение | absolute value |
| абсолютный, полный квадрат | perfect square |
| абсцисса | abscissa |
| алгебра | algebra |
| аксиома | axiom |
| аксиома завершенности | completeness axiom |
| аксиома поля | field axiom |
| аксиома порядка | order axiom |
| алгебраическая кривая | algebraic curve |
| анализ | analysis |
| антилогарифм | antilogarithm |
| аргумент, независимая переменная | argument |
| арифметика | arithmetic |
| арка, дуга | arc |
| апофема | apothem |
| ассоциативный | associative |

### -Б-

| | |
|---|---|
| безопасный | secure |
| бесконечный(о) | infinite(ly) |
| бесконечно малое приращение | increment |
| бесконечный предел | infinite limit |
| бесконечная производная | infinite derivative |
| бесконечный ряд | infinite series |
| боковой,   латеральный | lateral |

## -В-

| | |
|---|---|
| вводить | to introduce |
| величина, значение | value |
| вертикальный | vertical |
| вершина (вершины) | vertex (vertices) |
| ветвь (у гиперболы) | branch |
| внешний *(угол)* | exterior |
| вносить вклад | to contribute |
| внутренние точки | interior points |
| внутренний *(угол)* | interior |
| вогнутый | concave |
| воображаемый | fictitious |
| вписанный круг | inscribed circle |
| вращение | rotation |
| выбирать | to select |
| выбор | choice |
| выводить, получать, извлекать (о знании) | to derive |
| выдающийся | distinguished |
| выпуклый | convex |
| вырожденный | degenerating |
| вырождаться | to degenerate |
| высота  под  наклоном | slant height |
| высота треугольника | altitude |
| вычисление | computation |
| вычислять | compute |

## -Г-

| | |
|---|---|
| геометрическое место точек | locus (pl. loci) |
| горизонтальный | horizontal |
| гнуть, сгибать, изгибать | to bend (bent-bent) |
| грань | face |

| | |
|---|---|
| грань, фаска, ребро | edge |
| градиент | gradient |
| график | graph |

## -Д-

| | |
|---|---|
| двучленный, биноминальный | binomial |
| действовать | to operate |
| действительное число | real number |
| делать вывод | to conclude |
| делимое | dividend |
| делитель | divisor |
| десятичный логарифм | common logarithm |
| детерминант, определитель | determinant |
| директриса | directrix (pl. Dirextrices) |
| дискриминант | discriminant |
| дистрибутивный | distributive |
| дифференциальное исчисление | differential calculus |
| дифференцирование | differentiation |
| додекаэдр, двенадцатигранник | dodecahedron |
| дробь | fraction |
| доказывать | to prove |
| доказательство | proof |
| дуга, арка | arc |

## -Е-

| | |
|---|---|
| Евклидова геометрия | Euclidean geometry |

## -И-

| | |
|---|---|
| идентичный | identical |
| изгиб, наклон | slope |
| измерение, мера, предел, степень | measure |
| изнурение, истощение, исчерпание | exhaustion |
| изображение, образ, отражение | image |
| изобретать | to invent |
| икосаэдр, двадцатигранник | icosahedron |
| икосидодекаэдр, тридцатидвухгранник | icosidodecahedron |
| интеграл | integral |
| интегральное исчисление | integral calculation |

| | |
|---|---|
| интегрирование | integration |
| интервал | interval |
| интерпретация | interpetation |
| иррациональный | irrational |
| иррациональность | irrationality |
| исчисление | calculus |

## -З-

| | |
|---|---|
| замкнутый, закрытый | closed |
| закрытая кривая | closed curve |
| закрытый интервал | close interval |
| зеркальное отражение | mirror image |
| знаменатель | denominator |
| значительный | significant |

## -К-

| | |
|---|---|
| касательная | tangent |
| касательная плоскость | tangent plane |
| касаться | to concern |
| кратное число | multiple |
| квадрант, четверть круга | quadrant |
| квадратный (об уравнениях) | quadratic |
| коммутативный | commutative |
| комплексное | complex |
| комплексное число | complex nuber |
| комплектовать | to complete |
| конгруэнтный | congruent |
| конечный | finite |
| конический | conic |
| конфигурация, очертание | configuration |
| копланарный, расположеннный в одной плоскости | coplanar |
| кривая | curve |
| кривизна | curvature |
| круглый, круговой | circular |
| кубическое | cubic |
| кубооктаэдр, трехгранник | cuboctahedron |

## -Л-

| | |
|---|---|
| линейный | linear |
| линия отсчета | reference line |
| логарифм | logarithm |

## -M-

| | |
|---|---|
| мантисса | mantissa |
| математический | mathematical |
| мгновенный, моментальный | instantaneous |
| метод бесконечно малых величин | infinitesimal method |
| многогранник | polyhedron |
| многогранный, полиэдрический | polyhedral |
| многочленный | polynomial |
| многоугольник | polygon |
| множество | set |
| множитель, фактор | factor |
| момент инерции | moment of inertia |

## -Н-

| | |
|---|---|
| наклонная линия, косая линия | oblique |
| направление | direction |
| направленные числа | directed numbers |
| натуральный логарифм | natural logarithm |
| начало координат | origin |
| начальная ось | initial axis |
| независимый | independent |
| неизменный | unvarying |
| непрерывная, функция | continuous function |
| неопределенный | undefined |
| неуловимый | elusive |
| нулевой угол | null angle |

## -О-

| | |
|---|---|
| обобщать | to generalize |
| обозначать | to denote |
| обозревать | to review |
| общее значение | total |
| общий | in common |
| образующая поверхности | generator - generatrix |
| обратная величина | reciprocal |
| обратно | conversely |
| объем | volume |
| одновременный | simultaneous |

| | |
|---|---|
| однозначное соответствие, отображение | one-to-one mapping |
| однообразный | uniform |
| октаэдр, восьмигранник | octahedron |
| определять | to determine |
| ордината | ordinate |
| основной, главный | principal |
| ось | axis |
| открытый | open |
| открытая кривая | open curve |
| отношение | relation |
| отражать | to ret1ect |
| отрезок | segment |
| отрицательный | negative |
| очевидный | obvious |

## -П-

| | |
|---|---|
| параллелограмм | parallelogram |
| пентаграмма | pentagram |
| переменная величина, функция | fluent |
| пересекаться | to intersect |
| перпендикулярный | perpendicular |
| пирамида | pyramid |
| Платонов, относящийся к Платону | Platonic |
| плоскостная кривая | plane curve |
| плоскостной, плоский | planar |
| плоскость, плоскостной | plane |
| площадь всей поверхности | total surface area |
| поверхность | surface |
| подмножество | subset |
| подразделяться, распадаться на | to fall (fell,fallen) into |
| подразумевать | to imply |
| подчиняться правилам (законам) | to obey laws |
| познакомиться с | to be familiar (with) |
| полный угол | round angle |
| положительный | positive |
| полуправильный | semiregular |
| понятие | notion |
| понятие, концепт | concept |
| по часовой стрелке | clockwise |
| против часовой стрелки | anticlockwise |
| правильный | regular |

| | |
|---|---|
| *(о многоугольниках и т.д.)* | |
| предел отношения | limit of a ratio |
| предельный случай | limiting case |
| предполагать | to assume, to suppose |
| представлять | to imagine, to represent |
| преобразование | translation |
| призма | prism |
| применение | application |
| приписывать | to credit |
| проекция | projection |
| произведение | product |
| производная | derivative |
| производная, флюксия | fluxion |
| простой | simple |
| простое *(число)* | prime |
| противоречить | to contradict |
| противоречие | contradiction |
| процедура | procedure |
| прямой | straight |
| пятиугольник | pentagon |
| пятиугольный | pentagonal |

## -Р-

| | |
|---|---|
| равносторонний | equilateral |
| равноугольный | equiangular |
| радиус | radius |
| развернутый угол | flat angle |
| развитие | development |
| разлагать | to resolve |
| разложение множителей | factorization |
| располагаться между | to lie between |
| рассматривать | to regard |
| расстояние | distance |
| рациональный | rational |
| решать | to solve |
| рост | growth |

## -С-

| | |
|---|---|
| сводить в таблицу | tabulate |
| свойство | property |
| сечение | section |
| система прямоугольных координат | Cartesian coordinates |

| | |
|---|---|
| система записи | notation |
| скорость, быстрота | velociy |
| скорость изменения | rate of change |
| сложение | addition |
| сокращать | to cancel |
| сокращать, преобразовывать | to reduce |
| соответствовать | to correspond |
| средний | mean |
| средняя величина (значение) | average |
| ссылаться на | to refer (to) |
| степень | degree |
| стереографический | stereographic |
| сторона (в уравнении) | side |
| сфера | sphere |
| сумма | sum |
| существовать | to exist |

## -Т-

| | |
|---|---|
| таблица | table |
| твердое тело | solid |
| тетраэдр, четырехгранник | tetrahedron |
| точка | point |
| трансцендентный | transcendental |
| треугольный, трехсторонний | triangular |
| трехмерный, объемный, пространственный | three-dimensional |

## -У-

| | |
|---|---|
| увеличивать | to enlarge |
| угол понижения (падения) | depression angle |
| угол возвышения | elevation angle |
| удлиненный | elongated |
| удобство | convenience |
| удовлетворять | to satisfy |
| указывать | to indicate |
| умножение | multiplication |
| уравнение | equation |
| усекать, обрезать; отсекать верхушку | to truncate |
| ускорение | acceleration |
| установить | to estabIish |

## -Х-

| | |
|---|---|
| характеристика | characteristic |

## -Ц-

| | |
|---|---|
| целый, весь, полный | entire |
| целое число | integer |
| цель | purpose |
| центр массы (тяжести) | centroid |

## -Ч-

| | |
|---|---|
| частное | quotient |
| часть | unit |
| числа со знаками | signed numbers |
| числитель | numerator |
| числа со знаками | signed numbers |

## -Ш-

| | |
|---|---|
| шестиугольник | hexagon |
| шестиугольный | hexagonal |

## -Э-

| | |
|---|---|
| элемент, составная часть | element |
| элементарный | elementary |
| эллипс | ellipse |
| эллиптический | elliptical |