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Optical Phase Synchronization by Injection of Common Broadband Low-Coherent Light

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We demonstrate experimentally and numerically that, by injecting common broadband optical noise into two uncoupled lasers, the phases of the respective laser oscillations can be successfully synchronized. Experimental observation of the resulting phase dynamics is achieved using a heterodyne detection method. The present phase synchronization differs from the synchronization induced by coherent light signals or narrow band optical noise in the sense that it occurs without any frequency locking to the injection light. Moreover, it is revealed that there is an optimal intensity of the injection light that maximizes the quality of the phase synchronization. These results can open new perspectives for understanding and functional utilization of the laser phase dynamics.

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Synchronization is a key mechanism for the emergence of order in a variety of dynamical systems that consist of oscillatory units. Such systems can be found in physical, chemical, biological, and engineering contexts [1,2], and they are often subject to noise. A fundamental goal of nonlinear physics is to reveal the roles of noise in synchronization phenomena within real systems. Interestingly, recent studies have revealed that two independent dynamical systems can be synchronized with each other when they are driven by a common external noise [3]. This phenomenon is called *common noise-induced synchronization* (CNIS).

A theoretical study showed that CNIS can occur quite generally between two identical limit-cycle oscillators [4,5]. This result has been generalized to the case of oscillators with slightly different frequencies [6,7]. An essential feature of CNIS is phase synchronization without frequency locking: even in the case of oscillators with different frequencies, their phases become almost always locked to each other as noise intensity increases, even though the mean frequency difference between the oscillators remains constant [7]. These theoretical studies suggest the universality of experimental realization of CNIS in various real systems. However, experimental evidence is still limited and has been reported only for a few systems such as neurons [8,9], ecological systems [10], and electric circuits [11].

From a dynamical system point of view, a laser can be regarded as limit-cycle oscillator with a well-defined frequency: the phase variable of oscillation is given by the optical phase of the laser light. As such, lasers should offer good experimental stages to address fundamental issues of synchronization phenomena. However, the synchronization of the optical phase dynamics of lasers has not yet been well studied in an experimental context. In particular, there is no experimental evidence for phase synchronization in lasers driven by a common *broadband* noise, i.e., CNIS, although a numerical study suggests its possibility [12].

Several experiments have been conducted on the synchronization of lasers driven by a common noise [13,14]; however, all of these focused on the intensity dynamics of the lasers rather than their optical phase dynamics. More importantly, the phenomena addressed in these studies are essentially different from the CNIS of limit-cycle oscillators that was theoretically predicted in [4,5,7,12]. Experiments using Nd:YAG microchip lasers in [13] correspond to the CNIS of systems with stable fixed points. The synchronization reported in [14] can be essentially interpreted as being induced by a common periodic signal; as a narrow band random driving light with a characteristic frequency was in fact used, the synchronization occurring in this experiment was accompanied with frequency locking to the driving light.

In this Letter, we report on the first experimental demonstration of CNIS in the *optical phases* of lasers. Broadband low-coherence light, i.e., amplified spontaneous emission (ASE) light, is used as the common optical noise. As reported in [15], ASE light can be treated as classical broadband noise. The present experiment is a realization of CNIS by the broadband optical noise, and it is distinguished from the coherent light or narrow band random light induced synchronization in the sense that no frequency locking to the driving light is observed. The achievement of CNIS in the optical phases suggests new potential for the control of optical phase dynamics using ASE light, although such light has been so far considered as a source that disturbs laser coherence. This CNIS by ASE light could be useful for applications in communications, including a recently proposed secure key distribution scheme [16].

The system investigated in this Letter is illustrated in Fig. 1. Common ASE noise is injected into two lasers that

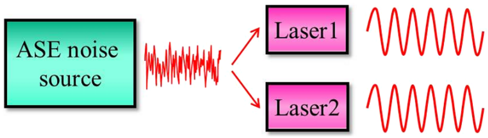

FIG. 1 (color online). Configuration of common ASE noise-driven lasers.

are not coupled to each other. Our interest here is in determining whether fast oscillations of the output light of the two lasers can be synchronized by injecting the common noise. A key technique for capturing the phase dynamics of the fast oscillations is to combine a heterodyne detection method with the Hilbert transform analysis. The proposed scheme works as follows. First, heterodyne signals are created by optical interference between a reference light with frequency ω_0, which is denoted as $A_0 \exp(-i\omega_0 t)$, where t is time, and the light signal from laser j ($j = 1, 2$), which is denoted as $A_j \exp[-i\omega_0 t + i\phi_j(t)]$. $\phi_j(t)$ and A_j denote the phase deviation from the reference light and the amplitude, respectively. The resulting heterodyne signal contains the interference term $I_j(t) = A_j A_0 \cos[\phi_j(t)]$. Then, the optical phase is extracted using the Hilbert transform $\phi_j(t) = \arg[I_j + iH(I_j)]$, where $H(I)$ denotes the Hilbert transform of I. Finally, a phase-unwrapping procedure is carried out, and the phase difference $\Phi = \phi_2 - \phi_1$ is calculated.

Figure 2 shows the experimental setup for implementing the above scheme. The setup is based on polarization-maintaining optical fiber components. The two lasers ($L1$ and $L2$) are distributed-feedback semiconductor lasers operating at almost the same frequency of 193 THz (corresponding to a wavelength of 1555 nm) above the threshold current $J_{\rm th} \approx 12$ mA. The side-mode suppression ratios of the two lasers were both more than 40 dB up to the injection current $J \approx 3J_{\rm th}$. A super luminescent diode (SLD) was used as an ASE light source, which has a broader spectrum than the lasers around 193 THz [17]. The ASE light from the SLD was divided into two via a 50/50 optical fiber coupler (FC) and injected into $L1$ and $L2$. The intensity of the ASE light was controlled with an optical attenuator (Att). A narrow (100 kHz) linewidth laser (REF) was used for the heterodyne detection. The frequency of REF was detuned from those of the lasers at ~5 GHz. The

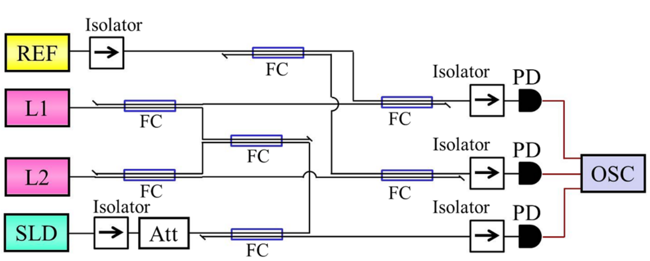

FIG. 2 (color online). Schematic of experimental setup.

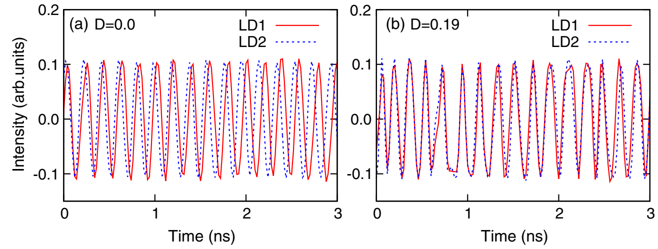

FIG. 3 (color online). ac components of heterodyne signals between the lasers and a reference for (a) $D = 0$ and (b) $D = 0.19$. $J/J_{\rm th}$ was fixed at 2.25.

created heterodyne signals were detected by photodetectors (PDs) with a bandwidth of 12.5 GHz and observed with an oscilloscope (OSC) having a bandwidth of 16 GHz at 50 GSample/s.

For convenience, the intensity of the injected ASE light is henceforth represented by the injection ratio $D = P_{\rm ASE}/P_L$, where P_L is the average output power of the lasers and $P_{\rm ASE}$ is the ASE power arriving at the lasers [17].

First, we consider the case when ASE light is not injected into the two lasers. Figure 3(a) shows the ac components of the heterodyne signals of $L1$ and $L2$ observed for $D = 0$ and $J = 2.25J_{\rm th}$. The difference between the phases extracted from the heterodyne signals $\Phi(= \phi_2 - \phi_1)$ is shown by the blue curve in Fig. 4. In this case, there is a slight frequency mismatch between $L1$ and $L2$ of about 43 MHz owing to the technical limitations of frequency tuning. The oscillations of the two signals are not matched, and Φ changes almost linearly with time. Consequently, the probability distribution of Φ is almost uniform [see Fig. 5(a)].

Next, we discuss the case when ASE light is injected into the two lasers. We observed that, as the injection power increases up to $D \approx 0.2$, the phase dynamics become more

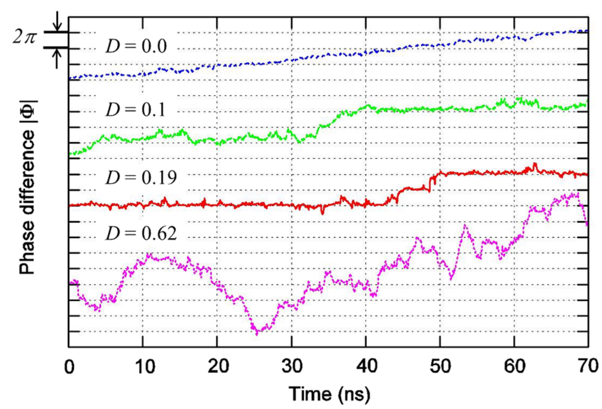

FIG. 4 (color online). Phase difference Φ versus time for $D = 0.0$ (blue dotted curve), $D = 0.1$ (green dashed curve), $D = 0.19$ (red solid curve), $D = 0.62$ (pink dotted curve). $J/J_{\rm th}$ was fixed at 2.25.

synchronous, although phase slip occurs because of the intrinsic noise in each laser and the frequency mismatch between the two lasers. The clearest result is obtained at $D = 0.19$. Figure 3(b) shows the ac components of the measured heterodyne signals. The phases of the respective oscillations are well matched. The phase difference Φ is shown by the red solid curve in Fig. 4. Φ is almost perfectly maintained for a long time of over 10 ns, which corresponds to about 2×10^6 times the period of laser oscillation for the lasing frequency of 193 THz. Consequently, the long entrainment time leads to a localized probability distribution of Φ [see Fig. 5(a)]. Hereafter, we refer to this state as the synchronized state.

It should be noted that the synchronized state is different from that induced by coherent light because the frequencies of response lasers are completely locked to the driving light in the latter case. By contrast, the two lasers in our experiments are not locked to the driving ASE light, and a frequency difference still remains between the two lasers. Figure 5(b) shows the temporal behaviors of $\Phi(t)$ and the short-term frequency difference, which was characterized by a short-term moving average of $d\Phi(t)/dt$, for $D = 0.19$. Although this frequency difference is almost always close to zero, it intermittently exhibits large absolute (and frequently positive) values when phase slips occur. Thus, the mean frequency difference is not zero ($\approx$40 MHz), but synchronization is still observed for long periods of time between successive phase slips. This phase synchronization with a frequency difference coincides well with the theoretical result for detuned limit-cycle oscillators perturbed by white noise [7].

When the injection power of the ASE light is further increased, however, we observe the transition from the synchronized state to a desynchronized one, where the phase difference is fluctuating [see the pink dotted curve in Fig. 4]. To evaluate the synchronization quality quantitatively, we used the synchronization index $\gamma^2 = \langle \cos \Phi(t) \rangle^2 + \langle \sin \Phi(t) \rangle^2$ [9], where the brackets denote the average over time. $\gamma = 1$ indicates complete synchronization while $\gamma = 0$ indicates a loss of synchrony. In

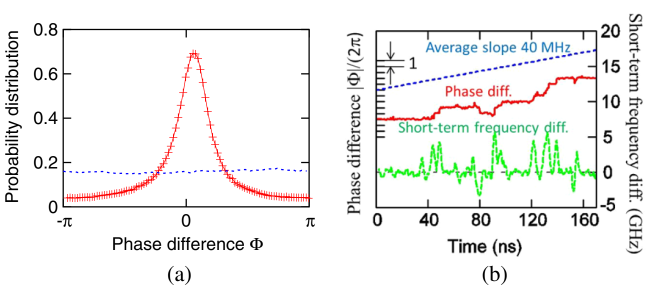

FIG. 5 (color online). (a) Probability distribution of Φ for $D = 0$ (blue dotted curve) and $D = 0.19$ (red crosses). (b) Temporal behaviors of Φ and the short-term frequency difference, which was characterized by 4 ns moving average of $d\Phi/dt$, for $D = 0.19$.

Fig. 6(a), γ is shown as a function of D, where the mean frequency difference is maintained at around 40 MHz [18]. There is the maximum of γ for an optimal injection power of the ASE light. In the weak injection region ($D < 0.05$), the γ value is low, and the synchronous dynamics is not observed, because the intrinsic noise in each laser is dominant. As D increases and the effect of the ASE light overcomes the intrinsic noise, the synchrony is enhanced. However, when D is too large ($D > 0.2$), γ decreases and the synchronous dynamics cannot be observed again. Consequently, high-quality synchronization can be observed only around the optimal value of the injection power.

The qualitatively same D dependence of γ was observed for different values of the mean frequency difference between the two lasers, but the maximum value of the synchronization index $\gamma_{\max} = \max \gamma(D)$ depended on the frequency difference values. In addition, we found that the injection current J is also a dominant parameter for affecting the synchronization quality. In Fig. 6(b), $\gamma_{\max}$ is plotted as a function of $J/J_{\rm th}$. One can clearly see that $\gamma_{\max}$ monotonically increases, with increasing J.

To understand the physical mechanism of the synchronization properties, we performed numerical calculations by using the rate equation model of a single-mode semiconductor laser [19]. The slowly varying complex amplitude of the optical field E_j and the carrier density N_j of laser ($j = 1, 2$) were described by the following set of dimensionless equations [20]:

$$\frac{d}{dt}E_j = -i\Delta_j E_j + (1 + i\alpha)g(N_j - 1)E_j + \xi + \eta_j, \quad (1)$$

$$\frac{d}{dt}N_j = \mu - N_j - N_j|E_j|^2. \quad (2)$$

In these equations, the time t is normalized to the carrier lifetime $\tau_s = 2.04$ ns. ξ and η_j denote the injected ASE light and intrinsic optical noise in laser j, respectively. They were modeled as complex white Gaussian noises of zero means such that $\langle \xi(t)\xi^*(t') \rangle = D_c \delta(t - t')$, $\langle \eta_j(t)\eta_k^*(t') \rangle = D_i \delta_{jk} \delta(t - t')$, and $\langle \xi(t)\eta_j(t') \rangle = 0$, where $D_i = 8.2 \times 10^{-3}$ and D_c characterize the intensities of the intrinsic noise and common noise, respectively. For

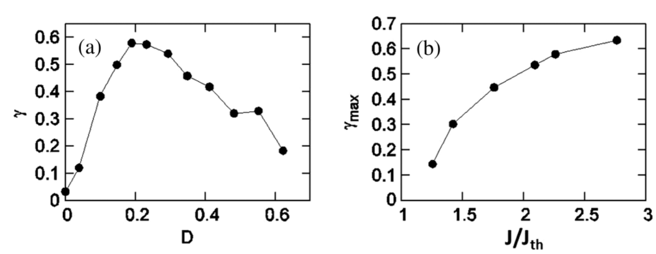

FIG. 6. (a) Injection ratio D dependence of γ at $J/J_{\rm th} = 2.25$. (b) $\gamma_{\max}$ versus $J/J_{\rm th}$.

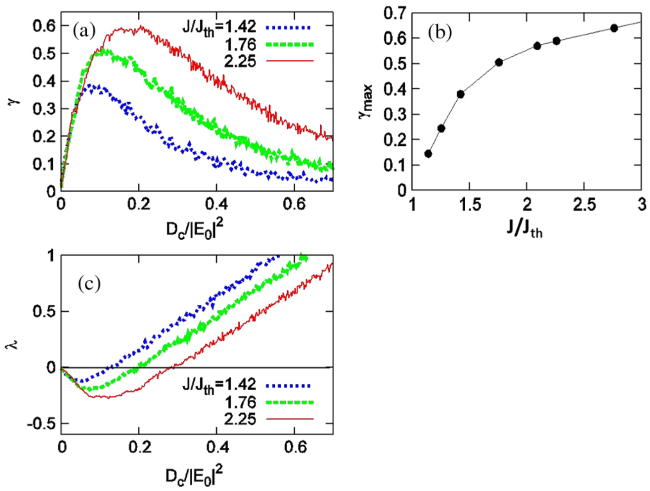

FIG. 7 (color online). Numerical results. (a) D_c dependence of γ for various values of $J/J_{\text{th}} = 1.42$, 1.76, and 2.25. (b) $\gamma_{\max}$ versus J/J_{th}. (c) D_c dependence of conditional Lyapunov exponent λ.

simplicity, the laser parameters except Δ_j are assumed to be identical for the two lasers. $\alpha = 5$ and $g \approx 630$ denote the linewidth enhancement factor and normalized differential gain, respectively. μ is proportional to the injection current $(J\tau_s - N_0)/(N_{\text{th}} - N_0)$, where $N_0 = 1.4 \times 10^{24}\ \text{m}^{-3}$ and $N_{\text{th}} = 2.018 \times 10^{24}\ \text{m}^{-3}$ denote the transparent carrier density and threshold carrier density, respectively. Δ_j is the frequency detuned from a reference frequency. For correspondence with the experiment, we set $(\Delta_2 - \Delta_1)/(2\pi\tau_s) = 40$ MHz.

If $D_c = D_i = 0$, the system of Eqs. (1) and (2) has a limit cycle solution $(E_j, N) = [E_0 e^{-i(\Delta_j t + \phi_j^0)}, 1]$ that emerges via Hopf bifurcation for $\mu > 1$, where $E_0 = \sqrt{\mu - 1}$ and ϕ_j^0 is an initial phase. This solution corresponds to the lasing state with the detuned frequency Δ_j. For $D_c \neq 0$ and $D_i \neq 0$, we numerically solved Eqs. (1) and (2) and calculated the phase difference between the two lasers as $\Phi(t) = \arg E_2 - \arg E_1$. Figures 7(a) and 7(b) show the numerical results for the D_c dependence of γ and J dependence of $\gamma_{\max}$, respectively. There exists the optimal noise intensity for the synchronization, and $\gamma_{\max}$ increases with increasing J. These numerical results qualitatively coincide with and confirm the experimental results.

We also calculated the maximum conditional Lyapunov exponents λ of Eqs. (1) and (2), as shown in Fig. 7(c). Compared with the result in Fig. 7(a), γ decreases in a large D_c region where the sign of λ changes from negative to positive. $\lambda > 0$ suggests a loss of synchrony between two uncoupled oscillators [12]. The desynchronization emerges when the state of each oscillator deviates considerably from the limit cycle by the perturbation of a noisy force and the fluctuation of the oscillation amplitude affects the phase dynamics [12,21]. The effect of the deviation is dominant when the relaxation rate to the limit cycle is small.

In this laser model, the injection current J characterizes not only the laser oscillation amplitude but also the relaxation rate to the oscillation state [19]. Therefore, for large values of J, the oscillation state is robust against noises, and the effect of the deviation becomes less dominant; i.e., the oscillation state may be localized near the limit cycle even for a large noisy perturbation. Consequently, as shown in Figs. 7(a) and 7(c), the noise intensity needed for the transition to the desynchronization becomes larger as J increases, and the synchronized state can be maintained even for larger values of D_c. This suggests that better synchronization can be observed as J increases and the relaxation rate becomes larger.

Lastly, we remark that the phase synchronization is accompanied by the synchronous behavior of the laser amplitudes, in the sense that the respective amplitudes can be well correlated when the phases coincide. For details, see Supplemental Material, Sec. III [22].

In conclusion, we have experimentally shown that common ASE noise can induce synchronization without frequency-locking in optical phase dynamics of lasers. The observed features can be explained by a simple model describing the interaction between laser oscillations and white noise. These results are convincing evidence for the universality of CNIS phenomenon and imply that the fast phase dynamics of laser oscillation in the hundred terahertz regime can be controlled with the ASE noise. For example, by injecting ASE noise repeatedly into a laser and using the consistency [13] of the response in the noise-driven laser, the phase of the laser oscillation may be reliably and reproducibly controlled. This could be useful for applications using optical phases.

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[17] See Supplemental Material, Sec. I at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.204101 for details on the ASE and laser spectra.

[18] See Supplemental Material, Sec. II at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.204101 for details on the experimental method and data acquisition.

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[22] See Supplemental Material, Sec. III at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.204101 for details on the intensity dynamics.