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DETERMINISTIC CHAOS AND NOISE IN THE DC VOLTAGE REFERENCE SOURCE SIGNALS

*Irena Nančovska**, *Franc Solina*†, *Anton Jeglič**, *Dušan Fefer**
*Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia
†Faculty of Computer and Information Science, University of Ljubljana, Ljubljana, Slovenia

Abstract: In this work different methods for distinguishing between low-dimensional dissipative dynamics and randomness in the time series are studied. Time series are generated by solid-state voltage reference elements (VRE), which compose a group DC voltage reference source (DCVRS).

Several tests to evaluate whether the correlation integral methods reflect the global geometry or the local fractal structure of the trajectories are applied. Tests are evaluated on the measured signals and on the self-affine sequences that exhibit a power law spectrum of the form $S_v(f) = f^{-\alpha}, 1 \le \alpha \le 2$ known as fractional Brownian motions (FBM). While the measured series pass almost all the tests, the FBM-s fail the test of time differentiation in which the series $\Delta x(t_i) = x(t_i) - x(t_{i-1})$ gave the evidence of their random nature. The presence and the significance of deterministic features in the changing of voltage are confirmed by singular value decomposition (SVD) analysis.

Keywords: time series, deterministic chaos, randomness

1 INTRODUCTION

Throughout scientific research, measured series are the basis for characterising an observed system. The interpretations of the measurements of physical systems often depend on the tools developed for processing the results of the measurements. This paper outlines the possibility of attaching the measures of the dynamical non-linear systems to the metrological systems' signals as a tool for interpretation of the observations of the complex systems behaviour.

The observed time series are generated by solid-state VRE-s, Zener diodes, of a group DCVRS. The goal of their analysis is to achieve long-term stability and to lower the measurement uncertainty of the generated voltage. The time traces of the measured voltage are usually »irregular« or chaotic. For characterisation of the phenomenon of »irregular« changing in time series could be used the fractal theory and the theory of dynamical non-linear systems. The measured time series are observed as mixtures of deterministic components and random noise. With their analyses we try to disentangle (if possible) this two components from the view of non-linear dynamical systems. The results of the analysis could be used for the reconstruction of the time series, prediction, control of the influence parameters etc. The possible benefit of this work could be the use of the results of the analyses in the modelling of the neural networks for their prediction. In the further work, by successful prediction we can lower the uncertainty of the measurement.

Through the paper we will discuss the methods for:

- extraction of the time series from the background and estimation of the statistical properties, which are important in the metrological meaning. In this context we explain the reasons why we need simulations;
- construction an appropriate phase space in which the full structure of the (possible) underlying attractor associated with the chaotic observations is unfolded;

- evaluating invariant properties of the dynamics: correlation dimension, Lyapunov exponents …;
- performing tests for distinguishing between low-dimensional dissipative dynamics and randomness in the measured time series (time and space separation, signal differentiation, phase randomisation, structure function) that confirm the significance of the calculated dimensions.

For purposes of comparison we generate self-affine sequences that exhibit a power law spectrum of the form $S_v(f) = f^{-\alpha}, 1 \leq \alpha \leq 2$ known as FBM-s. For some choices of α the statistical properties and the correlation dimensions of the measured signals are attained. The properties of the measured signals are compared to the properties of the signals generated by FBM also by using the tests for distinguishing between deterministic and random systems.

2 SIGNALS

In this section, we will explain the system, which generate the signals and we will give the reasons why we use FBM-s for simulations [5].

2.1 Measured signals of voltage

Signals are generated by group precision DCVRS which is, in our case, built of solid state voltage reference elements (VRE-s). DCVRS, placed in thermally stable environment, consists of a parallel group of four VRE-s (A, B, C, D). Reference elements are ultra stable Zener diodes LTZ 1000. The observed time series are produced by measuring the absolute values of voltage of each VRE, which is controlled by PC-computer. The PC communicates with DCVRS via serial port RS232. Measuring instrument is digital voltmeter HP3458A. The measured data is transferred to the PC by using IEEE-488 bus. The measurement results are saved in a file on the PC. The time series present 1000 hour measurements[1]. The length of each data set is 4096 samples which are taken in intervals of 15 minutes.

In many natural systems as well as in semiconductors $f^{-\alpha}$ noise was detected as a fluctuation of parameters or values. Therefore a simple white noise test for discrete time series data proposed by Von Newmann *et. al* and Allan [1] should be performed on the measured signals. A comparison of the results of white noise test to the power spectra of the measured time series makes the estimation of deviation from white noise behaviour feasible. The white noise test confirms the presence of the $f^{-\alpha}$ noise in the measured signals. Signals generated by FBM exhibit a power law spectrum of the form $f^{-\alpha}$. That's why they are good candidates for comparison with the measured signals[2].

2.2 Fractional Brownian motions or 1/f noises

FBM is a random function provided by Mandelbrot and Van Ness [5]. It is an extension of Brownian motion and it could be a good starting point for understanding of anomalous diffusions as random changes. The most important feature of FBM is that its increments $[B_H(t+T) - B_H(t)] = h^{-H} [B_H(t+hT) - B_H(t)]$ are stationary and statistically self-similar. Increments of FBM have Gaussian distribution with a standard deviation:

$$\sigma[BH(t+T) - BH(t)] = C_H T^H \quad (1)$$

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[1] Duration of that measurement is about one and a half months.

[2] The deviation of the spectrum of the measured signal from the $f^{-\alpha}$ form could possibly be ascribed to the environmental influences and the inherent characteristics of the solid state diode LTZ 1000 as can be noticed in repeated measurements.

where C_H is constant. This is usually called T^H law of FBM. The parameter H is directly related to the fractal (Hausdorff) dimension D; for FBM one variable D = 2-H. For generation of the FBM signals is used the method of spectral synthesis[3].

3 PHASE SPACE RECONSTRUCTION AND CORRELATION DIMENSION

In this section is discussed the phase space reconstruction based on method of delays and the use of mutual information. This follows the theory of estimation of correlation dimensions as a lower bound of the fractal dimension.

3.1 Embedding procedure

In the analysis of a scalar data set equally spaced in time (*i.e.* $x(n) = x(t_o + n\,t)$) the standard technique is based on the phase space reconstruction, such as time embedding procedure. The first step is to create time embedded vectors $\bar{x}(n) = [x(n), x(n+\tau), \ldots, x(n+(d_E-1)\tau]$ for various values of the embedding dimensions d_E or estimating the window $\tau_\omega = d_E * \tau$. The value of τ have to be large enough to introduce a degree of statistical independence between the components. The time delay is typically determined by the use of average mutual information. Mutual information answers the question: "given that x has been measured at time n, what is the average uncertainty in measurement of x at time $n+\tau$?" The aim of time series embedding analysis could be the determination of the dimensionallity of the subspace where the whole structure of the (possible) underlying attractor associated with the chaotic observations is unfolded. We use singular value decomposition (SVD) analysis proposed by Broomhead & King [2] to determine the dimension of the space where the phase portrait could be reconstructed. From the SVD-analyses of the time series VRE A is obvious that it is noisy and we need at least 9 dimensions for reconstruction of the non-linear dynamics. Only the 6 first singular values are significantly bigger then the others or the important dynamics could be confined to a 6-dimensional subspace of the embedding space.

3.2 Correlation integral

Effective method for estimation of correlation dimension deals with correlation integral. It is based on counting the points of attractor which well defined distances are less then some value. The correlation integral of N vectors $(\bar{x}(n), n = 1, \ldots, N)$, is defined in [3]:

$$C(r) = \frac{2}{N(N-1)} \sum_{m=1}^{N} \sum_{n=m+1}^{N} \Theta(r - \|\bar{x}(m) - \bar{x}(n)\|) \qquad (3)$$

where $\Theta(x)$ is the Heaviside step function and $\|x\|$ is any well defined vector norm[4]. The main property of $C(r)$ is that it behaves as power law of r for small r-s: $C(r) \cong r^{D_{cor}}$. The correlation dimension D_{cor} is then formally defined as the following limit:

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[3] FBM-s are often characterised by spectral densities $S_v = \frac{\|B_H\|^2}{\Delta f}$ with the general form $S_v = f^{-\alpha}$, α is directly connected to dimension H with $\alpha = 2H + 1$. Time series of length N are obtained inverse Fourier transformation (IFT) by the formula:

$$X_i = \sum_{k=1}^{N/2} [CK^{-\alpha} (\frac{2\pi}{N})^{1-\alpha}]^{1/2} \cos(\frac{2\pi i k}{N} + \Phi_i) \qquad (2)$$

where C is constant and φ_i are randomly distributed in $[0, 2\pi]$.

[4] The most appropriate choice for norm is L-infinity norm, defined as:

$$D_{cor} = \lim_{r \to 0} \frac{\log(C(r))}{\log(r)} \tag{4}$$

In practice, D_{cor} is obtained by plotting *C(r)* versus *r* on a log-log scale and reading off the slope from the portion of the graph.

As d_E increases, the value of D_{cor} saturates to the appropriate correlation dimension of the system. The minimum at which d_E this saturation occurs is known as embedding dimension (figure 1). The correlation function D_{cor} can be related to the theoretical fractal Hausdorff dimension by inequality $D_{cor} \leq D$ and for very large series D_{cor} almost reaches *D*.

4 TESTS FOR DISTINGUISHING BETWEEN LOW-DIMENSIONAL DYNAMICS AND RANDOMNESS

Some fractal and statistical parameters, that seem to be important either for stability in metrological or dynamical sense, of the measured series are compared to the same characteristics of the series generated by FBM. For some choices of α the statistical properties and even the correlation dimensions and the leading Lyapunov exponent λ_{max} [5] of the measured signals are attained. The best results for VRE-s A, B, C, D are obtained with the following spectrum functions respectively $f^{1.25}$, $f^{1.27}$, $f^{1.27}$, $f^{1.3}$. Although the correlation dimensions of FBM noises saturate, we can not make any reliable statement about their behaviour. The upper statement is confirmed by the results of the tests for distinguishing between dynamics and stochastics, performed on the both: measured and FBM series [6].

4.1 Signal differentiation

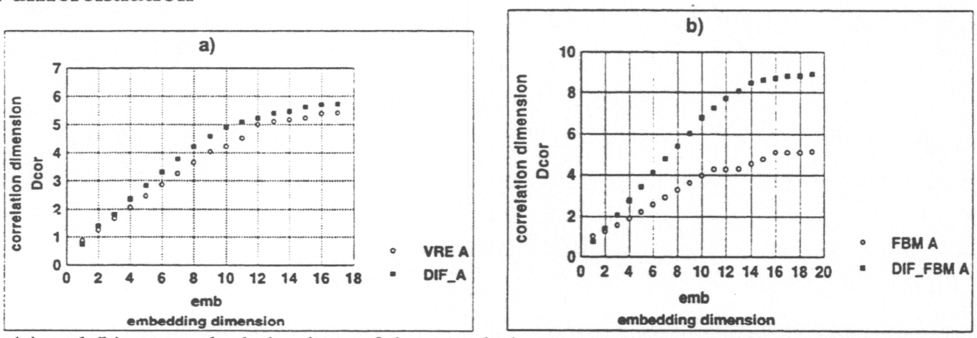

Figure 1: (a) and (b) report the behaviour of the correlation exponents versus the embedding dimension of the original (circles) and the difference (filled squares) signal for the measured and FBM signals respectively.

For a system governed by a dynamical system, the value of correlation dimension should be the same for the signal as well as for the first (or a higher) derivative. The first derivative of the signal $\Delta x(t_i) = x(t_i) - x(t_{i-1})$ has a correlation dimension which is often much larger then the one of the original signal. No saturation of the correlation exponent of the difference signal of the stochastic (FBM) process can be observed (figure 1). This is due to the fact that the increments $\Delta x(t_i)$ behave like pure noise as in the case of FBM-s. Consequently, if the results of correlation analysis do not change under differentiation, one has indication that the time

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$$\|x(\bar{m}) - x(\bar{n})\|_{\infty} \equiv \max_{1 \leq i \leq d_E} |y_i(m) - y_i(n)|$$

[5] λ_{max} is measuring the average convergence of the nearby trajectories in the phase space or it is measuring how predictable the system is.

series has been generated by a dynamical process which is true in the case of the measured signals.

4.2 Time and space separation

Through the calculation of correlation dimension, we can assume that the distance between pairs of points could possible be due to the geometry of the reconstruction, not because the points are dynamically correlated and their separation in space reflects their being neighbours in time. The test is based on plotting contour maps of the fraction of points closer then distance r at given time separation Δt for arbitrary t. This test reflects the temporal rather then spatial separation but still shows the different behaviour between the measured signals and the simulated signals with FBM-s.

4.3 Phase randomisation and other tests

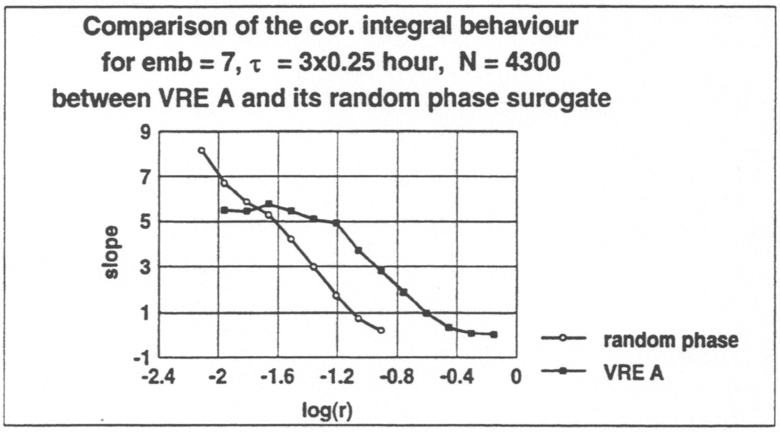

Figure 2. The correlation dimension of the VRE A saturates and the correlation dimension of the random phase signal doesn't.

From given measured series $x(t)$, thought to be chaotic, we generate surrogate signals obtained by inverting a power spectrum exactly equal to that of the signal under study and with random, independent and uniformly distributed Fourier phases. If the correlation dimension is invariant under phase randomisation, that strongly implies that this estimation is not product of low-dimensional dynamics. From the figure 2 is obvious that the correlation dimension of the measured signal converges and the correlation dimension of the random phase signal doesn't.

Other tests, like structure function, independent realisations [6]... confirm the presence of low-dimensional dynamics in the measured signals and distinguish them from random signals FBM-s.

5 CONCLUSIONS

The results of the analyses of the measures signals of VRE-s could be used for diagnostic purposes or for correction of the generated voltage[6]. The results of the analyses are given in the following statements:

- The estimated correlation dimensions of the measured signals are between 4.5 and 5.5. SVD analyses confirm that the whole dynamics happens in 5 to 7 dimensions.
- The tests for distinguishing low-dimensional dynamics from randomness confirm that the behaviour of VRE-s is similar to the behaviour of non-linear system, which consists of a few subsystems.

[6] The current voltage could be corrected by using the predicted value.

- Test proposed by Allan and Von Newmann [1,4] confirms the presence of coloured noise in the measured signals.
- It is difficult to disentangle the components of determinism and randomness in the observed signals and to determine the amounts of each of them. Clearly, in the low frequency part of the spectrum are present the following periodicities: half month, week, a few days, day, and a few hours, which play important roles in the appearance of low-dimensional dynamics. It is not clear which are the other sources of determinism (if any) present in the data.
- The results of the analyses could be used for construction of the model for prediction, which corrects the voltage and by this lowers the measurement stability.

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