Electromagnetic Waves in Three Media Which Contain a Linearly Varying Isotropic Medium

by

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Abstract

Radiation fields from a vertical electric dipole antenna are obtained when the regions I and II are arbitrary homogeneous media and when region III is a plane isotropic ionosphere whose the electron density varies linearly with height and the collision frequency is constant throughout the layer.

The point of observation is far away from the source point. So, if the integral solution is expanded in the infinite series and only the first term of the series is used, the integral solution can be shown by radiations from frour image sources. Moreover, as the argument of Airy function is large, the propagation mechanism becomes clear by using the asymptotic expansion.

In the case that one (I) of the three regions is sea water another (II) the atmosphere, and the other (III) the ionosphere, the characteristics of the field intensity versus distance is numerically calculated.

1 Introduction

On radio wave propagation in three plane layer media, integral solutions of electromagnetic fields in three homogeneous plane layer media from vertical and horizontal dipole antennae are obtained previously.[1] A.W. Biggs[2] has studied radiation fields from a horizontal dipole antenna of which the region I is sea water, the region II the ice, the region III the atomosphere.

However, the field intensity for inhomogeneous plane medium of the region III is not obtained. In this paper, when the region III is ionosphere where the electron density varies linearly with height, the integral solution of electromagnetic field from vertical electric dipole antenna is obtained. And when the point of observation is far away from the source point, the approximate solution is obtained.

2 Fundamental Equations

The Hertz vector Π is assumed to vary with time as $e^{j\omega t}$ in the medium. As shown in Fig. 1, we regard a vertical electric dipole as transmitter Q, locate the source at $z=h$,

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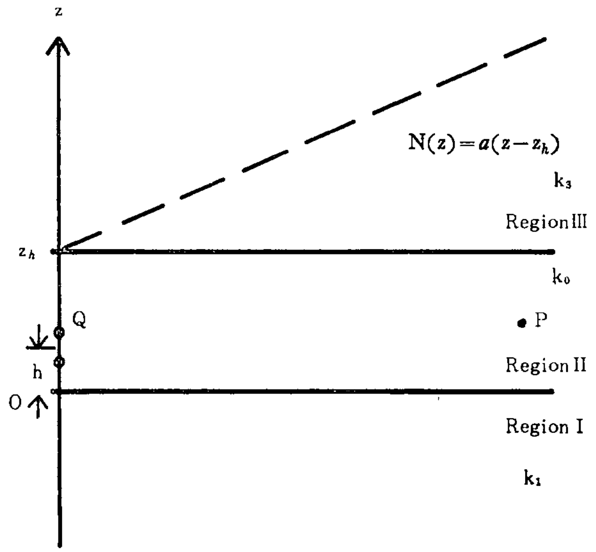

Fig. 1 Profile of three media which contain inhomogeneous medium. (Q : Source point, P : Observation point.)

and show the observation point P by using a cylindrical coordinate system (r, ϕ, z) in which $\overline{\mathrm{OQ}}$ direction is regarded as z axis. In such the case, the Hertz vector Π has only a z component and is invariant with respect to ϕ coordinate. So, hereafter the scalar component of Π is regarded as ψ, the homogeneous wave equation is given as follows:

$$\nabla^2\psi+k^2(z)\psi=0\,, \tag{1}$$

or by cylindrical coordinate,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{\partial^2\psi}{\partial z^2}+k^2(z)\psi=0\,, \tag{2}$$

where $k(z)=\omega\sqrt{\mu\bar{\varepsilon}(z)}\,,\ \bar{\varepsilon}(z)=\varepsilon(z)-j\sigma(z)/\omega.$

As the equation is separable, placing $\psi=R(r)v(z)$ one finds

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\kappa^2R=0\,, \tag{3}$$

$$\frac{d^2v}{dz^2}+\{k^2(z)-\kappa^2\}v=0\,, \tag{4}$$

where the parameter κ is a separation constant.

Eq. (3) has Bessel solutions of zeroth oder $J_o\ (\kappa r)$ and $N_o\ (\kappa r)$ as independent solutions, but $N_o\ (\kappa r)$ is inadequate because it becomes infinite at $r=0$. And also, we assume that the solution of eq. (4) is $v\ (z,\kappa)$. Then, the general solution ef eq. (1) is given as follows:

$$\psi=\int_o^\infty f(\kappa)J_o(\kappa r)v(z,\kappa)\kappa d\kappa. \tag{5}$$

The primary field ψ_o, which is radiated from the vertical electric dipole, satisfies the following inhomogeneous wave equation which takes the wave source into consideration,

$$\nabla^2\psi_o+k_0^2\psi_o=-4\pi\delta(R_1). \tag{6}$$

The solution of eq. (6) is given by

$$\psi_o=\frac{e^{-jk_0R_1}}{R_1}, \tag{7}$$

where $\quad R_1=\sqrt{r^2+(z-h)^2}$.

The expansion of eq. (7) in terms of cylindrical waves is well known and is given by eq. (5) [6]

$$\psi_0=\int_o^\infty\frac{J_o(\kappa r)}{\sqrt{\kappa^2-k_0^2}}\exp\{-\sqrt{\kappa^2-k_0^2}\,|z-h|\}\kappa d\kappa. \tag{8}$$

The secondary fields reflected from the upper and lower layers, which have z comonent only, are respectively expressed by ψ_u and ψ_g. In each regions, ψ is given as follows;

$$\begin{aligned}\psi&=\psi_1 && z<0,\\ &=\psi_o+\psi_u+\psi_g && 0<z<z_h,\\ &=\psi_3 && z>z_h.\end{aligned} \tag{9}$$

Where, ψ_1, ψ_u and ψ_g are given as follows, on account of eq. (4) and eq. (5).

$$\psi_1=\int_o^\infty f_o(\kappa)J_o(\kappa r)\exp\{\sqrt{\kappa^2-k_1^2}\,z\}\kappa d\kappa, \tag{10}$$

$$\psi_g=\int_o^\infty\frac{f_g(\kappa)}{\sqrt{\kappa^2-k_0^2}}J_o(\kappa r)\exp\{-\sqrt{\kappa^2-k_0^2}\,z\}\kappa d\kappa, \tag{11}$$

$$\psi_u=\int_o^\infty\frac{f_u(\kappa)}{\sqrt{\kappa^2-k_0^2}}J_o(\kappa r)\exp\{\sqrt{\kappa^2-k_0^2}\,z\}\kappa d\kappa, \tag{12}$$

$\sqrt{\kappa^2-k_0^2}$ and $\sqrt{\kappa^2-k_1^2}$ in eq. (8), eq. (10), eq. (11) and eq. (12) must be chosen such that the fields vanish as $z\to\infty$. we do this by making the real parts of those always positive. In the region III, the complex dielectric constant $\varepsilon(z)$ in the isotropic medium of which electron density is linearly distributed with height, $N(z)=a(z-z_h)$, is

$$\bar{\varepsilon}(z)=\varepsilon_0\{1-\alpha(z-z_h)\}, \tag{13}$$

where $\alpha=e^2(1+j\nu/\omega)a/m\varepsilon_0(\omega^2+\nu^2)$,

a: gradient of the electron density,

e: the charge of a free electron,

m: the mass of a free electron,

ν: the mean collision frequency of a free electron which collides with the other particles.

Substituting eq. (13) into eq. (4), we obtain

$$\frac{d^2v}{dz^2}+[\omega^2\varepsilon_0\mu_0\{1-\alpha(z-z_h)\}-\kappa^2]v=0\,. \tag{14}$$

By making variable tranformations

$$w_L=(k_0^2\alpha)^{1/3}(y-\beta/\alpha),$$

where $y=z-z_h$, $\beta=1-\kappa^2/k_0^2$,

the following Stoke's equation is obtained

$$\frac{d^2v}{dw_L{}^2}-w_Lv=0\,. \tag{15}$$

The solution of eq. (15) is expressed as the linear combination of the Airy functions $A_i(w_L)$ and $B_i(w_L)$;

$$v=D_1\,A_i(w_L)+D_2B_i(w_L). \tag{16}$$

Substituting eq. (16) into (5), we obtain

$$\psi_3=\int_0^\infty J_o(\kappa r)\{f_3(\kappa)A_i(w_L)+f_4(\kappa)B_i(w_L)\}\kappa d\kappa, \tag{17}$$

where $f_3(\kappa)=D_1f(\kappa)$, $f_4(\kappa)=D_2f(\kappa)$.

In eq. (17), $f_4(\kappa)$ must be zero in order to satisfy the radiation condition. Therefore eq. (17) is given as follows:

$$\psi_3=\int_0^\infty J_o(\kappa r)f_3(\kappa)A_i(w_L)\kappa d\kappa. \tag{18}$$

3 Integral Solution of the Field in the Region II

For ψ of eq. (9) in each region, ψ_1, ψ_g, ψ_u and ψ_3 are respectively expressed as eqs. (10), (11), (12) and (18). But those equations include the unknown coefficients f_1, f_g, f_u and f_3. Therefore, these unknown coefficients can be determined by the following boundary conditions.

$$\left.\begin{aligned}&\frac{\partial\psi_1}{\partial z}=\frac{\partial}{\partial z}(\psi_o+\psi_g+\psi_u) \qquad k_1^2\psi_1=k_0^2(\psi_o+\psi_g+\psi_u), \qquad (z=0)\\ &\frac{\partial}{\partial z}(\psi_o+\psi_g+\psi_u)=\frac{\partial\psi_3}{\partial z} \qquad k_0^2(\psi_o+\psi_g+\psi_u)=k_0^2\psi_3, \qquad (z=z_h)\end{aligned}\right\} \tag{19}$$

that is,

$$f_g=\frac{1}{\Delta}(e^{-A_o|z_h-h|}S+e^{A_o|z_h-h|}U), \tag{20}$$

$$f_u=\frac{1}{\Delta}(e^{-A_o|z_h-h|}T+e^{-A_o(z_h-h)}S) \quad (21)$$

where $\Delta=Me^{A_oz_h}-Se^{-A_oz_h}$,

$$M=A_oA_1A_i(w_{LOX})k_0^2-A_1A_i'(w_{LOX})(k_0^2\alpha)^{1/3}k_0^2+A_0^2A_i(w_{LOX})k_1^2-A_oA_i'(w_{LOX})k_1^2(k_0^2\alpha)^{1/3},$$

$$S=-A_oA_1A_i(w_{LOX})k_0^2-A_1A_i'(w_{LOX})(k_0^2\alpha)^{1/3}k_0^2+A_0^2A_i(w_{LOX})k_1^2+A_oA_i'(w_{LOX})k_1^2(k_0^2\alpha)^{1/3},$$

$$T=A_oA_1A_i(w_{LOX})k_0^2+A_1A_i'(w_{LOX})(k_0^2\alpha)^{1/3}k_0^2+A_0^2A_i(w_{LOX})k_1^2+A_oA_i'(w_{LOX})k_1^2(k_0^2\alpha)^{1/3},$$

$$U=-A_oA_1A_i(w_{LOX})k_0^2+A_1A_i'(w_{LOX})(k_0^2\alpha)^{1/3}k_0^2+A_0^2A_i(w_{LOX})k_1^2-A_oA_i'(w_{LOX})k_1^2(k_0^2\alpha)^{1/3},$$

$$A_o=\sqrt{\kappa^2-k_0^2},\quad A_1=\sqrt{\kappa^2-k_1^2},\quad w_{LOX}=-(k_0^2\alpha)^{1/3}(\beta/\alpha),\quad A_i'(w_{LOX})=\frac{\partial A_i(w_{LOX})}{\partial z}.$$

Now, let us transform $1/\Delta$ as follows:

$$\frac{1}{\Delta}=\frac{1}{Me^{A_oz_h}}\,\frac{1}{1-\frac{S}{Me}^{-2A_oz_h}}$$

As $|M|>|S|$*1 and $R_eA_o\geqq 0$, $|Se^{-2A_oz_h}/M|<1$, so $1/\Delta$ is expanded as follows:

$$\frac{1}{\Delta}=\frac{1}{Me^{A_oz_h}}\sum_{n=1}^{\infty}\left(\frac{S}{M}\right)^{n-1}e^{-(2n-2)A_oz_h}=\sum_{n=1}^{\infty}\frac{S^{n-1}}{M^n}e^{-(2n-1)A_oz_h}. \quad (22)$$

Substituting eq. (22) into eq. (20), eq. (21), we have

$$f_y=\sum_{n=1}^{\infty}\frac{S^{n-1}}{M^n}e^{-(2n-1)A_oz_h}(e^{-A_o|z_h-h|}S+e^{A_o(z_h-h)}U), \quad (23)$$

$$f_u=\sum_{n=1}^{\infty}\frac{S^{n-1}}{M^n}e^{-(2n-1)A_oz_h}(e^{-A_o|z_h-h|}T+e^{-A_o(z_h+h)}S). \quad (24)$$

Substituting eq. (23) and eq. (24) into eq. (11) and eq. (12), from the relation of eq. (9), integral solution ψ of Hertz vector in the region II is given as follows:

$$\psi=\frac{e^{-jk_0R_1}}{R_1}+\sum_{n=1}^{\infty}\Big[\int_0^{\infty}\frac{S^{n-1}}{M^n}U\frac{e^{-A_o\{(2n-2)z_h+h+z\}}}{A_0}J_o(\kappa r)\kappa d\kappa+\int_0^{\infty}\frac{S^n}{M^n}\frac{e^{-A_o\{2nz_h-h+z\}}}{A_o}J_o(\kappa r)\kappa d\kappa$$
$$+\int_0^{\infty}\frac{S^{n-1}}{M^n}T\frac{e^{-A_0\{2nz_h-h-z\}}}{A_o}J_0(\kappa r)\kappa d\kappa+\int_0^{\infty}\frac{S^n}{M^n}\frac{e^{-A_o\{2nz_h+h-z\}}}{A_0}J_o(\kappa r)\kappa d\kappa\Big]. \quad (25)$$

4 Approximate Solution When the Observation Point is Far Away from the Source Point

As the radiation field in the integral form from a vertical electric dipole antenna is obtained in ed. (25), we derive the approximate solution in this section when $kR_1\gg 1$ [9].

At first, the first term in the bracket of eq. (25) is examined. Because of $|S/M|<1$,

*1 Appendix

we may approximate the infinite series summed over n to the first term ($n=1$). By using following formulas,

$$J_\nu(\kappa r)=\frac{H_\nu^{(1)}(\kappa r)+H_\nu^{(2)}(\kappa r)}{2},\quad H_0^{(2)}(\kappa r)=-H_0^{(1)}(-\kappa r),$$

we replace the Bessel function by the Hankel function of the second kind as follows,

$$I=\int_0^\infty \frac{U}{M}\frac{e^{-A_0(h+z)}}{A_0}J_0(\kappa r)\kappa d\kappa=\frac{1}{2}\int_{-\infty}^\infty \frac{U}{M}\frac{e^{-A_0(h+z)}}{A_0}H_0^{(2)}(\kappa r)\kappa d\kappa. \tag{26}$$

The contour $-\infty$ to $+\infty$ is deformed to the contour $c+c_0+c_1$ as indicated in Fig. 2, where the contour c is a half circle of infinite radius, c_0 and c_1 are the contours around the two branch cuts from k_0 and k_1 to $-j\infty$ respectively. The integral along c can be shown to give 0. Now, let $I=I_{k0}+I_{k1}$, where I_{k0} and I_{k1} are the integrals along c_0 and c_1 respectively.

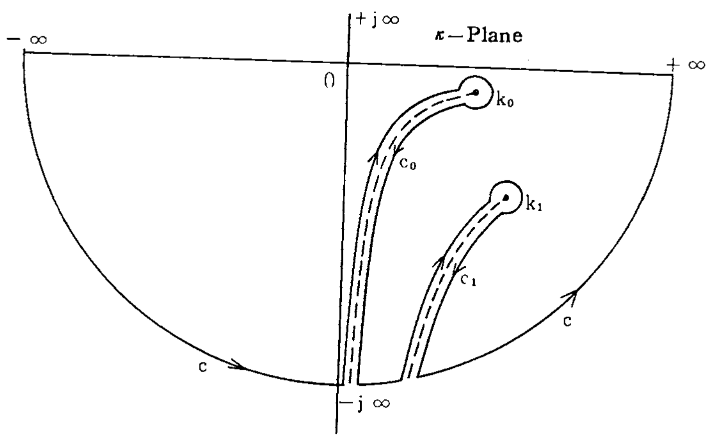

Fig. 2 Contour of integration in the κ plane
(k_0, k_1 : Branch point, Dotted line : Branch cut)

As the most of the value of the integral comes from the portion of the path of the integration very close to $\kappa^2=k_0^2$, U/M may be validly expanded in powers of $l=\sqrt{\kappa^2-k_0^2}$,

$$I_{k_0}=\frac{1}{2}\int_{c_0}(B_0+B_1 l+B_2 l^2+\cdots\cdots)H_0^{(2)}(\kappa r)\frac{e^{-\sqrt{\kappa^2-k_0^2}(z+h)}}{\sqrt{\kappa^2-k_0^2}}\kappa d\kappa \tag{27}$$

where B_0, B_1, $B_2\cdots\cdots$ are constants and independent of κ.

It is possible to replace each l of eq. (27) by $-\partial/\partial z$. Using eq. (7) and eq.(8), we have

$$I_{k_0}=\frac{1}{2}\left(B_0-B_1\frac{\partial}{\partial z}+B_2\frac{\partial^2}{\partial z^2}+\cdots\cdots\right)\int_{c_0}H_0^{(2)}(\kappa r)\frac{e^{-\sqrt{\kappa^2-k_0^2}(z+h)}}{\sqrt{\kappa^2-k_0^2}}\kappa d\kappa$$

$$=\left(B_0-B_1\frac{\partial}{\partial z}+B_2\frac{\partial^2}{\partial z^2}+\cdots\cdots\right)\left(\frac{e^{-jk_0R_2}}{R_2}\right) \tag{28}$$

Considering of $\partial/\partial z$ in eq. (28), we have

$$\frac{\partial}{\partial z}\left(\frac{e^{-jk_0R_2}}{R_2}\right)=-jk_0\cos\theta_2\frac{e^{-jk_0R_2}}{R_2}-\cos\theta_2\frac{e^{-jk_0R_2}}{R_2}$$

where $\frac{z+h}{R}=\cos\theta_2.$

The second term can be neglected when $k_0R_2\gg1$. Then,

$$\frac{\partial}{\partial z}=-jk_0\cos\theta_2=-jk_0\gamma_2, \tag{29}$$

where $\gamma_2=\cos\theta_2$.

Therefore, $l=\sqrt{\kappa^2-k_0^2}$ in U/M of eq. (28) may be replaced by $jk_0\gamma_2$,

$$I_{k_0}=\frac{-a_2-b_2+c_2+d_2}{a_2+b_2+c_2+d_2}\,\frac{e^{-jk_0R_2}}{R_2}. \tag{30}$$

In the same way, I_{k_1} is given as follows:

$$I_{k_1}=\left(\frac{UA_1}{MA_0}\right)_{\kappa^2=k_1^2-k_1^2\gamma_1^2}\;e^{\{jk_1\gamma_1-\sqrt{k_1^2(1-\gamma_1^2)-k_0^2}\}(z+h)}\,\frac{e^{-jk_1R_2}}{R_2}. \tag{31}$$

It is assumed here that k_1 has a negative imaginary part so that I_{k_1} of eq. (31) vanishes at large R_2. Therefore, $I\fallingdotseq I_{k_0}$.

Next, the rest terms in the bracket of eq. (25) are obtained in the same procedure of integration and eq. (25) is

$$\psi=\frac{e^{-jk_0R_1}}{R_1}+\frac{-a_2-b_2+c_2+d_2}{a_2+b_2+c_2+d_2}\,\frac{e^{-jk_0R_2}}{R_2}+\frac{-a_3+b_3+c_3-d_3}{a_3+b_3+c_3+d_3}\,\frac{e^{-jk_0R_3}}{R_3}$$
$$+\frac{a_4-b_4+c_4-d_4}{a_4+b_4+c_4+d_4}\,\frac{e^{-jk_0R_4}}{R_4}+\frac{-a_5+b_5+c_5-d_5}{a_5+b_5+c_5+d_5}\,\frac{e^{-jk_0R_5}}{R_5} \tag{32}$$

where $a_x=\gamma_xA_i(w_{LOX})\sqrt{n_g^2-1+\gamma_x^2}$ $\quad b_x=j\sqrt{n_g^2-1+\gamma_x^2}\,A_i'(w_{LO})(k_0^2\alpha)^{1/3}/k_0,$

$c_x=\gamma_x^2A_i(w_{LOX})n_g^2,$ $\quad d_x=j\gamma_xA_i'(w_{LOX})n_g^2(k_0^2\alpha)^{1/3}/k_0,$

$n_g=k_1/k_0,\ \gamma_x=\cos\theta_x,\ (x=2,\ 3,\ 4,\ 5).$

Moreover, as the argument w_{LOX} of the Airy function which is contained in a_x, b_x, c_x andd_x, is large, by using the following asymptotic expansions

$$A_i(z)=\frac{\pi^{-1/2}}{2}z^{-1/4}\{\exp\left(-\frac{2}{3}z^{3/2}\right)+j\exp\left(\frac{2}{3}z^{3/2}\right)\}\qquad\frac{2}{3}\pi\leq\arg z\leq\frac{4}{3}\pi, \tag{33}$$

$$A_i'(z)=\frac{\pi^{-1/2}}{2}z^{1/4}\{-\exp\left(-\frac{2}{3}z^{3/2}\right)+j\exp\left(\frac{2}{3}z^{3/2}\right)\}\qquad\frac{2}{3}\pi\leq\arg z\leq\frac{4}{3}\pi, \tag{34}$$

each coefficient except the first term of eq. (32) is obtained and eq. (32) is

$$\psi=\frac{e^{-jk_1R_1}}{R_1}+\frac{n_g^2\cos\theta_2-\sqrt{n_g^2-\sin^2\theta_2}}{n_g^2\cos\theta_2+\sqrt{n_g^2-\sin^2\theta_2}}\frac{e^{-jk_0R_2}}{R_2}+\frac{n_g^2\cos\theta_3-\sqrt{n_g^2-\sin^2\theta_3}}{n_g^2\cos\theta_3+\sqrt{n_g^2-\sin^2\theta_3}}je^{\frac{4}{3}w_{LO3}^{3/2}}\frac{e^{-jk_0R_3}}{R_3}$$

$$+je^{\frac{4}{3}w_{LO1}^{3/2}}\frac{e^{-jk_0R_4}}{R_4}+\frac{n_g^2\cos\theta_5-\sqrt{n_g^2-\sin^2\theta_5}}{n_g^2\cos\theta_5+\sqrt{n_g^2-\sin^2\theta_5}}je^{\frac{4}{3}w_{LO5}^{3/2}}\frac{e^{-jk_0R_5}}{R_5}, \tag{35}$$

where $R_1=\sqrt{r^2+(z-h)^2}$, $R_2=\sqrt{r^2+(z+h)^2}$, $R_3=\sqrt{r^2+(2z_h-h+z)^2}$,

$R_4=\sqrt{r^2+(2z_h-h-z)^2}$, $R_5=\sqrt{r^2+(2z_h+h-z)^2}$.

5 The Field Intensity and Its Consideration

It is found in eq. (32) that the radiation field in the case of large distance between source and observation point can be approximately expressed by radiations from four image sources. These relations are shown in Fig. 3.

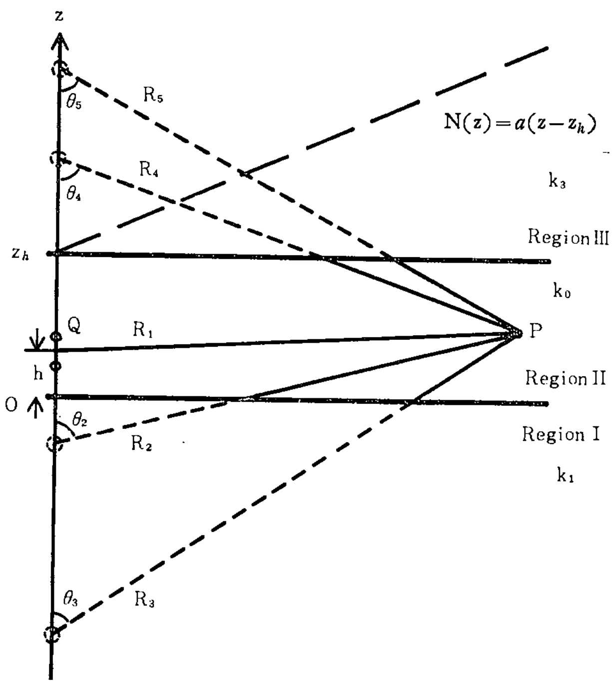

Fig. 3 Image source of three media which contain inhomogeneous medium. (○ : Image source.)

The coefficient of the second term of eq. (35) is the coefficient reflected only once from the lower layer and always satisfies Fresnel law. Also, the coefficient of the fourth term is the one reflected only once from the upper layer and its absolute value with $\nu=0$ is always 1. The coefficents of the third and the fifth term contain two reflections, that is, one is from the lower layer and the other is from the upper layer. As the results, the

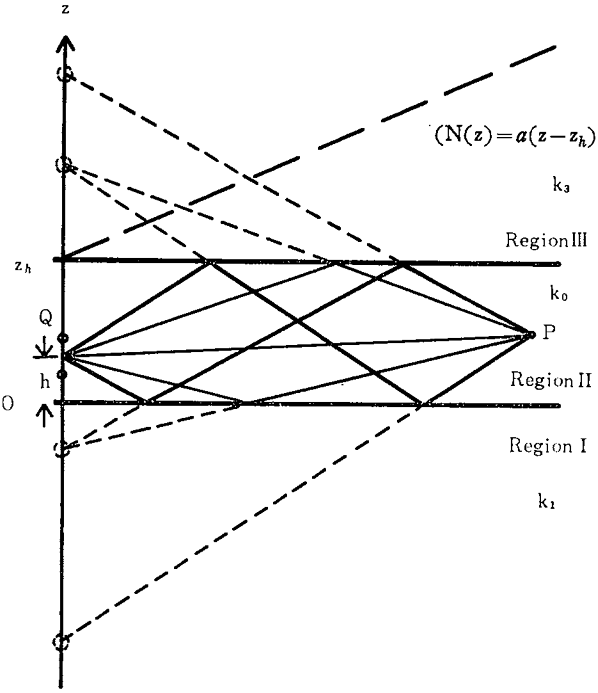

Fig. 4 Propagation path in three mldia which contain inhomogeneous medium. (○ : Image source)

propagation path for each term of eq. (35) is shown in Fig. 4.

Electric and magnetic fields ars given as follows:

$$E_z=\frac{\partial^2\psi}{\partial z^2}+k_0^2\psi\ ,\quad E_r=\frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial z}\right)\ ,\quad H_\varphi=-j\omega\varepsilon_0\frac{\partial\psi}{\partial r}\ . \tag{36}$$

Substituting eq. (35) into eqs. (36), we obtain electric and magnetic fields when observation point is far away from the source point. Now when we neglect some amount which is much smaller than r^{-2}, E_z is given as follows:

$$E_z\fallingdotseq k_0^2\psi=k_0^2\psi_1(\psi/\psi_1)\ . \tag{37}$$

E_z is in proportion to ψ.

Here, when we restore the constant factor of Hertz vector which is omitted in the above mentioning, electric field intensity is

$$E_z=\frac{-jIH}{4\pi\varepsilon\omega}k_0^2\frac{e^{-jk_0R_1+j\omega t}}{R_1}(\psi/\psi_1)=\frac{-j60\pi IH}{\lambda}\frac{e^{-jk_0R_1+j\omega t}}{R_1}(\psi/\psi_1)\ .$$

Now, $R_1\fallingdotseq r$, so $|E_z|$ is

$$|E_z|=\frac{60\pi IH}{\lambda r}|\psi/\psi_1|=\frac{120\pi IH}{\lambda r}\left|\frac{\psi}{2\psi_1}\right|=\frac{3\sqrt{10}\sqrt{P}}{r}\left|\frac{\psi}{2\psi_1}\right|\ , \tag{38}$$

where I: antenna current, H: effective height,

P: radiation power, r: distance from source point to observation point.

As an example, we assume that the region I is sea water, II the atmosphere, and III the ionosphere. And the numerical calculations are made in reference with the actual data of the ionospheric D layer. These results are shown in Fig. 5 and Fig. 6.

Electric field intensity decreases roughly with distance, arising the interference pattern. This is because the reflected waves add vectorially to the direct wave, producing domains of reinforcement and domains of cancellation. When f is constant and ν increases, the amplitude of the interference pattern decreases. Namely, it is noticed that the influences of reflected waves vanish and E_z attenuates with $1/r$. The field intensities in the ionosphere are able to be obtained easily by calculating ψ_8.

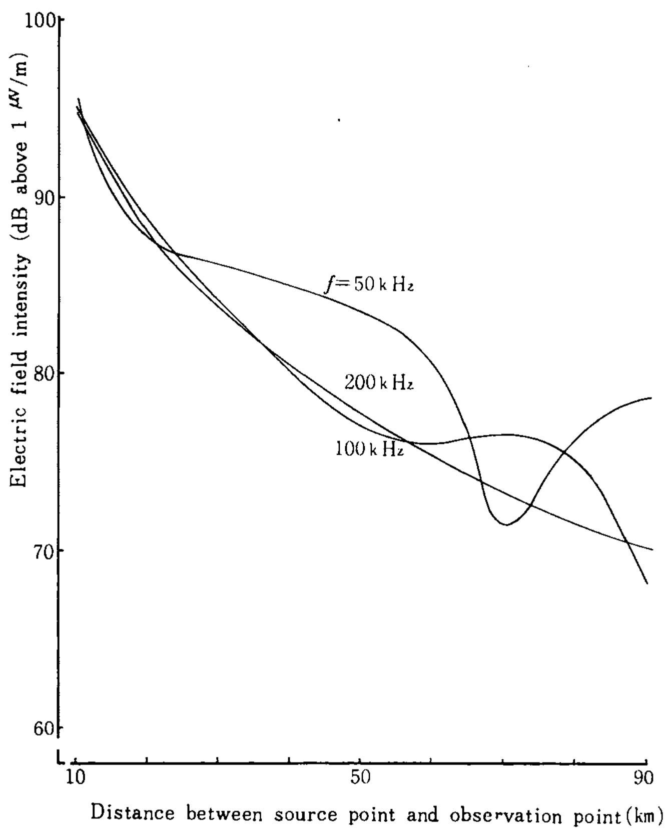

Fig. 5 **Electric field intensity over sea water.**
($\nu=10^6(\sec^{-1})$, $\sigma=4(℧/m)$, $a=2.5\times10^5$, $\varepsilon_1/\varepsilon_0=80$.
$h=100(m)$ $h_p=20(m)$, $P=1(kw)$.)

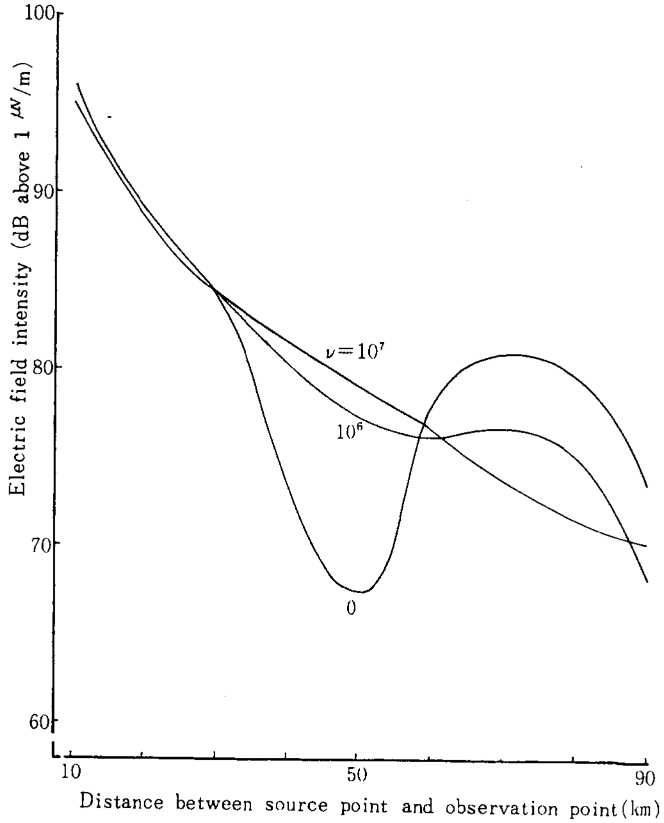

Fig. 6 Electric field intensity over sea water. (f=100(KHz), σ= 4 (℧/m), a=2.5×10⁵, $\varepsilon_1/\varepsilon_0$=80, h=100(m) h_P=20(m) P= 1 (kw).)

6 Conclusions

The integral solution of electromagnetic waves from vertical electric dipole antenna in three layer media which contain a linearly varying isotropic medium is obtained. Next, the approximate solution in the case of large distance between source and observing points is obtained.

Furthermore, as the argument of Airy function is very large, the propagation mechanism is able to be made clear by making use of the asymptotic expansion. And, also, the electric field intensity decreases roughly with distance, raising the interference pattern which consists of domains of reinforcement and domains of cancellation, and if f is constant and ν increases, the amplitude of the interference pattern decreaces and E_z attenuates with $1/r$.

We have following many similar unsolved problems on this study; problems on

1) not plane earth but spherical earth,
2) not large distance between source and observation points but small distance,

3) not isotropic in homogeneous ionosphere but anisotropic inhomogeneous ionosphere, etc.
FACOM 230-35 in Kanazawa Univ. was used for the numerical caluculations.

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Appendix

Substituting eq. (33) and eq. (34) into $A_i\,(w_{LOX})$ and $A'_i\,(w_{LOX})$ which contains the first and the fourth terms of M and S, we obtain

$$\frac{S}{M}=\frac{-A_0A_1\frac{\pi^{-1/2}}{2}w_{\mathrm{LOX}}^{-1/4}[\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_0^2-A_1A'_i\,(w_{LOX})\,k_0^2(k_0^2\alpha)^{1/3}}{A_0A_1\frac{\pi^{-1/2}}{2}w_{\mathrm{LOX}}^{-1/4}[\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_0^2-A_1A'_i\,(w_{LOX})\,k_0^2(k_0^2\alpha)^{1/3}}$$

$$\frac{+A_0^2A_i(w_{LOX})k_1^2+A_0\frac{\pi^{-1/2}}{2}w_{\mathrm{LOX}}^{1/4}[-\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_1^2(k_0^2\alpha)^{1/3}}{+A_0^2A_i(w_{LOX})k_1^2-A_0\frac{\pi^{-1/2}}{2}w_{\mathrm{LOX}}^{1/4}[-\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_1^2(k_0^2\alpha)^{1/3}}$$

As the second and third terms of numerator and denominator of the above equation are the same ones, we neglect them, and compare the first and the fourth terms.
The above equation is given as follows:

$$\frac{-A_1w_{\mathrm{LOX}}^{-1/4}[\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_0+j\gamma_x w_{\mathrm{LOX}}^{-1/4}[-\exp(-\frac{2}{3}w_{LOX})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_1^2}{A_1w_{\mathrm{LOX}}^{-1/4}[\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_0-j\gamma_x w_{\mathrm{LOX}}^{-1/4}[-\exp(-\frac{2}{3}w_{LOX})+j\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})]k_1^2}$$

$$=\frac{(-A_1k_0-j\gamma_x k_1^2)\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})-(-j\gamma_x k_1^2+jA_1k_0)\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})}{(A_1k_0+j\gamma_x k_1^2)\exp(-\frac{2}{3}w_{\mathrm{LOX}}^{3/2})+(\gamma_x k_1^2+jA_1k_0)\exp(\frac{2}{3}w_{\mathrm{LOX}}^{3/2})}$$

Therefore, $$|S/M|<1\ .$$

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