Dissertation Abstract

On a Coupled SPH-Rigid Body Method for the Surfing Problem

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In this work, we use the smoothed particle hydrodynamics (SPH) method coupled with a rigid body simulation to study the surfing problem. We simulate a surfing board on top of an ocean wave which moves at a constant velocity. A fluid-rigid body coupling is handled by using pure hydrodynamics-based force. External forces are applied to the board, representing a surfer trying to stabilize the board at a desired point along the uphill part of the ocean wave. An ordinary differential equation (ODE) control is used to manipulate the distribution of the external forces based on a position, velocity, and an inclination angle of the surfing board relative to the ocean wave. The control system successfully helps the surfing board to move and maintain its position at the desired point.

Dissertation Abstract

1 Introduction

In what we called the surfing problem, the goal is to maintain the position of the surfing board on top of the upslope part of the ocean wave as long as possible. In this work we propose an ODE control that is capable to control the movement of the surfing board and maintain the position of the surfing board to be at a given desired point. Here we validate the ODE control by performing a coupled fluid-rigid body SPH simulation for the surfing problem.

2 Governing Equations

2.1 Fluid Dynamics

The motion of fluid is governed by following conservation laws which are the Euler's equations of the fluid dynamics [1]:

1. Conservation of mass:

$$\frac{D\rho}{Dt} = -\rho \operatorname{div}(u),$$

2. Conservation of momentum:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\nabla p + b,$$

where ρ, p, and u are density, pressure, and velocity field, respectively, b is a body force per unit mass, and $\frac{D}{Dt}$ is a so-called *substantial derivative* defined as $\frac{Df}{Dt} = \frac{\partial f}{\partial t} + u \cdot \nabla f$ for any field function $f(x,t)$, with u is a velocity field.

2.2 Rigid Body Dynamics

Rigid body can only undergo a linear and a rotational transformation which follow the conservation of a linear momentum and the conservation of an angular momentum, respectively.

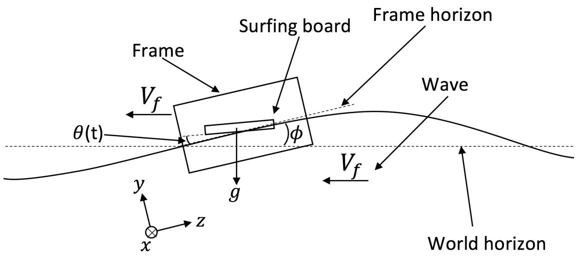
1. Conservation of linear momentum:

$$\frac{dG(t)}{dt} = F(t),$$

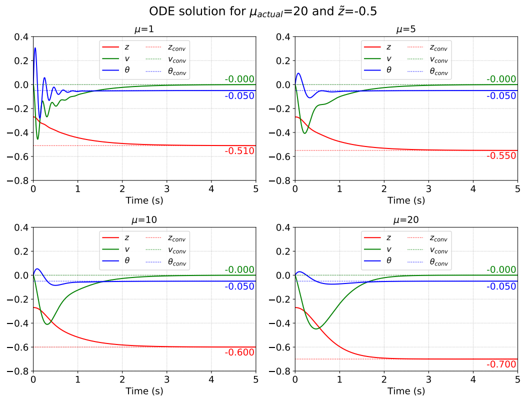
2. Conservation of angular momentum:

$$\frac{dL(t)}{dt} = K(t),$$

where $G(t) = MU(t)$ and $L(t) = \mathbf{J}(t)\omega(t)$ are a linear momentum and an angular momentum of the rigid body at a given time t, respectively, with M, U, $\mathbf{J}$, and ω are a mass, linear velocity of the rigid body, moment of inertia tensor, and an angular velocity of the rigid body, respectively. We can choose such a reference configuration of the rigid body so we can have a moment of inertia tensor to be a diagonal matrix $\hat{\mathbf{J}}$ called a principal moment of inertia tensor. Let $\mathbf{R}(t)$ be an orthogonal rotation matrix of the rigid body at a given



(A)



(B)

FIGURE 1: (A) Sideview of the illustration of the frame of system's domain, (B) Solution of the ODE by using direct ODE solver.

time t. The angular velocity is defined as a vector $\omega(t) = (\omega_1(t), \omega_2(t), \omega_3(t))$ satisfies

$$\frac{d}{dt}\mathbf{R}(t) = \mathbf{W}(t)\mathbf{R}(t), \qquad \mathbf{W}(t) = \begin{bmatrix} 0 & -\omega_3(t) & \omega_2(t) \\ \omega_3(t) & 0 & -\omega_1(t) \\ -\omega_2(t) & \omega_1(t) & 0 \end{bmatrix}.$$

This leads to the Euler's equation of the rigid body dynamics,

$$\hat{\omega}(t) \times \hat{\mathbf{J}}\hat{\omega}(t) + \hat{\mathbf{J}}\dot{\hat{\omega}}(t) = \hat{K}(t),$$

where $\hat{K}(t) = \mathbf{R}^T(t)K(t)$, $\hat{\omega}(t) = \mathbf{R}^T(t)\omega(t)$, and $\dot{\hat{\omega}}(t) = \frac{d\hat{\omega}(t)}{dt}$.

2.3 ODE Control

We choose the frame of the system to be parallel with the upslope part of the ocean wave and moves together with the ocean wave with a constant velocity (see Figure 1a).

The goal of the surfing problem is to control the position of the surfing board to be at the desired point. Here we want to control the surfing board on one axis only, in this case, the Z-axis. The only parameter that we can control is the inclination angle of the surfing board. We propose the following ODE control for the inclination angle,

$$\dot{\theta}(t) = a(Z(t) - \tilde{Z}) + b(V(t) - \tilde{V}) + c(\theta(t) - \tilde{\theta}), \tag{1}$$

where Z and V are the third component of position and linear velocity of the surfing board, respectively, θ is the inclination angle of the surfing board, $\tilde{Z}$, $\tilde{V}$, and $\tilde{\theta}$ are the desired position, velocity, and the desired inclination angle, respectively. Because of the choice of the frame, we set $\tilde{V} = 0$. $\tilde{Z}$ is given. Up to now we do not have any information about $\tilde{\theta}$. a, b, and c are given parameters that can make the system stable.

To find suitable parameters for a, b, and c, we consider a simplified linearized ODE model with the ODE control (1).

$$\begin{cases} \dot{Z}(t) &= V(t), \\ \dot{V}(t) &= -\mu\theta(t) - \mu_v V(t) - \mu_z Z(t) - \mu_0, \\ \dot{\theta}(t) &= a(Z(t) - \tilde{Z}) + bV(t) + c(\theta(t) - \tilde{\theta}), \end{cases} \tag{2}$$

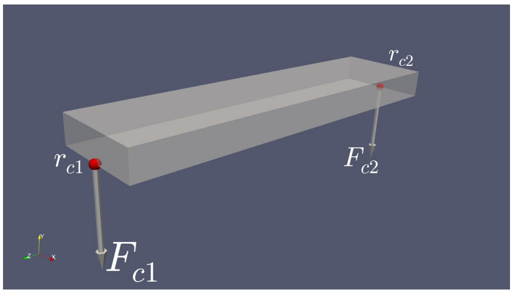


Figure 2: The locations of contact points.

where μ, μ_v, μ_z, and μ_0 are constants related to the drag and gravity forces. For now let us assume $\mu_v = \mu_z = 0$ for simplicity. Although the assumption is not correct, it does not seem to influence the stability of the system.

We can write (2) in a matrix form as

$$\dot{\xi}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -\mu \\ a & b & c \end{bmatrix} \xi(t) + \begin{bmatrix} 0 \\ -\mu_0 \\ 0 \end{bmatrix} =: \mathbf{A}\xi(t) + \zeta.$$

The stationary point will be stable if all eigenvalues of matrix $\mathbf{A}$ have negative real parts. The characteristic equation of $\mathbf{A}$ is

$$\det(\mathbf{A} - \lambda\mathbf{I}) = -\lambda^3 + c\lambda^2 - b\mu\lambda - a\mu = 0. \tag{3}$$

Let us set the roots of (3) to be $\lambda_1 = -3$, $\lambda_2 = -4$, and $\lambda_3 = -5$. This yields the equation

$$-\lambda^3 - 12\lambda^2 - 47\lambda - 60 = 0. \tag{4}$$

Comparing (4) with (3), we get $a = \frac{60}{\mu}$, $b = \frac{47}{\mu}$, and $c = -12$.

Since we do not know the actual value of μ in our case, our choice of μ might differ from the actual μ, leads to mismatch between a, b, and c with the correct ones. To observe the effect of the choice of μ to the behaviour of the ODE, we solve the ODE by using direct ODE solver with different values of μ. In the direct ODE solver, we set $\tilde{Z} = -0.5$, $\mu_{actual} = 20$, and $\tilde{\theta} = 0$. We can observe some oscillations occurred when we choose the value of μ far from μ_{actual}. The oscillation is decreasing when the μ is getting closer to μ_{actual}. We also observe that the stable position is shifted from the desired point $\tilde{Z}$ for some choices of μ. This problem happens because of the wrong choice of $\tilde{\theta}$.

The ODE control is implemented into the system by giving two external forces to each tip of the surfing board, mimicking the action of the surfer in an attempt to control the movement of the surfing board via their feet. The distribution of the forces is controlled by the inclination angle given from the ODE control,

$$F_{c1}(t) = T(t)W, \qquad F_{c2}(t) = (1 - T(t))W, \qquad T(t) = 0.5 - \sigma(\hat{\theta}(t) - \min(\theta(t), -0.05)),$$

where W is a weight of the surfer, $F_{c1}(t)$ and $F_{c2}(t)$ are the forces given at each tip of the surfing board at a given time t. $\hat{\theta}(t)$ is an observed inclination angle. σ is a constant. In this work we set $W = 10$ and $\sigma = 10$. We do not impose $T(t) \in [0, 1]$, which is possible in a real case if the surfer straps their feet to the surfing board.

3 Smoothed Particle Hydrodynamics

3.1 Basic Idea of the SPH Method

The basic idea of the SPH method comes from a convolution operation af any field function $f(x)$ with a sufficiently smooth mollifier $\psi \in C_c^k(\mathbb{R}^n)$,

$$(f * \psi)(x) := \int_{\mathbb{R}^n} f(y)\psi(x-y)\ dy.$$

We define

$$\psi_h(x) := \frac{1}{h^n}\psi\left(\frac{x}{h}\right), \quad h > 0,\ x \in \mathbb{R}^n.$$

For more detail, see [2].

For $\int_{\mathbb{R}^n} \psi(x)\ dx = 1$, a family of $\{\psi_h\}_{h>0}$ is called an approximate identity. If $f \in C^k(\mathbb{R}^n)$ for some $1 \leq k < \infty$, then we have $f * \psi_h \to f$ uniformly as $h \to 0$,

$$f(x) \approx \int_{\mathbb{R}^n} f(y)\psi_h(x-y)\ dy, \tag{5}$$

and by following the differentiation of a convolution in [2], the approximation for the derivative of f is

$$\partial^\alpha f(x) \approx \int_{\mathbb{R}^n} f(y)\partial^\alpha \psi_h(x-y)\ dy, \qquad |\alpha| \leq k, \tag{6}$$

for $\psi \in C_c^k(\mathbb{R}^n)$. Both (5) and (6) are SPH approximations for a field function and its derivative in an integral form. In this work we are using a compactly-supported piecewise cubic kernel function [3]

$$\psi(x) = \frac{\alpha_n}{6}\begin{cases} (2-|x|)^3 - 4(1-|x|)^3, & 0 \leq |x| < 1 \\ (2-|x|)^3, & 1 \leq |x| < 2 \\ 0, & 2 \leq |x| \end{cases} \tag{7}$$

where α_n is 1, $\frac{15}{7\pi}$, or $\frac{3}{2\pi}$ for $n = 1, 2, 3$ respectively. Note that $\psi \in C_c^2(\mathbb{R}^n)$.

We approximate the integral (5) and (6) by using discrete summations over N points,

$$f(x) \approx \sum_{i=1}^N f(r_i)\psi_h(x-r_i)V_i, \qquad \partial^\alpha f(x) \approx \sum_{i=1}^N f(r_i)\partial^\alpha \psi_h(x-r_i)V_i,$$

where V_i is a volume of the neighborhood around a point r_i. A common approximation for $V(E_i)$ is by using a mass and density at point r_i,

$$V(E_i) \approx \frac{m_i}{\sum_{j=1}^N m_j \psi_h(r_i - r_j)}.$$

3.2 Discretization of the Fluid Dynamics

Let us discretize the fluid into N points. The SPH approximations in their anti-symmetrized form can be written as

1. Conservation of mass [3]:

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} (u_i - u_j) \cdot \nabla\psi_h(r_i - r_j),$$

2. Conservation of momentum:

$$\frac{du_i}{dt} = -\sum_{j=1}^{N} \left(m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla\psi_h(r_i - r_j) \right) + b_i,$$

The SPH approximation for the conservation of mass in an anti-symmetrized form can be written as [3]

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} (u_i - u_j) \cdot \nabla\psi_h(r_i - r_j),$$

where m_i is a mass of a point r_i, $\rho_i(t)$ and $u_i(t)$ are density and velocity of a point r_i at a given time t, respectively.

The SPH approximation for the conservation of momentum in an anti-symmetrized form is

$$\frac{du_i}{dt} = -\sum_{j=1}^{N} \left(m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla\psi_h(r_i - r_j) \right) + b_i,$$

where $p_i(t)$ is a pressure of a point r_i at a given time t, and b_i is a body force per unit mass at a point r_i.

In this work we assume the fluid to be "slightly compressible", so we can approximate the pressure at any points as a function of the density [4][5]

$$p_i = \frac{c^2\rho_0}{\gamma} \left(\left(\frac{\rho_i}{\rho_0} \right)^{\gamma} - 1 \right),$$

where ρ_0 is a reference density of fluid, c is a speed of sound in a fluid, and $\gamma = 7$ for water-like fluid.

As is common in the SPH literature, we use the leapfrog time integrator scheme to evolve physical quantities of material points as follows

$$u_i\left(t + \frac{\tau}{2}\right) = u_i\left(t - \frac{\tau}{2}\right) + \frac{du_i}{dt}(t)\tau \quad (8)$$

$$r_i\left(t + \tau\right) = r_i(t) + u_i\left(t + \frac{\tau}{2}\right)\tau \quad (9)$$

$$u_i(t + \tau) = u_i\left(t + \frac{\tau}{2}\right) + \frac{du_i}{dt}(t)\frac{\tau}{2} \quad (10)$$

$$u_i(-\frac{\tau}{2}) = u_i(0) - \frac{du_i}{dt}(0)\frac{\tau}{2}, \quad (11)$$

where τ is a timestep.

3.3 Discretization of the Rigid Body Dynamics

The rigid body is discretized into a set of boundary points which can interact with other fluid points using a pure hydrodynamics-based force. The density of the rigid body points is set to be always equal to the reference density, $\rho_i = \rho_0$. Let us discretize the rigid body into N_b points. The force applied to the rigid body point i by the influence of all fluid points j is equal to

$$f_i = -m_i c_V \left(\left(\sum_{j=1}^{N} m_j \frac{p_j}{\rho_j^2} \nabla\psi_h(r_i - r_j) \right) + b_i \right),$$

where $c_V = \frac{h^3}{V_i}$. c_V is needed since there is a discrepancy between the "real volume" and the "SPH volume" of the rigid body point i.

The linear movement of the rigid body is done by calculating its linear acceleration

$$A(t) = \frac{1}{M}\left(\left(\sum_{i=1}^{N_b} f_i(t)\right) + F_{c1} + F_{c2}\right),$$

where M is a mass of the rigid body. After that, the position and linear velocity of the rigid body are updated using the leapfrog time integrator scheme similar with (8)–(11).

To evolve the rotational movement of the rigid body, we follow the algorithm from [6]. In the spirit of the leapfrog time integrator scheme, we can update the angular velocity of the rigid body by using

$$\hat{\omega}_\alpha^{(m+1)}\left(t+\frac{\tau}{2}\right) = \hat{\omega}_\alpha\left(t-\frac{\tau}{2}\right) + \frac{\tau}{J_\alpha}\left(\hat{K}_\alpha(t) \ + \hat{\omega}_\beta^{(m)}(t)\hat{\omega}_\gamma^{(m)}(t)\left(J_\beta - J_\gamma\right)\right),$$

where $(\alpha,\beta,\gamma) = (1,2,3)\,,(2,3,1)\,,$ and $(3,1,2)$, (m) is an iteration step, $\hat{K}(t) = \mathbf{R}^T(t)K(t)$, $\hat{\omega}(t) = \mathbf{R}^T(t)\omega(t)$, and $\dot{\hat{\omega}}(t) = \frac{d\hat{\omega}(t)}{dt}$. We set

$$\hat{\omega}_\alpha^{(0)}\left(t+\frac{\tau}{2}\right) = \hat{\omega}_\alpha\left(t-\frac{\tau}{2}\right),$$

and

$$\hat{\omega}_\beta^{(m)}(t)\hat{\omega}_\gamma^{(m)}(t) = \frac{1}{2}\left(\hat{\omega}_\beta\left(t-\frac{\tau}{2}\right)\hat{\omega}_\gamma\left(t-\frac{\tau}{2}\right) \ + \hat{\omega}_\beta^{(m)}\left(t+\frac{\tau}{2}\right)\hat{\omega}_\gamma^{(m)}\left(t+\frac{\tau}{2}\right)\right).$$

The moment of force $K(t)$ is calculated by using

$$K(t) = \left(\sum_{i=1}^{N_b}\left(r_i(t) - X(t)\right) \times f_i(t)\right) \ + \left(r_{c1}(t) - X(t)\right) \times F_{c1} \ + \left(r_{c2}(t) - X(t)\right) \times F_{c2},$$

After that we update the rotation matrix by using

$$\begin{aligned}\mathbf{R}(t+\tau) &= \mathbf{R}(t) + \tau\frac{d}{dt}\mathbf{R}\left(t+\frac{\tau}{2}\right)\\ &= \mathbf{R}(t) + \tau\mathbf{W}\left(t+\frac{\tau}{2}\right)\mathbf{R}\left(t+\frac{\tau}{2}\right)\\ &= \Theta\left(t+\frac{\tau}{2}\right)\mathbf{R}(t),\end{aligned}$$

where

$$\Theta\left(t+\frac{\tau}{2}\right) = \frac{\mathbf{I}\left(1-\frac{\tau^2}{4}\omega^2\left(t+\frac{\tau}{2}\right)\right) - \tau\mathbf{W}\left(t+\frac{\tau}{2}\right)}{1+\frac{\tau^2}{4}\omega^2\left(t+\frac{\tau}{2}\right)} + \frac{\frac{\tau^2}{2}\left(\omega\otimes\omega\right)\left(t+\frac{\tau}{2}\right)}{1+\frac{\tau^2}{4}\omega^2\left(t+\frac{\tau}{2}\right)},$$

Then we update the position and velocity of each rigid body point i by using

$$\begin{aligned}r_i(t+\tau) &= X(t+\tau) + \mathbf{R}(t+\tau)\mathbf{R}^T(t)\left(r_i(t) - X(t)\right),\\ u_i(t+\tau) &= U\left(t+\frac{\tau}{2}\right) + \frac{\tau}{2}A(t) + \omega(t+\tau)\times\left(r_i(t+\tau) - X(t+\tau)\right),\end{aligned}$$

where $X(t)$ is a position of the center of mass of the rigid body at time t.

3.4 Algorithm of the Simulation

The algorithm of the simulation can be seen on Figure 3.

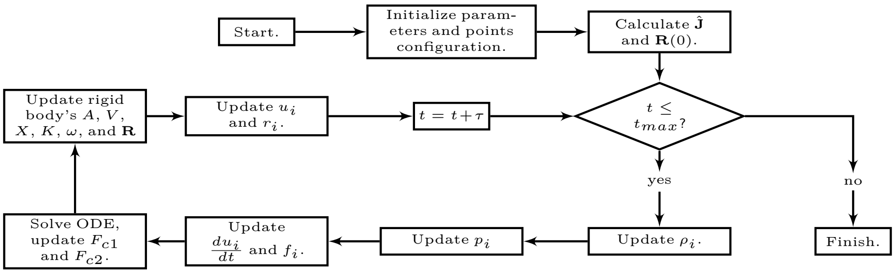


FIGURE 3: The algorithm of the simulation.

4 Results and Discussions

4.1 Simulation Set-up

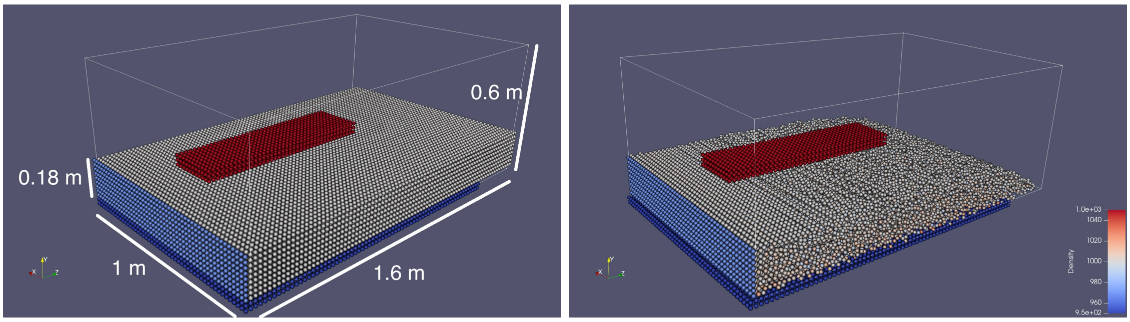
We choose the inclination angle of the ocean wave relative to the world horizon to be $\phi = 10°$ with the ocean wave moves with a constant velocity $V_f = 2.5$. The frame of the system's domain is located at the upslope part of the ocean wave and moves together with the ocean wave (see Figure 1a). Gravity acts as an external body force per unit mass is set to be $g = -9.81(0, \cos\phi, \sin\phi)$. The size of the domain is set to be $1 \times 0.6 \times 1.6$ with the depth of the water is 0.18. The system has a periodic boundary condition in x-axis, non-zero Dirichlet boundary on some parts of left boundary by using ghost points (rendered with light blue color in Figure 4a) for $-0.3 \leq y < -0.12$, bottom boundary is also set to be a non-zero Dirichlet boundary by using a non-moving boundary points (rendered with dark blue color in Figure 4a) for $-0.8 \leq z < 0.56$, and free boundary in right boundary, top boundary, left boundary for $-0.12 \leq y < 0.3$, and bottom boundary for $0.56 \leq z \leq 0.8$. Take a note that the origin point is located at the center of the domain. The size of the rigid body is $0.2 \times 0.06 \times 0.8$. The rigid body is represented by red points in Figure 4a. Initially, the center of mass of rigid body is positioned at $(0, -0.04, -0.27)$.

The density reference ρ_0 is set to be $\rho_0 = 1000$, while the density of the rigid body is $\rho_b = 100$. The fluid is initialized to have an initial velocity $u_i(0) = V_f = 2.5$ toward $+z$-axis, and initial density to be $\rho_i(0) = \rho_0$ for all fluid points r_i. By our choice of piecewise cubic kernel (7) as a mollifier function and choose the points to be initialized in a regular grid with a distance h in each axis, V_i and m_i for all points are $V_i = 8 \times 10^{-6}$ and $m_i = 0.008$. We set the parameter of the kernel function to be $h = 0.02$. Time step size is set to be $\tau = 0.0005$ with the speed of sound is chosen to be $c = 20$. The positions of contact point 1 and contact point 2 are $r_{c1} = (0.,\ -0.03,\ -0.4)$ and $r_{c2} = (0.,\ -0.03,\ 0.4)$, respectively, relative to the position of the center of mass of the rigid body.

Free boundary condition is implemented by changing the type of any fluid points leaving the domain into a ghost point which its velocity does not change with time and its density always equal to the reference density ρ_0, but still interacts with other points. If a ghost point enters the domain, it will be marked as a normal fluid point again. But if it leaves the domain farther than h, the point will be removed from the simulation.

Before we run the actual simulation, we run the "relaxation" process to stabilize the flow of the water up to $t = 1.5$. The initial condition after relaxation can be seen on Figure 4b.

To find the best combination of parameters, we run simulations with each combination of $\tilde{\theta} \in \{-0.05, -0.06, -0.07, -0.08, -0.09, -0.1\}$ and $\mu \in \{1, 2, 5, 10, 20, 50\}$ for each $\tilde{Z} \in \{-0.6, -0.5, -0.4, -0.3, -0.2\}$. We take



(A) (B)

Figure 4: (A) The initial configuration of the system, (B) The configuration of the system after the relaxation process.

the average of positional error for a given combination of parameters $(\mu, \tilde{\theta}, \tilde{Z})$ for the whole simulation time,

$$\overline{\epsilon_Z}(\mu, \tilde{\theta}, \tilde{Z}) := \frac{\sum_{m=0}^{\Upsilon} \left| Z(\mu, \tilde{\theta}, \tilde{Z}, m\tau) - \tilde{Z} \right|}{\Upsilon + 1},$$

where Υ is a number of time steps. The graph of the average positional error can be seen in Figure 5. Then we take the average of error over all parameters $\tilde{Z}$,

$$\overline{\overline{\epsilon_Z}}(\mu, \tilde{\theta}) := \frac{\sum_{\tilde{Z} \in \tilde{Z}_{\text{set}}} \overline{\epsilon_Z}(\mu, \tilde{\theta}, \tilde{Z})}{\# \tilde{Z}_{\text{set}}}, \qquad \tilde{Z}_{\text{set}} = \{-0.6, -0.5, -0.4, -0.3, -0.2\},$$

and calculate the cumulative error for each $\tilde{\theta}$ and μ. The average of error over all parameters $\tilde{Z}$ can be seen in Table1.

Table 1: Table of the average of error over all parameters $\tilde{Z}$ and its cumulative errors for each $\tilde{\theta}$ and μ.

$\mu \backslash \tilde{\theta}$	-0.05	-0.06	-0.07	-0.08	-0.09	-0.10	C.E.
1	5.74e-02	5.81e-02	5.33e-02	5.80e-02	5.98e-02	5.44e-02	3.41e-01
2	5.30e-02	5.13e-02	5.08e-02	4.67e-02	5.48e-02	5.94e-02	3.16e-01
5	5.02e-02	5.29e-02	4.62e-02	4.76e-02	4.53e-02	4.89e-02	2.91e-01
10	5.97e-02	5.13e-02	4.66e-02	4.65e-02	5.08e-02	6.29e-02	3.18e-01
20	8.47e-02	7.04e-02	5.07e-02	5.07e-02	6.92e-02	9.33e-02	4.19e-01
50	1.28e-01	1.08e-01	7.15e-02	7.00e-02	9.97e-02	1.72e-01	6.49e-01
C.E.	4.33e-01	3.92e-01	3.19e-01	3.20e-01	3.80e-01	4.91e-01	

As we can see from Table 1, $\tilde{\theta} = -0.07$ and $\mu = 5$ give the smallest cumulative errors for each $\tilde{\theta}$ and μ, respectively. Now let us see more in detail the simulation results for $\tilde{\theta} = -0.07$ in Figure 6.

As we can see from Figure 6, there are some oscillations of the position of the rigid body. As the value of μ is increasing, the amount of oscillations is dampened. The oscillation is occurred because of the wrong choice of the μ compared to the actual value μ of the system. As the value of μ is increasing (we assume that it is getting closer to the actual value of μ as it increases), the oscillation is dampened. Besides, our previous assumption that $\mu_v = \mu_z = 0$ is also not correct in the real case. As we choose the frame to be inclined relative to the world horizon, the gravity influences the velocity of the flow, yielding different velocities at different positions, and leading to the dependency of the drag to the position. Since now the flow is not constant, the drag also depends on the velocity of the flow, leading to the dependency of the drag to the velocity as well.

We also notice that we face another problem: shifted stable position. This problem is due to the wrong choice of $\tilde{\theta}$. We set $\tilde{\theta}$ to be the same value for the whole simulation. Adding an additional ODE control for $\tilde{\theta}$ will

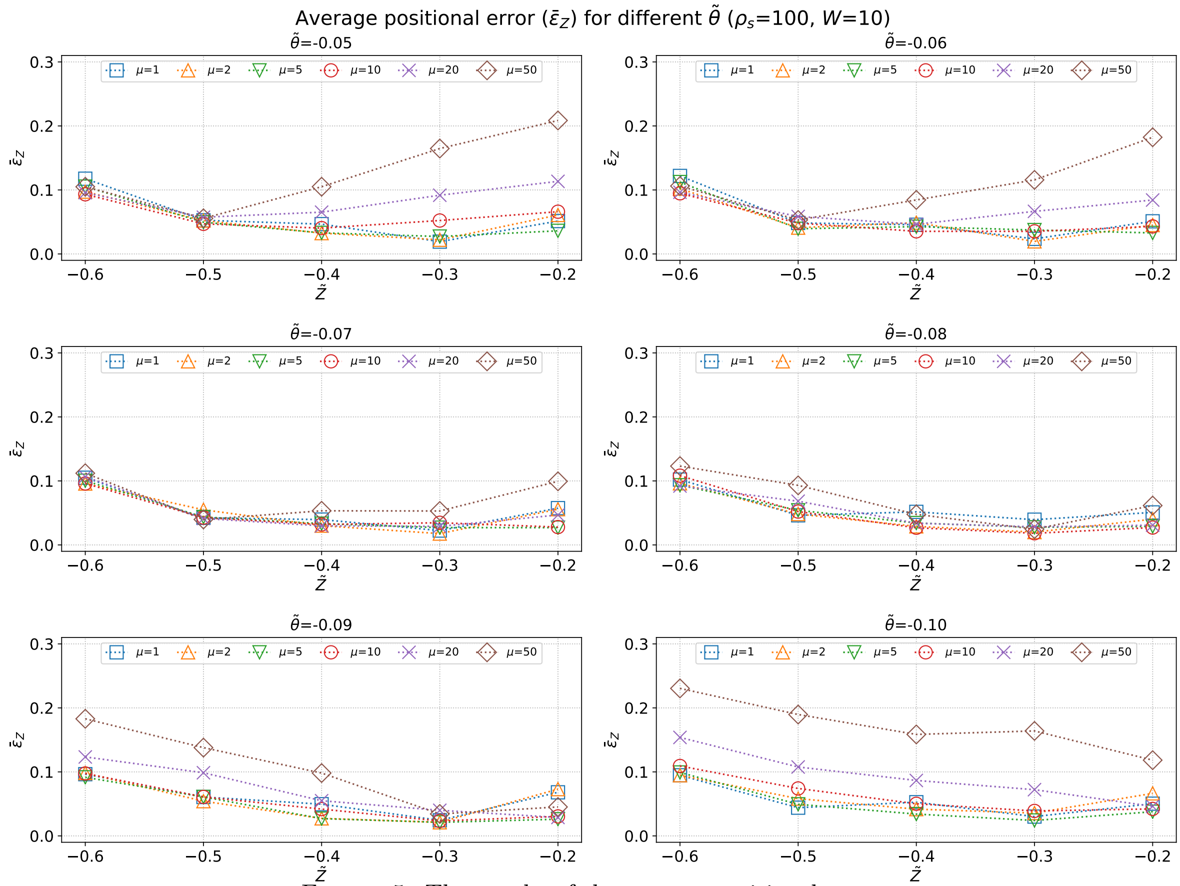


FIGURE 5: The graphs of the average positional error.

help the board to move slowly to the desired position after the "pre-stable condition" is achieved. In a real case, $\tilde{\theta}$ is learned by the surfer as the "target inclination angle" if they want to move the surfing board to a desired point along the ocean wave. Finding correct parameters for an additional ODE control for $\tilde{\theta}$ can be seen as an effort to learn those target inclination angle if the position of the surfing board is off from the desired position.

If we looks closer and compare Figure 1b with Figure 6, we can see that the oscillations on the direct solver is on the inclination angle and velocity, while in the SPH simulation the oscillations occur on the position. This problem happens because of the delayed response in the SPH simulation. The change of the inclination angle from the ODE control cannot be translated instantly into the change of the inclination angle in the SPH simulation.

5 Summary

An ODE control is successfully implemented into a coupled inviscid fluid-rigid body SPH simulation in an attempt to control the movement of the surfing board. For our system, the best values for $\tilde{\theta}$ and μ are $\tilde{\theta} = -0.07$ and $\mu = 5$. Although $\mu = 5$ does give the best result, it does not mean the actual μ of our system is equal to 5. Several problems still occur, such as oscillations and shifted stable position. The oscillation problem can be solved by finding correct values for all μ, μ_v, and μ_z. Adding an additional control for $\tilde{\theta}$ will solve the shifted stable position problem by nudging the surfing board slowly toward the desired position after the system is almost stable. The delayed response from the SPH simulation also has a responsibility on the oscillations problem, since the change of the inclination angle from the ODE control cannot be translated instantly into the SPH simulation.

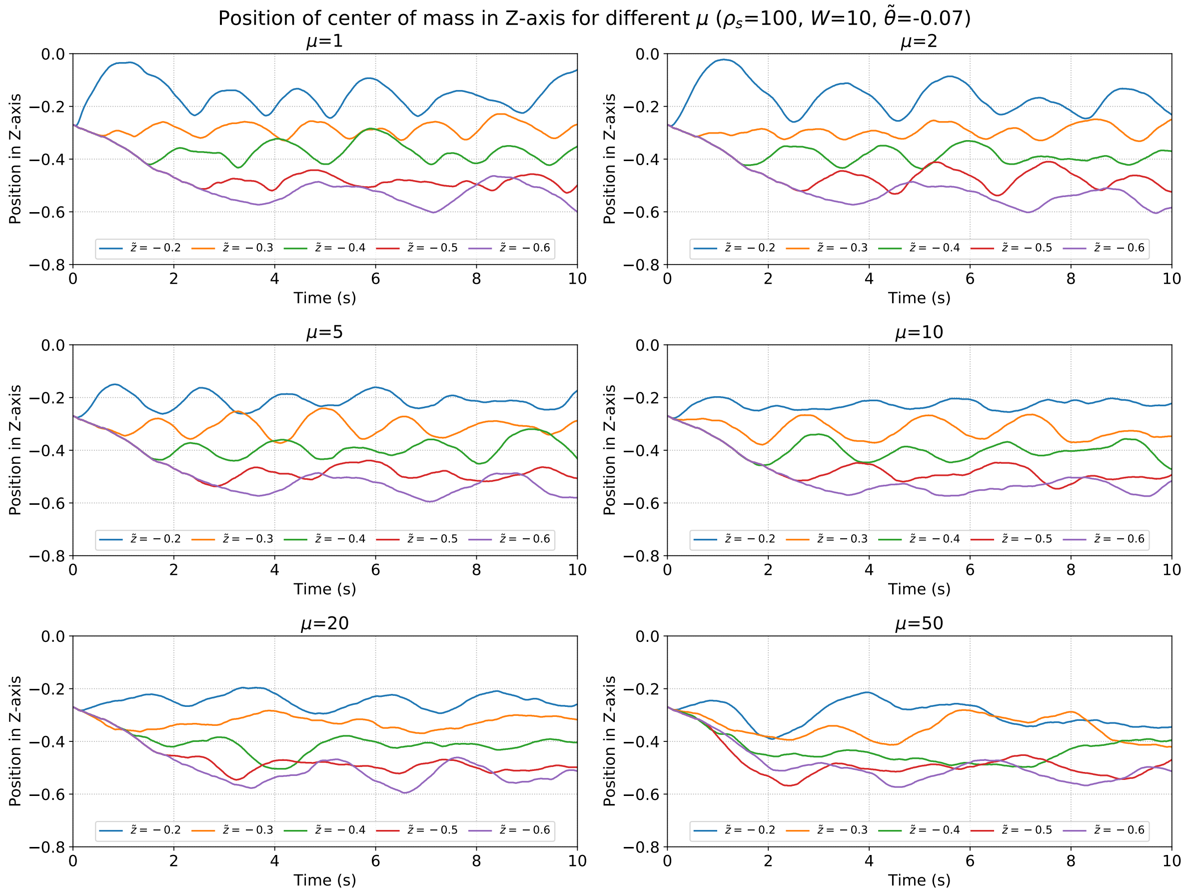


FIGURE 6: The z-axis-component position of surfing board for $\tilde{\theta} = -0.07$.

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学位論文審査報告書（甲）

1．学位論文題目（外国語の場合は和訳を付けること。）

On a Coupled SPH-Rigid Body Method for the Surfing Problem

SPH 法によるサーフィンモデル・個体液体連成問題の数値解法の開発について

2．論文提出者 (1) 所　属　数物科学専攻　　　専攻

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3．審査結果の要旨（600～650 字）

Reza Rendian Septiawan さんは、2015 年 10 月に自然科学研究科数物科学専攻に入学した(大学推薦の MEXT 奨学金給付生)。それ以来、剛体と流体との相互作用を解析するための数値解法の開発・研究を行ってきた。流体については大規模変形などに対応するため、Smoothed particle hydrodynamics (SPH) と呼ばれる粒子法を採用している。この利点の１つは、自由流体表面などの取り扱いに優れていることである。

Reza さんはこの方法を用いて剛体との相互作用を解析する効率的なソルバーの開発に成功した。これは、流体と複数個の剛体の相互作用を取り扱えるという意味で非常に有用である。Reza さんは、この連成解析を実現させるとともに、波乗り（サーフィン）を物理モデルに選択し、波の表面に上昇流が起きたときのサーフボードの運動を記述するプログラムの作成とサーフボードの制御のためのプログラムも作成した。Reza さんは波に対するボードの 2 点で角度をつけた力を加える制御方法を提案し、ボードを所定の位置で安定させることができるかどうか解析を試みた。この制御は、観測されたボードの位置、速度、および角度を入力とする常微分方程式 (ODE) を用いて行われている。さらに、彼は、このシステムの線形安定性解析を行い、サーフボードの安定性を保つパラメータを選択した。また、数値シミュレーションにより、提案された制御 ODE がボードを安定させる能力を持つことを検証した。これらにより、限られた場合ではあるが、流体に動かされるボードをサーファーの動作により制御するという複雑な現象の取り扱いに成功した。この結果は原著論文 1 報にまとめられた。以上により本論文は、博士(理学)を授与するに値すると判断した。

4．審査結果　(1) 判　定（いずれかに○印）　(合　格)・　不合格

(2) 授与学位　博　士（　理学　）