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A Numerical Method for Distinction between Blow-up and Global Solutions of the Nonlinear Heat Equation

By

Hideo Sakaguchi and Hitoshi Imai

Institute of Technology and Science, The University of Tokushima
Minami-josanjima, Tokushima 770-8506, JAPAN
e-mail address: saka@pm.tokushima-u.ac.jp
imai@pm.tokushima-u.ac.jp
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Abstract

The famous one-dimensional nonlinear heat equation is considered. To this equation a numerical method for distinction between blow-up and global solutions is proposed. Difficulty is in the treatment of the global solution which is defined in the infinite interval. The bounding transform is used to overcome this difficulty. Numerical experiments show the validity of our method.

2000 Mathematics Subject Classification. 65-05, 65M70

Introduction

Following the unique paper[7] there have been a lot of preceding researches on blow-up solutions for nonlinear heat equations. In the paper, the initial and boundary value problem governed by the famous one-dimensional nonlinear heat equation as follows:

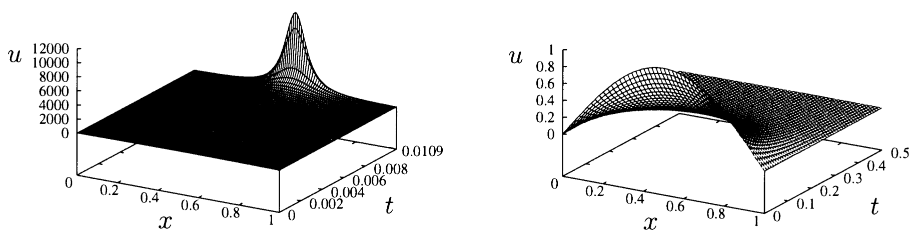
Problem 1 For two parameters $\alpha \geqq 0$, and $T > 0$, find $u(t, x)$ such that

$$
\begin{aligned}
&u_t = u_{xx} + u^2, && 0 < t < T,\ 0 < x < 1,\\
&u(t, 0) = 0, && 0 \leqq t < T,\\
&u(t, 1) = 0, && 0 \leqq t < T,\\
&u(0, x) = \alpha \sin \pi x, && 0 < x < 1.
\end{aligned}
$$

Numerical methods have been proposed to such a problem with the blow-up solution [14, 3, 9, 4]. They adopt the adaptive control on the time increment, i.e.

the time increment varies depending on the solution. This technique is useful for the computation of the blow-up time. The blow-up time T_b in Problem 1 with $\alpha = 100$ was computed to be approximately 0.01098[9].

By the way, it is well-known that Problem 1 has global solutions for small initial data and blow-up solutions for large initial data[7, 5, 6, 12, 13]. Numerical results by FDM$\oplus$EE mentioned in §1 shows these situations(Fig.1).



(a) Blow-up($\alpha = 100$). (b) No blow-up ($\alpha = 1$).

Fig. 1. Solution profiles.

For $\alpha = 100$ overflow easily occurs beyond $t = 0.0109$, so Fig.1(a) is recognized to show the profile of the blow-up solution. On the other hand, without any theoretical results it is vague that Fig.1(b) show the profile of the global decreasing solution because numerical computation is local.

In the paper a numerical method for distinction between blow-up and global solutions is proposed. Difficulty is global computation in time. To overcome this difficulty the bounding transform[10] is adopted. For precise numerical computation spectral collocation method is adopted. This method may offer new possibility of computation of the blow-up time.

1 Our numerical method

We consider the more complicated problem which is derived from Problem 1 by using the following transformations:

$$\tau = t/\beta, \quad \hat{u}(\tau, x) = \beta u(t, x)$$

where β is a positive constant. Then, Problem 2 is obtained.

Problem 2 For three parameters $\alpha \geqq 0,\ \beta > 0,$ and $T > 0$, find $\hat{u}(\tau, x)$ such that

$$\begin{aligned}
&\hat{u}_\tau = \beta \hat{u}_{xx} + \hat{u}^2, && 0 < \tau < T/\beta,\ 0 < x < 1,\\
&\hat{u}(\tau, 0) = 0, && 0 \leqq \tau < T/\beta,\\
&\hat{u}(\tau, 1) = 0, && 0 \leqq \tau < T/\beta,\\
&\hat{u}(0, x) = \beta\alpha \sin \pi x, && 0 < x < 1.
\end{aligned}$$

If $\beta = 1$ Problem 2 is equivalent to Problem 1. For small β the blow-up time $\tau_b (= T_b/\beta)$ becomes large. In this case numerical distinction that the solution is the global one or the blow-up one becomes difficult. For example, $T_b = 0.11$ in Problem 1 corresponds to $\tau_b = 0.11 \times 10^8$ in Problem 2 with $\beta = 10^{-8}$. The following bounding transform on τ is introduced for the treatment of the global solution[10].

$$\tau = \frac{s}{1 - s^2} \quad \left(s(\tau) = \frac{2\tau}{1 + \sqrt{1 + 4\tau^2}} \right).$$

The interval $[0, \infty)$ on τ is mapped onto the interval $[0, 1)$ on s. From this transform Problem 2 becomes the following Problem 3.

Problem 3 For three parameters $\alpha \geqq 0,\ \beta > 0,$ and $T > 0$, find $\tilde{u}(\tau, x)$ such that

$$\begin{aligned}
&\tilde{u}_s = \frac{1 + s^2}{(1 - s^2)^2}(\beta \tilde{u}_{xx} + \tilde{u}^2), && 0 < s < s(T/\beta),\ 0 < x < 1,\\
&\tilde{u}(s, 0) = 0, && 0 \leqq s < s(T/\beta),\\
&\tilde{u}(s, 1) = 0, && 0 \leqq s < s(T/\beta),\\
&\tilde{u}(0, x) = \beta\alpha \sin \pi x, && 0 < x < 1.
\end{aligned}$$

This problem is defined in the bounded domain and $s(T/\beta) = 1$ means $T = \infty$. Thus, the global solution in Problem 2 can be computed by solving Problem 3.

Our method for distinction between blow-up and global solutions is as follows.

Numerical computation is carried out by two types of discretization. One is FDM$\oplus$EE(second order finite difference method in space and first order explicit Euler method in time) and another is SCM(spectral collocation method) which is easily applicable to nonlinear problems[2]. In SCM Chebyshev-Gauss-Lobatto(CGL) collocation points are used in space, and Chebyshev-Gauss-Radau(CGR) collocation points or CGL points are used in time. Discretized equations by SCM are nonlinear, so Newton method is used for solving them. If exponential convergence of numerical solutions by SCM is obtained, then (converged) numerical solutions are very accurate and reliable[2, 11]. In FDM⊕EE the adaptive control on the time increment is not used for global computing. FDM⊕EE is so simple that it is firstly applied for rough numerical computation.

Concrete procedure is as follows:

0) Set $s_l = 0$ and choose the time increment $\Delta s (> 0)$ for EE adequately.

1) For $s_l \leqq s$ compute the solution profile by using FDM⊕EE. If necessary Δs may be varied or multiple precision is adopted. (If overflow occurs at $s = s_e (< 1)$ then the solution is probably of the blow-up type. If numerical computation works well until $s = s_e = 1$ then the solution is probably of the global type.)

2) Referring the solution profile obtained for $s_l \leqq s \leqq s_e$ in the above step 1), choose the interval $[s_l, s_r](s_r \leqq s_e)$ where the profile seems to be smooth and carry out numerical computation by SCM in this interval. $s_r (\leqq s_e)$ should be chosen for realizing exponential convergence. In this interval there is no blow-up solution. In the case where $s_r < 1$ and Newton method does not converge determination of the blow-up solution is done referring the solution profile.

3) If distinction between blow-up and global solutions can not be clear in the above step 2), set $s_l = s_r$ and go to the step 1) with the initial data at $s = s_r$ that is computed in the step 2).

The above procedure is not rigorous. In the practical situation trial and error is inevitable.

We should remark that our method is not perfect. For example, the grow-up solution in Problem 2 becomes the discontinuous solution at $s = 1$ in Problem 3. Numerical computation to such a solution is very difficult[16]. However, our method can realize global numerical computation and it may offer a new field of numerical analysis.

2 Numerical results

Numerical computation is basicly carried out in double precision. However, some results are computed in multiple precision[8].

In SCM N_s, N_x denote approximation orders on the time and space variables, respectively. In FDM⊕EE N_s, N_x denote devision numbers on the time and space variables, respectively. $\Delta s = 2/N_s, \Delta x = 2/N_x$. Moreover, $N_x = 20$ in FDM⊕EE because FDM⊕EE is used only for rough computation of the solution profile. The following Err_N is used for checking the convergence of solutions by SCM.

$$Err_N = \frac{\max\limits_{0 \leqq i,j \leqq 50} |v_N(s_j, x_i) - v_{N+10}(s_j, x_i)|}{\max\limits_{0 \leqq i,j \leqq 50} |v_{N+10}(s_j, x_i)|},$$

$$x_i = i/50,\ s_j = (s_e - s_l)j/50 + s_l,\ i, j = 0, \cdots, 50$$

where $v_N(s_j, x_i)$ is the interpolant for the data $\tilde{u}(s_j, x_i)$ which is computed in $0 \leqq x \leqq 1,\ s_l \leqq s \leqq s_e$ by SCM with $N = N_x = N_s$. s_j is the CGL or CGR points. x_i is the CGL points. In SCM discretized equations are nonlinear, so Newton method is used. The convergence of Newton method is determined whether the absolute relative difference of numerical solutions is smaller than $\varepsilon(\varepsilon = 10^{-13}$ in double precision, $\varepsilon = 10^{-47}$ in 50 digits) or not in 20 iterations.

(i) In the case of $\beta = 1$.
Numerical results for $\alpha = 1$ by FDM⊕EE are shown in Fig.2 and Table 1.

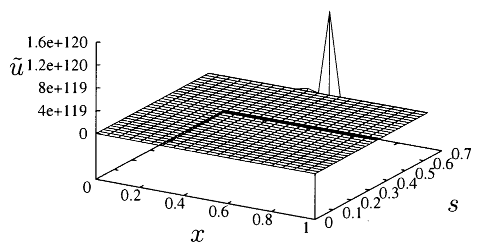
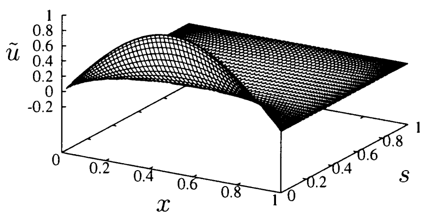


Fig. 2. Solution profile by FDM⊕EE ($\beta = 1,\ \alpha = 1,\ N_x = 20,\ \Delta s = 5 \times 10^{-4}$).

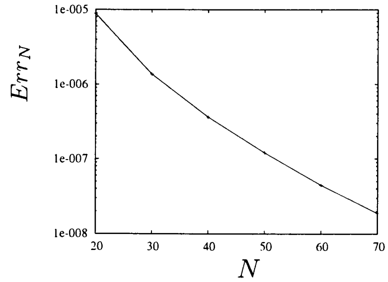
Table 1. Overflow time ($\beta = 1,\ \alpha = 1,\ N_x = 20$).

Δs	time
5×10^{-4}	0.6235
10^{-4}	0.8269
10^{-5}	0.94328
10^{-6}	0.981787

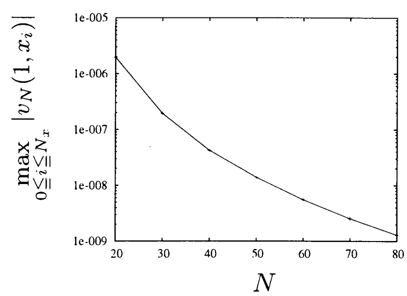
Table 1 shows that the overflow time approaches to 1 as Δs becomes small. So, the solution profile in Fig.2 seems not to be one of the blow-up solution. Then, numerical computation by SCM with CGL points on x and CGR points on s is carried out for $0 \leqq s < 1$(Fig.3).



(a) Solution profile($N = 80$).



(b) Behavior of Err_N.



(c) Behavior of $\max_{0 \leqq i \leqq N_x} |v_N(1, x_i)|$, x_i: CGL points.

Fig. 3. Numerical results by SCM with CGL$\oplus$CGR($\beta = 1$, $\alpha = 1$).

Fig.3(b) shows exponential convergence of numerical solutions. So, the converged numerical solution is recognized to be reliable. Fig.3(c) shows that values of numerical solutions at $s = 1$($T = \infty$ in Problem 1 or 2) converge to 0, then it suggests the existence of the global decreasing solution for $\beta = 1$, $\alpha = 1$.

Numerical results for $\alpha = 100$ by FDM$\oplus$EE are shown in Fig.4 and Table 2. From Table 2 the overflow time approaches to a constant $s_b < 1$ as Δs becomes small. This suggests the existence of the blow-up solution with the blow-up time s_b. Thus, numerical computation by SCM with CGL points is carried out for $0 \leqq s \leqq 0.0109$(Fig.5). The interval on s is divided in numerical computation for realizing exponential convergence(Figs.5(a-2),(b-2)). Fig.5(c) is obtained by joining Figs.5(a) and (b).

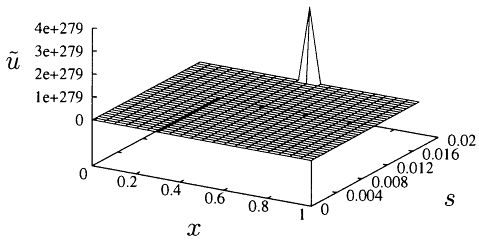
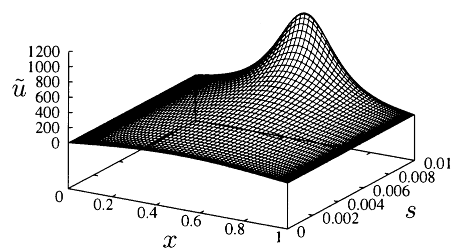


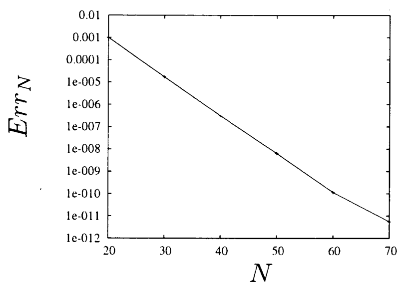
Fig. 4. Solution profile by FDM⊕EE ($\beta = 1,\ \alpha = 100,\ N_x = 20,\ \Delta s = 5 \times 10^{-4}$).

Table 2. Overflow time ($\beta = 1,\ \alpha = 100,\ N_x = 20$).

Δs	time
5×10^{-4}	0.0175
10^{-4}	0.0124
10^{-5}	0.01113
10^{-6}	0.010983
10^{-7}	0.0109666

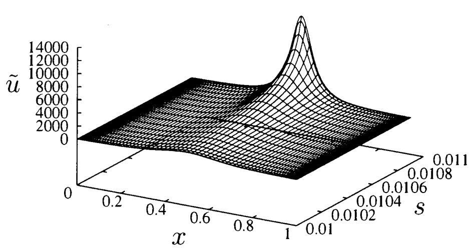


(a-1) Solution profile($N = 80$)

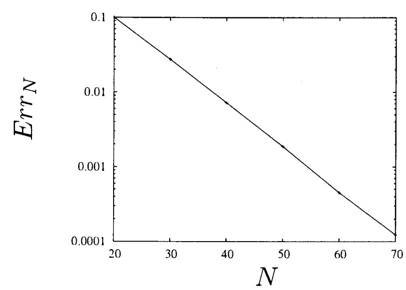


(a-2) Behavior of Err_N

(a) $0 \leqq s \leqq 0.01$.

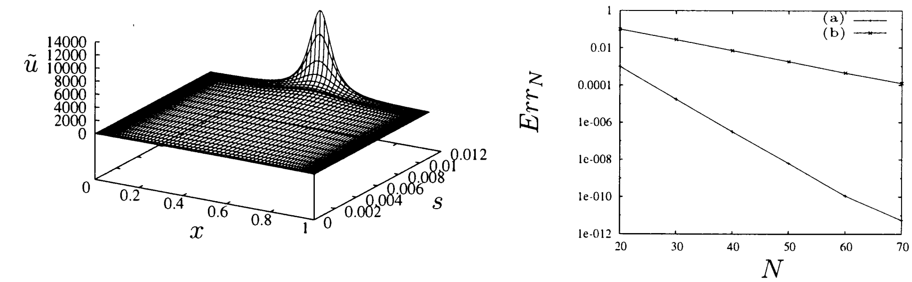


(b-1) Solution profile($N = 80$)



(b-2) Behavior of Err_N

(b) $0.01 \leqq s \leqq 0.0109$.



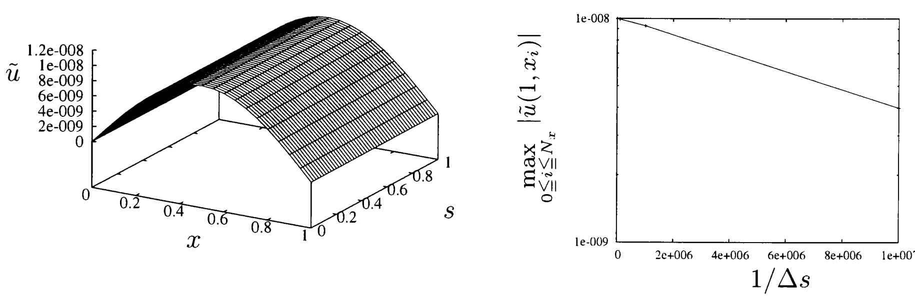
(c-1) Solution profile($N = 80$) (c-2) Behavior of Err_N

(c) $0 \leqq s \leqq 0.0109$.

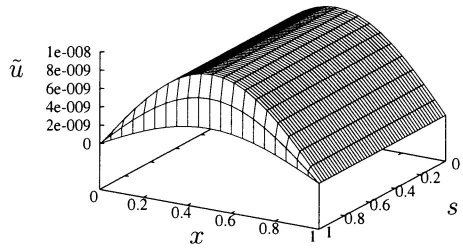
Fig. 5. Numerical results by SCM($\beta = 1,\ \alpha = 100$).

(ii) In the case of $\beta = 10^{-8}$.

Numerical results for $\alpha = 1$ by FDM$\oplus$EE are shown in Fig.6. Fig.6(a) with $\Delta s = 5 \times 10^{-4}$ suggests the global solution. From Fig.6(b) values of solutions at $s = 1(T = \infty)$ converge to 0 as Δs becomes small. Thus, the solution profile in Fig.6(a) is not precise. The solution profile with $\Delta s = 10^{-7}$ in Fig.6(c) may be precise and it suggests the existence of the global decreasing solution. In Fig.6(c) the view angle is different from that in Fig.6(a).

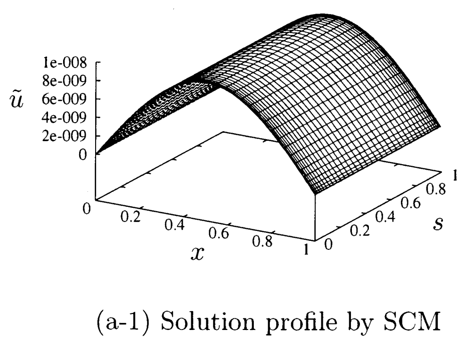


(a) Solution profile($\Delta s = 5 \times 10^{-4}$). (b) Behavior of $\max_{0 \leqq i \leqq N_x} |\tilde{u}(1, x_i)|$, $x_i = i\Delta x$.

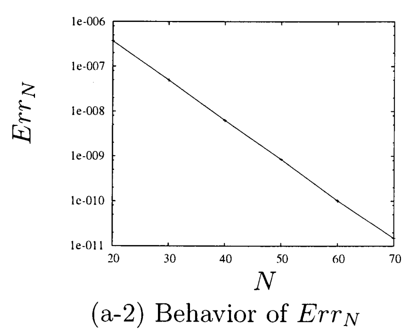


(c) Solution profile($\Delta s = 10^{-7}$).
Fig. 6. Solution profiles by FDM$\oplus$EE($\beta = 10^{-8}$, $\alpha = 1$, $0 \leqq s \leqq 1$).

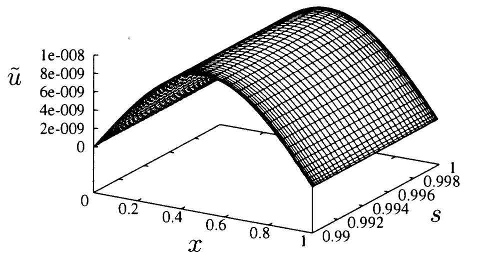
For more precise numerical computation SCM is applied in divided intervals for $0 \leqq s < 1$ referring rough numerical results by FDM$\oplus$EE in Fig.6. Numerical results are shown in Fig.7. Solution profiles for $0 \leqq s \leqq 0.9999$ by SCM in Figs.7(a-1),(a-3) are reliable because Figs.7(a-2),(a-4) show exponential convergence in each interval. For patching data across the interval CGL points are used on s. Fig.7(b-1) shows the rough solution profile for $0.9999 \leqq s \leqq 1$ by FDM$\oplus$EE. From Fig.7(b-1) the solution is smooth for $0.9999 \leqq s \leqq 0.999999$. Then, SCM is applied in this interval and it gives the solution profile in Fig.7(b-2) which is reliable due to exponential convergence in this interval(Fig.7(b-3)). Rough numerical computation for $0.999999 \leqq s \leqq 1$ by FDM$\oplus$EE is shown in Fig.7(c-1). Multiple precision is used because double precision induces oscillation. Referring this solution profile SCM in multiple precision is applied in divided intervals for $0.999999 \leqq s \leqq 1$(Figs.7(c-2)$\sim$(c-5)). Values of solutions at $s = 1$ in Fig.7(c-6) show the existence of the global decreasing solution for $\beta = 10^{-8}$, $\alpha = 1$. Fig.7(d) is obtained by joining Figs.7(a)$\sim$(c).



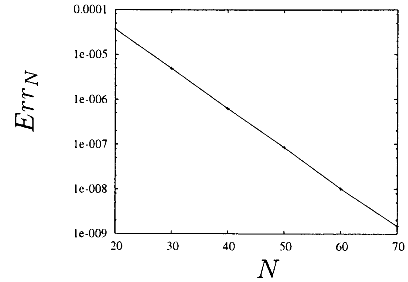
(a-1) Solution profile by SCM
($0 \leqq s \leqq 0.99$, $N = 80$).



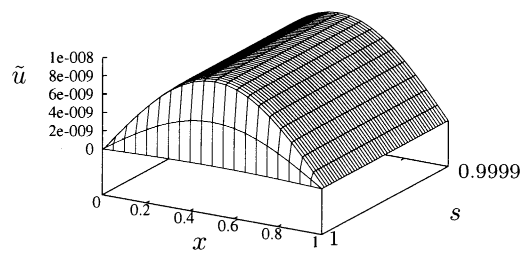
(a-2) Behavior of Err_N
($0 \leqq s \leqq 0.99$).



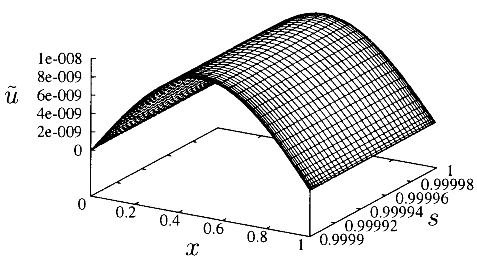
(a-3) Solution profile by SCM ($0.99 \leqq s \leqq 0.9999,\ N = 80$).



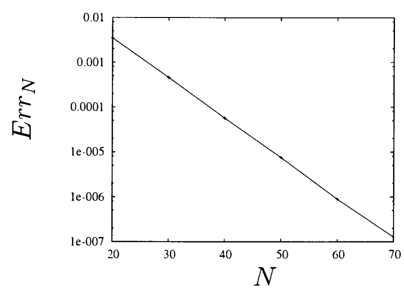
(a-4) Behavior of Err_N ($0.99 \leqq s \leqq 0.9999$).



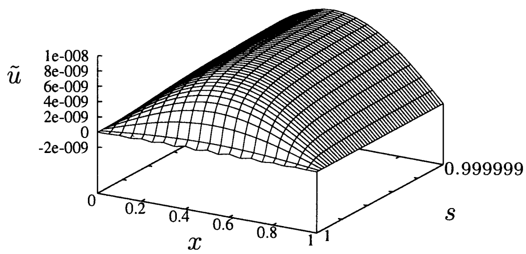
(b-1) Solution profile by FDM⊕EE($0.9999 \leqq s \leqq 1,\ \Delta s = 5 \times 10^{-8}$).



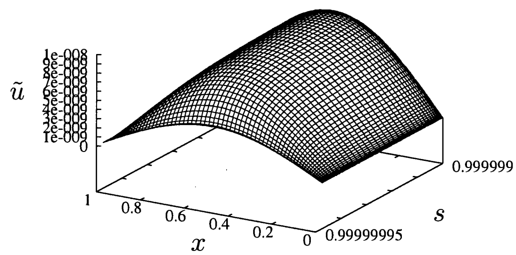
(b-2) Solution profile by SCM ($0.9999 \leqq s \leqq 0.999999,\ N = 80$).



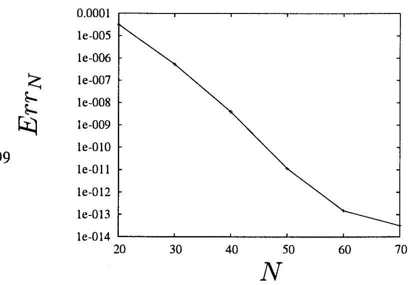
(b-3) Behavior of Err_N ($0.9999 \leqq s \leqq 0.999999$).



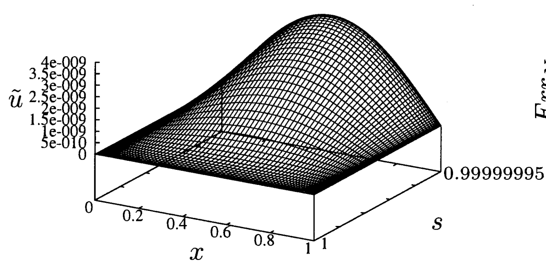
(c-1) Solution profile by FDM⊕EE($0.999999 \leqq s \leqq 1$, $\Delta s = 5 \times 10^{-10}$, 200digits).



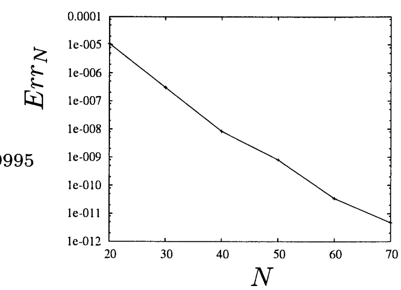
(c-2) Solution profile by SCM ($0.999999 \leqq s \leqq 0.99999995$, $N = 80$, 50digits).



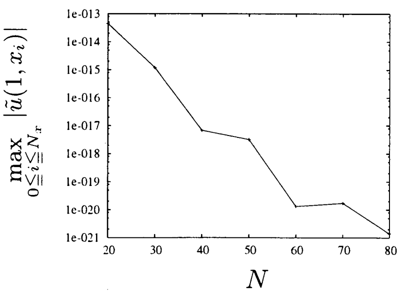
(c-3) Behavior of Err_N ($0.999999 \leqq s \leqq 0.99999995$).



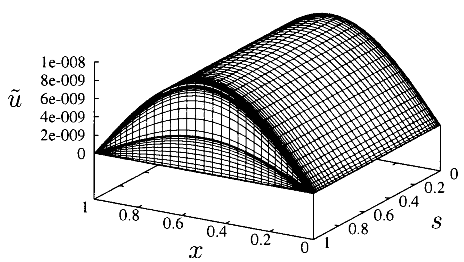
(c-4) Solution profile by SCM ($0.99999995 \leqq s \leqq 1$, $N = 80$, 50digits).



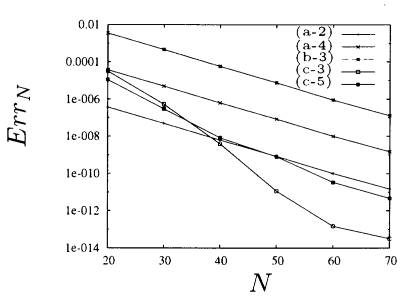
(c-5) Behavior of Err_N ($0.99999995 \leqq s \leqq 1$).



(c-6) Behavior of $\max_{0 \leqq i \leqq N_x} |\tilde{u}(1, x_i)|$, x_i: CGL points.



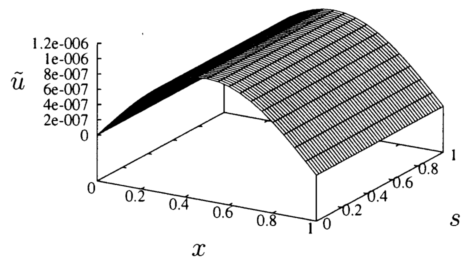
(d-1) Solution profile by SCM ($0 \leqq s \leqq 1$).



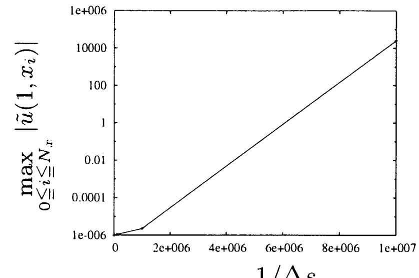
(d-2) Behavior of Err_N ($0 \leqq s \leqq 1$).

Fig. 7. Numerical results($\beta = 10^{-8}$, $\alpha = 1$).

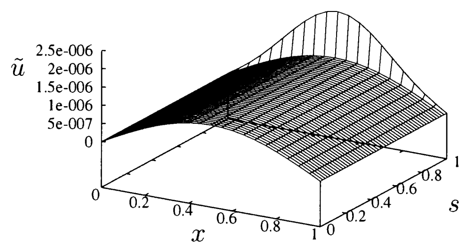
Numerical results for $\alpha = 100$ by FDM$\oplus$EE are shown in Fig.8. Fig.8(a) is obtained with $\Delta s (= 5 \times 10^{-4})$ which is not so small and it suggests that the solution is global. However, Fig.8(b) shows that the value of the solution at $s = 1 (T = \infty)$ grows as Δs becomes small. Fig.8(c) shows the solution profile with $\Delta s = 10^{-6}$ and it suggests the existence of the blow-up solution.



(a) Solution profile($\Delta s = 5 \times 10^{-4}$).



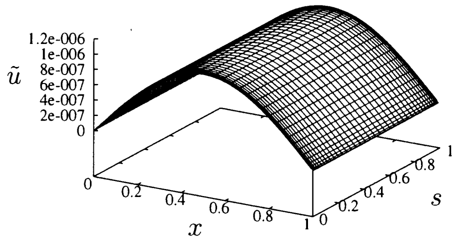
(b) Behavior of $\max_{0 \leqq i \leqq N_x} |\tilde{u}(1, x_i)|$, $x_i = i\Delta x$.



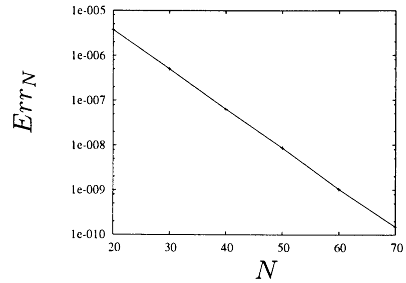
(c) Solution profile($\Delta s = 10^{-6}$).

Fig. 8. Solution profiles by FDM$\oplus$EE($\beta = 10^{-8}$, $\alpha = 100$, $0 \leqq s \leqq 1$).

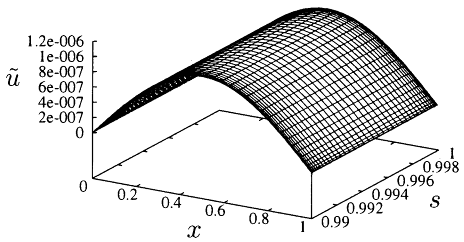
For more precise numerical computation SCM is applied in divided intervals for $0 \leqq s < 1$ referring rough numerical results by FDM$\oplus$EE in Fig.8. Numerical results are shown in Fig.9. The procedure is same as in Fig.7. Solution profiles for $0 \leqq s \leqq 0.99999954$ by SCM in Figs.9(a-1, 3, 5, 7, 9) are reliable because Figs.9(a-2, 4, 6, 8, 10) show exponential convergence in each interval. Fig.9(b) is obtained by joining Fig.9(a). Here, $s = 0.99999954$ corresponds to $\tau = 0.10869563 \times 10^7$. This means that numerical computation about the blow-up time is satisfactory.



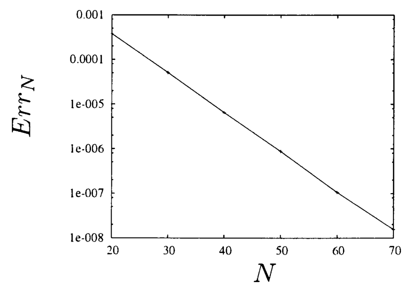
(a-1) Solution profile by SCM ($0 \leqq s \leqq 0.99,\ N = 80$).



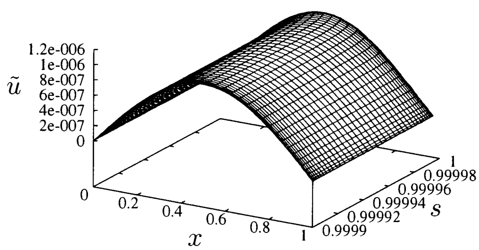
(a-2) Behavior of Err_N ($0 \leqq s \leqq 0.99$).



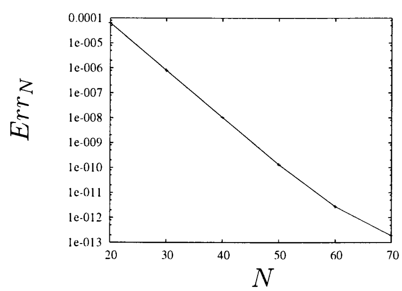
(a-3) Solution profile by SCM ($0.99 \leqq s \leqq 0.9999,\ N = 80$).



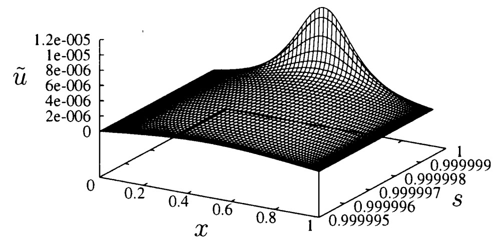
(a-4) Behavior of Err_N ($0.99 \leqq s \leqq 0.9999$).



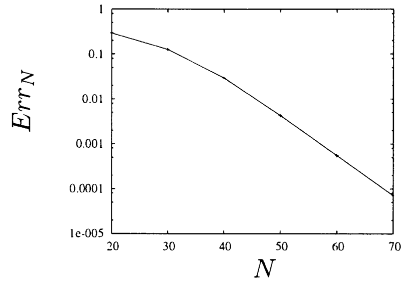
(a-5) Solution profile by SCM ($0.9999 \leqq s \leqq 0.999995,\ N = 80$).



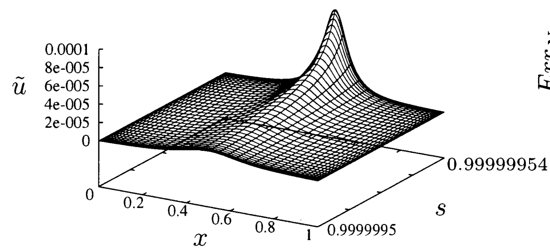
(a-6) Behavior of Err_N ($0.9999 \leqq s \leqq 0.999995$).



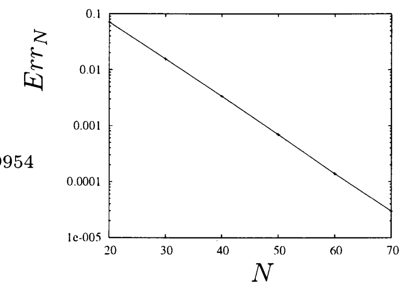
(a-7) Solution profile by SCM
($0.999995 \leqq s \leqq 0.9999995,\ N = 80$).



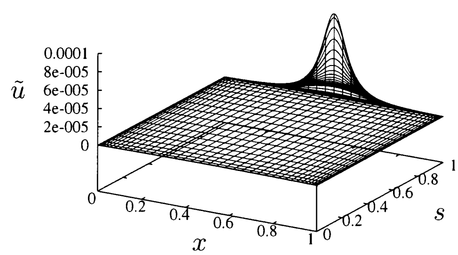
(a-8) Behavior of Err_N
($0.999995 \leqq s \leqq 0.9999995$).



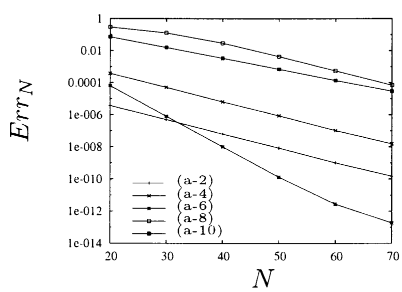
(a-9) Solution profile by SCM
($0.9999995 \leqq s \leqq 0.99999954,\ N = 80$).



(a-10) Behavior of Err_N
($0.9999995 \leqq s \leqq 0.99999954$).



(b-1) Solution profile by SCM
($0 \leqq s \leqq 0.99999954$).



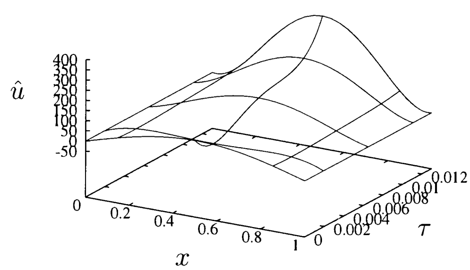
(b-2) Behavior of Err_N
($0 \leqq s \leqq 0.99999954$).

Fig. 9. Numerical results($\beta = 10^{-8},\ \alpha = 100$).

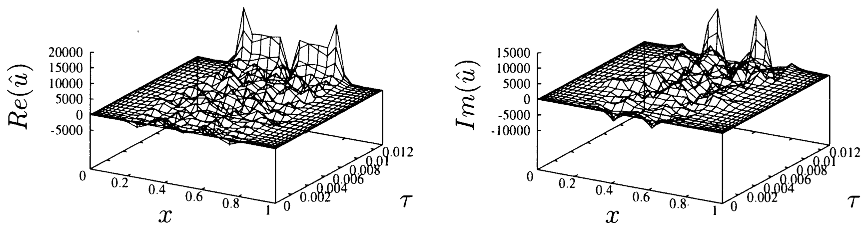
(iii) Blow-up time and complex Newton method

Former numerical methods[9, 14] for computing the blow-up time used the adaptive control on the time increment depending on the solution. From the view point of the solution profile we may propose the different idea for computing the blow-up time as follows. The profile of the blow-up solution has singularity in the bounded domain. On the other hand, the numerical method SCM determines the solution profile and it is very sensitive to singularity even if singularity exists outside the domain[15]. These suggest that SCM can feel the blow-up time as singularity. Together with numerical continuation[11]. SCM may offer a new approach for computing the blow-up time.

The solutions of problems considered here do not exist beyond the blow-up time[1]. This suggests that SCM fails in the time-space domain including the blow-up time. Problem 2 with $\beta = 1,\ \alpha = 100$ has the blow-up time $\tau_b \approx 0.0109$[9]. In the time-space domain as $0 \leqq \tau \leqq 0.013$ discretized equations by low order SCM($N_x = N_s = 4$) can be solved by Newton method. The solution profile is shown in Fig.10(a). On the other hand, discretized equations by higher order SCM cannot be solved by Newton method. If Newton method does not converge it gives apprehension rather than no information. Thus, application of complex Newton method is considered. It works well and it gives the solution profiles as Fig.10(b). From these numerical results and Fig.5(c) the blow-up time τ_b of Problem 2 with $\beta = 1,\ \alpha = 100$ is estimated as $0.010901\cdots \leqq \tau_b \leqq 0.013$. It is interesting that estimation from above is numerically obtained.



(a) Real Newton method, $N_x = N_s = 4$.



(b) Complex Newton method, $N_x = N_s = 30$.

Fig. 10. Solution profiles by SCM($\beta = 1,\ \alpha = 100, 0 \leqq \tau \leqq 0.013$).

3 conclusion

In the paper a numerical method for distinction between blow-up and global solutions is proposed. It consists of finite difference method, explicit Euler method, spectral collocation method, bounding transform, Newton method and multiple precision arithmetic. Our method is applied to a famous one-dimensional nonlinear heat equation. Numerical results are satisfactory. Moreover, the blow-up time is estimated from above in some case. In the paper complex Newton method is also used. Its new applicability is our future work.

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