Synthesis of Decentralized Variable Gain Robust Controllers for Uncertain Large-Scale Interconnected Systems

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March 2018

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Shunya Nagai

Graduate School of Advanced Technology and Science, Tokushima University

5015460042

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Thesis examiners;

Member (Chief)	:	Prof. Shinsuke KONAKA
Member	:	Prof. Atsushi TAKADA
Member	:	Prof. Tomohiro KUBO
Member	:	Prof. Hidetoshi OYA (Tokyo City Univ.)

まえがき

現代制御理論に基づいて制御系設計を行うには，モデリングによって制御対象の状態空間表現による数式モデルを得る必要がある．得られた数式モデルが制御対象を十分な精度で近似していれば，様々な理論を用いて望ましい制御性能を達成することができる．しかしながら，制御対象を正確にモデリングすることは非常に困難であり，制御対象とその数式モデルとの間には「不確かさ」と呼ばれるギャップが存在する．この不確かさには，線形化誤差，低次元化誤差等によるモデル化誤差，あるいは未知パラメータの変動が存在する．このような不確かさを無視して制御系設計を行うと，設計者の所望の制御性能が得られないばかりか，最悪の場合，システムが不安定となってしまう恐れがある．このため，この不確かさを陽に考慮した上で制御系設計を行う「ロバスト制御」が従来から盛んに研究されてきた．しかしながら，従来のロバスト制御の結果は固定的なゲインのみで構成されたコントローラに関する結果がほとんどであり，不確かさの最悪値を想定して設計された固定的なゲインを用いた従来のロバスト制御は，実際の不確かさの変動幅が想定されたものより小さい場合は保守的な制御系設計となってしまう．これに対し，可変ゲインロバストコントローラがいくつか提案されている．可変ゲインロバストコントローラを用いることにより，対象システムの利用可能な情報を用いて，コントローラのパラメータをオンラインで調整することができ，不確かさの影響による応答特性の劣化を抑制しつつ，過大な制御入力を避けることのできる柔軟な制御系設計が可能となる．

一方，近年の制御対象の特徴として，交通システムや電力システムなどのように大規模，かつ複雑化していることが挙げられる．このような対象システムは,「大規模複合システム」として考える必要がある．大規模複合システムに対する制御方式として，システムの全情報を一箇所に集め，単一のコントローラによって制御する「集中制御方式」が

ある．しかしながら，大規模複合システムに対して集中制御を考えた場合，情報量，計算量など，物理的な制約の面から適用できない場合が多い．これに対し，対象システムをいくつかのサブシステムに分割し，複数のコントローラを用いて各サブシステム毎に制御を行う「分散制御方式」が有効であり，これまで盛んに研究されている．さらに，ロバスト制御と関連して，不確かさを含む大規模複合システムに対する分散ロバスト制御についても従来から研究されており，多くの結果が報告されている．しかしながら，従来の分散ロバスト制御の結果は，集中制御におけるロバスト制御と同様に固定的なゲインのみで構成されたコントローラに関するものがほとんどであり，可変ゲインロバストコントローラに関する結果はあまり報告されていない．固定ゲインロバストコントローラを用いた分散ロバスト制御は，システムの次数が大きくなると，コントローラを設計するための解くべき制約式である線形行列不等式 (LMI：Linear Matrix Inequality) が高次元，複雑化してしまい，その解が存在しない，すなわちコントローラが設計できない場合もある．これに対して，可変ゲインロバストコントローラを用いた分散ロバスト制御では，固定ゲインコントローラの場合と比べ，LMI の低次元，簡単化が可能であるという特長があり，固定ゲインコントローラでは安定化できないシステムを安定化できる可能性がある．

本論文では，不確かさを含む大規模複合システムに対する分散可変ゲインロバストコントローラの構成法を提案する．

Chapter 1 では，制御理論の発展の歴史，ロバスト制御，および分散制御について述べるとともに，本研究の目的，新規性を述べる．また，本論文で用いる数学的記法，および補題を示す. Chapter 2 では，不確かさを含む大規模複合システムに対して，安定性を保証するだけでなく，設計者の望ましい応答特性を達成する分散可変ゲインロバストコントローラの構成法を提案する．ただし，制御対象となる大規模複合システムに含まれる不確かさ，および相互干渉はマッチング条件を満たすものとする．さらに，提案するコントローラの設計問題は，得られた LMI の可解性に帰着されることを示す. 最後に，提案する分散可変ゲインロバストコントローラの有用性を検証するための数値例を紹介する．Chapter 3 では，不確かさ，および外乱を含む大規模複合システムに対して，安定性を保証するだけでなく，「$\mathcal{L}_2$ ゲイン性能」と呼ばれる外乱抑圧性能を達成する分散可変ゲインロバストコントローラの

構成法を提案する。大規模複合システムに含まれる不確かさ，およびサブシステム間の相互干渉は，Chapter 2 と同様にマッチング条件ものとし，外乱入力は二乗可積分関数と仮定する．さらに，提案するコントローラの設計問題は，得られたLMI の可解性に帰着されることを示す．最後に，提案する $\mathcal{L}_2$ ゲイン性能を有する分散可変ゲインロバストコントローラの有用性を検証するための数値例を示す．Chapter 4 では，マッチング条件を満たさない不確かさを含む大規模複合システムに対し，安定性を保証する分散可変ゲインロバストコントローラの構成法を提案する．大規模複合システムに含まれる不確かさはマッチング条件を満たさないものとし，さらにその不確かさをマッチング条件を満たす部分（マッチ部）と満たさない部分（ミスマッチ部）に分割する．さらに，提案するコントローラの設計問題は，得られたLMIの可解性に帰着されることを示す．最後に，提案する分散可変ゲインロバストコントローラの有用性を検証するための数値例を紹介する．Chapter 5 では本論文で得られた成果をまとめ，今後の課題について述べる．

Abstruct

In order to design control systems via modern control theory, it is necessary to derive a mathematical model for controlled systems based on state-space representation. If the mathematical model describes the controlled system with sufficient accuracy, satisfactory control performance are achievable by using various controller design methods. However, there always exist some gaps referred to as "uncertainties" between the controlled system and the mathematical model. The uncertainties in the controlled system may cause deterioration of control performance or unstability of the controlled system. For this view point, many researchers have studied robust control for uncertain dynamical systems, and a lot of results for robust control have been developed. It is well known that controllers in most of the existing result for robust control have fixed gains only. Moreover, these robust controllers with fixed gains are designed by considering the worst case variations of uncertainties. Thus these design approaches are conservative when an actual perturbation regions of uncertainties is smaller than supposed ones. In contrast with these, several design methods of variable gain robust controllers have been proposed. By using these variable gain robust controllers, flexible controller design is possible such as avoidance of excessive control input and deterioration of response characteristics caused by the influence of uncertainties.

On the other hand, controlled systems become more complex because of the rapid development of modern industry. Such complex systems should be considered as "large-scale interconnected systems". However, as is well known, it is difficult to apply centralized control to such systems due to physical constraints, calculation amounts and so on. Therefore, in decentralized control, controlled systems are divided into several subsystems, and controllers are designed for each subsystems. Furthermore, many researchers have also studied decentralized robust control for uncertain large-scale interconnected systems. However, there are few results for decentralized robust controllers with variable gains. In the case of the conventional

decentralized robust controllers with fixed gains, the size of linear matrix inequalities (LMIs) which should be solved to design decentralized robust controller becomes large. In contrast with these, LMIs for decentralized variable gain robust controllers are more simple than ones of the conventional decentralized robust controllers with fixed gains only, namely, there is a possibility that decentralized variable gain controllers can stabilize uncertain large-scale interconnected systems which cannot be stabilize in the case of conventional fixed gain controllers.

In this thesis, for uncertain large-scale interconnected systems, we propose design methods of decentralized variable gain robust controllers (DVGRC).

First of all in chapter 1, we introduce the history of control theory, robust control and decentralized control. Moreover, the purpose and the originality in this thesis are described. Finally, notations and useful lemmas which are used in this thesis are shown. In chapter 2, we propose an LMI-based design method of a decentralized variable gain robust regulator for a class of uncertain large-scale interconnected systems. For the uncertain large-scale interconnected system, uncertainties and interactions satisfy so-called matching condition. Furthermore, a sufficient condition for the existence of proposed decentralized variable gain robust controller is given in terms of LMIs. Finally, we include a numerical example to show the effectiveness of the proposed decentralized robust controller. Next, chapter 3 describes a design method of decentralized variable gain robust controllers with guaranteed disturbance attenuation performance referred to as "$\mathcal{L}_2$ gain performance" for a class of uncertain large-scale interconnected systems. In chapter 3, we also assume that uncertainties and interactions satisfy matching condition, and disturbance inputs are square integrable functions. Additionally, we show that sufficient conditions for the existence of the proposed decentralized variable gain robust controller with guaranteed $\mathcal{L}_2$ gain performance are given in terms of LMIs. Finally, simple illustrative example is shown. Chapter 4 shows a decentralized variable gain robust controller for a class of large-scale interconnected systems with mismatched uncertainties. For the uncertain large-scale interconnected systems, uncertainties and interactions do not satisfy matching condition, and we divide them into the matched part and the mismatched one. A sufficient condition for the existence of the proposed decentralized variable gain robust controller is reduced to LMIs. Finally, we show a numerical example to validate the proposed design procedure. Chapter 5 describes conclusions in this thesis and future works.

Acknowledgments

Many people have contributed in various ways to this thesis. Firstly, I would like to express my sincere gratitude to Prof. Hidetoshi Oya with Tokyo City University for his entire guidance. He gave me a lot of valuable comments and suggestions through the discussions on this study. Moreover, he advised me to apply to the Conferences such as the Annual Conference of the IEEE Industrial Electronics Society (IECON) and so on to present some results of this work.

I also want to express my sincere gratitude to Prof. Tomohiro Kubo for variable and helpful advices that greatly contributed to my study and this thesis. Furthermore, I would like to thank Dr. Tsuyoshi Matsuki with National Institute of Technology, Niihama College for valuable advices. Also, I would like to thank Prof. Shinsuke Konaka and Prof. Atsushi Takada for valuable comments and suggestions for this thesis. Moreover, I'm very grateful to the laboratory members of Tokushima University and Tokyo City University.

Finally, special appreciation goes to my family for their moral support and great encouragement for their patience and understanding throughout the time that I spent for the dissertation.

Contents

List of Figures

Chapter 1

Introduction

Robust control for uncertain dynamical systems and decentralized robust control for uncertain large-scale interconnected systems have been well studied. In this chapter, we introduce the history of control theory. Namely, we look back the history of development for classical and modern control, robust control and decentralized control. Moreover, the purpose and the originality in this thesis are described. Finally, notations and useful lemmas which are used in this thesis are shown.

1.1 Classical Control and Modern Control

As you know, control systems can be found in diverse field of industry such as electric power systems, robotics, chemical plants, transportation systems, space systems and so on. In order to control such systems, many researchers have well studied various control strategies and when we design control systems, it is necessary to construct mathematical models for the controlled systems. For mathematical models, there are mainly two types of representation, i.e., transfer function representation (classical control theory) [1–3] and state-space representation (modern control theory) [4–6].

Classical control theory have been developed in the 1950's, and describe controlled systems by using the relation between inputs and outputs (i.e. transfer function representations and frequency responses). For the stability analysis based on classical control, Routh-Hurwitz stability criterion [7] and Nyquist criterion [8] are well known. Bode and Nichols proposed graphical design methods, which are well-known as "Bode diagram" [9] and "Nichols chart" [10], respectively. Moreover, some design methods based on classical control such as PID (Proportional, Integral and Derivative) controllers and phase lag-lead compensators have been suggested [11]. However in classical control theory, controlled systems are mainly linear and time-invariant, and have single input and single output only. Furthermore, experiences and trial and error are needed for design approaches based on classical control theory.

On the other hand, modern control theory has been presented by Kalman in the 1960's. Modern control theory describes not only the relation between inputs and outputs but also internal states of controlled systems by using state variables, i.e., controlled systems have been represented as state equation (i.e. state space representations). Kalman has proposed optimal regulator theory [12, 13] and optimal filtering one [14]. Namely, in modern control theory, controller design problems are reduced to optimization problems based on the concept of state variables. As mentioned above, classical control is a design theory of frequency domain and controlled systems are mainly linear and time-invariant, and have single input and single output only. In contrast, modern control is a design theory of time domain, and it is applicable to systems which are difficult to deal with by classical control (e.g. multi-input and multi-out systems and nonlinear systems), and thus there are a lot of results based on the state space representation for stability analysis and controller design problems [15–17]. The characteristics of classical control and modern control are summarized in Table 1.1.

Table 1.1: Characteristics of classical control and modern control

	Classical control	Modern control
Design space	Frequency region	Time domain
System representation	Transfer function	State equation
Design approach	Graphical methods	Matrix computation

In modern control theory, pole assignment [18, 19] and optimal control theory [20–22] are representative controller design methods. Pole assignment is that if controlled systems are linear and controllable, state feedback control laws can place the closed-loop poles for controlled systems at arbitrary locations in the complex plane. Moreover, optimal control is the problem of finding a control law which minimizes a certain cost function. Especially, for linear systems, linear quadratic optimal control is a controller design problem that minimizes a given quadratic cost function which includes state and control variables, and the control law derived by solving the optimization problem is referred to as linear-quadratic regulator (LQ regulator) [23]. It is well known that LQ regulator has a robustness and a low sensitivity for parameter variations of controlled systems. Furthermore, its state feedback control law can be designed easily by using the solution of an algebraic Riccati equation and stability for the closed-loop system is guaranteed [24].

1.2 Robust Control

When we design control systems, it is necessary to establish mathematical models for controlled systems. If the mathematical model describes the controlled system with sufficient accuracy, satisfactory control performance are achievable by using various controller design methods. However, there inevitably exist some gaps between the controlled systems and its mathematical model, and the gaps are referred to as "uncertainties". Uncertainties are caused by linearization for nonlinear systems, modelling error (e.g. model order reduction), variations of system parameters and so on. The uncertainties in the mathematical model may cause deterioration of control performance or unstability of control systems. Therefore, many researchers have well focused robust control problems for dynamical systems with uncertainties, and a large number of existing results for robust stability analysis and robust stabilization have already been obtained [25–27]. As an example of applications for robust control, let us consider the control for a space rocket such as Figure 1.1 [28].

Figure 1.1: Space rocket [28]

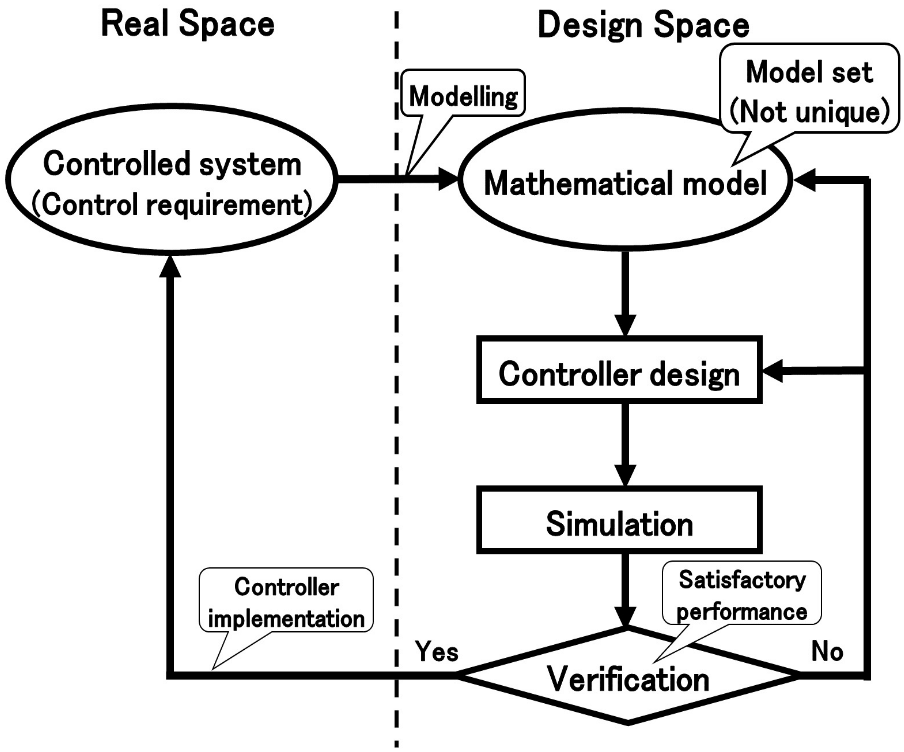

Figure 1.2: Overview of robust controller design

Although nominal parameters of the space rocket can be obtained by various tests, its true parameters are uncertain/unknown due to the effect of flight enviroment and so on. Therefore, designers should consider gaps (*uncertainties*) between nominal values and true ones. In particular, since launch and flight of the rocket are one-shot, robust control is very effective for such case.

In general, the mathematical model derived by modelling is unique. But, in robust control theory, the mathematical model for the controlled system is considered as a model set consisting of a nominal model and uncertainties, and controllers are designed such that robust stability and desired performance for all systems belonging to a model set are achieved. Namely, the controller design problem in robust control is defined as follows.

The controller design problem in robust control

Design a controller which stabilizes all systems belonging to a model set which includes a nominal model and variations of uncertainties.

This means that robust controller design is worst-case one. Figure 1.2 shows the process of controller design. In the 1980's, main problems of robust controller design was to ensure the robust stability of uncertain systems. From the 1990's, lots of controller design methods which achieve not only robust stability but also desired performance for uncertain systems have been established [29, 30]. One can see that quadratic stabilization based on Lyapunov stability criterion and $\mathcal{H}^\infty$ control are typical robust controller (e.g. [31–35]). Furthermore some researchers investigated quadratic stabilizing control with achievable performance level in reference to such as a quadratic cost function [36, 37], robust $\mathcal{H}^2$ control [38, 39] and robust $\mathcal{H}^\infty$-type disturbance attenuation [40, 41]. In addition, the results for robust stability analysis and robust controller design problems using parameter dependent Lyapunov functions (PDLFs) or piecewise Lyapunov functions (PLFs) have been presented [42–44]. Moreover, Chesi has proposed the design method of robust controllers based on homogeneous polynominal Lyapunov functions (HPLFs) for linear systems with polytopic time-varying uncertainty [45]. Additionally, for uncertain linear systems with exogenous disturbances, a linear state feedback controller which achieves not only robust stability but also minimization of the bound of invariant ellipsoidal set for the output has also been suggested [46]. However, most of robust controllers consist of fixed gain parameters which are designed by considering the worst case variations for uncertainties/unknown parameters. In contrast with such conventional robust control with fixed gains, several design methods of some robust controllers with variable gains have also been proposed (e.g. [47,48]). In the work of Maki and Hagino [47], by introducing time-varying adjustable parameters, adaptation mechanisms for improving transient behavior have been suggested. Moreover, for linear systems with matched uncertainties, Oya and Hagino [48] have introduced an adaptive compensation input which is determined so as to reduce the effect of uncertainties. In addition, a design method of robust controllers with variable gains based on LQ optimal control for a class of uncertain linear systems has also been shown [49]. These robust controllers have both fixed controller parameters and variable ones tuned by updating laws, and resultant control systems are more flexible and adaptive comparing with the conventional robust controllers with fixed gains only. Note that in this thesis, these robust controllers with time-varying adjustable parameters are referred to as "variable gain robust controller".

1.3 Decentralized Control

In recent years, owing to the rapid development of industry, controlled systems have become highly complex and large in dimension, and such dynamical systems are referred to as “large-scale interconnected systems” or “large-scale complex systems”. Although large-scale and complex systems can be seen in diverse fields such as traffic systems, economic systems, electrical systems and so on, it is difficult to apply centralized control strategies for such large-scale interconnected systems because of calculation amounts, physical communication constraints and so on. Therefore, decentralized control problems for large-scale interconnected systems have been well studied [50–54]. In the decentralized control strategy, large-scale interconnected systems are divided into several subsystems and controlled each one by more than one controller or decision maker involving decentralized computation. Figure 1.3 shows the overview of decentralized control. In Figure 1.3, the number of subsystems and controllers is 3, and d_{ij} denotes interactions from i-th subsystem to j-th subsystem. The major problem of large-scale interconnected systems is how to deal with

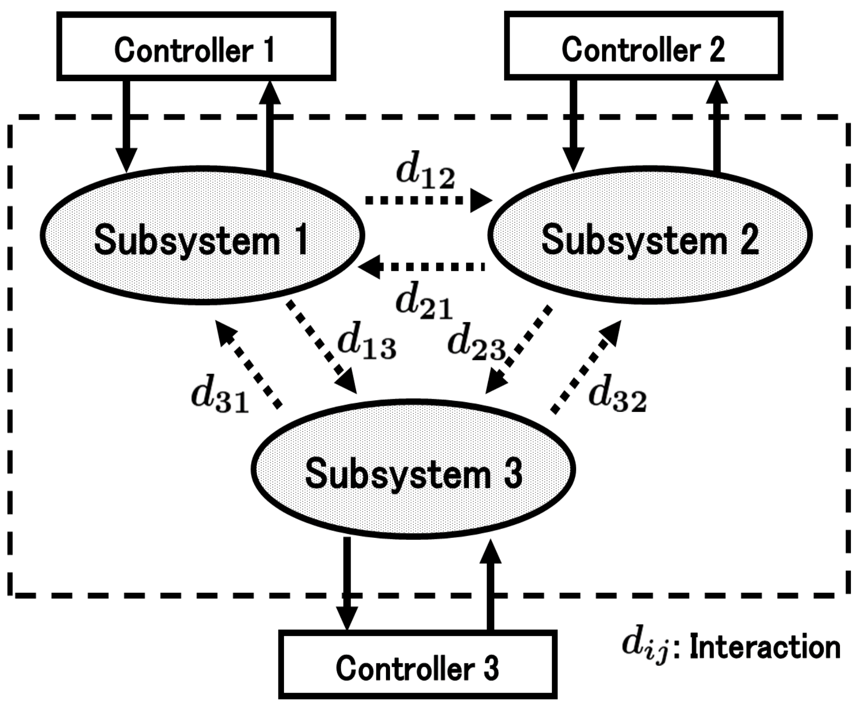

Figure 1.3: Overview of decentralized control

Figure 1.4: Traffic system [55]

the interactions among subsystems. Namely, each subsystem is controlled by the corresponding controller. For typical applications of decentralized control, a decentralized traffic signal control system based on decentralized intelligence (Figure 1.4) has been developed [55]. Furthermore, we find that smart grid (Figure 1.5), which control electric power supply and demand using networked decentralized small-scale power sources, receives much attention in recent years [56].

During the last three decades, various types of decentralized control problems have been studied, and a large number of results in decentralized control systems can be seen in the work of Šiljak [52]. Furthermore, a framework for the design of decentralized robust model reference adaptive control for interconnected time-delay systems has been considered in [57] and decentralized fault tolerant control problem has also been solved [58]. Lee et al. [59,60] have studied synchronization problems for complex dynamical network with randomly switching topology [59,60]. Additionally, stability analysis and decentralized controller design problems for fuzzy large-scale systems have been shown [61, 62]

For decentralized robust control for uncertain large-scale interconnected systems, many researchers have also considered various problems (e.g [63–67]). In the work of Mao and Lin [63], the aggregative derivation are tracked by using a model following technique with online improvement for large-scale interconnected systems with unmodelled interaction, and a sufficient condition for which the overall system when

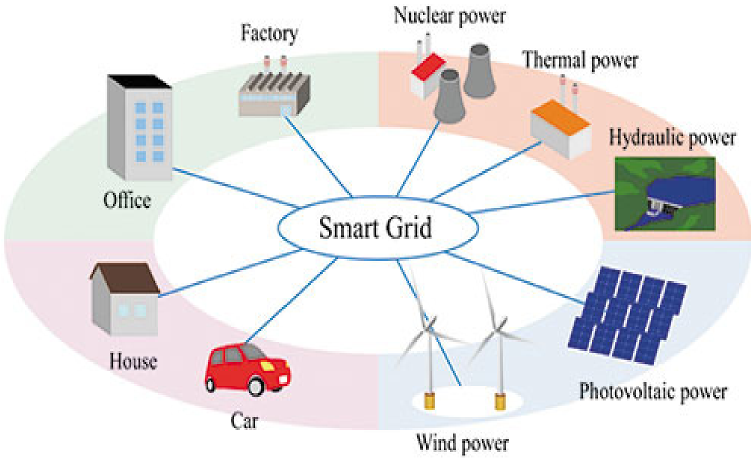

Figure 1.5: Smart grid [56]

controlled by the completely decentralized control is asymptotically stable has been presented. Chen et al. [64, 65] have considered a design problem of a decentralized controller for a class of interconnected nonlinear dynamical systems with uncertain parameters and input disturbances. Additionally, decentralized robust controllers which guarantee robust stability with prescribed degree of exponential convergence have been presented. For a class of uncertain interconnected systems with state and input delays, Zhang et al. [67] have proposed a design method of decentralized output feedback controllers based on Riccati equation. Furthermore, decentralized guaranteed cost controllers for uncertain large-scale interconnected systems have also been suggested [68–70]. In addition, the robust decentralized control problem for discrete-time singular large-scale systems with interval uncertainties has been investigated [71]. However, there are few results for decentralized variable gain robust controllers for large-scale interconnected systems. In the case of decentralized robust controllers with fixed gains, the size of linear matrix inequalities (LMIs) which should be solved to design decentralized robust controller becomes large. But, the size of derived LMIs for decentralized variable gain robust controllers are smaller comparing with the conventional decentralized robust controllers with fixed gains only, namely, there is a possibility that decentralized variable gain robust controller can stabilize uncertain large-scale interconnected systems which cannot be stabilized by the conventional decentralized robust controllers.

1.4 Purpose and Points of Originality

In this thesis, for uncertain large-scale interconnected systems, we propose LMI-based design methods for decentralized variable gain robust controllers. Furthermore, simple illustrative examples are included to show the effectiveness of the proposed decentralized variable gain robust control strategies.

First of all in chapter 2, an LMI-based design method of a decentralized variable gain robust regulator for a class of uncertain large-scale interconnected systems is proposed. Uncertainties and interactions which are included in the large-scale interconnected system satisfy so-called matching condition [31, 72]. Furthermore, a sufficient condition for the existence of proposed decentralized variable gain robust controller is given in terms of LMIs. The proposed decentralized variable gain robust controller achieves not only robust stability but also satisfactory transient behavior. Note that LMIs in the case of conventional decentralized fixed gain robust controllers may not feasible for large-scale interconnected systems with matched uncertainties. On the other hand, the proposed LMI condition is always feasible, namely, designers can derive the decentralized variable gain robust controller provided that some assumptions are satisfied.

Next, based on the result of chapter 2, we present a design method of decentralized variable gain robust controllers with guaranteed disturbance attenuation performance referred to as "$\mathcal{L}_2$ gain performance" for a class of uncertain large-scale interconnected systems in chapter 3. The proposed decentralized robust controller achieves not only internal stability but also $\mathcal{L}_2$ gain performance. The decentralized variable gain robust controller design method derived in chapter 3 is a natural extension of the result of chapter 2.

In chapter 4, we show a decentralized variable gain robust controller for a class of large-scale interconnected systems with mismatched uncertainties. For the uncertain large-scale interconnected systems, uncertainties and interactions do not satisfy matching condition. There is a possibility that the proposed decentralized variable gain robust controller can stabilize the large-scale interconnected systems with mismatched uncertainties, in the case that the conventional decentralized fixed gain robust controller cannot be designed. The effect of matched parts of uncertainties can be suppressed by the variable gain parameter in the proposed controller, and the size of LMIs which should be solved to design proposed variable gain robust controller is smaller than one for the conventional fixed gain robust controllers. Therefore, the proposed design method can be applied more larger class of uncertain large-scale interconnected systems, and the proposed decentralized robust control scheme is very useful.

Finally, in chapter 5, we summarize the result and the usefulness of the proposed decentralized variable gain robust control strategies in this thesis. Moreover, we describe future works to be carried out.

1.5 Notations and Lemmas

In this section, we show notations and useful and well-known lemmas (see [73–75] for details) which are used in this thesis.

For a matrix $\mathcal{A}$, the transpose of matrix $\mathcal{A}$ and the inverse of one are denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$, respectively. In addition, $H_e\{\mathcal{A}\}$ and I_n mean $\mathcal{A} + \mathcal{A}^T$ and n-dimensional identity matrix, respectively, and a block diagonal matrix composed of matrices $\mathcal{A}_i$ for $i = 1, \cdots, \mathcal{M}$ is represented as $\mathrm{diag}(\mathcal{A}_1, \cdots, \mathcal{A}_{\mathcal{M}})$. For real symmetric matrices $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} > \mathcal{B}$ (resp. $\mathcal{A} \geq \mathcal{B}$) means that $\mathcal{A} - \mathcal{B}$ is positive (resp. nonnegative) definite matrix. For a vector $\alpha \in \mathbb{R}^n$, $||\alpha||$ denotes standard Euclidian norm, and for a matrix $\mathcal{A}$, $||\mathcal{A}||$ represents its induced norm. The symbols "$\star$" and "$\stackrel{\triangle}{=}$" mean symmetric blocks in matrix inequalities and equality by definition, respectively.

Lemma 1.1 *For arbitrary vectors λ and ξ and the matrices $\mathcal{G}$ and $\mathcal{H}$ which have appropriate dimensions, the following inequality holds;*

$$H_e\left\{\lambda^T \mathcal{G}\Delta(t)\mathcal{H}\xi\right\} \leq 2\left\|\mathcal{G}^T\lambda\right\|\left\|\mathcal{H}\xi\right\|,$$

where $\Delta(t)$ with appropriate dimension is a time-varying unknown matrix satisfying $\|\Delta(t)\| \leq 1.0$.

Proof : By using Schwarz's inequality [74] and the relation $\|\Delta(t)\| \leq 1.0$, one can see that the relation

$$\begin{aligned} H_e\{\lambda^T \mathcal{G}\Delta(t)\mathcal{H}\xi\} &= 2\lambda^T \mathcal{G}\Delta(t)\mathcal{H}\xi \\ &\leq 2\|\mathcal{G}^T\lambda\|\|\Delta(t)\|\|\mathcal{H}\xi\| \\ &\leq 2\|\mathcal{G}^T\lambda\|\|\mathcal{H}\xi\| \end{aligned}$$

can easily be obtained. ■

Lemma 1.2 *(Schur complement) For a given constant real symmetric matrix Θ, the following items are equivalent.*

$$i).\ \Theta = \left(\begin{array}{c|c} \Theta_{11} & \Theta_{12} \\ \hline \Theta_{12}^T & \Theta_{22} \end{array}\right) > 0;$$

$$ii).\ \Theta_{11} > 0 \quad and \quad \Theta_{22} - \Theta_{12}^T \Theta_{11}^{-1} \Theta_{12} > 0;$$

$$iii).\ \Theta_{22} > 0 \quad and \quad \Theta_{11} - \Theta_{12} \Theta_{22}^{-1} \Theta_{12}^T > 0:$$

Proof : See Boyd et al. [73] for details. ■

Chapter 2

Decentralized Variable Gain Robust Controller for Large-Scale Interconnected Systems with Matched Uncertainties

In this chapter, for a class of large-scale interconnected systems with uncertainties which satisfy matching condition, a decentralized variable gain robust controller which achieves not only robust stability but also satisfactory transient behavior is proposed [76]. We show that the proposed variable gain robust controller design strategy is based on LMIs.

2.1 Problem Formulation

Let us consider the uncertain large-scale interconnected system composed of $\mathcal{N}$ subsystems represented by

$$\frac{d}{dt}x_i(t) = A_{ii}(t)x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} A_{ij}(t)x_j(t) + B_i u_i(t), \tag{2.1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ and $u_i(t) \in \mathbb{R}^{m_i}$ $(i = 1, \cdots, \mathcal{N})$ are the vectors of the state and the control input for the i-th subsystem, respectively and $x(t) = \left(x_1^T(t), \cdots, x_{\mathcal{N}}^T(t)\right)^T$ is the state of the overall system. The matrices $A_{ii}(t)$ and $A_{ij}(t)$ are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + B_i \Delta_{ii}(t) \mathcal{E}_{ii}, \\ A_{ij}(t) &= B_i \mathcal{D}_{ij} + B_i \Delta_{ij}(t) \mathcal{E}_{ij}. \end{aligned} \tag{2.2}$$

In (2.2), matrices $A_{ij}(t) \in \mathbb{R}^{n_i \times n_j}$ are coefficients for interactions among subsystems, and the matrices $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$ and $B_i \in \mathbb{R}^{n_i \times m_i}$ mean the nominal system matrix and the nominal input one. In addition, the matrices $\mathcal{D}_{ij}$, $\mathcal{E}_{ii}$ and $\mathcal{E}_{ij}$ with appropriate dimensions denote the structure of interactions or uncertainties. Moreover, matrices $\Delta_{ii}(t) \in \mathbb{R}^{m_i \times p_i}$ and $\Delta_{ij}(t) \in \mathbb{R}^{m_i \times q_{ij}}$ represent unknown parameters satisfying the relations $\|\Delta_{ii}(t)\| \leq 1.0$ and $\|\Delta_{ij}(t)\| \leq 1.0$, respectively. One can see from (2.2) that the uncertainties and the interaction terms satisfy so-called matching condition [31, 72].

Now, we introduce the following nominal subsystem which is obtained by ignoring uncertainties and interactions in (2.1);

$$\frac{d}{dt}\overline{x}_i(t) = A_{ii}\overline{x}_i(t) + B_i \overline{u}_i(t). \tag{2.3}$$

In (2.3), $\overline{x}_i(t) \in \mathbb{R}^{n_i}$ denote $\overline{u}_i(t) \in \mathbb{R}^{m_i}$ are the vectors of the state and the control input for the i-th nominal subsystem, respectively.

Firstly, the standard linear quadratic control problem is adopted for the i-th nominal subsystem of (2.3) in order to generate the desired trajectory in time response for the uncertain i-th subsystem of (2.1). Note that we can also adopt some other design methods for deriving the desirable response (e.g. pole assignment). It is well known that the optimal control input for the i-th nominal subsystem of (2.3) can be obtained as

$$\begin{aligned} \overline{u}_i(t) &= K_i \overline{x}_i(t), \\ K_i &\stackrel{\triangle}{=} -\mathcal{R}_i^{-1} B_i^T \mathcal{X}_i. \end{aligned} \tag{2.4}$$

In (2.4), $\mathcal{X}_i \in \mathbb{R}^{n_i \times n_i}$ is a symmetric positive define matrix which satisfies the algebraic Riccati equation

$$H_e\left\{A_{ii}^T \mathcal{X}_i\right\} - \mathcal{X}_i B_i \mathcal{R}_i^{-1} B_i^T \mathcal{X}_i + \mathcal{Q}_i = 0, \tag{2.5}$$

where $\mathcal{Q}_i \in \mathbb{R}^{n_i \times n_i}$ and $\mathcal{R}_i \in \mathbb{R}^{m_i \times m_i}$ are the weighting matrices, and these matrices are positive definite matrices. Note that $\mathcal{Q}_i \in \mathbb{R}^{n_i \times n_i}$ and $\mathcal{R}_i \in \mathbb{R}^{m_i \times m_i}$ are determined in advance so that the desirable transient behavior is achieved.

Let us introduce error vectors $e_i(t) \stackrel{\triangle}{=} x_i(t) - \overline{x}_i(t)$. Besides, for the i-th subsystem of (2.1), using the feedback gain matrix $K_i \in \mathbb{R}^{m_i \times n_i}$ of (2.4), we define the following control input [48];

$$u_i(t) \stackrel{\triangle}{=} K_i x_i(t) + v_i(t), \tag{2.6}$$

where $v_i(t) \in \mathbb{R}^{m_i}$ is the compensation input defined as

$$v_i(t) \stackrel{\triangle}{=} F_i e_i(t) + \mathcal{L}_i(x_i, e_i, t) e_i(t), \tag{2.7}$$

where, $F_i \in \mathbb{R}^{m_i \times n_i}$ and $\mathcal{L}_i(x_i, e_i, t) \in \mathbb{R}^{m_i \times n_i}$ denote the fixed compensation gain matrix and the variable one, respectively. From (2.1), (2.3), (2.6) and (2.7), the following uncertain error subsystem is derived;

$$\begin{aligned} \frac{d}{dt} e_i(t) = &\left(A_{K_i} + B_i F_i\right) e_i(t) + B_i \Delta_i(t) \mathcal{E}_{ii} x_i(t) + B_i \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} (\mathcal{D}_{ij} + \Delta_{ij}(t) \mathcal{E}_{ij}) x_j(t) \\ &+ B_i \mathcal{L}_i(x_i, e_i, t) e_i(t). \end{aligned} \tag{2.8}$$

In (2.8), $A_{K_i} \in \mathbb{R}^{n_i \times n_i}$ is the stable matrix described as $A_{K_i} = A_{ii} + B_i K_i$.

From the above discussion, our design objective in this chapter is to determine the decentralized variable gain controller of (2.6) such that the resultant overall system achieves not only robust stability but also satisfactory transient behavior.

2.2 Decentralized Variable Gain Robust Regulator

A sufficient condition for the existence of the proposed decentralized control system is shown as the following theorem [76];

Theorem 2.1 *Let us consider the uncertain error subsystem of (2.8) and the control input of (2.6).*

By using symmetric positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{n_i \times n_i}$ and $\mathcal{S}_i \in \mathbb{R}^{n_i \times n_i}$, matrices $\mathcal{W}_i \in \mathbb{R}^{m_i \times n_i}$ and positive constants ϵ_i which satisfy the LMIs

$$\left(\begin{array}{c|c} H_e\left\{A_{Ki}\mathcal{Y}_i + B_i \mathcal{W}_i\right\} & \Lambda_i\left(\mathcal{Y}_i\right) \\ \hline \star & -\Gamma_i\left(\epsilon_i\right) \end{array}\right) < 0, \tag{2.9}$$

$$\left(\begin{array}{c:c} H_e\left\{\mathcal{S}_i A_{K_i}\right\} & \Psi_i \\ \hdashline \star & -\Gamma_i\left(\epsilon_i\right) \end{array}\right) < 0, \tag{2.10}$$

the fixed gain matrix $F_i \in \mathbb{R}^{m_i \times n_i}$ *and the variable one* $\mathcal{L}_i(x_i, e_i, t) \in \mathbb{R}^{m_i \times n_i}$ *are determined as* $F_i = \mathcal{W}_i \mathcal{Y}_i^{-1}$ *and*

$$\mathcal{L}_i(x_i, e_i, t) \stackrel{\triangle}{=} \begin{cases} -\dfrac{\zeta_i(e_i, x_i, t) + \eta_i(e_i, t)}{\|B_i^T \mathcal{P}_i e_i(t)\|^2} B_i^T \mathcal{P}_i & (B_i^T \mathcal{P}_i e_i(t) \neq 0), \\ \mathcal{L}_i(x_i, e_i, t_\epsilon) & (B_i^T \mathcal{P}_i e_i(t) = 0). \end{cases} \tag{2.11}$$

In (2.9) – (2.11), matrices $\Lambda_i\left(\mathcal{Y}_i\right)$, Ψ_i *and* $\Gamma_i\left(\epsilon_i\right)$, *and positive scalar functions* $\zeta_i(e_i, x_i, t)$ *and* $\eta_i(e_i, t)$ *are given by*

$$\begin{aligned} \Lambda_i\left(\mathcal{Y}_i\right) \stackrel{\triangle}{=} \big(& \mathcal{Y}_i \mathcal{D}_{1i}^T \;\; \mathcal{Y}_i \mathcal{E}_{1i}^T \;\; \cdots \;\; \mathcal{Y}_i \mathcal{D}_{i-1i}^T \;\; \mathcal{Y}_i \mathcal{E}_{i-1i}^T \;\; \mathcal{Y}_i \mathcal{D}_{i+1i}^T \;\; \mathcal{Y}_i \mathcal{E}_{i+1i}^T \;\; \cdots \\ & \cdots \;\; \mathcal{Y}_i \mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i \mathcal{E}_{\mathcal{N}i}^T \big), \end{aligned} \tag{2.12}$$

$$\Psi_i \stackrel{\triangle}{=} \left(\mathcal{D}_{1i}^T \;\; \mathcal{E}_{1i}^T \;\; \cdots \;\; \mathcal{D}_{i-1\ i}^T \;\; \mathcal{E}_{i-1\ i}^T \;\; \mathcal{D}_{i+1\ i}^T \;\; \mathcal{E}_{i+1\ i}^T \;\; \cdots \;\; \mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{E}_{\mathcal{N}i}^T\right), \tag{2.13}$$

$$\begin{aligned} \Gamma_i\left(\epsilon_i\right) \stackrel{\triangle}{=} \operatorname{diag}\big(& \epsilon_1 I_{m_1},\ \epsilon_1 I_{q_{1i}},\ \cdots,\ \epsilon_{i-1} I_{m_{i-1}},\ \epsilon_{i-1} I_{q_{i-1i}},\ \epsilon_{i+1} I_{m_{i+1}}, \epsilon_i I_{q_{i+1i}},\ \cdots, \\ & \cdots,\ \epsilon_{\mathcal{N}} I_{m_{\mathcal{N}}},\ \epsilon_{\mathcal{N}} I_{q_{\mathcal{N}i}}\big), \end{aligned} \tag{2.14}$$

$$\zeta_i(e_i, x_i, t) \stackrel{\triangle}{=} \left\|B_i^T \mathcal{P}_i e_i(t)\right\| \left\|\mathcal{E}_{ii} x_i(t)\right\|, \tag{2.15}$$

$$\eta_i(e_i, x_i, t) \stackrel{\triangle}{=} 2\epsilon_i(\mathcal{N} - 1)\|B_i^T \mathcal{P}_i e_i(t)\|^2. \tag{2.16}$$

Moreover, t_ϵ *in (2.11) is given by* $t_\epsilon = \lim_{\epsilon > 0, \epsilon \to 0}(t - \epsilon)$ *[47].*

Then robust stability of the overall error system composed of the $\mathcal{N}$ *error subsystems of (2.8) is guaranteed.*

Proof : In order to prove **Theorem 2.1**, the following quadratic function is defined;

$$\mathcal{V}(e, \overline{x}, t) \stackrel{\triangle}{=} \sum_{i=1}^{\mathcal{N}} \mathcal{V}_{e_i}(e_i, t) + \sum_{i=1}^{\mathcal{N}} \mathcal{V}_{\overline{x}_i}(\overline{x}_i, t), \tag{2.17}$$

where $\mathcal{V}_{e_i}(e_i, t)$ and $\mathcal{V}_{\overline{x}_i}(\overline{x}_i, t)$ are given by

$$\mathcal{V}_{e_i}(e_i, t) \stackrel{\triangle}{=} e_i^T(t) \mathcal{P}_i e_i(t), \tag{2.18}$$

$$\mathcal{V}_{\overline{x}_i}(\overline{x}_i, t) \stackrel{\triangle}{=} \overline{x}_i^T(t) \mathcal{S}_i \overline{x}_i(t). \tag{2.19}$$

For the quadratic functions $\mathcal{V}_{e_i}(e_i, t)$ of (2.18), we have

$$\frac{d}{dt}\mathcal{V}_{e_i}(e_i, t) \leq e_i^T(t)\left[H_e\left\{(A_{Ki} + B_iF_i)^T\mathcal{P}_i\right\}\right]e_i(t) + 2\|B_i^TP_ie_i(t)\|\|\mathcal{E}_{ii}x_i(t)\| + 4\epsilon_i(\mathcal{N}-1)e_i^T(t)\mathcal{P}_iB_iB^T\mathcal{P}_ie_i(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij} + \mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)e_j(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij} + \mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)\overline{x}_j(t). \tag{2.20}$$

For derivation of (2.20), we have used **Lemma 1.1** and the well-known inequality

$$2\alpha^T\beta \leq \delta\alpha^T\alpha + \frac{1}{\delta}\beta^T\beta \tag{2.21}$$

for any vectors with appropriate dimensions and a positive scalar δ. Moreover, the following relation for the quadratic functions $\mathcal{V}_{\overline{x}_i}(\overline{x}_i, t)$ of (2.19) holds;

$$\frac{d}{dt}\mathcal{V}_{\overline{x}_i}(\overline{x}_i, t) = \overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}\right]\overline{x}_i(t). \tag{2.22}$$

Firstly, the case of $B_i^T\mathcal{P}_ie_i(t) \neq 0$ is considered. In this case, substituting the variable gain matrix of (2.11) into (2.20) and some algebraic manipulations give

$$\frac{d}{dt}\mathcal{V}_{e_i}(e_i, t) \leq e_i^T(t)\left[H_e\left\{(A_{Ki} + B_iF_i)^T\mathcal{P}_i\right\}\right]e_i(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij} + \mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)e_j(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij} + \mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)\overline{x}_j(t). \tag{2.23}$$

Thus, we can easily see that the following relation can be obtained;

$$
\begin{aligned}
\frac{d}{dt}\mathcal{V}(e,\overline{x},t) \leq & \sum_{i=1}^{\mathcal{N}} e_i^T(t)\left[H_e\left\{(A_{Ki}+B_iF_i)^T\mathcal{P}_i\right\}\right]e_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}\right]\overline{x}_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \left\{ \frac{1}{\epsilon_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)e_j(t) \right. \\
& \left. \qquad + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)\overline{x}_j(t) \right\}. \qquad (2.24)
\end{aligned}
$$

Since the inequality of (2.24) can be rewritten as

$$
\begin{aligned}
\frac{d}{dt}\mathcal{V}(e,\overline{x},t) \leq & \sum_{i=1}^{\mathcal{N}} e_i^T(t)\left[H_e\left\{(A_{Ki}+B_iF_i)^T\mathcal{P}_i\right\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\right]e_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\right]\overline{x}_i(t), \qquad (2.25)
\end{aligned}
$$

if the matrix inequality conditions

$$
H_e\left\{(A_{Ki}+B_iF_i)^T\mathcal{P}_i\right\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right) < 0, \qquad (2.26)
$$

$$
H_e\left\{A_{K_i}^T\mathcal{S}_i\right\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right) < 0 \qquad (2.27)
$$

holds, then the following inequality for the quadratic function $\mathcal{V}(e,\overline{x},t)$ is satisfied;

$$
\frac{d}{dt}\mathcal{V}(e,\overline{x},t) < 0 \quad \text{for} \quad \forall\xi(t) \neq 0, \qquad (2.28)
$$

where $\xi(t) \stackrel{\triangle}{=} \left(e_1^T(t),\cdots,e_{\mathcal{N}}^T(t),\overline{x}_1^T(t),\cdots,\overline{x}_{\mathcal{N}}^T(t)\right)^T$.

Next we consider the case of $B_i^T \mathcal{P}_i e_i(t) = 0$. In this case, from (2.20) and (2.22) the time derivative of the quadratic function $\mathcal{V}(e, \overline{x}, t)$ of (2.17) can be written as

$$\frac{d}{dt}\mathcal{V}(e, \overline{x}, t) = \sum_{i=1}^{\mathcal{N}} e_i^T(t) \left[H_e \left\{ (A_{Ki} + B_i F_i)^T \mathcal{P}_i \right\} \right] e_i(t) + \sum_{i=1}^{\mathcal{N}} \overline{x}_i^T(t) \left[H_e \left\{ A_{K_i}^T \mathcal{S}_i \right\} \right] \overline{x}_i(t). \tag{2.29}$$

Namely in the case of $B_i^T \mathcal{P}_i e_i(t) = 0$, the relation of (2.28) also holds.

From the above, the overall error system is clearly robust stable, because the nominal subsystem is asymptotically stable.

Finally, the matrix inequalities of (2.26) and (2.27) are considered. By introducing the matrices $\mathcal{Y}_i \triangleq \mathcal{P}_i^{-1}$ and $\mathcal{W}_i \triangleq F_i \mathcal{P}_i$ and pre- and post-multiplying both sides of the matrix inequality of (2.26) by $\mathcal{Y}_i$, the following matrix inequality can be obtained;

$$H_e \left\{ A_{Ki} \mathcal{Y}_i + B_i \mathcal{W}_i \right\} + \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j} \mathcal{Y}_i \left(\mathcal{D}_{ji}^T \mathcal{D}_{ji} + \mathcal{E}_{ji}^T \mathcal{E}_{ji} \right) \mathcal{Y}_i < 0. \tag{2.30}$$

Thus by applying **Lemma 1.2** (Schur complement) to (2.27) and (2.30), we find that the inequalities of (2.27) and (2.30) are equivalent to the LMIs of (2.10) and (2.9), respectively. Thus by solving the LMIs of (2.9) and (2.10), the fixed compensation gain matrix is determined as $F_i = \mathcal{W}_i \mathcal{Y}_i^{-1}$ and the variable one is given by (2.11). Hence the proof of **Theorem 2.1** is completed. ■

Remark 2.1 *The decentralized variable gain robust controller design method in this chapter can be applied to the uncertain large-scale interconnected systems with time delays (see [77]). Furthermore, although the uncertainties in the large-scale interconnected systems of (2.1) are described as structured uncertainties, in [78], the parameter structured uncertainties are considered. The proposed design method in this chapter can easily be extended to such control systems.*

Remark 2.2 *In order to derive the proposed decentralized variable gain robust controller, solutions of the LMIs of (2.9) and (2.10) are needed. In the LMIs of (2.9) and (2.10), LMI variables* $\epsilon_i \in \mathbb{R}^1$ *can arbitrarily be selected subject to* $\epsilon_i > 0$. *Therefore we find that there always exists the solutions of the LMIs of (2.9) and (2.10), i.e., the proposed decentralized robust controller can always be designed. Therefore, the proposed design method is very useful.*

2.3 Numerical Examples

In this example, the uncertain large-scale interconnected system consisting of three two-dimensional subsystems is considered, namely, $\mathcal{N} = 3$. The system parameters are given as

$$
\begin{aligned}
&A_{11} = \begin{pmatrix} -1.0 & 0.0 \\ 1.0 & 1.0 \end{pmatrix},\ A_{22} = \begin{pmatrix} 0.5 & -1.0 \\ 1.0 & -1.0 \end{pmatrix},\ A_{33} = \begin{pmatrix} -1.0 & 1.5 \\ 0.0 & 1.0 \end{pmatrix}, \\
&B_1 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ B_2 = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ B_3 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{11}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix}, \\
&\mathcal{E}_{22}^T = \begin{pmatrix} 1.0 \\ 2.0 \end{pmatrix},\ \mathcal{E}_{33}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{12}^T = \begin{pmatrix} 1.0 \\ 2.0 \end{pmatrix},\ \mathcal{D}_{13}^T = \begin{pmatrix} 2.0 \\ 2.0 \end{pmatrix}, \\
&\mathcal{D}_{21}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{23}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{31}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{32}^T = \begin{pmatrix} 0.0 \\ 2.0 \end{pmatrix}, \\
&\mathcal{E}_{12}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{13}^T = \begin{pmatrix} 2.0 \\ 2.0 \end{pmatrix},\ \mathcal{E}_{21}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{E}_{23}^T = \begin{pmatrix} 0.0 \\ 2.0 \end{pmatrix},
\end{aligned}
$$

$$
\mathcal{E}_{31}^T = \begin{pmatrix} 2.0 \\ 0.0 \end{pmatrix},\ \mathcal{E}_{32}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix}. \tag{2.31}
$$

Furthermore, we select the following initial values of the uncertain large-scale system of (2.31) and the nominal system;

$$
\begin{aligned}
x(0) &= \left(\begin{array}{cc|cc|cc} 2.0 & -1.0 & -0.5 & 1.5 & 1.0 & -2.0 \end{array} \right)^T, \\
\overline{x}(0) &= \left(\begin{array}{cc|cc|cc} 1.0 & -1.5 & -1.0 & 1.0 & 1.5 & -1.5 \end{array} \right)^T.
\end{aligned} \tag{2.32}
$$

Additionally, unknown parameters are selected as $\Delta_{ii}(t) = \cos(5\pi t)$ and $\Delta_{ij}(t) = -\sin(2\pi t)$, respectively.

In this example, for the weighting matrices $\mathcal{Q}_i \in \mathbb{R}^{2\times 2}$ and $\mathcal{R}_i \in \mathbb{R}^{1\times 1}$ $(i = 1, 2, 3)$ for the nominal subsystems, we consider the following two cases:

- **Type 1** : $\mathcal{Q}_1 = \mathrm{diag}(1.0,\ 2.0),\ \mathcal{Q}_2 = \mathrm{diag}(1.0,\ 1.0\times 10^1),\ \mathcal{Q}_3 = I_2,$
  $\mathcal{R}_1 = 1.0,\ \mathcal{R}_2 = 1.0\times 10^1,\ \mathcal{R}_3 = 1.0\times 10^1$

- **Type 2** : $\mathcal{Q}_1 = 1.0\times 10^1 I_2,\ \mathcal{Q}_2 = 1.0\times 10^1 I_2,\ \mathcal{Q}_3 = \mathrm{diag}(5.0,\ 1.0\times 10^1),$
  $\mathcal{R}_1 = 1.0\times 10^{-1},\ \mathcal{R}_2 = 1.0,\ \mathcal{R}_3 = 1.0$

Firstly, **Type 1** is considered. By solving the algebraic Riccati equation of (2.5), the symmetric positive definite matrices $\mathcal{X}_i \in \mathbb{R}^{2\times 2}$ and the optimal gain matrices $K_i \in \mathbb{R}^{1\times 2}$ of (2.33) for the i-th nominal subsystem are derived as

$$\begin{aligned}
&\mathcal{X}_1 = \begin{pmatrix} 2.6458 & 5.6458 \\ \star & 1.4937\times 10^{1} \end{pmatrix},\ \mathcal{X}_2 = \begin{pmatrix} 7.5663 & -3.4261 \\ \star & 7.5682 \end{pmatrix}, \\
&\mathcal{X}_3 = \begin{pmatrix} 4.9377 & 3.5301 \\ \star & 2.0981 \end{pmatrix} \times 10^{-1}, \\
&K_1 = \begin{pmatrix} -2.6458 & -5.6458 \end{pmatrix},\ K_2 = \begin{pmatrix} -4.1402 & -4.1421 \end{pmatrix} \times 10^{-1}, \\
&K_3 = \begin{pmatrix} -3.5301\times 10^{-2} & -2.0981 \end{pmatrix}.
\end{aligned} \tag{2.33}$$

Besides, by using **Theorem2.1**, we design the proposed decentralized variable gain robust controller. By solving LMIs of (2.9) and (2.10), we have

$$\begin{aligned}
&\mathcal{Y}_1 = \begin{pmatrix} 2.0399\times 10^{1} & -3.6077 \\ \star & 1.7958 \end{pmatrix},\ \mathcal{W}_1^T = \begin{pmatrix} -2.2706\times 10^{1} \\ -7.1943 \end{pmatrix}, \\
&\mathcal{Y}_2 = \begin{pmatrix} 8.0537 & 4.8515\times 10^{-1} \\ \star & 9.8791 \end{pmatrix},\ \mathcal{W}_2^T = \begin{pmatrix} -3.1187 \\ -1.5631 \end{pmatrix} \times 10^{1}, \\
&\mathcal{Y}_3 = \begin{pmatrix} 6.1443 & -3.1541 \\ -3.1541 & 1.1110\times 10^{1} \end{pmatrix},\ \mathcal{W}_3^T = \begin{pmatrix} -8.7819 \\ -5.1838\times 10^{1} \end{pmatrix}, \\
&\mathcal{S}_1 = \begin{pmatrix} 1.9194 & 3.5411 \\ \star & 1.6921 \end{pmatrix} \times 10^{1},\ \mathcal{S}_2 = \begin{pmatrix} 8.7701 & -5.1918 \\ \star & 8.5940 \end{pmatrix} \times 10^{1}, \\
&\mathcal{S}_3 = \begin{pmatrix} 4.7618 & 2.3655 \\ \star & 7.4323 \end{pmatrix} \times 10^{1}, \\
&\epsilon_1 = 1.7410\times 10^{1},\ \epsilon_2 = 1.2826\times 10^{1},\ \epsilon_3 = 4.2199\times 10^{1}.
\end{aligned} \tag{2.34}$$

Thus the symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{2\times 2}$ and the fixed gain matrices $F_i \in \mathbb{R}^{1\times 2}$ can be computed as

$$\begin{aligned}
&\mathcal{P}_1 = \begin{pmatrix} 7.6040\times 10^{-2} & 1.5276\times 10^{-1} \\ \star & 8.6376\times 10^{-1} \end{pmatrix}, \\
&\mathcal{P}_2 = \begin{pmatrix} 1.2453\times 10^{-1} & -6.1157\times 10^{-3} \\ \star & 1.0152\times 10^{-1} \end{pmatrix}, \\
&\mathcal{P}_3 = \begin{pmatrix} 1.9052\times 10^{-1} & 5.4085\times 10^{-2} \\ \star & 1.0536\times 10^{-1} \end{pmatrix}, \\
&F_1 = \begin{pmatrix} -5.7563 & -1.9542\times 10^{1} \end{pmatrix},\ F_2 = \begin{pmatrix} -2.5281 & 2.0481\times 10^{-1} \end{pmatrix}, \\
&F_3 = \begin{pmatrix} -2.8277 & -3.4977 \end{pmatrix}.
\end{aligned} \tag{2.35}$$

Next, **Type 2** is considered. As with **Type 1**, the symmetric positive definite matrices $\mathcal{X}_i \in \mathbb{R}^{2\times 2}$ and the optimal gain matrices $K_i \in \mathbb{R}^{1\times 2}$ of (2.36) for the i-th nominal subsystem are derived as

$$\begin{aligned}
&\mathcal{X}_1 = \begin{pmatrix} 1.1417 & 2.6595 \\ \star & 3.0364\times 10^{1} \end{pmatrix},\ \mathcal{X}_2 = \begin{pmatrix} 1.0128\times 10^{1} & -8.1942 \\ \star & 1.0511\times 10^{1} \end{pmatrix},\\
&\mathcal{X}_3 = \begin{pmatrix} 2.2373 & 7.2487\times 10^{-1} \\ \star & 4.6297 \end{pmatrix} \times 10^{-1},\\
&K_1 = \begin{pmatrix} -1.1417 & -2.6595 \end{pmatrix} \times 10^{1},\ K_2 = \begin{pmatrix} -1.9338 & -2.3166 \end{pmatrix},\\
&K_3 = \begin{pmatrix} -7.24871\times 10^{-1} & -4.6297 \end{pmatrix}.
\end{aligned} \tag{2.36}$$

By solving LMIs of (2.9) and (2.10), matrices $\mathcal{Y}_i \in \mathbb{R}^{2\times 2}$, $\mathcal{W}_i \in \mathbb{R}^{1\times 2}$ and $\mathcal{S}_i \in \mathbb{R}^{2\times 2}$, and positive scalars ϵ_i can be obtained as

$$\begin{aligned}
&\mathcal{Y}_1 = \begin{pmatrix} 9.3921 & -2.5581 \\ \star & 1.0929 \end{pmatrix},\ \mathcal{W}_1^T = \begin{pmatrix} 2.5430\times 10^{1} \\ -5.5057 \end{pmatrix},\\
&\mathcal{Y}_2 = \begin{pmatrix} 6.1941 & -1.5624 \\ \star & 7.6210 \end{pmatrix},\ \mathcal{W}_2^T = \begin{pmatrix} -9.0216 \\ 1.2692 \end{pmatrix} \times 10^{1},\\
&\mathcal{Y}_3 = \begin{pmatrix} 5.3346 & -2.0748 \\ \star & 6.1365 \end{pmatrix},\ \mathcal{W}_3^T = \begin{pmatrix} -9.3795 \\ -1.8179 \end{pmatrix},\\
&\mathcal{S}_1 = \begin{pmatrix} 1.0483 & 2.1948 \\ \star & 4.5887\times 10^{1} \end{pmatrix},\ \mathcal{S}_2 = \begin{pmatrix} 2.2637 & -1.8223 \\ \star & 2.4260 \end{pmatrix} \times 10^{1},\\
&\mathcal{S}_3 = \begin{pmatrix} 1.7343\times 10^{1} & 3.9504 \\ \star & 7.8018 \end{pmatrix},\\
&\epsilon_1 = 1.7141\times 10^{1},\ \epsilon_2 = 1.2878\times 10^{1},\ \epsilon_3 = 3.2954\times 10^{1}.
\end{aligned} \tag{2.37}$$

Consequently, we can derive

$$\begin{aligned}
&\mathcal{P}_1 = \begin{pmatrix} 2.9374\times 10^{-1} & 6.8755\times 10^{-1} \\ \star & 2.5243 \end{pmatrix},\\
&\mathcal{P}_2 = \begin{pmatrix} 1.7025\times 10^{-1} & 3.4904\times 10^{-2} \\ \star & 1.3837\times 10^{-1} \end{pmatrix},\\
&\mathcal{P}_3 = \begin{pmatrix} 2.1584\times 10^{-1} & 7.2975\times 10^{-2} \\ \star & 1.8763\times 10^{-1} \end{pmatrix},\\
&F_1 = \begin{pmatrix} 3.6844 & 3.5863 \end{pmatrix},\ F_2 = \begin{pmatrix} -1.0929 & 1.4414 \end{pmatrix},\\
&F_3 = \begin{pmatrix} -2.1571 & -1.0256 \end{pmatrix}.
\end{aligned} \tag{2.38}$$

Figures 2.1 – 2.8 show the simulation results of this numerical example. In these figures, $x_i^{(l)}(t)$ and $\overline{x}_i^{(l)}(t)$ $(l = 1, 2)$ are the l-th element of $x_i(t)$ for i-th subsystem and one of the state $\overline{x}_i(t)$ for i-th nominal subsystem, respectively. From these figures, one can find that the proposed decentralized variable gain robust controller stabilizes the uncertain large-scale systems with system parameters of (2.31) in spite of uncertainties and interactions. Moreover, the proposed decentralized variable gain robust controller achieves good transient response close to the desired transient behavior generated by the nominal subsystem. Additionally, in the result of **Type2**, one can see that the state variables of each subsystem converge faster than the result of **Type 1**. Namely, it can be confirmed that the transient behavior for each subsystem can be changed by adjusting the weighting matrices. Thus, we have shown the effectiveness of the proposed decentralized robust control system.

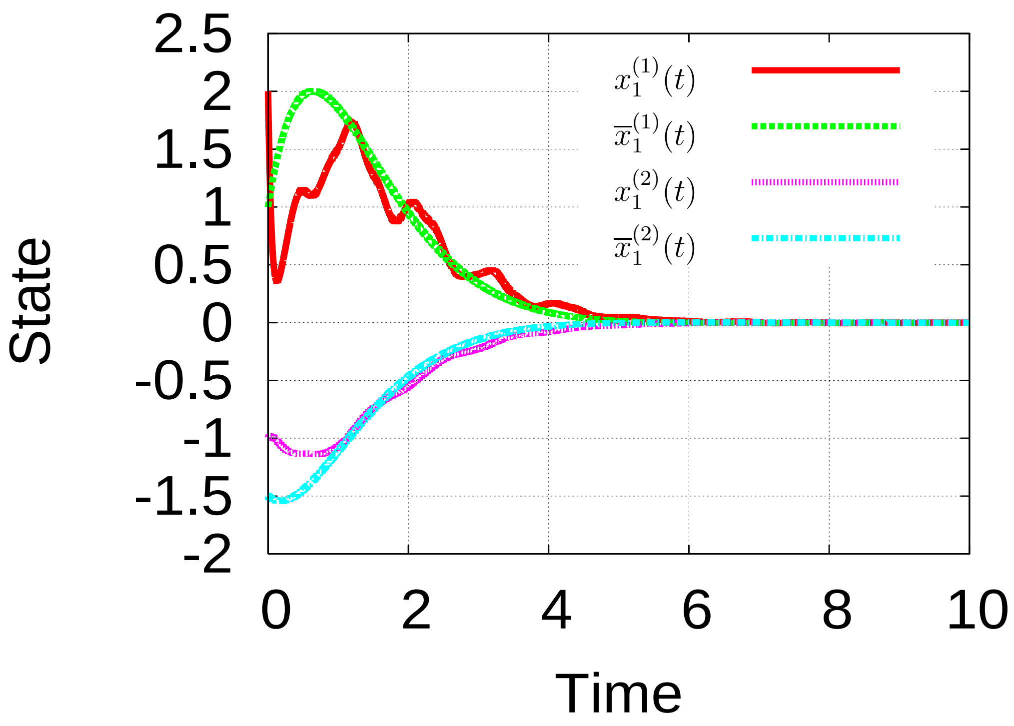

Figure 2.1: Time histories of $x_1(t)$ and $\overline{x}_1(t)$: **Type 1**

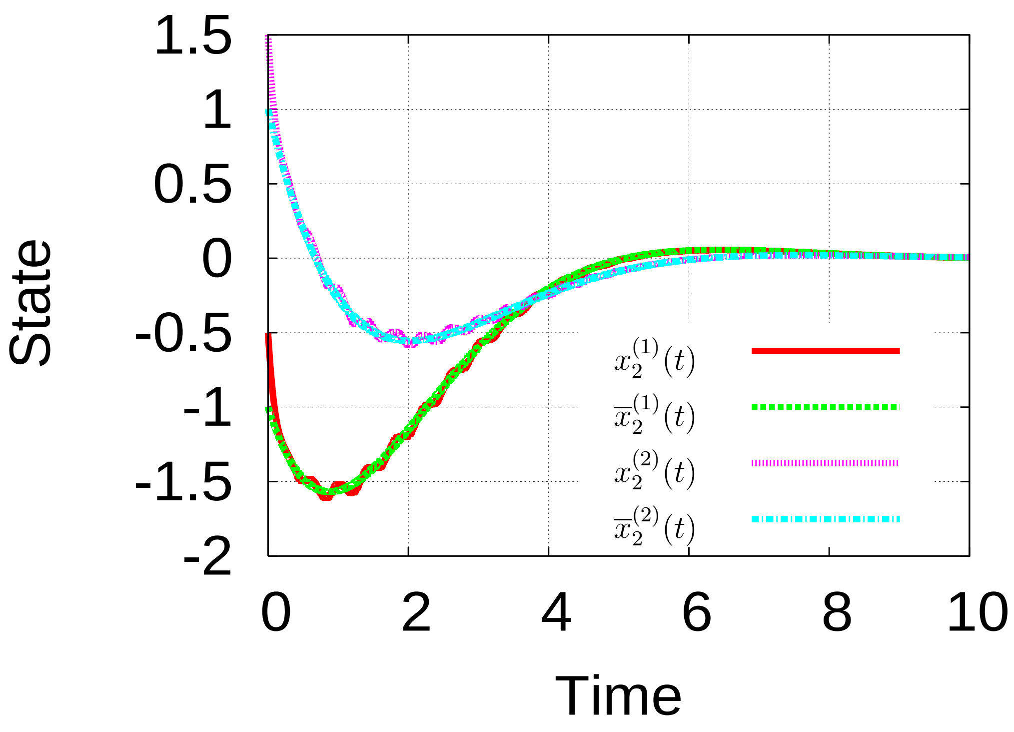

Figure 2.2: Time histories of $x_2(t)$ and $\overline{x}_2(t)$: **Type 1**

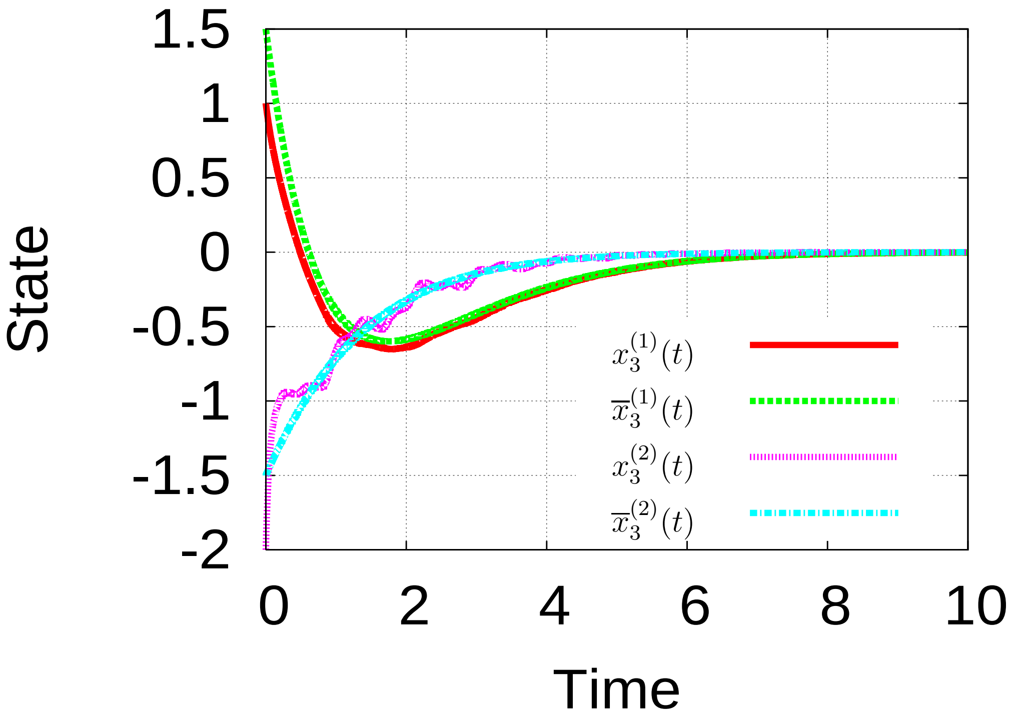

Figure 2.3: Time histories of $x_3(t)$ and $\overline{x}_3(t)$: **Type 1**

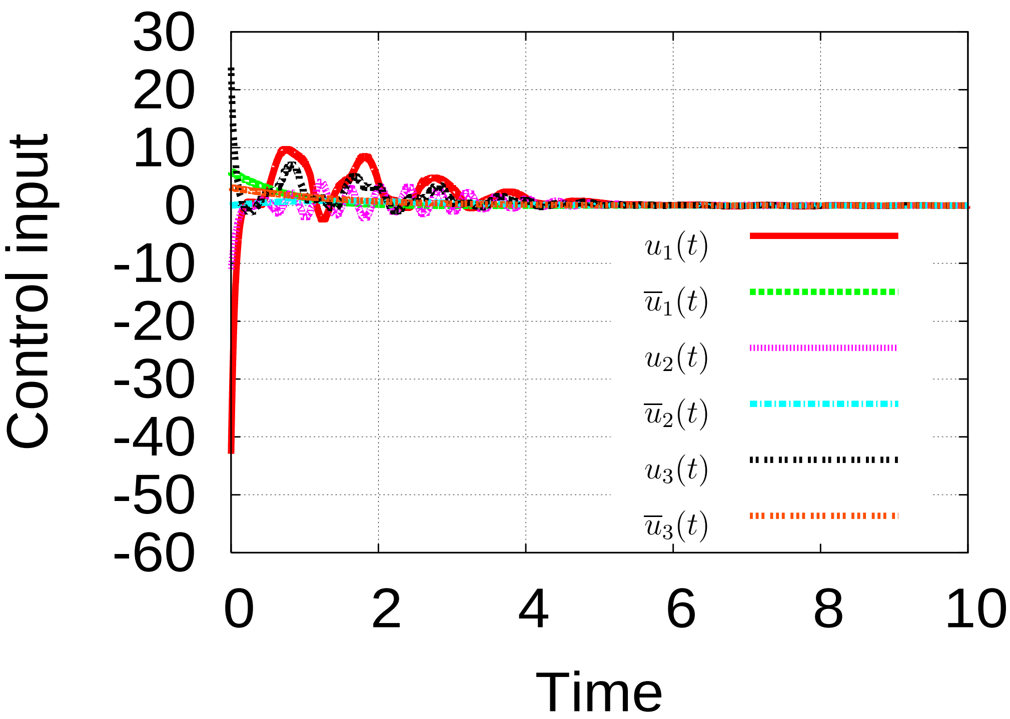

Figure 2.4: Time histories of $u(t)$ and $\overline{u}(t)$: **Type 1**

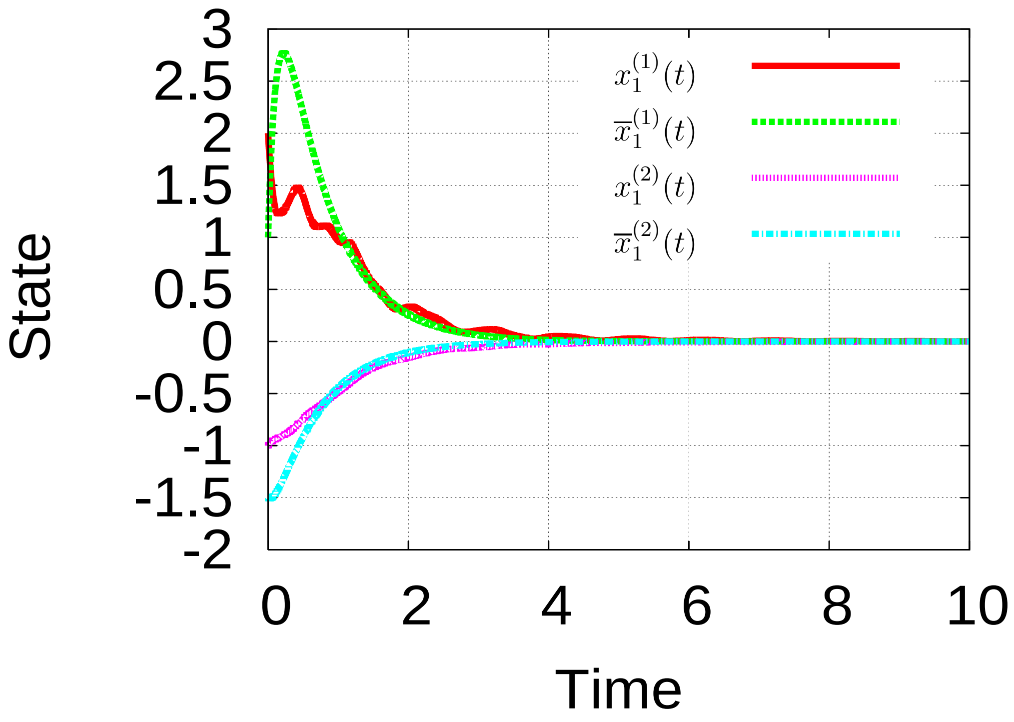

Figure 2.5: Time histories of $x_1(t)$ and $\overline{x}_1(t)$: **Type 2**

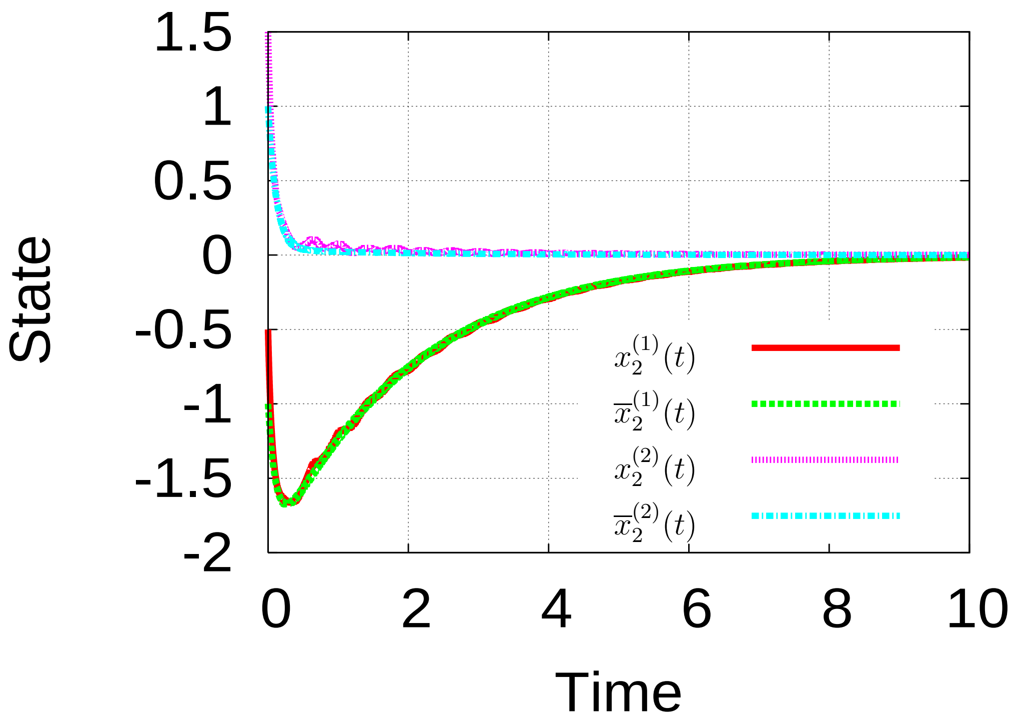

Figure 2.6: Time histories of $x_2(t)$ and $\overline{x}_2(t)$: **Type 2**

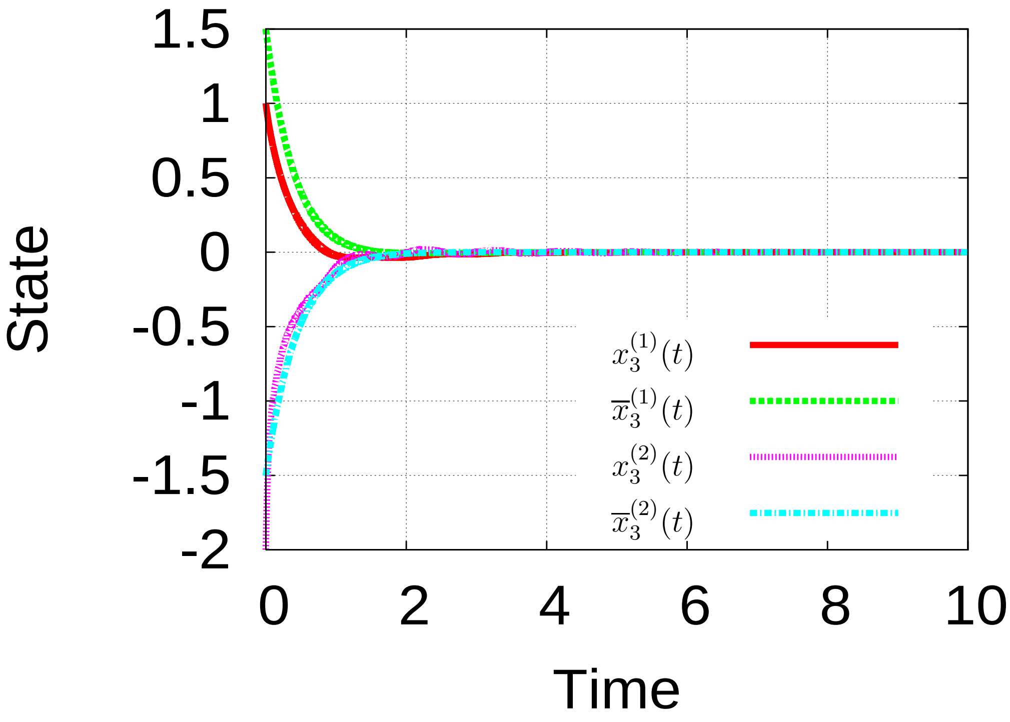

Figure 2.7: Time histories of $x_3(t)$ and $\overline{x}_3(t)$: **Type 2**

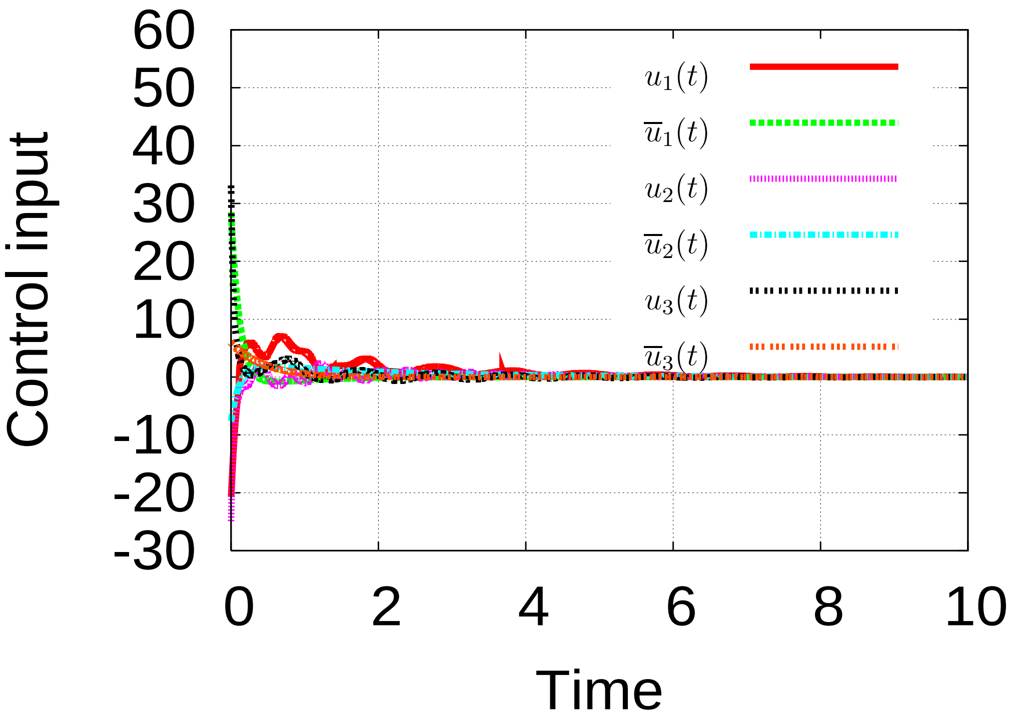

Figure 2.8: Time histories of $u(t)$ and $\overline{u}(t)$: **Type 2**

2.4 Summary

In this chapter, a decentralized variable gain robust controller for a class of large-scale interconnected systems with uncertainties and interactions which satisfy matching condition has been proposed. Furthermore, a numerical example have been illustrated to show the effectiveness of the proposed control strategies. The proposed decentralized variable gain robust controller achieves not only robust stability but also satisfactory transient behavior generated by the nominal subsystem. Moreover, the transient behavior for each subsystem can be adjusted by selecting the weighting matrices. The proposed LMI condition is always feasible, i.e., designers can derive the decentralized variable gain robust controller provided that some assumptions are satisfied. On the other hand, in the case of the conventional decentralized fixed gain robust controllers, derived LMIs may not feasible for large-scale interconnected systems with matched uncertainties. Thus, the proposed method in this chapter is very useful.

Chapter 3

Decentralized Variable Gain Robust Controllers with Guaranteed $\mathcal{L}_2$ Gain Performance for Uncertain Large-Scale Interconnected Systems

For a class of uncertain large-scale interconnected systems, an LMI-based design method of decentralized variable gain robust controller with guaranteed $\mathcal{L}_2$ gain performance is shown in this chapter [79]. Moreover, the effectiveness of the proposed controller is presented through simple numerical examples.

3.1 Problem Formulation

Consider the uncertain large-scale interconnected system composed of the following $\mathcal{N}$ subsystems;

$$\begin{aligned} \frac{d}{dt}x_i(t) &= A_{ii}(t)x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} A_{ij}(t)x_j(t) + B_i u_i(t) + \varGamma_{x_i}\omega_i(t), \\ z_i(t) &= C_{ii}x_i(t) + \varGamma_{z_i}\omega_i(t), \end{aligned} \tag{3.1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, $z_i(t) \in \mathbb{R}^{p_i}$ and $\omega_i(t) \in \mathbb{R}^{q_i}$ $(i = 1, \cdots, \mathcal{N})$ are the vectors of the state, the control input, the controlled output and the disturbance input for the i-th subsystem, respectively. The disturbance input $\omega_i(t) \in \mathbb{R}^{q_i}$ is assumed to be square integrable, that is, $\omega_i(t) \in \mathcal{L}_2[0, \infty)$. The matrices $A_{ii}(t)$ and $A_{ij}(t)$ in (3.1) are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + B_i \Delta_{ii}(t) \mathcal{E}_{ii}, \\ A_{ij}(t) &= B_i \mathcal{D}_{ij} + B_i \Delta_{ij}(t) \mathcal{E}_{ij}. \end{aligned} \tag{3.2}$$

In (3.1) and (3.2), the matrices $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $C_{ii} \in \mathbb{R}^{p_i \times n_i}$, $\mathit{\Gamma}_{x_i} \in \mathbb{R}^{n_i \times q_i}$, and $\mathit{\Gamma}_{z_i} \in \mathbb{R}^{p_i \times q_i}$ are known system parameters, and the matrices $\mathcal{D}_{ij}$, $\mathcal{E}_{ii}$ and $\mathcal{E}_{ij}$ with appropriate dimensions represent the structure of interactions or uncertainties. Moreover, the matrices $\Delta_{ii}(t) \in \mathbb{R}^{m_i \times r_i}$ and $\Delta_{ij}(t) \in \mathbb{R}^{m_i \times s_{ij}}$ are unknown time-varying parameters satisfying the relations $\|\Delta_{ii}(t)\| \le 1.0$ and $\|\Delta_{ij}(t)\| \le 1.0$, respectively, i.e., the uncertainties and the interaction terms satisfy the matching condition.

Now, we define the following control input for the i-th subsystem of (3.1);

$$\begin{aligned} u_i(t) &\stackrel{\triangle}{=} F_i x_i(t) + \psi_i(x_i, t), \\ \psi_i(x_i, t) &\stackrel{\triangle}{=} \mathcal{L}_i(x_i, t) x_i(t), \end{aligned} \tag{3.3}$$

where, $F_i \in \mathbb{R}^{m_i \times n_i}$ and $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ denote the fixed gain matrix and the compensation input for the i-th subsystem of (3.1). From (3.1), (3.2), and (3.3), we can derive the following closed-loop subsystem;

$$\begin{aligned} \frac{d}{dt} x_i(t) = &\left(A_{ii} + B_i F_i\right) x_i(t) + B_i \Delta_{ii} \mathcal{E}_{ii} x_i(t) + B_i \sum_{\substack{j=1 \\ j \ne i}}^{\mathcal{N}} \left(\mathcal{D}_{ij} + \Delta_{ij} \mathcal{E}_{ij}\right) x_j(t) \\ &+ B_i \mathcal{L}_i(x_i, t) x_i(t) + \mathit{\Gamma}_{x_i} \omega_i(t). \end{aligned} \tag{3.4}$$

Now a definition of the decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance is given as follows;

Definition 3.1 *The control input of (3.3) for the uncertain large-scale interconnected system of (3.1) is said to be a decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance $\gamma^\star > 0$ if the internal stability of the resultant closed-loop system of (3.4) is ensured, and $\mathcal{H}_\infty$-norm of the transfer function from the disturbance input $\omega(t) \stackrel{\triangle}{=} (\omega_1^T(t), \omega_2^T(t), \cdots, \omega_{\mathcal{N}}^T(t))^T$ to the controlled output $z(t) \stackrel{\triangle}{=} (z_1^T(t), z_2^T(t), \cdots, z_{\mathcal{N}}^T(t))^T$ is less than or equal to a positive constant $\gamma^\star$.*

By using symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i \times n_i}$, we consider the following quadratic function;

$$\mathcal{V}(x,t) \stackrel{\triangle}{=} \sum_{i=1}^{\mathcal{N}} \mathcal{V}_i(x_i,t), \tag{3.5}$$

where $\mathcal{V}_i(x_i,t)$ is

$$\mathcal{V}_i(x_i,t) \stackrel{\triangle}{=} x_i^T(t)\mathcal{P}_i x_i(t). \tag{3.6}$$

Moreover, we introduce the following Hamiltonian;

$$\mathcal{H}(x,t) \stackrel{\triangle}{=} \frac{d}{dt}\mathcal{V}(x,t) + \sum_{i=1}^{\mathcal{N}} \left\{ z_i^T(t)z_i(t) - (\gamma_i^*)^2 \omega_i^T(t)\omega_i(t) \right\}. \tag{3.7}$$

Then, for the uncertain large-scale interconnected system of (3.1) and the control input of (3.3), we have the following lemma for the decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance $\gamma^* > 0$;

Lemma 3.1 *Let us consider the uncertain large-scale interconnected system of (3.1) and the control input of (3.3).*

If there exist symmetric positive definite matrices $\mathcal{P}_i$ $(i = 1, \cdots, \mathcal{N})$ and positive scalars γ_i^* which satisfy the inequality

$$\mathcal{H}(x,t) < 0, \tag{3.8}$$

for the quadratic function $\mathcal{V}(x,t)$ and the signals $z(t)$ and $\omega(t)$, then the control input of (3.3) is a decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance γ^*, where γ^* is given by

$$\gamma^* = \max_i \gamma_i^* \quad (i = 1, \cdots, \mathcal{N}). \tag{3.9}$$

Proof : The following inequality can be obtained by integrating both sides of the inequality of (3.8) from 0 to ∞ with $x_i(0) = 0$;

$$\mathcal{V}(x,\infty) + \sum_{i=1}^{\mathcal{N}} \left\{ \int_0^\infty z_i^T(t)z_i(t)dt - (\gamma_i^*)^2 \int_0^\infty \omega_i^T(t)\omega_i(t)dt \right\} < 0. \tag{3.10}$$

One can see that the overall uncertain closed-loop system of (3.4) is robustly stable (internally stable) from the inequality of (3.7) and (3.8), i.e., robust stability of the overall uncertain closed-loop system with $\omega(t) = 0$ is guaranteed. Moreover, the

$\mathcal{H}_\infty$-norm of the transfer function from the disturbance input $\omega(t)$ to the controlled output $z(t)$ is less than a positive constant γ^*, because the inequality of (3.10) means the following relation;

$$\left\| z(t) \right\|_{\mathcal{L}_2} < \gamma^* \left\| \omega(t) \right\|_{\mathcal{L}_2}. \tag{3.11}$$

Thus the proof of **Lemma 3.1** is accomplished. ■

From the above discussion, in this chapter, the design objective is to design the decentralized variable gain robust controller of (3.3) such that the overall system achieves not only internal stability but also guaranteed $\mathcal{L}_2$ gain performance $\gamma^\star > 0$. That is to derive the symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i \times n_i}$, positive constants γ^*, the fixed gain matrices $F_i \in \mathbb{R}^{m_i \times n_i}$ and the compensation input $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ which satisfy the inequality of (3.8) for uncertainties $\Delta_{ii}(t) \in \mathbb{R}^{m_i \times r_i}$ and $\Delta_{ij}(t) \in \mathbb{R}^{m_i \times s_{ij}}$, and the disturbance input $\omega_i(t) \in \mathcal{L}_2[0, \infty)$.

3.2 Decentralize Variable Gain Controller with guaranteed $\mathcal{L}_2$ Gain Performance

The following theorem shows sufficient conditions for the existence of the proposed decentralized robust control system [79];

Theorem 3.1 *Let us consider the large-scale interconnected system of (3.1) and the control input of (3.3).*

By using symmetric positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{n_i \times n_i}$, the matrices $\mathcal{W}_i \in \mathbb{R}^{m_i \times n_i}$ and positive scalars ϵ_i and γ_i which satisfy the LMIs

$$\begin{pmatrix} H_e\left\{A_{ii}\mathcal{Y}_i + B_i\mathcal{W}_i\right\} & \varGamma_{x_i} + \mathcal{Y}_i C_{ii}^T \varGamma_{z_i} & \varLambda_i\left(\mathcal{Y}_i\right) \\ \star & \varGamma_{z_i}^T \varGamma_{z_i} - \gamma_i I_{q_i} & 0 \\ \star & \star & -\Omega_i\left(\epsilon_i\right) \end{pmatrix} < 0, \tag{3.12}$$

the fixed gain matrix $F_i \in \mathbb{R}^{m_i \times n_i}$ and the compensation input $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ are determined as $F_i \stackrel{\triangle}{=} \mathcal{W}_i \mathcal{Y}_i^{-1}$ and

$$\psi_i(x_i, t) \stackrel{\triangle}{=} \begin{cases} -\dfrac{\zeta_i(x_i, t) + \eta_i(x_i, t)}{\left\| B_i^T \mathcal{P}_i x_i(t) \right\|^2} B_i^T \mathcal{P}_i x_i(t) & \left(B_i^T \mathcal{P}_i x_i(t) \neq 0\right), \\ \psi_i(x_i, t_\epsilon) & \left(B_i^T \mathcal{P}_i x_i(t) = 0\right). \end{cases} \tag{3.13}$$

In (3.12) and (3.13), matrices $\mathit{\Lambda}_i(\mathcal{Y}_i)$ and $\Omega_i(\epsilon_i)$, and scalar functions $\zeta_i(x_i, t)$ and $\eta_i(x_i, t)$ are given by

$$\begin{aligned}
\mathit{\Lambda}(\mathcal{Y}_i) &\triangleq \big(\mathcal{Y}_i C_{ii}^T \;\; \mathcal{Y}_i\mathcal{D}_{1i}^T \;\; \mathcal{Y}_i\mathcal{E}_{1i}^T \;\; \mathcal{Y}_i\mathcal{D}_{2i}^T \;\; \mathcal{Y}_i\mathcal{E}_{2i}^T \;\; \cdots \\
&\qquad \cdots \;\; \mathcal{Y}_i\mathcal{D}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{D}_{i+1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i+1\ i}^T \cdots \\
&\qquad \cdots \;\; \mathcal{Y}_i\mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i\mathcal{E}_{\mathcal{N}i}^T\big), \qquad (3.14)\\
\Omega(\epsilon_i) &\triangleq \mathrm{diag}\left(I_{p_i}, \epsilon_1 I_{m_1}, \epsilon_1 I_{s_1 i}, \epsilon_2 I_{m_2}, \epsilon_2 I_{s_2 i}, \cdots\right. \\
&\qquad \cdots, \epsilon_{i-1} I_{m_{i-1}}, \epsilon_{i-1} I_{s_{i-1} i}, \epsilon_{i+1} I_{m_{i+1}}, \epsilon_{i+1} I_{s_{i+1} i}, \cdots \\
&\qquad \left.\cdots, \epsilon_{\mathcal{N}} I_{m_{\mathcal{N}}}, \epsilon_{\mathcal{N}} I_{s_{\mathcal{N}} i}\right), \qquad (3.15)\\
\zeta_i(x_i, t) &= \left\|B_i^T \mathcal{P}_i x_i(t)\right\| \left\|\mathcal{E}_{ii} x_i(t)\right\|, \qquad (3.16)\\
\eta_i(x_i, t) &= \epsilon_i(\mathcal{N} - 1)\left\|B_i^T \mathcal{P}_i x_i(t)\right\|^2. \qquad (3.17)
\end{aligned}$$

Note that t_ϵ in (3.13) is given by $t_\epsilon = \lim_{\epsilon>0, \epsilon\to 0}(t - \epsilon)$ [47].

Then the control input of (3.3) is the decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance $\gamma^* = \max_i \sqrt{\gamma_i}$.

Proof : In order to prove **Theorem 3.1**, we consider the quadratic function $\mathcal{V}(x, t)$ of (3.5), the Hamiltonian $\mathcal{H}(x, t)$ of (3.7) and the inequality of (3.8).

For the quadratic function $\mathcal{V}_i(x_i, t)$ of (3.6), its time derivative along the trajectory of the resultant closed-loop subsystem of (3.4) can be compute as

$$\begin{aligned}
\frac{d}{dt}\mathcal{V}_i(x_i, t) &= x_i^T(t)\left[H_e\left\{(A_{ii} + B_i F_i)^T \mathcal{P}_i\right\}\right] x_i + H_e\left\{x_i^T(t)\mathcal{P}_i B_i \Delta_{ii}(t)\mathcal{E}_{ii} x_i(t)\right\} \\
&\quad + H_e\left\{x_i^T(t)\mathcal{P}_i B_i \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \left(\mathcal{D}_{ij} + \Delta_{ij}\mathcal{E}_{ij}\right) x_j(t)\right\} \\
&\quad + H_e\left\{x_i^T(t)\mathcal{P}_i B_i \mathcal{L}_i(x_i, t) x_i(t)\right\} + H_e\left\{x_i^T(t)\mathcal{P}_i \mathit{\Gamma}_{x_i}\omega_i(t)\right\}. \qquad (3.18)
\end{aligned}$$

Moreover, by using **Lemma 1** and the well-known inequality

$$2\alpha^T\beta \le \delta\alpha^T\alpha + \frac{1}{\delta}\beta^T\beta, \qquad (3.19)$$

for any vectors α and β with appropriate dimensions and a positive scalar δ, we have the following relation for the function $\mathcal{V}_i(x_i, t)$;

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(x_i,t) \leq x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) + 2\left\|B_i^T\mathcal{P}_ix_i(t)\right\|\left\|\mathcal{E}_{ii}x_i(t)\right\| \\ + 2\epsilon_i(\mathcal{N}-1)x_i^T(t)\mathcal{P}_iB_iB_i^T\mathcal{P}_ix_i(t) \\ + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t) \\ + H_e\left\{x_i^T(t)\mathcal{P}_iB_i\mathcal{L}_i(x_i,t)x_i(t)\right\} + H_e\left\{x_i^T(t)\mathcal{P}_i\Gamma_{x_i}\omega_i(t)\right\}. \qquad (3.20)\end{aligned}$$

Firstly, the case of $B_i^T\mathcal{P}_ix_i(t) \neq 0$ is considered. In this case, substituting the compensation input of (3.13) into (3.20) and some algebraic manipulations derive the following inequality;

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(x_i,t) \leq x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) \\ + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t) \\ + H_e\left\{x_i^T(t)\mathcal{P}_i\Gamma_{x_i}\omega_i(t)\right\}. \qquad (3.21)\end{aligned}$$

Additionally, one can see from (3.1) that the relation

$$\begin{aligned}z_i^T(t)z_i(t) - (\gamma_i^*)^2\omega_i^T(t)\omega_i(t) = x_i^T(t)C_{ii}^TC_{ii}x_i(t) + H_e\left\{x_i^T(t)C_{ii}\Gamma_{z_i}\omega_i(t)\right\} \\ + \omega_i^T(t)\left(\Gamma_{z_i}^T\Gamma_{z_i} - \gamma_iI_{q_i}\right)\omega_i(t), \qquad (3.22)\end{aligned}$$

holds, where $(\gamma_i^*)^2 \triangleq \gamma_i$. Therefore from (3.5), (3.7), (3.21) and (3.22), we can obtain the following relation for the Hamiltonian $\mathcal{H}(x,t)$;

$$\begin{aligned}\mathcal{H}(x,t) \leq \sum_{i=1}^{\mathcal{N}}x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) \\ + \sum_{i=1}^{\mathcal{N}}\frac{1}{\epsilon_i}\left\{\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t)\right\}\end{aligned}$$

$$
+\sum_{i=1}^{\mathcal{N}} H_e\left\{x_i^T(t)\left(\mathcal{P}_i\Gamma_{x_i}+C_{ii}^T\Gamma_{z_i}\right)\omega_i(t)\right\}+\sum_{i=1}^{\mathcal{N}} x_i^T(t)C_{ii}^TC_{ii}x_i(t)
$$
$$
+\sum_{i=1}^{\mathcal{N}} \omega_i^T(t)\left(\Gamma_{z_i}^T\Gamma_{z_i}-\gamma_iI_{q_i}\right)\omega_i(t). \tag{3.23}
$$

The inequality of (3.23) can also be rewritten as

$$
\begin{aligned}
\mathcal{H}(x,t) \le\ & \sum_{i=1}^{\mathcal{N}} x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) \\
& +\sum_{i=1}^{\mathcal{N}} x_i^T(t)\left\{\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\right\}x_i(t) \\
& +\sum_{i=1}^{\mathcal{N}} H_e\left\{x_i^T(t)\mathcal{P}_i\Gamma_{x_i}\omega_i(t)\right\}+\sum_{i=1}^{\mathcal{N}} x_i^T(t)C_{ii}^TC_{ii}x_i(t) \\
& +\sum_{i=1}^{\mathcal{N}} H_e\left\{x_i^T(t)C_{ii}^T\Gamma_{z_i}\omega_i(t)\right\}+\sum_{i=1}^{\mathcal{N}} \omega_i^T(t)\left(\Gamma_{z_i}^T\Gamma_{z_i}-\gamma_iI_{q_i}\right)\omega_i(t) \\
=\ & \sum_{i=1}^{\mathcal{N}} x_i^T(t)\Bigg[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}+C_{ii}^TC_{ii} \\
& \qquad +\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\Bigg]x_i(t) \\
& +\sum_{i=1}^{\mathcal{N}} H_e\left\{x_i^T(t)\left(\mathcal{P}_i\Gamma_{x_i}+C_{ii}^T\Gamma_{z_i}\right)\omega_i(t)\right\} \\
& +\sum_{i=1}^{\mathcal{N}} \omega_i^T(t)\left(\Gamma_{z_i}^T\Gamma_{z_i}-\gamma_iI_{q_i}\right)\omega_i(t).
\end{aligned} \tag{3.24}
$$

Furthermore, some algebraic manipulations for (3.24) give the following inequality;

$$
\mathcal{H}(x,t) \le \sum_{i=1}^{\mathcal{N}} \begin{pmatrix} x_i(t) \\ \omega_i(t) \end{pmatrix}^T \Psi_i(\mathcal{P}_i,\epsilon_i,\gamma_i)\begin{pmatrix} x_i(t) \\ \omega_i(t) \end{pmatrix}, \tag{3.25}
$$

where $\Psi_i(\mathcal{P}_i, \epsilon_i, \gamma_i) \in \mathbb{R}^{(n_i+q_i)\times(n_i+q_i)}$ is given by

$$\Psi_i(\mathcal{P}_i, \epsilon_i, \gamma_i) \triangleq \left(\begin{array}{c|c} \Upsilon_i(\mathcal{P}_i, \epsilon_i) & \mathcal{P}_i\Gamma_{x_i} + C_{ii}^T\Gamma_{z_i} \\ \hline \star & \Gamma_{z_i}^T\Gamma_{z_i} - \gamma_i I_{q_i} \end{array}\right), \tag{3.26}$$

$$\begin{aligned}\Upsilon_i(\mathcal{P}_i, \epsilon_i) \triangleq\, & H_e\left\{(A_{ii} + B_iF_i)^T\mathcal{P}_i\right\} + C_{ii}^TC_{ii} \\ & + \sum_{i=1}^{\mathcal{N}}\frac{1}{\epsilon_i}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}\right).\end{aligned} \tag{3.27}$$

Hence if the matrix inequality

$$\Psi_i(\mathcal{P}_i, \epsilon_i, \gamma_i) < 0 \tag{3.28}$$

holds, then the inequality of (3.8) for the Hamiltonian is satisfied.

Next in the case of $B_i^T\mathcal{P}_ix_i(t) = 0$, one can see from (3.18) and (3.22) and the definition of the control input of (3.3) and the compensation input of (3.13) that if the matrix inequality of (3.28) holds, then the inequality of (3.8) is also satisfied.

Finally, we consider the matrix inequality of (3.28). By introducing the matrices $\mathcal{Y}_i \triangleq \mathcal{P}_i^{-1}$ and $\mathcal{W}_i \triangleq F_i\mathcal{Y}_i$ and pre- and post-multiplying both sides of the matrix inequality of (3.28) by $\mathrm{diag}(\mathcal{Y}_i, I_{q_i})$, we have the following inequality

$$\begin{aligned}\Phi_i(\mathcal{Y}_i, \mathcal{W}_i, \epsilon_i, \gamma_i) = & \left(\begin{array}{c|c} \Xi_i(\mathcal{Y}_i, \mathcal{W}_i, \epsilon_i) & \Gamma_{x_i} + \mathcal{Y}_iC_{ii}^T\Gamma_{z_i} \\ \hline \star & \Gamma_{z_i}^T\Gamma_{z_i} - \gamma_i I_{q_i} \end{array}\right) \\ & < 0,\end{aligned} \tag{3.29}$$

where $\Xi_i(\mathcal{Y}_i, \mathcal{W}_i, \epsilon_i) \in \mathbb{R}^{n_i\times n_i}$ is matrix described as

$$\begin{aligned}\Xi_i(\mathcal{Y}_i, \mathcal{W}_i, \epsilon_i) \triangleq\, & H_e\left\{A_{ii}\mathcal{Y}_i + B_i\mathcal{W}_i\right\} + \mathcal{Y}_iC_{ii}^TC_{ii}\mathcal{Y}_i \\ & + \sum_{i=1}^{\mathcal{N}}\frac{1}{\epsilon_i}\mathcal{Y}_i\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\mathcal{Y}_i.\end{aligned} \tag{3.30}$$

Thus by applying **Lemma 2** (Schur complement) to (3.29) we find that the matrix inequalities of (3.29) are equivalent to the LMIs of (3.12). In the LMIs of (3.12), scalar variables $\epsilon_i > 0$ and $\gamma_i > 0$ can arbitrarily be selected. Therefore we find that the LMIs of (3.12) are always feasible, i.e. there always exists the solution of the LMIs of (3.12). Therefore, by solving the LMIs of (3.12), the fixed gain matrix is determined as $F_i = \mathcal{W}_i\mathcal{Y}_i^{-1}$ and the compensation input is given by (3.13), and the proposed control input of (3.3) becomes a decentralized variable gain robust control with guaranteed $\mathcal{L}_2$ gain performance γ^* of (3.9). Therefore the proof of **Theorem 3.1** is accomplished. ■

Next, the conventional fixed gain decentralized robust controller for uncertain large-scale interconnected systems of (3.1) is provided. The next corollary gives an LMI-based design method of the conventional fixed gain decentralized robust controller with guaranteed $\mathcal{L}_2$ gain performance.

Collorary 3.1 *Consider the following control input instead of (3.3):*

$$u_i(t) \stackrel{\triangle}{=} K_i x_i(t), \tag{3.31}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}$ is the fixed gain matrix for the i-th subsystem of (3.1). In this case, the LMIs of (3.12) in **Theorem 3.1** *is transformed into following LMIs;*

$$\left(\begin{array}{c|c|c|c} \varTheta_i(\mathcal{S}_i, \mathcal{W}_i, \epsilon_i, \epsilon_{ij}) & \varPi_i & \varGamma_{x_i} & \varLambda_i(\mathcal{S}_i) \\ \hline \star & -I_{\sum_i^{\mathcal{N}} n_i} & 0 & 0 \\ \hline \star & \star & \varGamma_{z_i}^T \varGamma_{z_i} - \gamma_i I_{q_i} & 0 \\ \hline \star & \star & \star & \varOmega_i(\epsilon_i, \epsilon_{ij}) \end{array}\right) < 0, \tag{3.32}$$

$$\varTheta_i(\mathcal{S}_i, \mathcal{W}_i, \epsilon_i, \epsilon_{ij}) \stackrel{\triangle}{=} H_e\left\{A_{ii}\mathcal{S}_i + B_i\mathcal{W}_i\right\} + \epsilon_i B_i B_i^T + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \epsilon_{ij} B_i B_i^T, \tag{3.33}$$

$$\varPi_i \stackrel{\triangle}{=} \left(B_i\mathcal{D}_{i1} \quad B_i\mathcal{D}_{i2} \quad \cdots \quad B_i\mathcal{D}_{i\mathcal{N}}\right), \tag{3.34}$$

$$\varLambda_i(\mathcal{S}_i) \stackrel{\triangle}{=} \left(\mathcal{S}_i C_{ii}^T \quad \mathcal{S}_i \quad \mathcal{S}_i\mathcal{E}_{ii}^T \quad \mathcal{E}_{i1}^T \quad \mathcal{E}_{i2}^T \quad \cdots \quad \mathcal{E}_{i\mathcal{N}}^T\right), \tag{3.35}$$

$$\varOmega_i(\epsilon_i, \epsilon_{ij}) \stackrel{\triangle}{=} -\mathrm{diag}\left(I_{p_i}, \frac{1}{\mathcal{N}-1} I_{n_i}, \epsilon_i I_{r_i}, \epsilon_{i1} I_{s_{i1}}, \cdots, \epsilon_{i\mathcal{N}} I_{i\mathcal{N}}\right). \tag{3.36}$$

Namely, by solving the LMIs of (3.32), the fixed gain matrix is determined as $K_i = \mathcal{W}_i\mathcal{S}_i^{-1}$.

Proof : By using the similar way to the proof of **Theorem 3.1**, **Corollary 3.2** can easily be proved. ■

Remark 3.1 *In chapter 2, the nominal system is introduced so as to generate the desired trajectory of the state and the control input. Moreover, the proposed controller design method can be applied to the uncertain large-scale interconnected systems with state delays (see [82] for details). The proposed design method in this chapter can be easily extended to such control problem.*

Remark 3.2 *The decentralized robust controller synthesis proposed in this chapter is adaptable when some assumptions are satisfied. Namely, if the matching condition for uncertainties and interactions is satisfied, then the proposed decentralized variable gain robust controller is applicable, i.e. the LMIs of (3.12) are always feasible (see [77]). Additionally, the size of LMIs in the proposed design equals to* $n_i + 2q_i + \sum_{j=1, j\neq i}^{\mathcal{N}} (n_j + s_{ij})$. *On the other hand, for decentralized robust controllers with fixed gain matrices, the size of LMIs of (3.32) to be solved is* $2n_i + \sum_{j=1, j\neq i}^{\mathcal{N}} n_j + 2p_i + q_i + r_i + \sum_{j=1, j\neq i}^{\mathcal{N}} s_{ij}$. *Furthermore, the number of variables for LMIs of (3.12) is less than that of the decentralized robust controllers with fixed gain matrices. Therefore, one can see that the proposed decentralized robust controller design method in this chapter is very useful.*

Remark 3.3 *The proposed decentralized variable gain robust controller can be obtained by solving LMIs of (3.12). Since LMIs of (3.12) define convex solution sets of* $(\mathcal{Y}_i, \mathcal{W}_i, \epsilon_i, \gamma_i)$, *and thus various efficient convex optimization algorithms can be applied to test whether these LMIs are solvable and to generate particular solutions [80, 81]. In addition, these solutions parametrize the set of decentralized variable gain robust controllers with the* $\mathcal{L}_2$ *gain performance. Namely, one can see that the result in* **Theorem 3.1** *can easily be extended to the decentralized variable gain robust controller with suboptimal* $\mathcal{L}_2$ *gain performance (see* **Corollary3.2***).*

Collorary 3.2 *Since the LMIs of (3.12) define a convex solution set, we consider minimizing the parameter* γ_i, *because our interest is in establishing* $\mathcal{L}_2$ *gain performance. Furthermore in the LMIs of (3.12),* γ_i *has no correlation with* γ_j $(j \neq i)$. *Thus our design problem can be reduced to the following constrained convex optimization problem (see [80, 81]);*

$$\underset{\mathcal{Y}_i>0,\ \mathcal{W}_i,\ \epsilon_i>0,\ \gamma_i>0}{\text{Minimize}} \quad [\gamma_i] \quad \text{subject to (3.12).} \tag{3.37}$$

If the optimal solution $\mathcal{Y}_i > 0$, $\mathcal{W}_i$, $\epsilon_i > 0$ *and* $\gamma_i > 0$ *of the constrained optimization problem of (3.37) is obtained, then the control input of (3.3) with the fixed gain matrix* $F_i = \mathcal{W}_i\mathcal{Y}^{-1}$ *and the compensation input* $\psi_i(x_i, t)$ *of (3.13) is the decentralized variable gain robust control with suboptimal* $\mathcal{L}_2$ *gain performance* γ^* *of (3.9).*

3.3 Numerical Examples

In this example, the uncertain large-scale interconnected systems consisting of three two-dimensional subsystems is considered, i.e. $\mathcal{N} = 3$. The system parameters are given as

$$\begin{aligned}
&A_{11} = \begin{pmatrix} -1.5 & 1.0 \\ 1.0 & 0.5 \end{pmatrix},\ A_{22} = \begin{pmatrix} 1.0 & -1.0 \\ 1.0 & -1.5 \end{pmatrix},\ A_{33} = \begin{pmatrix} 0.5 & 1.0 \\ 1.5 & -1.0 \end{pmatrix},\\
&B_1 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ B_2 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ B_3 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{11}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\\
&\mathcal{E}_{22}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{33}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{12}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{13}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\\
&\mathcal{D}_{21}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{23}^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{31}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{32}^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\\
&\mathcal{E}_{12}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{13}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{E}_{21}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{E}_{23}^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\\
&\mathcal{E}_{31}^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{32}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix}.\ \varGamma_{x_1} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ \varGamma_{x_2} = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\\
&\varGamma_{x_3} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ C_{11}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ C_{22}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\ C_{33}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\\
&\varGamma_{z_1} = 1.0, \quad \varGamma_{z_2} = 1.0, \quad \varGamma_{z_3} = 1.0.
\end{aligned} \tag{3.38}$$

Firstly, we design the proposed decentralized variable gain robust controller on the basis of **Theorem 3.1**. By solving LMIs of (3.12), we have positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{2\times 2}$, matrices $\mathcal{W}_i \in \mathbb{R}^{1\times 2}$, and positive scalars ϵ_i and γ_i given by

$$\begin{aligned}
&\mathcal{Y}_1 = \begin{pmatrix} 1.0187 & -4.3846 \times 10^{-1} \\ \star & 3.9896 \times 10^{-1} \end{pmatrix},\ \mathcal{W}_1^T = \begin{pmatrix} -2.4004 \\ -1.8211 \end{pmatrix},\\
&\mathcal{Y}_2 = \begin{pmatrix} 1.4782 & -1.0391 \\ \star & 1.8897 \end{pmatrix},\ \mathcal{W}_2^T = \begin{pmatrix} -5.7365 \\ -3.4728 \times 10^{-1} \end{pmatrix},\\
&\mathcal{Y}_3 = \begin{pmatrix} 8.7797 \times 10^{-1} & -9.7835 \times 10^{-1} \\ \star & 1.7125 \end{pmatrix},\ \mathcal{W}_3^T = \begin{pmatrix} -2.9187 \\ -2.3220 \times 10^{-1} \end{pmatrix},\\
&\epsilon_1 = 6.1366, \quad \epsilon_2 = 6.9400, \quad \epsilon_3 = 6.1119,\\
&\gamma_1 = 3.0826, \quad \gamma_2 = 3.0208, \quad \gamma_3 = 3.0840.
\end{aligned} \tag{3.39}$$

Thus the symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{2\times 2}$ and the the fixed gain matrices $F_i \in \mathbb{R}^{1\times 2}$ can be computed as

$$
\begin{aligned}
\mathcal{P}_1 &= \begin{pmatrix} 1.8628 & 2.0472 \\ \star & 4.7564 \end{pmatrix}, \ \mathcal{P}_2 = \begin{pmatrix} 1.1027 & 6.0629 \times 10^{-1} \\ \star & 8.6254 \times 10^{-1} \end{pmatrix}, \\
\mathcal{P}_3 &= \begin{pmatrix} 3.1346 & 1.7908 \\ \star & 1.6071 \end{pmatrix}, \\
F_1 &= \begin{pmatrix} -8.1997 & -1.3576 \times 10^{1} \end{pmatrix}, \ F_2 = \begin{pmatrix} -6.5360 & -3.7775 \end{pmatrix}, \\
F_3 &= \begin{pmatrix} -1.0786 \times 10^{1} & -7.2254 \end{pmatrix}.
\end{aligned} \tag{3.40}
$$

Additionally, the positive scalars $\gamma_i^* = \sqrt{\gamma_i}$ can be obtained as

$$\gamma_1^* = 1.7557, \quad \gamma_2^* = 1.7380, \quad \gamma_3^* = 1.7561. \tag{3.41}$$

Therefore, the guaranteed $\mathcal{L}_2$ gain performance γ^* of (3.9) for the proposed controller is given by

$$\gamma^* = 1.7561. \tag{3.42}$$

In this example, the initial value of the uncertain large-scale system with system parameters of (3.38) is selected as follow;

$$x(0) = \left(\begin{array}{cc:cc:cc} 1.0 & -1.0 & -0.5 & 1.0 & 1.0 & -2.0 \end{array} \right)^T. \tag{3.43}$$

Furthermore, unknown parameters and disturbance inputs are given as

$$
\begin{aligned}
\Delta_{ii}(t) &= \cos(5\pi t), \\
\Delta_{ij}(t) &= -\sin(2\pi t), \\
\omega_i(t) &= 2.0 \exp(-t) \cos(5\pi t).
\end{aligned} \tag{3.44}
$$

Note that disturbance inputs $\omega_i \in \mathbb{R}^1$ $(i = 1, 2, 3)$ tend to 0 as t tends to infinity.

The simulation result of this numerical example is shown in Figures 3.1 – 3.4. In these figures, $x_i^{(l)}(t)$ denotes the l-th element of the state $x_i(t)$ for the i-th subsystem, respectively. From these figures, one can see that the proposed decentralized variable gain controller achieves internal stability for the uncertain large-scale systems with system parameters of (3.38) in spite of uncertainties and interactions. Therefore, the effectiveness of the proposed decentralized robust control system has been shown.

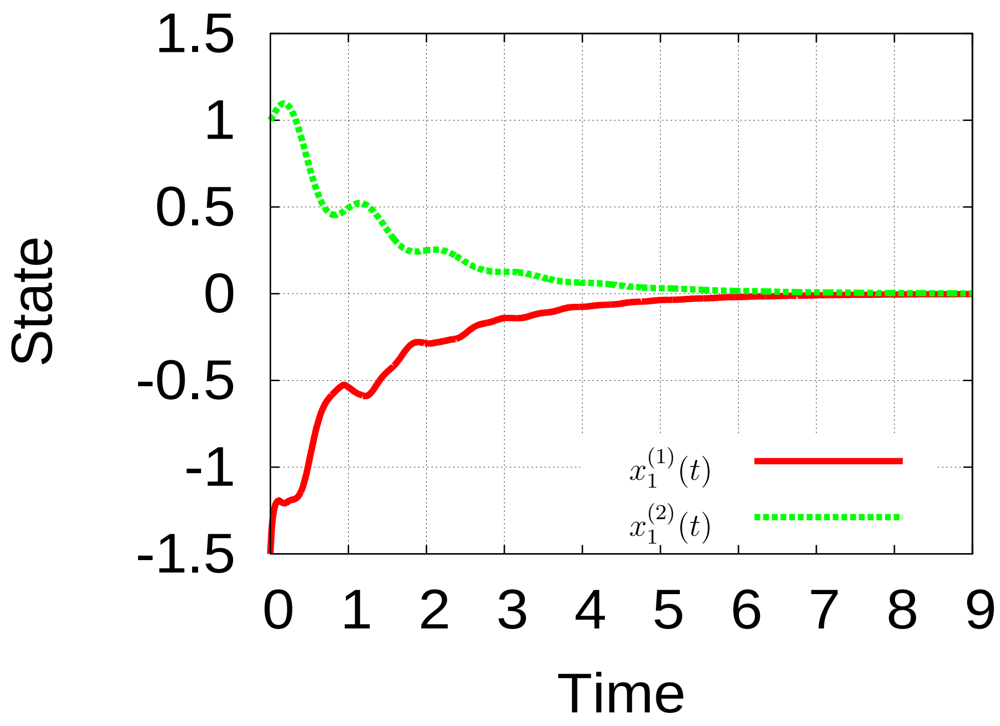

Figure 3.1: Time histories of $x_1(t)$

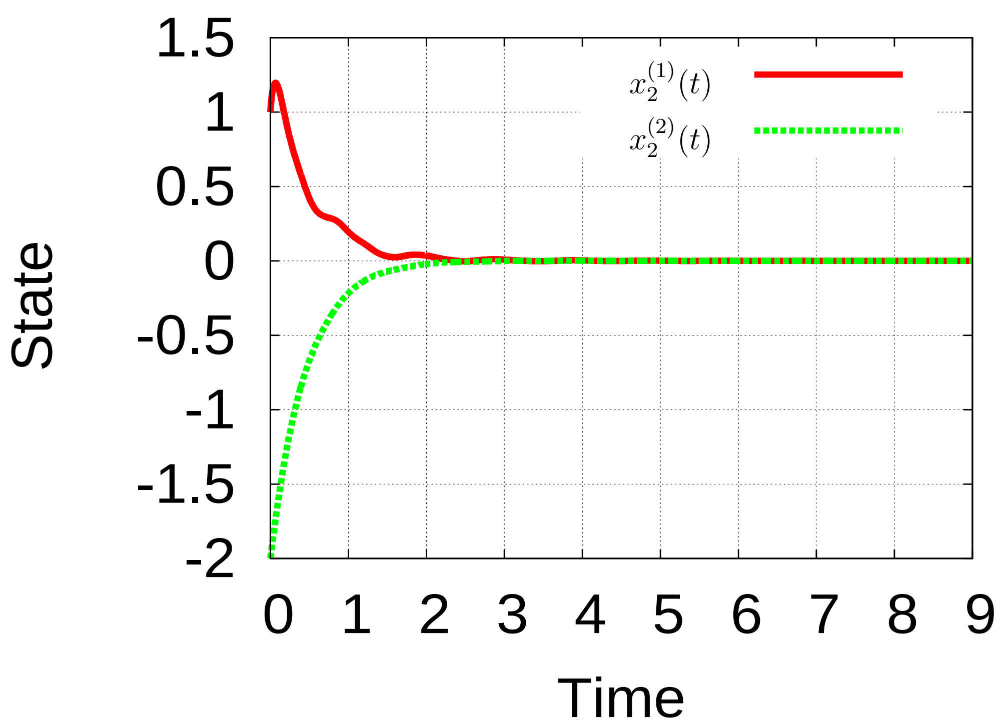

Figure 3.2: Time histories of $x_2(t)$

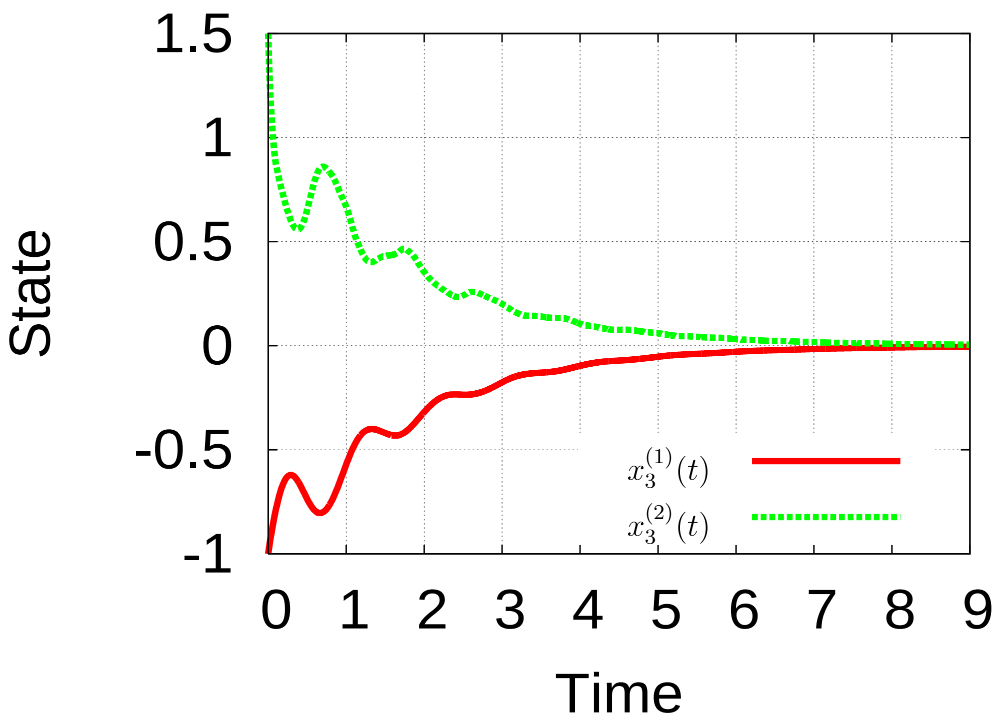

Figure 3.3: Time histories of $x_3(t)$

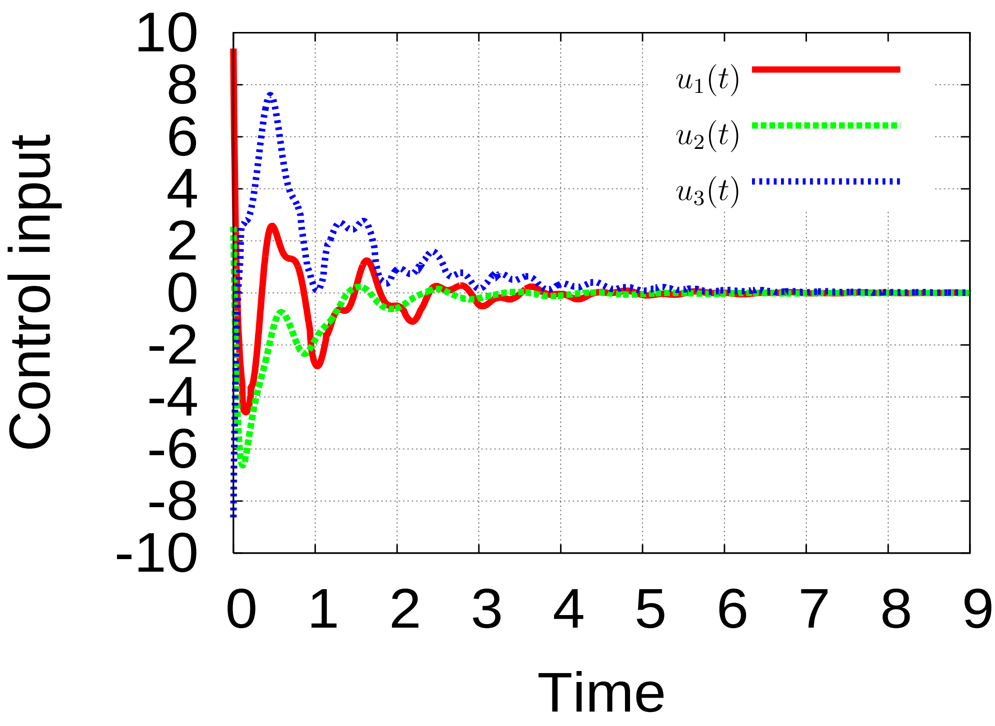

Figure 3.4: Time histories of $u(t)$

3.4 Summary

In this chapter, based on the result of chapter 2, we have presented an LMI-based design method of a decentralized variable gain robust controller with $\mathcal{L}_2$ gain performance for a class of uncertain large-scale interconnected systems. As with chapter 2, uncertainties and interactions which are included in the large-scale interconnected system satisfy matching condition. The proposed decentralized robust controller achieves not only internal stability but also $\mathcal{L}_2$ gain performance. Furthermore, the derived LMIs are always feasible, and the size and the number of variables of resultant LMIs are smaller than that of the conventional decentralized fixed gain robust controller. In addition, the proposed decentralized variable gain robust controller can easily be extended to one with suboptimal $\mathcal{L}_2$ gain performance by applying a convex constraint optimization problem. One can easily see that the result in this chapter is an extension of the result of chapter 2. Thus, the effectiveness of the proposed decentralized variable gain robust controller is presented.

Chapter 4

Decentralized Variable Gain Robust Controller for Large-Scale Interconnected Systems with Mismatched Uncertainties

In this chapter, a decentralized variable gain robust controller for a class of large-scale interconnected systems with mismatched uncertainties is shown [83]. The decentralized variable gain robust controller is natural extension of the result derived in chapter 2, and thus the controller design problem is reduced to the solvability of LMIs.

4.1 Problem Formulation

We consider the uncertain large-scale interconnected system composed of $\mathcal{N}$ subsystems as

$$\frac{d}{dt}x_i(t) = A_{ii}(t)x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} A_{ij}(t)x_j(t) + B_i u_i(t), \tag{4.1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ and $u_i(t) \in \mathbb{R}^{m_i}$ $(i = 1, \cdots, \mathcal{N})$ are the vectors of the state and the control input for the i-th subsystem, respectively. In (4.1), the matrices

$A_{ii}(t) \in \mathbb{R}^{n_i \times n_i}$ and $A_{ij}(t) \in \mathbb{R}^{n_i \times n_j}$ are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + B_i \Delta_{ii}(t) \mathcal{E}_{ii} + B_i^{\perp} \Delta_{ii}^{\perp}(t) \mathcal{E}_{ii}^{\perp}, \\ A_{ij}(t) &= A_{ij} + B_i \mathcal{D}_{ij} + B_i \Delta_{ij}(t) \mathcal{E}_{ij} + B_{ij}^{\perp} \Delta_{ij}^{\perp}(t) \mathcal{E}_{ij}^{\perp}. \end{aligned} \tag{4.2}$$

In (4.1) and (4.2), the matrices $A_{ii} \in \mathbb{R}^{n_i \times n_i}$ and $B_i \in \mathbb{R}^{n_i \times m_i}$ are known system parameters and the matrices $\Delta_{ii}(t) \in \mathbb{R}^{m_i \times r_i}$, $\Delta_{ij}(t) \in \mathbb{R}^{m_i \times s_{ij}}$, $\Delta_{ii}^{\perp}(t) \in \mathbb{R}^{p_{ii} \times q_{ii}}$ and $\Delta_{ij}^{\perp}(t) \in \mathbb{R}^{p_{ij} \times q_{ij}}$ are unknown time-varying parameters which satisfy $\|\Delta_{ii}(t)\| \leq 1.0$, $\|\Delta_{ij}(t)\| \leq 1.0$, $\|\Delta_{ii}^{\perp}(t)\| \leq 1.0$ and $\|\Delta_{ij}^{\perp}(t)\| \leq 1.0$, respectively. Moreover, the matrices $\mathcal{D}_{ij}$, $\mathcal{E}_{ii}$ and $\mathcal{E}_{ij}$ with appropriate dimensions represent the structure of matched interactions or uncertainties and the matrices A_{ij}, $B_i^{\perp}$, $B_{ij}^{\perp}$, $\mathcal{E}_{ii}^{\perp}$ and $\mathcal{E}_{ij}^{\perp}$ denote the structure of mismatched ones. Namely, the uncertainties and interactions in the large-scale interconnected system under consideration are treated separately divided into the matched part and the mismatched one.

Now, the control input is defined as

$$\begin{aligned} u_i(t) &\stackrel{\triangle}{=} F_i x_i(t) + \psi_i(x_i, t), \\ \psi_i(x_i, t) &\stackrel{\triangle}{=} \mathcal{L}_i(x_i, t) x_i(t). \end{aligned} \tag{4.3}$$

In (4.3), $F_i \in \mathbb{R}^{m_i \times n_i}$ and $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ are a fixed gain matrix and a compensation input for the i-th subsystem of (4.1). Note that $\mathcal{L}_i(x_i, t) \in \mathbb{R}^{m_i \times n_i}$ is a variable gain matrix for the i-th subsystem. From (4.1) – (4.3), the following closed-loop subsystem can be obtained;

$$\begin{aligned} \frac{d}{dt} x_i(t) = &\left(A_{ii} + B_i F_i\right) x_i(t) + \left(B_i \Delta_{ii}(t) \mathcal{E}_{ii} + B_i^{\perp} \Delta_{ii}^{\perp}(t) \mathcal{E}_{ii}^{\perp}\right) x_i(t) \\ &+ B_i \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \left(\mathcal{D}_{ij} + \Delta_{ij}(t) \mathcal{E}_{ij}\right) x_j(t) + \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \left(A_{ij} + B_{ij}^{\perp} \Delta_{ij}^{\perp}(t) \mathcal{E}_{ij}^{\perp}\right) x_j(t) \\ &+ B_i \mathcal{L}_i(x_i, t) x_i(t). \end{aligned} \tag{4.4}$$

From the above discussion, the design problem in this chapter is to derive the decentralized variable gain robust control input of (4.3) such that the overall closed-loop system with mismatched uncertainties achieves robust stability. More specifically, the fixed gain matrices $F_i \in \mathbb{R}^{m_i \times n_i}$ and the compensation input $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ are designed such that asymptotical stability of the overall closed-loop system composed of $\mathcal{N}$ subsystems of (4.4) is guaranteed.

4.2 Decentralize Variable Gain Robust Controller

In this section, we show an LMI-based design strategy for the decentralized variable gain robust controller for the overall closed-loop system with mismatched uncertainties composed of $\mathcal{N}$ subsystems represented by (4.4). A sufficient condition for the existence of the proposed decentralized variable gain robust control system is summarized as follows;

Theorem 4.1 *Consider the large-scale interconnected system of (4.1) and the control input of (4.3).*

If there exists the solution of LMIs

$$\left(\begin{array}{c:c} \Theta_i(\mathcal{Y}_i, \mathcal{W}_i, \sigma_i, \delta_{ij}) & \Lambda_i(\mathcal{Y}_i) \\ \hdashline \star & -\Omega_i(\sigma_i, \epsilon_i, \delta_{ji}) \end{array}\right) < 0, \tag{4.5}$$

then by using symmetric positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{n_i \times n_i}$, matrices $\mathcal{W}_i \in \mathbb{R}^{m_i \times n_i}$ and positive scalars σ_i, ϵ_i and δ_{ij} which satisfy the LMIs of (4.5), the fixed gain matrix $F_i \in \mathbb{R}^{m_i \times n_i}$ and the compensation input $\psi_i(x_i, t) \in \mathbb{R}^{m_i}$ are determined as $F_i = \mathcal{W}_i \mathcal{Y}_i^{-1}$ and

$$\psi_i(x_i, t) \triangleq \begin{cases} -\dfrac{\zeta_i(x_i, t) + \eta_i(x_i, t)}{\left\|B_i^T \mathcal{P}_i x_i(t)\right\|^2} B_i^T \mathcal{P}_i x_i(t) & \left(B_i^T \mathcal{P}_i x_i(t) \neq 0\right), \\ \psi_i(x_i, t_\epsilon) & \left(B_i^T \mathcal{P}_i x_i(t) = 0\right), \end{cases} \tag{4.6}$$

where t_ϵ in (4.6) is given by $t_\epsilon = \lim_{\epsilon > 0, \epsilon \to 0}(t - \epsilon)$ [47]. In (4.5) and (4.6), matrices $\Theta_i(\mathcal{Y}_i, \mathcal{W}_i, \sigma_i, \delta_{ij})$, $\Lambda_i(\mathcal{Y}_i)$ and $\Omega_i(\sigma_i, \epsilon_i, \delta_{ji})$, and scalar functions $\zeta_i(x_i, t)$ and $\eta_i(x_i, t)$

are given by

$$\begin{aligned}\Theta_i(\mathcal{Y}_i, \mathcal{W}_i, \sigma_i, \delta_{ij}) &\triangleq H_e\{A_{ii}\mathcal{Y}_i + B_i\mathcal{W}_i\} + \sigma_i B_i^{\perp}\left(B_i^{\perp}\right)^T + \sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \delta_{ij}A_{ij}A_{ij}^T \\ &+ \sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \delta_{ij}B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T, \end{aligned} \tag{4.7}$$

$$\begin{aligned}\Lambda_i\left(\mathcal{Y}_i\right) \triangleq \Big(&\mathcal{Y}_i\left(\mathcal{E}_{ii}^{\perp}\right)^T \;\; \mathcal{Y}_i\mathcal{D}_{1i}^T \;\; \mathcal{Y}_i\mathcal{E}_{1i}^T \;\; \mathcal{Y}_i\mathcal{D}_{2i}^T \;\; \mathcal{Y}_i\mathcal{E}_{2i}^T \;\; \cdots \\ &\cdots \;\; \mathcal{Y}_i\mathcal{D}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{D}_{i+1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i+1\ i}^T \cdots \\ &\cdots \;\; \mathcal{Y}_i\mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i\mathcal{E}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i \;\; \cdots \;\; \mathcal{Y}_i \;\; \mathcal{Y}_i\left(\mathcal{E}_{1i}^{\perp}\right)^T \;\; \cdots \\ &\cdots \;\; \mathcal{Y}_i\left(\mathcal{E}_{i-1\ i}^{\perp}\right)^T \;\; \mathcal{Y}_i\left(\mathcal{E}_{i+1\ i}^{\perp}\right)^T \cdots \;\; \mathcal{Y}_i\left(\mathcal{E}_{\mathcal{N}i}^{\perp}\right)^T\Big), \end{aligned} \tag{4.8}$$

$$\begin{aligned}\Omega_i(\sigma_i, \epsilon_i, \delta_{ij}) \triangleq \mathrm{diag}\,(&\sigma_i I_{q_{ii}}, \epsilon_1 I_{m_1}, \epsilon_1 I_{s_1 i}, \epsilon_2 I_{m_2}, \epsilon_2 I_{s_2 i}, \cdots \\ &\cdots, \epsilon_{i-1}I_{m_{i-1}}, \epsilon_{i-1}I_{s_{i-1}i}, \epsilon_{i+1}I_{m_{i+1}}, \epsilon_{i+1}I_{s_{i+1}i}, \cdots \\ &\cdots, \epsilon_{\mathcal{N}}I_{m_{\mathcal{N}}}, \epsilon_{\mathcal{N}}I_{s_{\mathcal{N}}i}, \delta_{1i}I_{n_i}, \cdots, \delta_{i-1i}I_{n_i}, \delta_{i+1i}I_{n_i}, \cdots \\ &\cdots, \delta_{\mathcal{N}i}I_{n_i}, \delta_{1i}I_{q_{1i}}, \cdots, \delta_{i-1i}I_{q_{i-1i}}, \delta_{i+1i}I_{q_{i+1i}}, \cdots, \delta_{\mathcal{N}i}I_{q_{\mathcal{N}i}}), \end{aligned} \tag{4.9}$$

$$\zeta_i(x_i, t) \triangleq \|B_i^T\mathcal{P}_i x_i(t)\|\|\mathcal{E}_{ii}x_i(t)\|, \tag{4.10}$$

$$\eta_i(x_i, t) \triangleq \epsilon_i(\mathcal{N} - 1)\|B_i^T\mathcal{P}_i x_i(t)\|^2. \tag{4.11}$$

Then the control law described by (4.3) is the decentralized variable gain robust controller which stabilizes the overall system.

Proof : Using symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i\times n_i}$, we define the Lyapunov function candidate as

$$\mathcal{V}(x, t) \triangleq \sum_{i=1}^{\mathcal{N}} \mathcal{V}_i(x_i, t), \tag{4.12}$$

where $\mathcal{V}_i(x_i, t)$ is the following quadratic function;

$$\mathcal{V}_i(x_i, t) \triangleq x_i^T(t)\mathcal{P}_i x_i(t). \tag{4.13}$$

The time derivative of $\mathcal{V}_i(x_i,t)$ along the trajectory of the closed-loop subsystem of (4.4) is given by

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(x_i,t) = {} & x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) + 2x_i^T(t)\mathcal{P}_iB_i\Delta_{ii}(t)\mathcal{E}_{ii}x_i(t)\\ & + 2x_i^T(t)\mathcal{P}_iB_i^{\perp}\Delta_{ii}^{\perp}(t)\mathcal{E}_{ii}^{\perp}x_i(t) + 2x_i^T(t)\mathcal{P}_iB_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\left(\mathcal{D}_{ij}+\Delta_{ij}(t)\mathcal{E}_{ij}\right)x_j(t)\\ & + 2x_i^T(t)\mathcal{P}_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\left(A_{ij}+B_{ij}^{\perp}\Delta_{ij}^{\perp}(t)\mathcal{E}_{ij}^{\perp}\right)x_j(t) + 2x_i^T(t)\mathcal{P}_iB_i\mathcal{L}_i(x_i,t)x_i(t).\end{aligned} \tag{4.14}$$

Additionally, by applying **Lemma 1** and the well-known inequality

$$2\alpha^T\beta \le \delta\alpha^T\alpha + \frac{1}{\delta}\beta^T\beta. \tag{4.15}$$

for any vectors α and β with appropriate dimensions and a positive scalar δ, the following relation for the quadratic function $\mathcal{V}_i(x_i,t)$ can be obtained;

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(x_i,t) \le {} & x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) + 2\left\|B_i^T\mathcal{P}_ix_i(t)\right\|\left\|\mathcal{E}_{ii}x_i(t)\right\|\\ & + \sigma_ix_i^T(t)\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_ix_i(t) + \frac{1}{\sigma_i}x_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}x_i(t)\\ & + 2\epsilon_i\left(\mathcal{N}-1\right)x_i^T(t)\mathcal{P}_iB_iB_i^T\mathcal{P}_ix_i(t)\\ & + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t)\\ & + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_ix_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)x_j(t)\\ & + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iB_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_ix_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}x_j(t)\\ & + 2x_i^T(t)\mathcal{P}_iB_i\mathcal{L}_i(x_i,t)x_i(t).\end{aligned} \tag{4.16}$$

Firstly, the case of $B_i^T\mathcal{P}_ix_i(t)\neq 0$ is taken into account. In this case, by substituting the variable gain matrix of (4.6) into (4.16) and some algebraic manipulations, we can derive the following inequality;

$$
\begin{aligned}
\frac{d}{dt}\mathcal{V}_i(x_i,t) \leq\ & x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) + \sigma_i x_i^T(t)\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_ix_i(t) \\
& + \frac{1}{\sigma_i}x_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}x_i(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t) \\
& + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_ix_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)x_j(t) \\
& + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iB_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_ix_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}x_j(t).
\end{aligned}
\tag{4.17}
$$

Therefore from (4.12) and (4.17), we obtain the following inequality for the quadratic function $\mathcal{V}(x,t)$;

$$
\begin{aligned}
\frac{d}{dt}\mathcal{V}(x,t) \leq & \sum_{i=1}^{\mathcal{N}}x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}}\sigma_i x_i^T(t)\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_ix_i(t) + \sum_{i=1}^{\mathcal{N}}\frac{1}{\sigma_i}x_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}}\frac{1}{\epsilon_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}x_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij}\right)x_j(t) \\
& + \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_ix_i(t) + \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)x_j(t) \\
& + \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}x_i^T(t)\mathcal{P}_iB_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_ix_i(t) \\
& + \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ij}}x_j^T(t)\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}x_j(t).
\end{aligned}
\tag{4.18}
$$

The inequality of (4.18) can be rewritten as

$$\begin{aligned}
\frac{d}{dt}\mathcal{V}(x,t) \leq\ & \sum_{i=1}^{\mathcal{N}} x_i^T(t)\left[H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\}\right]x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \sigma_i x_i^T(t)\mathcal{P}_i B_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i x_i(t) + \sum_{i=1}^{\mathcal{N}} \frac{1}{\sigma_i} x_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j} x_i^T(t)\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij} x_i^T(t)\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i x_i(t) + \sum_{i=1}^{\mathcal{N}} \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_{ji}} x_i^T(t)x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij} x_i^T(t)\mathcal{P}_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i x_i(t) \\
& + \sum_{i=1}^{\mathcal{N}} \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_{ji}} x_i^T(t)\left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp} x_i(t) \\
=\ & \sum_{i=1}^{\mathcal{N}} x_i^T(t)\Phi_i(\mathcal{P}_i,\sigma_i,\epsilon_i,\delta_{ij})x_i(t),
\end{aligned} \tag{4.19}$$

where $\Phi_i(\mathcal{P}_i,\sigma_i,\epsilon_i,\delta_{ij}) \in \mathbb{R}^{n_i\times n_i}$ is given by

$$\begin{aligned}
\Phi_i(\mathcal{P}_i,\sigma_i,\epsilon_i,\delta_{ij}) \triangleq\ & H_e\left\{(A_{ii}+B_iF_i)^T\mathcal{P}_i\right\} + \sigma_i\mathcal{P}_i B_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i + \frac{1}{\sigma_i}\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} \\
& + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij}\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i \\
& + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_{ji}} I_{n_i} + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij}\mathcal{P}_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_{ji}}\left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}.
\end{aligned} \tag{4.20}$$

Therefore, if the matrix inequality

$$\Phi_i(\mathcal{P}_i,\sigma_i,\epsilon_i,\delta_{ij}) < 0 \tag{4.21}$$

holds, then the following inequality for the time derivative of $\mathcal{V}(x,t)$ of (4.12) is satisfied;

$$\frac{d}{dt}\mathcal{V}(x,t) < 0, \quad \forall x(t) \neq 0. \tag{4.22}$$

Secondly, for the case of $B_i^T \mathcal{P}_i x_i(t) = 0$, one can see from (4.14) and the definition of the fixed gain and variable one of (4.6) that if the matrix inequality of (4.21) holds, then the inequality of (4.22) is also satisfied.

Finally, the matrix inequality of (4.21) is analyzed. By introducing the complementary matrices $\mathcal{Y}_i \triangleq \mathcal{P}_i^{-1}$ and $\mathcal{W}_i \triangleq F_i \mathcal{Y}_i$, and pre- and post-multiplying both sides of the matrix inequality of (4.21) by $\mathcal{Y}_i \in \mathbb{R}^{n_i \times n_i}$, it can be obtained that

$$\begin{aligned}
&H_e\{A_{ii}\mathcal{Y}_i + B_i\mathcal{W}_i\} + \sigma_i B_i^{\perp}\left(B_i^{\perp}\right)^T + \frac{1}{\sigma_i}\mathcal{Y}_i\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}\mathcal{Y}_i \\
&+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\epsilon_j}\mathcal{Y}_i\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)\mathcal{Y}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}A_{ij}A_{ij}^T + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ji}}\mathcal{Y}_i\mathcal{Y}_i \\
&+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\delta_{ij}B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}\frac{1}{\delta_{ji}}\mathcal{Y}_i\left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\mathcal{Y}_i < 0. \qquad (i = 1,\cdots,\mathcal{N})
\end{aligned} \tag{4.23}$$

Thus, from the inequality of (4.23) and **Lemma 1.2** (Schur complement), one can find that the matrix inequalities of (4.23) are equivalent to the LMIs of (4.5). Therefore, by solving the LMIs of (4.5), the fixed gain matrix and variable one are given by $F_i = \mathcal{W}_i\mathcal{Y}_i^{-1}$ and (4.6), respectively, and the proposed control input of (4.3) becomes a decentralized variable gain robust stabilizing control. Thus the proof of **Theorem 4.1** is completed. ■

Theorem 4.1 represents the proposed decentralized variable gain robust control strategy. Next we show the conventional fixed gain decentralized robust controller for large-scale interconnected systems with mismatched uncertainties of (4.1). The next corollary gives an LMI-based design method of the conventional fixed gain decentralized robust controller.

Collorary 4.1 *Consider the control input*

$$u_i(t) \triangleq K_i x_i(t), \tag{4.24}$$

instead of (4.3), where $K_i \in \mathbb{R}^{m_i \times n_i}$ is the fixed gain matrix for the i-th subsystem of (4.1). In this case, the LMIs of (4.5) in **Theorem 4.1** *is transformed into following*

LMIs;

$$\left(\begin{array}{c|c} \Upsilon_i(\mathcal{Y}_i, \mathcal{W}_i, \mu_i, \sigma_i, \epsilon_i, \delta_{ij}) & \Pi_i(\mathcal{Y}_i) \\ \hline \star & -\mathcal{Q}_i(\mu_i, \sigma_i, \epsilon_i, \delta_{ij}) \end{array}\right) < 0, \tag{4.25}$$

$$\begin{aligned} \Upsilon_i(\mathcal{Y}_i, \mathcal{W}_i, \mu_i, \sigma_i, \epsilon_i, \delta_{ij}) \triangleq\, & H_e \left\{ A_{ii}\mathcal{Y}_i + B_i\mathcal{W}_i \right\} + \mu_i B_i B_i^T + \sigma_i B_i^{\perp} \left(B_i^{\perp}\right)^T \\ & + 2\epsilon_i \left(\mathcal{N} - 1\right) B_i B_i^T + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij} A_{ij} A_{ij}^T + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \delta_{ij} B_{ij}^{\perp} \left(B_{ij}^{\perp}\right)^T , \end{aligned} \tag{4.26}$$

$$\begin{aligned} \Pi_i(\mathcal{Y}_i) \triangleq \Big(& \mathcal{Y}_i\mathcal{E}_{ii}^T \;\; \mathcal{Y}_i \left(\mathcal{E}_{ii}^{\perp}\right)^T \;\; \mathcal{Y}_i\mathcal{D}_{1i}^T \;\; \mathcal{Y}_i\mathcal{E}_{1i}^T \;\; \cdots \\ & \cdots \;\; \mathcal{Y}_i\mathcal{D}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i-1\ i}^T \;\; \mathcal{Y}_i\mathcal{D}_{i+1\ i}^T \;\; \mathcal{Y}_i\mathcal{E}_{i+1\ i}^T \cdots \\ & \cdots \;\; \mathcal{Y}_i\mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i\mathcal{E}_{\mathcal{N}i}^T \;\; \mathcal{Y}_i \;\; \cdots \;\; \mathcal{Y}_i \;\; \mathcal{Y}_i \left(\mathcal{E}_{1i}^{\perp}\right)^T \;\; \cdots \\ & \cdots \;\; \mathcal{Y}_i \left(\mathcal{E}_{i-1\ i}^{\perp}\right)^T \;\; \mathcal{Y}_i \left(\mathcal{E}_{i+1\ i}^{\perp}\right)^T \;\; \cdots \;\; \mathcal{Y}_i \left(\mathcal{E}_{\mathcal{N}i}^{\perp}\right)^T \Big), \end{aligned} \tag{4.27}$$

$$\begin{aligned} \mathcal{Q}_i(\mu_i, \sigma_i, \epsilon_i, \delta_{ij}) \triangleq \mathrm{diag}\, (& \mu_i I_{r_i}, \sigma_i I_{q_{ii}}, \epsilon_1 I_{m_1}, \epsilon_1 I_{s_1 i}, \cdots \\ & \cdots, \epsilon_{i-1} I_{m_{i-1}}, \epsilon_{i-1} I_{s_{i-1} i}, \epsilon_{i+1} I_{m_{i+1}}, \epsilon_{i+1} I_{s_{i+1} i}, \cdots \\ & \cdots, \epsilon_{\mathcal{N}} I_{m_{\mathcal{N}}}, \epsilon_{\mathcal{N}} I_{s_{\mathcal{N}} i}, \delta_{1i} I_{n_i}, \cdots, \delta_{i-1i} I_{n_i}, \delta_{i+1i} I_{n_i}, \cdots \\ & \cdots, \delta_{\mathcal{N}i} I_{n_i}, \delta_{1i} I_{q_{1i}}, \cdots, \delta_{i-1i} I_{q_{i-1i}}, \delta_{i+1i} I_{q_{i+1i}}, \cdots \\ & \cdots, \delta_{\mathcal{N}i} I_{q_{\mathcal{N}i}}). \end{aligned} \tag{4.28}$$

By solving the LMIs of (4.25), the fixed gain matrix is determined as $K_i = \mathcal{W}_i\mathcal{Y}_i^{-1}$.

Proof : By adopting the similar way to the proof of **Theorem 4.1**, **Corollary 4.1** can easily be proved. ■

Remark 4.1 *In this chapter, the uncertain large-scale interconnected system composed of* $\mathcal{N}$ *subsystems of (4.1) is considered. Moreover, the uncertainties and the interactions included in the controlled system are represented as (4.2), i.e., these terms consist of the matched part and the mismatched one. Note that this assumption holds without loss of generality, because it is well-known that the general structured uncertain terms can be dealt with the matched part and the mismatched one (e.g.* **Remark 5** *in [84]). By adopting such a dividing way, we can derive less con-*

servative LMI conditions for the proposed decentralized controller comparing with the existing result (e.g. [68])

Remark 4.2 *In this paper, the stabilization problem of the large-scale interconnected system with mismatched uncertainties is mainly concerned. On the other hand in chapter 2, the nominal subsystem is introduced to generate the desired trajectory of the state and the control input. Furthermore, although the uncertainty included in the controlled system of (4.1) is described as structured uncertainties, in the work of [78], the parameter structured uncertainties are considered. Note that the proposed design strategy can easily be extended to such control problems.*

Remark 4.3 *In the design method of the conventional fixed gain controller in* **Corollary 4.1**, *the size of LMIs to be solved equals to*

$$\mathcal{Z}_c = \mathcal{N} n_i + r_i + q_{ii} + \sum_{j=1, j\neq i}^{\mathcal{N}} (m_{ji} + s_{ji} + q_{ji}). \tag{4.29}$$

However, the size of LMIs in the proposed design method is

$$\mathcal{Z}_p = \mathcal{N} n_i + q_{ii} + \sum_{j=1, j\neq i}^{\mathcal{N}} (m_{ji} + s_{ji} + q_{ji}), \tag{4.30}$$

i.e. $\mathcal{Z}_c - \mathcal{Z}_p = r_i$. *Moreover, the number of variables in the LMIs of (4.5) is less than that of the conventional decentralized fixed gain robust controller. In consequence, the feasible region of the LMIs of (4.5) is large comparing with (4.25). Therefore, we find that the proposed decentralized robust controller design method is very useful and less conservative comparing with the conventional decentralized robust control.*

Remark 4.4 *In chapter 2, mismatched uncertainties have not been considered. Note that by eliminating some parameters corresponding to mismatched term in (4.5), the LMIs of (4.5) derived in this chapter can be reduced to the LMIs of (2.9). Therefore, one can see that the proposed design method for in this chapter is the natural extension of the result in chapter 2. Moreover, one can easily see from (4.6) and (4.17) that the effects of matched uncertainties and interactions can be reduced by the proposed variable gain controller. Therefore the size of LMIs becomes small compared with the case that uncertainties dealt with the mismatched part only as uncertainties, i.e., the proposed LMI condition is less conservative than the case that uncertainties are not divided into matched and mismatched* *parts. Furthermore, the compensation input $\psi_i(x_i,t)$ is bounded because the relation $\|\psi_i(x_i,t)\| = \left\|\mathcal{E}_{ii}x_i(t)\right\| + \epsilon_i\,(\mathcal{N}-1)\left\|B_i^T\mathcal{P}_i x_i(t)\right\|$ is satisfied. Additionally, in the case that there exist the mismatched uncertainties only, the compensation input $\psi_i(x_i,t)$ defined as (4.6) becomes $\psi_i(x_i,t) = \epsilon_i(\mathcal{N}-1)B_i^T\mathcal{P}_i x_i(t)$ because of $\mathcal{E}_{ii} = 0$.*

4.3 Numerical Examples

In this section, two numerical examples are run in order to demonstrate the efficiency of proposed decentralized variable gain robust controller.

4.3.1 Example 1

In the simulation study, the uncertain large-scale interconnected systems consisting of three two-dimensional subsystems is involved, i.e., $\mathcal{N} = 3$. The system parameters are supposed to

$$
\begin{aligned}
&A_{11} = \begin{pmatrix} -1.0 & 1.5 \\ 1.0 & 1.0 \end{pmatrix},\ A_{22} = \begin{pmatrix} 1.5 & -1.0 \\ 0.0 & -2.0 \end{pmatrix},\ A_{33} = \begin{pmatrix} -2.0 & 2.0 \\ 1.5 & 0.5 \end{pmatrix},\\
&B_1 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ B_2 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ B_3 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{11} = \begin{pmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{22} = \begin{pmatrix} 0.0 & 2.0 \\ 2.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{33} = \begin{pmatrix} 3.0 & 0.0 \\ 1.0 & 0.0 \end{pmatrix},\ B_1^{\perp} = \begin{pmatrix} 5.0\times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\\
&B_2^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 3.0\times 10^{-1} & 0.0 \end{pmatrix},\ B_3^{\perp} = \begin{pmatrix} 0.0 & 5.0\times 10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{11}^{\perp} = \begin{pmatrix} 4.0\times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{22}^{\perp} = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{33}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 5.0\times 10^{-1} & 0.0 \end{pmatrix},\\
&A_{12} = \begin{pmatrix} 3.0\times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ A_{13} = \begin{pmatrix} 0.0 & 3.0\times 10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\\
&A_{21} = \begin{pmatrix} 0.0 & 0.0 \\ 5.0\times 10^{-1} & 0.0 \end{pmatrix},\ A_{23} = \begin{pmatrix} 0.0 & 0.0 \\ 5.0\times 10^{-1} & 0.0 \end{pmatrix},\\
&A_{31} = \begin{pmatrix} 0.0 & 4.0\times 10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\ A_{32} = \begin{pmatrix} 5.0\times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},
\end{aligned}
$$

$$\mathcal{D}_{12}^T = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{13}^T = \begin{pmatrix} 2.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{21}^T = \begin{pmatrix} 1.0 \\ 3.0 \end{pmatrix},\ \mathcal{D}_{23}^T = \begin{pmatrix} 1.0 \\ 2.0 \end{pmatrix},$$
$$\mathcal{D}_{31}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{32}^T = \begin{pmatrix} 0.0 \\ 3.0 \end{pmatrix},\ \mathcal{E}_{12} = \begin{pmatrix} 1.0 & 1.0 \\ 0.0 & 3.0 \end{pmatrix},\ \mathcal{E}_{13} = \begin{pmatrix} 1.0 & 0.0 \\ 2.0 & 0.0 \end{pmatrix},$$
$$\mathcal{E}_{21} = \begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 2.0 \end{pmatrix},\ \mathcal{E}_{23} = \begin{pmatrix} 0.0 & 2.0 \\ 1.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{31} = \begin{pmatrix} 1.0 & 0.0 \\ 3.0 & 1.0 \end{pmatrix},$$
$$\mathcal{E}_{32} = \begin{pmatrix} 2.0 & 0.0 \\ 3.0 & 0.0 \end{pmatrix},\ B_{12}^{\perp} = \begin{pmatrix} 5.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ B_{13}^{\perp} = \begin{pmatrix} 3.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},$$
$$B_{21}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 5.0 \times 10^{-1} & 0.0 \end{pmatrix},\ B_{23}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 4.0 \times 10^{-1} & 0.0 \end{pmatrix},$$
$$B_{31}^{\perp} = \begin{pmatrix} 4.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ B_{32}^{\perp} = \begin{pmatrix} 5.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},$$
$$\mathcal{E}_{12}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 5.0 \times 10^{-1} & 0.0 \end{pmatrix},\ \mathcal{E}_{13}^{\perp} = \begin{pmatrix} 0.0 & 5.0 \times 10^{-1} \\ 0.0 & 0.0 \end{pmatrix},$$
$$\mathcal{E}_{21}^{\perp} = \begin{pmatrix} 3.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{23}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 5.0 \times 10^{-1} \end{pmatrix},$$
$$\mathcal{E}_{31}^{\perp} = \begin{pmatrix} 5.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{32}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 4.0 \times 10^{-1} \end{pmatrix}. \tag{4.31}$$

Firstly by solving LMIs of (4.5), we can obtain

$$\mathcal{Y}_1 = \begin{pmatrix} 8.9953 & -1.0453 \times 10^1 \\ \star & 2.3056 \times 10^1 \end{pmatrix},\ \mathcal{Y}_2 = \begin{pmatrix} 1.3156 \times 10^1 & -9.9780 \times 10^{-1} \\ \star & 1.2110 \times 10^1 \end{pmatrix},$$
$$\mathcal{Y}_3 = \begin{pmatrix} 1.7559 \times 10^1 & -1.2504 \times 10^1 \\ \star & 2.1249 \times 10^1 \end{pmatrix},$$
$$\mathcal{W}_1 = \begin{pmatrix} -1.4434 & -9.4126 \times 10^1 \end{pmatrix},\ \mathcal{W}_2 = \begin{pmatrix} -7.7177 & 1.3525 \end{pmatrix} \times 10^1,$$
$$\mathcal{W}_3 = \begin{pmatrix} -2.8256 & -6.9872 \end{pmatrix} \times 10^1,$$
$$\sigma_1 = 1.9919 \times 10^1,\ \sigma_2 = 3.0988 \times 10^1,\ \sigma_3 = 4.4852 \times 10^1,$$
$$\epsilon_1 = 1.8034 \times 10^2,\ \epsilon_2 = 1.4918 \times 10^2,\ \epsilon_3 = 1.6632 \times 10^2,$$
$$\delta_{12} = 3.2237 \times 10^1,\ \delta_{13} = 4.2751 \times 10^1,\ \delta_{21} = 2.1469 \times 10^1,$$
$$\delta_{23} = 2.2465 \times 10^1,\ \delta_{31} = 6.2652 \times 10^1,\ \delta_{32} = 5.3206 \times 10^1. \tag{4.32}$$

Therefore, symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{1\times 2}$ and fixed gain matrices

$F_i \in \mathbb{R}^{1\times 2}$ can be computed as

$$\begin{aligned}
\mathcal{P}_1 &= \begin{pmatrix} 2.3494\times 10^{-1} & 1.0651\times 10^{-1} \\ \star & 9.1662\times 10^{-2} \end{pmatrix}, \\
\mathcal{P}_2 &= \begin{pmatrix} 7.6490\times 10^{-2} & 6.3025\times 10^{-3} \\ \star & 8.3097\times 10^{-2} \end{pmatrix}, \\
\mathcal{P}_3 &= \begin{pmatrix} 9.8037 & 5.7692 \\ \star & 8.1012 \end{pmatrix} \times 10^{-2}, \\
F_1 &= \begin{pmatrix} -1.0365\times 10^{1} & -8.7815 \end{pmatrix}, \\
F_2 &= \begin{pmatrix} -5.8180 & 6.3744\times 10^{-1} \end{pmatrix}, \\
F_3 &= \begin{pmatrix} -6.8012 & -7.2907 \end{pmatrix}.
\end{aligned} \tag{4.33}$$

Now the conventional design method for the decentralized robust control with fixed gains in the work of [68] is applied to the uncertain large-scale system of (4.31). In the case of the conventional decentralized fixed gain robust controller, the LMIs of (4.25) cannot be solved, namely, the conventional fixed gain robust controller of (4.24) cannot be designed.

In this example, initial value of large-scale interconnected system of (4.31) is selected as $x(0) = \left(\begin{array}{cc:cc:cc} -1.5 & 1.0 & 1.0 & -2.0 & 1.5 & -1.0 \end{array} \right)^T$. Furthermore, unknown parameters are given as

$$\begin{aligned}
\Delta_{ii}(t) &= \begin{pmatrix} \cos(2.0\pi t) & 0 \end{pmatrix}, \\
\Delta_{ij}(t) &= \begin{pmatrix} 0 & \cos(-\pi t) \end{pmatrix}, \\
\Delta_{ii}^{\perp}(t) &= \mathrm{diag}\begin{pmatrix} \sin(-6.0\pi t), & \cos(-6.0\pi t) \end{pmatrix}, \\
\Delta_{ij}^{\perp}(t) &= \mathrm{diag}\begin{pmatrix} -\cos(\pi t), & \sin(\pi t) \end{pmatrix}.
\end{aligned} \tag{4.34}$$

Figures 4.1 – 4.4 show the simulation result of this example. In these figures, $x_i^{(l)}(t)$ denotes the l-th element of the state $x_i(t)$ for the i-th subsystem. From these figures, the proposed decentralized variable gain robust controller stabilizes the uncertain large-scale interconnected systems with system parameters of (4.31). Therefore, the effectiveness of the proposed design method of decentralized variable gain robust controller is shown.

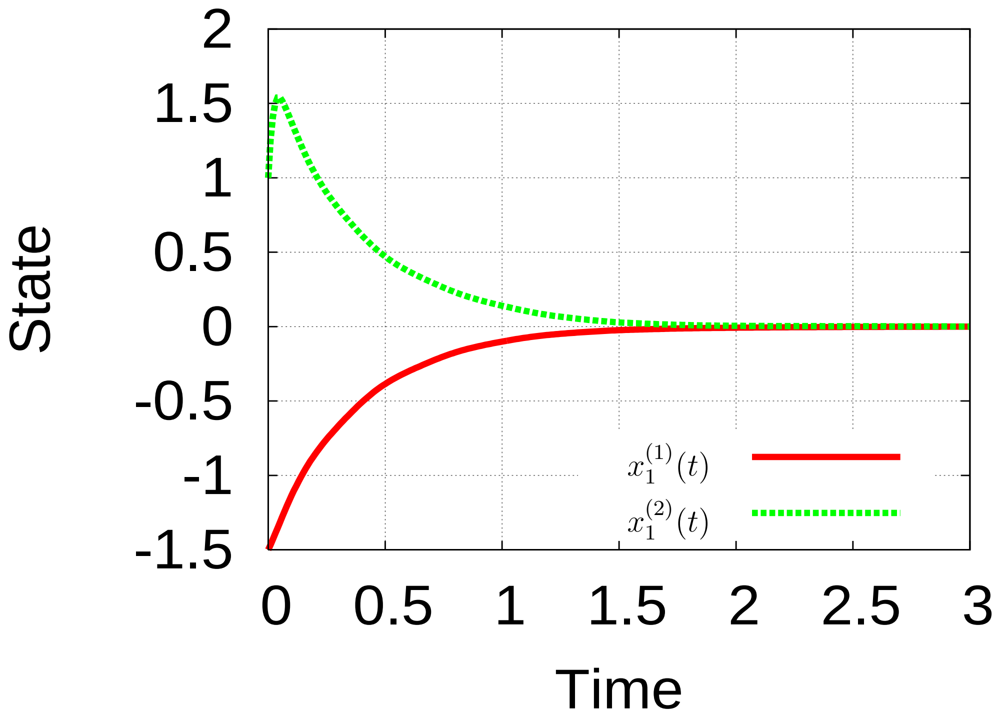

Figure 4.1: Time histories of $x_1(t)$ (**Example 1**)

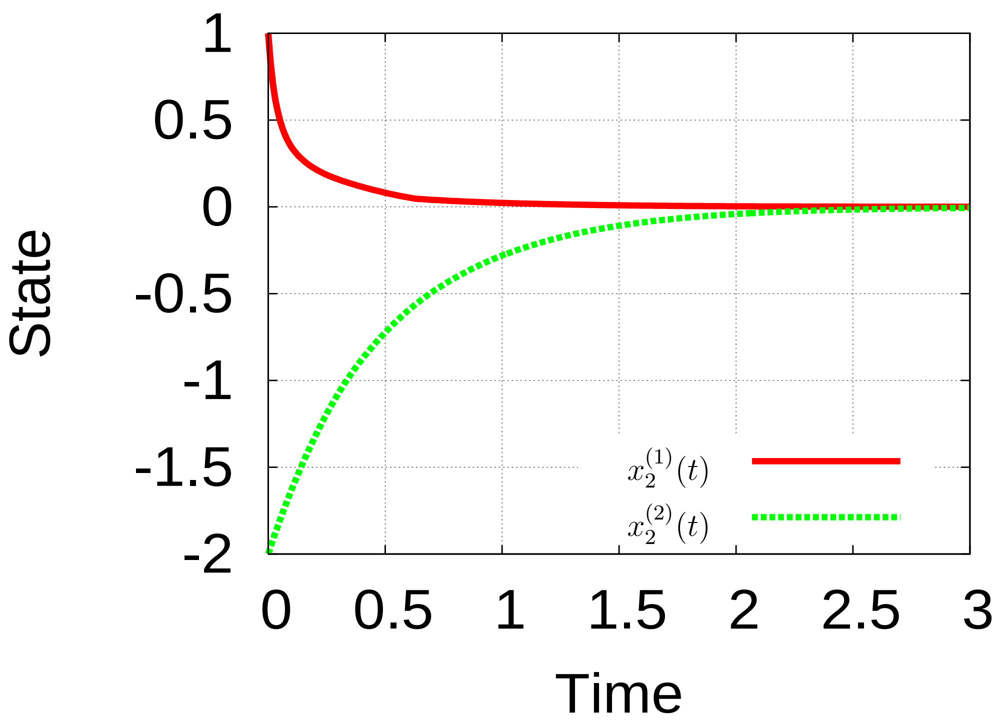

Figure 4.2: Time histories of $x_2(t)$ (**Example 1**)

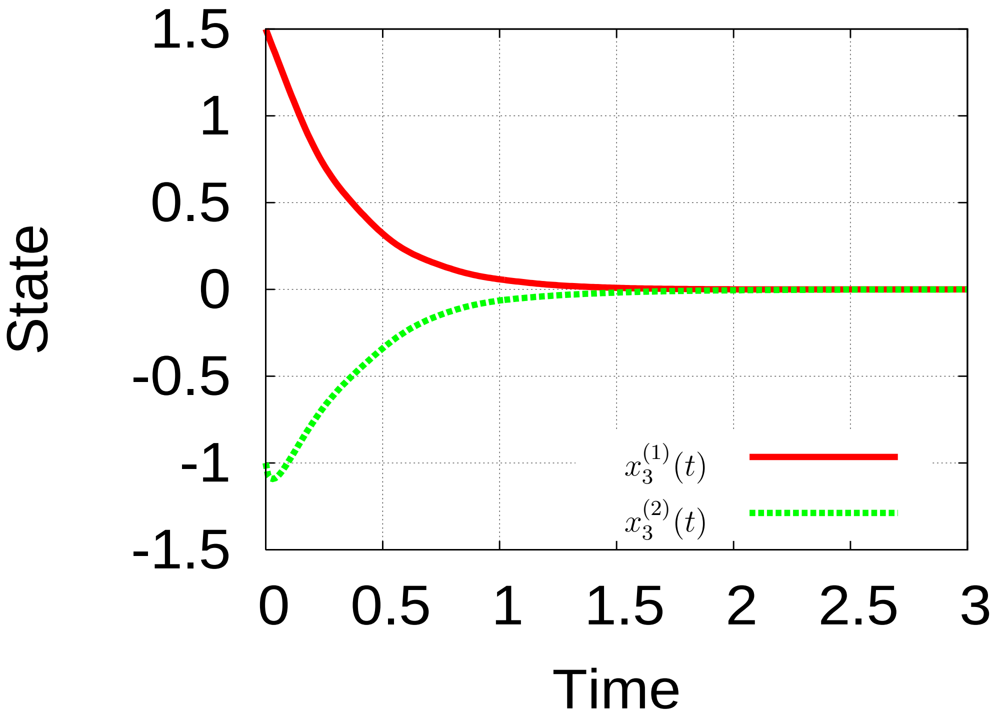

Figure 4.3: Time histories of $x_3(t)$ (**Example 1**)

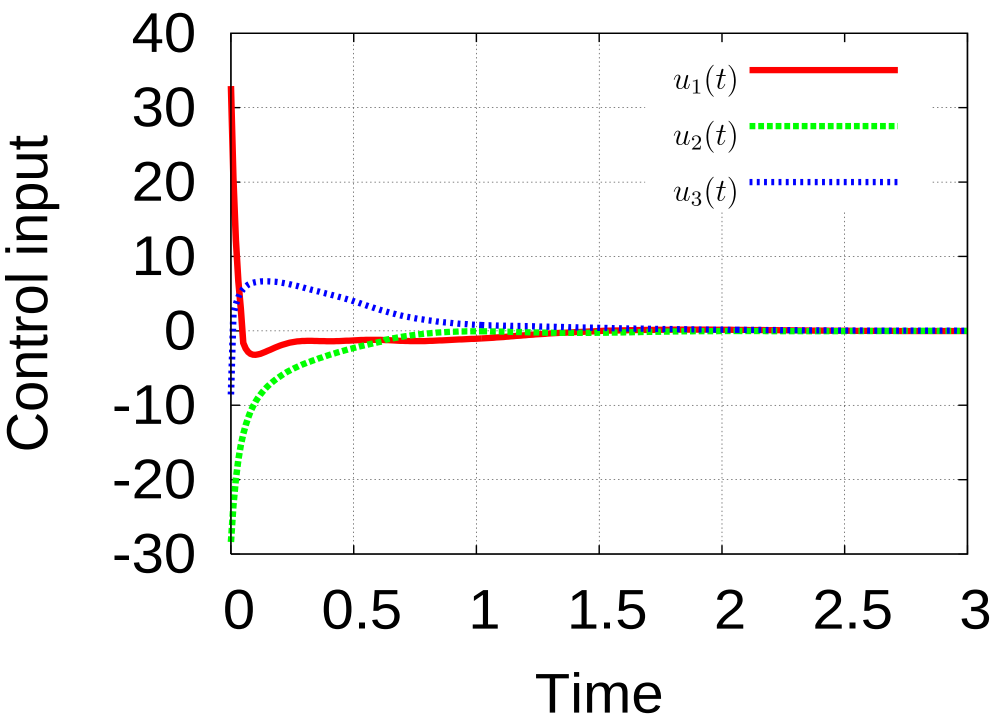

Figure 4.4: Time histories of $u(t)$ (**Example 1**)

4.3.2 Example 2

In this example, we consider three-machine infinite bus system model as shown in Figure 4.5 [85].

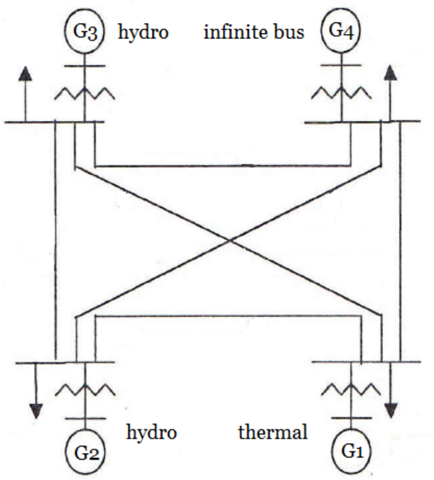

Figure 4.5: Three-machine infinite bus system model [85]

This three-machine infinite bus system model system can be represented as the large-scale interconnected system composed of three four-dementional subsystems with the following system parameters [86];

$$A_{11} = \begin{pmatrix} -9.2200 \times 10^{-1} & 1.000 & -2.6600 \times 10^{-1} & -9.0000 \times 10^{-3} \\ -2.7500 & -2.7800 & -1.3600 & -3.7000 \times 10^{-1} \\ 0.0 & 0.0 & 0.0 & 1.0000 \\ -4.9500 & 0.0 & -5.5500 \times 10^{1} & -3.0000 \times 10^{-1} \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} -2.1000 \times 10^{-1} & 1.0000 & -1.6000 & -5.0000 \times 10^{-3} \\ -1.9000 & -1.8000 & 9.3000 & -1.2000 \times 10^{-1} \\ 0.0 & 0.0 & 0.0 & 1.000 \\ -3.1000 & 0.0 & -5.6000 & 3.2000 \times 10^{-2} \end{pmatrix},$$

$$A_{33} = \begin{pmatrix} -1.9700 \times 10^{-1} & 1.0000 & -1.2000 & -3.0000 \times 10^{-3} \\ -5.4400 \times 10^{1} & 2.0000 \times 10^{1} & 7.0100 \times 10^{1} & -2.3700 \\ 0.0 & 0.0 & 0.0 & 1.0000 \\ -3.4000 & 0.0 & -2.1000 \times 10^{1} & -1.7000 \times 10^{-2} \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.0 \\ 3.6100 \\ 0.0 \\ 0.0 \end{pmatrix}, \; B_2 = \begin{pmatrix} 0.0 \\ 7.8900 \\ 0.0 \\ 0.0 \end{pmatrix}, \; B_3 = \begin{pmatrix} 0.0 \\ 5.6300 \\ 0.0 \\ 0.0 \end{pmatrix},$$

$$\mathcal{E}_{11}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 2.7700 \\ 5.5400 \end{pmatrix} \times 10^{-5}, \; \mathcal{E}_{22}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.2700 \times 10^{-5} \\ 1.2700 \times 10^{-6} \end{pmatrix},$$

$$\mathcal{E}_{33}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.8000 \\ 1.2700 \end{pmatrix} \times 10^{-6}, \; \left(\mathcal{E}_{11}^{\perp}\right)^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.0000 \\ 2.0000 \end{pmatrix} \times 10^{-2},$$

$$\left(\mathcal{E}_{22}^{\perp}\right)^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.0000 \times 10^{-2} \\ 1.0000 \times 10^{-3} \end{pmatrix}, \; \left(\mathcal{E}_{33}^{\perp}\right)^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.0000 \times 10^{-3} \\ 2.0000 \times 10^{-2} \end{pmatrix},$$

$$A_{12} = \begin{pmatrix} 2.4000 \times 10^{-2} & 0.0 & -8.7000 \times 10^{-2} & -2.0000 \times 10^{-3} \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 2.2200 \times 10^{-1} & 0.0 & 8.1700 \times 10^{-1} & 4.0000 \times 10^{-3} \end{pmatrix},$$

$$A_{13} = \begin{pmatrix} 7.2000 \times 10^{-1} & 0.0 & -2.5000 \times 10^{-2} & -3.0000 \times 10^{-3} \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 9.2400 \times 10^{-2} & 0.0 & 1.7500 \times 10^{-1} & 2.0000 \times 10^{-2} \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} 2.1000 \times 10^{-2} & 0.0 & 1.2100 \times 10^{-2} & 3.0000 \times 10^{-3} \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ -2.4300 \times 10^{-2} & 0.0 & 1.3700 \times 10^{-2} & -3.4000 \times 10^{-2} \end{pmatrix},$$

$$A_{23} = \begin{pmatrix} 6.0000\times10^{-2} & 0.0 & 4.6000\times10^{-2} & 2.0000\times10^{-3} \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 1.2000\times10^{-2} & 0.0 & 2.9800\times10^{-2} & -2.8000\times10^{-2} \end{pmatrix},$$

$$A_{31} = \begin{pmatrix} -2.0000\times10^{-3} & 0.0 & 8.3000\times10^{-2} & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ -1.2400\times10^{-1} & 0.0 & 4.9800\times10^{-2} & -1.7000\times10^{-2} \end{pmatrix},$$

$$A_{32} = \begin{pmatrix} 1.1000\times10^{-2} & 0.0 & 2.2000\times10^{-2} & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ -7.0000\times10^{-3} & 0.0 & 6.3700\times10^{-2} & -1.1000\times10^{-1} \end{pmatrix},$$

$$\mathcal{D}_{12}^T = \begin{pmatrix} -4.3767\times10^{-3} \\ 0.0 \\ 3.0471\times10^{-2} \\ -3.0471\times10^{-3} \end{pmatrix}, \ \mathcal{D}_{13}^T = \begin{pmatrix} -1.2742\times10^{-2} \\ 0.0 \\ 7.7562\times10^{-3} \\ -5.5402\times10^{-3} \end{pmatrix},$$

$$\mathcal{D}_{21}^T = \begin{pmatrix} -1.3942\times10^{-3} \\ 0.0 \\ -2.0532\times10^{-2} \\ -1.9011\times10^{-3} \end{pmatrix}, \ \mathcal{D}_{23}^T = \begin{pmatrix} -1.2674\times10^{-1} \\ 0.0 \\ 1.8889\times10^{-4} \\ -3.0697\times10^{-3} \end{pmatrix},$$

$$\mathcal{D}_{31}^T = \begin{pmatrix} -1.2043\times10^{-2} \\ 0.0 \\ -1.7940\times10^{-2} \\ -1.5986\times10^{-2} \end{pmatrix}, \ \mathcal{D}_{32}^T = \begin{pmatrix} -3.7300 \\ 0.0 \\ 3.0195 \\ -2.1847 \end{pmatrix} \times 10^{-3},$$

$$\mathcal{E}_{12}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 4.1600 \\ 1.3900 \end{pmatrix} \times 10^{-5}, \ \mathcal{E}_{13}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 8.3100\times10^{-5} \\ 2.7700\times10^{-7} \end{pmatrix},$$

$$\mathcal{E}_{21}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 1.2700\times10^{-5} \\ 6.3400\times10^{-8} \end{pmatrix}, \ \mathcal{E}_{23}^T = \begin{pmatrix} 0.0 \\ 0.0 \\ 2.5300\times10^{-5} \\ 1.2700\times10^{-6} \end{pmatrix},$$

$$
\begin{aligned}
\mathcal{E}_{31}^{T} &= \begin{pmatrix} 0.0 \\ 0.0 \\ 8.8800 \times 10^{-6} \\ 1.7800 \times 10^{-5} \end{pmatrix}, \ \mathcal{E}_{32}^{T} = \begin{pmatrix} 0.0 \\ 0.0 \\ 8.8800 \times 10^{-5} \\ 1.7800 \times 10^{-6} \end{pmatrix}, \\
\left(\mathcal{E}_{12}^{\perp}\right)^{T} &= \begin{pmatrix} 0.0 \\ 0.0 \\ 1.5000 \times 10^{-2} \\ 5.0000 \times 10^{-3} \end{pmatrix}, \ \left(\mathcal{E}_{13}^{\perp}\right)^{T} = \begin{pmatrix} 0.0 \\ 0.0 \\ 3.0000 \times 10^{-2} \\ 1.0000 \times 10^{-4} \end{pmatrix}, \\
\left(\mathcal{E}_{21}^{\perp}\right)^{T} &= \begin{pmatrix} 0.0 \\ 0.0 \\ 1.0000 \times 10^{-2} \\ 5.0000 \times 10^{-4} \end{pmatrix}, \ \left(\mathcal{E}_{23}^{\perp}\right)^{T} = \begin{pmatrix} 0.0 \\ 0.0 \\ 2.0000 \times 10^{-2} \\ 1.0000 \times 10^{-3} \end{pmatrix}, \\
\left(\mathcal{E}_{31}^{\perp}\right)^{T} &= \begin{pmatrix} 0.0 \\ 0.0 \\ 5.0000 \times 10^{-3} \\ 1.0000 \times 10^{-2} \end{pmatrix}, \ \left(\mathcal{E}_{32}^{\perp}\right)^{T} = \begin{pmatrix} 0.0 \\ 0.0 \\ 5.0000 \times 10^{-2} \\ 1.0000 \times 10^{-3} \end{pmatrix}, \\
B_i^{\perp} = B_{ij}^{\perp} &= \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 1.0000 \times 10^{-3} \end{pmatrix} \quad \{i, j = 1, 2, 3\} \ \ (i \neq j).
\end{aligned}
\tag{4.35}
$$

In order to obtain the proposed decentralized variable gain robust controller, we consider **Theorem 4.1**. By solving LMIs of (4.5), the following solutions can be obtained;

$$
\mathcal{Y}_1 = \begin{pmatrix} 3.7878 \times 10^{1} & -7.4806 & -3.2862 & 5.0962 \\ \star & 5.9707 \times 10^{1} & -3.5341 & 3.2705 \\ \star & \star & 7.7496 \times 10^{-1} & -3.3345 \times 10^{-1} \\ \star & \star & \star & 2.7495 \times 10^{1} \end{pmatrix},
$$

$$
\mathcal{Y}_2 = \begin{pmatrix} 7.4248 & -8.2387 & -1.5062 & 8.0610 \\ \star & 3.2510 \times 10^{1} & -3.5360 \times 10^{-1} & -5.3083 \times 10^{-1} \\ \star & \star & 3.8621 & -1.5011 \\ \star & \star & \star & 1.9168 \times 10^{1} \end{pmatrix},
$$

$$\mathcal{Y}_3 = \begin{pmatrix} 1.4578\times10^{1} & -1.0812\times10^{1} & -2.1242 & 4.4548 \\ \star & 3.9548\times10^{1} & -1.4665 & -1.7421\times10^{-1} \\ \star & \star & 1.0947 & -3.5470\times10^{-1} \\ \star & \star & \star & 1.6254\times10^{1} \end{pmatrix},$$
$$\mathcal{W}_1 = \begin{pmatrix} 7.0764 & 2.2199\times10^{1} & -5.0421 & -5.6077\times10^{1} \end{pmatrix},$$
$$\mathcal{W}_2 = \begin{pmatrix} -5.0647\times10^{-1} & -1.9396 & -4.9560 & 9.7925\times10^{-1} \end{pmatrix}, \tag{4.36}$$
$$\mathcal{W}_3 = \begin{pmatrix} 2.0315\times10^{2} & -2.3730\times10^{2} & -2.8862\times10^{1} & 4.3241\times10^{1} \end{pmatrix},$$
$$\sigma_1 = 6.5797\times10^{1},\ \sigma_2 = 6.5790\times10^{1},\ \sigma_3 = 6.5781\times10^{1},$$
$$\epsilon_1 = 6.5822\times10^{1},\ \epsilon_2 = 6.5893\times10^{1},\ \epsilon_3 = 6.5829\times10^{1},$$
$$\delta_{12} = 2.5875\times10^{1},\ \delta_{13} = 5.0776\times10^{1},\ \delta_{21} = 1.2289\times10^{2},$$
$$\delta_{23} = 8.4152\times10^{1},\ \delta_{31} = 1.1860\times10^{2},\ \delta_{32} = 7.7637\times10^{1}.$$

Therefore, symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{4\times4}$ and fixed gain matrices $F_i \in \mathbb{R}^{1\times4}$ can be computed as

$$\mathcal{P}_1 = \begin{pmatrix} 8.6264\times10^{-2} & 4.5044\times10^{-2} & 5.6498\times10^{-1} & -1.4495\times10^{-2} \\ \star & 4.6520\times10^{-2} & 3.9927\times10^{-1} & -9.0400\times10^{-3} \\ \star & \star & 5.4701 & -8.5871\times10^{-2} \\ \star & \star & \star & 3.9090\times10^{-2} \end{pmatrix},$$
$$\mathcal{P}_2 = \begin{pmatrix} 5.5408\times10^{-1} & 1.3843\times10^{-1} & 1.4407\times10^{-1} & -2.1791\times10^{-1} \\ \star & 6.5396\times10^{-2} & 3.9244\times10^{-2} & -5.3331\times10^{-2} \\ \star & \star & 3.0486\times10^{-1} & -3.5627\times10^{-2} \\ \star & \star & \star & 1.3955\times10^{-1} \end{pmatrix},$$
$$\mathcal{P}_3 = \begin{pmatrix} 2.1184\times10^{-1} & 7.6182\times10^{-2} & 4.9811\times10^{-1} & -4.6374\times10^{-2} \\ \star & 5.4023\times10^{-2} & 2.1515\times10^{-1} & -1.5606\times10^{-2} \\ \star & \star & 2.1400 & -8.7514\times10^{-2} \\ \star & \star & \star & 7.2156\times10^{-2} \end{pmatrix},$$
$$F_1 = \begin{pmatrix} -4.2552\times10^{-1} & -1.5477\times10^{-1} & -9.9039 & -2.0623 \end{pmatrix},$$
$$F_2 = \begin{pmatrix} -1.4765 & -4.4367\times10^{-1} & -1.6948 & 5.2702\times10^{-1} \end{pmatrix},$$
$$F_3 = \begin{pmatrix} 8.5758 & -4.2275 & -1.5412\times10^{1} & -7.1738\times10^{-2} \end{pmatrix}. \tag{4.37}$$

In this example, we choose the initial value of large-scale interconnected system of (4.35) as follows;

$$x(0) = \left(\begin{array}{cccc:cccc:cccc} 1.0 & -1.0 & 0.0 & 0.5 & 1.2 & -3.0 & 0.0 & -0.5 & 1.0 & -1.3 & 0.0 & 0.5 \end{array} \right)^T . \tag{4.38}$$

Furthermore, unknown parameters are given as

$$\begin{aligned} &\Delta_{11}(t) = \Delta_{1j}(t) = \Delta_{11}^{\perp}(t) = \Delta_{1j}^{\perp}(t) = \sin(60\pi t) \ \ \{j = 2, 3\}, \\ &\Delta_{22}(t) = \Delta_{2j}(t) = \Delta_{22}^{\perp}(t) = \Delta_{2j}^{\perp}(t) = 1 - \exp(-0.01t) \ \ \{j = 1, 3\}, \\ &\Delta_{33}(t) = \Delta_{3j}(t) = \Delta_{33}^{\perp}(t) = \Delta_{3j}^{\perp}(t) = \frac{1}{2}\sin(120\pi t) \ \ \{j = 1, 2\}. \end{aligned} \tag{4.39}$$

The simulation result of this example is shown in Figures 4.6 – 4.9. In these figures, $x_i^{(l)}(t)$ are the l-th element of the state $x_i(t)$ for the i-th subsystem. From these figures, the uncertain power system with system parameters of (4.35) is robust stable by the proposed decentralized variable gain robust controller. Therefore, the effectiveness of the proposed design method of decentralized variable gain robust controller is shown.

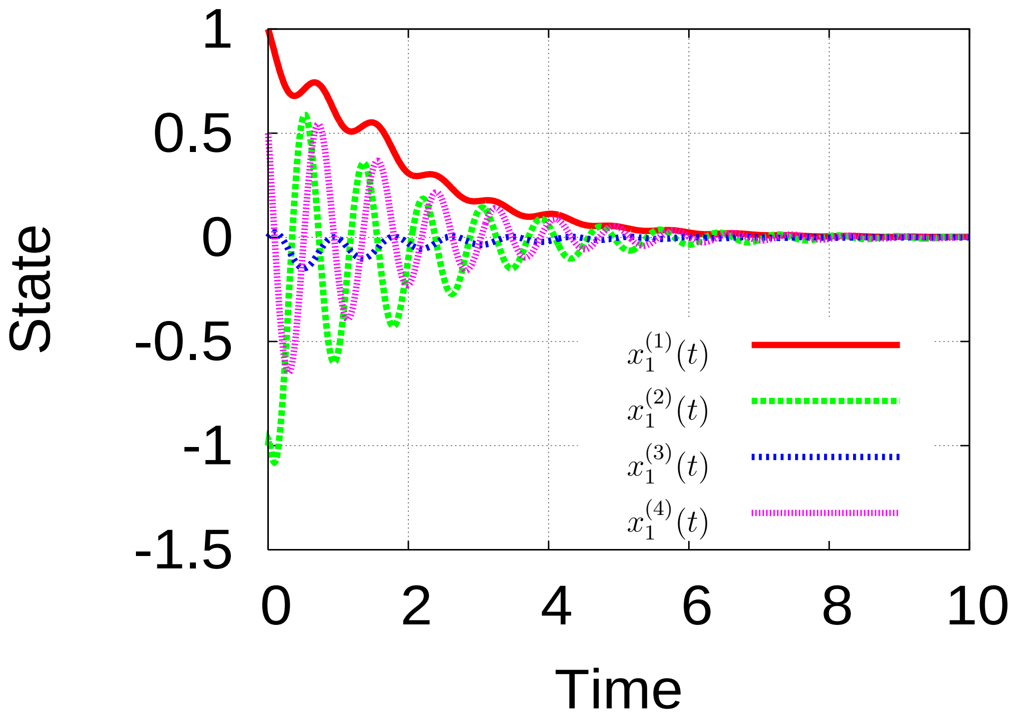

Figure 4.6: Time histories of $x_1(t)$ (**Example 2**)

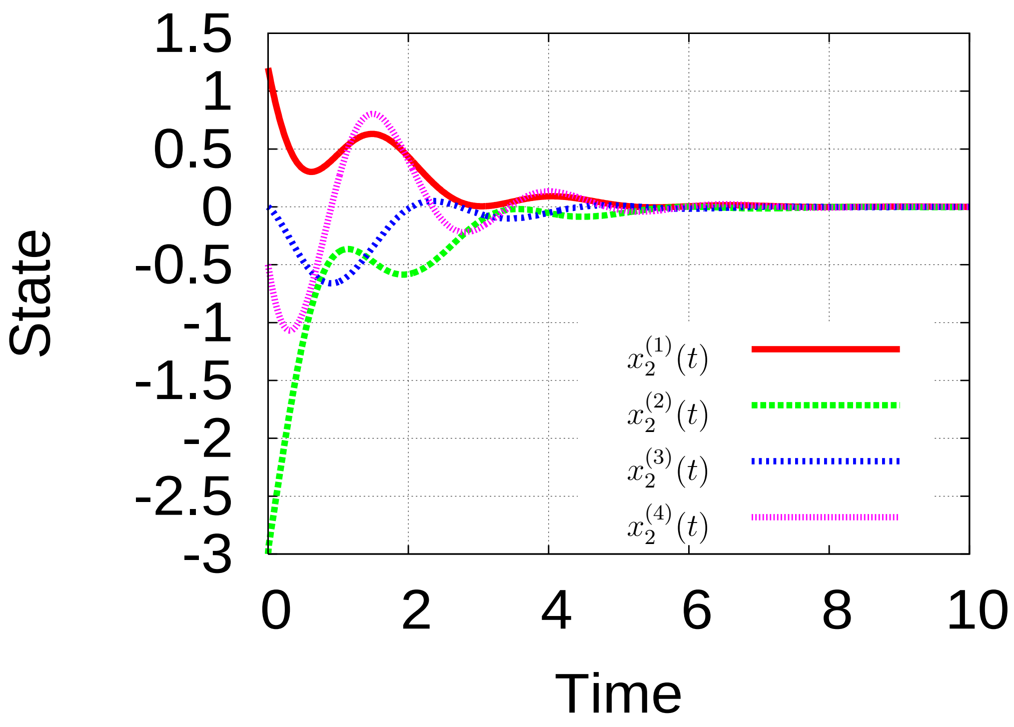

Figure 4.7: Time histories of $x_2(t)$ (**Example 2**)

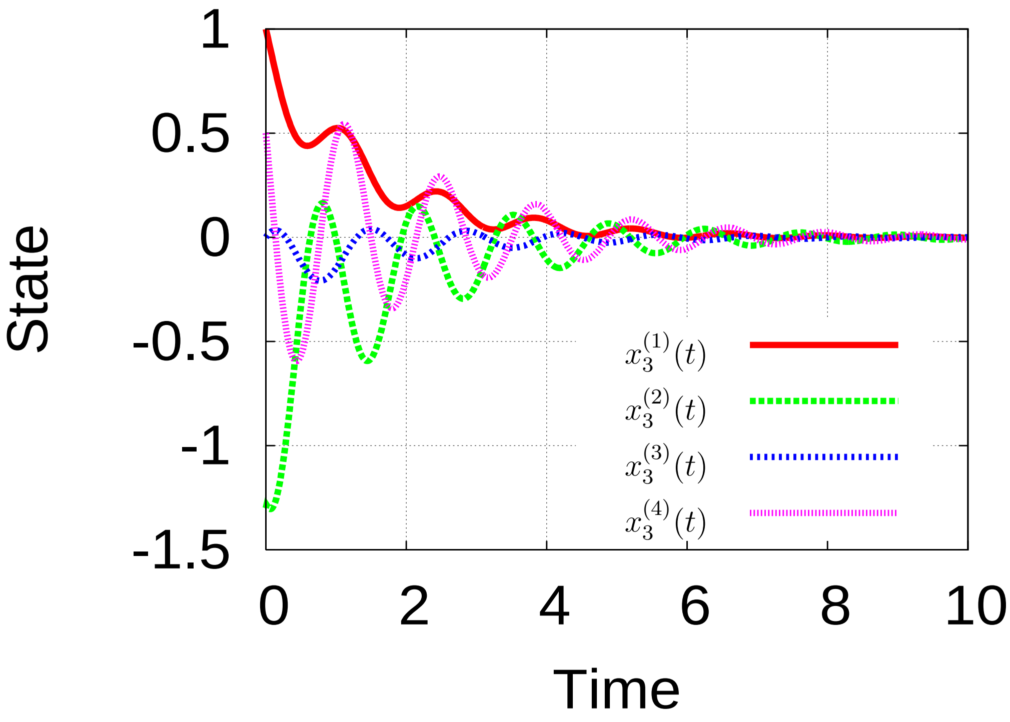

Figure 4.8: Time histories of $x_3(t)$ (**Example 2**)

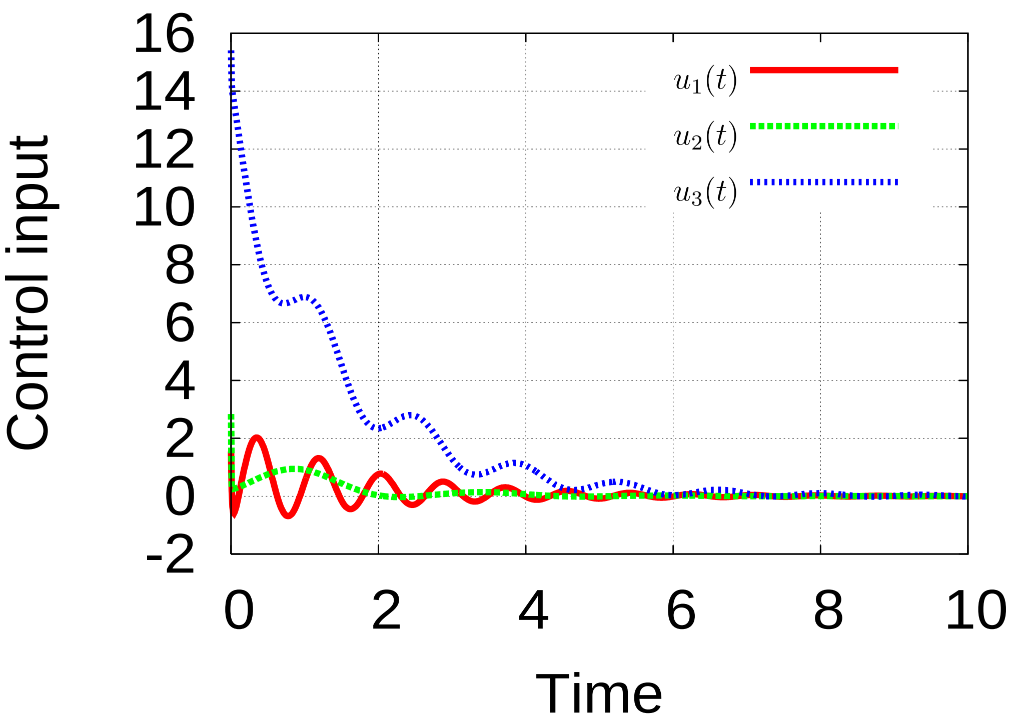

Figure 4.9: Time histories of $u(t)$ (**Example 2**)

4.4 Summary

This chapter has shown the decentralized variable gain robust controller for a class of large-scale interconnected systems with mismatched uncertainties. The uncertainties and interactions which are included in the large-scale interconnected system are divided into the matched part and the mismatched one, and the effect of matched parts can be reduced by the variable gain parameters in the proposed controller. Moreover, the size of the proposed LMIs is smaller than both the case that uncertainties and interactions are not divided into matched and mismatched part, and the conventional decentralized fixed gain robust controller. Namely, the feasible region of the LMIs derived in this chapter is large comparing with ones for the conventional decentralized fixed gain robust controller. In other words, the proposed decentralized variable gain robust controller can be applied to more larger class of uncertain large-scale interconnected systems comparing with the conventional decentralized fixed gain robust controller. Therefore, the result in this chapter is very useful.

Chapter 5

Conclusions and Future Works

5.1 Conclusions

In this thesis, we have proposed new decentralized variable gain robust controllers for uncertain large-scale interconnected systems. Additionally, the effectiveness of decentralized variable gain robust control strategies proposed in this thesis have been shown through simple numerical examples.

In chapter 2, for a class of large-scale interconnected systems with uncertainties and interactions which satisfy matching condition, a decentralized variable gain robust controller which achieves not only robust stability but also satisfactory transient behavior generated by the nominal subsystem has been proposed. The proposed LMI condition is always feasible, namely, designers can derive the decentralized variable gain robust controller provided that some assumptions are satisfied. In the case of the conventional decentralized fixed gain controllers, derived LMIs may not feasible for large-scale interconnected systems with matched uncertainties. Thus, the proposed decentralized robust control strategy is useful.

Chapter 3 is an extension of the result of chapter 2 and the design method of the decentralized variable gain robust controller with guaranteed $\mathcal{L}_2$ gain performance for a class of uncertain large-scale interconnected systems with disturbance inputs has been suggested. As with the result of chapter 2, if the matching condition for uncertainties and interactions is satisfied, then the resultant LMIs are always feasible, i.e., the proposed decentralized variable gain robust controller can be designed. The size and number of variables of resultant LMIs are smaller than that of the conventional decentralized fixed gain robust controller. In addition, the proposed decentralized variable gain robust controller can easily be extended to one with suboptimal $\mathcal{L}_2$ gain performance by applying a convex constraint optimization problem.

In chapter 4, the decentralized variable gain robust controller for a class of large-scale interconnected systems with mismatched uncertainties. The uncertainties in the large-scale interconnected system are divided into the matched part and the mismatched one, and the effect of matched parts can be reduced by the variable gain parameters in the proposed controller. Moreover, the size of LMIs which should be solved is smaller than the case that deals with the mismatched part only as uncertainties, i.e., the proposed LMI condition is less conservative than the case that uncertainties are not divided into the matched and the mismatched parts.

By the way, in the work of Hopp and Schmitendorf [87], the design methods of linear controllers which achieve practical tracking for linear systems with matched uncertainty and ϵ-tracking for linear systems with mismatched uncertainty have been suggested. The proposed decentralized variable gain robust control strategy can be extended to such control problems (see **Appendix**).

The design problems of decentralized variable gain robust controllers for uncertain large-scale interconnected systems considered in this thesis are reduced to the solvability of LMIs, and the size of LMIs are smaller than the case of the conventional fixed gain robust control. Therefore the proposed controller design methods of decentralized variable gain robust control systems are less conservative, and the proposed decentralized variable gain robust controller can easily be derived. Note that it is well known that LMI-conditions can easily be derived using various calculation tools (e.g. MATLAB Robust Control Toolbox and Scilab LMITOOL). Thus, we find that the proposed controller design methods are very useful.

5.2 Future Works

In the future research, we will extend the proposed variable gain robust controllers to such a broad class of systems as uncertain large-scale interconnected systems with time delays, discrete-time systems and so on. Furthermore, in this thesis, we have assumed that all state variables of uncertain large-scale interconnected systems are measurable directly. However, there are only a few case of such systems in general, and estimation of internal variables by using measurable input and output variables is needed. For such cases, observer-based controller and output feedback control are useful. Thus, we will apply the proposed controller design methods to such problems by using observer [88] or output feedback control [89].

On the other hand, in recent years, formation control for multi-agent systems (MASs) has attracted the attention of many researchers (e.g. [90–92]). A multi-agent system consists of several agents (e.g. vehicles and mobile robots) which interact with one-another by network. In control strategies for multi-agent systems, consensus problem [93–95] and coverage problem [96–98] are mainly considered. Since formation control problems for multi-agent systems are considered as one of decentralized control problems, we will also apply the proposed decentralized variable gain robust control strategies in this thesis to formation control problems for multi-agent systems with uncertainties.

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List of Publications Related to this Thesis

[1] S. Nagai and H. Oya, "Decentralized Variable Gain Robust Controllers for a Class of Uncertain Large-Scale Interconnected Systems," Proc. of the 33rd IASTED Int. Conf. on Modelling, Identification and Contr., pp.199-204, Innsbruck, AUSTRIA (2014) **(Related to Chapter 2)**

[2] S. Nagai and H. Oya, "Synthesis of Decentralized Variable Gain Robust Controllers for Large-Scale Interconnected Systems with Structured Uncertainties," J. Contr. Science and Engineering, vol.2014, Article ID 848465, 10 pages (2014) **(Related to Chapter 2)**

[3] S. Nagai and H. Oya, "Synthesis of Decentralized Variable Gain Robust Controllers with $\mathcal{L}_2$ Gain Performance for a Class of Uncertain Large-Scale Interconnected Systems," J. Contr. Science and Engineering, Vol.2015, Article ID 342867, 11 pages (2015) **(Related to Chapter 3)**

[4] S. Nagai, H. Oya, T. Kubo and T. Matsuki, "Decentralized Variable Gain Robust Controller Design for a Class of Large-Scale Interconnected Systems with Mismatched Uncertainties," Int. J. Systems Science, Vol.48, No.8, pp.1616–1623 (2017) **(Related to Chapter 4)**

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List of Other Publications

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Author's Biography

Shunya Nagai was born in Tokyo, Japan in October 21st, 1990. He received his Bachelor's degree in engineering in 2013 and Master's degree in engineering in 2015 at Tokushima University. He will also receive his Ph.D degree in March, 2018 at Tokushima University. His main research interest is the control theory and control engineering, particularly robust control for dynamical systems with uncertainties, decentralized control for large-scale interconnected systems and formation control for multi-agent systems. He is IEEE student member and the Society of Instrument and Control Engineers (SICE) student member.

Appendix

Decentralized Variable Gain Robust ϵ-Tracking for a Class of Uncertain Large-Scale Interconnected Systems

In this appendix, a decentralized variable gain robust controller which achieves ϵ-tracking for a class of large-scale interconnected systems with mismatched uncertainties is introduced [99]. Furthermore, we show that sufficient conditions for the existence of the proposed decentralized variable gain robust controller are reduced to the feasibility of linear matrix inequalities (LMIs). Finally, a simple numerical example is presented to demonstrate the effectiveness of the proposed decentralized robust control system.

A.1 Problem Formulation

Let us consider the uncertain large-scale interconnected system composed of $\mathcal{N}$ subsystems described as

$$\begin{aligned} \frac{d}{dt}x_i(t) &= A_{ii}(t)x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} A_{ij}(t)x_j(t) + B_i u_i(t), \\ y_i(t) &= C_i x_i(t), \end{aligned} \tag{A.1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^{l_i}$ $(i = 1, \cdots, \mathcal{N})$ are the vectors of the state, the control input and the output for the i-th subsystem, respectively and $x(t) = \left(x_1^T(t), \cdots, x_{\mathcal{N}}^T(t)\right)^T$ is the state of the overall system. The matrices $A_{ii}(t)$ and $A_{ij}(t)$ are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + B_i\Delta_{ii}(t)\mathcal{E}_{ii} + B_i^{\perp}\Delta_{ii}^{\perp}(t)\mathcal{E}_{ii}^{\perp}, \\ A_{ij}(t) &= A_{ij} + B_i\mathcal{D}_{ij} + B_i\Delta_{ij}(t)\mathcal{E}_{ij} + B_{ij}^{\perp}\Delta_{ij}^{\perp}(t)\mathcal{E}_{ij}^{\perp}. \end{aligned} \quad (\mathbf{A}.2)$$

In ($\mathbf{A}$.2), the matrices $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $B_i \in \mathbb{R}^{n_i \times m_i}$ and $C_i \in \mathbb{R}^{l_i \times n_i}$ denote the nominal system matrices. Additionally, the matrices $\mathcal{D}_{ij}$, $\mathcal{E}_{ii}$ and $\mathcal{E}_{ij}$ with appropriate dimensions represent the structure of matched interactions or uncertainties and the matrices A_{ij}, $B_i^{\perp}$, $B_{ij}^{\perp}$, $\mathcal{E}_{ii}^{\perp}$ and $\mathcal{E}_{ij}^{\perp}$ show the structure of mismatched ones [83]. Namely, the uncertainties and interactions are divided into the matched part and mismatched one. Besides, matrices $\Delta_{ii}(t) \in \mathbb{R}^{m_i \times r_i}$, $\Delta_{ij}(t) \in \mathbb{R}^{m_i \times s_{ij}}$, $\Delta_{ii}^{\perp}(t) \in \mathbb{R}^{p_{ii} \times q_{ii}}$ and $\Delta_{ij}^{\perp}(t) \in \mathbb{R}^{p_{ij} \times q_{ij}}$ denote unknown parameters satisfying the relations $\|\Delta_{ii}(t)\| \le 1.0$, $\|\Delta_{ij}(t)\| \le 1.0$, $\|\Delta_{ii}^{\perp}(t)\| \le 1.0$ and $\|\Delta_{ij}^{\perp}(t)\| \le 1.0$.

Now, the reference model which should be tracked by the subsystems of ($\mathbf{A}$.1) is given by

$$\begin{aligned} \frac{d}{dt}x_{r_i}(t) &= A_{r_i}x_{r_i}(t), \\ y_{r_i}(t) &= C_{r_i}x_{r_i}(t), \end{aligned} \quad (\mathbf{A}.3)$$

where $x_{r_i}(t) \in \mathbb{R}^{n_{r_i}}$ and $y_{r_i}(t) \in \mathbb{R}^{l_i}$ are the state and the output of the reference model, and we assume that there exist a finite positive scalar M_i such that

$$\|x_{r_i}(t)\| \le M_i, \ \forall t \ge 0. \quad (\mathbf{A}.4)$$

Furthermore for the reference model of ($\mathbf{A}$.3), there exist matrices $G_i \in \mathbb{R}^{n_i \times n_{r_i}}$ and $H_i \in \mathbb{R}^{m_i \times n_{r_i}}$ which satisfy [87]

$$\left(\begin{array}{c|c} A_{ii} & B_i \\ \hline C_i & O_{l_i \times m_i} \end{array}\right) \left(\begin{array}{c} G_i \\ \hline H_i \end{array}\right) = \left(\begin{array}{c} G_iA_{r_i} \\ \hline C_{r_i} \end{array}\right). \quad (\mathbf{A}.5)$$

The nominal subsystem, which is obtained by ignoring uncertainties and interactions in ($\mathbf{A}$.1), is shown as

$$\begin{aligned} \frac{d}{dt}\overline{x}_i(t) &= A_{ii}\overline{x}_i(t) + B_i\overline{u}_i(t), \\ \overline{y}_i(t) &= C_i\overline{x}_i(t). \end{aligned} \quad (\mathbf{A}.6)$$

In (**A**.6), $\overline{x}_i(t) \in \mathbb{R}^{n_i}$, $\overline{u}_i(t) \in \mathbb{R}^{m_i}$ and $\overline{y}_i(t) \in \mathbb{R}^{l_i}$ are the vectors of the state, the control input and the output for the i-th nominal subsystem. Next by introducing an error vector between the nominal system and the reference model $\overline{e}_i(t) \triangleq \overline{x}_i(t) - G_i x_{r_i}(t)$, the nominal error subsystem

$$\begin{aligned}\frac{d}{dt}\overline{e}_i(t) &= A_{ii}\overline{x}_i(t) + B_i\overline{u}_i(t) - G_i A_{r_i} x_{r_i}(t) \\ &= A_{ii}\overline{x}_i(t) + B_i\overline{u}_i(t) - (A_{ii}G_i + B_i H_i)\,\overline{x}_{r_i}(t) \\ &= A_{ii}\overline{e}_i(t) + B_i\omega_i(t), \end{aligned} \qquad (\mathbf{A}.7)$$

is derived. In (**A**.7), $\omega_i(t) \triangleq \overline{u}_i(t) - H_i x_{r_i}(t)$. If the vector $\omega_i(t)$ can be written as $\omega_i(t) = -K_i\overline{e}_i(t)$, then the error subsystem of (**A**.7) can be rewritten as

$$\begin{aligned}\frac{d}{dt}\overline{e}_i(t) &= (A_{ii} - B_i K_i)\overline{e}_i(t) \\ &= A_{K_i}\overline{e}_i(t), \end{aligned} \qquad (\mathbf{A}.8)$$

where $A_{K_i} \triangleq A_{ii} - B_i K_i$. Furthermore, the tracking error $\overline{e}_{y_i}(t) \triangleq \overline{y}_i(t) - y_{r_i}(t)$ can be represented as

$$\overline{e}_{y_i}(t) = C_i\overline{e}_i(t). \qquad (\mathbf{A}.9)$$

Therefore, if the matrix A_{K_i} is stable, then $\overline{e}_{y_i}(t)$ tends to 0 as t goes to infinity, i.e., the output of (**A**.6) tracks one of the reference model of (**A**.3). Then the nominal control input $\overline{u}_i(t)$ for the nominal subsystem is

$$\begin{aligned}\overline{u}_i(t) &= -\,K_i\overline{e}_i(t) + H_i x_{r_i}(t) \\ &= -\,K_i\overline{x}_i(t) + (K_i G_i + H_i)x_{r_i}(t). \end{aligned} \qquad (\mathbf{A}.10)$$

Now, by using the nominal control input $\overline{u}_i(t)$ of (**A**.10), we define the following control input for the large-scale interconnected system of (**A**.1):

$$u_i(t) \triangleq -\,K_i x_i(t) + (K_i G_i + H_i)x_{r_i}(t) + v_i(t). \qquad (\mathbf{A}.11)$$

In (**A**.11), $v_i(t) \in \mathbb{R}^{m_i}$ is the compensation input defined as [83]

$$v_i(t) \triangleq -F_i e_i(t) - \varphi_i(e_i, x_{r_i}, t). \qquad (\mathbf{A}.12)$$

where, $e_i(t) \stackrel{\triangle}{=} x_i(t) - G_i x_{r_i}(t)$ is the error vector between the controlled system and the reference model, and $F_i \in \mathbb{R}^{m_i \times n_i}$ and $\varphi_i(e_i, x_{r_i}, t) \in \mathbb{R}^{m_i}$ are a fixed compensation gain matrix and a nonlinear modification term. From (**A**.1), (**A**.2), (**A**.11) and (**A**.12), we have the uncertain closed-loop subsystem of (**A**.13)

$$\begin{aligned}\frac{d}{dt}x_i(t) = A_{K_i}x_i(t) + B_i(K_iG_i + H_i)x_{r_i}(t) + B_i\Delta_{ii}(t)\mathcal{E}_{ii}x_i(t) + B_i^{\perp}\Delta_{ii}^{\perp}(t)\mathcal{E}_{ii}^{\perp}x_i(t) \\ + B_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(\mathcal{D}_{ij} + \Delta_{ij}(t)\mathcal{E}_{ij})x_j(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(A_{ij} + B_{ij}^{\perp}\Delta_{ij}^{\perp}(t)\mathcal{E}_{ij}^{\perp})x_j(t) - B_iF_ie_i(t) \\ - B_i\varphi_i(e_i, x_{r_i}, t).\end{aligned} \quad (\mathbf{A}.13)$$

Hence from (**A**.13) and the definition of $e_i(t)$, the following error subsystem can be obtained:

$$\begin{aligned}\frac{d}{dt}e_i(t) = (A_{K_i} - B_iF_i)e_i(t) + B_i\Delta_{ii}(t)\mathcal{E}_{ii}x_i(t) + B_i^{\perp}\Delta_{ii}^{\perp}(t)\mathcal{E}_{ii}^{\perp}x_i(t) \\ + B_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(\mathcal{D}_{ij} + \Delta_{ij}(t)\mathcal{E}_{ij})x_j(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(A_{ij} + B_{ij}^{\perp}\Delta_{ij}^{\perp}(t)\mathcal{E}_{ij}^{\perp})x_j(t) \\ - B_i\varphi_i(e_i, x_{r_i}, t).\end{aligned} \quad (\mathbf{A}.14)$$

From the above, the design problem in this appendix is to determine the compensation input $v_i(t)$ of (**A**.12) such that the tracking error $e_y(t) = (e_{y_1}^T(t), \cdots, e_{y_{\mathcal{N}}}^T(t))^T$ is satisfactorily small, namely the overall system tracks the reference model as closely as possible.

A.2 Design Method of Variable Gain Robust ϵ-Tracking Controller

By using the symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i \times n_i}$, we introduce the following Lyapunov function candidate so as to derive the proposed decentralized robust controller:

$$\mathcal{V}(e, t) \stackrel{\triangle}{=} \sum_{i=1}^{\mathcal{N}} \mathcal{V}_i(e_i, t), \quad (\mathbf{A}.15)$$

$$\mathcal{V}_i(e_i, t) \stackrel{\triangle}{=} e_i^T(t)\mathcal{P}_ie_i(t). \quad (\mathbf{A}.16)$$

For the quadratic function $\mathcal{V}_i(e_i,t)$, its time derivative along the trajectory of the error subsystem of (**A**.14) is given by

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(e_i,t) = {} & e_i^T(t)H_e\{(A_{K_i}-B_iF_i)^T\mathcal{P}_i\}e_i(t) \\ & + 2e_i^T(t)\mathcal{P}_iB_i\Delta_{ii}(t)\mathcal{E}_{ii}(e_i(t)+G_ix_{r_i}(t)) \\ & + 2e_i^T(t)\mathcal{P}_iB_i^{\perp}\Delta_{ii}^{\perp}(t)\mathcal{E}_{ii}^{\perp}(e_i(t)+G_ix_{r_i}(t)) \\ & + 2e_i^T(t)\mathcal{P}_iB_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(\mathcal{D}_{ij}+\Delta_{ij}(t)\mathcal{E}_{ij})(e_j(t)+G_jx_{r_j}(t)) \\ & + 2e_i^T(t)\mathcal{P}_i\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}(A_{ij}+B_{ij}^{\perp}\Delta_{ij}^{\perp}(t)\mathcal{E}_{ij}^{\perp})(e_j(t)+G_jx_{r_j}(t)) \\ & - 2e_i^T(t)\mathcal{P}_iB_i\varphi_i(e_i,x_{r_i},t). \qquad (\mathbf{A}.17)\end{aligned}$$

For the relation of (**A**.17), applying the well-known inequality for any vectors with appropriate dimensions and a positive scalar δ_i

$$2\alpha\beta \leq \delta\alpha^T\alpha + \frac{1}{\delta_i}\beta\beta \qquad (\mathbf{A}.18)$$

and some algebraic manipulations give

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(e_i,t) \leq {} & e_i^T(t)H_e\{(A_{K_i}-B_iF_i)^T\mathcal{P}_i\}e_i(t) + 2\left\|B_i^T\mathcal{P}_ie_i(t)\right\|\left\|\mathcal{E}_{ii}e_i(t)\right\| \\ & + 2\left\|B_i^T\mathcal{P}_ie_i(t)\right\|\left\|\mathcal{E}_{ii}G_ix_{r_i}(t)\right\| + 4(\mathcal{N}-1)\delta_ie_i^T(t)\mathcal{P}_iB_iB_i^T\mathcal{P}_ie_i(t) \\ & + \frac{1}{\delta_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}e_j^T(t)(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})e_j(t) \\ & + \frac{1}{\delta_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}x_{r_j}^T(t)G_j^T(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})G_jx_{r_j}(t) \\ & + 2\delta_ie_i^T(t)\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_ie_i(t) + \frac{1}{\delta_i}e_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}e_i(t) \\ & + \frac{1}{\delta_i}x_{r_i}^T(t)G_i^T\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}G_ix_{r_i}(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}}2\delta_ie_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_ie_i(t)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i e_i(t)\\
&+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_i} e_j^T(t)\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}e_j(t)\\
&+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_i} x_{r_j}^T(t)G_j^T\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}G_j x_{r_j}(t)\\
&-2e_i^T(t)\mathcal{P}_i B_i\varphi_i(e_i,x_{r_i},t).
\end{aligned} \tag{A.19}
$$

Now we define the nonlinear modification term $\varphi_i(e_i, x_{r_i}, t)$ as

$$
\varphi_i(e_i,x_{r_i},t)\triangleq\frac{\mu_i(e_i,x_{r_i},t)+\eta_i(e_i,t)}{\|B_i^T\mathcal{P}_i e_i(t)\|^2}B_i^T\mathcal{P}_i e_i(t), \tag{A.20}
$$

where scalar functions $\mu_i(e_i, x_{r_i}, t)$ and $\eta_i(e_i, t)$ are given by

$$
\begin{aligned}
\mu_i(e_i,x_{r_i},t)&\triangleq\|B_i\mathcal{P}_i e_i(t)\|\left(\|\mathcal{E}_{ii}e_i(t)\|+\|\mathcal{E}_{ii}G_i x_{r_i}(t)\|\right),\\
\eta_i(e_i,t)&\triangleq 2\delta_i(\mathcal{N}-1)\|B_i\mathcal{P}_i e_i(t)\|^2.
\end{aligned} \tag{A.21}
$$

By substituting (**A**.20) into (**A**.19) and some algebraic manipulations, we can derive the following relation for the quadratic function $\mathcal{V}_i(e_i, t)$:

$$
\begin{aligned}
\frac{d}{dt}\mathcal{V}_i(e_i,t)\le{}& e_i^T(t)H_e\{(A_{K_i}-B_iF_i)^T\mathcal{P}_i\}e_i(t)+\frac{1}{\delta_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} e_j^T(t)(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})e_j(t)\\
&+\frac{1}{\delta_i}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} x_{r_j}^T(t)G_j^T(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})G_j x_{r_j}(t)\\
&+2\delta_i e_i^T(t)\mathcal{P}_i B_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i e_i(t)+\frac{1}{\delta_i}e_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}e_i(t)\\
&+\frac{1}{\delta_i}x_{r_i}^T(t)G_i^T\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}G_i x_{r_i}(t)+\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i e_i(t)
\end{aligned}
$$

$$+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i e_i(t)$$

$$+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \frac{1}{\delta_i} e_j^T(t)\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}e_j(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \frac{1}{\delta_i} x_{r_j}^T(t)G_j^T\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}G_j x_{r_j}(t). \tag{A.22}$$

Therefore, from (**A**.15), (**A**.16) and (**A**.22), the inequality of (**A**.23) for $\mathcal{V}(e,t)$ can be obtained

$$\begin{aligned}
\frac{d}{dt}\mathcal{V}(e,t) \leq\ & \sum_{i=1}^{\mathcal{N}} e_i^T(t)H_e\{(A_{K_i}-B_iF_i)^T\mathcal{P}_i\}e_i(t)\\
&+\sum_{i=1}^{\mathcal{N}}\frac{1}{\delta_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} e_j^T(t)(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})e_j(t)\\
&+\sum_{i=1}^{\mathcal{N}}\frac{1}{\delta_i}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} x_{r_j}^T(t)G_j^T(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{E}_{ij}^T\mathcal{E}_{ij})G_j x_{r_j}(t)\\
&+\sum_{i=1}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i B_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i e_i(t)+\sum_{i=1}^{\mathcal{N}}\frac{1}{\delta_i}e_i^T(t)\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}e_i(t)\\
&+\sum_{i=1}^{\mathcal{N}}\frac{1}{\delta_i}x_{r_i}^T(t)G_i^T\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}G_i x_{r_i}(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i e_i(t)\\
&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} 2\delta_i e_i^T(t)\mathcal{P}_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i e_i(t)\\
&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\delta_i}e_j^T(t)\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}e_j(t)\\
&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\delta_i}x_{r_j}^T(t)G_j^T\left\{I_n+\left(\mathcal{E}_{ij}^{\perp}\right)^T\mathcal{E}_{ij}^{\perp}\right\}G_j x_{r_j}(t).
\end{aligned} \tag{A.23}$$

The inequality of (**A**.23) can be rewritten as

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(e,t) \leq \sum_{i=1}^{\mathcal{N}} e_i^T(t)\Bigg\{& H_e\{(A_{K_i} - B_iF_i)^T\mathcal{P}_i\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}) \\ &+ 2\delta_i\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i + \frac{1}{\delta_i}\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i\mathcal{P}_iB_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\left\{I_n + \left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\right\}\Bigg\}e_i(t) \\ + \sum_{i=1}^{\mathcal{N}} x_{r_i}^T(t)G_i^T\Bigg\{& \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}) + \frac{1}{\delta_i}\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\left\{I_n + \left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\right\}\Bigg\}G_ix_{r_i}(t). \qquad (\mathbf{A}.24)\end{aligned}$$

From the assumption of (**A**.4) for the reference model, there exists a positive scalar σ_i satisfying the following relation:

$$\begin{aligned}x_{r_i}^T(t)G_i^T\Bigg\{& \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}) + \frac{1}{\delta_i}\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\left\{I_n + \left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\right\}\Bigg\}G_ix_{r_i}(t) \leq \sigma_i. \qquad (\mathbf{A}.25)\end{aligned}$$

Furthermore, we assume that for the symmetric positive definite matrix $\mathcal{P}_i \in \mathbb{R}^{n_i\times n_i}$, the fixed compensation gain matrix F_i and positive scalars α_i, δ_i and δ_j, the inequality

$$H_e\{(A_{K_i} - B_iF_i)^T\mathcal{P}_i\} + \sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji}) + 2\delta_i\mathcal{P}_iB_i^{\perp}\left(B_i^{\perp}\right)^T\mathcal{P}_i$$

$$+\frac{1}{\delta_i}\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp} + \sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} 2\delta_i\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i + \sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} 2\delta_i\mathcal{P}_iB_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T\mathcal{P}_i$$

$$+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\left\{I_n + \left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\right\} < -\alpha_iI_n \tag{A.26}$$

holds. Therefore, from (**A**.24) – (**A**.26), we have

$$\frac{d}{dt}\mathcal{V}(e,t) < -\sum_{i=1}^{\mathcal{N}}\alpha_ie_i^T(t)e_i(t) + \sum_{i=1}^{\mathcal{N}}\sigma_i$$
$$= -e^T(t)\mathcal{A}e(t) + \sigma. \tag{A.27}$$

In (**A**.27), $\mathcal{A} \triangleq \mathrm{diag}(\alpha_1, \cdots, \alpha_{\mathcal{N}})$ and $\sigma \triangleq \sum_{i=1}^{\mathcal{N}}\sigma_i$. One can easily see that the following relation is obvious:

$$e^T(t)\mathcal{A}e(t) \geq \lambda_{\min}(\mathcal{A})\|e(t)\|^2, \tag{A.28}$$

and thus the inequality of (**A**.27) can be rewritten as

$$\frac{d}{dt}\mathcal{V}(e,t) < -\min_i\{\alpha_i\}\|e(t)\|^2 + \sigma. \tag{A.29}$$

Therefore if the inequality of (**A**.26) is satisfied, then the upper bound of $e(t)$ is given by

$$\|e(t)\| < \sqrt{\frac{\sigma}{\min_i\{\alpha_i\}}}, \tag{A.30}$$

i.e., from (**A**.9) and (**A**.30), the upper bound of $e_y(t)$ is given by

$$\|e_y(t)\| < |C|\sqrt{\frac{\sigma}{\min_i\{\alpha_i\}}}, \tag{A.31}$$

where $C = \mathrm{diag}\left(C_1, \cdots, C_{\mathcal{N}}\right)$.

Finally, we consider the inequality of (**A**.26). By introducing the symmetric positive definite matrices $\mathcal{S}_i \stackrel{\triangle}{=} \mathcal{P}_i^{-1}$ and matrices $\mathcal{W}_i \stackrel{\triangle}{=} F_i\mathcal{S}_i$, and by pre- and post-multiplying both sides of the inequality of (**A**.26) by $\mathcal{S}_i$, we have

$$\begin{aligned} &H_e\{A_{K_i}\mathcal{S}_i - B_i\mathcal{W}_i\} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\mathcal{S}_i(\mathcal{D}_{ji}^T\mathcal{D}_{ji} + \mathcal{E}_{ji}^T\mathcal{E}_{ji})\mathcal{S}_i + 2\delta_i B_i^{\perp}\left(B_i^{\perp}\right)^T \\ &+ \frac{1}{\delta_i}\mathcal{S}_i\left(\mathcal{E}_{ii}^{\perp}\right)^T\mathcal{E}_{ii}^{\perp}\mathcal{S}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i A_{ij}A_{ij}^T + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \delta_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\delta_j}\mathcal{S}_i\left\{I_n + \left(\mathcal{E}_{ji}^{\perp}\right)^T\mathcal{E}_{ji}^{\perp}\right\}\mathcal{S}_i + \alpha_i\mathcal{S}_i\mathcal{S}_i < 0. \end{aligned} \qquad (\mathbf{A}.32)$$

Thus by applying **Lemma 1.2** (Schur complement) to (**A**.32), we find that the inequalities of (**A**.32) are equivalent to the following LMIs:

$$\left(\begin{array}{c:c} \Theta_i(\mathcal{S}_i, \mathcal{W}_i, \delta_i) & \Lambda_i(\mathcal{S}_i) \\ \hdashline \star & -\Omega_i(\beta_i, \delta_i) \end{array}\right) < 0. \qquad (\mathbf{A}.33)$$

where, matrices $\Theta_i(\mathcal{S}_i, \mathcal{W}_i, \delta_i)$, $\Lambda_i(\mathcal{S}_i)$ and $\Omega_i(\beta_i, \delta_i)$ are given by

$$\begin{aligned} \Theta_i(\mathcal{S}_i, \mathcal{W}_i, \delta_i) &\stackrel{\triangle}{=} H_e\{A_{K_i}\mathcal{S}_i - B_i\mathcal{W}_i\} + 2\delta_i B_i^{\perp}\left(B_i^{\perp}\right)^T + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i A_{ij}A_{ij}^T \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} 2\delta_i B_{ij}^{\perp}\left(B_{ij}^{\perp}\right)^T, \end{aligned} \qquad (\mathbf{A}.34)$$

$$\begin{aligned} \Lambda_i(\mathcal{S}_i) \stackrel{\triangle}{=} (&\mathcal{S}_i \;\; \mathcal{S}_i\mathcal{D}_{1i}^T \;\; \mathcal{S}_i\mathcal{E}_{1i}^T \;\; \cdots \;\; \mathcal{S}_i\mathcal{D}_{i-1i}^T \;\; \mathcal{S}_i\mathcal{E}_{i-1i}^T \;\; \mathcal{S}_i\mathcal{D}_{i+1i}^T \;\; \mathcal{S}_i\mathcal{E}_{i+1i}^T \;\; \cdots \\ &\cdots \;\; \mathcal{S}_i\mathcal{D}_{\mathcal{N}i}^T \;\; \mathcal{S}_i\mathcal{E}_{\mathcal{N}i}^T \;\; \mathcal{S}_i\left(\mathcal{E}_{ii}^{\perp}\right)^T \;\; \mathcal{S}_i \;\; \mathcal{S}_i\left(\mathcal{E}_{1i}^{\perp}\right)^T \;\; \mathcal{S}_i \;\; \mathcal{S}_i\left(\mathcal{E}_{2i}^{\perp}\right)^T \;\; \cdots \\ &\cdots \;\; \mathcal{S}_i \;\; \mathcal{S}_i\left(\mathcal{E}_{\mathcal{N}i}^{\perp}\right)^T), \end{aligned} \qquad (\mathbf{A}.35)$$

$$\begin{aligned} \Omega_i(\beta_i, \delta_i) \stackrel{\triangle}{=} \mathrm{diag}(&\beta_i I_{n_i},\; \delta_1 I_{m_i},\; \delta_1 I_{s_{1i}}, \cdots,\; \delta_{i-1}I_{m_i},\; \delta_{i-1}I_{s_{i-1\,i}},\; \delta_{i+1}I_{m_i},\; \delta_{i+1}I_{s_{i+1\,i}}, \\ &\cdots,\; \delta_{\mathcal{N}}I_{m_i},\; \delta_{\mathcal{N}}I_{s_{\mathcal{N}\,i}},\; \delta_i I_{q_{ii}},\; \delta_1 I_{n_i},\; \delta_1 I_{q_{1i}}, \cdots,\; \delta_{i-1}I_{n_i},\; \delta_{i-1}I_{q_{i-1\,i}}, \\ &\delta_{i+1}I_{n_i},\; \delta_{i+1}I_{q_{i+1\,i}}, \cdots,\; \delta_{\mathcal{N}}I_{n_i},\; \delta_{\mathcal{N}}I_{q_{\mathcal{N}\,i}}), \end{aligned} \qquad (\mathbf{A}.36)$$

and a scalar β_i is defined as $\beta_i \stackrel{\triangle}{=} \alpha_i^{-1}$. Therefore by using the solution of the LMIs of

(**A**.33), the fixed compensation gain matrix F_i and the nonlinear modification term $\varphi_i(e_i, x_{r_i}, t)$ are determined as $F_i \stackrel{\triangle}{=} \mathcal{W}_i \mathcal{S}_i^{-1}$ and (**A**.20), respectively.

Summarizing the above, we obtain the following theorem.

Theorem A.1 *Consider the large-scale interconnected system of (**A**.1) and the control input of (**A**.9).*

*If there exist symmetric positive definite matrices $\mathcal{S}_i \in \mathbb{R}^{n_i \times n_i}$, matrices $\mathcal{W}_i \in \mathbb{R}^{m_i \times n_i}$ and positive constants β_i and δ_i which satisfy the LMIs of (**A**.33), the fixed compensation gain matrix $F_i \in \mathbb{R}^{m_i \times n_i}$ and the nonlinear modification term $\varphi_i(e_i, x_{r_i}, t) \in \mathbb{R}^{m_i}$ are determined as $F_i \stackrel{\triangle}{=} \mathcal{W}_i \mathcal{S}_i^{-1}$ and (**A**.20). Then the upper bound of $e_y(t)$ is given by (**A**.31).*

Remark A.1 *The nonlinear modification term $\varphi_i(e_i, x_{r_i}, t)$ of (**A**.20) is bounded, because one can easily see that the norm of the function $\varphi_i(e_i, x_i, t)$ can be represented as $\left\|\varphi_i(e_i, x_i, t)\right\| = \|\mathcal{E}_{ii} e_i(t)\| + \|\mathcal{E}_{ii} G_i x_{r_i}(t)\| + 2\delta_i(\mathcal{N} - 1)\|B_i \mathcal{P}_i e_i(t)\|$.*

Remark A.2 *The tracking performance of the proposed decentralized controller is high comparing with the conventional decentralized control scheme, because the feasible region of the LMIs of (**A**.33) is more larger than one of the LMIs corresponding to the conventional decentralized controller design strategies, and the relation $\sigma_i < \sigma_i^{\mathcal{C}}$ for the parameters σ_i of (**A**.25) and $\sigma_i^{\mathcal{C}}$ corresponding to σ_i is satisfied. Therefore, we find that the proposed decentralized variable gain robust ϵ-tracking controller is useful.*

A.3 Numerical Examples

A numerical example is provided to demonstrate the efficiency of the proposed robust controller.

The uncertain large-scale interconnected systems consisting of three two-dimensional subsystems ($\mathcal{N} = 3$) is involved. The parameters of the controlled system are given by

$$A_{11} = \begin{pmatrix} -1.0 & 1.0 \\ 1.0 & 0.1 \end{pmatrix}, \ A_{22} = \begin{pmatrix} 1.0 & 3.0 \\ 0.0 & -2.0 \end{pmatrix}, \ A_{33} = \begin{pmatrix} -1.0 & 1.0 \\ 2.0 & 0.0 \end{pmatrix},$$
$$A_{12} = \begin{pmatrix} 3.0 \times 10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix}, \ A_{13} = \begin{pmatrix} 0.0 & 3.0 \times 10^{-1} \\ 0.0 & 0.0 \end{pmatrix},$$

$$
\begin{aligned}
&A_{21} = \begin{pmatrix} 0.0 & 0.0 \\ 2.0\times10^{-1} & 0.0 \end{pmatrix},\ A_{23} = \begin{pmatrix} 0.0 & 0.0 \\ 3.0\times10^{-1} & 0.0 \end{pmatrix},\\
&A_{31} = \begin{pmatrix} 0.0 & 2.0\times10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\ A_{32} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\\
&B_1 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ B_2 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ B_3 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ C_1^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\\
&C_2^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ C_3^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{E}_{11} = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{22} = \begin{pmatrix} 0.0 & 1.0 \\ 1.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{33} = \begin{pmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{pmatrix},\ B_1^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ B_2^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 3.0\times10^{-1} & 0.0 \end{pmatrix},\\
&B_3^{\perp} = \begin{pmatrix} 0.0 & 5.0\times10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{11}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{22}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{33}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 2.0\times10^{-1} & 0.0 \end{pmatrix},\ \mathcal{D}_{12}^T = \begin{pmatrix} 0.0 \\ 2.0 \end{pmatrix},\\
&\mathcal{D}_{13}^T = \begin{pmatrix} 2.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{21}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{D}_{23}^T = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix},\ \mathcal{D}_{31}^T = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix},\\
&\mathcal{D}_{32}^T = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix},\ \mathcal{E}_{12} = \begin{pmatrix} 1.0 & 1.0 \\ 0.0 & 1.0 \end{pmatrix},\ \mathcal{E}_{13} = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{21} = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix},\\
&\mathcal{E}_{23} = \begin{pmatrix} 0.0 & 2.0 \\ 1.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{31} = \begin{pmatrix} 1.0 & 0.0 \\ 1.0 & 1.0 \end{pmatrix},\ \mathcal{E}_{32} = \begin{pmatrix} 0.0 & 0.0 \\ 1.0 & 0.0 \end{pmatrix},\\
&B_{12}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ B_{13}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\\
&B_{21}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 1.0\times10^{-1} & 0.0 \end{pmatrix},\ B_{23}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 1.0\times10^{-1} & 0.0 \end{pmatrix},\\
&B_{31}^{\perp} = \begin{pmatrix} 2.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ B_{32}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{12}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 2.0\times10^{-1} & 0.0 \end{pmatrix},\ \mathcal{E}_{13}^{\perp} = \begin{pmatrix} 0.0 & 1.0\times10^{-1} \\ 0.0 & 0.0 \end{pmatrix},\\
&\mathcal{E}_{21}^{\perp} = \begin{pmatrix} 3.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{23}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 2.0\times10^{-1} \end{pmatrix},\\
&\mathcal{E}_{31}^{\perp} = \begin{pmatrix} 1.0\times10^{-1} & 0.0 \\ 0.0 & 0.0 \end{pmatrix},\ \mathcal{E}_{32}^{\perp} = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 2.0\times10^{-1} \end{pmatrix},
\end{aligned}
\tag{A.37}
$$

and the parameters of the reference models for each subsystem are given by

$$\begin{aligned} &A_{r_1} = \begin{pmatrix} 0.0 & 1.0 \\ -1.0 & 0.0 \end{pmatrix}, \ A_{r_2} = \begin{pmatrix} 0.0 & -1.0 \\ 1.0 & 0.0 \end{pmatrix}, \ A_{r_3} = \begin{pmatrix} 0.0 & 1.0 \\ -1.0 & 0.0 \end{pmatrix}, \\ &C_{r_1} = \begin{pmatrix} 0.0 & 1.0 \end{pmatrix}, \ C_{r_2} = \begin{pmatrix} 0.0 & 1.0 \end{pmatrix}, \ C_{r_3} = \begin{pmatrix} 0.0 & 1.0 \end{pmatrix}. \end{aligned} \tag{\mathbf{A}.38}$$

Firstly, by solving the the matrix equation of (**A**.5), the solutions $G_i \in \mathbb{R}^{2\times 2}$ and $H_i \in \mathbb{R}^{1\times 2}$ are obtained as

$$\begin{aligned} &G_1 = \begin{pmatrix} 0.0 & 1.0 \\ -1.0 & 0.1 \end{pmatrix}, \ G_2 = \begin{pmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{pmatrix}, \ G_3 = \begin{pmatrix} 0.4 & -0.2 \\ 1.0 & 0.0 \end{pmatrix}, \\ &H_1 = \begin{pmatrix} -0.9 & -2.1 \end{pmatrix}, \ H_2 = \begin{pmatrix} 1.0 & -1.0 \end{pmatrix}, \ H_3 = \begin{pmatrix} -0.8 & 1.4 \end{pmatrix}. \end{aligned} \tag{\mathbf{A}.39}$$

Now, in order to design the feedback gain matrices $K_i \in \mathbb{R}^{1\times 2}$, we consider the standard LQ optimal control problem. By selecting weighting matrices $\mathcal{Q}_i \in \mathbb{R}^{2\times 2}$ and $\mathcal{R}_i$ as $\mathcal{Q}_1 = \mathcal{Q}_2 = \mathcal{Q}_3 = I_2$ and $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = 1$, respectively, we solve the algebraic Riccati equation of (2.5). Then we can obtain the LQ optimal gain matrices $K \in \mathbb{R}^{1\times 2}$ as

$$\begin{aligned} K_1 &= \begin{pmatrix} 1.0361 & 1.8556 \end{pmatrix}, \\ K_2 &= \begin{pmatrix} 2.4142 & 2.1213 \end{pmatrix}, \\ K_3 &= \begin{pmatrix} 2.1481 & 2.3014 \end{pmatrix}. \end{aligned} \tag{\mathbf{A}.40}$$

Next, we consider **Theorem A.1**. By solving LMIs of (**A**.33), we can obtain the following solutions:

$$\begin{aligned} &\mathcal{S}_1 = \begin{pmatrix} 5.5310 & -5.5213 \\ \star & 1.0599 \end{pmatrix} \times 10^{-1}, \ \mathcal{W}_1^T = \begin{pmatrix} 1.0540 \\ 6.5998 \times 10^{-1} \end{pmatrix}, \\ &\mathcal{S}_2 = \begin{pmatrix} 6.0460 & -7.0482 \\ \star & 5.7199 \end{pmatrix} \times 10^{-1}, \ \mathcal{W}_2^T = \begin{pmatrix} 5.7658 \\ 6.3485 \end{pmatrix} \times 10^{-1}, \\ &\mathcal{S}_3 = \begin{pmatrix} 3.7559 & -3.2534 \\ \star & 8.1638 \end{pmatrix} \times 10^{-1}, \ \mathcal{W}_3^T = \begin{pmatrix} 8.7626 \\ 1.3804 \end{pmatrix} \times 10^{-1}, \\ &\beta_1 = 3.4846, \ \beta_2 = 3.0113, \ \beta_3 = 3.0437, \\ &\delta_1 = 2.5598, \ \delta_2 = 2.8465, \ \delta_3 = 2.6082. \end{aligned} \tag{\mathbf{A}.41}$$

Therefore, the symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{2\times 2}$, the positive scalars

α_i and the fixed compensation gain matrices $F_i \in \mathbb{R}^{1\times 2}$ can be computed as

$$\begin{aligned}
&\mathcal{P}_1 = \begin{pmatrix} 3.7667 & 1.9621 \\ \star & 1.9656 \end{pmatrix}, \ \mathcal{P}_2 = \begin{pmatrix} 1.6781 & 2.0678 \times 10^{-1} \\ \star & 1.7738 \end{pmatrix}, \\
&\mathcal{P}_3 = \begin{pmatrix} 4.0661 & 1.6204 \\ \star & 1.8707 \end{pmatrix}, \\
&\alpha_1 = 3.4846, \ \alpha_2 = 3.0113, \ \alpha_3 = 3.0437, \\
&F_1 = \begin{pmatrix} 5.2651 & 3.3654 \end{pmatrix}, \ F_2 = \begin{pmatrix} 1.0988 & 1.2453 \end{pmatrix}, \ F_3 = \begin{pmatrix} 3.7867 & 1.6781 \end{pmatrix}.
\end{aligned} \tag{\textbf{A}.42}$$

In this example, initial values of large-scale interconnected system of (**A**.37) and reference models of (**A**.38) are selected as

$$x(0) = \begin{pmatrix} -1.0 & -2.0 & \vdots & -1.0 & 0.0 & \vdots & 0.0 & 2.0 \end{pmatrix}^T, \tag{\textbf{A}.43}$$

and $x_{r_1}(0) = x_{r_2}(0) = x_{r_3}(0) = (1.0\ 1.0)^T$. Furthermore, unknown parameters are given by

$$\begin{aligned}
&\Delta_{ii}(t) = \begin{pmatrix} \cos(2.0\pi t) & 0 \end{pmatrix}, \ \Delta_{ij}(t) = \begin{pmatrix} 0 & \cos(-\pi t) \end{pmatrix}, \\
&\Delta_{ii}^{\perp}(t) = \mathrm{diag}\begin{pmatrix} \sin(-6.0\pi t), & \cos(-6.0\pi t) \end{pmatrix}, \\
&\Delta_{ij}^{\perp}(t) = \mathrm{diag}\begin{pmatrix} -\cos(\pi t), & \sin(\pi t) \end{pmatrix}.
\end{aligned} \tag{\textbf{A}.44}$$

Figures **A**.1–**A**.5 are the simulation results of this numerical example. These figures show that the time histories of the output $y(t)$ and $\overline{y}(t)$, the norm of the error $e_y(t) = y(t) - \overline{y}(t)$ and the proposed decentralized variable gain robust controller $u(t)$.

From these figures, we can see that the proposed decentralized variable gain robust controller achieves good tracking performance for each subsystem with uncertainties. Therefore, the effectiveness of the proposed decentralized variable gain robust controller have been shown.

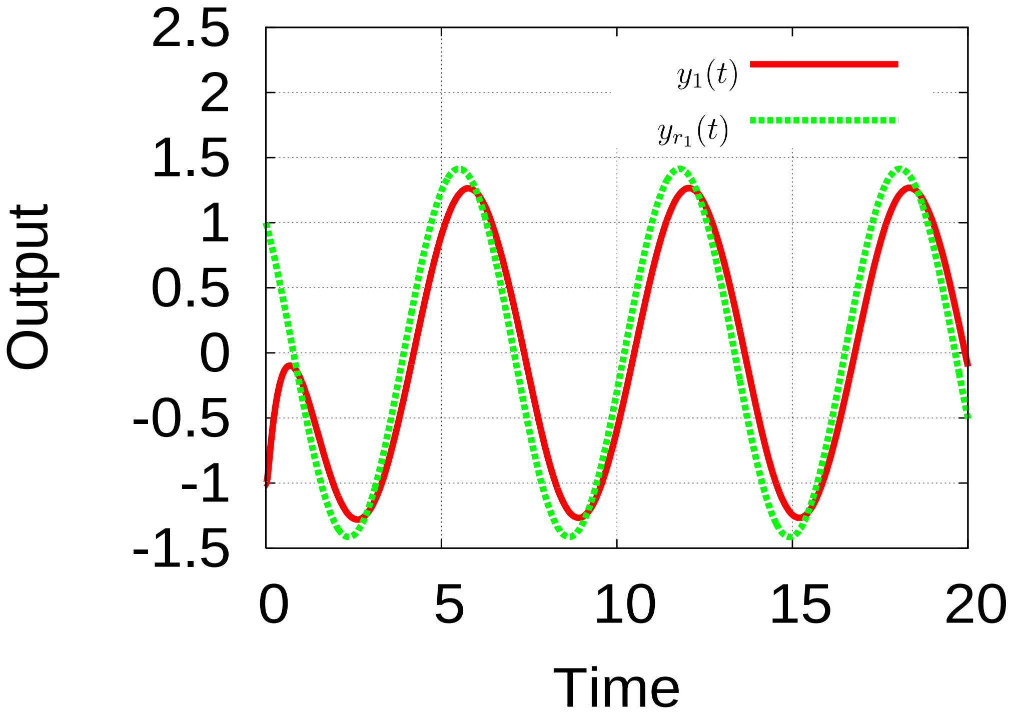

Figure **A**.1: Time histories of $y_1(t)$ and $\overline{y}_1(t)$

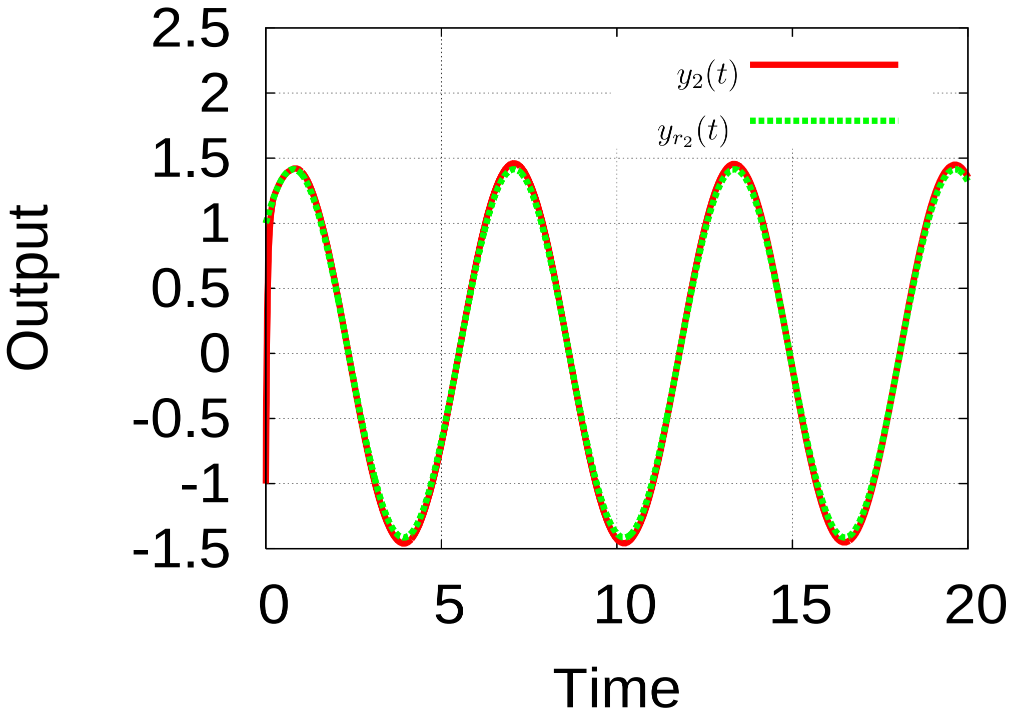

Figure **A**.2: Time histories of $y_2(t)$ and $\overline{y}_2(t)$

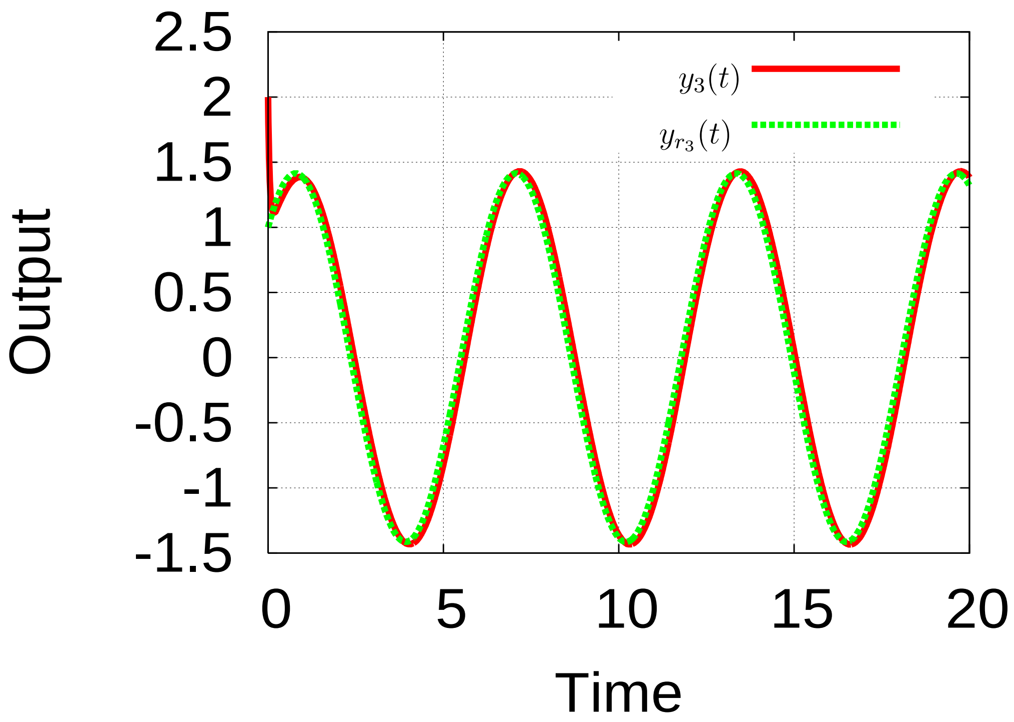

Figure **A**.3: Time histories of $y_3(t)$ and $\overline{y}_3(t)$

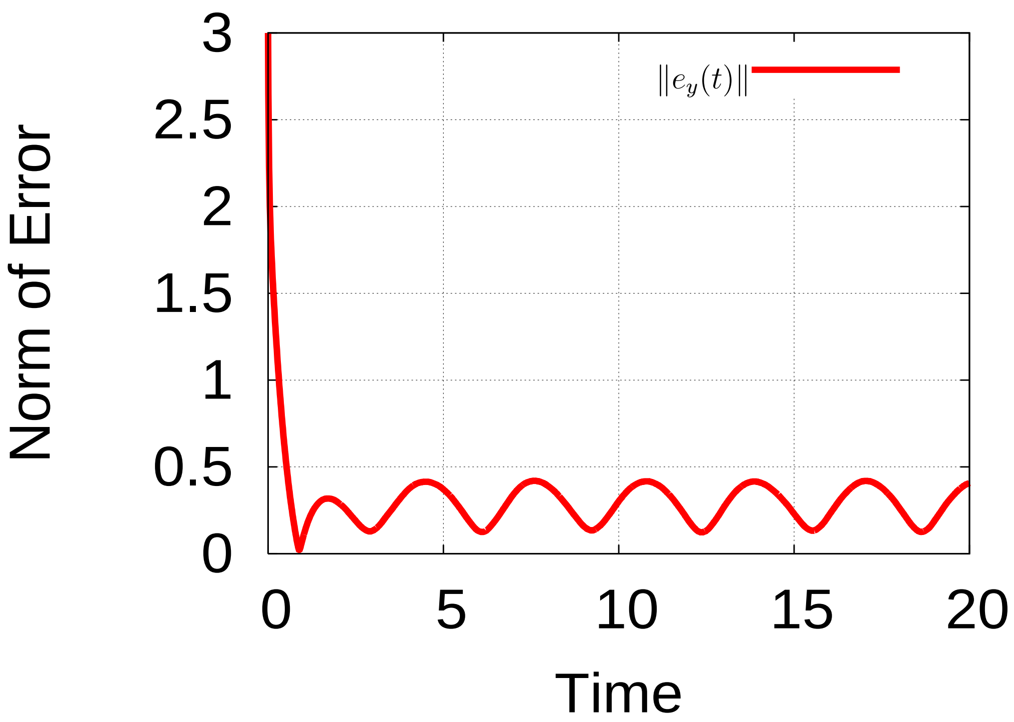

Figure **A**.4: Time histories of $\|e_y(t)\|$

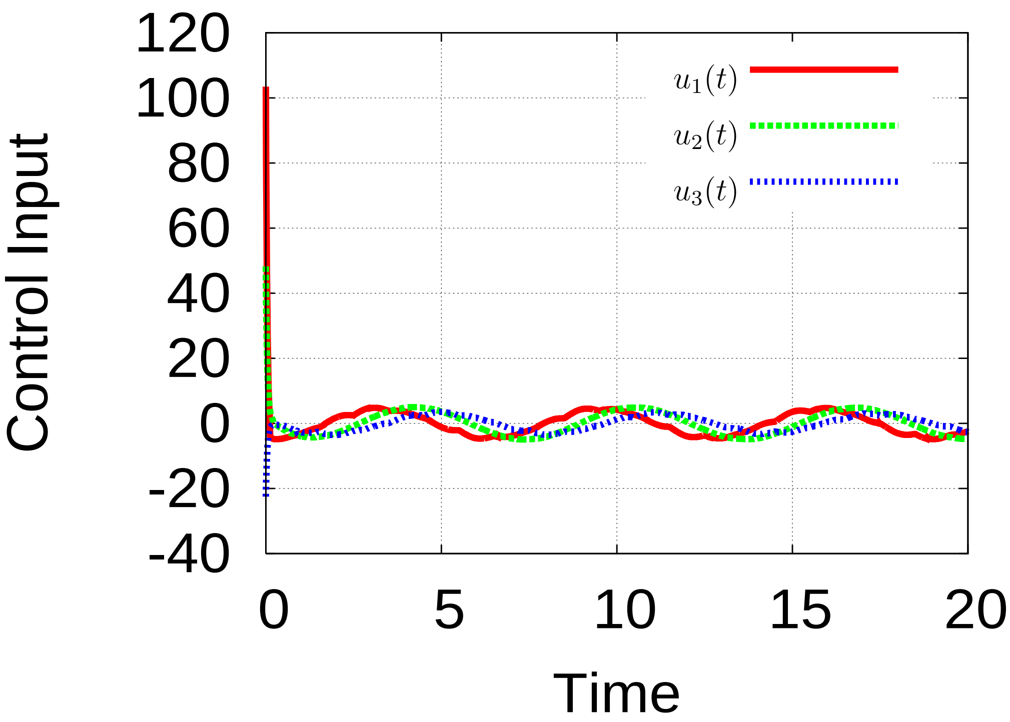

Figure **A**.5: Time histories of $u(t)$