1975

A preliminary analysis of short-rotation Aspen management

Alan R. Ek

J.D. Brodie

Recommended Citation
Ek, Alan R.; Brodie, J. D. 1974. A preliminary analysis of short-rotation Aspen management. Canadian Journal of Forest Research 5(2):245-258.

Sup of 134
COPY 2

A Preliminary Analysis of Short-rotation Aspen Management[1]

ALAN R. EK AND J. D. BRODIE[2]
Department of Forestry, College of Agriculture and Life Sciences, University of Wisconsin, Madison, Wisconsin

Received June 26, 1974
Accepted December 20, 1974

EK, A. R., and BRODIE, J. D. 1975. A preliminary analysis of short-rotation aspen management. Can. J. For. Res. **5**, 245–258.

Stand-yield models were developed for stem and branch wood, stem wood and conventional utilization standards for various sites and initial densities. A model of sucker reproduction following harvesting is also presented. These models were then subjected to conventional economic analyses and long-term simulation comparisons. Results indicate that aspen rotations may be moderately shortened, with substantial increases in yields if utilization standards are increased. Greatest potential lies with the best sites, but more complete utilization standards may also allow operations on sites currently considered marginal. Rotations based on the usual soil-expectation value criteria could be reduced from the current 35 to 45-year range (at 5% discount rate) down to 20–30 years. Extremely short rotations (*e.g.*, < 15 years) appear undesirable due to sustained rapid volume and value growth rates well into the third decade.

EK, A. R., et BRODIE, J. D. 1975. A preliminary analysis of short-rotation aspen management. Can. J. For. Res. **5**, 245–258.

Des modèles de peuplement ont été conçus pour exprimer, d'une part le rendement en matière ligneuse du tronc et des branches, et d'autre part le rendement en matière ligneuse du tronc uniquement, le tout lié à des normes d'utilisation conventionnelles pour des sites et des densités initiales différents. Un modèle de regénération après coupe par rejet de souche est aussi présenté. Ces modèles ont été soumis à une analyse économique conventionnelle ainsi qu'à une simulation à long terme. Les résultats indiquent qu'une diminution modérée des révolutions de peuplier ainsi qu'une augmentation importante des rendements pourraient être atteintes si les normes d'utilisation étaient accrues. La plus grande capacité productive repose certes sur les meilleurs sites mais des normes d'utilisation plus complète pourraient permettre des opérations sur des sites actuellement classés marginaux. Les révolutions basées sur les critères usuels de valeurs du fonds pourraient passer de la classe 35–45 ans à la classe 20–30 ans, avec un taux d'escompte égal à 5%. Les très courtes révolutions, i.e. inférieures à 15 ans, ne semblent pas souhaitables à cause des rapides taux de croissance soutenue, et en volume et en valeur, trouvés dans la troisième décennie. [Traduit par le journal]

Introduction

Research on short rotation management has considered two different approaches, agronomic management with improved genetic stock and intensive crop-tending inputs, and harvest and regeneration of natural sucker stands with utilization of all stem and branch wood. The latter approach is considered here for aspen in the Lake States.

The first step in the study was the development of mathematical models for sucker production and various yield components. The second involved analysis of economic implications of alternative utilization standards, rotation lengths and management strategies.

Yield and Reproduction Models

Yield-prediction Methodology

Several yield tables are available for aspen stands in the Lake States (Kittredge and Gevorkiantz 1929; Anderson 1936; Gevorkiantz and Duerr 1938; Graham *et al.* 1963; Schlaegel 1971), but these alone were not considered adequate as a base for economic analyses in this study. The recent growth model by Schlaegel was constructed only with remeasured plot data from north-central Minnesota, and is limited to projections from known or specified initial stand conditions. The other tables also have geographical limitations and/or are based on temporary-plot data and "normal" or "vari-

[1]Based on a paper presented at the 6th TAPPI Forest Biology Conference May 1–3, 1972, Appleton, Wisconsin.

[2]This research was supported by the College of Agriculture and Life Sciences, University of Wisconsin, Madison, the University of Wisconsin Graduate School, and the McIntire-Stennis Cooperative Forestry Research Program, Project 1675.

TABLE 1. Aspen stand yield and reproduction models[a]

Dependent variable	Model[b]	R^2	Standard error	Basis (no. of observations)	Equation
(*a*) Yield models					
Total number of trees per acre ≥1.0 ft (0.30 cm) tall	$N_t = N_{t-1}\, e^{-(0.39318\, S\, A_{t-1}^{0.93042} N_{t-1}\, 10^{-7})}$	0.96	443	88	1
Average dominant height, ft	$H = S(1.46 - 1.4337\, e^{-0.227A})$	(from Lundgren and Dolid (1970))			2
Total stem volume, ft³/acre, from 0.5 ft (0.15 m) stump to tip of all trees ≥1.0 ft (0.30 m) tall (excluding brush species)	$V_s = 0.4972(H - 4.5)^{1.9139} N^{0.1439}$	0.88	283	133	3
Stem- and branch-wood volume, ft³/acre, from 0.5-ft stump to branch tips	$V_{sb} = V_s\,(1.1400 + H^{-1.1172})$	0.96	321	133	4
Conventional utilization standards volume, ft³/acre, for trees ≥5.0 in. (12.8 cm), from 0.5 ft stump to 3.0 in. (7.8 cm) top diameter inside bark	$V_c = V_s\,[\{0.1848(\mathrm{Dbh} - 1.7)\}\, R_1 + \{0.6150 + 0.3277\,(1.0 - e^{-0.5400(\mathrm{Dbh} - 5.0)})\}\, R_2]$, where if average stand diameter Dbh ≤1.7 in., $R_1 = 0$, $R_2 = 0$; 1.7 in. <Dbh ≤5.0 in., $R_1 = 1$, $R_2 = 0$; Dbh >5.0 in., $R_1 = 0$, $R_2 = 1$	0.95	187	133	5
Stand basal area, ft²/acre	$B = 0.0623\,(H - 4.5)^{1.3826}\, N^{0.3015}$	0.85	7.9	133	6
(*b*) Reproduction model					
Sucker density per acre at age 2 following parent-stand cutting, stems ≥1.0 ft (0.30 m) tall; bigtooth aspen included as minor component	$N_2 = 5.9638\, S\, B_a\, e^{-0.0197\, B_a}(1.0 - e^{-41.4665/(R + 1.0)})\, D\, T$	0.50	2926	58	7

[a]Metric equivalents: 1 acre = 0.405 ha; 1 ft = 0.3048 m; 1 ft²/acre = 0.2296 m²/ha; 1 ft³/acre = 0.0670 m³/ha.
[b]Definition of terms. Yield models (subscript t indicates variable value at time t): N = trees per acre; A = stand age; H = average dominant height; S = site index. Reproduction model (B_a = parent stand aspen basal area, ft²/acre): R = residual basal area (all trees), ft²/acre; D = date of treatment effect (see text); T = site treatment effect (see text).

able-density" yield table construction methodology (see Spurr 1952). Consequently, the stocking levels and mortality rates in these tables are likely to be only crude approximations of actual stand development. Other criticisms are vague merchantability standards, dated or incompletely specified curve-fitting methodology and a lack of information on stands less than 20 years of age.

An examination of available data then led to the construction of yield models based on tree height and stand-density development and methodology that would avoid much of the above-mentioned criticisms. The basic elements of the approach are: (*i*) prediction of changes in stand density as a function of initial stand conditions and time from permanent plot records; (*ii*) prediction of average dominant height growth as a function of age and site index; (*iii*) expression of future product volumes as functions of predicted future stand density and average dominant height obtained from steps *i–ii* above. The models developed are described in Table 1*a*. They are referenced later by equation numbers given in the table.

These models were developed to predict stand yield for ages 2–80 years. Initial density was evaluated at age 2, since previous studies indicated that ingrowth and mortality during the first two years following harvesting is erratic (Sandberg 1951). Beyond the end of the second growing season, however, ingrowth is minimal and mortality rates have stabilized. Fitting methodology involved nonlinear regression techniques (see Draper and Smith 1966) and the testing of numerous promising model forms.

Data Base

Sources of data used for model construction are indicated in Table 2*a* along with basic data characteristics. Some of these data were obtained from plot descriptions given in the literature. Data from upper Michigan, Minnesota and Wisconsin were included in all phases of the study. Some of the data (52 observations) for the construction of equations 3–6 were also taken from published yield tables (Kittredge and Gevorkiantz 1929; Gevorkiantz and Duerr 1938), and thus were smoothed average values representing a number of plots. Most plots were 75% or more trembling aspen (*Populus tremuloides* Mich.) by basal area. Bigtooth aspen (*Populus grandidentata* Michx.) was also present on a number of plots.

Since plots of many sizes were included in the data, each was weighted by the square root of its respective size according to suggestions by Freese (1962) and Wensel and John (1969). The R^2 and standard-error values given in Table 1 refer to weighted residuals. The standard errors noted may be underestimates, since some plot locations were clustered due to experimental designs used for earlier studies from which the data were drawn. Plot size for observations based on an average of a known number of plots was assumed to be equal to the aggregate study area (*e.g.*, an average reportedly based on five 0.1-acre (0.04-ha) plots was assumed to have arisen from a single 0.5-acre (0.2-ha) plot). Where information on plot size was absent, 0.1 acre was assumed.

Stand Density

Changes in stand density may be estimated by an iterative yearly solution of equation 1 (see Table 1*a*). Stands cut in June, July or August are assumed to be 2 years of age at the end of the next growing season.

Although outwardly complicated, this model is but a modification of the well known exponential decay function. The equation was based on weighted observations of mortality from 24 permanent plots. Records on more than one growth period were available for some plots. In addition to weighting by the square root of plot size, observations for this model were also weighted by the number of years in the growth period.

Volume

Some plot records did not indicate actual stem-wood volumes (V_s), although all of them did provide average and/or dominant height and basal area information. Consequently, some stem-wood volumes for fitting model 3 were derived from estimated form factors. Form factors were synthesized from height–diameter relationships given by Kittredge and Gevorkiantz (1929) and volume tables by Gevorkiantz and Olsen (1955). The stand form factor was $F = V_s/BH$, where B = stand basal area per acre in square feet. Representative values for F were 0.59, 0.44 and 0.40 for stands with dominant heights of 10, 20 and 30+ ft (3.0, 6.1 and 9.1+ m) respectively.

TABLE 2. Sources and description of data bases for aspen yield and reproduction equations given in Table 1[a]

Dependent variable	Equation	No. of observations	Sources	Site index	Stand age (years)	Parent stand age (years)	Basal area (ft^2/acre)	Sucker density per acre (stems ≥ 1.0 ft tall)[b]
(*a*) Yield models								
Total number of trees per acre, N_t	1	2	Strothmann and Heinselman (1957)	73	3– 5			
		2	Zehngraff (1946, 1949)	75–80	13–39			
		2	Sorenson (1968)	73	5–15			
		1	Day (1958), Mich. State Univ.	80	20–27			
		3	Steneker (1964), Steneker and Jarvis (1966)	53–69	14–33			
		5	Owens-Illinois, Inc.	60–66	30–58			
		1	Wis. Dep. Nat. Res.	58	57–75			
		3	Consolidated Papers, Inc.	63–83	10–24			
		69	Boise Cascade	50–90	2–32			
Average dominant height, H	2		Kittredge and Gevorkiantz (1929)					
Total stem volume, V	3	11	Person *et al.* (1971)	75	2–12		0– 76	
		21	Gevorkiantz and Duerr (1938)	60–70	20–80		69–127	
		29	Boise Cascade, Inc.	50–90	7–34		23–129	
		31	Kittredge and Gevorkiantz (1929)	40–80	20–80		38–165	
		15	Day (1958), Mich. State Univ.	60–80	10–28		58–144	
		6	Zehngraff (1946, 1949)	70–80	13–39		54–103	
		1	Sorenson (1952)	73	15		51	
		1	Bella and Jarvis (1967)	64	13		47	
		11	Steneker (1964), Steneker and Jarvis (1966)	53–70	11–45		54–162	
		7	Institute of Paper Chemistry	60–80	8–18		25– 74	

TABLE 2. (*Concluded*)

Dependent variable		Equation	No. of observations	Sources	Site index	Stand age (years)	Parent stand age (years)	Basal area (ft^2/acre)	Sucker density per acre (stems ≥1.0 ft tall)[b]
Stem- and branch-wood volume,	V_{sb}	4		Same as for equation 3					
Conventional utilization standards volume,	V_c	5		Same as for equation 3					
Stand basal area,	B	6		Same as for equation 3					
(*b*) Reproduction model									
Sucker density per acre at age 2,	N_2	7	13*	Consolidated Papers, Inc.	63–85	2– 7	37–57	12–130**	3 600 – 22 300
			14*	Wis. Dep. Nat. Res.	57–80	2– 5	6–47	2–142**	800 – 18 200
			10*	U.S. Forest Service, Chequamegon, N.F.	60–74	2– 4	39–53	8– 53**	2 850 – 22 500
			4	U.S. Forest Service, Nicolet, N.F.	45	5	10	15– 19**	1 560 – 6 153
			9*	Owens-Illinois, Inc.	58–87	2–11	38–53	8– 76**	2 900 – 24 500
			3*	Institute of Paper Chemistry	65–83	2– 8	18–37	33– 65**	9 695 – 21 300
			5	Boise Cascade	75–90	2– 7	31–37	122–129**	2 700 – 17 600

[a]Metric equivalents: 1 ft^2/acre = 0.2296 m^2/ha; 1 acre = 0.405 ha; 1 ft = 0.3048 m.
[b]Aspen sucker density only at observed age.
*Plots partially or completely observed by A. R. Ek, 1971.
**Parent-stand aspen basal area only.

TABLE 3. Aspen stand characteristics and yields for various sites, ages and initial sucker densities at age 2

		Per acre stand characteristics and yield[a]														
		Initial density, 1250/acre					Initial density, 5000/acre					Initial density, 20 000 / acre				
Site index	Age	Number of trees (N)	Basal area (B) (ft²)	Average Dbh(D) (in.)	Stem- and branch-wood volume (V_{sb}) (ft³)	Conventional volume (V_c) (ft³)	N	B	D	V_{sb}	V_c	N	B	D	V_{sb}	V_c
50	2	1250	0	0.0	0	0	5000	0	0.0	0	0	20000	0	0.0	0	0
	5	1217	4	0.8	30	0	4509	6	0.5	37	0	13733	9	0.3	43	0
	10	1124	15	1.6	170	0	3428	21	1.1	200	0	6863	26	0.8	221	0
	20	874	37	2.8	618	106	1814	46	2.1	687	49	2437	50	1.9	717	27
	40	475	65	5.0	1589	845	658	71	4.5	1665	742	723	73	4.3	1688	713
60	2	1250	0	0.3	1	0	5000	1	0.2	2	0	20000	1	0.1	2	0
	5	1211	7	1.0	57	0	4420	10	0.6	69	0	12873	14	0.4	80	0
	10	1101	21	1.9	270	7	3221	29	1.3	315	0	6035	35	1.0	345	0
	20	823	48	3.3	920	234	1605	59	2.6	1013	147	2067	64	2.4	1051	115
	40	423	82	6.0	2290	1491	560	90	5.4	2384	1411	606	92	5.3	2411	1383
	60	238	96	8.6	3278	2554	275	100	8.2	3349	2573	286	101	8.1	3367	2576
												Initial density = 30 000				
70	2	1250	1	0.4	5	0	5000	2	0.3	7	0	30000	3	0.1	9	0
	5	1205	10	1.2	91	0	4335	14	0.8	109	0	14912	20	0.5	131	0
	10	1079	27	2.2	390	28	3036	37	1.5	453	0	5828	45	1.2	498	0
	20	779	61	3.8	1276	428	1439	73	3.1	1394	304	1837	79	2.8	1444	257
	40	380	101	7.0	3101	2237	487	109	6.4	3213	2202	524	111	6.2	3248	2186
	80	130	118	12.9	5192	4247	140	121	12.6	5249	4289	143	122	12.5	5265	4301
80	2	1250	2	0.6	12	0	5000	3	0.3	14	0	30000	6	0.2	18	0
	5	1198	13	1.4	133	0	4252	18	0.9	159	0	13818	26	0.6	188	0
	10	1059	34	2.4	531	62	2870	46	1.7	613	1	5201	55	1.4	668	0
	20	738	74	4.3	1683	700	1304	87	3.5	1827	532	1617	93	3.3	1884	472
	40	346	119	8.0	4018	3065	431	128	7.4	4148	3074	460	130	7.2	4186	3072
	80	115	139	14.9	6669	5476	123	142	14.5	6737	5528	125	142	14.4	6750	5541
90	2	1250	3	0.7	20	0	5000	5	0.4	24	0	30000	8	0.2	31	0
	5	1192	16	1.6	181	0	4171	23	1.0	217	0	12858	33	0.7	255	0
	10	1039	41	2.7	693	111	2721	55	1.9	796	29	4694	65	1.6	861	0
	20	702	87	4.8	2141	1060	1192	102	4.0	2311	844	1444	108	3.7	2375	769
	40	317	138	8.9	5038	3968	387	147	8.3	5185	4016	409	149	8.2	5227	4026
	80	103	159	16.8	8304	6831	110	162	16.5	8376	6889	111	163	16.4	8394	6904

[a]Gross volumes, no reduction for defect. Metric equivalents: 1 ha = 2.471 acres; 1 in. = 2.54 cm; 1 ft²/acre = 0.2296 m²/ha; 1 ft³/acre = 0.0670 m³/ha.

The stem- and branch-wood volume (V_{sb}) equation (4) was fitted by two-stage least squares, as described in Johnston (1963),[3] with V_s obtained from equation 3. This equation indicates that branches comprise 27% of stand volume for a stand with a 5-ft (1.52-m) average dominant height and that this percentage diminishes asymptotically to 14% as H increases. Since few of the plots utilized had actual measurements of V_{sb}, however, a graphical representation of the V_{sb}/V_s – dominant height relationship was derived by extrapolating from Keays (1971). This graph was then used to obtain missing V_{sb} values. A combination of actual and graphically estimated values was then used as the data base for fitting model 4.

The equation (5) for volume with conventional utilization standards (V_c) involves two expressions on the right-hand side. The first is the V_c/V_s ratio for stands in which Dbh $\leq$ 5.0 in. (12.8 cm). The second is the corresponding ratio for stands of larger Dbh. These expressions were first formulated from a graphical analysis of stand volume – average stand diameter information in yield tables from Kittredge and Gevorkiantz (1929), Gevorkiantz and Duerr (1938), Brown and Gevorkiantz (1934), tree size–volume relationships given by Gevorkiantz and Olsen (1955) and field data from cooperators. The resulting graphical approximation of the V_c/V_s – average stand diameter relationship was then used to derive conventional merchantable volumes from stem-wood volumes for plots for which the former data were not available. Model 5 was then fitted to a combination of actual and graphically estimated observations with predicted basal areas from equation 6 to determine the necessary Dbh. Actual rather than predicted numbers of trees (N) were used as independent variables in this fitting process. This was necessitated by the fact that few plots with volume records also had records of stocking at age 2 on which to base predictions of future N from equation 1.

Yields

Yield information based on equations 1–6 for a range of sites, ages and initial sucker densities is presented in Table 3. If merchantability standards should change or more definitive information on top volume becomes available, it would be a simple matter to alter the conversion factors applied to stem-wood volume. Tree-defect deductions, which were ignored here, could also be handled by an alteration of conversion factors. These equations can be used for growth projections from any initial condition or age, since beginning density (N_{t-1}) and age (A_{t-1}) values in equation 1 are not restricted to values at age 2.

These yields depend to a large extent on the site-index curves utilized. Their accuracy in application depends on the degree to which actual stand-height growth follows these curves. Because of the variability in height growth of sucker stands and the narrow site-curve separation at young ages, yield predictions are expected to be less precise in the 1–20 year range than for more advanced ages.

Reproduction-prediction Methodology

Suckering data were limited, since records from earlier studies by Sandberg (1951) and Stoeckler and Macon (1956) could not be located. Consequently, 49 of the plots used were visited in the fall of 1971. Some cutting dates and treatment of permanent plot records were obtained from cooperators. Missing data on parent-stand characteristics or sucker density were then tallied. Usual plot sizes were 0.1 acre (0.04 ha) for the parent stand and 0.001 acre (0.0004 ha) for suckers. Ten of the latter were usually located systematically over the parent-stand plot. Parent-stand basal area was determined by converting stump diameter to Dbh using conversion tables from Horn and Keller (1957). Only 20 of the sucker observations were actually measured at age 2, but all were less than 12 years old. Sucker densities for age 2 for older stands were obtained by working backwards to age 2, iteratively, with equation 1. Weighting used in model fitting was by parent-stand plot size only. Additional information on the data set is given in Table 2*b*.

[3]Two-stage least squares was used here to improve consistency in yield predictions and to obtain better estimates of true variances in cases where independent variables may themselves be predictions rather than measured values. Johnston's text covers only applications involving linear models. The application of this methodology to nonlinear models was based on intuition, since the authors were unable to locate references on the subject.

TABLE 4. Aspen-sucker density at age 2 as a function of site quality, parent-stand aspen density and residual density for date class 1 (summer) and treatment class 1 (kill parent aspen stand)[a]

Site index	Residual basal area (ft²/acre)	Parent-stand aspen basal area at time of harvest (ft²/acre) 10	30	50	70	90	110	130
		Trees per acre ≥ 1.0 ft tall						
50	0	2449	4953	5566	5254	4555	3754	2991
	20	2109	4265	4793	4525	3923	3233	2576
	40	1558	3151	3542	3343	2898	2388	1903
	70	1083	2191	2462	2324	2015	1660	1323
70	0	3428	6934	7792	7356	6377	5255	4188
	20	2952	5971	6711	6335	5492	4526	3606
	40	2181	4412	4958	4680	4057	3344	2664
	70	1516	3067	3447	3254	2821	2325	1852
90	0	4407	8915	10019	9457	8199	6757	5384
	20	3796	7678	8628	8144	7061	5819	4637
	40	2804	5673	6375	6018	5217	4299	3426
	70	1950	3944	4432	4184	3627	2989	2382

[a]Metric equivalents: 1 ft²/acre = 0.2296 m²/ha; 1 ha = 2.471 acres.

The model developed for aspen-sucker density at age 2 following parent-stand cutting is shown in Table 1*b*. The nature and magnitude of the date (D) and site treatment (T) effects are as follows, with harvest and site treatment assumed to be within the same date class.

Date class	Effect multiplier
1. Dormant season cutting	$D = 1.17$
2. Summer (June, July, August)	$D = 1.00$
Treatment class	
1. Kill parent aspen stand (*e.g.*, hypohatchet, no timber removal)	$T = 1.00$
2. Harvest cut, no site treatment	$T = 1.32$
3. Harvest cut, residual knocked down with heavy equipment (*e.g.*, dozer blade, KG blade)	$T = 1.62$
4. Harvest cut, residual felled, burned	$T = 3.05$
5. Harvest cut, disked	$T = 3.46$

Table 4 gives equation results for several site indices and residual density levels for summer cutting and treatment class 1. Results for other date or treatment classes can be obtained by applying the class effect multipliers. For treatment class 5, for example, table values for number of suckers would be multiplied by 3.46.

The class multipliers were obtained by grouping months and treatments into classes and assigning tentative multipliers as dummy variables. These dummy variables were then adjusted by an exponential parameter assigned to them in the model-fitting process. Multipliers were revised and the process repeated until no further improvement was evident. Resulting treatment ranking appears to be according to the degree of site disturbance. Sandberg (1951) indicates a basis for such ordering.

Sucker production was originally thought to be asymptotic with respect to increasing parent-stand basal area, but much better fits were obtained with model 7 than with asymptotic expressions. Equation 7 has a maximum sucker reproduction at approximately 50 ft²/acre (11.5 m²/ha) of parent-stand aspen basal area. This result may be an age effect, or it could be a feature of the sample. An age effect could be explained by diminishing individual tree sprouting coinciding with decreased basal area growth as the stand matures. The fact that the function is relatively flat-topped should minimize concern for this effect in interpretation. Attempts to introduce an explicit age term in the model produced fits consistently inferior to equation 7. This lack of improvement is attributed to the fact that age and basal area are correlated and to the apparent almost overwhelming effect of site treatment.

Economic Analysis

Analysis of the yield models were carried out for site indices 60, 75 and 90 with initial densities of 1 250, 10 000 and 30 000 suckers per acre (3 089, 24 710 and 74 132/ha) for conventional utilization volume (V_c) and full-tree volume (V_{sb}). The primary analysis considered a range of rotation lengths and corresponding physical and financial yields. The sensitivity of these yields and soil-expectation values to site quality, sucker density and changes in rotation lengths was then examined. Implications of conversion from current 40-year rotations to shorter rotations were studied in a 200-year simulation of operations on a hypothetical forest.

The financial results tabulated here are based on roadside values. The latter were derived from a roadside value for aspen of \$0.105/ft^3 (\$3.71/m^3) with logging costs of \$10.00/acre (\$24.71/ha) plus \$0.075/ft^3 (\$2.65/m^3) and a site treatment cost of \$8.00/acre (\$19.77/ha). Results of experiments done with different cost levels and harvesting costs variable with volume per acre are also reported below.

Rotation Lengths

Table 5 indicates optimal rotation lengths for conventional and full-tree utilization for various evaluation criteria: current annual increment (CAI), mean annual increment (MAI) and soil expectation value. Significant points to note are that (*i*) optimal rotations for these criteria are substantially shorter for full tree utilization than for conventional harvest practices; (*ii*) rotation lengths decrease as site quality increases, the greatest effect being noted for conventional utilization standards; (*iii*) rotations based on soil-expectation criteria are much more sensitive to site-quality changes than those based on CAI or MAI; (*iv*) initial sucker density has only a slight effect on rotation lengths (higher densities shortened rotations by 1 or 2 years at most).

Perala (1973) suggests that full-tree mean annual increment culminates at 27 or more years, but this was well beyond the maximum age in his data base. The present analysis (Table 5) indicates that it lies around age 36 for good sites and 39 for poorer sites. For conventional yield, the range for maximum mean annual increment is 46–58 years.

The apparent negligible effect of initial sucker density is understandable upon examination of both the mortality equation (1), which tends to normalize stand density with increasing age, and the relatively small exponent for N in volume equation 3.

Physical and Financial Yields

Selected yield and financial variables are given in Table 6 for site index 90. Significant results are as follows. (*1*) Both physical and financial yields are substantially higher for full-tree than for conventional utilization, especially at younger stand ages. For example, the maximum MAI in cubic feet on site index 90 for full-tree is 28% greater than that for conventional utilization. (*2*) The maximum soil expectation value for full-tree utilization occurs earlier and is more than twice as high as that for conventional harvesting practices. For site index 60 (not shown), maximum soil-expectation values (5%) were \$2.61 for full-tree *versus* −\$3.65 for conventional utilization. (*3*) Both soil-expectation and mean annual-increment curves are relatively flatter in the region of culmination for stem- and branchwood utilization than for conventional. This indicates that financial and physical rotations are less sensitive decisions on the higher utilization standard.

For the site-stocking example of Table 6, the internal rate of return on a maximum soil-expectation rotation (5%) of 22 years was 10.4%. Increases in discount rate tend to reduce optimal rotation length. The use of an 8% rate reduces rotation in this example from 22 to 19 years. Internal rate of return peaks at 10.975% on a rotation of 17 years. This sets a lower economic bound on rotation length and an upper bound on discount rate of 10.975%. This follows from the definition of the maximum internal rate of return as the highest discount rate that can be applied and still yield a non-negative soil expectation. A discount rate of zero (forest rent) yields a rotation of 40 years.

These results suggest why previous attempts to extrapolate from data gathered only from younger age classes have underestimated the culmination point of MAI. Natural aspen stands have the rapid early growth necessary for productive short rotations; however, since this growth rates does not fall off rapidly, financial or physical rotations occur at more

TABLE 5. Calculated rotation lengths for various evaluation criteria for full-tree and conventional utilization standards

Site index	Sucker density at age 2 years (stems per acre)[b]	Utilization-level and evaluation criteria[a]							
		Full-tree maximum CAI	Conventional maximum CAI	Full-tree maximum MAI	Conventional maximum MAI	Maximum soil-expectation value, full-tree		Maximum soil-expectation value, conventional	
						5%	8%	5%	8%
		Rotation length (years)							
60	1 250	21	36	39	58	29	25	46	42
	10 000	21	38	39	58	29	25	46	42
75	1 250	20	31	38	51	25	21	39	35
	30 000	20	31	37	51	25	21	39	35
90	1 250	20	26	37	46	22	19	34	30
	30 000	20	26	36	46	22	19	34	31

[a]CAI = current annual increment, MAI = mean annual increment.
[b]Equivalent: 1 ha = 2.471 acres.

TABLE 6. Selected yield and financial variables for full-tree and conventional aspen-utilization standards, site 90, initial sucker density at age 2, 10 000 stems per acre

Age	Volume per acre (ft^3)[a]	CAI (ft^3)[b]	MAI (ft^3)	Costs per ft^3 (\$)	Costs per acre (\$)[c]	Soil-expectation value (5%) (\$)	Gross harvest value (\$)[d]
Full-tree							
10	853	140	85	0.10	87.00	4.07	89.56
20	2369	156[e]	118	0.09	208.91	24.10	248.75
22	2681	155	121	0.09	234.52	24.43[e]	281.56
26	3296	150	126	0.09	285.64	23.65	346.07
34	4448	134	131[e]	0.09	385.63	19.14	467.04
46	5915	106	129	0.09	529.07	10.90	621.03
Conventional							
10	—	20	—	—	—	−36.62	—
20	776	141	39	0.12	89.39	−4.81	81.43
22	1070	164	49	0.11	113.67	−0.67	112.38
26	1792	190[e]	69	0.10	172.86	5.99	188.18
34	3185	151	93	0.09	290.94	10.24[e]	334.48
46	4713	102	102[e]	0.09	438.98	6.63	494.90

[a]Metric equivalents: 1 ft^3/acre = 0.0670 m^3/ha; 1 ha = 2.471 acres.
[b]CAI = current annual increment, MAI = mean annual increment.
[c]Comprised of a variable cost of \$0.075/ft^3 (\$2.65/m^3), harvesting set-up cost of \$10.00/acre (\$24.71/ha) and an \$8.00/acre (\$19.71/ha site-preparation cost compounded to harvest.
[d]Equivalent: \$0.105/ft^3 (\$3.71/m^3).
[e]Maximum in a series.

advanced ages than shorter-range studies have considered. Site 60 produces only 40 ft^3/year conventional yield on an optimal soil-expectation rotation (5%) of 46 years, whereas the comparable yield for full-tree is 59 ft^3 on a rotation of 29 years.

Cost Sensitivity

Additional studies with cost assumptions other than the values noted above and in Table 6 indicate that large increases in set-up and regeneration costs increase full-tree rotation length slightly. A fairly steep reduction in variable cost related to volume per acre has a similar slight effect on full-tree rotation length. The conclusion was, however, that optimal full-tree rotation length is relatively insensitive to these costs over a range of reasonable values. The soil expectation value itself, however, is quite sensitive to cost assumptions.

It is possible that advances in harvesting technology (*i.e.*, cost reductions) may eventually favor shorter rotations. Table 7 describes the level to which costs must fall to favor rotations less than that determined by current soil expectation values (22 years for site index 90). As an example, if costs on an 8-year rotation were to fall from \$65.92 to \$50.07, the soil-expectation value on an 8-year rotation would be equivalent to the current maximum at age 22. For this cost reduction to be strictly meaningful, the new technology that induced the saving would have to be applicable to only the shorter rotation. It is apparent from Table 7, however, that the relative cost reductions necessary to reach the current maximum soil-expectation values are substantial for rotations of less than 12 years.

Implications for Conversion to Shorter Rotations

Analysis of a real or hypothetically stocked forest manipulated over a finite-time horizon is a substantial aid in the formulation of a conversion strategy. Almost all initial stocking conditions that include stands over the specified rotation age will yield higher values of physical and financial comparison variables in the early years of analysis on shorter *versus* longer rotations. This is due to the larger conversion-period harvests[4] with the shorter rotations. Longer rotations begin to catch up with the shorter rotations after conversion because of higher equilibrium-period harvests.

The simulated conversion analysis was based

[4]For a time horizon of T and rotation length t in years, simulation consists of t area-control conversion harvests followed by sustained yield harvests for the remaining $T - t$ years.

TABLE 7. Harvesting costs required to achieve the current maximum soil-expectation value on shorter rotations, full-tree utilization site 90, initial sucker density at age 2, 10 000 stems per acre[a]

Rotation length (years)	Cost required to equal current maximum soil-expectation value ($)	Cost per acre ($)[b]
2	0.61	21.05
4	11.39	31.62
8	50.07	65.92
12	100.04	109.92
16	154.20	158.25
20	208.36	208.91
22[c]	234.52	234.52

[a]Equivalent: 1 ha = 2.471 acres.
[b]Consists of set-up ($10.00/acre ($24.71/ha)), variable harvesting ($0.075/ft^3 ($2.65/m^3)) and regeneration cost ($8.00/acre ($19.71/ha)) compounded at 5% to harvest. Price is $0.105/ft^3 ($3.71/m^3).
[c]Maximum soil-expectation rotation; 5% discount rate.

TABLE 8. A simulated comparison of selected rotations from 2 years to maximum soil expectation based on conversio of 1000 acres uniform stocking, ages 1–40, with 200 years conversion and sustained-yield harvest, site 90, initial sucker density at age 2, 10 000 stems per acre, full-tree utilization[a]

Rotation length (years)[b]	Cumulated volume (200 years) (MM ft^3)[c]	Cumulated net revenue (200 years) (MM dollars)[d]	Cumulated compounded (5%) net revenue (200 years) (MMM dollars)[d]	Volume intersection (year)[e]	Net-revenue intersection (year)[e]	Compounded net-revenue (5% intersection (year)[e]
2	5.49			12	6	11
4	10.50			15	9	18
8	17.30		18.36	18	13	31
12	21.55	0.226	24.53	20	17	48
16	24.34	0.365	26.30	19	18	59
20	26.19	0.443	26.10	17	11	23
22	27.20	0.474	27.08	—	—	—

[a]Equivalent: 1 ha = 2.471 acres.
[b]Maximum soil expectation is at 22 years for 5% interest.
[c]Includes terminal growing stock.
[d]Negative values omitted.
[e]Intersection indicates the year in which the shorter rotation series falls behind the corresponding maximum soil-expectation series.

on the assumption of 1000 acres uniformly stocked with age classes 1–40. This forest base was then converted to a sustained yield basis and harvested over a 200-year time horizon for all rotations from maximum soil expectation to 1 year. Comparison variables were cumulative harvest volume, cumulative net revenue (cash flow) and compounded net revenue.

Results in Table 8 represent only the full tree comparisons for the previously discussed site index 90, $N = 10\,000$. For the 8- *versus* 22-year rotation, a superiority in volume and value for the longer rotation emerged in the second decade (year 18 and 13, respectively), and for compounded value, in the third decade (31 years). The values at the end of 200 yea have the greatest relative difference in n revenue and substantial difference in volum and compounded net revenue. The 22-ye maximum soil-expectation rotation is superi for all variables by year 59.

These results indicate that only small shor run gains would be obtained by conversion t rotations less than 22 years. The relatively la emergence of superiority for the 22- *versus* 1 year rotation in terms of compounded n revenue is due to the effect of compounding o early returns from the shorter rotation. A higher interest rates, emergence of superiorit occurs much later or not at all.

Results and Discussion

The usual approach to a growth and yield study involves substantial time and field effort in collecting a "clean" data set. Given the preliminary nature of interest in short-rotation aspen management, however, such effort did not seem warranted. Further, to suggest that 60 years of research had not already provided the basis for meaningful economic analyses would be a gross indictment of past work. Consequently, the authors adapted a somewhat unorthodox approach involving a synthesis of data from numerous sources. Although some extrapolation is evident in the yield and reproduction section of the paper, it was felt that each such step was relatively small and any resulting cumulative bias is unlikely to be large and significant in analyses.

Although not exhaustive, the economic analysis suggests that substantial shortening of aspen rotations for a fiber-production objective may be brought about by utilization of small stem and branch wood, *i.e.*, material presently considered nonmerchantable. Calculated best rotations are reduced from the 30- to 45-year range down to the 20- to 30-year range. Further, reproduction is not a critical factor here, since suckering appears substantial under minimal-treatment programs and rotation lengths are only slightly affected over a broad range of initial stand densities.

These shorter rotation lengths are within the realm of current experience, thus technological and policy adjustments would seem to require only evolutionary change. Management problems induced by pathological rotations — early stand deterioration and breakup — would also appear to be reduced, since such conditions usually develop after age 30. Of course, managers may have other product alternatives available, such as veneer or sawbolt material. These alternatives would probably lengthen rotation, but such opportunities are not expected to be extensive (Hughes and Brodie 1972).

Assistance from staff of the U.S. Forest Service North Central Forest Experiment Station, Timber Management Staff of the Chequamegon and Nicolet National Forests, the Wisconsin Department of Natural Resources Divisions of Forestry and Game Research, Owens-Illinois, Inc., Consolidated Papers, Inc., Boise Cascade, the Institute of Paper Chemistry, University of Minnesota College of Forestry, and the Michigan State University Dunbar Forest Experiment Station was invaluable in the synthesis of a data base. Special thanks are extended to Richard C. Doolittle, Research Forester, Consolidated Papers, Inc., and A. B. Johnson, Technical Forester, Owens-Illinois, Inc., for assistance with field studies in northern Wisconsin.

ANDERSON, R. T. 1936. Yields of aspen in the lake states. Lake States For. Expt. Stn. For. Res. Digest, May–June. pp. 3–7.

BELLA, I. E., and JARVIS, J. M. 1967. High total productivity of young aspen stand—Manitoba. Pulp Paper Mag. Can. Woodlands Rev. (October) **432**, 437.

BROWN, R. M., and GEVORKIANTZ, S. R. 1934. Volume, yield and stand tables for tree species in the Lake States. Univ. of Minn. Agric. Exp. Stn. Tech. Bull. 39 (revised).

DAY, M. W. 1958. Thinning aspen in upper Michigan. Mich. Agric. Exp. Stn. Q. Bull. 41, pp. 311–320.

DRAPER, N. E., and SMITH, H. 1966. Applied regression analysis. John Wiley & Sons, Inc., New York.

FREESE, F. 1962. Elementary forest sampling. U.S.D.A., Forest Service, Agric. Handb. 232.

GEVORKIANTZ, S. R., and OLSEN, L. P. 1955. Composite volume tables for timber and their application in the Lake States. U.S.D.A. Tech. Bull. 1104.

GEVORKIANTZ, S. R., and DUERR, W. A. 1938. Methods of predicting growth of forest stands. U.S. For. Serv. Lake States For. Exp. Stn. Econ. Note 9.

GRAHAM, S. A., HARRISON, R. P., JR., and WESTELL, C. E., JR. 1963. Aspens, phoenix trees of the Great Lakes Region. University of Michigan Press, Ann Arbor.

HORN, A. G., and KELLER, R. C. 1957. Tree diameter at breast height in relation to stump diameter by species group. U.S. For. Serv. Lake States For. Exp. Stn. Tech. Note 507.

HUGHES, J. M., and BRODIE, J. D. 1972. Selected economic aspects of aspen. *In* Aspen. Symp. Proc., U.S. For. Serv. Gen. Tech. Rep. NC-1, pp. 27–34.

JOHNSTON, J. 1963. Econometric methods. McGraw–Hill Book Co., New York.

KEAYS, J. L. 1971. Complete Tree Utilization: An analysis of the literature. In five parts. Can. Dep. of Fish. and For., For. Prod. Lab., Inf. Reps. VP-X-69, 70, 71, 77, 79. Vancouver, B.C.

KITTREDGE, J., JR. 1938. The interrelations of habitat, growth rate and associated vegetation in the aspen community of Minnesota and Wisconsin. Ecol. Monogr. **8**, pp. 151–246.

KITTREDGE, J., JR., and GEVORKIANTZ, S. R. 1929. Forest possibilities of aspen lands in the Lake States. Univ. Minn. Agric. Exp. Stn. Tech. Bull. 60.

LUNDGREN, A. L., and DOLID, W. A. 1970. Biological growth functions describe published site index curves for Lake States timber species. U.S. For. Serv. Res. Pap. NC-36.

PERALA, D. A. 1973. Stand equations for estimating aerial biomass net productivity, and stem survival of young aspen suckers on good sites. Can. J. For. Res. **3**, 288–292.

PERSON, R. A., HALLGREN, A. R., and HUBBARD, J. W. 1971. Yields from short-rotation aspen suckers. Univ. Minn. Sch. For., Minn. For. Res. Note 224.

SANDBERG, D. 1951. The regeneration of quaking aspen by root suckering. M.S. thesis, Univ. Minn. Sch. For., St. Paul.

SHIRLEY, H. L. 1931. Part I: Does light burning stimulate aspen suckers? J. For. **29**, 524–525.

——— 1932. Part II: Does light burning stimulate aspen suckers? J. For. **30**, 419–420.

SCHLAEGEL, B. E. 1971. Growth and yield of quaking aspen in north central Minnesota. U.S.D.A. For. Serv. Res. Pap. NC-58.

SORENSON, R. W. 1968. Size of aspen crop trees little affected by initial sucker density. U.S. For. Serv. Res. Note NC-51.

SPURR, S. H. 1952. Forest inventory. The Ronald Press Co., New York.

STENEKER, G. A. 1964. Ten-year results of thinning 14-, 19- and 23-year-old aspen to different spacings. Can. Dep. For. Pub. 1038.

STENEKER, G. A., and JARVIS, J. M. 1966. Thinning in trembling aspen stands in Manitoba and Alberta. Can. Dep. For. Pub. 1140.

STOECKLER, J. H., and MACON, J. W. 1956. Regeneration of aspen cutover areas in northern Wisconsin. J. For. **54**, 13–16.

STROTHMAN, R. O., and HEINSELMAN, M. L. 1957. Five-year results in an aspen sucker density study. U.S. For. Serv. Lake States For. Exp. Stn. Tech. Note 490.

WENSEL, L. C., and JOHN, H. 1969. A statistical procedure for combining different types of sampling units in forest inventory. For. Sci. **15**, 307–317.

ZEHNGRAFF, P. J. 1946. Thinning young aspen can pay. U.S. For. Serv. Lake States For. Exp. Stn. Tech. Note 263.

——— 1949. Aspen as a crop in the Lake States. J. For. **47**, 555–565.