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Configuration Interaction Matrix Elements

Masatoshi YAMAZAKI
Department of Physics, Faculty of Science, Kanazawa University
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The formulae of configuration interaction matrix elements are collected in the present paper for singlet and triplet state wave functions, which cover up to the case when the unpaired electrons are at most six, and for doublet and quartet state wave functions, which cover up to the unpaired electrons are at most five.

Usually in atomic and molecular calculations we use a scheme in which each electron is given its individual quantum state, and the atom and the molecule as a whole are described by a configuration, i. e., by a set of quantum numbers for the individual electrons. Two-particle correlations are included in the "configuration interaction". In the present paper tables useful for the configuration interaction calculations are collected in the general forms ready to be able to apply them to any particular problem.

The total electronic wave function is assumed to have the following form:

$$\Phi = \sum_{Ai} C_{Ai}\, \Phi_{Ai} \quad ,$$

where C_{Ai} are the coeficients to be determined by the configuration interaction calculation and Φ_A have the form:

$$\Phi_A = [M!]^{-1/2} \sum_P (-1)^P P\{\varphi_1(r_1)\,\theta_1(\sigma_1)\,\varphi_2(r_2)\,\theta_2(\sigma_2)\cdots\varphi_M(r_M)\,\theta_M(\sigma_M)\} \quad ,$$

$$A = \Psi_A^0 = \varphi_1(r_1)\,\varphi_2(r_2)\cdots\varphi_M(r_M) \quad .$$

The letter A (B) denotes the space orbital configuration (i. e., a set of space orbital quantum numbers for the individual electrons) with definite space symmetry and i (j) is the numbering of the independent spin functions with the same space orbital configuration. The wave functions are assumed to be made from products of single-particle wave functions (wave functions of the individual particle). It is assumed that

$$\int \bar{\varphi}_m \varphi_n \, dv = \delta_{mn}$$

(orthogonality of the single-particle space orbital functions).
In order to evaluate the coeficients C_{Ai} by the configuration interaction calculation, the secular equation must be solved, the matrix elements of which are

$$H_{B_j}^{A_i} = \int \bar{\Phi}_{Ai} H \Phi_{Bj} \, dv.$$

In the present paper the formulae of configuration interaction matrix elements $H_{B_j}^{A_i}$ are tabulated for singlet, doublet, triplet, and quartet state wave functions.

The explicit forms of the spin functions $\Theta_{S,M,k}^{N}$ used in the present paper are shown in the preceeding paper[1]. It is convenient to explain the definition of the wave functions Φ_{Ai} by taking the simple example which is shown in the following: for example in the case of that 1) the number of the unpaired electrons is five ($N=5$), 2) the total spin quantum number is one half ($S=\frac{1}{2}$), 3) z-component of the total spin quantum number is one half ($M=\frac{1}{2}$), 4) the numbering of the independent spin functions with the same definite S and M is one ($k=1$), and 5) the space orbital configuration is $A=\Psi_A^0=abcde$ (five electrons in five different space orbitals, i. e., five unpaired electrons),

$$\Phi_{Ai} = \Phi_{A1} = \frac{1}{2}(\Phi[a\alpha, b\alpha, c\beta, d\alpha, e\beta] - \Phi[a\alpha, b\alpha, c\beta, d\beta, e\alpha] - \Phi[a\alpha, b\beta, c\alpha, d\alpha, e\beta] + \Phi[a\alpha, b\beta, c\alpha, d\beta, e\alpha]),$$

where $\Phi[\varphi_1\theta_1, \varphi_2\theta_2, \cdots, \varphi_N\theta_N]$ is the Slater determinant (Φ_{A1} is the linear combination of four Slater determinant), $\Phi[\varphi_1\theta_1, \varphi_2\theta_2, \cdots, \varphi_M\theta_M]$

$$= \frac{1}{\sqrt{M!}} \begin{vmatrix} \varphi_1(r_1)\theta_1(\sigma_1) & \varphi_2(r_1)\theta_2(\sigma_1) & \cdots & \varphi_M(r_1)\theta_M(\sigma_1) \\ \varphi_1(r_2)\theta_1(\sigma_2) & \varphi_2(r_2)\theta_2(\sigma_2) & \cdots & \varphi_M(r_2)\theta_M(\sigma_2) \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_1(r_M)\theta_1(\sigma_M) & \varphi_2(r_M)\theta_2(\sigma_M) & \cdots & \varphi_M(r_M)\theta_M(\sigma_M) \end{vmatrix}$$

and $\Theta_{S,M,k}^{N} = \Theta_{1/2,1/2,1}^{5}$ has the form as shown in the preceeding paper[1],

$$\Theta_{1/2,1/2,1}^{5} = \frac{1}{2}[\alpha(1)\alpha(2)\beta(3)\alpha(4)\beta(5) - \alpha(1)\alpha(2)\beta(3)\beta(4)\alpha(5) - \alpha(1)\beta(2)\alpha(3)\alpha(4)\beta(5) + \alpha(1)\beta(2)\alpha(3)\beta(4)\alpha(5)].$$

When there exist the paired electrons in the space orbital configuration A, in writing down the Ψ_A^0, the simple product of single-particle space orbital functions corresponding to the space orbital configuration A, WE WRITE ALWAYS IN DEFINITE ORDER THE UNPAIRED SPACE ORBITAL FUNCTIONS AT FIRST, AND THEN THE PAIRED ONES. For example in the case of the above 1), 2), 3), and 4), and particularily in the case of that 5) the space orbital configuration is

$$A = \Psi_A^0 = abcde\, \underbrace{ff\, gg \cdots ll}_{M \text{ pair}},$$

$$\Phi_{A1}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^M\Big(\Phi\,[a\alpha, b\alpha, c\beta, d\alpha, e\beta, ff(\alpha\beta-\beta\alpha), gg(\alpha\beta-\beta\alpha),\cdots, ll(\alpha\beta-\beta\alpha)]$$
$$-\Phi\,[a\alpha, b\alpha, c\beta, d\beta, e\alpha, ff(\alpha\beta-\beta\alpha), gg(\alpha\beta-\beta\alpha),\cdots, ll(\alpha\beta-\beta\alpha)]$$
$$-\Phi\,[a\alpha, b\beta, c\alpha, d\alpha, e\beta, ff(\alpha\beta-\beta\alpha), gg(\alpha\beta-\beta\alpha),\cdots, ll(\alpha\beta-\beta\alpha)]$$
$$+\Phi\,[a\alpha, b\beta, c\alpha, d\beta, e\alpha, ff(\alpha\beta-\beta\alpha), gg(\alpha\beta-\beta\alpha),\cdots, ll(\alpha\beta-\beta\alpha)]\Big).$$

Φ_{A1} is the linear combination of 4×2^M Slater determinants. After we have decided the recipe to write down the $\Theta^N_{S,M,k}$, which are shown in the preceeding paper[1], and the Ψ^0_A, which are explained in the just above, we can define uniquely our wave functions Φ_{Ai}. The tables of the formulae of configuration interaction matrix elements $H^{A_i}_{B_j}=\int\bar{\Phi}_{Ai}H\Phi_{Bj}\,dv$ obtained from the wave functions Φ_{Ai}, which we have just defined in the above, are collected in two parts, A. Diagonal Part and B. Nondiagonal Part. In the formulae of the diagonal matrix elements $H^{A_i}_{A_i}$ only the exchange integrals between the unpaired electrons are written, and the exchange integrals between the paired electrons, the exchange integrals between the paired and the unpaired electron, all coulomb integrals, and all kinetic and nuclear integrals are not written, because they can be obtained very easily.

Our wave functions Φ_{Ai} which are defined in the present paper by putting to use the Slater determinant can also be written in the following form:[2]

$$\Phi_{Ai}=\sum_k \Psi^{Ai}_{S,M,k}\,\Theta_{S,M,k}\quad ,$$

where $\Theta_{S,M,k}$ are the same spin functions as defined in the preceeding paper[1] and $\Psi^{Ai}_{S,M,k}$ are some linear combinations of the products of single-particle space orbital functions. We can construct the irreducible representation matrices $V(P)$, the bases of which are the spin functions $\Theta_{S,M,k}$,

$$P\,\Theta_{S,M,k}=\sum_h v^{S,M}_{hk}(P)\,\Theta_{S,M,k}\quad .$$

We make the new irreducible representation matrices $U(P)$ from the inverse matrices of $V(P)$ multiplied by $\varepsilon_P=(-1)^P$

$$U(P)=\varepsilon_P\bar{V}(P)\quad .$$

Then it follows that

$$P\,\Psi_{S,M,k}=\sum_h u^{S,M}_{hk}(P)\,\Psi_{S,M,k}\quad ,$$

because of the fact that the total wave function must be always antisymmetric in the permutations of electrons. Using the irreducible representation matrices $U(P)$, Our matrix elements $H^{A_i}_{B_j}$ can be written as

$$H^{A_i}_{B_j}=\sum_P u_{ji}(P)\,H^A_B(P),\qquad H^A_B(P)=\int P\,\bar{\Psi}^0_A\,H\,\Psi^0_B\,dv.$$

In section C of the present tables some irreducible representation matrices $U(P)$ are collected for singlet and triplet wave functions, which cover up to the case when the unpaired electrons are at most six, and for doublet and quarted state wave functions, which cover up to the unpaired electrons are at most five. The matrix elements $H_{B_j}^{A_i}$ shown in the present paper have been practically obtained by making use of these irreducible representation matrices $U(P)$.

Explanation of Symbols Used in Tables

N : number of the unpaired electrons

k : numbering of the independent spin functions with definite total spin quantum number S and its z-component M

A, B : $A=\Psi_A^0$, $B=\Psi_B^0$, the simple product of single-particle space orbital functions φ_m 's (space orbital configuration)

$a, b, c, d, e, f, \& g$: single-particle space orbital functions φ_m 's

$K(ab)$: exchange integral $(ab|ba)$

$(ab|cd)$: $\iint \bar{a}(1)\,\bar{c}(2)\left(\frac{1}{r_{12}}\right) b(1)\,d(2)\,dv_1\,dv_2$

$(a|h|b)$: kinetic and nuclear integrals

$$\int \bar{a}(1)\left(-\frac{1}{2}\Delta_1-\sum_h \frac{Z_h}{R_{h1}}\right) b(1)\,dv_1$$

P_a : the PAIRED single-particle space orbital functions

References

1) M. Yamazaki : Sci. Rep. Kanazawa Univ. Vol 8, No 2. pp 371–395, September 1963

2) Kotani, Amemiya, Ishiguro, and Kimura : Table of Molecular Integrals (1955) Maruzen, Tokyo

Contents of Tables

A. DIAGONAL PART

I. Formulae of configuration interaction matrix elements for Singlet state wave function ($S=0$)

1. $N=2$ $(k=1)$ (Omitted)
2. $N=4$ $(k=2)$ $A=abcd$

$$H^{A_1}_{A_1}=+1\,K(ab)-\frac{1}{2}K(ac)-\frac{1}{2}K(ad)-\frac{1}{2}K(bc)-\frac{1}{2}K(bd)+1\,K(cd)$$

$$H^{A_2}_{A_2}=-1\quad+\frac{1}{2}\quad+\frac{1}{2}\quad+\frac{1}{2}\quad+\frac{1}{2}\quad-1$$

$$H^{A_1}_{A_2}=\frac{\sqrt{3}}{2}\Big[K(ca)-K(da)-K(cb)+K(db)\Big]=H^{A_2}_{A_1}$$

3. $N=6$ $(k=5)$ $A=abcdef$

	$K(ab)$	$K(ac)$	$K(ad)$	$K(ae)$	$K(af)$	$K(bc)$	$K(bd)$	$K(be)$	$K(bf)$	$K(cd)$	$K(ce)$	$K(cf)$	$K(de)$	$K(df)$	$K(ef)$
$H^{A_1}_{A_1}=$	$+1$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$
$H^{A_2}_{A_2}=$	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$
$H^{A_3}_{A_3}=$	-1	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1	-1	-1
$H^{A_4}_{A_4}=$	-1	$+\frac{1}{2}$	$-\frac{5}{6}$	$+\frac{1}{6}$	$+\frac{1}{6}$	$+\frac{1}{2}$	$-\frac{5}{6}$	$+\frac{1}{6}$	$+\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1
$H^{A_5}_{A_5}=$	$+1$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1

$$H^{A_1}_{A_2}=\frac{\sqrt{3}}{2}\Big[K(ca)-K(da)-K(cb)+K(db)\Big]$$

$$H^{A_1}_{A_3}=\frac{1}{\sqrt{2}}\Big[K(ea)-K(fa)-K(eb)+K(fb)\Big]$$

$$H^{A_1}_{A_4}=-\frac{1}{2}\Big[K(ea)-K(fa)-K(eb)+K(fb)\Big]=H^{A_2}_{A_5}$$

$$H^{A_1}_{A_5}=\frac{\sqrt{3}}{2}\Big[K(ec)-K(fc)-K(ed)+K(fd)\Big]$$

$$H^{A_2}_{A_3}=\frac{1}{\sqrt{6}}\Big[K(ea)-K(fa)+K(eb)-K(fb)-2K(ec)+2K(fc)\Big]$$

$$H^{A_2}_{A_4}=\frac{1}{\sqrt{3}}\Big[K(ea)-K(fa)+K(eb)-K(fb)-\frac{1}{2}K(ec)+\frac{1}{2}K(fc)$$
$$-\frac{3}{2}K(ed)+\frac{3}{2}K(fd)\Big]$$

$$H^{A_3}_{A_4}=\frac{\sqrt{2}}{3}\Big[K(da)-\frac{1}{2}K(ea)-\frac{1}{2}K(fa)+K(db)-\frac{1}{2}K(eb)$$
$$-\frac{1}{2}K(fb)-2K(dc)+K(ec)+K(fc)\Big]$$

$$H^{A_3}_{A_5}=\frac{1}{\sqrt{6}}\Big[2K(da)-K(ea)-K(fa)-2K(db)+K(eb)$$
$$+K(fb)\Big]$$

$$H^{A_4}_{A_5} = \frac{1}{\sqrt{3}}\Big[\frac{3}{2}K(ca) + \frac{1}{2}K(da) - K(ea) - K(fa) - \frac{3}{2}K(cb) - \frac{1}{2}K(ab) + K(eb) + K(fb)\Big]$$

$$H^{A_i}_{A_j} \equiv H^{A_j}_{A_i}$$

II. Formulae of configuration interaction matrix elements for Doublet state wave function $(S = \frac{1}{2})$

1. $N = 1$ $(k = 1)$ (Omitted)
2. $N = 3$ $(k = 2)$ $A = abc$

$$H^{A_1}_{A_1} = -\frac{1}{2}K(ab) - \frac{1}{2}K(ac) + 1K(bc)$$

$$H^{A_2}_{A_2} = +\frac{1}{2} \qquad + \frac{1}{2} \qquad - 1$$

$$H^{A_1}_{A_2} = \frac{\sqrt{3}}{2}\Big[K(ba) - K(ca)\Big] = H^{A_2}_{A_1}$$

3. $N = 5$ $(k = 5)$ $A = abcde$

	$K(ab)$	$K(ac)$	$K(ad)$	$K(ae)$	$K(bc)$	$K(bd)$	$K(be)$	$K(cd)$	$K(ce)$	$K(de)$
$H^{A_1}_{A_1} =$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$
$H^{A_2}_{A_2} =$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$
$H^{A_3}_{A_3} =$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1
$H^{A_4}_{A_4} =$	$+\frac{1}{2}$	$-\frac{5}{6}$	$+\frac{1}{6}$	$+\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1
$H^{A_5}_{A_5} =$	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1	-1	-1

$$H^{A_1}_{A_2} = \frac{\sqrt{3}}{2}\Big[K(ba) - K(ca)\Big]$$

$$H^{A_1}_{A_3} = -\frac{\sqrt{3}}{2}\Big[K(db) - K(eb) - K(dc) + K(ec)\Big]$$

$$H^{A_1}_{A_4} = -\frac{1}{2}\Big[K(da) - K(ea)\Big] = -H^{A_2}_{A_3}$$

$$H^{A_1}_{A_5} = \frac{1}{\sqrt{2}}\Big[K(da) - K(ea)\Big]$$

$$H^{A_2}_{A_4} = \frac{1}{\sqrt{3}}\Big[K(da) - K(ea) - \frac{1}{2}K(db) + \frac{1}{2}K(eb) - \frac{3}{2}K(dc) + \frac{3}{2}K(ec)\Big]$$

$$H^{A_2}_{A_5} = \frac{1}{\sqrt{6}}\Big[K(da) - K(ea) - 2K(db) + 2K(eb)\Big]$$

$$H^{A_3}_{A_4} = \frac{1}{\sqrt{3}}\Big[-\frac{3}{2}K(ba) - \frac{1}{2}K(ca) + K(da) + K(ea)\Big]$$

Doublet

$$H_{A_5}^{A_3} = \frac{1}{\sqrt{6}}\left[-2K(ca) + K(da) + K(ea)\right]$$

$$H_{A_5}^{A_4} = \frac{\sqrt{2}}{3}\left[K(ca) - \frac{1}{2}K(da) - \frac{1}{2}K(ea) - 2K(cb) + K(db) + K(eb)\right]$$

$$H_{A_j}^{A_i} \equiv H_{A_i}^{A_j}$$

III. Formulae of configuration interaction matrix elements for Triplet state wave function ($S=1$)

1. $N=2$ $(k=1)$ (Omitted)
2. $N=4$ $(k=3)$ $A = abcd$

$H_{A_1}^{A_1}$ =	$-1K(ab)$	$-\frac{1}{2}K(ac)$	$-\frac{1}{2}K(ad)$	$-\frac{1}{2}K(bc)$	$-\frac{1}{2}K(bd)$	$+1K(cd)$
$H_{A_2}^{A_2}$ =	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1
$H_{A_3}^{A_3}$ =	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	-1	-1	-1

$$H_{A_2}^{A_1} = \frac{\sqrt{3}}{2}\left[-\frac{1}{3}K(ca) + \frac{1}{3}K(da) - K(cb) + K(db)\right]$$

$$H_{A_3}^{A_1} = -\frac{\sqrt{6}}{3}\left[K(ca) - K(da)\right]$$

$$H_{A_3}^{A_2} = \frac{\sqrt{2}}{3}\left[-2K(ba) + K(ca) + K(da)\right]$$

$$H_{A_j}^{A_i} \equiv H_{A_i}^{A_j}$$

3. $N=6$ $(k=9)$ $A = abcdef$

	K^* (ab)	K (ac)	K (ad)	K (ae)	K (af)	K (bc)	K (bd)	K (be)	K (bf)	K (cd)	K (ce)	K (cf)	K (de)	K (df)	K (ef)
$H_{A_1}^{A_1}$ =	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1
$H_{A_2}^{A_2}$ =	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1
$H_{A_3}^{A_3}$ =	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1
$H_{A_4}^{A_4}$ =	-1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	-1	-1	-1	-1	-1	-1
$H_{A_5}^{A_5}$ =	$\frac{1}{3}$	$-\frac{11}{12}$	$\frac{7}{36}$	$\frac{7}{36}$	$\frac{7}{36}$	$-\frac{1}{4}$	$-\frac{11}{12}$	$-\frac{11}{12}$	$-\frac{11}{12}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1
$H_{A_6}^{A_6}$ =	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{7}{9}$	$\frac{1}{18}$	$\frac{1}{18}$	-1	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$H_{A_7}^{A_7}$ =	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{7}{9}$	$-\frac{7}{9}$	$-\frac{7}{9}$	-1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1
$H_{A_8}^{A_8}$ =	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{7}{18}$	$-\frac{13}{18}$	$-\frac{13}{18}$	$\frac{1}{2}$	$-\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$H_{A_9}^{A_9}$ =	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1

* K(ab)'s are printed as $\underset{(ab)}{K}$'s

Triplet

$$H_{A_2}^{A_1} = \frac{\sqrt{3}}{2}\left[-\frac{1}{3} K(ca) + \frac{1}{3} K(da) - K(cb) + K(db) \right]$$

$$H_{A_3}^{A_1} = -\frac{\sqrt{6}}{3}\left[K(ca) - K(da) \right]$$

$$H_{A_4}^{A_1} = H_{A_9}^{A_4} = 0$$

$$H_{A_5}^{A_1} = -\frac{2}{3}\left[K(ea) - K(fa) \right]$$

$$H_{A_6}^{A_1} = \frac{\sqrt{2}}{3}\left[K(ea) - K(fa) \right] = H_{A_9}^{A_3}$$

$$H_{A_7}^{A_1} = \frac{1}{\sqrt{2}}\left[-\frac{1}{3} K(ea) + \frac{1}{3} K(fa) - K(eb) + K(fb) \right]$$

$$H_{A_8}^{A_1} = \frac{1}{2}\left[\frac{1}{3} K(ea) - \frac{1}{3} K(fa) + K(eb) - K(fb) \right]$$

$$H_{A_9}^{A_1} = \frac{\sqrt{3}}{2}\left[K(ec) - K(fc) - K(ed) + K(fd) \right]$$

$$H_{A_3}^{A_2} = \frac{\sqrt{2}}{3}\left[-2K(ba) + K(ca) + K(da) \right]$$

$$H_{A_4}^{A_2} = -\frac{\sqrt{5}}{3}\left[K(ea) - K(fa) \right]$$

$$H_{A_5}^{A_2} = \frac{\sqrt{3}}{9}\left[K(ea) - K(fa) \right] = H_{A_7}^{A_3}$$

$$H_{A_6}^{A_2} = \frac{\sqrt{6}}{9}\left[K(ea) - K(fa) \right] = H_{A_8}^{A_3}$$

$$H_{A_7}^{A_2} = \frac{1}{\sqrt{6}}\left[-\frac{1}{3} K(ea) + \frac{1}{3} K(fa) + K(eb) - K(fb) - 2K(ec) + 2K(fc) \right]$$

$$H_{A_8}^{A_2} = \frac{1}{\sqrt{3}}\left[-\frac{1}{3} K(ea) + \frac{1}{3} K(fa) + K(eb) - K(fb) - \frac{1}{2} K(ec) + \frac{1}{2} K(fc) - \frac{3}{2} K(ed) + \frac{3}{2} K(fd) \right]$$

$$H_{A_9}^{A_2} = \frac{1}{6}\left[K(ea) - K(fa) + 3K(eb) - 3K(fb) \right]$$

$$H_{A_4}^{A_3} = \frac{\sqrt{10}}{12}\left[K(ea) - K(fa) - 3K(eb) + 3K(fb) \right]$$

$$H_{A_5}^{A_3} = \frac{\sqrt{6}}{12}\left[\frac{5}{3} K(ea) - \frac{5}{3} K(fa) - K(eb) + K(fb) - 4K(ec) + 4K(fc) \right]$$

$$H_{A_6}^{A_3} = \frac{\sqrt{3}}{6}\left[\frac{5}{3} K(ea) - \frac{5}{3} K(fa) - K(eb) + K(fb) - K(ec) + K(fc) - 3K(ed) + 3K(fd) \right]$$

$$H_{A_5}^{A_4} = \frac{\sqrt{15}}{12}\left[K(ca) - \frac{1}{3} K(da) - \frac{1}{3} K(ea) - \frac{1}{3} K(fa) - 3K(cb) + K(db) + K(eb) + K(fb) \right]$$

$$H_{A_6}^{A_4} = \frac{\sqrt{30}}{12}\left[\frac{2}{3} K(da) - \frac{1}{3} K(ea) - \frac{1}{3} K(fa) - 2K(db) + K(eb) + K(fb) \right]$$

Triplet

$$H_{A_7}^{A_4} = \frac{\sqrt{30}}{18}\left[-3K(ca) + K(da) + K(ea) + K(fa) \right]$$

$$H_{A_8}^{A_4} = \frac{\sqrt{15}}{9}\left[-2K(da) + K(ea) + K(fa) \right]$$

$$H_{A_6}^{A_5} = \frac{\sqrt{2}}{12}\left[\frac{10}{3}K(da) - \frac{5}{3}K(ea) - \frac{5}{3}K(fa) - 2K(db) + K(eb) + K(fb) - 8K(dc) + 4K(ec) + 4K(fc) \right]$$

$$H_{A_7}^{A_5} = \frac{5\sqrt{2}}{18}\left[-\frac{12}{5}K(ba) - \frac{3}{5}K(ca) + K(da) + K(ea) + K(fa) \right]$$

$$H_{A_8}^{A_5} = \frac{1}{9}\left[2K(da) - K(ea) - K(fa) \right] = H_{A_7}^{A_6}$$

$$H_{A_9}^{A_5} = \frac{2\sqrt{3}}{9}\left[-2K(da) + K(ea) + K(fa) \right]$$

$$H_{A_8}^{A_6} = \frac{\sqrt{2}}{3}\left[-2K(ba) + K(ca) - \frac{1}{3}K(da) + \frac{2}{3}K(ea) + \frac{2}{3}K(fa) \right]$$

$$H_{A_9}^{A_6} = \frac{2\sqrt{6}}{9}\left[-\frac{3}{2}K(ca) - \frac{1}{2}K(da) + K(ea) + K(fa) \right]$$

$$H_{A_8}^{A_7} = \frac{\sqrt{2}}{3}\left[-\frac{1}{3}K(da) + \frac{1}{6}K(ea) + \frac{1}{6}K(fa) + K(db) - \frac{1}{2}K(eb) - \frac{1}{2}K(fb) - 2K(dc) + K(ec) + K(fc) \right]$$

$$H_{A_9}^{A_7} = \frac{1}{\sqrt{6}}\left[-\frac{2}{3}K(da) + \frac{1}{3}K(ea) + \frac{1}{3}K(fa) - 2K(db) + K(eb) + K(fb) \right]$$

$$H_{A_9}^{A_8} = \frac{1}{\sqrt{3}}\left[-\frac{1}{2}K(ca) - \frac{1}{6}K(da) + \frac{1}{3}K(ea) + \frac{1}{3}K(fa) - \frac{3}{2}K(cb) - \frac{1}{2}K(db) + K(eb) + K(fb) \right]$$

$$H_{A_j}^{A_i} \equiv H_{A_i}^{A_j}$$

IV. Formulae of configuration interaction matrix elements for Quartet state wave function $(S=\frac{3}{2})$

1. $N=3$ $(k=1)$ (Omitted)
2. $N=5$ $(k=4)$ $A = abcde$

	$K(ab)$	$K(ac)$	$K(ad)$	$K(ae)$	$K(bc)$	$K(bd)$	$K(be)$	$K(cd)$	$K(ce)$	$K(de)$
$H_{A_1}^{A_1} =$	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$
$H_{A_2}^{A_2} =$	-1	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$H_{A_3}^{A_3} =$	$-\frac{1}{4}$	$-\frac{11}{12}$	$-\frac{11}{12}$	$-\frac{11}{12}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	-1
$H_{A_4}^{A_4} =$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	-1	-1	-1	-1	-1	-1

$$H_{A_2}^{A_1} = \frac{\sqrt{3}}{6}\left[-K(da) + K(ea) - K(db) + K(eb) - 3K(dc) + 3K(ec) \right]$$

Quartet

$$H_{A_3}^{A_1} = \frac{\sqrt{6}}{12}\left[-K(da)+K(ea)-4K(db)+4K(eb)\right]$$

$$H_{A_4}^{A_1} = -\frac{\sqrt{10}}{4}\left[K(da)-K(ea)\right]$$

$$H_{A_3}^{A_2} = \frac{\sqrt{2}}{3}\left[-\frac{1}{2}K(ca)+\frac{1}{4}K(da)+\frac{1}{4}K(ea)-2K(cb)+K(db)+K(eb)\right]$$

$$H_{A_4}^{A_2} = \frac{\sqrt{30}}{12}\left[-2K(ca)+K(da)+K(ea)\right]$$

$$H_{A_4}^{A_3} = \frac{\sqrt{15}}{12}\left[-3K(ba)+K(ca)+K(da)+K(ea)\right]$$

$$H_{A_j}^{A_i} \equiv H_{A_i}^{A_j}$$

B. NONDIAGONAL PART

I. Formulae of configuration interaction matrix elements for Singlet state wave function ($S=0$)

1. $N_A=2$ or 0, $N_B=2$ or 0 ($k_A=1$, $k_B=1$)

$A=abcc$, $B=abdd$ $\quad H_B^A = (cd \mid cd)$

$A=abcc$, $B=cdaa$ $\quad H_B^A = -(bd \mid ca)+2(cd \mid ba)$

$A=abcc$, $B=cdbb$ $\quad H_B^A = -(ad \mid cb)+2(cd \mid ab)$

$A=abcc$, $B=cbdd$ $\quad H_B^A = -(ad \mid cd)$

$A=abcc$, $B=cbaa$ $\quad H_B^A = -\left\{(c \mid h \mid a)+(ca \mid aa)+(ca \mid bb)+(ca \mid cc)+2(ca \mid P_aP_a)-(P_aa \mid cP_a)\right\}+2(cb \mid ba)$

$A=ab$, $B=ad$ $\quad H_B^A = \left\{(b \mid h \mid d)+(bd \mid aa)+2(bd \mid P_aP_a)-(P_ad \mid bP_a)\right\}+(ba \mid ad)$

$A=ab$, $B=db$ $\quad H_B^A = \left\{(a \mid h \mid d)+(ad \mid bb)+2(ad \mid P_aP_a)-(P_ad \mid aP_a)\right\}+2(ab \mid bd)$

$A=aa$, $B=de$ $\quad H_B^A = \sqrt{2}(ad \mid ae)$

$A=abcc$, $B=dcbb$ $\quad H_B^A = -(ad \mid cb)+2(cd \mid ab)$

$A=abcc$, $B=adbb$ $\quad H_B^A = -(cd \mid cb)$

$A=abcc$, $B=dbaa$ $\quad H_B^A = -(cd \mid ca)$

$A=ab$, $B=de$ $\quad H_B^A = (ad \mid be)+(bd \mid ae)$

$A=abcc$, $B=dcaa$ $\quad H_B^A = -(bd \mid ca)+2(cd \mid ba)$

$A=abcc$, $B=bdaa$ $\quad H_B^A = -(cd \mid ca)$

Singlet 1×1

$A = abccdd$, $B = cdaabb$ $\quad H_B^A = (ca \mid db) + (da \mid cb)$

$A = abcc$, $B = acbb$ $\quad H_B^A = -\Big\{(c \mid h \mid b) + (cb \mid aa) + (cb \mid bb) + (cb \mid cc) + 2\,(cb \mid P_a P_a) - (P_a b \mid cP_a)\Big\} + 2\,(ca \mid ab)$

$A = abcc$, $B = bcaa$ $\quad H_B^A = -\Big\{(c \mid h \mid a) + (ca \mid aa) + (ca \mid bb) + (ca \mid cc) + 2\,(ca \mid P_a P_a) - (P_a a \mid cP_a)\Big\} + 2\,(cb \mid ba)$

$A = ab$, $B = da$ $\quad H_B^A = \Big\{(b \mid h \mid d) + (bd \mid aa) + 2\,(bd \mid P_a P_a) - (P_a d \mid bP_a)\Big\} + (ba \mid ad)$

$A = ab$, $B = aa$ $\quad H_B^A = \sqrt{2}\Big\{(b \mid h \mid a) + (ba \mid aa) + 2\,(ba \mid P_a P_a) - (P_a a \mid bP_a)\Big\}$

$A = aa$, $B = dd$ $\quad H_B^A = (ad \mid ad)$

$A = aa$, $B = da$ $\quad H_B^A = \sqrt{2}\Big\{(a \mid h \mid d) + (ad \mid aa) + 2\,(ad \mid P_a P_a) - (P_a d \mid aP_a)\Big\}$

$A = abcc$, $B = dcaa$ $\quad H_B^A = -(bd \mid ca) + 2\,(cd \mid ba)$

$A = abcc$, $B = aabb$ $\quad H_B^A = -\sqrt{2}\,(ca \mid cb)$

$A = abcc$, $B = dbaa$ $\quad H_B^A = -(cd \mid ca)$

$A = abcc$, $B = dcbb$ $\quad H_B^A = -(ad \mid cb) + 2\,(cd \mid ab)$

$A = abcc$, $B = adbb$ $\quad H_B^A = -(cd \mid cb)$

$A = abcc$, $B = dabb$ $\quad H_B^A = -(cb \mid cd)$

$A = abcc$, $B = acdd$ $\quad H_B^A = -(bd \mid cd)$

$A = abcc$, $B = bcdd$ $\quad H_B^A = -(ad \mid cd)$

2. $N_A = 2$ or 0, $\quad N_B = 4 \quad (k_A = 1, \quad k_B = 2)$

$A = abcc$, $B = abdc$ $\quad H_{B_1}^A = \sqrt{2}\Big[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) - \frac{1}{2}(cb \mid bd)\Big]$

$$H_{B_2}^A = \sqrt{2}\Big[\frac{\sqrt{3}}{2}(ca \mid ad) - \frac{\sqrt{3}}{2}(cb \mid bd)\Big]$$

$A = abcc$, $B = abcd$ $\quad H_{B_1}^A = \sqrt{2}\Big[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) - \frac{1}{2}(cb \mid bd)\Big]$

$$H_{B_2}^A = \sqrt{\frac{3}{2}}\Big[-(ca \mid ad) + (cb \mid bd)\Big]$$

$A = abcc$, $B = debc$ $\quad H_{B_1}^A = -\frac{1}{\sqrt{2}}\Big[(ad \mid ce) + (cd \mid ae)\Big]$

$$H_{B_2}^A = \frac{\sqrt{6}}{2}\Big[-(ad \mid ce) + (cd \mid ae)\Big]$$

$A = abcc$, $B = decb$ $\quad H_{B_1}^A = -\frac{1}{\sqrt{2}}\Big[(ad \mid ce) + (cd \mid ae)\Big]$

$$H_{B_2}^A = \sqrt{\frac{3}{2}}\Big[(ad \mid ce) - (cd \mid ae)\Big]$$

Singlet 1×2

$A = abcc$
$B = adcb$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c\mid h\mid d) + (cd\mid aa) + (cd\mid bb) + (cd\mid cc) + 2(cd\mid P_aP_a) - (P_a d\mid cP_a)\} + (cb\mid bd) - \frac{1}{2}(ca\mid ad)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{2}(ca\mid ad)\Big]$$

$A = abccdd$
$B = bcedaa$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid da) - (de\mid ca)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce\mid da)$$

$A = abccdd$
$B = adecbb$

$$H^A_{B_1} = \sqrt{2}\Big[-(ce\mid db) + \frac{1}{2}(de\mid cb)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(de\mid cb)$$

$A = abccdd$
$B = caedbb$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid db) - (de\mid ca)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(ce\mid db)$$

$A = abccdd$
$B = acedbb$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid db) - (de\mid cb)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce\mid db)$$

$A = abccdd$
$B = cadebb$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid db) - (de\mid cb)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce\mid db)$$

$A = abccdd$
$B = acdebb$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid db) - (de\mid cb)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(ce\mid db)$$

$A = abccdd$
$B = bcdeaa$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(ce\mid da) - (de\mid ca)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(ce\mid da)$$

$A = abcc$
$B = bdec$

$$H^A_{B_1} = \sqrt{2}\Big[(ad\mid ce) - \frac{1}{2}(cd\mid ae)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd\mid ae)$$

$A = abcc$
$B = dbec$

$$H^A_{B_1} = \sqrt{2}\Big[(ad\mid ce) - \frac{1}{2}(cd\mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(cd\mid ae)$$

$A = abcc$
$B = cbde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ad\mid ce) + (cd\mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[(ad\mid ce) - (cd\mid ae)\Big]$$

$A = abcc$
$B = acde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(bd\mid ce) + (cd\mid be)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[-(bd\mid ce) + (cd\mid be)\Big]$$

Singlet 1×2

$A = abcc$
$B = bcde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ad \mid ce) + (cd \mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[-(ad \mid ce) + (cd \mid ae)\Big]$$

$A = abcc$
$B = bdce$

$$H^A_{B_1} = \sqrt{2}\Big[(ad \mid ce) - \frac{1}{2}(cd \mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(cd \mid ae)$$

$A = abcc$
$B = dbce$

$$H^A_{B_1} = \sqrt{2}\Big[(ad \mid ce) - \frac{1}{2}(cd \mid ae)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd \mid ae)$$

$A = aabb$
$B = abcd$

$$H^A_{B_1} = -(ac \mid bd) - (bc \mid ad)$$

$$H^A_{B_2} = -\sqrt{3}(ac \mid bd) + \sqrt{3}(bc \mid ad)$$

$A = abcc$
$B = adeb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(cd \mid ce)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(cd \mid ce)$$

$A = abcc$
$B = daeb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(cd|ce)$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd|ce)$$

$A = abccdd$
$B = ecdbaa$

$$H^A_{B_1} = \sqrt{2}\Big[-(ce \mid da) + \frac{1}{2}(de \mid ca)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(de \mid ca)$$

$A = abccdd$
$B = cedbaa$

$$H^A_{B_1} = \sqrt{2}\Big[-(ce \mid da) + \frac{1}{2}(de \mid ca)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(de \mid ca)$$

$A = abccdd$
$B = cdebaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(ce \mid da) + (de \mid ca)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[(ce \mid da) - (de \mid ca)\Big]$$

$A = abcc$
$B = dabc$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid c P_a)\} - \frac{1}{2}(ca \mid ad) + (cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{2}(ca \mid ad)\Big]$$

$A = abcc$
$B = dace$

$$H^A_{B_1} = \sqrt{2}\Big[(bd \mid ce) - \frac{1}{2}(cd \mid be)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd \mid be)$$

$A = abcc$
$B = dcae$

$$H^A_{B_1} = \sqrt{2}\Big[(cd \mid be) - \frac{1}{2}(bd \mid ce)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(bd|ce)$$

Singlet 1×2

$A = abcc$
$B = cade$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[(bd \mid ce) - (cd \mid be)\right]$$

$A = abcc$
$B = cdae$

$$H^A_{B_1} = \sqrt{2}\left[(cd \mid be) - \frac{1}{2}(bd \mid ce)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(bd \mid ce)$$

$A = abcc$
$B = deac$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[-(bd \mid ce) + (cd \mid be)\right]$$

$A = abcc$
$B = adec$

$$H^A_{B_1} = \sqrt{2}\left[(bd \mid ce) - \frac{1}{2}(cd \mid be)\right]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd|be)$$

$A = abcc$
$B = daec$

$$H^A_{B_1} = \sqrt{2}\left[(bd \mid ce) - \frac{1}{2}(cd \mid be)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(cd|be)$$

$A = abcc$
$B = deab$

$$H^A_{B_1} = \sqrt{2}\left[(cd \mid ce)\right]$$

$$H^A_{B_2} = 0$$

$A = aabb$
$B = deab$

$$H^A_{B_1} = -(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = -\sqrt{3}(ad \mid be) + \sqrt{3}(bd|ae)$$

$A = aabb$
$B = daeb$

$$H^A_{B_1} = 2(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = \sqrt{3}(bd \mid ae)$$

$A = aabb$
$B = dabe$

$$H^A_{B_1} = 2(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = -\sqrt{3}(bd \mid ae)$$

$A = aabb$
$B = adeb$

$$H^A_{B_1} = 2(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = -\sqrt{3}(bd \mid ae)$$

$A = aabb$
$B = abde$

$$H^A_{B_1} = -(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = -\sqrt{3}(ad \mid be) + \sqrt{3}(bd \mid ae)$$

$A = aabb$
$B = adbe$

$$H^A_{B_1} = 2(ad \mid be) - (bd \mid ae)$$

$$H^A_{B_2} = \sqrt{3}(bd \mid ae)$$

$A = abcc$
$B = cade$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[(bd \mid ce) - (cd \mid be)\right]$$

$A = abccdd$
$B = acdbee$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (ce \mid de)$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}} (ce \mid de)$$

$A = abccdd$
$B = cabdee$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (ce \mid de)$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}} (ce \mid de)$$

$A = abccdd$
$B = abcdee$

$$H^A_{B_1} = -\sqrt{2} (ce \mid de)$$

$$H^A_{B_2} = 0$$

$A = abccdd$
$B = cdabee$

$$H^A_{B_1} = -\sqrt{2} (ce \mid de)$$

$$H^A_{B_2} = 0$$

$A = abcc$
$B = adbc$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) + (cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{2}(ca \mid ad)\Big]$$

$A = abcc$
$B = acbd$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + (ca \mid ad) - \frac{1}{2}(cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{2}(cb \mid bd)\Big]$$

$A = abcc$
$B = dacb$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) + (cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{2}(ca \mid ad)\Big]$$

$A = abcc$
$B = dcab$

$$H^A_{B_1} = \sqrt{2}\Big[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) - \frac{1}{2}(cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[(ca \mid ad) - (cb \mid bd)\Big]$$

$A = abcc$
$B = cadb$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + (ca \mid ad) - \frac{1}{2}(cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{2}(cb \mid bd)\Big]$$

$A = abcc$
$B = cdab$

$$H^A_{B_1} = \sqrt{2}\Big[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(cb \mid bd) - \frac{1}{2}(ca \mid ad)\Big]$$

$$H^A_{B_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{2}(ca \mid ad) + \frac{\sqrt{3}}{2}(cb \mid bd)\Big]$$

Singlet 1×2

3. $N_A = 2,\quad N_B = 6\quad (k_A = 1,\quad k_B = 5)$

$A = abccdd$
$B = abefcd$

$$H^A_{B_1} = -(ce\mid df) - (de\mid cf)$$
$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = 0$$
$$H^A_{B_5} = -\sqrt{3}\,(ce\mid df) + \sqrt{3}\,(de\mid cf)$$

$A = abccdd$
$B = acefdb$

$$H^A_{B_1} = \frac{1}{2}(ce\mid df) + \frac{1}{2}(de\mid cf)$$
$$H^A_{B_2} = -\frac{\sqrt{3}}{2}(ce\mid df) + \frac{\sqrt{3}}{2}(de\mid cf)$$
$$H^A_{B_3} = -\sqrt{2}\,(de\mid cf)$$
$$H^A_{B_4} = \frac{3}{2}(ce\mid df) - \frac{1}{2}(de\mid cf)$$
$$H^A_{B_5} = -\frac{\sqrt{3}}{2}(ce\mid df) + \frac{\sqrt{3}}{2}(de\mid cf)$$

$A = abccdd$
$B = caefbd$

$$H^A_{B_1} = \frac{1}{2}(ce\mid df) + \frac{1}{2}(de\mid cf)$$
$$H^A_{B_2} = \frac{\sqrt{3}}{2}(ce\mid df) - \frac{\sqrt{3}}{2}(de\mid cf)$$
$$H^A_{B_3} = -\sqrt{2}\,(de\mid cf)$$
$$H^A_{B_4} = \frac{3}{2}(ce\mid df) - \frac{1}{2}(de\mid cf)$$
$$H^A_{B_5} = \frac{\sqrt{3}}{2}(ce\mid df) - \frac{\sqrt{3}}{2}(de\mid cf)$$

$A = abccdd$
$B = cdefab$

$$H^A_{B_1} = -(ce\mid df) - (de\mid cf)$$
$$H^A_{B_2} = -\sqrt{3}\,(ce\mid df) + \sqrt{3}\,(de\mid cf)$$
$$H^A_{B_3} = H^A_{B_4} = H^A_{B_5} = 0$$

$A = abccdd$
$B = aedfbc$

$$H^A_{B_1} = \frac{1}{2}(de\mid cf) - (ce\mid df)$$
$$H^A_{B_2} = -\frac{\sqrt{3}}{2}(de\mid cf)$$
$$H^A_{B_3} = -\sqrt{2}\,(ce\mid df)$$
$$H^A_{B_4} = -\frac{3}{2}(de\mid cf) + (ce\mid df)$$
$$H^A_{B_5} = \frac{\sqrt{3}}{2}(de\mid cf)$$

$A = abccdd$
$B = eadfcb$

$$H^A_{B_1} = \frac{1}{2}(de\mid cf) - (ce\mid df)$$
$$H^A_{B_2} = \frac{\sqrt{3}}{2}(de\mid cf)$$
$$H^A_{B_3} = -\sqrt{2}\,(ce\mid df)$$
$$H^A_{B_4} = -\frac{3}{2}(de\mid cf) + (ce\mid df)$$
$$H^A_{B_5} = -\frac{\sqrt{3}}{2}(de\mid cf)$$

Singlet 1×5

$A = abccdd$
$B = acdebf$

$$H^A_{B_1} = \frac{1}{2}(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = \frac{\sqrt{3}}{2}(ce \mid df)$$
$$H^A_{B_3} = \sqrt{2}\,(ce \mid df) - \sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = \frac{1}{2}(ce \mid df) + (de \mid cf)$$
$$H^A_{B_5} = -\frac{\sqrt{3}}{2}(ce \mid df)$$

$A = abccdd$
$B = cadefb$

$$H^A_{B_1} = \frac{1}{2}(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = -\frac{\sqrt{3}}{2}(ce \mid df)$$
$$H^A_{B_3} = \sqrt{2}\,(ce \mid df) - \sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = \frac{1}{2}(ce \mid df) + (de \mid cf)$$
$$H^A_{B_5} = \frac{\sqrt{3}}{2}(ce \mid df)$$

$A = abccdd$
$B = efabcd$

$$H^A_{B_1} = -(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = H^A_{B_5} = 0$$
$$H^A_{B_3} = -\sqrt{2}\,(ce \mid df) + \sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = (ce \mid df) - (de \mid cf)$$

$A = abccdd$
$B = ecabdf$

$$H^A_{B_1} = 2\,(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = H^A_{B_5} = 0$$
$$H^A_{B_3} = -\sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = (de \mid cf)$$

$A = abccdd$
$B = ceabfd$

$$H^A_{B_1} = 2\,(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = H^A_{B_5} = 0$$
$$H^A_{B_3} = -\sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = (de \mid cf)$$

$A = abccdd$
$B = cdabef$

$$H^A_{B_1} = -(ce \mid df) - (de \mid cf)$$
$$H^A_{B_2} = H^A_{B_5} = 0$$
$$H^A_{B_3} = -\sqrt{2}\,(ce \mid df) + \sqrt{2}\,(de \mid cf)$$
$$H^A_{B_4} = (ce \mid df) - (de \mid cf)$$

$A = abccdd$
$B = aefdbc$

$$H^A_{B_1} = \frac{1}{2}(de \mid cf) - (ce \mid df)$$

Singlet 1×5

$$H^{A}_{B_2} = \frac{\sqrt{3}}{2}\,(de \mid cf)$$

$$H^{A}_{B_3} = \sqrt{2}\,(de \mid cf) - \sqrt{2}\,(ce \mid df)$$

$$H^{A}_{B_4} = \frac{1}{2}\,(de \mid cf) + (ce \mid df)$$

$$H^{A}_{B_5} = -\frac{\sqrt{3}}{2}\,(de \mid cf)$$

$A = abccdd$
$B = eafdcb$

$$H^{A}_{B_1} = \frac{1}{2}\,(de \mid cf) - (ce \mid df)$$

$$H^{A}_{B_2} = -\frac{\sqrt{3}}{2}\,(de \mid cf)$$

$$H^{A}_{B_3} = \sqrt{2}\,(de \mid cf) - \sqrt{2}\,(ce \mid df)$$

$$H^{A}_{B_4} = \frac{1}{2}\,(de \mid cf) + (ce \mid df)$$

$$H^{A}_{B_5} = \frac{\sqrt{3}}{2}\,(de \mid cf)$$

$A = abccdd$
$B = acedbf$

$$H^{A}_{B_1} = \frac{1}{2}\,(ce \mid df) - (de \mid cf)$$

$$H^{A}_{B_2} = -\frac{\sqrt{3}}{2}\,(ce \mid df)$$

$$H^{A}_{B_3} = -\sqrt{2}\,(de \mid cf)$$

$$H^{A}_{B_4} = -\frac{3}{2}\,(ce \mid df) + (de \mid cf)$$

$$H^{A}_{B_5} = \frac{\sqrt{3}}{2}\,(ce \mid df)$$

$A = abccdd$
$B = caedfb$

$$H^{A}_{B_1} = \frac{1}{2}\,(ce \mid df) - (de \mid cf)$$

$$H^{A}_{B_2} = \frac{\sqrt{3}}{2}\,(ce \mid df)$$

$$H^{A}_{B_3} = -\sqrt{2}\,(de \mid cf)$$

$$H^{A}_{B_4} = -\frac{3}{2}\,(ce \mid df) + (de \mid cf)$$

$$H^{A}_{B_5} = -\frac{\sqrt{3}}{2}\,(ce \mid df)$$

4. $N_A = 4$, $N_B = 4$ $(k_A = 2, \quad k_B = 2)$

$A = abcd$
$B = eacd$

$$H^{A_2}_{B_2} = -\Big\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + (be \mid dd) + 2\,(be \mid P_a P_a) - (P_a e \mid bP_a)\Big\} + (ba \mid ae) - \frac{1}{2}\,(bc \mid ce) - \frac{1}{2}\,(bd \mid de)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\,(bc \mid ce) + \frac{\sqrt{3}}{2}\,(bd \mid de) = -\,H^{A_1}_{B_2}$$

$$H^{A_1}_{B_1} = \Big\{\text{the above}\Big\} + (ba \mid ae) - \frac{1}{2}\,(bc \mid ce) - \frac{1}{2}\,(bd \mid de)$$

$A = abcd$
$B = ebcd$

$$H^{A_2}_{B_2} = \Big\{(a \mid h \mid e) + (ae \mid bb) + (ae \mid cc) + (ae \mid dd) + 2\,(ae \mid P_a P_a) - (P_a e \mid aP_a)\Big\} - (ab \mid be) + \frac{1}{2}\,(ac \mid ce) + \frac{1}{2}\,(ad \mid de)$$

$$H^{A_2}_{B_1} = H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\,(ac \mid ce) - \frac{\sqrt{3}}{2}\,(ad \mid de)$$

$$H^{A_1}_{B_1} = \Big\{\text{the above}\Big\} + (ab \mid be) - \frac{1}{2}\,(ac \mid ce) - \frac{1}{2}\,(ad \mid de)$$

Singlet 2×2

$A = abcdee$
$B = becdaa$

$$H_{B_2}^{A_2} = \Big\{(e \mid h \mid a) + (ea \mid aa) + (ea \mid bb) + (ea \mid cc) + (ea \mid dd) + (ea \mid ee) + 2\,(ea \mid P_a P_a) - (P_a a \mid eP_a)\Big\} - \frac{3}{2}\,(ec \mid ca) - \frac{3}{2}\,(ed \mid da)$$

$$H_{B_1}^{A_2} = -\,H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\,(ec \mid ca) - \frac{\sqrt{3}}{2}\,(ed \mid da)$$

$$H_{B_1}^{A_1} = -\,\Big\{\text{the above}\Big\} + 2\,(eb \mid ba) + \frac{1}{2}\,(ec \mid ca) + \frac{1}{2}\,(ed \mid da)$$

$A = abcdee$
$B = aecdbb$

$$H_{B_2}^{A_2} = -\,\Big\{(e \mid h \mid b) + (eb \mid aa) + (eb \mid bb) + (eb \mid cc) + (eb \mid dd) + (eb \mid ee) + 2\,(eb \mid P_a P_a) - (P_a b \mid eP_a)\Big\} + \frac{3}{2}\,(ec \mid cb) + \frac{3}{2}\,(ed \mid db)$$

$$H_{B_1}^{A_2} = H_{B_2}^{A_1} = -\,\frac{\sqrt{3}}{2}\,(ec \mid cb) + \frac{\sqrt{3}}{2}\,(ed \mid db)$$

$$H_{B_1}^{A_1} = -\,\Big\{\text{the above}\Big\} + 2\,(ea \mid ab) + \frac{1}{2}\,(ec \mid cb) + \frac{1}{2}\,(ed \mid db)$$

$A = abcdee$
$B = acefbb$

$$H_{B_2}^{A_2} = -\,\frac{1}{2}\,(df \mid eb)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\,(df \mid eb) - \sqrt{3}\,(ef \mid db)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\,(df \mid eb)$$

$$H_{B_1}^{A_1} = \frac{1}{2}\,(df \mid eb) - (ef \mid db)$$

$A = abcdee$
$B = aebfcc$

$$H_{B_2}^{A_2} = -\,\frac{1}{2}\,(df \mid ec)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\,(df \mid ec)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\,(df \mid ec) - \sqrt{3}\,(ef \mid dc)$$

$$H_{B_1}^{A_1} = \frac{1}{2}\,(df \mid ec) - (ef \mid dc)$$

$A = abcdee$
$B = acedbb$

$$H_{B_2}^{A_2} = -\,\frac{1}{2}\,\Big\{(e \mid h \mid b) + (eb \mid aa) + (eb \mid bb) + (eb \mid cc) + (eb \mid dd) + (eb \mid ee) + 2\,(eb \mid P_a P_a) - (P_a b \mid eP_a)\Big\} + \frac{3}{2}\,(ec \mid cb)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\,\Big\{\text{the above}\Big\} - \frac{\sqrt{3}}{2}\,(ec \mid cb) - \sqrt{3}\,(ed \mid db)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\,\Big\{\text{the above}\Big\} - \sqrt{3}\,(ea \mid ab) - \frac{\sqrt{3}}{2}\,(ec \mid cb)$$

$$H_{B_1}^{A_1} = \frac{1}{2}\,\Big\{\text{the above}\Big\} - (ea \mid ab) + \frac{1}{2}\,(ec \mid cb) - (ed \mid db)$$

$A = abcd$
$B = abce$

$$H_{B_1}^{A_1} = \Big\{(d \mid h \mid e) + (de \mid aa) + (de \mid bb) + (de \mid cc) + 2\,(de \mid P_a P_a) - (P_a e \mid dP_a)\Big\} - \frac{1}{2}\,(da \mid ae) - \frac{1}{2}\,(db \mid be) + (dc \mid ce)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = -\,\frac{\sqrt{3}}{2}\,(da \mid ae) + \frac{\sqrt{3}}{2}\,(db \mid be)$$

$$H_{B_2}^{A_2} = \Big\{\text{the above}\Big\} + \frac{1}{2}\,(da \mid ae) + \frac{1}{2}\,(db \mid be) - (dc \mid ce)$$

$A = abcdee$
$B = fecdaa$

$$H_{B_1}^{A_1} = -\,(bf \mid ea) + 2\,(ef \mid ba)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$$H_{B_2}^{A_2} = (bf \mid ea)$$

Singlet 2×2

$A = abcdee$
$B = fecdbb$

$$H_{B_1}^{A_1} = -(af \mid eb) + 2(ef \mid ab)$$
$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$
$$H_{B_2}^{A_2} = -(af \mid eb)$$

$A = abcdee$
$B = efcdaa$

$$H_{B_1}^{A_1} = -(bf \mid ea) + 2(ef \mid ba)$$
$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$
$$H_{B_2}^{A_2} = -(bf \mid ea)$$

$A = abcdee$
$B = acfedd$

$$H_{B_1}^{A_1} = \frac{1}{2}(bf \mid ed) - (ef \mid bd)$$
$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(bf \mid ed)$$
$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(bf \mid ed) - \sqrt{3}(ef \mid bd)$$
$$H_{B_2}^{A_2} = -\frac{1}{2}(bf \mid ed)$$

$A = abcdee$
$B = bdfecc$

$$H_{B_1}^{A_1} = \frac{1}{2}(af \mid ec) - (ef \mid ac)$$
$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(af \mid ec)$$
$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(af \mid ec) - \sqrt{3}(ef \mid ac)$$
$$H_{B_2}^{A_2} = -\frac{1}{2}(af \mid ec)$$

$A = abcdee$
$B = acfebb$

$$H_{B_1}^{A_1} = \frac{1}{2}(df \mid eb) - (ef \mid db)$$
$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(df \mid eb)$$
$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(df \mid eb) - \sqrt{3}(ef \mid db)$$
$$H_{B_2}^{A_2} = \frac{1}{2}(df \mid eb)$$

$A = abcdee$
$B = bdfeaa$

$$H_{B_1}^{A_1} = \frac{1}{2}(cf \mid ea) - (ef \mid ca)$$
$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(cf \mid ea)$$
$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(cf \mid ea) - \sqrt{3}(ef \mid ca)$$
$$H_{B_2}^{A_2} = \frac{1}{2}(cf \mid ea)$$

$A = abcdee$
$B = abcdff$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = (ef \mid ef)$$
$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdee$
$B = afbecc$

$$H_{B_1}^{A_1} = \frac{1}{2}(df \mid ec) - (ef \mid dc)$$
$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(df \mid ec) - \sqrt{3}(ef \mid dc)$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(df\mid ec)$$

$$H_{B_2}^{A_2} = \frac{1}{2}(df\mid ec)$$

$A = abcdee$
$B = faebcc$

$$H_{B_1}^{A_1} = \frac{1}{2}(df\mid ec) - (ef\mid dc)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(df\mid ec) - \sqrt{3}\,(ef\mid dc)$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(df\mid ec)$$

$$H_{B_2}^{A_2} = \frac{1}{2}(df\mid ec)$$

$A = abcdee$
$B = eafbcc$

$$H_{B_1}^{A_1} = \frac{1}{2}(df\mid ec) - (ef\mid dc)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(df\mid ec) - \sqrt{3}\,(ef\mid dc)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(df\mid ec)$$

$$H_{B_2}^{A_2} = -\frac{1}{2}(df\mid ec)$$

5. $N_A = 4, \quad N_B = 6 \quad (k_A = 2, \quad k_B = 5)$

$A = abcdee$
$B = eacdfb$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[-\frac{1}{2}\{(e\mid h\mid f) + (ef\mid aa) + (ef\mid bb) + (ef\mid cc) + (ef\mid dd) + (ef\mid ee) + 2\,(ef\mid P_a P_a) - (P_a f\mid eP_a)\} + (ea\mid af) + \frac{1}{4}(ec\mid cf) - \frac{1}{2}(eb\mid bf) + \frac{1}{4}(ed\mid df)\Big]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4}\Big[(ec\mid cf) - (ed\mid df)\Big]$$

$$H_{B_3}^{A_1} = \sqrt{2}\Big[-\frac{1}{\sqrt{2}}\{\text{the above}\} + \frac{1}{\sqrt{2}}(eb\mid bf) + \frac{1}{\sqrt{2}}(ed\mid df)\Big]$$

$$H_{B_4}^{A_1} = \sqrt{2}\Big[\frac{1}{2}\{\text{the above}\} - \frac{3}{4}(ec\mid cf) - \frac{1}{2}(eb\mid bf) + \frac{1}{4}(ed\mid df)\Big]$$

$$H_{B_5}^{A_1} = \sqrt{2}\Big[-\frac{\sqrt{3}}{4}(ec\mid cf) + \frac{\sqrt{3}}{4}(ed\mid df)\Big] = H_{B_1}^{A_2}$$

$$H_{B_2}^{A_2} = \sqrt{2}\Big[\frac{1}{2}\{\text{the above}\} - \frac{3}{4}(ec\mid cf) + \frac{1}{2}(eb\mid bf) - \frac{3}{4}(ed\mid df)\Big]$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{3}}\{\text{the above}\} - (eb\mid bf) - (ed\mid df)\Big]$$

$$H_{B_4}^{A_2} = \sqrt{2}\Big[\frac{\sqrt{3}}{3}\{\text{the above}\} - \frac{\sqrt{3}}{4}(ec\mid cf) - \frac{\sqrt{3}}{3}(eb\mid bf) - \frac{\sqrt{3}}{12}(ed\mid df)\Big]$$

$$H_{B_5}^{A_2} = \sqrt{2}\Big[-\frac{1}{2}\{\text{the above}\} + (ea\mid af) - \frac{1}{4}(ec\mid cf) + \frac{1}{2}(eb\mid bf) - \frac{1}{4}(ed\mid df)\Big]$$

$A = abcdee$
$B = abefcd$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[\{(e\mid h\mid f) + (ef\mid aa) + (ef\mid bb) + (ef\mid cc) + (ef\mid dd) + (ef\mid ee) + 2\,(ef\mid P_a P_a) - (P_a f\mid eP_a)\} - \frac{1}{2}(ea\mid af) - \frac{1}{2}(eb\mid bf) - \frac{1}{2}(ec\mid cf) - \frac{1}{2}(ed\mid df)\Big]$$

$$H_{B_2}^{A_1} = \sqrt{\frac{3}{2}}\Big[-(ea\mid af) + (eb\mid bf)\Big]$$

Singlet 2×5

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = 0 = H_{B_1}^{A_2}$$

$$H_{B_5}^{A_1} = \sqrt{\frac{3}{2}}\left[-(ec \mid cf) + (ed \mid df)\right]$$

$$H_{B_2}^{A_2} = \frac{1}{\sqrt{2}}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H_{B_3}^{A_2} = \frac{2}{\sqrt{3}}\left[\{\text{the above}\} - (ec \mid cf) - (ed \mid df)\right]$$

$$H_{B_4}^{A_2} = \sqrt{2}\left[-\frac{\sqrt{3}}{3}\{\text{the above}\} + \frac{\sqrt{3}}{2}(ea \mid af) + \frac{\sqrt{3}}{2}(eb \mid bf) - \frac{\sqrt{3}}{6}(ec \mid cf) - \frac{\sqrt{3}}{6}(ed \mid df)\right]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{2}}\left[(ea \mid af) - (eb \mid bf)\right]$$

$A = abcdee$
$B = facdeb$

$$H_{B_1}^{A_1} = \sqrt{2}\left[-\frac{1}{2}\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} - \frac{1}{2}(ea \mid af) + (eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$$H_{B_2}^{A_1} = - H_{B_5}^{A_1} = - H_{B_1}^{A_2} = \frac{\sqrt{6}}{4}\left[-(ec \mid cf) + (ed \mid df)\right]$$

$$H_{B_3}^{A_1} = -\{\text{the above}\} + (ea \mid af) + (ec \mid cf)$$

$$H_{B_4}^{A_1} \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(ea \mid af) + \frac{1}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\right]$$

$$H_{B_2}^{A_2} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(ea \mid af) - (eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{3}}\left[\{\text{th eabove}\} - (ea \mid af) - (ec \mid cf)\right]$$

$$H_{B_4}^{A_2} = \sqrt{2}\left[\frac{\sqrt{3}}{3}\{\text{the above}\} - \frac{\sqrt{3}}{3}(ea \mid af) - \frac{\sqrt{3}}{12}(ec \mid cf) - \frac{\sqrt{3}}{4}(ed \mid df)\right]$$

$$H_{B_5}^{A_2} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(ca \mid af) + \frac{3}{4}(ec \mid cf) + \frac{3}{4}(ed \mid df)\right]$$

$A = abcdee$
$B = aecdbf$

$$H_{B_1}^{A_1} = \sqrt{2}\left[-\frac{1}{2}\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} + (ea \mid af) - \frac{1}{2}(eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$$H_{B_2}^{A_1} = - H_{B_5}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{6}}{4}\left[-(ec \mid cf) + (ed \mid df)\right]$$

$$H_{B_3}^{A_1} = -\{\text{the above}\} + (eb \mid bf) + (ed \mid df)$$

$$H_{B_4}^{A_1} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(eb \mid bf) - \frac{3}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$$H_{B_2}^{A_2} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(eb \mid bf) + \frac{3}{4}(ec \mid cf) + \frac{3}{4}(ed \mid df)\right]$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{3}}\left[\{\text{the above}\} - (eb \mid bf) - (ed \mid df)\right]$$

Singlet 2×5

$$H^{A_2}_{B_4} = \sqrt{2}\left[\frac{\sqrt{3}}{3}\{\text{the above}\} - \frac{\sqrt{3}}{3}(eb \mid bf) - \frac{\sqrt{3}}{4}(ec \mid cf) - \frac{\sqrt{3}}{12}(ed \mid df)\right]$$

$$H^{A_2}_{B_5} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - (ea \mid af) - \frac{1}{2}(eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$A = abcdee$
$B = abfecd$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} - \frac{1}{2}(ea \mid af) - \frac{1}{2}(eb \mid bf) - \frac{1}{2}(ec \mid cf) - \frac{1}{2}(ed \mid df)\right]$$

$$H^{A_1}_{B_2} = \sqrt{\frac{3}{2}}\left[(ea \mid af) - (eb \mid bf)\right]$$

$$H^{A_1}_{B_3} = H^{A_1}_{B_4} = 0 = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_5} = \sqrt{\frac{3}{2}}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H^{A_2}_{B_2} = \frac{1}{\sqrt{2}}\left[-(ec \mid cf) + (ed \mid df)\right]$$

$$H^{A_2}_{B_3} = \frac{2}{\sqrt{3}}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{2}\left[-\frac{\sqrt{3}}{3}\{\text{the above}\} - \frac{\sqrt{3}}{6}(ea \mid af) - \frac{\sqrt{3}}{6}(eb \mid bf) + \frac{\sqrt{3}}{2}(ec \mid cf) + \frac{\sqrt{3}}{2}(ed \mid df)\right]$$

$$H^{A_2}_{B_5} = \frac{1}{\sqrt{2}}\left[-(ea \mid af) + (eb \mid bf)\right]$$

$A = abcdee$
$B = afcdbe$

$$H^{A_1}_{B_1} = \sqrt{2}\left[-\frac{1}{2}\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} - \frac{1}{2}(ea \mid af) + (eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\right]$$

$$H^{A_1}_{B_2} = -H^{A_1}_{B_5} = H^{A_2}_{B_1} = \frac{\sqrt{6}}{4}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H^{A_1}_{B_3} = -\{\text{the above}\} + (ea \mid af) - (ec \mid cf)$$

$$H^{A_1}_{B_4} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - \frac{1}{2}(ea \mid af) + \frac{1}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_2} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} + \frac{1}{2}(ea \mid af) + (eb \mid bf) - \frac{1}{4}(ec \mid cf) - \frac{1}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_3} = \frac{1}{\sqrt{3}}\left[\{\text{the above}\} - (ea \mid af) - (ec \mid cf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{2}\left[\frac{\sqrt{3}}{3}\{\text{the above}\} - \frac{\sqrt{3}}{3}(ea \mid af) - \frac{\sqrt{3}}{12}(ec \mid cf) - \frac{\sqrt{3}}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_5} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} + \frac{1}{2}(ea \mid af) - \frac{3}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\right]$$

6. $N_A = 6$, $N_B = 6$ $(k_A = 5,\ k_B = 5)$ (Omitted) **Singlet 2×5**

II. Formulae of configuration interaction matrix elements for Doublet state wave function $(S=\frac{1}{2})$

1. $N_A = 1,\quad N_B = 1\quad (k_A = 1,\quad k_B = 1)$

$A = abb$, $B = daa$: $H^A_B = -(bd \mid ba)$

$A = abb$, $B = bdd$: $H^A_B = -(ad \mid bd)$

$A = abb$, $B = baa$: $H^A_B = -\left\{(b \mid h \mid a) + (ba \mid aa) + (ba \mid bb) + 2\,(ba \mid P_a P_a) - (P_a a \mid bP_a)\right\}$

$A = abb$, $B = add$: $H^A_B = (bd \mid bd)$

$A = abb$, $B = dbb$: $H^A_B = (a \mid h \mid d) + 2\,(ad \mid P_a P_a) - (P_a d \mid aP_a)$

2. $N_A = 1,\quad N_B = 3\quad (k_A = 1,\quad k_B = 2)$

$A = abb$, $B = abd$:

$$H^A_{B_1} = \sqrt{2}\left[\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(ba \mid P_a P_a) - (P_a d \mid bP_a)\} - \frac{1}{2}(ba \mid ad)\right]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\,(ba \mid ad)$$

$A = abb$, $B = bde$:

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(ad \mid be) + (bd \mid ae)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[(ad \mid be) - (bd \mid ae)\right]$$

$A = abb$, $B = adb$:

$$H^A_{B_1} = \sqrt{2}\left[\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(bd \mid P_a P_a) - (P_a d \mid bP_a)\} - \frac{1}{2}(ba \mid ad)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\,(ba \mid ad)$$

$A = abb$, $B = ade$:

$$H^A_{B_1} = \sqrt{2}\,(bd \mid be)$$

$$H^A_{B_2} = 0$$

$A = abb$, $B = deb$:

$$H^A_{B_1} = \sqrt{2}\left[(ad \mid be) - \frac{1}{2}(bd \mid ae)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\,(bd \mid ae)$$

$A = abb$, $B = dae$:

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\,(bd \mid be)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\,(bd \mid be)$$

$A = abb$, $B = dbe$:

$$H^A_{B_1} = \sqrt{2}\left[(ad \mid be) - \frac{1}{2}(bd \mid ae)\right]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\,(bd \mid ae)$$

$A = abbcc$, $B = dbcaa$:

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(cd \mid ba) + (bd \mid ca)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[-(cd \mid ba) + (bd \mid ca)\right]$$

$A = abbcc$
$B = bdcaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(bd \mid ca) - 2\,(cd \mid ba)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\,(bd \mid ca)$$

$A = abb$
$B = dab$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(bd \mid P_aP_a) - (P_ad \mid bP_a)\} + 2\,(ba \mid ad)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[\{\text{the above}\}\Big]$$

$A = abb$
$B = dba$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(bd \mid P_aP_a) - (P_ad \mid bP_a)\} + 2\,(ba \mid ad)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[-\{\text{the above}\}\Big]$$

$A = abb$
$B = bad$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(bd \mid P_aP_a) - (P_ad \mid bP_a)\} + (bc \mid cd)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[\{\text{the above}\} - (ba \mid ad)\Big]$$

$A = abbcc$
$B = bcdaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(bd \mid ca) - 2\,(cd \mid ba)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\,(bd \mid ca)$$

$A = abbcc$
$B = abcdd$

$$H^A_{B_1} = -\sqrt{2}\,(bd \mid cd)$$

$$H^A_{B_2} = 0$$

$A = abbcc$
$B = bcadd$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(bd \mid cd)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\,(bd \mid cd)$$

$A = abb$
$B = dea$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\,(bd \mid be)$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\,(bd \mid be)$$

$A = abb$
$B = bde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ad \mid be) + (bd \mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[(ad \mid be) - (bd \mid ae)\Big]$$

$A = abb$
$B = bda$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(b \mid h \mid d) + (bd \mid aa) + (bd \mid bb) + 2\,(bd \mid P_aP_a) - (P_ad \mid bP_a)\} - (ba \mid ad)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[-\{\text{the above}\} + (ba \mid ad)\Big]$$

Doublet 1×2

$A = abbcc$
$B = bacdd$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (bd \mid cd)$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}} (bd \mid cd)$$

$A = abbcc$
$B = dcabb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} \Big[- \{(c \mid h \mid d) + (cd \mid aa) + 2\,(cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + 2\,(ca \mid ad) + (cb \mid bd) \Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}} \Big[- \{\text{the above}\} + (cb \mid bd) \Big]$$

3. $N_A = 1$, $N_B = 5$ $(k_A = 1,\quad k_B = 5)$

$A = abbcc$
$B = deabc$

$$H^A_{B_1} = \frac{1}{2} (bd \mid ce) + \frac{1}{2} (cd \mid be)$$

$$H^A_{B_2} = \frac{\sqrt{3}}{2} \Big[(bd \mid ce) + (cd \mid be) \Big]$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2} \Big[(bd \mid ce) - (cd \mid be) \Big]$$

$$H^A_{B_4} = \frac{1}{2} \Big[(bd \mid ce) - (cd \mid be) \Big]$$

$$H^A_{B_5} = \sqrt{2} \Big[(bd \mid ce) - (cd \mid be) \Big]$$

$A = abbcc$
$B = abcde$

$$H^A_{B_1} = - (bd \mid ce) - (cd \mid be)$$

$$H^A_{B_2} = H^A_{B_4} = H^A_{B_5} = 0$$

$$H^A_{B_3} = \sqrt{3} \Big[(bd \mid ce) - (cd \mid be) \Big]$$

$A = abbcc$
$B = dabec$

$$H^A_{B_1} = - (bd \mid ce) + \frac{1}{2} (cd \mid be)$$

$$H^A_{B_2} = \sqrt{3}\, (bd \mid ce) - \frac{\sqrt{3}}{2} (cd \mid be)$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2} (cd \mid be)$$

$$H^A_{B_4} = \frac{3}{2} (cd \mid be)$$

$$H^A_{B_5} = 0$$

$A = abbcc$
$B = badec$

$$H^A_{B_1} = - (bd \mid ce) + \frac{1}{2} (cd \mid be)$$

$$H^A_{B_2} = \sqrt{3}\, (bd \mid ce) - \frac{\sqrt{3}}{2} (cd \mid be)$$

$$H^A_{B_3} = - \frac{\sqrt{3}}{2} (cd \mid be)$$

$$H^A_{B_4} = - \frac{3}{2} (cd \mid be)$$

$$H^A_{B_5} = 0$$

$A = abbcc$
$B = dabce$

$$H^A_{B_1} = - (bd \mid ce) + \frac{1}{2} (cd \mid be)$$

$$H^A_{B_2} = \sqrt{3}\, (bd \mid ce) - \frac{\sqrt{3}}{2} (cd \mid be)$$

$$H^{A}_{B_3} = -\frac{\sqrt{3}}{2}(cd \mid be)$$
$$H^{A}_{B_4} = -\frac{3}{2}(cd \mid be)$$
$$H^{A}_{B_5} = 0$$

$A = abbcc$
$B = dbaec$

$$H^{A}_{B_1} = -(bd \mid ce) + \frac{1}{2}(cd \mid be)$$
$$H^{A}_{B_2} = -\sqrt{3}\,(bd \mid ce) + \frac{\sqrt{3}}{2}(cd \mid be)$$
$$H^{A}_{B_3} = -\frac{\sqrt{3}}{2}(cd \mid be)$$
$$H^{A}_{B_4} = -\frac{1}{2}(cd \mid be)$$
$$H^{A}_{B_5} = -\sqrt{2}\,(cd \mid be)$$

$A = abbcc$
$B = daebc$

$$H^{A}_{B_1} = \frac{1}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_2} = -\frac{\sqrt{3}}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_3} = \frac{\sqrt{3}}{2}\left[-(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_4} = \frac{3}{2}\left[-(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_5} = 0$$

$A = abbcc$
$B = dbace$

$$H^{A}_{B_1} = -(bd \mid ce) + \frac{1}{2}(cd \mid be)$$
$$H^{A}_{B_2} = -\sqrt{3}\,(bd \mid ce) + \frac{\sqrt{3}}{2}(cd \mid be)$$
$$H^{A}_{B_3} = \frac{\sqrt{3}}{2}(cd \mid be)$$
$$H^{A}_{B_4} = \frac{1}{2}(cd \mid be)$$
$$H^{A}_{B_5} = \sqrt{2}\,(cd \mid be)$$

$A = abbcc$
$B = abdec$

$$H^{A}_{B_1} = 2\,(bd \mid ce) - (cd \mid be)$$
$$H^{A}_{B_2} = H^{A}_{B_4} = H^{A}_{B_5} = 0$$
$$H^{A}_{B_3} = \sqrt{3}\,(cd \mid be)$$

$A = abbcc$
$B = adbce$

$$H^{A}_{B_1} = 2\,(bd \mid ce) - (cd \mid be)$$
$$H^{A}_{B_2} = H^{A}_{B_4} = H^{A}_{B_5} = 0$$
$$H^{A}_{B_3} = \sqrt{3}\,(cd \mid be)$$

$A = abbcc$
$B = bcade$

$$H^{A}_{B_1} = \frac{1}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_2} = \frac{\sqrt{3}}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$
$$H^{A}_{B_3} = \frac{\sqrt{3}}{2}\left[(bd \mid ce) - (cd \mid be)\right]$$

$$H^A_{B_4} = \frac{1}{2}\left[(bd \mid ce) - (cd \mid be)\right]$$

$$H^A_{B_5} = \sqrt{2}\left[(bd \mid ce) - (cd \mid be)\right]$$

$A = abbcc$
$B = badce$

$$H^A_{B_1} = -(bd \mid ce) + \frac{1}{2}(cd \mid be)$$

$$H^A_{B_2} = \sqrt{3}(bd \mid ce) - \frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^A_{B_4} = \frac{3}{2}(cd \mid be)$$

$$H^A_{B_5} = 0$$

$A = abbcc$
$B = adebc$

$$H^A_{B_1} = -(bd \mid ce) - (cd \mid be)$$

$$H^A_{B_2} = H^A_{B_4} = H^A_{B_5} = 0$$

$$H^A_{B_3} = \sqrt{3}(bd \mid ce) - \sqrt{3}(cd \mid be)$$

$A = abbcc$
$B = dbcea$

$$H^A_{B_1} = \frac{1}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^A_{B_2} = \frac{\sqrt{3}}{2}\left[(bd \mid ce) - (cd \mid be)\right] = H^A_{B_3}$$

$$H^A_{B_4} = -\frac{3}{2}(bd \mid ce) + \frac{1}{2}(cd \mid be)$$

$$H^A_{B_5} = \sqrt{2}(cd \mid be)$$

$A = abbcc$
$B = bcdea$

$$H^A_{B_1} = \frac{1}{2}(bd \mid ce) - (cd \mid be)$$

$$H^A_{B_2} = H^A_{B_3} = -\frac{\sqrt{3}}{2}(bd \mid ce)$$

$$H^A_{B_4} = \frac{1}{2}(bd \mid ce) + (cd \mid be)$$

$$H^A_{B_5} = \sqrt{2}\left[(bd \mid ce) - (cd \mid be)\right]$$

$A = abbcc$
$B = cbeda$

$$H^A_{B_1} = -(be \mid cd) + \frac{1}{2}(ce \mid bd)$$

$$H^A_{B_2} = H^A_{B_3} = -\frac{\sqrt{3}}{2}(ce \mid bd)$$

$$H^A_{B_4} = (be \mid cd) + \frac{1}{2}(ce \mid bd)$$

$$H^A_{B_5} = \sqrt{2}\left[-(be \mid cd) + (ce \mid bd)\right]$$

$A = abbcc$
$B = debca$

$$H^A_{B_1} = -(cd \mid be) + \frac{1}{2}(bd \mid ce)$$

$$H^A_{B_2} = H^A_{B_3} = -\frac{\sqrt{3}}{2}(bd \mid ce)$$

$$H^A_{B_4} = (cd \mid be) + \frac{1}{2}(bd \mid ce)$$

$$H^A_{B_5} = \sqrt{2}\left[-(cd \mid be) + (bd \mid ce)\right]$$

$A = abbcc$
$B = edcba$

$$H^A_{B_1} = \frac{1}{2}(ce \mid bd) - (be \mid cd)$$

$$H^{A}_{B_2} = H^{A}_{B_3} = -\frac{\sqrt{3}}{2}(ce \mid bd)$$

$$H^{A}_{B_4} = \frac{1}{2}(ce \mid bd) + (be \mid cd)$$

$$H^{A}_{B_5} = \sqrt{2}\left[(ce \mid bd) - (be \mid cd)\right]$$

$A = abbcc$
$B = bdeca$

$$H^{A}_{B_1} = \frac{1}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_2} = H^{A}_{B_3} = \frac{\sqrt{3}}{2}\left[(bd \mid ce) - (cd \mid be)\right]$$

$$H^{A}_{B_4} = -\frac{3}{2}(bd \mid ce) + \frac{1}{2}(cd \mid be)$$

$$H^{A}_{B_5} = \sqrt{2}(cd \mid be)$$

$A = abbcc$
$B = bdaec$

$$H^{A}_{B_1} = -(bd \mid ce) + \frac{1}{2}(cd \mid be)$$

$$H^{A}_{B_2} = -\sqrt{3}(bd \mid ce) + \frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^{A}_{B_3} = \frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^{A}_{B_4} = \frac{1}{2}(cd \mid be)$$

$$H^{A}_{B_5} = \sqrt{2}(cd \mid be)$$

$A = abbcc$
$B = bacde$

$$H^{A}_{B_1} = -\frac{1}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_2} = -\frac{\sqrt{3}}{2}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_3} = \frac{\sqrt{3}}{2}\left[-(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_4} = \frac{3}{2}\left[-(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_5} = 0$$

$A = abbcc$
$B = bdace$

$$H^{A}_{B_1} = -(bd \mid ce) + \frac{1}{2}(cd \mid be)$$

$$H^{A}_{B_2} = -\sqrt{3}(bd \mid ce) + \frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^{A}_{B_3} = -\frac{\sqrt{3}}{2}(cd \mid be)$$

$$H^{A}_{B_4} = -\frac{1}{2}(cd \mid be)$$

$$H^{A}_{B_5} = -\sqrt{2}(cd \mid be)$$

4. $N_A = 3, \quad N_B = 3 \quad (k_A = 2, \quad k_B = 2)$

$A = abcdd$
$B = bedcc$

$$H^{A_1}_{B_1} = \frac{1}{2}(ae \mid dc) - (de \mid ac)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}(ae \mid dc)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(ae \mid dc) + \sqrt{3}(de \mid ac)$$

$$H^{A_2}_{B_2} = -\frac{1}{2}(ae \mid dc)$$

Doublet 2×2

$A = abc$
$B = aef$

$$H^{A_1}_{B_1} = (be \mid cf) + (ce \mid bf)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$$H^{A_2}_{B_2} = (be \mid cf) - (ce \mid bf)$$

$A = abcdd$
$B = dceaa$

$$H^{A_1}_{B_1} = -(be \mid da) + \frac{1}{2}(de \mid ba)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(de \mid ba)$$

$$H^{A_2}_{B_2} = (be \mid da) - \frac{3}{2}(de \mid ba)$$

$A = abcdd$
$B = aedcc$

$$H^{A_1}_{B_1} = -(be \mid dc) + 2(de \mid bc)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$$H^{A_2}_{B_2} = -(be \mid dc)$$

$A = abcdd$
$B = debcc$

$$H^{A_1}_{B_1} = \frac{1}{2}\Big[(ae \mid dc) + (ac \mid de)\Big]$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[-(ae \mid dc) + (ac \mid de)\Big]$$

$$H^{A_2}_{B_2} = \frac{1}{2}(ae \mid dc) - \frac{3}{2}(ac \mid de)$$

$A = abc$
$B = aec$

$$H^{A_1}_{B_1} = \Big\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + 2(be \mid P_a P_a) - (P_a e \mid bP_a)\Big\} - \frac{1}{2}(ba \mid ae) + (bc \mid ce)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(ba \mid ae)$$

$$H^{A_2}_{B_2} = \Big\{\text{the above}\Big\} + \frac{1}{2}(ba \mid ae) - (bc \mid ce)$$

$A = abc$
$B = aeb$

$$H^{A_1}_{B_1} = \Big\{(c \mid h \mid e) + (ce \mid aa) + (ce \mid bb) + 2(ce \mid P_a P_a) - (P_a e \mid cP_a)\Big\} - \frac{1}{2}(ca \mid ae) + (cb \mid be)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(ca \mid ae)$$

$$H^{A_2}_{B_2} = -\Big\{\text{the above}\Big\} - \frac{1}{2}(ca \mid ae) + (cb \mid be)$$

$A = abcdd$
$B = dbcaa$

$$H^{A_1}_{B_1} = -\Big\{(d \mid h \mid a) + (da \mid aa) + (da \mid bb) + (da \mid cc) + (da \mid dd) + 2(da \mid P_a P_a) - (P_a a \mid dP_a)\Big\} + \frac{1}{2}(db \mid ba) + \frac{1}{2}(dc \mid ca)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[(db \mid ba) - (dc \mid ca)\Big]$$

$$H^{A_2}_{B_2} = -\Big\{\text{the above}\Big\} + \frac{3}{2}(db \mid ba) + \frac{3}{2}(dc \mid ca)$$

$A = abcdd$
$B = edcaa$

$$H^{A_1}_{B_1} = \frac{1}{2}\Big[(de \mid ba) + (be \mid da)\Big]$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[(de \mid ba) - (be \mid da)\Big]$$

$$H^{A_2}_{B_2} = \frac{3}{2}(de \mid ba) - \frac{1}{2}(be \mid da)$$

$A = abcdd$
$B = decaa$

$$H^{A_1}_{B_1} = -(be \mid da) + \frac{1}{2}(de \mid ba)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(de \mid ba)$$

$$H_{B_2}^{A_2} = -(be \mid da) + \frac{3}{2}(de \mid ba)$$

$A = abcdd$
$B = debaa$

$$H_{B_1}^{A_1} = -(ce \mid da) + \frac{1}{2}(de \mid ca)$$

$$H_{B_2}^{A_1} = -H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(de \mid ca)$$

$$H_{B_2}^{A_2} = (ce \mid da) - \frac{3}{2}(de \mid ca)$$

$A = abcddee$
$B = adebbcc$

$$H_{B_1}^{A_1} = (db \mid ec) + (dc \mid eb)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$$H_{B_2}^{A_2} = (db \mid ec) - (dc \mid eb)$$

$A = abcdd$
$B = adebb$

$$H_{B_1}^{A_1} = -(ce \mid db) + 2(de \mid cb)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$$H_{B_2}^{A_2} = -(ce \mid db)$$

$A = abcdd$
$B = bdeaa$

$$H_{B_1}^{A_1} = \frac{1}{2}(ce \mid da) - (de \mid ca)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(ce \mid da)$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(ce \mid da) + \sqrt{3}(de \mid ca)$$

$$H_{B_2}^{A_2} = -\frac{1}{2}(ce \mid da)$$

$A = abcdd$
$B = dbeaa$

$$H_{B_1}^{A_1} = -(ce \mid da) + \frac{1}{2}(de \mid ca)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(de \mid ca)$$

$$H_{B_2}^{A_2} = -(ce \mid da) + \frac{3}{2}(de \mid ca)$$

$A = abcdd$
$B = becaa$

$$H_{B_1}^{A_1} = \frac{1}{2}(de \mid da) = -H_{B_2}^{A_2}$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(de \mid da)$$

$A = abcddee$
$B = decaabb$

$$H_{B_1}^{A_1} = (da \mid eb) - \frac{1}{2}(ea \mid db)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(ea \mid db)$$

$$H_{B_2}^{A_2} = (da \mid eb) + \frac{1}{2}(ea \mid db)$$

$A = abcdd$
$B = dbcee$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = -(ae \mid de)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdd$
$B = cedbb$

$$H_{B_1}^{A_1} = -(de \mid ab) + \frac{1}{2}(ae \mid db)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(ae \mid db)$$

$$H^{A_2}_{B_1} = -\sqrt{3}\ (de \mid ab) + \frac{\sqrt{3}}{2}\ (ae \mid db)$$

$$H^{A_2}_{B_2} = \frac{1}{2}\ (ae \mid db)$$

$A = abcdd$
$B = bcdee$

$$H^{A_1}_{B_1} = H^{A_2}_{B_2} = \frac{1}{2}\ (ae \mid de)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\ (ae \mid de)$$

$A = abcdd$
$B = bedcc$

$$H^{A_1}_{B_1} = \frac{1}{2}\ (ae \mid dc) - (de \mid ac)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}\ (ae \mid dc)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\ (ae \mid dc) + \sqrt{3}\ (de \mid ac)$$

$$H^{A_2}_{B_2} = -\frac{1}{2}\ (ae \mid dc)$$

$A = abcddee$
$B = cdeaabb$

$$H^{A_1}_{B_1} = -\frac{1}{2}\left[(da \mid eb) + (db \mid ea)\right]$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\left[(da \mid eb) - (db \mid ea)\right]$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\left[(da \mid eb) + (db \mid ea)\right]$$

$$H^{A_2}_{B_2} = \frac{1}{2}\left[-(da \mid eb) + (db \mid ea)\right]$$

$A = abcddee$
$B = dbeaacc$

$$H^{A_1}_{B_1} = (da \mid ec) - \frac{1}{2}\ (dc \mid ea)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\ (dc \mid ea)$$

$$H^{A_2}_{B_2} = (da \mid ec) + \frac{1}{2}\ (dc \mid ea)$$

$A = abcddee$
$B = deabbcc$

$$H^{A_1}_{B_1} = -\frac{1}{2}\ (db \mid ec) - \frac{1}{2}\ (dc \mid eb)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}\left[(db \mid ec) + (dc \mid eb)\right]$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\left[(db \mid ec) - (dc \mid eb)\right]$$

$$H^{A_2}_{B_2} = -\frac{1}{2}\ (db \mid ec) + \frac{1}{2}\ (dc \mid eb)$$

$A = abcddee$
$B = dceaabb$

$$H^{A_1}_{B_1} = (da \mid eb) - \frac{1}{2}\ (db \mid ea)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\ (db \mid ea)$$

$$H^{A_2}_{B_2} = -(da \mid eb) - \frac{1}{2}\ (db \mid ea)$$

$A = abcdd$
$B = daebb$

$$H^{A_1}_{B_1} = \frac{1}{2}\ (ce \mid db) - (de \mid cb)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}\ (ce \mid db) + \sqrt{3}\ (de \mid cb)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}\ (ce \mid db)$$

$$H^{A_2}_{B_2} = -\frac{1}{2}\ (ce \mid db)$$

$A = abcdd$
$B = deabb$

$$H_{B_1}^{A_1} = \frac{1}{2}(ce \mid db) - (de \mid cb)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(ce \mid db) - \sqrt{3}\,(de \mid cb)$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(ce \mid db)$$

$$H_{B_2}^{A_2} = \frac{1}{2}(ce \mid db)$$

$A = abcdd$
$B = aecbb$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = -(de \mid db)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdd$
$B = eadbb$

$$H_{B_1}^{A_1} = \frac{1}{2}(ce \mid db) - (de \mid cb)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(ce \mid db) + \sqrt{3}\,(de \mid cb)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(ce \mid db)$$

$$H_{B_2}^{A_2} = \frac{1}{2}(ce \mid db)$$

$A = abcdd$
$B = dbcaa$

$$H_{B_1}^{A_1} = \Big[-\{(d \mid h \mid a) + (da \mid aa) + (da \mid bb) + (da \mid cc) + (da \mid dd) + 2\,(da \mid P_a P_a) - (P_a a \mid dP_a)\} + \frac{1}{2}(db \mid ba) + \frac{1}{2}(dc \mid ca)\Big]$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\Big[(db \mid ba) - (dc \mid ca)\Big]$$

$$H_{B_2}^{A_2} = -\Big\{\text{the above}\Big\} + \frac{3}{2}(db \mid ba) + \frac{3}{2}(dc \mid ca)$$

$A = abcdd$
$B = daecc$

$$H_{B_1}^{A_1} = \frac{1}{2}(be \mid dc) - (de \mid bc)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(be \mid dc) + \sqrt{3}\,(de \mid bc)$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(be \mid dc)$$

$$H_{B_2}^{A_2} = \frac{1}{2}(be \mid dc)$$

$A = abcddee$
$B = bdeccaa$

$$H_{B_1}^{A_1} = -\frac{1}{2}\Big[(dc \mid ea) + (ec \mid da)\Big]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\Big[-(dc \mid ea) + (ec \mid da)\Big]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\Big[(dc \mid ea) + (ec \mid da)\Big]$$

$$H_{B_2}^{A_2} = \frac{1}{2}\Big[-(dc \mid ea) + (ec \mid da)\Big]$$

$A = abcddee$
$B = edcbbaa$

$$H_{B_1}^{A_1} = (db \mid ea) - \frac{1}{2}(eb \mid da)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}(eb \mid da)$$

$$H_{B_2}^{A_2} = (db \mid ea) + \frac{1}{2}(eb \mid da)$$

$A = abc$
$B = bae$

$$H_{B_1}^{A_1} = -\frac{1}{2}\Big\{(c \mid h \mid e) + (ce \mid aa) + (ce \mid bb) + 2\,(ce \mid P_a P_a) - (P_a e \mid cP_a)\Big\} - \frac{1}{2}(ca \mid ae) - \frac{1}{2}(cb \mid be)$$

Doublet 2×2

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\left\{\text{the above}\right\} - \frac{\sqrt{3}}{2}(ca \mid ae) + \frac{\sqrt{3}}{2}(cb \mid be)$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\left\{\text{the above}\right\} + \frac{\sqrt{3}}{2}(ca \mid ae) - \frac{\sqrt{3}}{2}(cb \mid be)$$

$$H^{A_2}_{B_2} = \frac{1}{2}\left\{\text{the above}\right\} - \frac{1}{2}(ca \mid ae) - \frac{1}{2}(cb \mid be)$$

$A = abcdd$
$B = adcbb$

$$H^{A_1}_{B_1} = -\Big\{(d \mid h \mid b) + (db \mid aa) + (db \mid bb) + (db \mid cc) + (db \mid dd) + 2(db \mid P_aP_a) - (P_a b \mid dP_a)\Big\} + \frac{1}{2}(da \mid ab) + 2(dc \mid cb)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(da \mid ab)$$

$$H^{A_2}_{B_2} = -\left\{\text{the above}\right\} + \frac{3}{2}(da \mid ab)$$

$A = abcdd$
$B = acdbb$

$$H^{A_1}_{B_1} = -\Big\{(d \mid h \mid b) + (db \mid aa) + (db \mid bb) + (db \mid cc) + (db \mid dd) + 2(db \mid P_aP_a) - (P_a b \mid dP_a)\Big\} + \frac{1}{2}(da \mid ab) + 2(dc \mid cb)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(da \mid ab)$$

$$H^{A_2}_{B_2} = \left\{\text{the above}\right\} - \frac{3}{2}(da \mid ab)$$

$A = abcdd$
$B = ecdbb$

$$H^{A_1}_{B_1} = -(ae \mid db) + \frac{1}{2}(de \mid ab)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(de \mid ab)$$

$$H^{A_2}_{B_2} = (ae \mid db) - \frac{3}{2}(de \mid ab)$$

$A = abcdd$
$B = edbcc$

$$H^{A_1}_{B_1} = -(ae \mid dc) + \frac{1}{2}(de \mid ac)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(de \mid ac)$$

$$H^{A_2}_{B_2} = (ae \mid dc) - \frac{3}{2}(de \mid ac)$$

$A = abcdd$
$B = edbaa$

$$H^{A_1}_{B_1} = \frac{1}{2}\Big[(de \mid ca) + (ce \mid da)\Big]$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[(de \mid ca) - (ce \mid da)\Big]$$

$$H^{A_2}_{B_2} = -\frac{3}{2}(de \mid ca) + \frac{1}{2}(ce \mid da)$$

$A = abcdd$
$B = adbcc$

$$H^{A_1}_{B_1} = -\Big\{(d \mid h \mid c) + (dc \mid aa) + (dc \mid bb) + (dc \mid cc) + (dc \mid dd) + 2(dc \mid P_aP_a) - (P_a c \mid dP_a)\Big\} + \frac{1}{2}(da \mid ac) + 2(db \mid bc)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(da \mid ac)$$

$$H^{A_2}_{B_2} = \left\{\text{the above}\right\} - \frac{3}{2}(da \mid ac)$$

$A = abcdd$
$B = aedbb$

$$H^{A_1}_{B_1} = 2(de \mid cb) - (ce \mid db)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$$H^{A_2}_{B_2} = (ce \mid db)$$

Doublet 2×2

$A = abcdd$
$B = acdee$

$$H_{B_1}^{A_1} = - H_{B_2}^{A_2} = - (be \mid de)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdd$
$B = adbee$

$$H_{B_1}^{A_1} = - H_{B_2}^{A_2} = - (ce \mid de)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdd$
$B = abcee$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = (de \mid de)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcdd$
$B = adcee$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = - (be \mid de)$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$A = abcddee$
$B = daebbcc$

$$H_{B_1}^{A_1} = - \frac{1}{2} \left[(db \mid ec) + (eb \mid dc) \right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2} \left[(db \mid ec) + (eb \mid dc) \right]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2} \left[(db \mid ec) - (eb \mid dc) \right]$$

$$H_{B_2}^{A_2} = \frac{1}{2} \left[(db \mid ec) - (eb \mid dc) \right]$$

$A = abcddee$
$B = bdeaacc$

$$H_{B_1}^{A_1} = - \frac{1}{2} \left[(da \mid ec) + (ea \mid dc) \right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2} \left[(da \mid ec) - (ea \mid dc) \right]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2} \left[(da \mid ec) + (ea \mid dc) \right]$$

$$H_{B_2}^{A_2} = \frac{1}{2} \left[(da \mid ec) - (ea \mid dc) \right]$$

$A = abcdd$
$B = bedaa$

$$H_{B_1}^{A_1} = \frac{1}{2} (ce \mid da) - (de \mid ca)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2} (ce \mid da)$$

$$H_{B_1}^{A_2} = - \frac{\sqrt{3}}{2} (ce \mid da) + \sqrt{3} (de \mid ca)$$

$$H_{B_2}^{A_2} = \frac{1}{2} (ce \mid da)$$

$A = abcdd$
$B = ecdaa$

$$H_{B_1}^{A_1} = \frac{1}{2} \left[(be \mid da) + (de \mid ba) \right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2} \left[(be \mid da) - (de \mid ba) \right]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2} \left[- (be \mid da) + (de \mid ba) \right]$$

$$H_{B_2}^{A_2} = \frac{1}{2} (be \mid da) - \frac{3}{2} (de \mid ba)$$

$A = abcdd$
$B = ebdaa$

$$H_{B_1}^{A_1} = \frac{1}{2} \left[(ce \mid da) + (de \mid ca) \right]$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = \frac{\sqrt{3}}{2} \left[(ce \mid da) - (de \mid ca) \right]$$

$$H_{B_2}^{A_2} = - \frac{1}{2} (ce \mid da) + \frac{3}{2} (de \mid ca)$$

Doublet 2×2

$A = abcdd$
$B = cedaa$

$$H^{A_1}_{B_1} = \frac{1}{2} (be \mid da) - (de \mid ba)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2} (be \mid da)$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2} (be \mid da) - \sqrt{3}\, (de \mid ba)$$

$$H^{A_2}_{B_2} = -\frac{1}{2} (be \mid da)$$

$A = abcdd$
$B = bdcaa$

$$H^{A_1}_{B_1} = \frac{1}{2} \Big\{(d \mid h \mid a) + (da \mid aa) + (da \mid bb) + (da \mid cc) + (da \mid dd) + 2\,(da \mid P_a P_a) - (P_a a \mid dP_a)\Big\} + \frac{1}{2} (db \mid ba) - (dc \mid ca)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2} \Big\{\text{the above}\Big\} + \frac{\sqrt{3}}{2} (db \mid ba)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} \Big\{\text{the above}\Big\} + \frac{\sqrt{3}}{2} (db \mid ba) + \sqrt{3}\, (dc \mid ca)$$

$$H^{A_2}_{B_2} = -\frac{1}{2} \Big\{\text{the above}\Big\} + \frac{3}{2} (db \mid ba)$$

$A = abcdd$
$B = eadcc$

$$H^{A_1}_{B_1} = \frac{1}{2} (be \mid dc) - (de \mid bc)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2} (be \mid dc) + \sqrt{3}\, (de \mid bc)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (be \mid dc)$$

$$H^{A_2}_{B_2} = -\frac{1}{2} (be \mid dc)$$

$A = abcdd$
$B = edcbb$

$$H^{A_1}_{B_1} = -(ae \mid db) + \frac{1}{2} (de \mid ab)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2} (de \mid ab)$$

$$H^{A_2}_{B_2} = -(ae \mid db) + \frac{3}{2} (de \mid ab)$$

$A = abcdd$
$B = eacbb$

$$H^{A_1}_{B_1} = -H^{A_2}_{B_2} = \frac{1}{2} (de \mid db)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (de \mid db)$$

$A = abcdd$
$B = ebcaa$

$$H^{A_1}_{B_1} = H^{A_2}_{B_2} = -(de \mid da)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$A = abcdd$
$B = abdcc$

$$H^{A_1}_{B_1} = -\Big\{(d \mid h \mid c) + (dc \mid aa) + (dc \mid bb) + (dc \mid cc) + (dc \mid dd) + 2\,(dc \mid P_a P_a) - (P_a c \mid dP_a)\Big\} + \frac{1}{2} (da \mid ac) + 2\,(db \mid bc)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (da \mid ac)$$

$$H^{A_2}_{B_2} = -\Big\{\text{the above}\Big\} + \frac{3}{2} (da \mid ac)$$

$A = abcdd$
$B = edabb$

$$H^{A_1}_{B_1} = \frac{1}{2} (ce \mid db) - (de \mid cb)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2} (ce \mid db) - \sqrt{3}\, (de \mid cb)$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2} (ce \mid db)$$

$$H^{A_2}_{B_2} = -\frac{1}{2} (ce \mid db)$$

$A = abcdd$
$B = abecc$

$$H^{A_1}_{B_1} = H^{A_2}_{B_1} = -(de \mid dc)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$A = abcdd$
$B = eabcc$

$$H^{A_1}_{B_1} = H^{A_2}_{B_2} = \frac{1}{2} (de \mid dc)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (de \mid dc)$$

$A = abcdd$
$B = abdee$

$$H^{A_1}_{B_1} = H^{A_2}_{B_2} = -(ce \mid de)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$A = abc$
$B = eac$

$$H^{A_1}_{B_1} = -\frac{1}{2} \left\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + 2 (be \mid P_a P_a) - (P_a e \mid bP_a)\right\} + (ba \mid ae) - \frac{1}{2} (bc \mid ce)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2} \left\{\text{the above}\right\} + \frac{\sqrt{3}}{2} (bc \mid ce)$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2} \left\{\text{the above}\right\} - \frac{\sqrt{3}}{2} (bc \mid ce)$$

$$H^{A_2}_{B_2} = \frac{1}{2} \left\{\text{the above}\right\} + (ba \mid ae) - \frac{1}{2} (bc \mid ce)$$

$A = abcdd$
$B = dbecc$

$$H^{A_1}_{B_1} = \frac{1}{2} \left[(ae \mid dc) + (de \mid ac)\right]$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2} \left[(ae \mid dc) - (de \mid ac)\right]$$

$$H^{A_2}_{B_2} = -\frac{1}{2} (ae \mid dc) + \frac{3}{2} (de \mid ac)$$

$A = abcdd$
$B = adecc$

$$H^{A_1}_{B_1} = -(be \mid dc) + 2 (de \mid bc)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$$H^{A_2}_{B_2} = (be \mid dc)$$

$A = abcdd$
$B = bdecc$

$$H^{A_1}_{B_1} = \frac{1}{2} (ae \mid dc) - (de \mid ac)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2} (ae \mid dc)$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (ae \mid dc) + \sqrt{3} (de \mid ac)$$

$$H^{A_2}_{B_2} = \frac{1}{2} (ae \mid dc)$$

$A = abcdd$
$B = bdcee$

$$H^{A_1}_{B_1} = -H^{A_2}_{B_2} = \frac{1}{2} (ae \mid de)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2} (ae \mid de)$$

$A = abcdd$
$B = ebdcc$

$$H^{A_1}_{B_1} = -(ae \mid dc) + \frac{1}{2} (de \mid ac)$$

Doublet 2×2

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(de \mid ac)$$

$$H^{A_2}_{B_2} = -(ae \mid dc) + \frac{3}{2}(de \mid ac)$$

$A = abc$
$B = abe$

$$H^{A_1}_{B_1} = \Big\{(c \mid h \mid e) + (ce \mid aa) + (ce \mid bb) + 2(ce \mid P_a P_a) - (P_a e \mid cP_a)\Big\} - \frac{1}{2}(ca \mid ae) + (cb \mid be)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = -\frac{\sqrt{3}}{2}(ca \mid ae)$$

$$H^{A_2}_{B_2} = \Big\{\text{the above}\Big\} + \frac{1}{2}(ca \mid ae) - (cb \mid be)$$

$A = abc$
$B = ebc$

$$H^{A_1}_{B_1} = \Big\{(a \mid h \mid e) + (ae \mid bb) + (ae \mid cc) + 2(ae \mid P_a P_a) - (P_a e \mid aP_a)\Big\} - \frac{1}{2}(ab \mid be) - \frac{1}{2}(ac \mid ce)$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[(ab \mid be) - (ac \mid ce)\Big]$$

$$H^{A_2}_{B_2} = \Big\{\text{the above}\Big\} + \frac{1}{2}(ab \mid be) + \frac{1}{2}(ac \mid ce)$$

$A = abcdd$
$B = ecdaa$

$$H^{A_1}_{B_1} = \frac{1}{2}\Big[(de \mid ba) + (be \mid da)\Big]$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\Big[-(de \mid ba) + (be \mid da)\Big]$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\Big[(de \mid ba) - (be \mid ba)\Big]$$

$$H^{A_2}_{B_2} = -\frac{3}{2}(de \mid ba) + \frac{1}{2}(be \mid da)$$

5. $N_A = 3$, $N_B = 5$ $(k_A = 2,$ $k_B = 5)$

$A = abcddee$
$B = edbfcaa$

$$H^{A_1}_{B_1} = \sqrt{2}\Big[\frac{1}{2}(df \mid ea) - \frac{1}{4}(ef \mid da)\Big]$$

$$H^{A_1}_{B_2} = \frac{\sqrt{6}}{4}(ef \mid da) = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}\Big[-(df \mid ea) + \frac{1}{2}(ef \mid da)\Big]$$

$$H^{A_1}_{B_4} = \frac{3\sqrt{2}}{4}(ef \mid da)$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_2} = -\frac{1}{\sqrt{2}}\Big[(df \mid ea) + \frac{1}{2}(ef \mid da)\Big]$$

$$H^{A_2}_{B_3} = -\frac{\sqrt{2}}{4}(ef \mid da)$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\Big[(df \mid ea) + \frac{1}{2}(ef \mid da)\Big]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\Big[(df \mid ea) - (ef \mid da)\Big]$$

$A = abcddee$
$B = ebdcfaa$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\Big[(df \mid ea) - \frac{1}{2}(ef \mid da)\Big]$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{6}}{4}(ef \mid da) = H^{A_2}_{B_1}$$

$$H_{B_3}^{A_1} = -\sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_4}^{A_1} = \frac{\sqrt{2}}{4}(ef \mid da) = -H_{B_3}^{A_2}$$

$$H_{B_5}^{A_1} = (ef \mid da)$$

$$H_{B_2}^{A_2} = \frac{1}{\sqrt{2}}\left[(df \mid ea) + \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_4}^{A_2} = \sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{5}{6}(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{3}}(ef \mid da)$$

$A = abcddee$
$B = ebfcdaa$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\left[(df \mid ea) - \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4}(ef \mid da) = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = -\sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_4}^{A_1} = -\frac{\sqrt{2}}{4}(ef \mid da) = -H_{B_3}^{A_2}$$

$$H_{B_5}^{A_1} = -(ef \mid da)$$

$$H_{B_2}^{A_2} = \frac{1}{\sqrt{2}}\left[(df \mid ea) - \frac{3}{2}(ef \mid da)\right]$$

$$H_{B_4}^{A_2} = \sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{1}{6}(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = -\frac{1}{\sqrt{3}}(ef \mid da)$$

$A = abcddee$
$B = efbdcaa$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\left[(df \mid ea) - \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{6}}{4}(ef \mid da) = -H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = -\sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{1}{2}(ef \mid da)\right]$$

$$H_{B_4}^{A_1} = -\frac{3\sqrt{2}}{4}(ef \mid da)$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = -\frac{1}{\sqrt{2}}\left[(df \mid ea) - \frac{3}{2}(ef \mid da)\right]$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{4}(ef \mid da)$$

$$H_{B_4}^{A_2} = \frac{1}{\sqrt{6}}\left[(df \mid ea) - \frac{3}{2}(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}(df \mid ea)$$

$A = abcddee$
$B = fdebcaa$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\left[(df \mid ea) + (da \mid ef)\right]$$

Doublet 2×5

$$H_{B_2}^{A_1} = \sqrt{\frac{3}{2}}\left[(df \mid ea) - (da \mid ef)\right]$$

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_1}^{A_2} = H_{B_2}^{A_2} = 0$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{2}}\left[(df \mid ea) - (da \mid ef)\right]$$

$$H_{B_4}^{A_2} = \sqrt{\frac{3}{2}}\left[-(df \mid ea) + \frac{1}{3}(da \mid ef)\right]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}(da \mid ef)$$

$A = abcddee$
$B = fdbceaa$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}\left[(ef \mid da) + (df \mid ea)\right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4}\left[(ef \mid da) - (df \mid ea)\right] = -H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = -\frac{\sqrt{6}}{4}\left[(ef \mid da) + (df \mid ea)\right]$$

$$H_{B_4}^{A_1} = \frac{3\sqrt{2}}{4}\left[-(ef \mid da) + (df \mid ea)\right]$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = -\frac{\sqrt{2}}{4}\left[(ef \mid da) - 3(df \mid ea)\right]$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{4}\left[(ef \mid da) - (df \mid ea)\right]$$

$$H_{B_4}^{A_2} = \frac{\sqrt{6}}{4}\left[-\frac{1}{3}(ef \mid da) + (df \mid ea)\right]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}(ef \mid da)$$

$A = abcddee$
$B = fbdecaa$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}\left[(df \mid ea) + (ef \mid da)\right]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4}\left[(df \mid ea) - (ef \mid da)\right] = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = -\frac{\sqrt{6}}{4}\left[(df \mid ea) + (ef \mid da)\right]$$

$$H_{B_4}^{A_1} = \frac{\sqrt{2}}{4}\left[(df \mid ea) - (ef \mid da)\right]$$

$$H_{B_5}^{A_1} = (df \mid ea) - (ef \mid da)$$

$$H_{B_2}^{A_2} = \frac{\sqrt{2}}{4}\left[-3(df \mid ea) + (ef \mid da)\right]$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{4}\left[-(df \mid ea) + (ef \mid da)\right]$$

$$H_{B_4}^{A_2} = \frac{\sqrt{6}}{12}\left[(df \mid ea) + 5(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{3}}\left[(df \mid ea) - (ef \mid da)\right]$$

$A = abcddee$
$B = fbcdeaa$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ea) + (ef \mid da)\right] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_3} = H^{A_2}_{B_1} = 0$$

$$H^{A_1}_{B_4} = \frac{1}{\sqrt{2}}\left[-(df \mid ea) + (ef \mid da)\right] = - H^{A_2}_{B_3}$$

$$H^{A_1}_{B_5} = (df \mid ea) - (ef \mid da)$$

$$H^{A_2}_{B_4} = \sqrt{\frac{2}{3}}\left[(df \mid ea) - (ef \mid da)\right]$$

$$H^{A_2}_{B_5} = \sqrt{\frac{1}{3}}\left[(df \mid ea) - (ef \mid da)\right]$$

$A = abcddee$
$B = abcdeff$

$$H^{A_1}_{B_1} = -\sqrt{2}\,(df \mid ef) = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_3} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = H^{A_2}_{B_4} = H^{A_2}_{B_5} = 0$$

$A = abcddee$
$B = adbceff$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}(df \mid ef) = - H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}(df \mid ef)$$

$$H^{A_2}_{B_4} = -\frac{1}{\sqrt{6}}(df \mid ef)$$

$$H^{A_2}_{B_5} = -\frac{2}{\sqrt{3}}(df \mid ef)$$

$A = abcddee$
$B = abdecff$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}(df \mid ef) = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}(df \mid ef) = - H^{A_2}_{B_4}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = H^{A_2}_{B_5} = 0$$

$A = abcddee$
$B = baedcff$

$$H^{A_1}_{B_1} = -\frac{\sqrt{2}}{4}(df \mid ef) = - H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = - H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}(df \mid ef) = H^{A_2}_{B_1} = - H^{A_2}_{B_4}$$

$$H^{A_1}_{B_4} = -\frac{3\sqrt{2}}{4}(df \mid ef) = - H^{A_2}_{B_3}$$

$$H^{A_1}_{B_5} = H^{A_2}_{B_5} = 0$$

$A = abcddee$
$B = dabecff$

$$H^{A_1}_{B_1} = -\frac{\sqrt{2}}{4}(df \mid ef) = H^{A_2}_{B_2} = H^{A_2}_{B_3}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}(df \mid cf) = - H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = \frac{3\sqrt{2}}{4}(df \mid ef)$$

Doublet 2×5

$$H_{B_5}^{A_1} = 0$$

$$H_{B_4}^{A_2} = \frac{\sqrt{6}}{12}(df \mid ef)$$

$$H_{B_5}^{A_2} = -\frac{2}{\sqrt{3}}(df \mid ef)$$

$A = abcddee$
$B = adebcff$

$$H_{B_1}^{A_1} = -\sqrt{2}\,(df \mid ef)$$

$$H_{B_2}^{A_1} = H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_1}^{A_2} = H_{B_2}^{A_2} = H_{B_3}^{A_2} = 0$$

$$H_{B_4}^{A_2} = \sqrt{\frac{2}{3}}(df \mid ef)$$

$$H_{B_5}^{A_2} = -\frac{2}{\sqrt{3}}(df \mid ef)$$

$A = abcddee$
$B = deabfcc$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}\Big[(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{6}}{4}\Big[(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{4}\Big[(df \mid ce) - (ef \mid dc)\Big] = H_{B_1}^{A_2}$$

$$H_{B_4}^{A_1} = \frac{\sqrt{2}}{4}\Big[(df \mid ec) - (ef \mid dc)\Big] = - H_{B_2}^{A_2}$$

$$H_{B_5}^{A_1} = (df \mid ec) - (ef \mid dc)$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{4}\Big[(df \mid ec) - 3\,(ef \mid dc)\Big]$$

$$H_{B_4}^{A_2} = \frac{\sqrt{6}}{12}\Big[5\,(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{3}}\Big[-(df \mid ec) + (ef \mid dc)\Big]$$

$A = abcddee$
$B = edbafcc$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\Big[-\frac{1}{2}(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4}(df \mid ec) = - H_{B_3}^{A_1} = - H_{B_1}^{A_2}$$

$$H_{B_4}^{A_1} = \frac{1}{\sqrt{2}}\Big[\frac{1}{2}(df \mid ec) + (ef \mid dc)\Big] = - H_{B_2}^{A_2}$$

$$H_{B_5}^{A_1} = (df \mid ec) - (ef \mid dc)$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{2}}\Big[-\frac{3}{2}(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_4}^{A_2} = \sqrt{\frac{2}{3}}\Big[-\frac{1}{4}(df \mid ec) + (ef \mid dc)\Big]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{3}}\Big[-(df \mid ec) + (ef \mid dc)\Big]$$

$A = abcddee$
$B = dabefcc$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\Big[-\frac{1}{2}(df \mid ec) + (ef \mid dc)\Big] = H_{B_2}^{A_2}$$

Doublet 2×5

$$H^{A_1}_{B_2} = \sqrt{\frac{3}{2}}\left[+\frac{1}{2}(df \mid ec) - (ef \mid dc)\right] = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{4}(df \mid ec)$$

$$H^{A_1}_{B_4} = -\frac{3\sqrt{2}}{4}(df \mid ec)$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_3} = \frac{\sqrt{2}}{4}(df \mid ec)$$

$$H^{A_2}_{B_4} = -\frac{\sqrt{6}}{12}(df \mid ec)$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = adebfcc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right]$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A_2}_{B_2} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - 3(ef \mid dc)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = abdefcc$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ec) - (ef \mid dc)\right] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = H^{A_2}_{B_5} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}(df \mid ec) = -H^{A_2}_{B_4}$$

$A = abcddee$
$B = baedfcc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right] = -H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right] = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}(ef \mid dc) = H^{A_2}_{B_4}$$

$$H^{A_1}_{B_4} = \frac{3\sqrt{2}}{4}(ef \mid dc) = -H^{A_2}_{B_3}$$

$$H^{A_1}_{B_5} = H^{A_2}_{B_5} = 0$$

$A = abcddee$
$B = deabcff$

$$H^{A_1}_{B_1} = H^{A_2}_{B_3} = \frac{1}{\sqrt{2}}(df \mid ef)$$

$$H^{A_1}_{B_2} = -H^{A_2}_{B_4} = \sqrt{\frac{3}{2}}(df \mid ef)$$

Doublet 2×5

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_1}^{A_2} = H_{B_2}^{A_2} = H_{B_5}^{A_2} = 0$$

$A = abcddee$
$B = edbacff$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}(df \mid ef) = -H_{B_3}^{A_2}$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{6}}{4}(df \mid ef) = -H_{B_3}^{A_1} = -H_{B_1}^{A_2} = H_{B_4}^{A_2}$$

$$H_{B_4}^{A_1} = -\frac{3\sqrt{2}}{4}(df \mid ef) = -H_{B_2}^{A_2}$$

$$H_{B_5}^{A_1} = H_{B_5}^{A_2} = 0$$

$A = abcddee$
$B = dfebcaa$

$$H_{B_1}^{A_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ea) - (ef \mid da)\right]$$

$$H_{B_2}^{A_1} = \sqrt{\frac{3}{2}}(df \mid ea)$$

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_1}^{A_2} = H_{B_2}^{A_2} = 0$$

$$H_{B_3}^{A_2} = \frac{1}{\sqrt{2}}(df \mid ea)$$

$$H_{B_4}^{A_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ea) - \frac{2}{3}(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = -\frac{2}{\sqrt{3}}(ef \mid da)$$

$A = abcddee$
$B = defbcaa$

$$H_{B_1}^{A_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ea) - (ef \mid da)\right]$$

$$H_{B_2}^{A_1} = -\sqrt{\frac{3}{2}}(df \mid ea)]$$

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_1}^{A_2} = H_{B_2}^{A_2} = 0$$

$$H_{B_3}^{A_2} = -\frac{1}{\sqrt{2}}(df \mid ea)$$

$$H_{B_4}^{A_2} = \frac{1}{\sqrt{6}}\left[(df \mid ea) + 2(ef \mid da)\right]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}\left[(df \mid ea) - (ef \mid da)\right]$$

$A = abcddee$
$B = fadecbb$

$$H_{B_1}^{A_1} = \frac{1}{\sqrt{2}}\left[(df \mid eb) - \frac{1}{2}(db \mid ef)\right]$$

$$H_{B_2}^{A_1} = -\sqrt{\frac{3}{2}}\left[(df \mid eb) - \frac{1}{2}(db \mid ef)\right]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{4}(db \mid ef) = -H_{B_1}^{A_2}$$

$$H_{B_4}^{A_1} = \frac{3\sqrt{2}}{4}(db \mid ef)$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = -\frac{\sqrt{2}}{4}(db \mid ef)$$

$$H^{A_2}_{B_3} = -\frac{1}{\sqrt{2}}\left[(df \mid eb) + \frac{1}{2}(db \mid ef)\right]$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\left[(df \mid eb) + \frac{1}{2}(db \mid ef)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[(df \mid eb) - (db \mid ef)\right]$$

$A = abcddee$
$B = adefcbb$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid eb) + (ef \mid db)\right]$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^{A_2}_{B_2} = -\frac{1}{\sqrt{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\left[(df \mid eb) - 3(ef \mid db)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}(df \mid eb)$$

$A = abcddee$
$B = acdfebb$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid eb) - (ef \mid db)\right] = -H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_5} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}(df \mid eb) = H^{A_2}_{B_4}$$

$A = abcddee$
$B = afcdebb$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid eb) + (ef \mid db)\right] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^{A_2}_{B_4} = -\frac{1}{\sqrt{6}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^{A_2}_{B_5} = -\frac{2}{\sqrt{3}}\left[(df \mid eb) - (ef \mid db)\right]$$

$A = abcddee$
$B = acdefbb$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid eb) - (ef \mid db)\right] = -H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = H^{A_2}_{B_5} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}(df \mid eb) = H^{A_2}_{B_4}$$

$A = abcddee$
$B = abfdecc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = H^{A_2}_{B_5} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}\left[(df \mid ec) - (ef \mid dc)\right] = -H^{A_2}_{B_4}$$

Doublet 2×5

$A = abcddee$
$B = dafbecc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_1}_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}(ef \mid dc) = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = \frac{3\sqrt{2}}{4}(ef \mid dc)$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_2} = -\frac{\sqrt{2}}{4}(ef \mid dc)$$

$$H^{A_2}_{B_3} = -\frac{1}{\sqrt{2}}\left[(df \mid ec) - \frac{3}{2}(ef \mid dc)\right]$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - \frac{3}{2}(ef \mid dc)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = fadebcc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - \frac{1}{2}(dc \mid ef)\right]$$

$$H^{A_1}_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - \frac{1}{2}(dc \mid ef)\right]$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}(dc \mid ef) = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = \frac{3\sqrt{2}}{4}(dc \mid ef)$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_2} = \frac{\sqrt{2}}{4}(dc \mid ef)$$

$$H^{A_2}_{B_3} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + \frac{1}{2}(dc \mid ef)\right]$$

$$H^{A_2}_{B_4} = -\frac{1}{\sqrt{6}}\left[(df \mid ec) + \frac{1}{2}(dc \mid ef)\right]$$

$$H^{A_2}_{B_5} = -\frac{2}{\sqrt{3}}\left[(df \mid ec) - (dc \mid ef)\right]$$

$A = abcddee$
$B = fadbecc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_1}_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{4}(ef \mid dc) = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = -\frac{3\sqrt{2}}{4}(ef \mid dc)$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_2} = \frac{\sqrt{2}}{4}(ef \mid dc)$$

$$H^{A_2}_{B_3} = -\frac{1}{\sqrt{2}}\left[(df \mid ec) + \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}\left[(df \mid ec) + \frac{1}{2}(ef \mid dc)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$A = abcddee$
$B = adbfecc$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ec) - (dc \mid ef)\right] = -H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}(df \mid ec)$$

$$H^{A_2}_{B_4} = \frac{1}{\sqrt{6}}(df \mid ec)$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = adbefcc$

$$H^{A_1}_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ec) - (ef \mid dc)\right] = -H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}(df \mid ec)$$

$$H^{A_2}_{B_4} = -\frac{1}{\sqrt{6}}(df \mid ec)$$

$$H^{A_2}_{B_5} = -\frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = adefbcc$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (dc \mid ef)\right]$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_4} = H^{A_1}_{B_5} = H^{A_2}_{B_1} = H^{A_2}_{B_3} = 0$$

$$H^{A_1}_{B_3} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H^{A_2}_{B_2} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H^{A_2}_{B_4} = \sqrt{\frac{3}{2}}\left[-\frac{1}{3}(df \mid ec) + (dc \mid ef)\right]$$

$$H^{A_2}_{B_5} = -\frac{2}{\sqrt{3}}(df \mid ec)$$

$A = abcddee$
$B = fabdecc$

$$H^{A_1}_{B_1} = -\frac{\sqrt{2}}{4}\left[(df \mid ec) + (dc \mid ef)\right] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = \frac{\sqrt{6}}{4}\left[(df \mid ec) + (dc \mid ef)\right] = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H_{B_4}^{A_1} = \frac{3\sqrt{2}}{4}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_3}^{A_2} = -\frac{\sqrt{2}}{4}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H_{B_4}^{A_2} = \frac{\sqrt{6}}{12}\left[(df \mid ec) - (dc \mid ef)\right]$$

$$H_{B_5}^{A_2} = -\frac{2}{\sqrt{3}}\left[(df \mid ec) - (dc \mid ef)\right]$$

$A = abcdd$
$B = fadbc$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd)$$
$$+ 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} + (da \mid af) + \frac{1}{4}(db \mid bf)$$
$$+ \frac{1}{4}(dc \mid cf)\Big]$$

$$H_{B_2}^{A_1} = \sqrt{2}\left[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{4}(db \mid bf) - \frac{\sqrt{3}}{4}(dc \mid cf)\right]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = -H_{B_1}^{A_2}$$

$$H_{B_4}^{A_1} = \frac{3\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right]$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = -\frac{\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right]$$

$$H_{B_3}^{A_2} = \sqrt{2}\left[\frac{1}{2}\{\text{the above}\} - \frac{3}{4}(db \mid bf) - \frac{3}{4}(dc \mid cf)\right]$$

$$H_{B_4}^{A_2} = \sqrt{2}\Big[-\frac{\sqrt{3}}{6}\{\text{the above}\} - \frac{\sqrt{3}}{3}(da \mid af) + \frac{\sqrt{3}}{4}(db \mid bf)$$
$$+ \frac{\sqrt{3}}{4}(dc \mid cf)\Big]$$

$$H_{B_5}^{A_2} = \sqrt{2}\left[-\frac{\sqrt{6}}{3}\{\text{the above}\} + \frac{\sqrt{6}}{3}(da \mid af)\right]$$

$A = abcdd$
$B = abcfd$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd)$$
$$+ 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2}(da \mid af) - \frac{1}{2}(db \mid bf)$$
$$- \frac{1}{2}(dc \mid cf) = H_{B_2}^{A_2}$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$$H_{B_3}^{A_1} = -\sqrt{\frac{3}{2}}\left[(db \mid bf) - (dc \mid cf)\right]$$

$$H_{B_4}^{A_1} = -\frac{1}{\sqrt{2}}(da \mid af) = -H_{B_3}^{A_2}$$

$$H_{B_5}^{A_1} = (da \mid af)$$

$$H_{B_4}^{A_2} = \frac{1}{\sqrt{6}}\left[2(da \mid af) - (db \mid bf) - 3(dc \mid cf)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[\frac{1}{2}(da \mid af) - (db \mid bf)\right]$$

$A = abcdd$
$B = fdabc$

$$H^{A_1}_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} + (da \mid af) + \frac{1}{4}(db \mid bf) + \frac{1}{4}(dc \mid cf)\Big]$$

$$H^{A_1}_{B_2} = \sqrt{2}\left[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{4}(db \mid bf) + \frac{\sqrt{3}}{4}(dc \mid cf)\right]$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = -\frac{\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = - H^{A_2}_{B_2}$$

$$H^{A_1}_{B_5} = -(db \mid bf) + (dc \mid cf)$$

$$H^{A_2}_{B_3} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} + \frac{3}{4}(db \mid bf) + \frac{3}{4}(dc \mid cf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{2}\left[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{3}(da \mid af) - \frac{\sqrt{3}}{12}(db \mid bf) - \frac{\sqrt{3}}{12}(dc \mid cf)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[(da \mid af) - \frac{1}{2}(db \mid bf) - \frac{1}{2}(dc \mid cf)\right]$$

$A = abcdd$
$B = dfabc$

$$H^{A_1}_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2}(da \mid af) + \frac{1}{4}(db \mid bf) + \frac{1}{4}(dc \mid cf)\Big]$$

$$H^{A_1}_{B_2} = \sqrt{2}\left[-\frac{\sqrt{3}}{2}\{\text{the above}\} + \frac{\sqrt{3}}{2}(da \mid af) + \frac{\sqrt{3}}{4}(db \mid bf) + \frac{\sqrt{3}}{4}(dc \mid cf)\right]$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = \frac{\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = - H^{A_2}_{B_2}$$

$$H^{A_1}_{B_5} = (db \mid bf) - (dc \mid cf)$$

$$H^{A_2}_{B_3} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} + \frac{1}{2}(da \mid af) - \frac{1}{4}(db \mid bf) - \frac{1}{4}(dc \mid cf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{2}\left[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{6}(da \mid af) - \frac{5\sqrt{3}}{12}(db \mid bf) - \frac{5\sqrt{3}}{12}(dc \mid cf)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[-(da \mid af) + \frac{1}{2}(db \mid bf) + \frac{1}{2}(dc \mid cf)\right]$$

$A = abcdd$
$B = abcdf$

$$H^{A_1}_{B_1} = \sqrt{2}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2}(da \mid af) - \frac{1}{2}(db \mid bf) - \frac{1}{2}(dc \mid cf)\Big] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_2} = H^{A_2}_{B_1} = 0$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}\left[(db \mid bf) - (dc \mid cf)\right]$$

Doublet 2×5

$$H^{A_1}_{B_4} = \frac{1}{\sqrt{2}}(da \mid af) = - \; H^{A_2}_{B_3}$$

$$H^{A_1}_{B_5} = -\,(da \mid af)$$

$$H^{A_2}_{B_4} = \sqrt{\frac{3}{2}}\left[-\frac{2}{3}(da \mid af) + \frac{1}{3}(db \mid bf) + (dc \mid cf)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[-\frac{1}{2}(da \mid af) + (db \mid bf)\right]$$

$A = abcdd$
$B = dafbc$

$$H^{A_1}_{B_1} = \sqrt{2}\left[-\frac{1}{2}\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2}(da \mid af) + \frac{1}{4}(db \mid bf) + \frac{1}{4}(dc \mid cf)\right]$$

$$H^{A_1}_{B_2} = \sqrt{2}\left[\frac{\sqrt{3}}{2}\{\text{the above}\} - \frac{\sqrt{3}}{2}(da \mid af) - \frac{\sqrt{3}}{4}(db \mid bf) - \frac{\sqrt{3}}{4}(dc|cf)\right]$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{4}\left[(db \mid bf) - (dc \mid cf)\right] = - \; H^{A_2}_{B_1}$$

$$H^{A_1}_{B_4} = -\frac{3\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right]$$

$$H^{A_1}_{B_5} = 0$$

$$H^{A_2}_{B_2} = \frac{\sqrt{2}}{4}\left[(db \mid bf) - (dc \mid cf)\right]$$

$$H^{A_2}_{B_3} = \frac{1}{\sqrt{2}}\left[\{\text{the above}\} - (da \mid af) + \frac{1}{2}(db \mid bf) + \frac{1}{2}(dc \mid cf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{\frac{3}{2}}\left[-\frac{1}{3}\{\text{the above}\} + (da \mid af) - \frac{1}{6}(db \mid bf) - \frac{1}{6}(dc \mid cf)\right]$$

$$H^{A_2}_{B_5} = \frac{2}{\sqrt{3}}\left[-\{\text{the above}\} + (db \mid bf) + (dc \mid cf)\right]$$

$A = abcdd$
$B = abfcd$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[-\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} + \frac{1}{2}(da \mid af) - (db \mid bf) + 2(dc \mid cf)\right]$$

$$H^{A_1}_{B_2} = \frac{\sqrt{6}}{4}(da \mid af) = H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = \sqrt{\frac{3}{2}}\left[\{\text{the above}\} - \frac{1}{2}(da \mid af) - (db \mid bf)\right]$$

$$H^{A_1}_{B_4} = -\frac{\sqrt{2}}{4}(da \mid af) = - \; H^{A_2}_{B_3}$$

$$H^{A_1}_{B_5} = -\,(da \mid af)$$

$$H^{A_2}_{B_2} = \sqrt{2}\left[-\frac{1}{2}\{\text{the above}\} - \frac{1}{4}(da \mid af) + \frac{1}{2}(db \mid bf) + (dc \mid cf)\right]$$

$$H^{A_2}_{B_4} = \sqrt{\frac{3}{2}}\left[-\{\text{the above}\} + \frac{5}{6}(da \mid af) + \frac{1}{3}(db \mid bf)\right]$$

Doublet 2×5

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}\Big[-\frac{1}{2}(da\mid af)+(db\mid bf)\Big]$$

$A = abcdd$
$B = abdcf$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d\mid h\mid f)+(df\mid aa)+(df\mid bb)+(df\mid cc)+(df\mid dd)+2(df\mid P_aP_a)-(P_af\mid dP_a)\}+\frac{1}{4}(da\mid af)+(db\mid bf)-\frac{1}{2}(dc\mid cf)\Big]$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{6}}{4}(da\mid af) = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = \sqrt{\frac{3}{2}}\Big[\{\text{the above}\}-\frac{1}{2}(da\mid af)-(dc\mid cf)\Big]$$

$$H_{B_4}^{A_1} = \frac{\sqrt{2}}{4}(da\mid af) = -H_{B_3}^{A_2}$$

$$H_{B_5}^{A_1} = (da\mid af)$$

$$H_{B_2}^{A_2} = \sqrt{2}\Big[-\frac{1}{2}\{\text{the above}\}+\frac{3}{4}(da\mid af)-\frac{1}{2}(dc\mid cf)\Big]$$

$$H_{B_4}^{A_2} = \sqrt{\frac{3}{2}}\Big[-\{\text{the above}\}+\frac{1}{6}(da|af)+\frac{2}{3}(db\mid bf)+(dc\mid cf)\Big]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{3}}\Big[(da\mid af)-2(db\mid bf)\Big]$$

$A = abcdd$
$B = afbdc$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d\mid h\mid f)+(df\mid aa)+(df\mid bb)+(df\mid cc)+(df\mid dd)+2(df\mid P_aP_a)-(P_af\mid dP_a)\}+\frac{1}{4}(da\mid af)-\frac{1}{2}(db\mid bf)+(dc\mid cf)\Big]$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{6}}{4}(da\mid af) = -H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = \sqrt{\frac{3}{2}}\Big[\{\text{the above}\}-\frac{1}{2}(da\mid af)-(db\mid bf)\Big]$$

$$H_{B_4}^{A_1} = -\frac{3\sqrt{2}}{4}(da\mid af)$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = \sqrt{2}\Big[\frac{1}{2}\{\text{the above}\}+\frac{1}{4}(da|af)-\frac{1}{2}(db\mid bf)-(dc\mid cf)\Big]$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{4}(da\mid af)$$

$$H_{B_4}^{A_2} = \sqrt{\frac{3}{2}}\Big[-\frac{1}{3}\{\text{the abovo}\}-\frac{1}{6}(da\mid af)+(db\mid bf)\Big]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}}\Big[-\{\text{the above}\}+(da\mid af)\Big]$$

$A = abcdd$
$B = adbfc$

$$H_{B_1}^{A_1} = \sqrt{2}\Big[-\frac{1}{2}\{(d\mid h\mid f)+(df\mid aa)+(df\mid bb)+(df\mid cc)+(df\mid dd)+2(df\mid P_aP_a)-(P_af\mid dP_a)\}+\frac{1}{4}(da\mid af)+(db\mid bf)-\frac{1}{2}(dc\mid cf)\Big]$$

Doublet 2×5

$$H_{B_2}^{A_1} = \frac{\sqrt{6}}{4} (da \mid af) = - H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = \sqrt{\frac{3}{2}} \left[\{\text{the above}\} - \frac{1}{2} (da \mid af) - (dc \mid cf) \right]$$

$$H_{B_4}^{A_1} = \frac{3\sqrt{2}}{4} (da \mid af)$$

$$H_{B_5}^{A_1} = 0$$

$$H_{B_2}^{A_2} = \frac{1}{\sqrt{2}} \left[\{\text{the above}\} - \frac{3}{2} (da \mid af) + (dc \mid cf) \right]$$

$$H_{B_3}^{A_2} = - \frac{\sqrt{2}}{4} (da \mid af)$$

$$H_{B_4}^{A_2} = \frac{1}{\sqrt{6}} \left[- \{\text{the above}\} + \frac{3}{2} (da \mid af) - 2 (db \mid bf) + (dc \mid cf) \right]$$

$$H_{B_5}^{A_2} = \frac{2}{\sqrt{3}} \left[- \{\text{the above}\} + (db \mid bf) + (dc \mid cf) \right]$$

6. $N_A = 5$, $N_B = 5$ $(k_A = 5,\ k_B = 5)$

$A = abcdeff$
$B = fbcdeaa$

$$H_{B_1}^{A_1} = - \Big\{ (f \mid h \mid a) + (fa \mid aa) + (fa \mid bb) + (fa \mid cc) + (fa \mid dd) + (fa \mid ee)$$
$$+ (fa \mid ff) + 2 (fa \mid P_a P_a) - (P_a a \mid f P_a) \Big\} + \frac{1}{2} (fb \mid ba)$$
$$+ \frac{1}{2} (fc \mid ca) + \frac{1}{2} (fd \mid da) + \frac{1}{2} (fe \mid ea) = H_{B_3}^{A_3}$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2} \left[(fb \mid ba) - (fc \mid ca) \right]$$

$$H_{B_3}^{A_1} = 0$$

$$H_{B_4}^{A_1} = - \frac{1}{2} \left[(fd \mid da) - (fe \mid ea) \right]$$

$$H_{B_4}^{A_1} = \frac{1}{\sqrt{2}} \left[(fd \mid da) - (fe \mid ea) \right]$$

$$H_{B_2}^{A_2} = - \Big\{\text{the above}\Big\} + \frac{3}{2} (fb \mid ba) + \frac{3}{2} (fc \mid ca) + \frac{1}{2} (fd \mid da)$$
$$+ \frac{1}{2} (fe \mid ea)$$

$$H_{B_3}^{A_2} = \frac{1}{2} \left[(fd \mid da) - (fe \mid ea) \right]$$

$$H_{B_4}^{A_2} = \frac{1}{\sqrt{3}} \left[(fd \mid da) - (fe \mid ea) \right]$$

$$H_{B_5}^{A_2} = \frac{1}{\sqrt{6}} \left[(fd \mid da) - (fe \mid ea) \right]$$

$$H_{B_4}^{A_3} = - \frac{\sqrt{3}}{2} \left[(fb \mid ba) + \frac{1}{3} (fc \mid ca) - \frac{2}{3} (fd \mid da) - \frac{2}{3} (fe \mid ea) \right]$$

$$\ _{B_5}^{A_3} = \frac{1}{\sqrt{6}} \left[- 2 (fc \mid ca) + (fd \mid da) + (fe \mid ea) \right]$$

$$H_{B_4}^{A_4} = - \Big\{\text{the above}\Big\} + \frac{3}{2} (fb \mid ba) + \frac{1}{6} (fc \mid ca) + \frac{7}{6} (fd \mid da)$$
$$+ \frac{7}{6} (fe \mid ea)$$

Doublet 5×5

$$H_{B_5}^{A_4} = \frac{\sqrt{2}}{3}\Big[(fc \mid ca) - \frac{1}{2}(fd \mid da) - \frac{1}{2}(fe \mid ea)\Big]$$

$$H_{B_5}^{A_5} = -\Big\{\text{the above}\Big\} + \frac{4}{3}(fc \mid ca) + \frac{4}{3}(fd \mid da) + \frac{4}{3}(fe \mid ea)$$

$$H_{B_j}^{A_i} \equiv H_{B_i}^{A_j}$$

$A = abcdeff$
$B = acfdebb$

$$H_{B_1}^{A_1} = -\Big\{(f \mid h \mid b) + (fb \mid aa) + (fb \mid bb) + (fb \mid cc) + (fb \mid dd) + (fb \mid ee) + (fb \mid ff) + 2(fb \mid P_a P_a) - (P_a b \mid fP_a)\Big\} + \frac{1}{2}(fa \mid ab) + 2(fc \mid cb) + \frac{1}{2}(fd \mid db) + \frac{1}{2}(fe \mid eb)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(fa \mid ab) = -H_{B_1}^{A_2} = -H_{B_3}^{A_4}$$

$$H_{B_3}^{A_1} = \frac{\sqrt{3}}{2}\Big[(fd \mid db) - (fe \mid eb)\Big] = H_{B_4}^{A_2} = -H_{B_1}^{A_3}$$

$$H_{B_4}^{A_1} = H_{B_5}^{A_1} = H_{B_3}^{A_2} = H_{B_5}^{A_2} = H_{B_2}^{A_3} = H_{B_1}^{A_4} = H_{B_1}^{A_5} = H_{B_3}^{A_5} = 0$$

$$H_{B_2}^{A_2} = \Big\{\text{the above}\Big\} - \frac{3}{2}(fa \mid ab) - \frac{1}{2}(fd \mid db) - \frac{1}{2}(fe \mid eb)$$

$$H_{B_3}^{A_3} = \Big\{\text{the above}\Big\} - \frac{1}{2}(fa \mid ab) - \frac{3}{2}(fd \mid db) - \frac{3}{2}(fe \mid eb)$$

$$H_{B_4}^{A_3} = \frac{\sqrt{3}}{6}(fa \mid ab)$$

$$H_{B_5}^{A_3} = \frac{\sqrt{6}}{3}(fa \mid ab)$$

$$H_{B_2}^{A_4} = \frac{\sqrt{3}}{6}\Big[(fd \mid db) - (fe \mid eb)\Big]$$

$$H_{B_4}^{A_4} = \frac{1}{3}\Big\{\text{the above}\Big\} - \frac{1}{2}(fa \mid ab) + \frac{2}{3}(fc \mid cb) - \frac{1}{2}(fd \mid db) - \frac{1}{2}(fe \mid eb)$$

$$H_{B_5}^{A_4} = \frac{2\sqrt{2}}{3}\Big\{\text{the above}\Big\} - \sqrt{2}(fa \mid ab) - \frac{2\sqrt{2}}{3}(fc \mid cb)$$

$$H_{B_2}^{A_5} = \frac{\sqrt{6}}{3}\Big[(fd \mid db) - (fe \mid eb)\Big]$$

$$H_{B_4}^{A_5} = \frac{2\sqrt{2}}{3}\Big\{\text{the above}\Big\} - \frac{2\sqrt{2}}{3}(fc \mid cb) - \sqrt{2}(fd \mid db) - \sqrt{2}(fe \mid eb)$$

$$H_{B_5}^{A_5} = -\frac{1}{3}\Big\{\text{the above}\Big\} + \frac{4}{3}(fc \mid cb)$$

$A = abcdeff$
$B = abcdfee$

$$H_{B_1}^{A_1} = -\Big\{(f \mid h \mid e) + (fe \mid aa) + (fe \mid bb) + (fe \mid cc) + (fe \mid dd) + (fe \mid ee) + (fe \mid ff) + 2(fe \mid P_a P_a) - (P_a e \mid fP_a)\Big\} + \frac{1}{2}(fa \mid ae) + \frac{1}{2}(fb \mid be) + \frac{1}{2}(fc \mid ce) + 2(fd \mid de) = H_{B_2}^{A_2}$$

$$H_{B_2}^{A_1} = H_{B_1}^{A_2} = 0$$

$$H_{B_3}^{A_1} = -\frac{\sqrt{3}}{2}\Big[(fc \mid ce) - (fb \mid be)\Big]$$

$$H_{B_4}^{A_1} = \frac{1}{2}(fa \mid ae) = -H_{B_3}^{A_2}$$

Doublet 5×5

$$H^{A_1}_{B_5} = -\frac{1}{\sqrt{2}}(fa \mid ae)$$

$$H^{A_2}_{B_4} = \frac{\sqrt{3}}{2}(fc \mid ce) + \frac{\sqrt{3}}{6}(fb \mid be) - \frac{\sqrt{3}}{3}(fa \mid ae)$$

$$H^{A_2}_{B_5} = \frac{1}{\sqrt{6}}\left[2(fb \mid be) - (fa \mid ae)\right]$$

$$H^{A_3}_{B_3} = H^{A_1}_{B_1} + (fb \mid be) + (fc \mid ce) - 2(fd \mid de)$$

$$H^{A_3}_{B_4} = \frac{1}{\sqrt{3}}(fa \mid ae)$$

$$H^{A_3}_{B_5} = \frac{1}{\sqrt{6}}(fa \mid ae)$$

$$H^{A_4}_{B_4} = H^{A_1}_{B_1} + \frac{2}{3}(fa \mid ae) - \frac{1}{3}(fb \mid be) + (fc \mid ce) - 2(fd \mid de)$$

$$H^{A_4}_{B_5} = \frac{\sqrt{2}}{3}\left[(fb \mid be) - \frac{1}{2}(fa \mid ae)\right]$$

$$H^{A_5}_{B_5} = H^{A_1}_{B_1} + \frac{5}{6}(fa \mid ae) + \frac{5}{6}(fb \mid be) - \frac{1}{2}(fc \mid ce) - 2(fd \mid de)$$

$$H^{A_i}_{B_j} \equiv H^{A_j}_{B_i}$$

III. Formulae of configuration interaction matrix elements for Triplet state wave function ($S = 1$)

1. $N_A = 2,\quad N_B = 2\quad (k_A = 1,\quad k_B = 1)$

$A = abcc$, $B = abdd$: $\quad H^A_B = (cd \mid cd)$

$A = abcc$, $B = cbaa$: $\quad H^A_B = -\left\{(c \mid h \mid a) + (ca \mid aa) + (ca \mid bb) + (ca \mid cc) + 2(ca \mid P_a P_a) - (P_a a \mid cP_a)\right\}$

$A = abcc$, $B = cdaa$: $\quad H^A_B = -(bd \mid ca)$

$A = abcc$, $B = cdbb$: $\quad H^A_B = (ad \mid cb)$

$A = abcc$, $B = cbdd$: $\quad H^A_B = -(ad \mid cd)$

$A = ab$, $B = ad$: $\quad H^A_B = \left\{(b \mid h \mid d) + (bd \mid aa) + 2(bd \mid P_a P_a) - (P_a d \mid bP_a)\right\} - (ba \mid ad)$

$A = ab$, $B = db$: $\quad H^A_B = \left\{(a \mid h \mid d) + (ad \mid bb) + 2(ad \mid P_a P_a) - (P_a d \mid aP_a)\right\} - (ab \mid bd)$

$A = ab$, $B = da$: $\quad H^A_B = -\left\{(b \mid h \mid d) + (bd \mid aa) + 2(bd \mid P_a P_a) - (P_a d \mid bP_a)\right\} + (ba \mid ad)$

$A = abcc$, $B = bcaa$: $\quad H^A_B = \left\{(c \mid h \mid a) + (ca \mid aa) + (ca \mid bb) + (ca \mid cc) + 2(ca \mid P_a P_a) - (P_a a \mid cP_a)\right\}$

$A = abcc$, $B = acbb$: $\quad H^A_B = -\left\{(c \mid h \mid b) + (cb \mid aa) + (cb \mid bb) + (cb \mid cc) + 2(cb \mid P_a P_a) - (P_a b \mid cP_a)\right\}$

$A = abcc$, $B = bdaa$: $\quad H^A_B = (cd \mid ca)$

$A = ab$, $B = de$: $\quad H^A_B = (ad \mid be) - (bd \mid ae)$

Triplet 1×1

$A = abccdd$ $B = cdaabb$ $\quad H^A_B = (ca \mid db) - (da \mid cb)$

$A = abcc$ $B = dcaa$ $\quad H^A_B = (bd \mid ca)$

$A = abcc$ $B = dbaa$ $\quad H^A_B = -(cd \mid ca)$

$A = abcc$ $B = abdd$ $\quad H^A_B = (cd \mid cd)$

$A = abcc$ $B = dcbb$ $\quad H^A_B = -(ad \mid cb)$

$A = abcc$ $B = adbb$ $\quad H^A_B = -(cd \mid cb)$

$A = abcc$ $B = dabb$ $\quad H^A_B = (cd \mid cb)$

$A = abcc$ $B = acdd$ $\quad H^A_B = -(bd \mid cd)$

$A = abcc$ $B = bcdd$ $\quad H^A_B = (ad \mid cd)$

2. $N_A = 2, \quad N_B = 4 \quad (k_A = 1, \quad k_B = 3)$

$A = abcc$ $B = daeb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (cd \mid ce)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}} (cd \mid ce)$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}} (cd \mid ce)$$

$A = abccdd$ $B = ecdbaa$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}} (de \mid ca)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}} \left[2(ce \mid da) + (de \mid ca) \right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}} \left[-(ce \mid da) + (de \mid ca) \right]$$

$A = abccdd$ $B = cedbaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (de \mid ca)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}} \left[\frac{2}{3} (ce \mid da) - (de \mid ca) \right]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}} (ce \mid da)$$

$A = abccdd$ $B = cdebaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} \left[-(ce \mid da) + (de \mid ca) \right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}} \left[(ce \mid da) - 3(de \mid ca) \right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}} (ce \mid da)$$

$A = abcc$ $B = dacb$

$$H^A_{B_1} = \sqrt{2} \left[\frac{1}{2} \{ (c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid c P_a) \} - \frac{1}{2} (ca \mid ad) - (cb \mid bd) \right]$$

Triplet 1×3

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + 3\,(ca \mid ad)\Big]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}\Big\{\text{the above}\Big\}$$

$A = abcc$
$B = dcab$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ca \mid ad) - (cb \mid bd)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[-\frac{2}{3}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_aP_a) - (P_a d \mid cP_a)\} + (ca \mid ad) + (cb \mid bd)\Big]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\Big\{\text{the above}\Big\}$$

$A = abcc$
$B = bdec$

$$H^A_{B_1} = \sqrt{2}\Big[-(ad \mid ce) + \frac{1}{2}(cd \mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(cd \mid ae)$$

$$H^A_{B_3} = 0$$

$A = abcc$
$B = dbec$

$$H^A_{B_1} = \sqrt{2}\Big[(ad \mid ce) - \frac{1}{2}(cd \mid ae)\Big]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(cd \mid ae)$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(cd \mid ae)$$

$A = abcc$
$B = cbde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ad \mid ce) + (cd \mid ae)\Big]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\Big[(ad \mid ce) - (cd \mid ae)\Big]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}\Big[(ad \mid ce) - (cd \mid ae)\Big]$$

$A = abcc$
$B = acde$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(bd \mid ce) + (cd \mid be)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\Big[(bd \mid ce) - (cd \mid be)\Big]$$

$$H^A_{B_3} = 0$$

$A = abcc$
$B = bcde$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(ad \mid ce) + (cd \mid ae)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\Big[(ad \mid ce) - (cd \mid ae)\Big]$$

$$H^A_{B_3} = 0$$

$A = abcc$
$B = bdce$

$$H^A_{B_1} = \sqrt{2}\Big[-(ad \mid ce) + \frac{1}{2}(cd \mid ae)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(cd \mid ae)$$

$$H^A_{B_3} = 0$$

$A = abcc$
$B = dbce$

$$H^A_{B_1} = \sqrt{2}\left[(ad \mid ce) - \frac{1}{2}(cd \mid ae)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}(cd \mid ae)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}(cd \mid ae)$$

$A = abcc$
$B = decb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(ad \mid ce) - (cd \mid ae)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[(ad \mid ce) - \frac{1}{3}(cd \mid ae)\right]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(cd \mid ae)$$

$A = abcc$
$B = cdab$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(ca \mid ad) - (cb \mid bd)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[-2\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - (ca \mid ad) - (cb \mid bd)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\left[\{\text{the above}\} - (ca \mid ad) - (cb \mid bd)\right]$$

$A = abcc$
$B = adbc$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + \frac{1}{2}(ca \mid ad) + (cb \mid bd)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[-3\{\text{the above}\} + (ca \mid ad)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}(ca \mid ad)$$

$A = abcc$
$B = acbd$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(cb \mid bd)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[-\{\text{the above}\} + \frac{2}{3}(ca \mid ad) + (cb \mid bd)\right]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(ca \mid ad)$$

$A = abcc$
$B = abcd$

$$H^A_{B_1} = \sqrt{2}\left[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) + (cb \mid bd)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}(ca \mid ad)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}(ca \mid ad)$$

$A = abccdd$
$B = bcedaa$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}(ce \mid da) + (de \mid ca)\right]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(ce \mid da)$$

$$H^A_{B_3} = 0$$

Triplet 1 × 3

$A = abccdd$
$B = acedbb$

$$H^A_{B_1} = \sqrt{2}\left[\frac{1}{2}(ce \mid db) - (de \mid cb)\right]$$
$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce \mid db)$$
$$H^A_{B_3} = 0$$

$A = abccdd$
$B = caedbb$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}(ce \mid db) + (de \mid cb)\right]$$
$$H^A_{B_2} = \frac{1}{\sqrt{6}}(ce \mid db)$$
$$H^A_{B_3} = \frac{2}{\sqrt{3}}(ce \mid db)$$

$A = abccdd$
$B = bcdeaa$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}(ce \mid da) + (de \mid ca)\right]$$
$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce \mid da)$$
$$H^A_{B_3} = 0$$

$A = abccdd$
$B = acdebb$

$$H^A_{B_1} = \sqrt{2}\left[\frac{1}{2}(ce \mid db) - (de \mid cb)\right]$$
$$H^A_{B_2} = \sqrt{\frac{3}{2}}(ce \mid db)$$
$$H^A_{B_3} = 0$$

$A = abccdd$
$B = cadebb$

$$H^A_{B_1} = \sqrt{2}\left[-\frac{1}{2}(ce \mid db) + (de \mid cb)\right]$$
$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(ce \mid db)$$
$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(ce \mid db)$$

$A = abccdd$
$B = abcdee$

$$H^A_{B_1} = -\sqrt{2}(ce \mid de)$$
$$H^A_{B_2} = H^A_{B_3} = 0$$

$A = abccdd$
$B = acdbee$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}(ce \mid de)$$
$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ce \mid de)$$
$$H^A_{B_3} = 0$$

$A = abcc$
$B = abdc$

$$H^A_{B_1} = \sqrt{2}\left[\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2(cd \mid P_a P_a) - (P_a d \mid cP_a)\} - \frac{1}{2}(ca \mid ad) - \frac{1}{2}(cb \mid bd)\right]$$
$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\left[\frac{1}{3}(ca \mid ad) + (cb \mid bd)\right]$$
$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(ca \mid ad)$$

$A = abccdd$
$B = cabdee$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}(ce \mid de)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}(ce \mid de)$$

$$H^{A}_{B_3} = -\frac{2}{\sqrt{3}}(ce \mid de)$$

$A = abccdd$
$B = cdabee$

$$H^{A}_{B_1} = 0$$

$$H^{A}_{B_2} = \sqrt{\frac{2}{3}}(ce \mid de)$$

$$H^{A}_{B_3} = -\frac{2}{\sqrt{3}}(ce \mid de)$$

$A = abcc$
$B = debc$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ad \mid ce) - (cd \mid ae)\Big]$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}}\Big[-3\,(ad \mid ce) + (cd \mid ae)\Big]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}(cd \mid ae)$$

$A = abcc$
$B = cadb$

$$H^{A}_{B_1} = \sqrt{2}\Big[-\frac{1}{2}\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + \frac{1}{2}(cb \mid bd)\Big]$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} - 2\,(ca \mid ad) + (cb \mid bd)\Big]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}\Big[-\{\text{the above}\} + (ca \mid ad) + (cb \mid bd)\Big]$$

$A = abcc$
$B = cdab$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}}\Big[(ca \mid ad) - (cb \mid bd)\Big]$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\Big[2\,\{(c \mid h \mid d) + (cd \mid aa) + (cd \mid bb) + (cd \mid cc) + 2\,(cd \mid P_a P_a) - (P_a d \mid cP_a)\} + (ca \mid ad) + (cb \mid bd)\Big]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\} - (ca \mid ad) - (cb \mid bd)\Big]$$

$A = abcc$
$B = deab$

$$H^{A}_{B_1} = 0$$

$$H^{A}_{B_2} = -\sqrt{\frac{2}{3}}(cd \mid ce)$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}(cd \mid ce)$$

$A = abcc$
$B = dace$

$$H^{A}_{B_1} = \sqrt{2}\Big[-(bd \mid ce) + \frac{1}{2}(cd \mid be)\Big]$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}(cd \mid be)$$

$$H^{A}_{B_3} = -\frac{2}{\sqrt{3}}(cd \mid be)$$

$A = abcc$
$B = dcae$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}}(bd \mid ce)$$

Triplet 1×3

$$H^{A}_{B_2} = \sqrt{\frac{3}{2}}\left[(bd \mid ce) - \frac{2}{3}(cd \mid be)\right]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}(cd \mid be)$$

$A = abcc$
$B = cade$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}}\left[(bd \mid ce) + (cd \mid be)\right]$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}}\left[(bd \mid ce) - (cd \mid be)\right]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}\left[(bd \mid ce) - (cd \mid be)\right]$$

$A = abcc$
$B = cdae$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}(bd \mid ce)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[(bd \mid ce) + 2(cd \mid be)\right]$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}\left[-(bd \mid ce) + (cd \mid be)\right]$$

$A = abcc$
$B = deac$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}\left[(be \mid cd) + (ce \mid bd)\right]$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[(be \mid cd) - 3(ce \mid bd)\right]$$

$$H^{A}_{B_3} = -\frac{2}{\sqrt{3}}(be \mid cd)$$

$A = abcc$
$B = adec$

$$H^{A}_{B_1} = \sqrt{2}\left[(bd \mid ce) - \frac{1}{2}(cd \mid be)\right]$$

$$H^{A}_{B_2} = -\sqrt{\frac{3}{2}}(cd \mid be)$$

$$H^{A}_{B_3} = 0$$

$A = abcc$
$B = daec$

$$H^{A}_{B_1} = \sqrt{2}\left[-(bd \mid ce) + \frac{1}{2}(cd \mid be)\right]$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}}(cd \mid be)$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}}(cd \mid be)$$

$A = abcc$
$B = adeb$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}(cd \mid ce)$$

$$H^{A}_{B_2} = \sqrt{\frac{3}{2}}(cd \mid ce)$$

$$H^{A}_{B_3} = 0$$

3. $N_A = 2$, $N_B = 6$ $(k_A = 1,\quad k_B = 9)$

$A = abccdd$
$B = deafbc$

$$H^{A}_{B_1} = \frac{1}{2}(ce \mid df)$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{3}}\left[(de \mid cf) + \frac{1}{2}(ce \mid df)\right]$$

Triplet 1×9

$$H^A_{B_3} = -\sqrt{\frac{2}{3}}\left[(de\mid cf) - (ce\mid df)\right]$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}(ce\mid df)$$

$$H^A_{B_5} = \frac{1}{3}(ce\mid df)$$

$$H^A_{B_6} = -\sqrt{2}\left[(de\mid cf) - \frac{1}{3}(ce\mid df)\right]$$

$$H^A_{B_7} = \frac{\sqrt{2}}{3}(ce\mid df)$$

$$H^A_{B_8} = (de\mid cf) - \frac{5}{6}(ce\mid df)$$

$$H^A_{B_9} = -\frac{\sqrt{3}}{6}(ce\mid df)$$

$A = abccdd$
$B = defacb$

$$H^A_{B_1} = \frac{1}{2}(ce\mid df)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{3}}\left[(de\mid cf) + \frac{1}{2}(ce\mid df)\right]$$

$$H^A_{B_3} = \frac{\sqrt{6}}{3}\left[(de\mid cf) - (ce\mid df)\right]$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}(ce\mid df)$$

$$H^A_{B_5} = -\frac{4}{3}(de\mid cf) + \frac{1}{3}(ce\mid df)$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}\left[(de\mid cf) - (ce\mid df)\right]$$

$$H^A_{B_7} = \frac{2\sqrt{2}}{3}\left[(de\mid cf) - (ce\mid df)\right]$$

$$H^A_{B_8} = \frac{1}{3}(de\mid cf) + \frac{1}{6}(ce\mid df)$$

$$H^A_{B_9} = -\frac{\sqrt{3}}{6}(ce\mid df)$$

$A = abccdd$
$B = deacbf$

$$H^A_{B_1} = -\frac{1}{2}(ce\mid df)$$

$$H^A_{B_2} = \frac{1}{\sqrt{3}}\left[-\frac{3}{2}(ce\mid df) + (de\mid cf)\right]$$

$$H^A_{B_3} = -\frac{\sqrt{6}}{3}(de\mid cf)$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}(ce\mid df)$$

$$H^A_{B_5} = -\frac{1}{3}(ce\mid df)$$

$$H^A_{B_6} = \sqrt{2}\left[\frac{2}{3}(ce\mid df) - (de\mid cf)\right]$$

$$H^A_{B_7} = -\frac{\sqrt{2}}{3}(ce\mid df)$$

Triplet 1×9

$$H^A_{B_8} = -\frac{1}{6}(ce \mid df) + (de \mid cf)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}(ce \mid df)$$

$A = abccdd$
$B = decafb$

$$H^A_{B_1} = -\frac{1}{2}(ce \mid df)$$

$$H^A_{B_2} = \frac{\sqrt{3}}{2}\left[(ce \mid df) - \frac{2}{3}(de \mid cf)\right]$$

$$H^A_{B_3} = \frac{\sqrt{6}}{3}(de \mid cf)$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}(ce \mid df)$$

$$H^A_{B_5} = (ce \mid df) - \frac{4}{3}(de \mid cf)$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}(de \mid cf)$$

$$H^A_{B_7} = \frac{2\sqrt{2}}{3}(de \mid cf)$$

$$H^A_{B_8} = -\frac{1}{2}(ce \mid df) + \frac{1}{3}(de \mid cf)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}(ce \mid df)$$

$A = abccdd$
$B = efabcd$

$$H^A_{B_1} = 0$$

$$H^A_{B_2} = \frac{1}{\sqrt{3}}\left[(ce \mid df) + (de \mid cf)\right]$$

$$H^A_{B_3} = -\frac{\sqrt{6}}{3}\left[(ce \mid df) + (de \mid cf)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_5} = -\frac{1}{3}(ce \mid df) + \frac{1}{3}(de \mid cf)$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_7} = -\frac{\sqrt{2}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_8} = -\frac{2}{3}(ce \mid df) + \frac{2}{3}(de \mid cf)$$

$$H^A_{B_9} = -\frac{1}{\sqrt{3}}\left[(ce \mid df) - (de \mid cf)\right]$$

$A = abccdd$
$B = efacdb$

$$H^A_{B_1} = \frac{1}{2}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_2} = \frac{\sqrt{3}}{2}\left[\frac{1}{3}(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_3} = \frac{\sqrt{6}}{3}(ce \mid df)$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}\Big[-(ce\,|\,df) + (de\,|\,cf)\Big]$$

$$H^A_{B_5} = -\frac{1}{3}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}\Big[(ce\,|\,df) + 2\,(de\,|\,cf)\Big]$$

$$H^A_{B_7} = -\frac{\sqrt{2}}{3}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$$H^A_{B_8} = \frac{5}{6}(ce\,|\,df) + \frac{1}{6}(de\,|\,cf)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$A = abccdd$
$B = efcabd$

$$H^A_{B_1} = \frac{1}{2}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$$H^A_{B_2} = \frac{\sqrt{3}}{2}\Big[-\frac{1}{3}(ce\,|\,df) + (de\,|\,cf)\Big]$$

$$H^A_{B_3} = -\frac{\sqrt{6}}{3}(ce\,|\,df)$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}\Big[-(ce\,|\,df) + (de\,|\,cf)\Big]$$

$$H^A_{B_5} = -\frac{1}{3}(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}(ce\,|\,df)$$

$$H^A_{B_7} = \frac{2\sqrt{2}}{3}(ce\,|\,df)$$

$$H^A_{B_8} = -\frac{1}{6}(ce\,|\,df) + \frac{1}{2}(de\,|\,cf)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$A = abccdd$
$B = efcdab$

$$H^A_{B_1} = H^A_{B_2} = H^A_{B_3} = 0$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$$H^A_{B_5} = -\frac{1}{3}(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_6} = H^A_{B_7} = \frac{2\sqrt{2}}{3}(ce\,|\,df)$$

$$H^A_{B_8} = \frac{1}{3}(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_9} = -\frac{1}{\sqrt{3}}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

$A = abccdd$
$B = abefcd$

$$H^A_{B_1} = -(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_9} = -\sqrt{3}\Big[(ce\,|\,df) - (de\,|\,cf)\Big]$$

Triplet 1×9

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = H^A_{B_5} = H^A_{B_6} = H^A_{B_7} = H^A_{B_8} = 0$$

$A = abccdd$
$B = abecdf$

$$H^A_{B_1} = 2\,(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_9} = -\sqrt{3}\,(de\,|\,cf)$$

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = H^A_{B_5} = H^A_{B_6} = H^A_{B_7} = H^A_{B_8} = 0$$

$A = abccdd$
$B = abcefd$

$$H^A_{B_1} = 2\,(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_9} = -\sqrt{3}\,(de\,|\,cf)$$

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = H^A_{B_5} = H^A_{B_6} = H^A_{B_7} = H^A_{B_8} = 0$$

$A = abccdd$
$B = abcdef$

$$H^A_{B_1} = -(ce\,|\,df) - (de\,|\,cf)$$

$$H^A_{B_9} = -\sqrt{3}\left[(ce\,|\,df) - (de\,|\,cf)\right]$$

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = H^A_{B_5} = H^A_{B_6} = H^A_{B_7} = H^A_{B_8} = 0$$

$A = abccdd$
$B = edafbc$

$$H^A_{B_1} = -\frac{1}{2}\,(ce\,|\,df)$$

$$H^A_{B_2} = \frac{1}{\sqrt{3}}\left[(de\,|\,cf) - \frac{3}{2}\,(ce\,|\,df)\right]$$

$$H^A_{B_3} = -\frac{\sqrt{6}}{3}\,(de\,|\,cf)$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}\,(ce\,|\,df)$$

$$H^A_{B_5} = -\frac{1}{3}\,(ce\,|\,df)$$

$$H^A_{B_6} = -\sqrt{2}\left[(de\,|\,cf) - \frac{2}{3}\,(ce\,|\,df)\right]$$

$$H^A_{B_7} = -\frac{\sqrt{2}}{3}\,(ce\,|\,df)$$

$$H^A_{B_8} = (de\,|\,cf) - \frac{1}{6}\,(ce\,|\,df)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}\,(ce\,|\,df)$$

$A = abccdd$
$B = edfacb$

$$H^A_{B_1} = -\frac{1}{2}\,(ce\,|\,df)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{3}}\left[(de\,|\,cf) - \frac{3}{2}\,(ce\,|\,df)\right]$$

$$H^A_{B_3} = \frac{\sqrt{6}}{3}\,(de\,|\,cf)$$

$$H^A_{B_4} = -\frac{\sqrt{15}}{3}\,(ce\,|\,df)$$

$$H^A_{B_5} = -\frac{4}{3}\,(de\,|\,cf) + (ce\,|\,df)$$

$$H^A_{B_6} = -\frac{\sqrt{2}}{3}\,(de\,|\,cf)$$

$$H^A_{B_7} = \frac{2\sqrt{2}}{3}(de \mid cf)$$

$$H^A_{B_8} = \frac{1}{3}(de \mid cf) - \frac{1}{2}(ce \mid df)$$

$$H^A_{B_9} = \frac{\sqrt{3}}{6}(ce \mid df)$$

$A = abccdd$
$B = edacbf$

$$H^A_{B_1} = \frac{1}{2}(ce \mid df)$$

$$H^A_{B_2} = \frac{1}{\sqrt{3}}\left[\frac{1}{2}(ce \mid df) + (de \mid cf)\right]$$

$$H^A_{B_3} = \frac{\sqrt{6}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}(ce \mid df)$$

$$H^A_{B_5} = \frac{1}{3}(ce \mid df)$$

$$H^A_{B_6} = \frac{\sqrt{2}}{3}(ce \mid df) - \sqrt{2}(de \mid cf)$$

$$H^A_{B_7} = \frac{\sqrt{2}}{3}(ce \mid df)$$

$$H^A_{B_8} = -\frac{5}{6}(ce \mid df) + (de \mid cf)$$

$$H^A_{B_9} = -\frac{\sqrt{3}}{6}(ce \mid df)$$

$A = abccdd$
$B = edcafb$

$$H^A_{B_1} = \frac{1}{2}(ce \mid df)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{3}}\left[\frac{1}{2}(ce \mid df) + (de \mid cf)\right]$$

$$H^A_{B_3} = -\frac{\sqrt{6}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_4} = \frac{\sqrt{15}}{3}(ce \mid df)$$

$$H^A_{B_5} = \frac{1}{3}(ce \mid df) - \frac{4}{3}(de \mid cf)$$

$$H^A_{B_6} = \frac{\sqrt{2}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_7} = -\frac{2\sqrt{2}}{3}\left[(ce \mid df) - (de \mid cf)\right]$$

$$H^A_{B_8} = \frac{1}{6}(ce \mid df) + \frac{1}{3}(de \mid cf)$$

$$H^A_{B_9} = -\frac{\sqrt{3}}{6}(ce \mid df)$$

4. $N_A = 4$, $N_B = 4$ $(k_A = 3, \quad k_B = 3)$

$A = abcd$
$B = eacd$

$$H^{A_1}_{B_1} = -\Big\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + (be \mid dd) + 2(be \mid P_a P_a) - (P_a e \mid bP_a)\Big\} + (ba \mid ae) + \frac{1}{2}(bc \mid ce) + \frac{1}{2}(bd \mid de)$$

Triplet 3×3

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{6}\Big[(bc \mid ce) - (bd \mid de)\Big]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{3}\Big[(bc \mid ce) - (bd \mid de)\Big]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\Big[(bc \mid ce) - (bd \mid de)\Big]$$

$$H_{B_2}^{A_2} = -\frac{1}{3}\Big\{\text{the above}\Big\} + (ba \mid ae) - \frac{1}{6}(bc \mid ce) - \frac{1}{6}(bd \mid de)$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}\Big\{\text{the above}\Big\} - \frac{\sqrt{2}}{3}(bc \mid ce) - \frac{\sqrt{2}}{3}(bd \mid de)$$

$$H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_3} = -\frac{2\sqrt{2}}{3}\Big\{\text{the above}\Big\} + \frac{2\sqrt{2}}{3}(bc \mid ce) + \frac{2\sqrt{2}}{3}(bd \mid de)$$

$$H_{B_3}^{A_3} = \frac{1}{3}\Big\{\text{the above}\Big\} + (ba \mid ae) - \frac{1}{3}(bc \mid ce) - \frac{1}{3}(bd \mid de)$$

$A = abcd$
$B = ebcd$

$$H_{B_1}^{A_1} = \Big\{(a \mid h \mid e) + (ae \mid bb) + (ae \mid cc) + (ae \mid dd) + 2\,(ae \mid P_a P_a) - (P_a e \mid a P_a)\Big\} - (ab \mid be) - \frac{1}{2}(ac \mid ce) - \frac{1}{2}(ad \mid de)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{6}\Big[(ac \mid ce) - (ad \mid de)\Big] = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = -\frac{\sqrt{6}}{3}\Big[(ac \mid ce) - (ad \mid de)\Big] = H_{B_1}^{A_3}$$

$$H_{B_2}^{A_2} = \Big\{\text{the above}\Big\} - \frac{1}{3}(ab \mid be) - \frac{5}{6}(ac \mid ce) - \frac{5}{6}(ad \mid de)$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}\Big[(ab \mid be) - \frac{1}{2}(ac \mid ce) - \frac{1}{2}(ad \mid de)\Big] = H_{B_2}^{A_3}$$

$$H_{B_3}^{A_3} = \Big\{\text{the above}\Big\} + \frac{1}{3}(ab \mid be) + \frac{1}{3}(ac \mid ce) + \frac{1}{3}(ad \mid de)$$

$A = abcdee$
$B = efcdaa$

$$H_{B_1}^{A_1} = -(bf \mid ea)$$

$$H_{B_2}^{A_1} = H_{B_3}^{A_1} = H_{B_1}^{A_2} = H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_2} = -(bf \mid ea) + \frac{2}{3}(ef \mid ba)$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}(ef \mid ba) = H_{B_2}^{A_3}$$

$$H_{B_3}^{A_3} = -(bf \mid ea) + \frac{4}{3}(ef \mid ba)$$

$A = abcdee$
$B = fecdaa$

$$H_{B_1}^{A_1} = (bf \mid ea)$$

$$H_{B_2}^{A_1} = H_{B_3}^{A_1} = H_{B_1}^{A_2} = H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_2} = \frac{1}{3}(bf \mid ea) + \frac{2}{3}(ef \mid ba)$$

$$H_{B_3}^{A_2} = \frac{2\sqrt{2}}{3}\Big[(bf \mid ea) - (ef \mid ba)\Big] = H_{B_2}^{A_3}$$

$$H_{B_3}^{A_3} = -\frac{1}{3}(bf \mid ea) + \frac{4}{3}(ef \mid ba)$$

$A = abcdee$
$B = fecdbb$

$$H_{B_1}^{A_1} = -(af \mid eb)$$

$$H_{B_2}^{A_1} = H_{B_3}^{A_1} = H_{B_1}^{A_2} = H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_2} = -(af \mid eb) + \frac{2}{3}(ef \mid ab)$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}(ef \mid ab) = H_{B_2}^{A_3}$$

$$H_{B_3}^{A_3} = -(af \mid eb) + \frac{4}{3}(ef \mid ab)$$

$A = abcdee$
$B = acfedd$

$$H_{B_1}^{A_1} = \frac{1}{2}(bf \mid ed) - (ef \mid bd)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(bf \mid ed)$$

$$H_{B_3}^{A_1} = H_{B_3}^{A_2} = H_{B_1}^{A_3} = H_{B_2}^{A_3} = 0$$

$$H_{B_1}^{A_2} = \sqrt{3}\left[\frac{1}{2}(bf \mid ed) - (ef \mid bd)\right]$$

$$H_{B_2}^{A_2} = -\frac{1}{2}(bf \mid ed)$$

$$H_{B_3}^{A_3} = (bf \mid ed)$$

$A = abcdee$
$B = bdfecc$

$$H_{B_1}^{A_1} = -\frac{1}{2}(af \mid ec) + (ef \mid ac)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(af \mid ec)$$

$$H_{B_3}^{A_1} = 0$$

$$H_{B_1}^{A_2} = \frac{1}{\sqrt{3}}\left[\frac{1}{2}(af \mid ec) - (ef \mid ac)\right]$$

$$H_{B_2}^{A_2} = -\frac{1}{6}(af \mid ec)$$

$$H_{B_3}^{A_2} = \frac{2\sqrt{2}}{3}(af \mid ec)$$

$$H_{B_1}^{A_3} = \frac{\sqrt{6}}{3}\left[(af \mid ec) - 2(ef \mid ac)\right]$$

$$H_{B_2}^{A_3} = -\frac{\sqrt{2}}{3}(af \mid ec)$$

$$H_{B_3}^{A_3} = -\frac{1}{3}(af \mid ec)$$

$A = abcdee$
$B = acfebb$

$$H_{B_1}^{A_1} = \frac{1}{2}(df \mid eb) - (ef \mid db)$$

$$H_{B_2}^{A_1} = -\frac{\sqrt{3}}{2}(df \mid eb)$$

$$H_{B_3}^{A_1} = H_{B_3}^{A_2} = H_{B_1}^{A_3} = H_{B_2}^{A_3} = 0$$

$$H_{B_1}^{A_2} = \sqrt{3}\left[\frac{1}{2}(df \mid eb) - (ef \mid db)\right]$$

$$H_{B_2}^{A_2} = \frac{1}{2}(df \mid eb)$$

Triplet 3×3

$$H_{B_3}^{A_3} = -\,(df \mid eb)$$

$A = abcdee$
$B = bdfeaa$

$$H_{B_1}^{A_1} = -\frac{1}{2}\,(cf \mid ea) + (ef \mid ca)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\,(cf \mid ea)$$

$$H_{B_3}^{A_1} = 0$$

$$H_{B_1}^{A_2} = \frac{1}{\sqrt{3}}\left[\frac{1}{2}\,(cf \mid ea) - (ef \mid ca)\right]$$

$$H_{B_2}^{A_2} = \frac{1}{6}\,(cf \mid ea)$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}\,(cf \mid ea)$$

$$H_{B_1}^{A_3} = \frac{\sqrt{6}}{3}\left[(cf \mid ea) - 2\,(ef \mid ca)\right]$$

$$H_{B_2}^{A_3} = \frac{\sqrt{2}}{3}\,(cf \mid ea)$$

$$H_{B_3}^{A_3} = \frac{1}{3}\,(cf \mid ea)$$

$A = abcdee$
$B = becdaa$

$$H_{B_1}^{A_1} = \Big\{(e \mid h \mid a) + (ea \mid aa) + (ea \mid bb) + (ea \mid cc) + (ea \mid dd) + (ea \mid ee) + 2\,(ea \mid P_a P_a) - (P_a a \mid eP_a)\Big\} - \frac{1}{2}\,(ec \mid ca) - \frac{1}{2}\,(ed \mid da)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\left[(ec \mid ca) - (ed \mid da)\right]$$

$$H_{B_3}^{A_1} = 0$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{6}\left[(ec \mid ca) - (ed \mid da)\right]$$

$$H_{B_2}^{A_2} = -\frac{1}{3}\,\Big\{\text{the above}\Big\} + \frac{2}{3}\,(eb \mid ba) - \frac{1}{2}\,(ec \mid ca) - \frac{1}{2}\,(ed \mid da)$$

$$H_{B_3}^{A_2} = \frac{2\sqrt{2}}{3}\,\Big\{\text{the above}\Big\} - \frac{2\sqrt{2}}{3}\,(eb \mid ba)$$

$$H_{B_1}^{A_3} = \frac{\sqrt{6}}{3}\left[(ec \mid ca) - (ed \mid da)\right]$$

$$H_{B_2}^{A_3} = \frac{2\sqrt{2}}{3}\left[\{\text{the above}\} - (eb \mid ba) - \frac{3}{2}\,(ec \mid ca) - \frac{3}{2}\,(ed \mid da)\right]$$

$$H_{B_3}^{A_3} = -\frac{1}{3}\,\Big\{\text{the above}\Big\} + \frac{4}{3}\,(eb \mid ba)$$

$A = abcdee$
$B = aecdbb$

$$H_{B_1}^{A_1} = -\Big\{(e \mid h \mid b) + (eb \mid aa) + (eb \mid bb) + (eb \mid cc) + (eb \mid dd) + (eb \mid ee) + 2\,(eb \mid P_a P_a) - (P_a b \mid eP_a)\Big\} + \frac{1}{2}\,(ec \mid cb) + \frac{1}{2}\,(ed \mid db)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\left[-\,(ec \mid cb) + (ed \mid db)\right] = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = H_{B_1}^{A_3} = 0$$

Triplet 3×3

$$H^{A_2}_{B_2} = -\left\{\text{the above}\right\} + \frac{2}{3}(ea \mid ab) + \frac{3}{2}(ec \mid cb) + \frac{3}{2}(ed \mid db)$$

$$H^{A_2}_{B_3} = -\frac{2\sqrt{2}}{3}(ea \mid ab) = H^{A_3}_{B_2}$$

$$H^{A_3}_{B_3} = -\left\{\text{the above}\right\} + \frac{4}{3}(ea \mid ab)$$

$A = abcdee$
$B = acedbb$

$$H^{A_1}_{B_1} = \frac{1}{2}\Big\{(e \mid h \mid b) + (eb \mid aa) + (eb \mid bb) + (eb \mid cc) + (eb \mid dd) + (eb \mid ee) + 2(eb \mid P_a P_a) - (P_a b \mid eP_a)\Big\} + \frac{1}{2}(ec \mid cb) - (ed \mid db)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\left\{\text{the above}\right\} - \frac{\sqrt{3}}{2}(ec \mid cb)$$

$$H^{A_1}_{B_3} = 0$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}\left[\{\text{the above}\} - \frac{2}{3}(ea \mid ab) - (ec \mid cb) - 2(ed \mid db)\right]$$

$$H^{A_2}_{B_2} = -\frac{1}{2}\left\{\text{the above}\right\} + \frac{1}{3}(ea \mid ab) + \frac{3}{2}(ec \mid cb)$$

$$H^{A_2}_{B_3} = \frac{2\sqrt{2}}{3}(ea \mid ab)$$

$$H^{A_3}_{B_1} = \frac{\sqrt{6}}{3}(ea \mid ab)$$

$$H^{A_3}_{B_2} = -\frac{\sqrt{2}}{3}(ea \mid ab)$$

$$H^{A_3}_{B_3} = \left\{\text{the above}\right\} - \frac{4}{3}(ea \mid ab)$$

$A = abcdee$
$B = acefbb$

$$H^{A_1}_{B_1} = \frac{1}{2}(df \mid eb) - (ef \mid db)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}(df \mid eb)$$

$$H^{A_1}_{B_3} = H^{A_2}_{B_3} = H^{A_3}_{B_1} = H^{A_3}_{B_2} = 0$$

$$H^{A_2}_{B_1} = \sqrt{3}\left[\frac{1}{2}(df \mid eb) - (ef \mid db)\right]$$

$$H^{A_2}_{B_2} = -\frac{1}{2}(df \mid eb)$$

$$H^{A_3}_{B_3} = (df \mid eb)$$

$A = abcdee$
$B = aebfcc$

$$H^{A_1}_{B_1} = \frac{1}{2}(df \mid ec) - (ef \mid dc)$$

$$H^{A_1}_{B_2} = \sqrt{3}\left[\frac{1}{2}(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A_1}_{B_3} = H^{A_2}_{B_3} = H^{A_3}_{B_1} = H^{A_3}_{B_2} = 0$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(df \mid ec)$$

$$H^{A_2}_{B_2} = -\frac{1}{2}(df \mid ec)$$

Triplet 3×3

$$H_{B_3}^{A_3} = (df \mid ec)$$

$A = abcd$
$B = abce$

$$H_{B_1}^{A_1} = \Big\{(d \mid h \mid e) + (de \mid aa) + (de \mid bb) + (de \mid cc) + 2\,(de \mid P_a P_a) - (P_a e \mid dP_a)\Big\} - \frac{1}{2}\,(da \mid ae) - \frac{1}{2}\,(db \mid be) + (dc \mid ce)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}\Big[\frac{1}{3}\,(da \mid ae) + (db \mid be)\Big] = H_{B_1}^{A_2}$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{3}\,(da \mid ae) = H_{B_1}^{A_3}$$

$$H_{B_2}^{A_2} = \Big\{\text{the above}\Big\} - \frac{5}{6}\,(da \mid ae) + \frac{1}{2}\,(db \mid be) - (dc \mid ce)$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{3}\,(da \mid ae) = H_{B_2}^{A_3}$$

$$H_{B_3}^{A_3} = \Big\{\text{the above}\Big\} + \frac{1}{3}\,(da \mid ae) - (db \mid be) - (dc \mid ce)$$

$A = abcdee$
$B = abcdff$

$$H_{B_1}^{A_1} = H_{B_2}^{A_2} = H_{B_3}^{A_3} = (ef \mid ef)$$

$$H_{B_2}^{A_1} = H_{B_3}^{A_1} = H_{B_1}^{A_2} = H_{B_3}^{A_2} = H_{B_1}^{A_3} = H_{B_2}^{A_3} = 0$$

$A = abcdee$
$B = afbecc$

$$H_{B_1}^{A_1} = \frac{1}{2}\,(df \mid ec) - (ef \mid dc)$$

$$H_{B_2}^{A_1} = \sqrt{3}\,\Big[\frac{1}{2}\,(df \mid ec) - (ef \mid dc)\Big]$$

$$H_{B_3}^{A_1} = H_{B_3}^{A_2} = H_{B_1}^{A_3} = H_{B_2}^{A_3} = 0$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}\,(df \mid ec)$$

$$H_{B_2}^{A_2} = \frac{1}{2}\,(df \mid ec)$$

$$H_{B_3}^{A_3} = -(df \mid ec)$$

$A = abcdee$
$B = faebcc$

$$H_{B_1}^{A_1} = -\frac{1}{2}\,(df \mid ec) + (ef \mid dc)$$

$$H_{B_2}^{A_1} = \frac{1}{\sqrt{3}}\Big[\frac{1}{2}\,(df \mid ec) - (ef \mid dc)\Big]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{3}\Big[(df \mid ec) - 2\,(ef \mid dc)\Big]$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{2}\,(df \mid ec)$$

$$H_{B_2}^{A_2} = \frac{1}{6}\,(df \mid ec)$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{3}\,(df \mid ec)$$

$$H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_3} = -\frac{2\sqrt{2}}{3}\,(df \mid ec)$$

$$H_{B_3}^{A_3} = \frac{1}{3}\,(df \mid ec)$$

Triplet 3×3

$A = abcdee$
$B = eafbcc$

$$H_{B_1}^{A_1} = -\frac{1}{2}(df \mid ec) + (ef \mid dc)$$

$$H_{B_2}^{A_1} = \frac{1}{\sqrt{3}}\left[-\frac{1}{2}(df \mid ec) - (ef \mid dc)\right]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{3}\left[(df \mid ec) - 2(ef \mid dc)\right]$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(df \mid ec)$$

$$H_{B_2}^{A_2} = -\frac{1}{6}(df \mid ec)$$

$$H_{B_3}^{A_2} = -\frac{\sqrt{2}}{3}(df \mid ec)$$

$$H_{B_1}^{A_3} = 0$$

$$H_{B_2}^{A_3} = \frac{2\sqrt{2}}{3}(df \mid ec)$$

$$H_{B_3}^{A_3} = -\frac{1}{3}(df \mid ec)$$

5. $N_A = 4,\quad N_B = 6\qquad (k_A = 3,\quad k_B = 9)$

$A = abcdee$
$B = cdeafb$

$$H_{B_1}^{A_1} = \frac{\sqrt{2}}{4}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H_{B_2}^{A_1} = \frac{1}{\sqrt{6}}\Big[-\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} - (eb \mid bf) - \frac{1}{2}(ec \mid cf) + \frac{3}{2}(ed \mid df)\Big]$$

$$H_{B_3}^{A_1} = \frac{1}{\sqrt{3}}\left[\{\text{the above}\} + (eb \mid bf) - (ec \mid cf)\right]$$

$$H_{B_4}^{A_1} = \sqrt{\frac{5}{6}}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H_{B_5}^{A_1} = \frac{2\sqrt{2}}{3}\Big[-\{\text{the above}\} + (ea \mid af) + (eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{3}{4}(ed \mid df)\Big]$$

$$H_{B_6}^{A_1} = \frac{1}{3}\left[-\{\text{the above}\} - 2(ea \mid af) + (eb \mid bf) + (ec \mid cf)\right]$$

$$H_{B_7}^{A_1} = \frac{2}{3}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf) - (ec \mid cf)\right]$$

$$H_{B_8}^{A_1} = \frac{\sqrt{2}}{6}\Big[\{\text{the above}\} + 2(ea \mid af) - (eb \mid bf) + \frac{1}{2}(ec \mid cf) - \frac{3}{2}(ed \mid df)\Big]$$

$$H_{B_9}^{A_1} = \frac{\sqrt{6}}{12}\left[-(ec \mid cf) + (ed \mid df)\right]$$

$$H_{B_1}^{A_2} = \sqrt{\frac{2}{3}}\Big[-\frac{1}{2}\{\text{the above}\} - (ea \mid af) + \frac{1}{2}(eb \mid bf) - \frac{3}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\Big]$$

Triplet 3×9

$$H^{A_2}_{B_2} = \sqrt{2}\left[-\frac{1}{3}\ \{\text{the above}\} - \frac{1}{3}(eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{3}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_3} = -\frac{1}{3}\left[\{\text{the above}\} + (eb \mid bf) - 3\,(ec \mid cf)\right]$$

$$H^{A_2}_{B_4} = \frac{\sqrt{10}}{6}\left[(ec \mid cf) + (ed \mid df)\right]$$

$$H^{A_2}_{B_5} = \frac{2\sqrt{6}}{9}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf) + \frac{1}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_6} = \frac{\sqrt{3}}{9}\left[\{\text{the above}\} + 2\,(ea \mid af) - (eb \mid bf) + (ec \mid cf)\right]$$

$$H^{A_2}_{B_7} = \frac{4\sqrt{3}}{9}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf) - \frac{1}{2}(ec \mid cf)\right]$$

$$H^{A_2}_{B_8} = \frac{\sqrt{6}}{9}\left[\{\text{the above}\} + 2\,(ea \mid af) - (eb \mid bf) + \frac{1}{4}(ec \mid cf) + \frac{3}{4}(ed \mid df)\right]$$

$$H^{A_2}_{B_9} = \frac{1}{\sqrt{2}}\left[\{\text{the above}\} - (eb \mid bf) - \frac{1}{6}(ec \mid cf) - \frac{1}{6}(ed \mid df)\right]$$

$$H^{A_3}_{B_1} = \frac{1}{\sqrt{3}}\left[-\ \{\text{the above}\} + 2\,(ea \mid af) - (eb \mid bf)\right]$$

$$H^{A_3}_{B_2} = -\frac{1}{3}\left[\{\text{the above}\} + (eb \mid bf)\right]$$

$$H^{A_3}_{B_3} = -\frac{\sqrt{2}}{6}\left[\{\text{the above}\} + (eb \mid bf)\right]$$

$$H^{A_3}_{B_4} = \frac{\sqrt{5}}{6}\left[(ec \mid cf) + (ed \mid df)\right]$$

$$H^{A_3}_{B_5} = \frac{2\sqrt{3}}{9}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf) + \frac{1}{4}(ec \mid cf) - \frac{3}{4}(ed \mid df)\right]$$

$$H^{A_3}_{B_6} = \frac{\sqrt{6}}{18}\left[\{\text{the above}\} + 2\,(ea \mid af) - (eb \mid bf) - 8\,(ec \mid cf)\right]$$

$$H^{A_3}_{B_7} = \frac{2\sqrt{6}}{9}\left[\{\text{the above}\} - (ea \mid af) - (eb \mid bf) - \frac{1}{2}(ec \mid cf)\right]$$

$$H^{A_3}_{B_8} = \frac{\sqrt{3}}{9}\left[\{\text{the above}\} + 2\,(ea \mid af) - (eb \mid bf) - 2\,(ec \mid cf) - 6\,(ed \mid df)\right]$$

$$H^{A_3}_{B_9} = -\left\{\text{the above}\right\} + (eb \mid bf) + \frac{2}{3}(ec \mid cf) + \frac{2}{3}(ed \mid df)$$

$A = abcdee$
$B = cdaebf$

$$H^{A_1}_{B_1} = \frac{\sqrt{2}}{4}\left[(ec \mid cf) - (ed \mid df)\right]$$

$$H^{A_1}_{B_2} = \frac{1}{\sqrt{6}}\left[\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2\,(ef \mid P_a P_a) - (P_a f \mid eP_a)\} + (eb \mid bf) + \frac{1}{2}(ec \mid cf) - \frac{3}{2}(ed \mid df)\right]$$

Triplet 3×9

$$H^{A_1}_{B_3} = -\frac{1}{\sqrt{3}}\Big[\{\text{the above}\} + (eb \mid bf) - (ec \mid cf)\Big]$$

$$H^{A_1}_{B_4} = \sqrt{\frac{5}{6}}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H^{A_1}_{B_5} = \frac{\sqrt{2}}{3}\Big[-2(ea \mid af) + \frac{1}{2}(ec \mid cf) - \frac{1}{2}(ed \mid df)\Big]$$

$$H^{A_1}_{B_6} = -\Big\{\text{the above}\Big\} + \frac{2}{3}(ea \mid af) + (eb \mid bf) + \frac{1}{3}(ec \mid cf) + \frac{2}{3}(ed \mid df)$$

$$H^{A_1}_{B_7} = \frac{2}{3}(ea \mid af) + \frac{1}{3}(ec \mid cf) - \frac{1}{3}(ed \mid df)$$

$$H^{A_1}_{B_8} = \frac{1}{\sqrt{2}}\Big[\{\text{the above}\} - \frac{2}{3}(ea \mid af) - (eb \mid bf) - \frac{5}{6}(ec \mid cf) - \frac{1}{6}(ed \mid df)\Big]$$

$$H^{A_1}_{B_9} = -\frac{\sqrt{6}}{12}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H^{A_2}_{B_1} = \frac{1}{\sqrt{6}}\Big[\{\text{the above}\} - 2(ea \mid af) + (eb \mid bf) - \frac{3}{2}(ec \mid cf) - \frac{3}{2}(ed \mid df)\Big]$$

$$H^{A_2}_{B_2} = \frac{\sqrt{2}}{3}\Big[\{\text{the above}\} + (eb \mid bf) - \frac{3}{4}(ec \mid cf) - \frac{9}{4}(ed \mid df)\Big]$$

$$H^{A_2}_{B_3} = \frac{1}{3}\Big[\{\text{the above}\} + (eb \mid bf) - 3(ec \mid cf)\Big]$$

$$H^{A_2}_{B_4} = \frac{\sqrt{10}}{6}\Big[(ec \mid cf) + (ed \mid df)\Big]$$

$$H^{A_2}_{B_5} = \frac{\sqrt{6}}{18}\Big[4(ea \mid af) + (ec \mid cf) + (ed \mid df)\Big]$$

$$H^{A_2}_{B_6} = \frac{1}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{2}{3}(ea \mid af) - (eb \mid bf) + \frac{1}{3}(ec \mid cf) - (ed \mid df)\Big]$$

$$H^{A_2}_{B_7} = \frac{\sqrt{3}}{9}\Big[4(ea \mid af) + (ec \mid cf) + (ed \mid df)\Big]$$

$$H^{A_2}_{B_8} = \sqrt{\frac{2}{3}}\Big[\{\text{the above}\} - \frac{2}{3}(ea \mid af) - (eb \mid bf) - \frac{5}{12}(ec \mid cf) + \frac{1}{12}(ed \mid df)\Big]$$

$$H^{A_2}_{B_9} = \frac{1}{\sqrt{2}}\Big[\{\text{the above}\} - (eb \mid bf) - \frac{1}{6}(ec \mid cf) - \frac{1}{6}(ed \mid df)\Big]$$

$$H^{A_3}_{B_1} = \frac{1}{\sqrt{3}}\Big[-\{\text{the above}\} + 2(ea \mid af) - (eb \mid bf)\Big]$$

$$H^{A_3}_{B_2} = \frac{1}{3}\Big[\{\text{the above}\} + (eb \mid bf)\Big]$$

$$H^{A_3}_{B_3} = \frac{\sqrt{2}}{6}\Big[\{\text{the above}\} + (eb \mid bf)\Big]$$

Triplet 3×9

$$H_{B_4}^{A_3} = \frac{\sqrt{5}}{6}\Big[(ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_5}^{A_3} = \frac{2\sqrt{3}}{9}\Big[(ea \mid af) - \frac{11}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\Big]$$

$$H_{B_6}^{A_3} = \frac{1}{\sqrt{6}}\Big[\{\text{the above}\} - \frac{2}{3}(ea \mid af) - (eb \mid bf) - \frac{2}{3}(ec \mid cf) - \frac{2}{3}(ed \mid df)\Big]$$

$$H_{B_7}^{A_3} = \frac{2\sqrt{6}}{9}\Big[(ea \mid af) - \frac{1}{2}(ec \mid cf) - 2(ed \mid df)\Big]$$

$$H_{B_8}^{A_3} = \frac{1}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{2}{3}(ea \mid af) - (eb \mid bf) - \frac{2}{3}(ec \mid cf) - \frac{2}{3}(ed \mid df)\Big]$$

$$H_{B_9}^{A_3} = -\Big\{\text{the above}\Big\} + (eb \mid bf) + \frac{2}{3}(ec \mid cf) + \frac{2}{3}(ed \mid df)$$

$A = abcdee$
$B = cdafbe$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_2}^{A_1} = \frac{1}{\sqrt{6}}\Big[\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} - (ea \mid af) - 2(eb \mid bf) - \frac{3}{2}(ec \mid cf) + \frac{1}{2}(ed \mid df)\Big]$$

$$H_{B_3}^{A_1} = \frac{1}{\sqrt{3}}\Big[-\{\text{the above}\} + (ea \mid af) + 2(eb \mid bf) + (ed \mid df)\Big]$$

$$H_{B_4}^{A_1} = \sqrt{\frac{5}{6}}\Big[-(ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_5}^{A_1} = \frac{\sqrt{2}}{6}\Big[4(ea \mid af) - (ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_6}^{A_1} = -\Big\{\text{the above}\Big\} + \frac{1}{3}(ea \mid af) + \frac{2}{3}(ec \mid cf) + \frac{1}{3}(ed \mid df)$$

$$H_{B_7}^{A_1} = -\frac{2}{3}\Big[(ea \mid af) + \frac{1}{2}(ec \mid cf) - \frac{1}{2}(ed \mid df)\Big]$$

$$H_{B_8}^{A_1} = \frac{1}{\sqrt{2}}\Big[\{\text{the above}\} - \frac{1}{3}(ea \mid af) - \frac{1}{6}(ec \mid cf) - \frac{5}{6}(ed \mid df)\Big]$$

$$H_{B_9}^{A_1} = \frac{\sqrt{6}}{12}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_1}^{A_2} = \frac{1}{\sqrt{6}}\Big[\{\text{the above}\} + (ea \mid af) - 2(eb \mid bf) + \frac{1}{2}(ec \mid cf) + \frac{1}{2}(ed \mid df)\Big]$$

$$H_{B_2}^{A_2} = \frac{\sqrt{2}}{3}\Big[\{\text{the above}\} - (ea \mid af) - 2(eb \mid bf) - \frac{1}{4}(ec \mid cf) + \frac{5}{4}(ed \mid df)\Big]$$

$$H_{B_3}^{A_2} = \frac{1}{3}\Big[\{\text{the above}\} - (ea \mid af) - 2(eb \mid bf) + 2(ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_4}^{A_2} = -\frac{\sqrt{10}}{6}\Big[(ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_5}^{A_2} = -\frac{2\sqrt{6}}{9}\Big[(ea \mid af) + \frac{1}{4}(ec \mid cf) + \frac{1}{4}(ed \mid df)\Big]$$

$$H_{B_6}^{A_2} = \frac{1}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{1}{3}(ea \mid af) - \frac{4}{3}(ec \mid cf) - \frac{1}{3}(ed \mid df)\Big]$$

$$H_{B_7}^{A_2} = -\frac{\sqrt{3}}{9}\Big[4(ea \mid af) + (ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_8}^{A_2} = \sqrt{\frac{2}{3}}\Big[\{\text{the above}\} - \frac{1}{3}(ea \mid af) - \frac{7}{12}(ec \mid cf) - \frac{13}{12}(ed \mid df)\Big]$$

$$H_{B_9}^{A_2} = \frac{1}{\sqrt{2}}\Big[\{\text{the above}\} - (ea \mid af) - \frac{5}{6}(ec \mid cf) - \frac{5}{6}(ed \mid df)\Big]$$

$$H_{B_1}^{A_3} = \frac{1}{\sqrt{3}}\Big[-\{\text{the above}\} - (ea \mid af) + 2(eb \mid bf) + (ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_2}^{A_3} = \frac{1}{3}\Big[\{\text{the above}\} - (ea \mid af) - 2(eb \mid bf) - (ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_3}^{A_3} = \frac{\sqrt{2}}{6}\Big[\{\text{the above}\} - (ea \mid af) - 2(eb \mid bf) - (ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_4}^{A_3} = -\frac{\sqrt{5}}{6}\Big[(ec \mid cf) + (ed \mid df)\Big]$$

$$H_{B_5}^{A_3} = \frac{2\sqrt{3}}{9}\Big[-(ea \mid af) + \frac{11}{4}(ec \mid cf) - \frac{1}{4}(ed \mid df)\Big]$$

$$H_{B_6}^{A_3} = \frac{1}{\sqrt{6}}\Big[\{\text{the above}\} - \frac{1}{3}(ea \mid af) - \frac{1}{3}(ec \mid cf) - \frac{1}{3}(ed \mid df)\Big]$$

$$H_{B_7}^{A_4} = \frac{\sqrt{6}}{9}\Big[-2(ea \mid af) + (ec \mid cf) + 4(ed \mid df)\Big]$$

$$H_{B_8}^{A_4} = \frac{1}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{1}{3}(ea \mid af) - \frac{1}{3}(ec \mid cf) - \frac{1}{3}(ed \mid df)\Big]$$

$$H_{B_9}^{A_4} = -\Big\{\text{the above}\Big\} + (ea \mid af) + \frac{1}{3}(ec \mid cf) + \frac{1}{3}(ed \mid df)$$

$A = abcdee$
$B = cdfaeb$

$$H_{B_1}^{A_1} = -\frac{\sqrt{2}}{4}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_2}^{A_1} = \frac{1}{\sqrt{6}}\Big[-\{(e \mid h \mid f) + (ef \mid aa) + (ef \mid bb) + (ef \mid cc) + (ef \mid dd) + (ef \mid ee) + 2(ef \mid P_a P_a) - (P_a f \mid eP_a)\} + (ea \mid af) + 2(eb \mid bf) + \frac{3}{2}(ec \mid cf) - \frac{1}{2}(ed \mid df)\Big]$$

$$H_{B_3}^{A_1} = \frac{1}{\sqrt{3}}\Big[\{\text{the above}\} - (ea \mid af) - 2(eb \mid bf) - (ed \mid df)\Big]$$

$$H_{B_4}^{A_1} = -\sqrt{\frac{5}{6}}\Big[(ec \mid cf) - (ed \mid df)\Big]$$

$$H_{B_5}^{A_1} = \sqrt{2}\Big[-\frac{2}{3}\{\text{the above}\} + \frac{1}{2}(ec \mid cf) + \frac{1}{6}(ed \mid df)\Big]$$

$$H_{B_6}^{A_1} = \frac{1}{3}\Big[-\{\text{the above}\} + 3(ea \mid af) + (ed \mid df)\Big]$$

$$H_{B_7}^{A_1} = \frac{2}{3}\Big[\{\text{the above}\} - (ed \mid df)\Big]$$

Triplet 3×9

$$H^{A_1}_{B_8} = \sqrt{2}\left[\frac{1}{6}\{\text{the above}\} - \frac{1}{2}(ea\mid af) - \frac{1}{4}(ec\mid cf) + \frac{1}{12}(ed\mid df)\right]$$

$$H^{A_1}_{B_9} = \frac{\sqrt{6}}{12}\left[(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_2}_{B_1} = \frac{1}{\sqrt{6}}\left[\{\text{the above}\} + (ea\mid af) - 2(eb\mid bf) + \frac{1}{2}(ec\mid cf) + \frac{1}{2}(ed\mid df)\right]$$

$$H^{A_2}_{B_2} = \frac{\sqrt{2}}{3}\left[-\{\text{the above}\} + (ea\mid af) + 2(eb\mid bf) + \frac{1}{4}(ec\mid cf) - \frac{5}{4}(ed\mid df)\right]$$

$$H^{A_2}_{B_3} = \frac{1}{3}\left[-\{\text{the above}\} + (ea\mid af) + 2(eb\mid bf) - 2(ec\mid cf) + (ed\mid df)\right]$$

$$H^{A_2}_{B_4} = -\frac{\sqrt{10}}{6}\left[(ec\mid cf) + (ed\mid df)\right]$$

$$H^{A_2}_{B_5} = \frac{2\sqrt{6}}{9}\left[\{\text{the above}\} - \frac{5}{4}(ec\mid cf) - \frac{1}{4}(ed\mid df)\right]$$

$$H^{A_2}_{B_6} = \frac{\sqrt{3}}{9}\left[\{\text{the above}\} - 3(ea\mid af) - 2(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_2}_{B_7} = \frac{4\sqrt{3}}{9}\left[\{\text{the above}\} - \frac{1}{2}(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_2}_{B_8} = \frac{\sqrt{6}}{9}\left[\{\text{the above}\} - 3(ea\mid af) - \frac{5}{4}(ec\mid cf) - \frac{7}{4}(ed\mid df)\right]$$

$$H^{A_2}_{B_9} = \frac{1}{\sqrt{2}}\left[\{\text{the above}\} - (ea\mid af) - \frac{5}{6}(ec\mid cf) - \frac{5}{6}(ed\mid df)\right]$$

$$H^{A_3}_{B_1} = \frac{1}{\sqrt{3}}\left[-\{\text{the above}\} - (ea\mid af) + 2(eb\mid bf) + (ec\mid cf) + (ed\mid df)\right]$$

$$H^{A_3}_{B_2} = \frac{1}{3}\left[-\{\text{the above}\} + (ea\mid af) + 2(eb\mid bf) + (ec\mid cf) + (ed\mid df)\right]$$

$$H^{A_3}_{B_3} = \frac{\sqrt{2}}{6}\left[-\{\text{the above}\} + (ea\mid af) + 2(eb\mid bf) + (ec\mid cf) + (ed\mid df)\right]$$

$$H^{A_3}_{B_4} = \frac{\sqrt{5}}{6}\left[-(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_3}_{B_5} = \frac{2\sqrt{3}}{9}\left[\{\text{the above}\} - \frac{5}{4}(ec\mid cf) - \frac{1}{4}(ed\mid df)\right]$$

$$H^{A_3}_{B_6} = \frac{\sqrt{6}}{18}\left[\{\text{the above}\} - 3(ea\mid af) + 7(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_3}_{B_7} = \frac{2\sqrt{6}}{9}\left[\{\text{the above}\} - \frac{1}{2}(ec\mid cf) - (ed\mid df)\right]$$

$$H^{A_3}_{B_8} = \frac{\sqrt{3}}{9}\left[\{\text{the above}\} - 3(ea\mid af) + (ec\mid cf) + 5(ed\mid df)\right]$$

$$H^{A_3}_{B_9} = -\left\{\text{the above}\right\} + (ea\mid af) + \frac{1}{3}(ec\mid cf) + \frac{1}{3}(ed\mid df)$$

$A = abcdee$
$B = efabcd$

$$H^{A_1}_{B_1} = \frac{1}{\sqrt{2}}\left[(ea\mid af) - (eb\mid bf)\right]$$

Triplet 3×9

$$H_{B_2}^{A_1} = \sqrt{\frac{2}{3}}\Big[-\{(e\mid h\mid f)+(ef\mid aa)+(ef\mid bb)+(ef\mid cc)+(ef\mid dd)$$
$$+(ef\mid ee)+2(ef\mid P_aP_a)-(P_af\mid eP_a)\}-\frac{1}{2}(ea\mid af)$$
$$-\frac{1}{2}(eb\mid bf)+\frac{1}{2}(ec\mid cf)+\frac{1}{2}(ed\mid df)\Big]$$

$$H_{B_3}^{A_1} = \frac{1}{\sqrt{3}}\Big[2\,\{\text{the above}\}-2(ea\mid af)-(eb\mid bf)-(ec\mid cf)-(ed\mid df)\Big]$$

$$H_{B_4}^{A_1} = -\sqrt{\frac{5}{6}}\Big[(ec\mid cf)-(ed\mid df)\Big]$$

$$H_{B_5}^{A_1} = -\frac{\sqrt{2}}{6}\Big[(ec\mid cf)-(ed\mid df)\Big] = H_{B_3}^{A_3}$$

$$H_{B_6}^{A_1} = -\frac{1}{3}\Big[(ec\mid cf)-(ed\mid df)\Big] = H_{B_7}^{A_1} = H_{B_3}^{A_2} = H_{B_2}^{A_3}$$

$$H_{B_8}^{A_1} = -\frac{\sqrt{2}}{3}\Big[(ec\mid cf)-(ed\mid df)\Big] = H_{B_2}^{A_2}$$

$$H_{B_9}^{A_1} = -\frac{1}{\sqrt{6}}\Big[(ec\mid cf)-(ed\mid df)\Big] = H_{B_1}^{A_2}$$

$$H_{B_4}^{A_2} = \frac{\sqrt{10}}{6}\Big[-2(eb\mid bf)+(ec\mid cf)+(ed\mid df)\Big]$$

$$H_{B_5}^{A_2} = \frac{\sqrt{6}}{9}\Big[-(eb\mid bf)+\frac{1}{2}(ec\mid cf)+\frac{1}{2}(ed\mid df)\Big]$$

$$H_{B_6}^{A_2} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\}-(ea\mid af)-\frac{1}{3}(eb\mid bf)-\frac{5}{6}(ec\mid cf)$$
$$-\frac{5}{6}(ed\mid df)\Big]$$

$$H_{B_7}^{A_2} = \frac{2\sqrt{3}}{9}\Big[-(eb\mid bf)+\frac{1}{2}(ec\mid cf)+\frac{1}{2}(ed\mid df)\Big]$$

$$H_{B_8}^{A_2} = \sqrt{\frac{2}{3}}\Big[-\{\text{the above}\}-\frac{1}{2}(ea\mid af)+\frac{5}{6}(eb\mid bf)-\frac{1}{6}(ec\mid cf)$$
$$-\frac{1}{6}(ed\mid df)\Big]$$

$$H_{B_9}^{A_2} = \frac{1}{\sqrt{2}}\Big[(ea\mid af)+\frac{1}{3}(eb\mid bf)-\frac{2}{3}(ec\mid cf)-\frac{2}{3}(ed\mid df)\Big]$$

$$H_{B_1}^{A_3} = \frac{1}{\sqrt{3}}\Big[(ec\mid cf)-(ed\mid df)\Big]$$

$$H_{B_4}^{A_3} = \frac{\sqrt{5}}{2}\Big[-(ea\mid af)+\frac{1}{3}(eb\mid bf)+\frac{1}{3}(ec\mid cf)+\frac{1}{3}(ed\mid df)\Big]$$

$$H_{B_5}^{A_3} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\}-\frac{1}{4}(ea\mid af)-\frac{11}{12}(eb\mid bf)-\frac{11}{12}(ec\mid cf)$$
$$-\frac{11}{12}(ed\mid df)\Big]$$

$$H_{B_6}^{A_3} = \frac{\sqrt{6}}{9}\Big[-(eb\mid bf)+\frac{1}{2}(ec\mid cf)+\frac{1}{2}(ed\mid df)\Big]$$

$$H_{B_7}^{A_3} = \sqrt{\frac{2}{3}}\Big[-\{\text{the above}\}+(ea\mid af)-\frac{1}{3}(eb\mid bf)-\frac{1}{3}(ec\mid cf)$$
$$-\frac{1}{3}(ed\mid df)\Big]$$

Triplet 3×9

$$H^{A_3}_{B_8} = \frac{\sqrt{3}}{9}\Big[-2\,(eb\mid bf) + (ec\mid cf) + (ed\mid df)\Big]$$

$$H^{A_3}_{B_9} = \frac{2}{3}\,(eb\mid bf) - \frac{1}{3}\,(ec\mid cf) - \frac{1}{3}\,(ed\mid df)$$

$A = abcdee$
$B = feabcd$

$$H^{A_1}_{B_1} = -\frac{1}{\sqrt{2}}\Big[(ea\mid af) - (eb\mid bf)\Big]$$

$$H^{A_1}_{B_2} = \sqrt{\frac{2}{3}}\Big[-\{(e\mid h\mid f) + (ef\mid aa) + (ef\mid bb) + (ef\mid cc) + (ef\mid dd) + (ef\mid ee) + 2\,(ef\mid P_aP_a) - (P_af\mid eP_a)\} + \frac{3}{2}\,(ea\mid af) + \frac{3}{2}\,(eb\mid bf) + \frac{1}{2}\,(ec\mid cf) + \frac{1}{2}\,(ed\mid df)\Big]$$

$$H^{A_1}_{B_3} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{1}{2}\,(ec\mid cf) - \frac{1}{2}\,(ed\mid df)\Big]$$

$$H^{A_1}_{B_4} = \sqrt{\frac{5}{6}}\Big[(ec\mid cf) - (ed\mid df)\Big]$$

$$H^{A_1}_{B_5} = \frac{\sqrt{2}}{6}\Big[(ec\mid cf) - (ed\mid df)\Big] = H^{A_3}_{B_3}$$

$$H^{A_1}_{B_6} = \frac{1}{3}\Big[(ec\mid cf) - (ed\mid df)\Big] = H^{A_1}_{B_7} = H^{A_2}_{B_3} = H^{A_3}_{B_2}$$

$$H^{A_1}_{B_8} = \frac{\sqrt{2}}{3}\Big[(ec\mid cf) - (ed\mid df)\Big] = H^{A_2}_{B_2}$$

$$H^{A_1}_{B_9} = \frac{1}{\sqrt{6}}\Big[(ec\mid cf) - (ed\mid df)\Big] = H^{A_2}_{B_1}$$

$$H^{A_2}_{B_4} = \frac{\sqrt{10}}{6}\Big[2\,(eb\mid bf) - (ec\mid cf) - (ed\mid df)\Big]$$

$$H^{A_2}_{B_5} = \frac{\sqrt{6}}{9}\Big[(eb\mid bf) - \frac{1}{2}\,(ec\mid cf) - \frac{1}{2}\,(ed\mid df)\Big]$$

$$H^{A_2}_{B_6} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{2}{3}\,(eb\mid bf) - \frac{1}{6}\,(ec\mid cf) - \frac{1}{6}\,(ed\mid df)\Big]$$

$$H^{A_2}_{B_7} = \frac{2\sqrt{3}}{9}\Big[(eb\mid bf) - \frac{1}{2}\,(ec\mid cf) - \frac{1}{2}\,(ed\mid df)\Big]$$

$$H^{A_2}_{B_8} = \sqrt{\frac{2}{3}}\Big[-\{\text{the above}\} + \frac{3}{2}\,(ea\mid af) + \frac{1}{6}\,(eb\mid bf) + \frac{7}{6}\,(ec\mid cf) + \frac{7}{6}\,(ed\mid df)\Big]$$

$$H^{A_2}_{B_9} = \frac{1}{\sqrt{2}}\Big[-(ea\mid af) - \frac{1}{3}\,(eb\mid bf) + \frac{2}{3}\,(ec\mid cf) + \frac{2}{3}\,(ed\mid df)\Big]$$

$$H^{A_3}_{B_1} = \frac{1}{\sqrt{3}}\Big[-(ec\mid cf) + (ed\mid df)\Big]$$

$$H^{A_3}_{B_4} = \frac{\sqrt{5}}{2}\Big[(ea\mid af) - \frac{1}{3}\,(eb\mid bf) - \frac{1}{3}\,(ec\mid cf) - \frac{1}{3}\,(ed\mid df)\Big]$$

$$H^{A_3}_{B_5} = \frac{2}{\sqrt{3}}\Big[\{\text{the above}\} - \frac{3}{4}\,(ea\mid af) - \frac{1}{12}\,(eb\mid bf) - \frac{1}{12}\,(ec\mid cf) - \frac{1}{12}\,(ed\mid df)\Big]$$

$$H^{A_3}_{B_6} = \frac{\sqrt{6}}{9}\Big[(eb\mid bf) - \frac{1}{2}\,(ec\mid cf) - \frac{1}{2}\,(ed\mid df)\Big]$$

Triplet 3×9

$$H^{A_3}_{B_7} = \sqrt{\frac{2}{3}}\left[-\{\text{the above}\} + \frac{4}{3}(eb \mid bf) + \frac{4}{3}(ec \mid cf) + \frac{4}{3}(ed \mid df)\right]$$

$$H^{A_3}_{B_8} = \frac{2\sqrt{3}}{9}\left[(eb \mid bf) - \frac{1}{2}(ec \mid cf) - \frac{1}{2}(ed \mid df)\right]$$

$$H^{A_3}_{B_9} = -\frac{2}{3}(eb \mid bf) + \frac{1}{3}(ec \mid cf) + \frac{1}{3}(ed \mid df)$$

6. $N_A = 6$, $N_B = 6$ $(k_A = 9,$ $k_B = 9)$ (Omitted)

IV. Formulae of configuration interaction matrix elements for Quartet state wave function $(S = \frac{3}{2})$

1. $N_A = 3$, $N_B = 3$ $(k_A = 1,$ $k_B = 1)$

$A = abcdd$, $B = bdeaa$ $\quad H^A_B = (ce \mid da)$

$A = abcdd$, $B = adebb$ $\quad H^A_B = -(ce \mid db)$

$A = abcdd$, $B = daecc$ $\quad H^A_B = -(be \mid dc)$

$A = abcdd$, $B = abcee$ $\quad H^A_B = (de \mid de)$

$A = abcdd$, $B = becaa$ $\quad H^A_B = (de \mid da)$

$A = abcdd$, $B = bdcaa$ $\quad H^A_B = (d \mid h \mid a) + (da \mid aa) + (da \mid bb) + (da \mid cc) + (da \mid dd) + 2(da \mid P_a P_a) - (P_a a \mid dP_a)$

$A = abc$, $B = efc$ $\quad H^A_B = (ae \mid bf) - (be \mid af)$

$A = abcdd$, $B = edcbb$ $\quad H^A_B = -(ae \mid db)$

$A = abcdd$, $B = eacbb$ $\quad H^A_B = (de \mid db)$

$A = abcdd$, $B = ebdcc$ $\quad H^A_B = -(ae \mid dc)$

$A = abc$, $B = ebc$ $\quad H^A_B = \left\{(a \mid h \mid e) + (ae \mid bb) + (ae \mid cc) + 2(ae \mid P_a P_a) - (P_a e \mid aP_a)\right\} - (ab \mid be) - (ac \mid ce)$

$A = abcdd$, $B = aedcc$ $\quad H^A_B = -(be \mid dc)$

$A = abcdd$, $B = aedbb$ $\quad H^A_B = (ce \mid db)$

$A = abcddee$, $B = edcbbaa$ $\quad H^A_B = (db \mid ea) - (eb \mid da)$

$A = abc$, $B = bae$ $\quad H^A_B = -\left\{(c \mid h \mid e) + (ce \mid aa) + (ce \mid bb) + 2(ce \mid P_a P_a) - (P_a e \mid cP_a)\right\} + (cb \mid be) + (ca \mid ae)$

$A = abc$, $B = eac$ $\quad H^A_B = -\left\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + 2(be \mid P_a P_a) - (P_a e \mid bP_a)\right\} + (ba \mid ae) + (bc \mid ce)$

$A = abcdd$, $B = dacee$ $\quad H^A_B = (be \mid de)$

$A = abcdd$, $B = abdcc$ $\quad H^A_B = -\left\{(d \mid h \mid c) + (dc \mid aa) + (dc \mid bb) + (dc \mid cc) + (dc \mid dd) + 2(dc \mid P_a P_a) - (P_a c \mid dP_a)\right\}$

Quartet 1×1

$A = abcdd$ $B = bedcc$	$H_B^A = (ae \mid dc)$
$A = abcdd$ $B = bedaa$	$H_B^A = -(ce \mid da)$
$A = abcdd$ $B = bcdee$	$H_B^A = -(ae \mid de)$
$A = abcdd$ $B = abdee$	$H_B^A = -(ce \mid de)$
$A = abcdd$ $B = abcee$	$H_B^A = (de \mid de)$
$A = abcdd$ $B = bedcc$	$H_B^A = (ae \mid dc)$
$A = abcdd$ $B = dbcee$	$H_B^A = -(ae \mid de)$
$A = abcdd$ $B = adcee$	$H_B^A = -(be \mid de)$
$A = abcdd$ $B = dbecc$	$H_B^A = (ae \mid dc)$
$A = abcdd$ $B = adecc$	$H_B^A = (be \mid dc)$
$A = abcdd$ $B = bdecc$	$H_B^A = -(ae \mid dc)$
$A = abcdd$ $B = bdcee$	$H_B^A = (ae \mid de)$
$A = abcdd$ $B = eadcc$	$H_B^A = (be \mid dc)$
$A = abcdd$ $B = eadbb$	$H_B^A = -(ce \mid db)$
$A = abcdd$ $B = ecdaa$	$H_B^A = -(be \mid da)$
$A = abcdd$ $B = daebb$	$H_B^A = (ce \mid db)$
$A = abc$ $B = eac$	$H_B^A = -\left\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + 2\,(be \mid P_a P_a) - (P_a e \mid bP_a)\right\} + (ba \mid ae) + (bc \mid ce)$
$A = abcdd$ $B = eabcc$	$H_B^A = -(de \mid dc)$
$A = abcdd$ $B = abecc$	$H_B^A = -(de \mid dc)$
$A = abcdd$ $B = edcaa$	$H_B^A = (be \mid da)$
$A = abcdd$ $B = dbcaa$	$H_B^A = -\left\{(d \mid h \mid a) + (da \mid aa) + (da \mid bb) + (da \mid cc) + (da \mid dd) + 2\,(da \mid P_a P_a) - (P_a a \mid dP_a)\right\}$
$A = abcddee$ $B = adebbcc$	$H_B^A = (db \mid ec) - (eb \mid dc)$
$A = abcddee$ $B = cdeaabb$	$H_B^A = (da \mid eb) - (ea \mid db)$
$A = abcddee$ $B = bdeaacc$	$H_B^A = -(da \mid ec) + (ea \mid dc)$

Quartet 1 × 1

$A = abcddee$ $B = dbeaacc$ $\quad H_B^A = (da \mid ec) - (ea \mid dc)$

$A = abcddee$ $B = daebbcc$ $\quad H_B^A = -(db \mid ec) + (eb \mid dc)$

$A = abcddee$ $B = dceaabb$ $\quad H_B^A = -(da \mid eb) + (ea \mid db)$

$A = abcdd$ $B = acdbb$ $\quad H_B^A = \Big\{(d \mid h \mid b) + (db \mid aa) + (db \mid bb) + (db \mid cc) + (db \mid dd) + 2\,(db \mid P_a P_a) - (P_a b \mid dP_a)\Big\}$

$A = abcdd$ $B = adcbb$ $\quad H_B^A = -\Big\{(d \mid h \mid b) + (db \mid aa) + (db \mid bb) + (db \mid cc) + (db \mid dd) + 2\,(db \mid P_a P_a) - (P_a b \mid dP_a)\Big\}$

$A = abcddee$ $B = decaabb$ $\quad H_B^A = (da \mid eb) - (ea \mid db)$

$A = abcdd$ $B = cedbb$ $\quad H_B^A = -(ae \mid db)$

$A = abcdd$ $B = edabb$ $\quad H_B^A = (ce \mid db)$

$A = abcdd$ $B = deabb$ $\quad H_B^A = -(ce \mid db)$

$A = abcdd$ $B = aecbb$ $\quad H_B^A = -(de \mid db)$

$A = abcdd$ $B = ebdaa$ $\quad H_B^A = (ce \mid da)$

$A = abcdd$ $B = ebcaa$ $\quad H_B^A = -(de \mid da)$

$A = abcdd$ $B = cedaa$ $\quad H_B^A = (be \mid da)$

$A = abcdd$ $B = edbaa$ $\quad H_B^A = -(ce \mid da)$

$A = abcdd$ $B = decaa$ $\quad H_B^A = -(be \mid da)$

$A = abcdd$ $B = dbeaa$ $\quad H_B^A = -(ce \mid da)$

$A = abcdd$ $B = debaa$ $\quad H_B^A = (ce \mid da)$

$A = abcddee$ $B = decaabb$ $\quad H_B^A = (da \mid eb) - (ea \mid db)$

$A = abcddee$ $B = deabbcc$ $\quad H_B^A = (db \mid ec) - (eb \mid dc)$

$A = abc$ $B = aec$ $\quad H_B^A = \Big\{(b \mid h \mid e) + (be \mid aa) + (be \mid cc) + 2\,(be \mid P_a P_a) - (P_a e \mid bP_a)\Big\} - (ba \mid ae) - (bc \mid ce)$

$A = abc$ $B = abe$ $\quad H_B^A = \Big\{(c \mid h \mid c) + (ce \mid aa) + (ce \mid bb) + 2\,(ce \mid P_a P_a) - (P_a e \mid cP_a)\Big\} - (ca \mid ae) - (cb \mid be)$

$A = abc$ $B = aeb$ $\quad H_B^A = -\Big\{(c \mid h \mid e) + (ce \mid aa) + (ce \mid bb) + 2\,(ce \mid P_a P_a) - (P_a e \mid cP_a)\Big\} - (ca \mid ae) - (cb \mid be)$

Quartet 1×1

2. $N_A = 3, \quad N_B = 5 \quad (k_A = 1, \quad k_B = 4)$

$A = abcddee$
$B = daefbcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - 3\,(ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\,(df \mid ec)$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcddee$
$B = adefcbb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid eb) - 3\,(ef \mid db)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\,(df \mid eb)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = adebfcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - 3\,(ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\,(df \mid ec)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = deabfcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[(df \mid ec) - 4\,(ef \mid dc)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcddee$
$B = daebfcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid ec) - 3\,(ef \mid dc)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\,(df \mid ec)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcdd$
$B = afcbd$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} + (da \mid af) + 2\,(db \mid bf) + (dc \mid cf)\Big]$$

Quartet 1×4

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + (da \mid af) + 3\,(dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-4\,\{\text{the above}\} + (da \mid af)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(da \mid af)$$

$A = abcdd$
$B = facdb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} - (da \mid af) - 2\,(db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + (da \mid af) + 3\,(dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-\{\text{the above}\} + 4\,(da \mid af)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\Big\{\text{the above}\Big\}$$

$A = abcddee$
$B = afebdcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(df \mid ec)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[(df \mid ec) + 2\,(ef \mid dc)\Big]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\Big[(df \mid ec) - (ef \mid dc)\Big]$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = faedbcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\,(df \mid ec)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[(df \mid ec) + 2\,(ef \mid dc)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[(df \mid ec) - (ef \mid dc)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\Big[(df \mid ec) - (ef \mid dc)\Big]$$

$A = abcdd$
$B = dacfb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} + (db \mid bf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + (db \mid bf) - 2\,(dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-\{\text{the above}\} - 3\,(da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\Big[-\{\text{the above}\} + (da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$A = abcdd$
$B = adcbf$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[-\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} - (db \mid bf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + (db \mid bf) - 2\,(dc \mid cf)\Big]$$

Quartet 1×4

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\left[-\{\text{the above}\} + \frac{3}{4}(da \mid af) + (db \mid bf) + (dc \mid cf)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(da \mid af)$$

$A = abcddee$
$B = decabff$

$$H^A_{B_1} = H^A_{B_2} = 0$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2}(df \mid ef)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid ef)$$

$A = abcddee$
$B = abcdeff$

$$H^A_{B_1} = -\sqrt{2}\,(df \mid ef)$$

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = defabcc$

$$H^A_{B_1} = 0$$

$$H^A_{B_2} = -\sqrt{\frac{2}{3}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[(df \mid ec) - 4\,(ef \mid dc)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid ec)$$

$A = abcddee$
$B = dafbecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}(ef \mid dc)$$

$$H^A_{B_2} = -\sqrt{\frac{2}{3}}\left[(df \mid ec) - \frac{3}{2}(ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}(df \mid ec)$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid ec)$$

$A = abcddee$
$B = adfebcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(ef \mid dc)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[(ef \mid dc) - \frac{2}{3}(df \mid ec)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}(df \mid ec)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = abfdecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = dacbeff$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(df \mid ef)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(df \mid ef)$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}(df \mid ef)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid ef)$$

$A = abcddee$
$B = adcebff$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}(df \mid ef)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(df \mid ef)$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(df \mid ef)$$

$$H^A_{B_4} = 0$$

$A = abcdd$
$B = afbdc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - (da \mid af) - (db \mid bf) - 2(dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} + (da \mid af) + 3(db \mid bf)\Big]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\Big[-\{\text{the above}\} + \frac{1}{4}(da \mid af)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(da \mid af)$$

$A = abcdd$
$B = adbfc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} + (dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-\{\text{the above}\} - 2(db \mid bf) + (dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\Big[-\{\text{the above}\} + \frac{3}{4}(da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(da \mid af)\Big]$$

$A = abcddee$
$B = abdefcc$

$$H^A_{B_1} = \sqrt{2}\Big[\frac{1}{2}(df \mid ec) - (ef \mid dc)\Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}(df \mid ec)$$

$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = baedfcc$

$$H^A_{B_1} = \sqrt{2}\Big[(df \mid ec) - \frac{1}{2}(ef \mid dc)\Big]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(ef \mid dc)$$

$$H^A_{B_3} = H^A_{B_4} = 0$$

Quartet 1×4

$A = abcddee$
$B = dabefcc$

$$H^A_{B_1} = \sqrt{2}\left[\frac{1}{2}(df \mid ec) - (ef \mid dc)\right]$$
$$H^A_{B_2} = \frac{1}{\sqrt{6}}(df \mid ec)$$
$$H^A_{B_3} = \frac{1}{\sqrt{12}}(df \mid ec)$$
$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid ec)$$

$A = abcddee$
$B = dabecff$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}(df \mid ef)$$
$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(df \mid ef)$$
$$H^A_{B_3} = -\frac{1}{\sqrt{12}}(df \mid ef)$$
$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid ef)$$

$A = abcddee$
$B = edbacff$

$$H^A_{B_1} = H^A_{B_2} = 0$$
$$H^A_{B_3} = -\frac{\sqrt{3}}{2}(df \mid ef)$$
$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid ef)$$

$A = abcddee$
$B = edbafcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$
$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - (ef \mid dc)\right]$$
$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[4\,(df \mid ec) - (ef \mid dc)\right]$$
$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(ef \mid dc)$$

$A = abcddee$
$B = abdecff$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}(df \mid ef)$$
$$H^A_{B_2} = -\sqrt{\frac{3}{2}}(df \mid ef)$$
$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = baedcff$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(df \mid ef)$$
$$H^A_{B_2} = \sqrt{\frac{3}{2}}(df \mid cf)$$
$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcdd$
$B = dafbc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(db \mid bf) - (dc \mid cf)\right]$$

Quartet 1×4

$$H^A_{B_2} = \sqrt{\frac{2}{3}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} + \frac{1}{2}(db \mid bf) + \frac{1}{2}(dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-\{\text{the above}\} - 3(da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\Big[-\{\text{the above}\} + (da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$A = abcdd$
$B = fadbc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{\sqrt{2}}{\sqrt{3}}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - (da \mid af) - \frac{3}{2}(db \mid bf) - \frac{3}{2}(dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-\{\text{the above}\} + 4(da \mid af)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\Big\{\text{the above}\Big\}$$

$A = abcddee$
$B = dfabecc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(ef \mid dc)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(ef \mid dc)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\Big[\frac{3}{4}(df \mid ec) - (ef \mid dc)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid ec)$$

$A = abcddee$
$B = adbefcc$

$$H^A_{B_1} = \sqrt{2}\Big[-\frac{1}{2}(df \mid ec) + (ef \mid dc)\Big]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}(df \mid ec)$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(df \mid ec)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = fadebcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}(ef \mid dc)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[2(df \mid ec) + (ef \mid dc)\Big]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\Big[(df \mid ec) - (ef \mid dc)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\Big[(df \mid ec) - (ef \mid dc)\Big]$$

$A = abcddee$
$B = daebcff$

$$H^A_{B_1} = 0$$

Quartet 1×4

$$H^A_{B_2} = -\sqrt{\frac{2}{3}}\,(df \mid ef)$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\,(df \mid ef)$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid ef)$$

$A = abcdd$
$B = fdabc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[-2\,(da \mid af) + (db \mid bf) + (dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2}\Big[-\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} + \frac{4}{3}\,(da \mid af) + \frac{4}{3}\,(db \mid bf) + \frac{4}{3}\,(dc \mid cf)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\Big\{\text{the above}\Big\}$$

$A = abcdd$
$B = dfabc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[2\,(da \mid af) - (db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\Big[-3\,\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2\,(df \mid P_a P_a) - (P_a f \mid dP_a)\} - (da \mid af) - (db \mid bf) - (dc \mid cf)\Big]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\Big[\{\text{the above}\} - (da \mid af) - (db \mid bf) - (dc \mid cf)\Big]$$

$A = abcddee$
$B = deacfbb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\Big[(df \mid eb) - (ef \mid db)\Big]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\Big[(df \mid eb) - (ef \mid db)\Big]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\Big[(df \mid eb) - 4\,(ef \mid db)\Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\,(df \mid eb)$$

$A = abcddee$
$B = daecfbb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\Big[(df \mid eb) - (ef \mid db)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\Big[(df \mid eb) - 3\,(ef \mid db)\Big]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\,(df \mid eb)$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid eb)$$

$A = abcddee$
$B = fdabecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}} (ef \mid dc)$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}} \left[3\,(df \mid ec) + (ef \mid dc) \right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2} \left[-(df \mid ec) + (ef \mid dc) \right]$$

$A = abcddee$
$B = defbcaa$

$$H^{A}_{B_1} = 0$$

$$H^{A}_{B_2} = \sqrt{\frac{2}{3}} \left[-(df \mid ea) + (ef \mid da) \right]$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}} \left[(df \mid ea) - 4\,(ef \mid da) \right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2} (df \mid ea)$$

$A = abcddee$
$B = fdacebb$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}} (ef \mid db)$$

$$H^{A}_{B_3} = -\frac{1}{\sqrt{12}} \left[3\,(df \mid eb) + (ef \mid db) \right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2} \left[(df \mid eb) - (ef \mid db) \right]$$

$A = abcddee$
$B = dfacebb$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}} (ef \mid db)$$

$$H^{A}_{B_3} = \frac{2}{\sqrt{3}} \left[-\frac{3}{4} (df \mid eb) + (ef \mid db) \right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2} (df \mid eb)$$

$A = abcddee$
$B = fdaecbb$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^{A}_{B_2} = \frac{1}{\sqrt{6}} (ef \mid db)$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}} \left[3\,(df \mid eb) + (ef \mid db) \right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2} \left[-(df \mid eb) + (ef \mid db) \right]$$

$A = abcddee$
$B = dfaecbb$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}} (ef \mid db)$$

$$H^{A}_{B_3} = \frac{\sqrt{3}}{2} \left[(df \mid eb) - \frac{4}{3} (ef \mid db) \right]$$

$$H^{A}_{B_4} = -\frac{\sqrt{5}}{2} (df \mid eb)$$

Quartet 1×4

$A = abcddee$
$B = adecfbb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid eb) - 3\,(ef \mid db)\right]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}\,(df \mid eb)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = fdaebcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\,(ef \mid dc)$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\left[3\,(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\left[(df \mid ec) - (ef \mid dc)\right]$$

$A = abcddee$
$B = dafebcc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^A_{B_2} = \sqrt{\frac{2}{3}}\left[(df \mid ec) - \frac{3}{2}\,(ef \mid dc)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\,(df \mid ec)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcddee$
$B = dfaebcc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\,(ef \mid dc)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\left[-\frac{3}{4}\,(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcddee$
$B = dfebcaa$

$$H^A_{B_1} = 0$$

$$H^A_{B_2} = \sqrt{\frac{2}{3}}\,(ef \mid da)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\left[\frac{3}{4}\,(df \mid ea) - (ef \mid da)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\,(df \mid ea)$$

$A = abcddee$
$B = dafbecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}}\left[-\frac{2}{3}\,(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\,(df \mid ec)$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid ec)$$

$A = abcddee$
$B = abfdecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_2} = -\sqrt{\frac{3}{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = afcdebb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid eb) + (ef \mid db)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = fabdecc$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\left[(df \mid ec) - (ef \mid dc)\right]$$

$A = abcddee$
$B = fbcdeaa$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ea) + (ef \mid da)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid ea) - (ef \mid da)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}\left[(df \mid ea) - (ef \mid da)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\left[(df \mid ea) - (ef \mid da)\right]$$

$A = abcddee$
$B = fdebcaa$

$$H^A_{B_1} = 0$$

$$H^A_{B_2} = -\sqrt{\frac{2}{3}}(ef \mid da)$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[3(df \mid ea) + (ef \mid da)\right]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}\left[(df \mid ea) - (ef \mid da)\right]$$

$A = abcddee$
$B = adebcff$

$$H^A_{B_1} = 0 = H^A_{B_4}$$

$$H^A_{B_2} = \sqrt{\frac{2}{3}}(df \mid ef)$$

$$H^A_{B_3} = -\frac{2}{\sqrt{3}}(df \mid ef)$$

Quartet 1×4

$A = abcddee$
$B = deabcff$

$$H^A_{B_1} = H^A_{B_2} = 0$$

$$H^A_{B_3} = \frac{\sqrt{3}}{2} (df \mid ef)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2} (df \mid ef)$$

$A = abcdd$
$B = abcfd$

$$H^A_{B_1} = \sqrt{2} \Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2 (df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2} (da \mid af) - \frac{1}{2} (db \mid bf) - \frac{1}{2} (dc \mid cf) \Big]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}} \Big[(da \mid af) + (db \mid bf) + 3 (dc \mid cf) \Big]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}} \Big[(da \mid af) + 4 (db \mid bf) \Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2} (da \mid af)$$

$A = abcddee$
$B = acdfebb$

$$H^A_{B_1} = \sqrt{2} \Big[-\frac{1}{2} (df \mid eb) + (ef \mid db) \Big]$$

$$H^A_{B_2} = \sqrt{\frac{3}{2}} (df \mid eb)$$

$$H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = abcdeff$

$$H^A_{B_1} = -\sqrt{2} (df \mid ef)$$

$$H^A_{B_2} = H^A_{B_3} = H^A_{B_4} = 0$$

$A = abcddee$
$B = adbfecc$

$$H^A_{B_1} = \sqrt{2} \Big[-\frac{1}{2} (df \mid ec) + (ef \mid dc) \Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}} (df \mid ec)$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}} (df \mid ec)$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = fadcebb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}} \Big[2 (df \mid eb) + (ef \mid db) \Big]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}} \Big[(df \mid eb) - (ef \mid db) \Big]$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2} \Big[(df \mid eb) - (ef \mid db) \Big]$$

$A = abcddee$
$B = fadecbb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}} (ef \mid db)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[2\,(df \mid eb) + (ef \mid db)\right]$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2}\left[(df \mid eb) - (ef \mid db)\right]$$

$A = abcddee$
$B = dafecbb$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}\,(ef \mid db)$$

$$H^{A}_{B_2} = \sqrt{\frac{2}{3}}\left[-(df \mid eb) + \frac{3}{2}(ef \mid db)\right]$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}}\,(df \mid eb)$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2}\,(df \mid eb)$$

$A = abcddee$
$B = acdefbb$

$$H^{A}_{B_1} = \sqrt{2}\left[-\frac{1}{2}(df \mid eb) + (ef \mid db)\right]$$

$$H^{A}_{B_2} = -\sqrt{\frac{3}{2}}\,(df \mid eb)$$

$$H^{A}_{B_3} = H^{A}_{B_4} = 0$$

$A = abcddee$
$B = adefbcc$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid ec) - 3\,(ef \mid dc)\right]$$

$$H^{A}_{B_3} = -\frac{2}{\sqrt{3}}\,(df \mid ec)$$

$$H^{A}_{B_4} = 0$$

$A = abcddee$
$B = deafbcc$

$$H^{A}_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A}_{B_3} = -\frac{1}{\sqrt{12}}\left[(df \mid ec) - 4\,(ef \mid dc)\right]$$

$$H^{A}_{B_4} = -\frac{\sqrt{5}}{2}\,(df \mid ec)$$

$A = abcddee$
$B = fadbecc$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}\,(ef \mid dc)$$

$$H^{A}_{B_2} = -\frac{1}{\sqrt{6}}\left[2\,(df \mid ec) + (ef \mid dc)\right]$$

$$H^{A}_{B_3} = \frac{1}{\sqrt{12}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^{A}_{B_4} = \frac{\sqrt{5}}{2}\left[(df \mid ec) - (ef \mid dc)\right]$$

$A = abcddee$
$B = dafcebb$

$$H^{A}_{B_1} = -\frac{1}{\sqrt{2}}\,(ef \mid db)$$

Quartet 1×4

$$H^A_{B_2} = \sqrt{\frac{2}{3}}\left[(df \mid eb) - \frac{3}{2}(ef \mid db)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}(df \mid eb)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid eb)$$

$A = abcddee$
$B = afbdecc$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid ec) + (ef \mid dc)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_3} = \frac{2}{\sqrt{3}}\left[(df \mid ec) - (ef \mid dc)\right]$$

$$H^A_{B_4} = 0$$

$A = abcddee$
$B = deafcbb$

$$H^A_{B_1} = -\frac{1}{\sqrt{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[(df \mid eb) - 4(ef \mid db)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(df \mid eb)$$

$A = abcddee$
$B = daefcbb$

$$H^A_{B_1} = \frac{1}{\sqrt{2}}\left[(df \mid eb) - (ef \mid db)\right]$$

$$H^A_{B_2} = -\frac{1}{\sqrt{6}}\left[(df \mid eb) - 3(ef \mid db)\right]$$

$$H^A_{B_3} = -\frac{1}{\sqrt{12}}(df \mid eb)$$

$$H^A_{B_4} = -\frac{\sqrt{5}}{2}(df \mid eb)$$

$A = abcdd$
$B = abcdf$

$$H^A_{B_1} = \sqrt{2}\Big[\{(d \mid h \mid f) + (df \mid aa) + (df \mid bb) + (df \mid cc) + (df \mid dd) + 2(df \mid P_a P_a) - (P_a f \mid dP_a)\} - \frac{1}{2}(da \mid af) - \frac{1}{2}(db \mid bf) - \frac{1}{2}(dc \mid cf)\Big]$$

$$H^A_{B_2} = \frac{1}{\sqrt{6}}\left[(da \mid af) + (db \mid bf) + 3(dc \mid cf)\right]$$

$$H^A_{B_3} = \frac{1}{\sqrt{12}}\left[(da \mid af) + 4(db \mid bf)\right]$$

$$H^A_{B_4} = \frac{\sqrt{5}}{2}(da \mid af)$$

3. $N_A = 5$, $N_B = 5$ $(k_A = 4$, $k_B = 4)$

$A = abcdeff$
$B = abfdecc$

$$H^{A_1}_{B_1} = -\Big\{(f \mid h \mid c) + (fc \mid aa) + (fc \mid bb) + (fc \mid cc) + (fc \mid dd) + (fc \mid ee) + (fc \mid ff) + 2(fc \mid P_a P_a) - (P_a c \mid fP_a)\Big\} + \frac{1}{2}(fd \mid dc) + \frac{1}{2}(fe \mid ec)$$

Quartet 4×4

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}\left[(fd\mid dc)-(fe\mid ec)\right]$$

$$H^{A_1}_{B_3} = H^{A_1}_{B_4} = 0$$

$$H^{A_2}_{B_2} = -\left\{\text{the above}\right\} + \frac{2}{3}(fa\mid ac) + \frac{2}{3}(fb\mid bc) + \frac{3}{2}(fd\mid dc) + \frac{3}{2}(fe\mid ec)$$

$$H^{A_2}_{B_3} = -\frac{\sqrt{2}}{6}\left[(fa\mid ac)+4(fb\mid bc)\right]$$

$$H^{A_2}_{B_4} = -\frac{\sqrt{30}}{6}(fa\mid ac)$$

$$H^{A_3}_{B_3} = -\left\{\text{the above}\right] + \frac{1}{12}(fa\mid ac) + \frac{4}{3}(fb\mid bc)$$

$$H^{A_3}_{B_4} = \frac{\sqrt{15}}{12}(fa\mid ac)$$

$$H^{A_4}_{B_4} = -\left\{\text{the above}\right\} + \frac{5}{4}(fa\mid ac)$$

$$H^{A_i}_{B_j} \equiv H^{A_j}_{B_i}$$

$A = abcdeffgg$
$B = abcfgddee$

$$H^{A_1}_{B_1} = (fd\mid ge) + (gd\mid fe)$$

$$H^{A_2}_{B_2} = H^{A_3}_{B_3} = H^{A_4}_{B_4} = (fd\mid ge) - (gd\mid fe)$$

$$H^{A_1}_{B_2} = H^{A_1}_{B_3} = H^{A_1}_{B_4} = H^{A_2}_{B_3} = H^{A_2}_{B_4} = H^{A_3}_{B_4} = 0$$

$$H^{A_i}_{B_j} \equiv H^{A_j}_{B_i}$$

$A = abcdeffgg$
$B = afbgeccdd$

$$H^{A_1}_{B_1} = -(fc\mid gd) + \frac{1}{2}(gc\mid fd)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{6}(gc\mid fd)$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{3}(gc\mid fd)$$

$$H^{A_1}_{B_4} = 0 = H^{A_2}_{B_4} = H^{A_3}_{B_1} = H^{A_3}_{B_4} = H^{A_4}_{B_1} = H^{A_4}_{B_2} = H^{A_4}_{B_3}$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(gc\mid fd)$$

$$H^{A_2}_{B_2} = -\frac{1}{3}\left[(fc\mid gd)+\frac{1}{2}(gc\mid fd)\right]$$

$$H^{A_2}_{B_3} = -\frac{2\sqrt{2}}{3}\left[(fc\mid gd)+\frac{1}{2}(gc\mid fd)\right]$$

$$H^{A_3}_{B_2} = -\frac{2\sqrt{2}}{3}\left[(fc\mid gd)-(gc\mid fd)\right]$$

$$H^{A_3}_{B_3} = \frac{1}{3}\left[(fc\mid gd)-(gc\mid fd)\right]$$

$$H^{A_4}_{B_4} = -(fc\mid gd) + (gc\mid fd)$$

Quartet 4×4

$A = abcdeffgg$
$B = afbegccdd$

$$H^{A_1}_{B_1} = -(fc \mid gd) + \frac{1}{2}(gc \mid fd)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{6}(gc \mid fd)$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{3}(gc \mid fd)$$

$$H^{A_1}_{B_4} = H^{A_2}_{B_4} = H^{A_3}_{B_1} = H^{A_3}_{B_4} = H^{A_4}_{B_1} = H^{A_4}_{B_2} = H^{A_4}_{B_3} = 0$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{2}(gc \mid fd)$$

$$H^{A_2}_{B_2} = \frac{1}{3}\left[(fc \mid gd) + \frac{1}{2}(gc \mid fd)\right]$$

$$H^{A_2}_{B_3} = \frac{2\sqrt{2}}{3}\left[(fc \mid gd) + \frac{1}{2}(gc \mid fd)\right]$$

$$H^{A_3}_{B_2} = \frac{2\sqrt{2}}{3}\left[(fc \mid gd) - (gc \mid fd)\right]$$

$$H^{A_3}_{B_3} = -\frac{1}{3}\left[(fc \mid gd) - (gc \mid fd)\right]$$

$$H^{A_4}_{B_4} = (fc \mid gd) - (gc \mid fd)$$

$A = abcdeffgg$
$B = acfdgbbee$

$$H^{A_1}_{B_1} = -(fb \mid ge) + \frac{1}{2}(gb \mid fe)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}(gb \mid fe)$$

$$H^{A_1}_{B_3} = H^{A_1}_{B_4} = H^{A_2}_{B_4} = H^{A_3}_{B_4} = H^{A_4}_{B_1} = H^{A_4}_{B_2} = H^{A_4}_{B_3} = 0$$

$$H^{A_2}_{B_1} = -\frac{\sqrt{3}}{6}(gb \mid fe)$$

$$H^{A_2}_{B_2} = -\frac{1}{3}\left[(fb \mid ge) + \frac{1}{2}(gb \mid fe)\right]$$

$$H^{A_2}_{B_3} = -\frac{2\sqrt{2}}{3}\left[(fb \mid ge) - (gb \mid fe)\right]$$

$$H^{A_3}_{B_1} = -\frac{\sqrt{6}}{3}(gb \mid fe)$$

$$H^{A_3}_{B_2} = -\frac{2\sqrt{2}}{3}\left[(fb \mid ge) + \frac{1}{2}(gb \mid fe)\right]$$

$$H^{A_3}_{B_3} = \frac{1}{3}\left[(fb \mid ge) - (gb \mid fe)\right]$$

$$H^{A_4}_{B_4} = -(fb \mid ge) + (gb \mid fe)$$

$A = abcdeffgg$
$B = afbdgccee$

$$H^{A_1}_{B_1} = -(fc \mid ge) + \frac{1}{2}(gc \mid fe)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{6}(gc \mid fe)$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{3}(gc \mid fe)$$

$$H_{B_4}^{A_1} = H_{B_4}^{A_2} = H_{B_1}^{A_3} = H_{B_4}^{A_3} = H_{B_1}^{A_4} = H_{B_2}^{A_4} = H_{B_3}^{A_4} = 0$$

$$H_{B_1}^{A_2} = -\frac{\sqrt{3}}{2}(gc \mid fe)$$

$$H_{B_2}^{A_2} = -\frac{1}{3}\left[(fc \mid ge) + \frac{1}{2}(gc \mid fe)\right]$$

$$H_{B_3}^{A_2} = -\frac{2\sqrt{2}}{3}\left[(fc \mid ge) + \frac{1}{2}(gc \mid fe)\right]$$

$$H_{B_2}^{A_3} = -\frac{2\sqrt{2}}{3}\left[(fc \mid ge) - (gc \mid fe)\right]$$

$$H_{B_3}^{A_3} = \frac{1}{3}\left[(fc \mid ge) - (gc \mid fe)\right]$$

$$H_{B_4}^{A_4} = -(fc \mid ge) + (gc \mid fe)$$

$A = abcdeff$
$B = abcdfee$

$$H_{B_1}^{A_1} = \Big[-\{(f \mid h \mid e) + (fe \mid aa) + (fe \mid bb) + (fe \mid cc) + (fe \mid dd) + (fe \mid ee) + (fe \mid ff) + 2(fe \mid P_a P_a) - (P_a e \mid f P_a)\} + \frac{1}{2}(fa \mid ae) + \frac{1}{2}(fb \mid be) + \frac{1}{2}(fc \mid ce) + 2(fd \mid de)\Big]$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{6}\left[(fa \mid ae) + (fb \mid be) + 3(fc \mid ce)\right]$$

$$H_{B_3}^{A_1} = \frac{\sqrt{6}}{12}\left[(fa \mid ae) + 4(fb \mid be)\right]$$

$$H_{B_4}^{A_1} = \frac{\sqrt{10}}{4}(fa \mid ae)$$

$$H_{B_2}^{A_2} = -\left\{\text{the above}\right\} + \frac{1}{6}(fa \mid ae) + \frac{1}{6}(fb \mid be) + \frac{3}{2}(fc \mid ce)$$

$$H_{B_3}^{A_2} = \frac{\sqrt{2}}{12}\left[(fa \mid ae) + 4(fb \mid be)\right]$$

$$H_{B_4}^{A_2} = \frac{\sqrt{30}}{12}(fa \mid ae)$$

$$H_{B_3}^{A_3} = -\left\{\text{the above}\right\} + \frac{1}{12}(fa \mid ae) + \frac{4}{3}(fb \mid be)$$

$$H_{B_4}^{A_3} = \frac{\sqrt{15}}{12}(fa \mid ae)$$

$$H_{B_4}^{A_4} = -\left\{\text{the above}\right\} + \frac{5}{4}(fa \mid ae)$$

$$H_{B_j}^{A_i} \equiv H_{B_i}^{A_j}$$

$A = abcdeffgg$
$B = acfgebbdd$

$$H_{B_1}^{A_1} = -(fb \mid gd) + \frac{1}{2}(gb \mid fd)$$

$$H_{B_2}^{A_1} = \frac{\sqrt{3}}{2}(gb \mid fd)$$

$$H_{B_3}^{A_1} = H_{B_4}^{A_1} = H_{B_4}^{A_2} = H_{B_4}^{A_3} = H_{B_1}^{A_4} = H_{B_2}^{A_4} = H_{B_3}^{A_4} = 0$$

$$H_{B_1}^{A_2} = \frac{\sqrt{3}}{6}(gb \mid fd)$$

Quartet 4×4

$$H^{A_2}_{B_2} = -\frac{1}{3}\left[(fb\mid gd) + \frac{1}{2}(gb\mid fd)\right]$$

$$H^{A_2}_{B_3} = -\frac{2\sqrt{2}}{3}\left[(fb\mid gd) - (gb\mid fd)\right]$$

$$H^{A_3}_{B_1} = \frac{\sqrt{6}}{3}(gb\mid fd)$$

$$H^{A_3}_{B_2} = -\frac{2\sqrt{2}}{3}\left[(fb\mid gd) + \frac{1}{2}(gb\mid fd)\right]$$

$$H^{A_3}_{B_3} = \frac{1}{3}\left[(fb\mid gd) - (gb\mid fd)\right]$$

$$H^{A_4}_{B_4} = -(fb\mid gd) + (gb\mid fd)$$

$A = abcdeffgg$
$B = acfegbbdd$

$$H^{A_1}_{B_1} = -(fb\mid gd) + \frac{1}{2}(gb\mid fd)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{2}(gb\mid fd)$$

$$H^{A_1}_{B_3} = H^{A_1}_{B_4} = H^{A_2}_{B_4} = H^{A_3}_{B_4} = H^{A_4}_{B_1} = H^{A_4}_{B_2} = H^{A_4}_{B_3} = 0$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{6}(gb\mid fd)$$

$$H^{A_2}_{B_2} = \frac{1}{3}\left[(fb\mid gd) + \frac{1}{2}(gb\mid fd)\right]$$

$$H^{A_2}_{B_3} = \frac{2\sqrt{2}}{3}\left[(fb\mid gd) - (gb\mid fd)\right]$$

$$H^{A_3}_{B_1} = \frac{\sqrt{6}}{3}(gb\mid fd)$$

$$H^{A_3}_{B_2} = \frac{2\sqrt{2}}{3}\left[(fb\mid gd) + \frac{1}{2}(gb\mid fd)\right]$$

$$H^{A_3}_{B_3} = -\frac{1}{3}\left[(fb\mid gd) - (gb\mid fd)\right]$$

$$H^{A_4}_{B_4} = (fb\mid gd) - (gb\mid fd)$$

$A = abcdeff$
$B = abcefdd$

$$H^{A_1}_{B_1} = -\Big\{(f\mid h\mid d) + (fd\mid aa) + (fd\mid bb) + (fd\mid cc) + (fd\mid dd) + (fd\mid ee) + (fd\mid ff) + 2(fd\mid P_aP_a) - (P_ad\mid fP_a)\Big\} + \frac{1}{2}(fa\mid ad) + \frac{1}{2}(fb\mid bd) + \frac{1}{2}(fc\mid cd) + 2(fe\mid ed)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{6}\left[(fa\mid ad) + (fb\mid bd) + 3(fc\mid cd)\right] = -H^{A_2}_{B_1}$$

$$H^{A_1}_{B_3} = \frac{\sqrt{6}}{12}\left[(fa\mid ad) + 4(fb\mid bd)\right] = -H^{A_3}_{B_1}$$

$$H^{A_1}_{B_4} = \frac{\sqrt{10}}{4}(fa\mid ad) = -H^{A_4}_{B_1}$$

$$H^{A_2}_{B_2} = \Big\{\text{the above}\Big\} - \frac{1}{6}(fa\mid ad) - \frac{1}{6}(fb\mid bd) - \frac{3}{2}(fc\mid cd)$$

$$H^{A_2}_{B_3} = -\frac{\sqrt{2}}{12}\left[(fa\mid ad) + 4(fb\mid bd)\right] = H^{A_3}_{B_2}$$

Quartet 4×4

$$H^{A_2}_{B_4} = -\frac{\sqrt{30}}{12}(fa \mid ad) = H^{A_4}_{B_2}$$

$$H^{A_3}_{B_3} = \left\{\text{the above}\right\} - \frac{1}{12}(fa \mid ad) - \frac{4}{3}(fb \mid bd)$$

$$H^{A_3}_{B_4} = -\frac{\sqrt{15}}{12}(fa \mid ad) = H^{A_4}_{B_3}$$

$$H^{A_4}_{B_4} = \left\{\text{the above}\right\} - \frac{5}{4}(fa \mid ad)$$

$A = abcdeff$
$B = acfdebb$

$$H^{A_1}_{B_1} = \Big\{(f \mid h \mid b) + (fb \mid aa) + (fb \mid bb) + (fb \mid cc) + (fb \mid dd) + (fb \mid ee) + (fb \mid ff) + 2(fb \mid P_a P_a) - (P_a b \mid fP_a)\Big\} - \frac{1}{2}(fd \mid db) - \frac{1}{2}(fe \mid eb)$$

$$H^{A_1}_{B_2} = \frac{\sqrt{3}}{2}\Big[(fd \mid db) - (fe \mid eb)\Big]$$

$$H^{A_1}_{B_3} = H^{A_1}_{B_4} = H^{A_2}_{B_4} = H^{A_4}_{B_1} = 0$$

$$H^{A_2}_{B_1} = \frac{\sqrt{3}}{6}\Big[(fd \mid db) - (fe \mid eb)\Big]$$

$$H^{A_2}_{B_2} = \frac{1}{3}\left\{\text{the above}\right\} + \frac{2}{3}(fc \mid cb) - \frac{1}{2}(fd \mid db) - \frac{1}{2}(fe \mid eb)$$

$$H^{A_2}_{B_3} = \frac{2\sqrt{2}}{3}\Big[\{\text{the above}\} - (fc \mid cb)\Big]$$

$$H^{A_3}_{B_1} = \frac{\sqrt{6}}{3}\Big[(fd \mid db) - (fe \mid eb)\Big]$$

$$H^{A_3}_{B_2} = \frac{2\sqrt{2}}{3}\Big[\{\text{the above}\} - \frac{3}{4}(fa \mid ab) - (fc \mid cb) - \frac{3}{2}(fd \mid db) - \frac{3}{2}(fe \mid eb)\Big]$$

$$H^{A_3}_{B_3} = -\frac{1}{3}\left\{\text{the above}\right\} + \frac{1}{4}(fa \mid ab) + \frac{4}{3}(fc \mid cb)$$

$$H^{A_3}_{B_4} = \frac{\sqrt{15}}{4}(fa \mid ab)$$

$$H^{A_4}_{B_2} = \frac{\sqrt{30}}{6}(fa \mid ab)$$

$$H^{A_4}_{B_3} = -\frac{\sqrt{15}}{12}(fa \mid ab)$$

$$H^{A_4}_{B_4} = \left\{\text{the above}\right\} - \frac{5}{4}(fa \mid ab)$$

$A = abcdeff$
$B = abcfedd$

$$H^{A_1}_{B_1} = -\Big\{(f \mid h \mid d) + (fd \mid aa) + (fd \mid bb) + (fd \mid cc) + (fd \mid dd) + (fd \mid ee) + (fd \mid ff) + 2(fd \mid P_a P_a) - (P_a d \mid fP_a)\Big\} + \frac{1}{2}(fa \mid ad) + \frac{1}{2}(fb \mid bd) + \frac{1}{2}(fc \mid cd) + 2(fe \mid ed)$$

$$H^{A_1}_{B_2} = -\frac{\sqrt{3}}{6}\Big[(fa \mid ad) + (fb \mid bd) + 3(fc \mid cd)\Big]$$

$$H^{A_1}_{B_3} = -\frac{\sqrt{6}}{12}\Big[(fa \mid ad) + 4(fb \mid bd)\Big]$$

Quartet 4×4

$$H^{A_1}_{B_4} = -\frac{\sqrt{10}}{4}(fa \mid ad)$$

$$H^{A_2}_{B_2} = -\left\{\text{the above}\right\} + \frac{1}{6}(fa \mid ad) + \frac{1}{6}(fb \mid bd) + \frac{3}{2}(fc \mid cd)$$

$$H^{A_2}_{B_3} = \frac{\sqrt{2}}{12}\left[(fa \mid ad) + 4(fb \mid bd)\right]$$

$$H^{A_2}_{B_4} = \frac{\sqrt{30}}{12}(fa \mid ad)$$

$$H^{A_3}_{B_3} = -\left\{\text{the above}\right\} + \frac{1}{12}(fa \mid ad) + \frac{4}{3}(fb \mid bd)$$

$$H^{A_3}_{B_4} = \frac{\sqrt{15}}{12}(fa \mid ad)$$

$$H^{A_4}_{B_4} = -\left\{\text{the above}\right\} + \frac{5}{4}(fa \mid ad)$$

$$H^{A_i}_{B_j} \equiv H^{A_j}_{B_i}$$

C. IRREDUCIBLE REPRESENTATION MATRICES U(P)

I. Irreducible representation matrices U(P) corresponding to $N=6$, $S=0$

(12)

	1	2	3	4	5
1	1				
2		-1			
3			-1		
4				-1	
5					1

(13)

	1	2	3	4	5
1	$-1/2$	$\sqrt{3}/2$			
2	$\sqrt{3}/2$	$1/2$			
3			-1		
4				$1/2$	$\sqrt{3}/2$
5				$\sqrt{3}/2$	$-1/2$

(14)

	1	2	3	4	5
1	$-1/2$	$-\sqrt{3}/2$			
2	$-\sqrt{3}/2$	$1/2$			
3			$1/3$	$\sqrt{2}/3$	$\sqrt{6}/3$
4			$\sqrt{2}/3$	$-5/6$	$\sqrt{3}/6$
5			$\sqrt{6}/3$	$\sqrt{3}/6$	$-1/2$

(15)

	1	2	3	4	5
1	$-1/2$		$1/\sqrt{2}$	$-1/2$	
2		$-1/2$	$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/2$
3	$1/\sqrt{2}$	$1/\sqrt{6}$	$1/3$	$-\sqrt{2}/6$	$-1/\sqrt{6}$
4	$-1/2$	$1/\sqrt{3}$	$-\sqrt{2}/6$	$1/6$	$-1/\sqrt{3}$
5		$-1/2$	$-1/\sqrt{6}$	$-1/\sqrt{3}$	$-1/2$

(16)

	1	2	3	4	5
1	$-1/2$		$-1/\sqrt{2}$	$1/2$	
2		$-1/2$	$-1/\sqrt{6}$	$-1/\sqrt{3}$	$1/2$
3	$-1/\sqrt{2}$	$-1/\sqrt{6}$	$1/3$	$-\sqrt{2}/6$	$-1/\sqrt{6}$
4	$1/2$	$-1/\sqrt{3}$	$-\sqrt{2}/6$	$1/6$	$-1/\sqrt{3}$
5		$1/2$	$-1/\sqrt{6}$	$-1/\sqrt{3}$	$-1/2$

(23)

	1	2	3	4	5
1	$-1/2$	$-\sqrt{3}/2$			
2	$-\sqrt{3}/2$	$1/2$			
3			-1		
4				$1/2$	$-\sqrt{3}/2$
5				$-\sqrt{3}/2$	$-1/2$

(24)

	1	2	3	4	5
1	$-1/2$	$\sqrt{3}/2$			
2	$\sqrt{3}/2$	$1/2$			
3			$1/3$	$\sqrt{2}/3$	$-\sqrt{6}/3$
4			$\sqrt{2}/3$	$-5/6$	$-\sqrt{3}/6$
5			$-\sqrt{6}/3$	$-\sqrt{3}/6$	$-1/2$

(25)

	1	2	3	4	5
1	$-1/2$		$-1/\sqrt{2}$	$1/2$	
2		$-1/2$	$1/\sqrt{6}$	$1/\sqrt{3}$	$1/2$
3	$-1/\sqrt{2}$	$1/\sqrt{6}$	$1/3$	$-\sqrt{2}/6$	$1/\sqrt{6}$
4	$1/2$	$1/\sqrt{3}$	$-\sqrt{2}/6$	$1/6$	$1/\sqrt{3}$
5		$1/2$	$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/2$

(26)

	1	2	3	4	5
1	$-1/2$		$1/\sqrt{2}$	$-1/2$	
2		$-1/2$	$-1/\sqrt{6}$	$-1/\sqrt{3}$	$-1/2$
3	$1/\sqrt{2}$	$-1/\sqrt{6}$	$1/3$	$-\sqrt{2}/6$	$1/\sqrt{6}$
4	$-1/2$	$-1/\sqrt{3}$	$-\sqrt{2}/6$	$1/6$	$1/\sqrt{3}$
5		$-1/2$	$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/2$

(34)

	1	2	3	4	5
1	1				
2		-1			
3			$1/3$	$-2\sqrt{2}/3$	
4			$-2\sqrt{2}/3$	$-1/3$	
5					-1

$N=6,\ S=0$

(35)

	1	2	3	4	5
1	$-1/2$				$\sqrt{3}/2$
2		$-1/2$	$-\sqrt{6}/3$	$-\sqrt{3}/6$	
3		$-\sqrt{6}/3$	$1/3$	$\sqrt{2}/3$	
4		$-\sqrt{3}/6$	$\sqrt{2}/3$	$-5/6$	
5	$\sqrt{3}/2$				$1/2$

(36)

	1	2	3	4	5
1	$-1/2$				$-\sqrt{3}/2$
2		$-1/2$	$\sqrt{6}/3$	$\sqrt{3}/6$	
3		$\sqrt{6}/3$	$1/3$	$\sqrt{2}/3$	
4		$\sqrt{3}/6$	$\sqrt{2}/3$	$-5/6$	
5	$-\sqrt{3}/2$				$1/2$

(45)

	1	2	3	4	5
1	$-1/2$				$-\sqrt{3}/2$
2		$-1/2$		$-\sqrt{3}/2$	
3			-1		
4		$-\sqrt{3}/2$		$1/2$	
5	$-\sqrt{3}/2$				$1/2$

(46)

	1	2	3	4	5
1	$-1/2$				$\sqrt{3}/2$
2		$-1/2$		$\sqrt{3}/2$	
3			-1		
4		$\sqrt{3}/2$		$1/2$	
5	$\sqrt{3}/2$				$1/2$

(56)

	1	2	3	4	5
1	1				
2		1			
3			-1		
4				-1	
5					-1

II. Irreducible representation matrices U(P) corresponding to $N=5$, $S=\frac{1}{2}$

(12)

	1	2	3	4	5
1	$-1/2$	$\sqrt{3}/2$			
2	$\sqrt{3}/2$	$1/2$			
3			$-1/2$	$-\sqrt{3}/2$	
4			$-\sqrt{3}/2$	$1/2$	
5					-1

(13)

	1	2	3	4	5
1	$-1/2$	$-\sqrt{3}/2$			
2	$-\sqrt{3}/2$	$1/2$			
3			$-1/2$	$-\sqrt{3}/6$	$-\sqrt{6}/3$
4			$-\sqrt{3}/6$	$-5/6$	$\sqrt{2}/3$
5			$-\sqrt{6}/3$	$\sqrt{2}/3$	$1/3$

(14)

	1	2	3	4	5
1	$-1/2$			$-1/2$	$1/\sqrt{2}$
2		$-1/2$	$1/2$	$1/\sqrt{3}$	$1/\sqrt{6}$
3		$1/2$	$-1/2$	$1/\sqrt{3}$	$1/\sqrt{6}$
4	$-1/2$	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/6$	$-\sqrt{2}/6$
5	$1/\sqrt{2}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$-\sqrt{2}/6$	$1/3$

(15)

	1	2	3	4	5
1	$-1/2$			$1/2$	$-1/\sqrt{2}$
2		$-1/2$	$-1/2$	$-1/\sqrt{3}$	$-1/\sqrt{6}$
3		$-1/2$	$-1/2$	$1/\sqrt{3}$	$1/\sqrt{6}$
4	$1/2$	$-1/\sqrt{3}$	$1/\sqrt{3}$	$1/6$	$-\sqrt{2}/6$
5	$-1/\sqrt{2}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$-\sqrt{2}/6$	$1/3$

(23)

	1	2	3	4	5
1	1				
2		-1			
3			-1		
4				$-1/3$	$-2\sqrt{2}/3$
5				$-2\sqrt{2}/3$	$1/3$

(24)

	1	2	3	4	5
1	$-1/2$		$-\sqrt{3}/2$		
2		$-1/2$		$-\sqrt{3}/6$	$-\sqrt{6}/3$
3	$-\sqrt{3}/2$		$1/2$		
4		$-\sqrt{3}/6$		$-5/6$	$\sqrt{2}/3$
5		$-\sqrt{6}/3$		$\sqrt{2}/3$	$1/3$

$N=5$, $S=\frac{1}{2}$

(25)

	1	2	3	4	5
1	$-1/2$		$\sqrt{3}/2$		
2		$-1/2$		$\sqrt{3}/6$	$\sqrt{6}/3$
3	$\sqrt{3}/2$		$1/2$		
4		$\sqrt{3}/6$		$-5/6$	$\sqrt{2}/3$
5		$\sqrt{6}/3$		$\sqrt{2}/3$	$1/3$

(34)

	1	2	3	4	5
1	$-1/2$		$\sqrt{3}/2$		
2		$-1/2$		$-\sqrt{3}/2$	
3	$\sqrt{3}/2$		$1/2$		
4		$-\sqrt{3}/2$		$1/2$	
5					-1

(35)

	1	2	3	4	5
1	$-1/2$		$-\sqrt{3}/2$		
2		$-1/2$		$\sqrt{3}/2$	
3	$-\sqrt{3}/2$		$1/2$		
4		$\sqrt{3}/2$		$1/2$	
5					-1

(45)

	1	2	3	4	5
1	1				
2		1			
3			-1		
4				-1	
5					-1

$N=5, \ S=\frac{1}{2}$

III Irreducible representation matrices U(P) corresponding to $N=6$, $S=1$

(12)

	1	2	3	4	5	6	7	8	9
1	-1								
2		$-1/3$	$-2\sqrt{2}/3$						
3		$-2\sqrt{2}/3$	$1/3$						
4				-1					
5					$1/3$		$-2\sqrt{2}/3$		
6						$1/3$		$-2\sqrt{2}/3$	
7					$-2\sqrt{2}/3$		$-1/3$		
8						$-2\sqrt{2}/3$		$-1/3$	
9									-1

(13)

	1	2	3	4	5	6	7	8	9
1	$-1/2$	$-\sqrt{3}/6$	$-\sqrt{6}/3$						
2	$-\sqrt{3}/6$	$-5/6$	$\sqrt{2}/3$						
3	$-\sqrt{6}/3$	$\sqrt{2}/3$	$1/3$						
4				$1/4$	$\sqrt{15}/12$		$-\sqrt{30}/6$		
5				$\sqrt{15}/12$	$-11/12$		$-\sqrt{2}/6$		
6						$1/3$		$\sqrt{2}/3$	$-\sqrt{6}/3$
7				$-\sqrt{30}/6$	$-\sqrt{2}/6$		$-1/3$		
8						$\sqrt{2}/3$		$-5/6$	$-\sqrt{3}/6$
9						$-\sqrt{6}/3$		$-\sqrt{3}/6$	$-1/2$

(14)

	1	2	3	4	5	6	7	8	9
1	$-1/2$	$\sqrt{3}/6$	$\sqrt{6}/3$						
2	$\sqrt{3}/6$	$-5/6$	$\sqrt{2}/3$						
3	$\sqrt{6}/3$	$\sqrt{2}/3$	$1/3$						
4				$1/4$	$-\sqrt{15}/36$	$\sqrt{30}/18$	$\sqrt{30}/18$	$-2\sqrt{15}/9$	
5				$-\sqrt{15}/36$	$7/36$	$5\sqrt{2}/18$	$5\sqrt{2}/18$	$2/9$	$-4\sqrt{3}/9$
6				$\sqrt{30}/18$	$5\sqrt{2}/18$	$-7/9$	$2/9$	$-\sqrt{2}/9$	$-\sqrt{6}/9$
7				$\sqrt{30}/18$	$5\sqrt{2}/18$	$2/9$	$-7/9$	$-\sqrt{2}/9$	$-\sqrt{6}/9$
8				$-2\sqrt{15}/9$	$2/9$	$-\sqrt{2}/9$	$-\sqrt{2}/9$	$-7/18$	$-\sqrt{3}/18$
9					$-4\sqrt{3}/9$	$-\sqrt{6}/9$	$-\sqrt{6}/9$	$-\sqrt{3}/18$	$-1/2$

(15)

	1	2	3	4	5	6	7	8	9
1	$-1/2$				$-2/3$	$\sqrt{2}/3$	$-\sqrt{2}/6$	$1/6$	
2		$-1/2$		$-\sqrt{5}/3$	$\sqrt{3}/9$	$\sqrt{6}/9$	$-\sqrt{6}/18$	$-\sqrt{3}/9$	$1/6$
3			$-1/2$	$\sqrt{10}/12$	$5\sqrt{6}/36$	$5\sqrt{3}/18$	$\sqrt{3}/9$	$\sqrt{6}/9$	$\sqrt{2}/3$
4		$-\sqrt{5}/3$	$\sqrt{10}/12$	$1/4$	$-\sqrt{15}/36$	$-\sqrt{30}/36$	$\sqrt{30}/18$	$\sqrt{15}/9$	
5	$-2/3$	$\sqrt{3}/9$	$5\sqrt{6}/36$	$-\sqrt{15}/36$	$7/36$	$-5\sqrt{2}/36$	$5\sqrt{2}/18$	$-1/9$	$2\sqrt{3}/9$
6	$\sqrt{2}/3$	$\sqrt{6}/9$	$5\sqrt{3}/18$	$-\sqrt{30}/36$	$-5\sqrt{2}/36$	$1/18$	$-1/9$	$2\sqrt{2}/9$	$2\sqrt{6}/9$
7	$-\sqrt{2}/6$	$-\sqrt{6}/18$	$\sqrt{3}/9$	$\sqrt{30}/18$	$5\sqrt{2}/18$	$-1/9$	$-7/9$	$\sqrt{2}/18$	$\sqrt{6}/18$
8	$1/6$	$-\sqrt{3}/9$	$\sqrt{6}/9$	$\sqrt{15}/9$	$-1/9$	$2\sqrt{2}/9$	$\sqrt{2}/18$	$-13/18$	$\sqrt{3}/9$
9		$1/6$	$\sqrt{2}/3$		$2\sqrt{3}/9$	$2\sqrt{6}/9$	$\sqrt{6}/18$	$\sqrt{3}/9$	$-1/2$

$N=6, \ S=1$

(16)

	1	2	3	4	5	6	7	8	9
1	$-1/2$				$2/3$	$-\sqrt{2}/3$	$\sqrt{2}/6$	$-1/6$	
2		$-1/2$		$\sqrt{5}/3$	$-\sqrt{3}/9$	$-\sqrt{6}/9$	$\sqrt{6}/18$	$\sqrt{3}/9$	$-1/6$
3			$-1/2$	$-\sqrt{10}/12$	$-5\sqrt{6}/36$	$-5\sqrt{3}/18$	$-\sqrt{3}/9$	$-\sqrt{6}/9$	$-\sqrt{2}/3$
4		$\sqrt{5}/3$	$-\sqrt{10}/12$	$1/4$	$-\sqrt{15}/36$	$-\sqrt{30}/36$	$\sqrt{30}/18$	$\sqrt{15}/9$	
5	$2/3$	$-\sqrt{3}/9$	$-5\sqrt{6}/36$	$-\sqrt{15}/36$	$7/36$	$-5\sqrt{2}/36$	$5\sqrt{2}/18$	$-1/9$	$2\sqrt{3}/9$
6	$-\sqrt{2}/3$	$-\sqrt{6}/9$	$-5\sqrt{3}/18$	$-\sqrt{30}/36$	$-5\sqrt{2}/36$	$1/18$	$-1/9$	$2\sqrt{2}/9$	$2\sqrt{6}/9$
7	$\sqrt{2}/6$	$\sqrt{6}/18$	$-\sqrt{3}/9$	$\sqrt{30}/18$	$5\sqrt{2}/18$	$-1/9$	$-7/9$	$\sqrt{2}/18$	$\sqrt{6}/18$
8	$-1/6$	$\sqrt{3}/9$	$-\sqrt{6}/9$	$\sqrt{15}/9$	$-1/9$	$2\sqrt{2}/9$	$\sqrt{2}/18$	$-13/18$	$\sqrt{3}/9$
9		$-1/6$	$-\sqrt{2}/3$		$2\sqrt{3}/9$	$2\sqrt{6}/9$	$\sqrt{6}/18$	$\sqrt{3}/9$	$-1/2$

(23)

	1	2	3	4	5	6	7	8	9
1	$-1/2$	$-\sqrt{3}/2$							
2	$-\sqrt{3}/2$	$1/2$							
3			-1						
4				$1/4$	$-\sqrt{15}/4$				
5				$-\sqrt{15}/4$	$-1/4$				
6						-1			
7							-1		
8								$1/2$	$-\sqrt{3}/2$
9								$-\sqrt{3}/2$	$-1/2$

$N=6,\ S=1$

(24)

	1	2	3	4	5	6	7	8	9
1	$-1/2$	$\sqrt{3}/2$							
2	$\sqrt{3}/2$	$1/2$							
3			-1						
4				$1/4$	$\sqrt{15}/12$	$-\sqrt{30}/6$			
5				$\sqrt{15}/12$	$-11/12$	$-\sqrt{2}/6$			
6				$-\sqrt{30}/6$	$-\sqrt{2}/6$	$-1/3$			
7							$1/3$	$\sqrt{2}/3$	$-\sqrt{6}/3$
8							$\sqrt{2}/3$	$-5/6$	$-\sqrt{3}/6$
9							$-\sqrt{6}/3$	$-\sqrt{3}/6$	$-1/2$

(25)

	1	2	3	4	5	6	7	8	9
1	$-1/2$						$-1/\sqrt{2}$	$1/2$	
2		$-1/2$					$1/\sqrt{6}$	$1/\sqrt{3}$	$1/2$
3			$-1/2$	$-\sqrt{10}/4$	$-\sqrt{6}/12$	$-\sqrt{3}/6$			
4			$-\sqrt{10}/4$	$1/4$	$\sqrt{15}/12$	$\sqrt{30}/12$			
5			$-\sqrt{6}/12$	$\sqrt{15}/12$	$-11/12$	$\sqrt{2}/12$			
6			$-\sqrt{3}/6$	$\sqrt{30}/12$	$\sqrt{2}/12$	$-5/6$			
7	$-1/\sqrt{2}$	$1/\sqrt{6}$					$1/3$	$-\sqrt{2}/6$	$1/\sqrt{6}$
8	$1/2$	$1/\sqrt{3}$					$-\sqrt{2}/6$	$1/6$	$1/\sqrt{3}$
9		$1/2$					$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/2$

N = 6, S = 1

(26)

	1	2	3	4	5	6	7	8	9
1	$-1/2$						$1/\sqrt{2}$	$-1/2$	
2		$-1/2$					$-1/\sqrt{6}$	$-1/\sqrt{3}$	$-1/2$
3			$-1/2$	$\sqrt{10}/4$	$\sqrt{6}/12$	$\sqrt{3}/6$			
4			$\sqrt{10}/4$	$1/4$	$\sqrt{15}/12$	$\sqrt{30}/12$			
5			$\sqrt{6}/12$	$\sqrt{15}/12$	$-11/12$	$\sqrt{2}/12$			
6			$\sqrt{3}/6$	$\sqrt{30}/12$	$\sqrt{2}/12$	$-5/6$			
7	$1/\sqrt{2}$	$-1/\sqrt{6}$					$1/3$	$-\sqrt{2}/6$	$1/\sqrt{6}$
8	$-1/2$	$-1/\sqrt{3}$					$-\sqrt{2}/6$	$1/6$	$1/\sqrt{3}$
9		$-1/2$					$1/\sqrt{6}$	$1/\sqrt{3}$	$-1/2$

(34)

	1	2	3	4	5	6	7	8	9
1	1								
2		-1							
3			-1						
4				-1					
5					$1/3$	$-2\sqrt{2}/3$			
6					$-2\sqrt{2}/3$	$-1/3$			
7							$1/3$	$-2\sqrt{2}/3$	
8							$-2\sqrt{2}/3$	$-1/3$	
9									-1

N = 6, S = 1

(35)

	1	2	3	4	5	6	7	8	9
1	$-1/2$								$\sqrt{3}/2$
2		$-1/2$					$-\sqrt{6}/3$	$-\sqrt{3}/6$	
3			$-1/2$		$-\sqrt{6}/3$	$-\sqrt{3}/6$			
4				-1					
5			$-\sqrt{6}/3$		$1/3$	$\sqrt{2}/3$			
6			$-\sqrt{3}/6$		$\sqrt{2}/3$	$-5/6$			
7		$-\sqrt{6}/3$					$1/3$	$\sqrt{2}/3$	
8		$-\sqrt{3}/6$					$\sqrt{2}/3$	$-5/6$	
9	$\sqrt{3}/2$								$1/2$

(36)

	1	2	3	4	5	6	7	8	9
1	$-1/2$								$-\sqrt{3}/2$
2		$-1/2$					$\sqrt{6}/3$	$\sqrt{3}/6$	
3			$-1/2$		$\sqrt{6}/3$	$\sqrt{3}/6$			
4				-1					
5			$\sqrt{6}/3$		$1/3$	$\sqrt{2}/3$			
6			$\sqrt{3}/6$		$\sqrt{2}/3$	$-5/6$			
7		$\sqrt{6}/3$					$1/3$	$\sqrt{2}/3$	
8		$\sqrt{3}/6$					$\sqrt{2}/3$	$-5/6$	
9	$-\sqrt{3}/2$								$1/2$

(45)

	1	2	3	4	5	6	7	8	9
1	$-1/2$								$-\sqrt{3}/2$
2		$-1/2$						$-\sqrt{3}/2$	
3			$-1/2$			$-\sqrt{3}/2$			
4				-1					
5					-1				
6			$-\sqrt{3}/2$			$1/2$			
7							-1		
8		$-\sqrt{3}/2$						$1/2$	
9	$-\sqrt{3}/2$								$1/2$

(46)

	1	2	3	4	5	6	7	8	9
1	$-1/2$								$\sqrt{3}/2$
2		$-1/2$						$\sqrt{3}/2$	
3			$-1/2$			$\sqrt{3}/2$			
4				-1					
5					-1				
6			$\sqrt{3}/2$			$1/2$			
7							-1		
8		$\sqrt{3}/2$						$1/2$	
9	$\sqrt{3}/2$								$1/2$

$N=6,\ S=1$

(56)

	1	2	3	4	5	6	7	8	9
1	1								
2		1							
3			1						
4				− 1					
5					− 1				
6						− 1			
7							− 1		
8								− 1	
9									− 1

IV. Irreducible representation matrices U(P) corresponding to $N=5$, $S=3/2$

(12)

	1	2	3	4
1	− 1			
2		− 1		
3			− 1 / 4	$-\sqrt{15}/4$
4			$-\sqrt{15}/4$	1 / 4

(13)

	1	2	3	4
1	− 1			
2		− 1 / 3	$-\sqrt{2}/6$	$-\sqrt{30}/6$
3		$-\sqrt{2}/6$	−11/12	$\sqrt{15}/12$
4		$-\sqrt{30}/6$	$\sqrt{15}/12$	1 / 4

(14)

	1	2	3	4
1	− 1 / 2	$-\sqrt{3}/6$	$-\sqrt{6}/12$	$-\sqrt{10}/4$
2	$-\sqrt{3}/6$	− 5 / 6	$\sqrt{2}/12$	$\sqrt{30}/12$
3	$-\sqrt{6}/12$	$\sqrt{2}/12$	−11/12	$\sqrt{15}/12$
4	$-\sqrt{10}/4$	$\sqrt{30}/12$	$\sqrt{15}/12$	1 / 4

(15)

	1	2	3	4
1	− 1 / 2	$\sqrt{3}/6$	$\sqrt{6}/12$	$\sqrt{10}/4$
2	$\sqrt{3}/6$	− 5 / 6	$\sqrt{2}/12$	$\sqrt{30}/12$
3	$\sqrt{6}/12$	$\sqrt{2}/12$	−11/12	$\sqrt{15}/12$
4	$\sqrt{10}/4$	$\sqrt{30}/12$	$\sqrt{15}/12$	1 / 4

(23)

	1	2	3	4
1	-1			
2		$-1/3$	$-2\sqrt{2}/3$	
3		$-2\sqrt{2}/3$	$1/3$	
4				-1

(24)

	1	2	3	4
1	$-1/2$	$-\sqrt{3}/6$	$-\sqrt{6}/3$	
2	$-\sqrt{3}/6$	$-5/6$	$\sqrt{2}/3$	
3	$-\sqrt{6}/3$	$\sqrt{2}/3$	$1/3$	
4				-1

(25)

	1	2	3	4
1	$-1/2$	$\sqrt{3}/6$	$\sqrt{6}/3$	
2	$\sqrt{3}/6$	$-5/6$	$\sqrt{2}/3$	
3	$\sqrt{6}/3$	$\sqrt{2}/3$	$1/3$	
4				-1

(34)

	1	2	3	4
1	$-1/2$	$-\sqrt{3}/2$		
2	$-\sqrt{3}/2$	$1/2$		
3			-1	
4				-1

(35)

	1	2	3	4
1	$-1/2$	$\sqrt{3}/2$		
2	$\sqrt{3}/2$	$1/2$		
3			-1	
4				-1

(45)

	1	2	3	4
1	1			
2		-1		
3			-1	
4				-1

$N=5,\ S=\frac{3}{2}$