Group chase and escape with some fast chasers

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Group chase and escape with some fast chasers

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We study group chase and escape with some fast chasers. In our model chasers look for the nearest target and move to one of the nearest sites in order to catch the target. On the other hand, targets try to escape from the nearest chaser. When a chaser catches a target, the target is removed from the system and the number of targets decreases. The lifetime of targets, at which all targets caught, decreases as t^{α} with increasing the number of chasers. When there are no fast chasers and the total number of chasers is small, the exponent α is large. When the total number of chasers is large, α becomes small. There is an optimal number of chasers to minimize the cost used in order to catch all targets. However, when we add a few fast chasers, the region with the large α vanishes. The optimal number of chasers vanishes, and the cost monotonically increases with increasing the number of chasers.

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I. INTRODUCTION

Problems of chasing and escaping have been studied for a long time [1–3]. Chasing and escaping between a target and a chaser, which is a model of a warship chasing a pirate vessel, is one of the simplest cases. One of the more complicated cases is the problem of chasing and escaping with a target and some chasers [4–7]. In that case, if chasers walk randomly and a target does not move, the probability $p_{\mathrm{imm}}(t)$ with which the target survives until time t is given by $p_{\mathrm{imm}}(t) \sim \mathrm{e}^{-\alpha\rho t}$, where α is the positive constant and ρ is the density of chasers. The dependence of the density of chasers changes when both chasers and targets can move. If chasers and targets search each other in the region which is shorter than the critical length, the survival probability is given by $p_{\mathrm{imm}}(t) \sim \mathrm{e}^{-\alpha\rho^3 t}$.

Recently, Kamimura and Ohira [8] studied a more complicated case: group chase and escape. In their model, although each chaser independently moves in order to catch one of the nearest targets, some groups of chasers are simultaneously formed. A lifetime T, at which the last target is caught, depends on the number of chasers. The lifetime T decreases with the number of chasers as $N_{\mathrm{c}}^{-\alpha}$. When there is only a few chasers, the value of the exponent α is about 3. On the other hand, with sufficiently many chasers, the value of α decreases and becomes 0.75. The dependence of an average lifetime τ on the number of targets was also studied. When the number of targets is small, the lifetime τ increases with increasing the number of targets. With too many targets, however, they prevent each other from escaping from chasers, and the lifetime τ decreases with increasing the number of targets. Thus, although T increases monotonically with increasing the number of targets, there is an optimal density of targets, which gives the longest average lifetime. The authors also evaluated the cost to catch all targets and showed the existence of an optimal number of chasers, which minimize the cost to catch all targets.

The model used in a previous study is so simple that many various modified versions of the model are conceivable. For example, Nishi and coworkers studied the group chase and escapes under a conversion law [9], and we studied the group chase and escape with three species [10]. In the previous paper [8], some variations of the original model, which are valuable to study, are suggested. One of them is the model in which chasers or targets with fast velocity are present. If only one fast chaser is added to system, the chaser can catch targets without cooperation with other slow chasers, so that the existence of a fast chaser probably changes some features of the system. In this report we add fast chasers in the system and study how the dependence of the lifetimes τ and T on the number of chasers change owing to the presence of a few fast chasers. We also investigate the influence of fast chasers to the cost for catching all targets. In Sec. II we introduce our model. In Sec. III we show the results of simulations. In Sec. IV we summarize our results and give brief discussions.

II. MODEL

Our model is the modified model of previous studies [8,10]. We consider a system of two-dimensional square lattice with periodic boundary condition. Initially we put N_{c} chasers and N_{T} targets randomly on sites. In a trial we choose a particle from targets and chasers. When the chosen particle is a chaser, it looks for the nearest target and tries to move to one of the nearest neighboring sites in order to decrease the distance from the nearest target. If there are two paths by which the chaser decreases the distance to the target, the chaser selects a site with a probability 1/2 and tries to move. For example, if the position of chaser is (i,j) and that of target is $(i+1,j+1)$, there are two paths, $(i,j) \rightarrow (i+1,j) \rightarrow (i+1,j+1)$ and $(i,j) \rightarrow (i,j+1) \rightarrow (i+1,j+1)$, for the chaser to catch the target. The target selects $(i+1,j)$ or $(i,j+1)$ with the probability 1/2. If the selected site is already occupied by another chaser, the chaser does not move and stays in the same site. If the selected cite is occupied by a target, the chaser moves to the site, and the target is removed from the system. When the chosen particle is a target, the target looks for the nearest chaser and escapes from it. The rule for targets to escape from chasers is similar to that for chasers to chase

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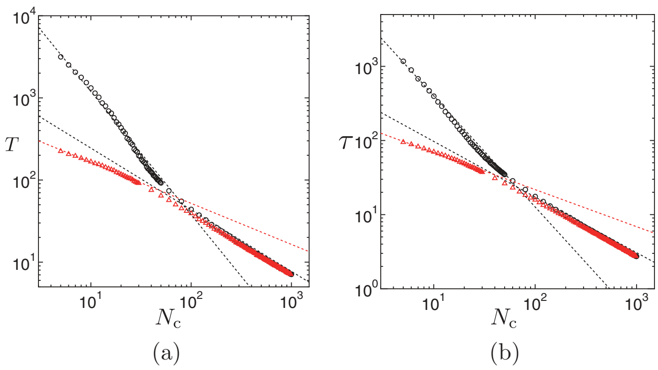


FIG. 1. (Color online) Dependence of lifetimes (a) T and (b) τ on the number of chasers $N_{\rm c}$. Black circles and red triangles show the dependence without fast chasers and that with four fast chasers, respectively. The simulation is carried out with the system size 160 × 120, and the data are averaged over 1000 individual runs.

targets. A target tries to escape from the nearest chaser by moving in the opposite direction. If there are two or more nearest chasers, the target selects one of them and escapes from the chaser. Note that the movement rules we used in this study correspond to asynchronous updating, which is different from the asynchronous updating in Ref. [8].

In our simulation, we add some fast chasers, which can move M sites in a trial. When we select an active chaser in a trial, it can move to M sites in the direction to decrease the distance between the nearest neighboring target. If it collides with another chaser or catches the nearest neighboring target before moving M sites, the chaser stops moving there. In our simulation, we set $M = 2$. Hereafter, we refer to the numbers of fast chasers, slow chasers (normal chasers), and the total number as $N_{\rm c,fast}$, $N_{\rm c,slow}$, and $N_{\rm c}$, respectively.

III. RESULTS OF SIMULATION

Figure 1 shows the dependence of lifetimes T and τ on the total number of chasers N_c. The system size is $L_x \times L_y = 160 \times 120$. The initial number of targets is 20. The lifetimes T and τ are averaged on 1000 individual runs. The dependence of the lifetimes on $N_{\rm c}$ without fast chasers is shown by black circles in Fig. 1. Since we neglect fast chasers, $N_{\rm c} = N_{\rm c,slow}$. The relation between T and $N_{\rm c}$ is expressed as $N_{\rm c}^{-\alpha}$.

When there are a few chasers, the exponent $\alpha = 1.5$. which is smaller than that in the original model [8]. The difference may be caused by differences in the way to move chasers and targets between our model and the model in a previous study [8]: The positions of targets and chasers are updated asynchronously in our model but simultaneously in Ref. [8]. With sufficiently many chasers, the difference of α from the previous model vanishes, and the exponent becomes $\alpha = 0.75$. The dependence of T on $N_{\rm c}$ with some fast chasers is expressed by red triangles. The number of the fast chasers $N_{\rm c,fast} = 4$. By adding normal chasers and keeping the number of fast chasers constant ($N_{\rm N,fast} = 4$), we increase $N_{\rm c}$. In this region, the exponent, $\alpha = 0.5$, is much smaller than without fast chasers. With sufficiently many chasers, we observe $\alpha = 0.75$, which agrees with the result in Ref. [8]. The existence of fast chasers changes the dependence of T in the region with small $N_{\rm c}$,

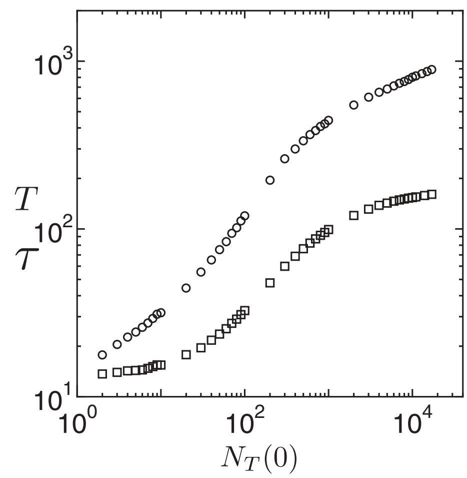


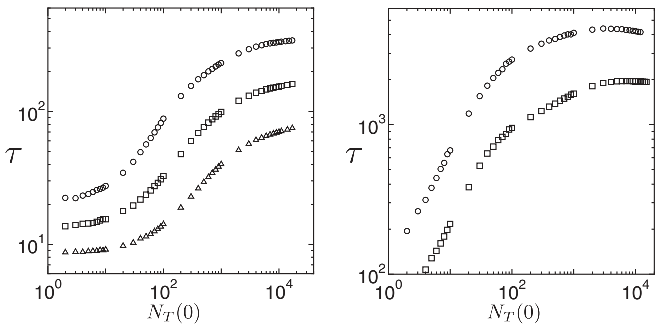
FIG. 2. Dependence of lifetimes T and τ on $N_{\rm T}$. $N_{\rm c} = 100$ and the number of fast chasers, $N_{\rm c,fast} = 0$. Circles and squares show the change of T and τ, respectively. The system size and the number of runs for averaging data are the same as those in Fig. 1

where the capture of targets may be completely dominated by the fast chasers. However, with a large number of slow chasers, the capture by collective action of slow chasers is more effective than that of a few fast chasers. Thus, the difference from the original model vanishes.

From Fig. 1, we found that the difference in updating already changes the dependence of T on $N_{\rm c}$ in our model in comparison to Ref. [8] even without fast chasers. Thus, we first neglect the fast chasers and investigate the dependence of τ and T. Then we study how the behaviors change owing to the presence of a few fast chasers. Figure 2 shows the dependence of the lifetimes T and τ on the initial number of targets $N_{\rm T}(0)$, in which fast chasers are absent. The dependence of T and τ is expressed by circles and triangles, respectively. The numbers of chasers are $N_{\rm c,fast} = 0$ and $N_{\rm c,slow} = N_{\rm c} = 100$. The averaged lifetime τ is defined as

$$\tau = \sum_t t \frac{\Delta N_{\rm T}(t)}{N_{\rm T}(0)}, \tag{1}$$

where $\Delta N_{\rm T}(t)$ represents the decrease of the number of targets at time t. In the region with a few $N_{\rm T}(0)$, both T and τ increase gradually with increasing $N_{\rm T}(0)$. Then the increase of the lifetimes accelerates. When $N_{\rm T}(0)$ is much larger than $N_{\rm c}$, the increase of the lifetimes T and τ becomes gradual again. Figure 3 shows the dependence of τ on $N_T(0)$ with various values of $N_{\rm c}$, in which the fast chasers are absent: $N_{\rm c,fast} = 0$ and $N_{\rm c,slow} = N_{\rm c}$. In Fig. 3(a), the dependence with 50, 100, and 200 chasers is shown. In each case, similarly to the result in Fig. 1, τ monotonically increases with increasing $N_{\rm T}(0)$. When $N_{\rm c}$ is much smaller than that in Fig. 3(a), the dependence of τ on $N_{\rm T}(0)$ changes. In Fig. 3(b), we show the dependence of τ with 5 and 10 chasers. In the region with small $N_{\rm T}(0)$, τ increases with increasing $N_{\rm T}(0)$. However, with too many targets, τ decreases with increasing $N_{\rm T}(0)$, and a maximum of an average lifetime, $\tau_{\rm max}$, appears. $N_{\rm T,max}(0)$, which is $N_{\rm T}(0)$ giving $\tau_{\rm max}(0)$, is about 1.1×10^4 with 10 chasers and 3.0×10^3 with 5 chasers, so that $N_{\rm T,max}(0)$ becomes large with increasing $N_{\rm c}$.



(a) (b)

FIG. 3. Dependence of τ on the $N_\mathrm{T}(0)$ (a) with 50 (circles), 100 (squares), and 200 (triangles) chasers, and (b) with 5 (circles) and 10 (squares) chasers, in which the fast chasers are absent. The system size and the number of runs for averaging data are the same as those in Fig. 1

Figure 3 shows that the behavior of τ for large N_c without fast chasers in our model differs from that reported in Ref. [8]. Next, we add a few fast chasers and study how the existence of fast chasers changes the dependence of T and τ on $N_\mathrm{T}(0)$. Figure 4 shows the dependence of the lifetimes T and τ on targets; $N_\mathrm{c} = 100$ with various values of $N_{\mathrm{c,fast}}$. Without fast chasers, the increase of the lifetimes slows down in the region with many chasers. If fast chasers are added, the amplitude of slowing down becomes small. In Fig. 4 we show the results with 100 chasers. Though we did not show results, we also carried out a simulation with $N_\mathrm{c} = N_{\mathrm{c,fast}} = 5$. In that case, $\tau_{\max}$, which appears in Fig. 3(b), does not appear, and τ keeps increasing with increasing $N_\mathrm{T}(0)$. Figure 5 shows the relation between $N_\mathrm{T}(0)$ and the number of targets caught by one fast chaser, N_f. Except for in the region with sufficiently large $N_\mathrm{T}(0)$, N_f increases as $N_\mathrm{f} = \beta N_\mathrm{T}(0)$. The velocity of fast chasers is twice as fast as that of normal chasers, so that the number of sites which a fast chaser can search is 12 in a trial, which is 3 times larger than the number that a slow chaser can search. Thus, the expected value of β is about 3×10^{-2} because $N_\mathrm{c} = 100$. However, we obtain $\beta = 4.2 \times 10^{-2}$, so

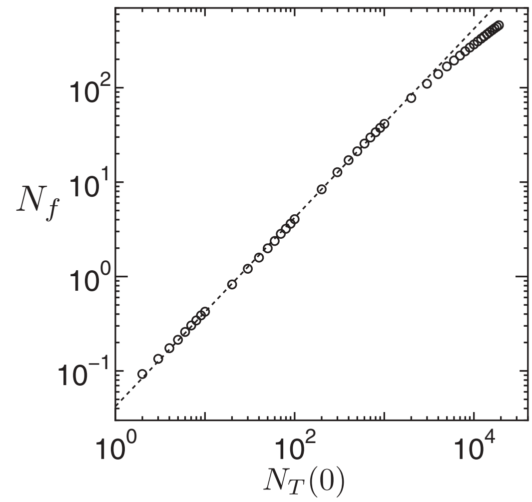


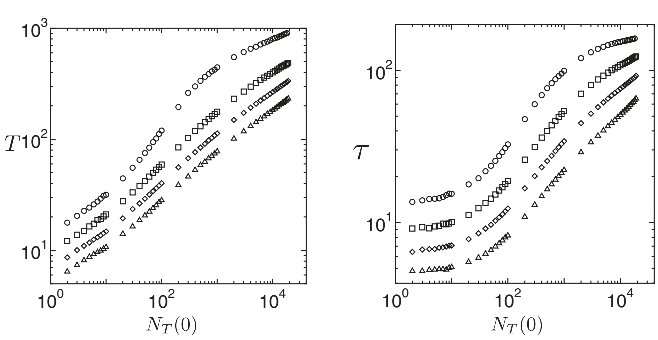
FIG. 5. Dependence of N_f on $N_\mathrm{T}(0)$. $N_\mathrm{c} = 100$ and $N_{\mathrm{c,fast}} = 10$. The system size and the number of runs for averaging data are the same as those in Fig. 1.

that the fast chasers appear to catch more targets than our expectation.

We also study the dependence of the cost required to catch all targets on N_c. Figure 6 shows the dependence of the cost on N_c. The definition of the the cost c is given by

$$c = \frac{T}{4N_\mathrm{T}(0)} \sum_{i=1}^{N_\mathrm{c}} v_i . \tag{2}$$

We assumed that the cost per a chaser is proportional to its moving range. In a trial, the number of sites which a chaser can reach is 4 for a slow chaser and 12 for a fast chaser. Thus, we use $v_i = 4$ for a slow chaser and $v_i = 12$ for a fast chaser. In the simulation, $N_\mathrm{T}(0)$ is 10 and the minimum number of chasers is 2. We investigate the dependence of c on N_c without fast chasers and with a single fast chaser. Circles show the dependence without fast chasers. With increasing the number of chasers, initially, c decreases. Then a minimum of c appears



(a) (b)

FIG. 4. Dependence of τ and T on $N_\mathrm{T}(0)$. N_c is kept at 100 with various values of $N_{\mathrm{c,fast}}$. Circles, squares, diamonds, and triangles show the dependence with 0, 20, 50, and 100 fast chasers, respectively. The system size and the number of runs for averaging data are the same as those in Fig. 1.

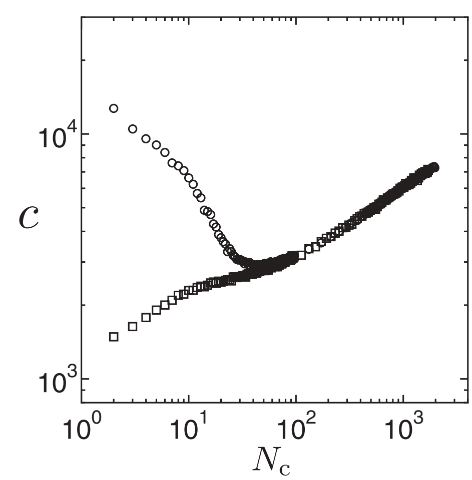


FIG. 6. Dependence of cost on the number of chasers. Circles and squares show the result without chasers and with a single fast chaser, respectively. The system size and the number of runs for averaging data are the same as those in Fig. 1

and the cost again increases, so that there is an optimal number of chasers in order to catch targets. When one chaser is added in the system, the dependence drastically changes: The minimum of c vanishes and c increases monotonically.

IV. SUMMARY AND DISCUSSIONS

In this paper we studied group chase and escape with some fast chasers. The lifetime T, at which all targets vanish, decreases with increasing N_c as $N_c^{-\alpha}$. Without fast chasers, $\alpha = 1.5$ in the region with small N_c, and $\alpha = 0.75$ in the region with large N_c. The exponent α in the region with small N_c is smaller than that in a previous study, which is probably caused by the difference in the way chasers and targets are moved. In the original model, all chasers are moved at same time. Then the positions of all targets are updated simultaneously. On the other hand, we select and move a particle. Thus, the chasers in the original model enclose the targets more easily than those in our model. When the density of chasers is large, i.e., N_c is large, the targets meet chasers frequently. The difference between our model and previous model is less important, and we obtain α, which is the same as that in the original model [8]. In previous studies [11–13], the difference in the method of updating the system, synchronous updating and asynchronous updating, affects the spatiotemporal pattern. Since it is suggested that asynchronous updating is a good approximation of real continuous time, the model used in the report is better than that in the previous model [8].

When we added a few fast chasers, the exponent α with small N_c became smaller than that without fast chasers, while α with large N_c hardly changed. In our simulation, we added normal chasers and did not add the fast chasers in order to increase the number of chasers. With small N_c, chasers need to move for a long distance in order to catch targets because surrounding and catching targets is difficult. However, the normal chasers do not move as fast as the fast chasers. Thus, the effect of the increase of N_c on the decrease of the lifetime T is small, and the exponent α in the region with a few chasers becomes smaller than that without the fast chasers. The dependence of the lifetimes τ and T on $N_T(0)$ was also affected by fast chasers. When we added a few fast chasers, the slowing down of the increase of lifetimes with large $N_T(0)$ did not occur. In our simulation, the ability for fast chasers to catch targets is about four times larger than that of normal chasers. The existence of fast chasers is equivalent to the increase of N_c. Thus, the slowing down of the increase of lifetimes probably occurs if we carry out simulations with larger $N_T(0)$.

The fast chasers drastically changed the dependence of the cost c. Without fast chasers, the maximum of c appeared, which means that there is a suitable number of chasers to catch the targets. However, if we added a few fast chasers, c increased monotonically and the minimum of c did not appear. In our simulation, to increase N_c, we added normal chasers. When N_c is small, the added normal chasers did not work as well as the fast chasers which were already present, so that the increase of chasers did not cause the decrease of c and the minimum of c vanished.

In previous studies and this paper, a two-dimensional lattice was considered, and targets and chasers moved on the sites. In that case the lattice probably gives a significant impact to behaviors of targets and chasers. In previous papers [14–18], collective motions of interacting entities were studied by using particles moving off-lattice space. Thus, we intend to study the problem of group chase and escape on an off-lattice space.

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