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A Note on Non-Abelian Gauge Fields

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Abstract Two points on non-abelian gauge fields are presented in this note. One point is that there are conserved axial vector currents, besides the well-known conserved vector currents, with the same transformation property of global group. The other is that the self-forces of non-abelian gauge fields always vanish. These two results hold gauge-invariantly. Further remarks are also presented.

§ 1 Introduction

The non-abelian gauge fields[1)-4)] (or Yang-Mills fields) A^a_μ and tensor fields $G^a_{\mu\nu}$ are not gauge invariant quantities. Besides, non-abelian gauge fields have the charges and are self-interacting and therefore Maxwell equations for non-abelian gauge fields are non-linear equations and the superposition principle must be abandoned. Up to the present day no one has succeeded in obtaining of the exact classical solutions of non-abelian gauge field Maxwell equations[5)- 6)]. Of course the Lagrangian is gauge invariant, but even the Hamiltonian is not gauge invariant. When we define the observables are gauge invariant quantities, the observables are the integrals T^a (defined in eq. (64)) and T^{aA} (defined in eq. (63)), Lorentz forces K_μ (defined in eq. (36)), Poynting vectors (defined in eq. (58)), Maxwell stress tensors (defined in eqs. (59) and (60)), free field energy densities (defined in eq. (61)), and the quantities $\sum_a J^a_\mu J^a_\nu$ (J^a_μ are defined in eq. (8)) and $\sum_a G^a_{\mu\nu} G^a_{\lambda\rho}$. The following quantities are not the observables: the fields A^a_μ, the field strengths $G^a_{\mu\nu}$, Lorentz forces $\tilde{\tilde{K}}_\mu$, and K^A_μ (defined in eqs. (37), and (35), respectively), and various currents J^a_μ, $\tilde{\tilde{J}}^a_\mu$, $\tilde{J}^a_\mu$, and J^{aA}_μ (defined in eqs. (8), (18), (9) and (19), respectively).

In this note it will be shown that there are conserved axial vector currents, besides the well-known conserved vector currents, with the same transformation property of global group. It will also be shown that the self-forces of non-abelian gauge fields always vanish, which are the sum of electric Lorentz forces and magnetic Lorentz forces (defined in § 4, eq. (35)). These two results hold gauge-invariantly.

Some notations are explained in § 2. Axial vector currents are defined and discussed in § 3. Magnetic Lorentz forces are defined and the self-forces of non-abelian gauge fields are discussed in § 4. The contents in § 3 and § 4 are gauge independent ones. Lorentz condition is used solely in § 5. In § 5 the Hamiltonian, Poynting vectors, Maxwell stress tensors, and energy-momentum conservation expressions are referred.

§ 2 Notations

We consider a fermion field ψ with internal degrees of freedom (a, b, c : internal indices). The generator of internal symmetry group is F^a and f^{abc} is the structure constant of the group. Yang and Mills introduced gauge fields A^a_μ (λ, μ, ν:Lorentz indices) which must accompany the fermion field ψ[1]. We consider the gauge transformation

$$(\psi)' = u\psi u^{-1},\ u(x) = \exp i\ \{F^a\omega^{a\cdot}(x)\} \tag{1}$$

Gauge fields are transformed as

$$(A^a_\mu)' = A^a_\mu + f^{abc}A^b_\mu\delta\omega^c + \frac{1}{g}\ \partial_\mu\delta\omega^a \tag{2}$$

$$(G^a_{\mu\nu})' = G^a_{\mu\nu} + f^{abc}G^b_{\mu\nu}\delta\omega^c, \tag{3}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ and g is the coupling constant.Covariant derivatives are

$$D_\mu = \partial_\mu - igA^a_\mu\ [F^a,\ ]\ . \tag{4}$$

Because

$$D_\nu\ (F^aG^a_{\mu\nu}) = \partial_\nu F^aG^a_{\mu\nu} - igA^b_\nu\ [F^b, F^a]\ G^a_{\mu\nu}$$
$$= \partial_\nu F^aG^a_{\mu\nu} + gA^b_\nu f^{bac}F^cG^a_{\mu\nu} = \partial_\nu F^aG^a_{\mu\nu} + gf^{bca}A^b_\nu F^aG^c_{\mu\nu},$$

$D_\nu G^a_{\mu\nu} = \partial_\nu G^a_{\mu\nu} + gf^{abc}A^b_\nu G^c_{\mu\nu}$ are covariant derivatives of $G^a_{\mu\nu}$ tensor field. But $D_\mu A^a_\nu = \partial_\mu A^a_\nu + gf^{abc}A^b_\mu A^c_\nu$ are not covariant derivatives of A^a_ν fields and are merely mathematical symbols. Covariant derivatives of fermion field ψ are

$$D_\mu\psi = \partial_\mu\psi - igA^a_\mu\ [F^a, \psi] = \partial_\mu\psi - igA^a_\mu\lambda^a\psi. \tag{5}$$

The gauge invariant Lagrangian density is

$$L = -\ \frac{1}{4}\ G^a_{\mu\nu}G^a_{\mu\nu} - \bar{\psi}\gamma_\mu D_\mu\psi - \bar{\psi}m\psi. \tag{6}$$

The equations of motion are

$$\gamma_\mu (\partial_\mu - ig\lambda^a A^a_\mu)\, \psi + m\psi = 0 \tag{7}$$

$$\partial_\nu G^a_{\mu\nu} + gf^{abc} A^b_\nu G^c_{\mu\nu} = J^a_\mu, \tag{8}$$

where $J^a_\mu = ig\bar{\psi}\gamma_\mu\lambda^a\psi$. The fermion currents J^a_μ are not conserved. $\partial_\mu J^a_\mu = -gf^{abc}A^b_\mu J^c_\mu \neq 0$. When we define total vector currents

$$\tilde{J}^a_\mu = J^a_\mu - gf^{abc}A^b_\nu G^c_{\mu\nu} = J^a_\mu + \tilde{\tilde{J}}^a_\mu, \tag{9}$$

then they are conserved $\partial_\nu \tilde{J}^a_\nu = 0$ and $T^a = \int d^3x \tilde{J}^a_4 = \int d^3x\, \partial_i G^a_{4i}$ are independent of time and Lorentz scalars.

§ 3 Conserved Axial Vector Currents J^{aA}_μ

Maxwell equations for non-abelian gauge fields are

$$D_\nu G^a_{\mu\nu} = J^a_\mu \tag{10}$$

$$\varepsilon_{\mu\nu\lambda\rho} D_\nu G^a_{\lambda\rho} = 0, \tag{11}$$

where $D_\nu G^a_{\lambda\rho} = \partial_\nu G^a_{\lambda\rho} + gf^{abc}A^b_\nu G^c_{\lambda\rho}$. Eq. (11) are gauge-Bianchi identities. In eqs. (10) and (11) covariant derivatives D_μ for non-abelian gauge fields play the roles of derivatives ∂_μ for electromagnetic fields. But we have $G^a_{\mu\nu} \neq D_\mu A^a_\nu - D_\nu A^a_\mu$. When we use the notation $G^a_{23} = H^a_1$ (cyclic) and $G^a_{j4} = -iE^a_j$, Maxwell equations (10) and (11) are

$$\text{rot}\, \boldsymbol{E}^a = -\frac{1}{c}\frac{\partial}{\partial t} \boldsymbol{H}^a - gf^{abc}\boldsymbol{A}^b \times \boldsymbol{E}^c + gf^{abc}A^b_0 \boldsymbol{H}^c \tag{12}$$

$$\text{rot}\, \boldsymbol{H}^a = \frac{1}{c}\frac{\partial}{\partial t} \boldsymbol{E}^a + \boldsymbol{J}^a - gf^{abc}\boldsymbol{A}^b \times \boldsymbol{H}^c - gf^{abc}A^b_0 \boldsymbol{E}^c \tag{13}$$

$$\text{div}\, \boldsymbol{E}^a = c\rho^a - gf^{abc}\boldsymbol{A}^b \cdot \boldsymbol{E}^c \tag{14}$$

$$\text{div}\, \boldsymbol{H}^a = -gf^{abc}\boldsymbol{A}^b \cdot \boldsymbol{H}^c. \tag{15}$$

We have

$$\boldsymbol{E}^a = -\text{grad}\, A^a_0 - \frac{1}{c}\frac{\partial}{\partial t} \boldsymbol{A}^a + gf^{abc}A^b_0 \boldsymbol{A}^c \tag{16}$$

$$\boldsymbol{H}^a = \text{rot}\, \boldsymbol{A}^a + \frac{1}{2} gf^{abc}\boldsymbol{A}^b \times \boldsymbol{A}^c, \tag{17}$$

where $A^a_\mu = (\boldsymbol{A}^a, A^a_4) = (\boldsymbol{A}^a, iA^a_0)$. Eqs. (12)~(15) contains explicitly gauge fields A^a_μ and therefore each term of eqs. (12)~(15) is not gauge covariant. Of course the equation itself is gauge covariant.

We define both vector currents (electric currents) $\tilde{\tilde{J}}{}^a_\mu$ and axial vector currents (magnetic currents) J^{aA}_μ as

$$\tilde{\tilde{J}}{}^a_\mu = -gf^{abc}A^b_\nu G^c_{\mu\nu} = (\tilde{\tilde{\boldsymbol{J}}}{}^a, ic\tilde{\tilde{\rho}}{}^a)$$

$$= (-gf^{abc}\boldsymbol{A}^b \times \boldsymbol{H}^c - gf^{abc}A^b_0\boldsymbol{E}^c, -igf^{abc}\boldsymbol{A}^b\cdot\boldsymbol{E}^c)$$

$$= \partial_\nu G^a_{\mu\nu} - J^a_\mu \tag{18}$$

$$J^{aA}_\mu = -(1/2)\,\varepsilon_{\mu\nu\lambda\rho}gf^{abc}A^b_\nu G^c_{\lambda\rho} = (i\boldsymbol{J}^{aA}, -c\rho^{aA})$$

$$= (i \times (gf^{abc}\boldsymbol{A}^b \times \boldsymbol{E}^c - gf^{abc}A^b_0\boldsymbol{H}^c), gf^{abc}\boldsymbol{A}^b\cdot\boldsymbol{H}^c)$$

$$= (1/2)\,\varepsilon_{\mu\nu\lambda\rho}\,\partial_\nu G^a_{\lambda\rho}, \tag{19}$$

where $\varepsilon_{1234} = 1$. Currents $\tilde{\tilde{J}}{}^a_\mu$ and J^{aA}_μ are not gauge covariant. They are transformed as

$$(\tilde{\tilde{J}}{}^a_\mu)' = \tilde{\tilde{J}}{}^a_\mu + f^{abc}\tilde{\tilde{J}}{}^b_\mu\delta\omega^c - f^{abc}\,(\partial_\nu\delta\omega^b)\,G^c_{\mu\nu} \tag{20}$$

$$(J^{aA}_\mu)' = J^{aA}_\mu + f^{abc}J^{bA}_\mu\delta\omega^c - \varepsilon_{\mu\nu\lambda\rho}f^{abc}\,(\partial_\nu\delta\omega^b)\,G^c_{\lambda\rho}\,. \tag{21}$$

Maxwell equations (10) and (11) can be written as

$$\text{rot}\,\boldsymbol{E}^a = -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{H}^a - \boldsymbol{J}^{aA} \tag{22}$$

$$\text{rot}\,\boldsymbol{H}^a = \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{E}^a + \boldsymbol{J}^a + \tilde{\tilde{\boldsymbol{J}}}{}^a \tag{23}$$

$$\text{div}\,\boldsymbol{E}^a = c\rho^a + c\tilde{\tilde{\rho}}{}^a \tag{24}$$

$$\text{div}\,\boldsymbol{H}^a = c\rho^{aA}\,. \tag{25}$$

The axial vector currents (magnetic currents) J^{aA}_μ are, together with the well-known conserved currents $\tilde{J}^a_\mu$ $(= J^a_\mu + \tilde{\tilde{J}}{}^a_\mu)$, conserved currents.

$$\partial_\mu\tilde{J}^a_\mu = 0 \tag{26}$$

$$\partial_\mu J^{aA}_\mu = 0 \tag{27}$$

The conservation of axial currents J^{aA}_μ is due to Jacobi identity of the structure constant f^{abc} of gauge group. The quantities J^a_μ, $\partial_\mu J^a_\mu$, $\partial_\mu\tilde{\tilde{J}}{}^a_\mu$, $\partial_\mu\tilde{J}^a_\mu$ and $\partial_\mu J^{aA}_\mu$ are transformed as follows:

$$(J^a_\mu)' = J^a_\mu + f^{abc}J^b_\mu\delta\omega^c \tag{28}$$

$$(\partial_\mu J^a_\mu)' = \partial_\mu J^a_\mu + f^{abc}(\partial_\mu J^b_\mu)\,\delta\omega^c - f^{abc}\,(\partial_\mu\delta\omega^b)\,J^c_\mu \tag{29}$$

$$(\partial_\mu \widetilde{\widetilde{J}}{}^a_\mu)' = \partial_\mu \widetilde{\widetilde{J}}{}^a_\mu + f^{abc}\,(\partial_\mu \widetilde{\widetilde{J}}{}^b_\mu)\,\delta\omega^c + f^{abc}\,(\partial_\mu \delta\omega^b)\,J^c_\mu \tag{30}$$

$$(\partial_\mu \widetilde{J}^a_\mu)' = \partial_\mu \widetilde{J}^a_\mu + f^{abc}\,(\partial_\mu \widetilde{J}^b_\mu)\,\delta\omega^c \tag{31}$$

$$(\partial_\mu J^{aA}_\mu)' = \partial_\mu J^{aA}_\mu + f^{abc}\,(\partial_\mu J^{bA}_\mu)\,\delta\omega^c. \tag{32}$$

Because the quantities $\partial_\mu \widetilde{J}^a_\mu$ and $\partial_\mu J^{aA}_\mu$ are gauge covariant, eqs. (26) and (27) are gauge invariant.

$$(\partial_\mu \widetilde{J}^a_\mu)' = \partial_\mu \widetilde{J}^a_\mu = 0 \tag{33}$$

$$(\partial_\mu J^{aA}_\mu)' = \partial_\mu J^{aA} = 0. \tag{34}$$

§ 4 Vanishing Self-Force $\widetilde{\widetilde{K}}_\mu + K^A_\mu$

We assume and define the magnetic Lorentz force as

$$\begin{aligned} K^A_\mu &= -\,(1/2)\,\varepsilon_{\mu\nu\lambda\rho} G^a_{\lambda\rho} J^{aA}_\nu \\ &= (c\rho^{aA}\boldsymbol{H}^a - \boldsymbol{J}^{aA} \times \boldsymbol{E}^a, i\boldsymbol{J}^{aA}\cdot\boldsymbol{H}^a). \end{aligned} \tag{35}$$

The minus sign in front of ε symbol ($\varepsilon_{1234} = 1$) in eq. (35) is necessary in conformity with the definition of the axial vector currents J^{aA}_μ in eq. (19). We denote usual Lorentz forces K_μ, and $\widetilde{\widetilde{K}}_\mu$ as

$$K_\mu = \sum_a G^a_{\mu\nu} J^a_\nu = (c\rho^a \boldsymbol{E}^a + \boldsymbol{J}^a \times \boldsymbol{H}^a,\ i\boldsymbol{J}^a\cdot\boldsymbol{E}^a) \tag{36}$$

$$\widetilde{\widetilde{K}}_\mu = \sum_a G^a_{\mu\nu} \widetilde{\widetilde{J}}{}^a_\nu \tag{37}$$

They are transformed as

$$(K_\mu)' = K_\mu \tag{38}$$

$$(\widetilde{\widetilde{K}}_\mu)' = \widetilde{\widetilde{K}}_\mu - f^{abc} G^a_{\mu\nu}\, G^c_{\nu\lambda}\,(\partial_\lambda \delta\omega^b) \tag{39}$$

$$(K^A_\mu)' = K^A_\mu + f^{abc}\,(1/2)\,\varepsilon_{\mu\nu\kappa\rho}\, G^a_{\kappa\rho}\,(1/2)\varepsilon_{\mu\lambda\tau\chi}\, G^c_{\tau\chi}\,(\partial_\lambda \delta\omega^b) \tag{40}$$

Lorentz force K_μ is gauge invariant and Lorentz forces $\widetilde{\widetilde{K}}_\mu$, and K^A are not gauge covariant (and of course not gauge invariant). But we have

$$(\widetilde{\widetilde{K}}_\mu + K^A_\mu)' = \widetilde{\widetilde{K}}_\mu + K^A_\mu\ . \tag{41}$$

After some calculations we have

$$\sum_a (c\rho^{aA}\,\boldsymbol{H}^a - \boldsymbol{J}^{aA} \times \boldsymbol{E}^a) \doteq -\,[\textstyle\sum_a (c\widetilde{\widetilde{\rho}}{}^a \boldsymbol{E}^a + \widetilde{\widetilde{\boldsymbol{J}}}{}^a \times \boldsymbol{H}^a)] \tag{42}$$

$$\Sigma_a(\boldsymbol{J}^{aA}\cdot\boldsymbol{H}^a) = -\,[\Sigma_a(\tilde{\tilde{\boldsymbol{J}}}^a\cdot\boldsymbol{E}^a)] \tag{43}$$

namely, $K^A_\mu = -\,\tilde{\tilde{K}}_\mu$. (44)

Therefore we have

$$(\tilde{\tilde{K}}_\mu + K^A_\mu)' = \tilde{\tilde{K}}_\mu + K^A_\mu = 0. \tag{45}$$

Eq. (44) states the self-force of non-abelian gauge fields always vanishes. Furthermore eq. (41) shows this statement is gauge invariant.

§ 5 Further Remarks

Up to here we have not fixed the gauge. From now on we fix the gauge by Lorentz condition $\partial_\mu A^a_\mu = 0$. The quantity $\partial_\mu A^a_\mu$ is transformed as

$$(\partial_\mu A^a_\mu)' = \partial_\mu A^a_\mu + f^{abc}(\partial_\mu A^b_\mu)\,\delta\omega^c + f^{abc}A^b_\mu\partial_\mu\delta\omega^c + \frac{1}{g}\,\Box\,\delta\omega^a. \tag{46}$$

Parameter $\delta\omega^a(x)$ for gauge transformation must satisfy

$$\Box\,\delta\omega^a + gf^{abc}A^b_\mu\partial_\mu\delta\omega^c = D_\mu\partial_\mu\delta\omega^a = 0. \tag{47}$$

We consider the Lagrangian density

$$L = -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{2}(\partial_\mu A^a_\mu)(\partial_\nu A^a_\nu) - \bar{\psi}\gamma_\mu D_\mu\psi - \bar{\psi}m\psi. \tag{48}$$

Maxwell equations are

$$D_\nu G^a_{\mu\nu} = J^a_\mu + \partial_\mu\chi^a \tag{49}$$

$$D_\lambda G^a_{\mu\nu} + D_\mu G^a_{\nu\lambda} + D_\nu G^a_{\lambda\mu} = 0, \tag{50}$$

where $\chi^a = \partial_\mu A^a_\mu$. We have

$$\Box\chi^a + gf^{abc}A^b_\nu\partial_\nu\chi^c = D_\nu\partial_\nu\chi^a = 0 \tag{51}$$

We consider the subsidiary condition

$$\chi^a = 0 \tag{52}$$

$$\dot{\chi}^a = 0, \tag{53}$$

then we recover the expression (10). Canonical conjugate momenta Π^a_μ are

$$\Pi^a_\mu = (1/ic)\,(G^a_{\mu4} - \delta_{\mu4}\chi^a). \tag{54}$$

The Hamiltonian density is

$$H = \frac{1}{2} c^2 (| \Pi^a |^2 - \Pi_4^a)^2 + \frac{1}{2} \mathrm{rot} | A^a |^2 + ic \{ \Pi^a \cdot \nabla A_4^a - \Pi_4^a \mathrm{div} A^a \}$$
$$+ \bar{\psi}\gamma_i \partial_i \psi + \bar{\psi} m \psi + ic \Pi_i^a g f^{abc} A_i^b A_4^c$$
$$+ \partial_i A_j^a g f^{abc} A_i^b A_j^c + \frac{1}{4} (g f^{abc} A_i^b A_j^c)^2 - A_\mu^a J_\mu^a. \qquad (55)$$

Adding divergence term $-ic\ div\ (\Pi^a A_4^a)$, we have

$$H = A_4^a (\partial_i G_{4i}^a + g f^{abc} A_i^b G_{4i}^c - J_4^a) + \text{terms without } A_4^a \text{ and } \partial_i A_4^a.$$

We have $\dot{\Pi}_4^a = - (\delta \bar{H} / \delta A_4^a) = - (\partial_i G_{4i}^a + g f^{abc} A_i^b G_{4i}^c - J_4^a)$. Taking account of subsidiary condition (53), we obtain

$$H = \frac{1}{2} c^2 | \Pi^a |^2 + \frac{1}{2} | \mathrm{rot} A^a |^2 + \bar{\psi}\gamma_i \partial_i \psi + \bar{\psi} m \psi$$
$$- A_i^a J_i^a + \partial_i A_j^a g f^{abc} A_i^b A_j^c + \frac{1}{4} (g f^{abc} A_i^b A_j^c)^2. \qquad (56)$$

Gauge fields A_μ^a satisfy (when $\partial_\mu A_\mu^a = 0$)

$$\Box A_\mu^a + g f^{abc} A_\nu^b \partial_\nu A_\mu^c = - \tilde{J}_\mu^a. \qquad (57)$$

From symmetrized energy-momentum tensors $\theta_{\mu\nu}$ we obtain the expressions for momentum density θ_{4i}^{G} and Maxwell stress tensor density θ_{ij}^{G} $(= - T_{ij}^{\mathrm{G}})$ for non-abelian gauge fields as

$$\theta_{4i}^{\mathrm{G}} = i\, (\boldsymbol{E}^a \times \boldsymbol{H}^a)_i \qquad (58)$$

$$T_{11}^{\mathrm{G}} = - \theta_{11}^{\mathrm{G}} = E_1^{a^2} + H_1^{a^2} - \frac{1}{2} | \boldsymbol{E}^a |^2 - \frac{1}{2} | \boldsymbol{H}^a |^2 \qquad (59)$$

$$T_{12}^{\mathrm{G}} = - \theta_{12}^{\mathrm{G}} = E_1^a E_2^a + H_1^a H_2^a, \text{ etc. .} \qquad (60)$$

The energy and momentum conservation is expressed by

$$\mathrm{div}\, (\boldsymbol{E}^a \times \boldsymbol{H}^a) + \frac{1}{c} \frac{\partial}{\partial t} \frac{1}{2} (| \boldsymbol{E}^a |^2 + | \boldsymbol{H}^a |^2) = - \boldsymbol{J}^a \cdot \boldsymbol{E}^a \qquad (61)$$

$$\mathrm{div}\ T^{\mathrm{G}} - \frac{1}{c} \frac{\partial}{\partial t} (\boldsymbol{E}^a \times \boldsymbol{H}^a) = c \rho^a \boldsymbol{E}^a + \boldsymbol{J}^a \times \boldsymbol{H}^a. \qquad (62)$$

They are identical (except internal index a) with the expressions for electromagnetic field.

§ 6 Summary

We have obtained one kind of integrals

$$T^{aA} = \int d^3x J_4^{aA} = \int d^3x\, (\partial_1 G_{23}^a + \partial_2 G_{31}^a + \partial_3 G_{12}^a) = \int d^3x\ \mathrm{div} H^a, \qquad (63)$$

besides the well known integrals

$$T^a = \int d^3x \tilde{J}_4^a = \int d^3x\, \partial_i G_{4i}^a = i \int d^3x \,\mathrm{div}\, \boldsymbol{E}^a. \tag{64}$$

They are independent of time and Lorentz scalars. The conservation of the quantities T^{aA} is due to kinematical origins (Jacobi identities).

We have also shown that the self-force of non-abelian gauge fields always vanishes.

The above two results are gauge invariant. We have assumed the magnetic Lorentz force defined in eq. (35), without which the self force neither be gauge invariant (and therefore not the observable) nor vanishes. In § 5 we have used Lorentz condition. Eq. (47) contains the fields A_μ^a and so it is rather not clear how to fix the gauge by Lorentz condition. As referred in § 1 Introduction it is very difficult to obtain the exact classical solutions (even the classical ones!) of Maxwell equations of non-abelian gauge fields. Such problems still remain to challenge us.

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