Sci. Rep. Kanazawa Univ.
Vol. 21. No. 1, pp. 1-8
June 1976

On Pseudoconvexity of Fibre Bundles

Kazuyoshi UENO*

Department of Mathematics, Faculty of Science, Kanazawa University

(Received April 30, 1976)

Introduction.

Y. Matsushima and A. Morimoto [5] solved the problem raised by J. P. Serre: Owing to them, every complex analytic fibre bundle whose base and fibre are both Stein manifolds is a Stein manifold if it's structure group is connected. Replacing "Stein" with "weakly 1-complete" in this problem, we shall investigate some situations in which complex analytic fibre bundles form weakly 1-complete manifolds.

In § 1 we shall see that every complex analytic principal fibre bundle over a Stein manifold with connected structure group is weakly 1-complete. This fact was already announced in H. Kazama [2]. Moreover he studied strong q-completeness of complex abelian Lie groups in [3], and of complex Lie groups in [4]. He obtained stronger result in case of abelian groups. We are to be assured that just the same result as in case of abelian groups is valid even in general case.

In § 2 we shall give a condition for holomorphic vector bundles to make weakly 1-complete manifolds. It is clear that closed submanifold of a weakly 1-complete manifold is also weakly 1-complete. Therefore we may well start with the assumption that the base space is weakly 1-complete, since it can be regarded as the 0-section. We shall see that a holomorphic vector bundle over a weakly 1-complete manifold is weakly 1-complete, if it is weakly negative in the sense of S. Nakano [7].

This note is a rewrite of master's thesis of the author. He would like to express his cordial gratitude to Professors E. Sakai and C. Watanabe under whom he studied in the last two years.

§ 0. Preliminaries.

A complex manifold M is said to be *weakly 1-complete* if there exists a real-valued function φ of class C^∞ on M with the following two properties:

(1) For any real number c, $\{x \in M \mid \varphi(x) < c\}$ is a relatively compact subset of M.

* Present address: Toyama College of Technology.

(2) The Levi Form $L(\varphi)$ of φ is positive semi-definite.

Replacing the condition (2) with the following (3), we call M a *strongly q-complete* manifold:

(3) $L(\varphi)$ has at least $n-q+1$ positive eigenvalues, where n denotes the complex dimension of M.

Namely, a complex manifold which admits a smooth plurisubharmonic (or strongly q-pseudoconvex) function φ diverging to ∞ at the boundary is called a weakly 1-complete (or strongly q-complete) manifold with the exhausting function φ.

0.1. LEMMA 1. *Let f be a holomorphic function on a connected complex Lie group. If f is bounded, then f is a constant. Moreover a bounded plurisubharmonic function on a connected complex abelian Lie group is reduced to a constant.*

Proof. Since the exponential mapping is a holomorphic mapping of the complex number field into the Lie group for every left-invariant vector field, it is clear by Liouville's theorem that f is constant on any 1-parameter subgroup. Therefore f is constant in some neighbourhood of the unit element. By the uniqueness of analytic continuation, f is reduced to a constant.

We can prove similarly the second half, observing that the complex number field admits no bounded subharmonic function except for constants and that abelian Lie group is entirely covered with 1-parameter subgroups.

0.2. The two complex Lie subgroups K_0 and G^0.

Let G be a complex Lie group, and K be the maximal compact subgroup of G. The Lie algebras of G, K are denoted by $\mathfrak{g}$, $\mathfrak{k}$ respectively. We consider the complex Lie subgroup K_0 corresponding to the complex Lie subalgebra $\mathfrak{k} \cap \sqrt{-1}\,\mathfrak{k}$. Since the components of the adjoint representation of G are holomorphic on G and bounded on K_0, the above lemma concludes that K_0 is an abelian group contained in the center of G, which is nothing else but the kernel of the adjoint representation.

K_0 is independent of the choice of K, for maximal compact subgroups are mutually conjugate (Iwasawa [1]). It is easy to see that K_0 is a characteristic subgroup of G, i.e. every holomorphic automorphism of G leaves K_0 invariant.

A. Morimoto [6] introduced a subset G^0 of G as follows:

$$G^0 = \{x \in G \mid f(x) = f(e) \text{ for all } f \in \mathcal{O}(G)\},$$

where e denotes the unit element of G, and $\mathcal{O}(G)$ the ring of all holomorphic functions on G. He proved

THEOREM 1. *The following assertions hold*:

(1) *G^0 is a complex Lie subgroup closed in G and contained in the center of G. G^0 is a characteristic subgroup of G.*

(2) *The factor group G/G^0 is a Stein manifold. If N is a normal complex Lie subgroup such that G/N is a Stein manifold, then $N \supset G^0$.*

(3) *G^0 is connected and all holomorphic functions on G^0 are necessarily constant.*

As for the relation between the two central characteristic subgroups K_0 and G^0 of G, $K_0 \subset G^0$ is clear from the above lemma. When does it occur that $K_0 = G^0$? A. Morimoto answered as follows:

THEOREM 2. *The following conditions are mutually equivalent*:

(1) *G is holomorphically convex.*

(2) *G^0 is compact.*

(3) *$K_0 = G^0$.*

(4) *K_0 is closed in G.*

The Lie groups without non-constant holomorphic functions, such as G^0, are called (H. C)-groups. We see that an (H. C)-group is necessarily abelian, considering it's adjoint representation. Of course all compact groups are (H. C)-groups, while (H. C)-groups are not always compact. In fact, A. Morimoto constructed an (H. C)-group of arbitrary dimension $n \geqq 2$, which contains no complex torus of positive dimension. This construction contains simultaneously such an example that $K_0 \neq G^0$, since $G^0 (=G)$ is not compact.

§1. Complex analytic principal fibre bundles over a Stein manifold.

1.1. A function on an (H. C)-group.

Let G be an (H. C)-group of dimension n. Since every (H. C)-group is abelian, G is isomorphic to $\boldsymbol{C}^n/\Gamma$ for some discrete subgroup Γ of $\boldsymbol{C}^n$. Let Γ be generated by vectors $d_1, \cdots, d_s$ linearly independent over $\boldsymbol{R}$. Then the generators span $\boldsymbol{C}^n$ over $\boldsymbol{C}$, or otherwise G contains a complex line as a direct product factor. Therefore after suitable change of coordinates we may put

$$\Gamma = \langle e_1, \cdots, e_n, u_1, \cdots, u_q \rangle, \qquad (q = 2n - s),$$

where e_i denotes the i-th unit vector of $\boldsymbol{C}^n$ for $i=1, 2, \ldots, n$. Let K be the maximal compact subgroup of G and $\mathfrak{k}$ the Lie algebra of K. Then

$$\begin{aligned} \mathfrak{k} &= \langle e_1, \cdots, e_n, u_1, \cdots, u_q \rangle_{\boldsymbol{R}} \\ &= \langle e_1, \cdots, e_n, \sqrt{-1}\,\mathrm{Im}\, u_1, \cdots, \sqrt{-1}\,\mathrm{Im}\, u_q \rangle_{\boldsymbol{R}}. \end{aligned}$$

From this representation of $\mathfrak{k}$, we can see easily,

$$\mathfrak{k} \cap \sqrt{-1}\,\mathfrak{k} = \langle \mathrm{Im}\, u_1, \cdots, \mathrm{Im}\, u_q \rangle_C.$$

Let us choose suitable vectors $v_1, \cdots, v_{n-q}$ in $\boldsymbol{R}^n$ such that

$$\boldsymbol{C}^n = \langle e_1, \cdots, e_n, \sqrt{-1}\,\mathrm{Im}\, u_1, \cdots, \sqrt{-1}\,\mathrm{Im}\, u_q, \sqrt{-1}\, v_1, \cdots, \sqrt{-1}\, v_{n-q} \rangle_{\boldsymbol{R}}.$$

Then there exists a non-degenerate real (n, n)-matrix (a_i^j) such that

$$e_i = \sum_{j=1}^{q} a_i^j\, \mathrm{Im}\, u_j + \sum_{j=1}^{n-q} a_i^{j+q}\, v_j.$$

Every point $(z^1, \cdots, z^n) \in \boldsymbol{C}^n$ has the following representation:

$$(z^1, \cdots, z^n) = \sum_{i=1}^{n} x^i e_i + \sqrt{-1} \sum_{j=1}^{q} \sum_{i=1}^{n} y^i a_i^j\, \mathrm{Im}\, u_j + \sqrt{-1} \sum_{j=1}^{n-q} \sum_{i=1}^{n} y^i a_i^{j+q} v_j,$$

where $z^i = x^i + \sqrt{-1}\, y^i$ for $i = 1, 2, \cdots, n$.

Let us consider the following mapping:

$$\boldsymbol{C}^n \ni (z^1, \cdots, z^n) \longmapsto \left(\sum_{i=1}^{n} y^i a_i^{1+q}, \cdots, \sum_{i=1}^{n} y^i a_i^n \right) \in \boldsymbol{R}^{n-q}.$$

This mapping can be regarded as defined on G because it is Γ-invariant. Let us denote this mapping by $\varphi : G \longrightarrow \boldsymbol{R}^{n-q}$. It is clear that φ satisfies the conditions of the following lemma:

LEMMA 2. *Let G be an* (H. C)*-group of dimension n, and $\mathfrak{k}$ be the Lie algebra of the maximal compact subgroup of G. If the complex dimension of $\mathfrak{k} \cap \sqrt{-1}\,\mathfrak{k}$ is q, there exists a mapping $\varphi : G \longrightarrow \boldsymbol{R}^{n-q}$, satisfying the following conditions*:

(1) *φ is a homomorphism of G into the additive group $\boldsymbol{R}^{n-q}$.*

(2) *For any real number c, $\{x \in G \mid \|\varphi(x)\| < c\}$ is a relatively compact subset of G, where $\|\cdot\|$ is the Euclidean norm of $\boldsymbol{R}^{n-q}$.*

(3) *If we choose a suitable system (z^i) of local coordinates of G, $(\partial\varphi/\partial z^1, \cdots, \partial\varphi/\partial z^n)$ is a matrix of maximal rank, whose entries are all constants independent of coordinate neighbourhoods. Especially, all the derivatives of second order of φ with respect to these coordinates vanish.*

1.2. Principal fibre bundles over a Stein manifold.

The aim of this section is to prove the following theorem.

THEOREM 3. *Let $P(B, G)$ be a complex analytic principal fibre bundle over a Stein manifold B with connected structure group G. Let G^0 denote the* (H. C)*-subgroup of G introduced by A. Morimoto, and K^0, $\mathfrak{k}^0$ the maximal compact subgroup of G^0, it's Lie algebra, respectively. Let K_0^0 be the complex Lie subgroup corresponding to $\mathfrak{k}^0 \cap \sqrt{-1}\,\mathfrak{k}^0$, and q denote the complex dimension of K_0^0.*

Then, the total space P is weakly 1*-complete and strongly* $(q+1)$*-complete.*

Proof. The total space P can be regarded as a principal fibre bundle with structure group G^0 over P_1, which stands for a principal fibre bundle over B with structure group G/G^0. Since G/G^0 is a Stein manifold by Theorem 1, the conclusion of Matsushima-Morimoto [5] affirms that P_1 is a Stein manifold.

Accordingly, we have only to prove for a principal fibre bundle whose structure group is an (H. C)-group and whose base is a Stein manifold. We replace $P(B, G)$ with $P(M, G^0)$ anew, where the base $M(=P_1)$ is a Stein manifold.

Let us take a locally finite open covering $\{U_\lambda\}_{\lambda\in\Lambda}$ of M such that P is trivial on each U_λ. Suppose that P is defined by 1-cocycle

$$e_{\lambda\mu}: U_\lambda \cap U_\mu \longrightarrow G^0, \qquad (\lambda, \mu \in \Lambda).$$

We denote by n, q the complex dimension of G^0, K_0^0 respectively. Let

$$\varphi: G^0 \longrightarrow \boldsymbol{R}^{n-q}$$

be a function satisfying the conditions of Lemma 2. Then the composition $\{\varphi\circ e_{\lambda\mu}\}$ is also 1-cocycle, since φ is a homomorphism. Therefore $\{\varphi\circ e_{\lambda\mu}\}$ can be reduced to a coboundary of some 0-cochain $\{\gamma_\lambda\}$, if we use a partition of unity subordinate to $\{U_\lambda\}$. Namely, there exist functions

$$\gamma_\lambda: U_\lambda \longrightarrow \boldsymbol{R}^{n-q}, \qquad (\lambda \in \Lambda),$$

such that

$$\varphi\circ e_{\lambda\mu} = \gamma_\mu - \gamma_\lambda, \qquad (\lambda, \mu \in \Lambda).$$

Let us consider the mapping on each $P\mid U_\lambda$ defined as follows,

$$U_\lambda \times G^0 \ni (p, \xi_\lambda) \longmapsto \gamma_\lambda(p) + \varphi(\xi_\lambda) \in \boldsymbol{R}^{n-q}.$$

Since φ is a homomorphism, this mapping is well defined globally on P, and will be denoted by $\Phi: P \longrightarrow \boldsymbol{R}^{n-q}$.

As Stein manifolds are characterized to be strongly 1-complete, there exists a strongly pseudoconvex function Ψ on M which diverges to ∞ at the boundary of M. Let $\pi: P \longrightarrow M$ be the canonical projection. Using a function $f(x) = \sqrt{1+\|x\|}$, we put

$$\psi = \chi\circ\Psi\circ\pi + f\circ\Phi,$$

where χ is a function $\chi: \boldsymbol{R} \longrightarrow \boldsymbol{R}$, $(\chi, \chi', \chi'' > 0)$ suitably chosen later.

Then the Levi form $L(\psi)$ of ψ is as follows, where (w_λ^α) is local coordinates of M on U_λ, and (z^μ) of G^0 satisfying Lemma 2.

$$\begin{aligned} L(\psi) = L(\chi\circ\psi\circ\pi) &+ \sum_{\alpha,\beta}\sum_{i=1}^{n-q} \frac{\partial f}{\partial x^i}\frac{\partial^2\gamma^i}{\partial w_\lambda^\alpha \partial \bar{w}_\lambda^\beta} dw_\lambda^\alpha d\bar{w}_\lambda^\beta \\ &+ \sum_{i,j=1}^{n-q}\frac{\partial^2 f}{\partial x^i \partial x^j}\left(\sum_\alpha \frac{\partial\gamma^i}{\partial w_\lambda^\alpha} dw_\lambda^\alpha + \sum_{\mu=1}^{n}\frac{\partial\varphi^i}{\partial z^\mu} dz^\mu\right) \\ &\cdot\left(\sum_\beta \frac{\partial\gamma^j}{\partial \bar{w}_\lambda^\beta} d\bar{w}_\lambda^\beta + \sum_{\nu=1}^{n}\frac{\partial\varphi^j}{\partial \bar{z}^\nu} d\bar{z}^\nu\right). \end{aligned}$$

Considering a compact refinement if necessary, we may assume that $(\gamma_\lambda^i)_i$ and their partial derivative in the above formula are all bounded on each coordinate neighbourhood. Besides, the partial derivatives of first order of f are all bounded. Therefore, if we choose suitably a function χ in the well known way, the sum of the first and second terms is positive definite on every U_λ. The third term is non-negative since the Hessian of f is positive definite everywhere. Consequently, $L(\psi)$ is positive semi-definite. Now let us estimate the rank of $L(\psi)$. Since $(\partial\varphi/\partial z^1, \cdots, \partial\varphi/\partial z^n)$ is a matrix of maximal rank, we can verify that $L(\psi)$ degenerates by q from the maximal rank.

It is clear that $\{\psi < c\}$ is a relatively compact subset of P for any real c, if we observe boundedness of γ_λ^i.

Thus we have proved that $P(B, G)$ is weakly 1-complete and strongly $(q+1)$-complete with the exhausting function ψ.

A complex Lie group can be regarded as a principal fibre bundle with structure group G^0 over G/G^0, which is a Stein manifold by Theorem 1. Therefore we obtain

COROLLARY. Every complex Lie group G is strongly $(q+1)$-complete with an exhausting plurisubharmonic function, where q denotes the complex dimension of K_0^0.

REMARK. This corollary guarantees the existence of a plurisubharmonic function ψ on a complex Lie group G such that rank $L(\psi) =$ codim K_0^0. On the other hand, rank $L(\psi) \leqq$ codim K_0, since ψ is constant on K_0 because of Lemma 1. Consequently, it follows that $K_0 = K_0^0$. This fact is proved more directly, for we can find out a maximal compact subgroup K^0 of G^0 such that $(G^0 \supset)\ K^0 \supset K_0$.

H. Kazama [2], [4] proved that $P(B, G)$ (or G) is a strongly (dim G^0+1)-complete manifold with plurisubharmonic exhausting function. Moreover, according to his paper [3], if G is commutative, then $K_0 = K_0^0$ and G admits a plurisubharmonic function ψ such that rank $L(\psi) =$ codim K_0.

The result which holds good in case of abelian groups is established even in general case by our theorem.

It would be clear that our result is the precisest one concerning to strong q-completeness with respect to plurisubharmonic functions.

§2. Holomorphic vector bundles.

Let $\pi : E \longrightarrow M$ be a holomorphic vector bundle over a complex manifold M of dimension m, with fibre C^n. We suppose M weakly 1-complete, by reason mentioned in the introduction. We take an open covering $\{U_i\}$ of M such that E is trivial on each U_i. Let E be defined by 1-cocycle

$$e_{ij} : U_i \cap U_j \longrightarrow GL(n, C),$$

in such a way that $(p_i, \xi_i) \in U_i \times C^n$ and $(p_j, \xi_j) \in U_j \times C^n$ are identified if and only if $p_i = p_j$, $\xi_i = e_{ij}(p_j)\, \xi_j$.

We introduce an Hermitian metric h on the fibres of E. This is represented on each U_i as a positive definite Hermitian matrix:

$$h_i : U_i \longrightarrow GL(n, C),$$

whose (λ, μ)-component will be denoted by $h_{i\lambda\bar{\mu}}$.

The ∂-connection θ with respect to the metric h is defined locally by $\theta_i = h_i^{-1} \partial h_i$ on each U_i. This is an (n, n)-matrix whose entries are differential forms of type (1, 0). The (λ, μ)-component is as follows;

$$\theta_i{}^\lambda{}_\mu = \sum_{\alpha=1}^{n} \sum_{\tau=1}^{m} h_i^{\bar{\tau}\lambda} \frac{\partial h_{i\mu\bar{\tau}}}{\partial z_i^\alpha}\, dz_i^\alpha,$$

where (z_i^α) is the local coordinates on U_i.

The curvature form is given by $\Theta_i = \sqrt{-1}\ \bar{\partial}\theta_i$, whose (λ, μ)-component is as follows:

$$\Theta_i{}^\lambda{}_\mu = \sqrt{-1} \sum_{\alpha,\beta=1}^{m} \Theta_i{}^\lambda{}_{\mu\alpha\bar{\beta}}\, dz_i^\alpha \wedge d\bar{z}_i{}^\beta,$$
$$\Theta_i{}^\lambda{}_{\mu\alpha\bar{\beta}} = \sum_{\rho,\tau,\sigma} h_i^{\bar{\sigma}\lambda}\left(h_i^{\bar{\tau}\rho} \frac{\partial h_{i\mu\bar{\tau}}}{\partial z_i^\alpha} \frac{\partial h_{i\rho\bar{\sigma}}}{\partial \bar{z}_i^\beta} - \frac{\partial^2 h_{i\mu\bar{\sigma}}}{\partial z_i^\alpha \partial \bar{z}_i{}^\beta}\right)$$

The following definition is due to S. Nakano [7].

DEFINITION. A holomorphic vector bundle is said to be *positive* (or *weakly negative*) if there exists a fibre metric h such that the (mn, mn)-matrix

$$H_i = (H_{i\bar{\nu}\mu\alpha\bar{\beta}}), \quad H_{i\bar{\nu}\mu\alpha\bar{\beta}} = \sum_{\lambda=1}^{n} h_{i\lambda\bar{\nu}} \Theta_i{}^\lambda{}_{\mu\alpha\bar{\beta}},$$

is positive definite (or negative semi-definite) everywhere.

Let M be weakly 1-complete with the exhausting function φ. We consider the function ψ on E defined by

$$\psi = \varphi \circ \pi + h.$$

Of course $\{x \in E \mid \psi(x) < c\}$ is a relatively compact subset of E for any $c \in \boldsymbol{R}$. Let us seek a condition under which the Levi form of ψ is positive semi-definite. Denoting by (z^α), (ξ^λ) the local coordinates of base, fibre respectively, we can compute (where the subscript i is omitted for simplicity);

$$\frac{\partial^2 \psi}{\partial z^\alpha \partial \bar{z}^\beta} = \frac{\partial^2 \varphi}{\partial z^\alpha \partial \bar{z}^\beta} + \sum_{\lambda,\mu} \frac{\partial^2 h_{\lambda\bar{\mu}}}{\partial z^\alpha \partial \bar{z}^\beta} \xi^\lambda \bar{\xi}^\mu,$$
$$\frac{\partial^2 \psi}{\partial z^\alpha \partial \bar{\xi}^\mu} = \sum_\lambda \frac{\partial h_{\lambda\bar{\mu}}}{\partial z^\alpha} \xi^\lambda, \quad \frac{\partial^2 \psi}{\partial \bar{z}^\beta \partial \xi^\lambda} = \sum_\mu \frac{\partial h_{\lambda\bar{\mu}}}{\partial \bar{z}^\beta} \bar{\xi}^\mu,$$
$$\frac{\partial^2 \psi}{\partial \xi^\lambda \partial \bar{\xi}^\mu} = h_{\lambda\bar{\mu}},$$
$$H_{\bar{\nu}\mu\alpha\bar{\beta}} = \sum_{\tau,\rho} h^{\bar{\tau}\rho} \frac{\partial h_{\mu\bar{\tau}}}{\partial z^\alpha} \frac{\partial h_{\rho\bar{\nu}}}{\partial \bar{z}^\beta} - \frac{\partial^2 h_{\mu\bar{\nu}}}{\partial z^\alpha \partial \bar{z}^\beta}.$$

By these preperations we can write

$$L(\psi) = \sum_{\alpha,\beta} \frac{\partial^2 \varphi}{\partial z^\alpha \partial \bar{z}^\beta} dz^\alpha d\bar{z}^\beta - \sum_{\alpha,\beta} \sum_{\mu,\nu} H_{\bar{\nu}\mu\alpha\bar{\beta}} \xi^\mu \bar{\xi}^\nu dz^\alpha d\bar{z}^\beta$$
$$+ \sum_{\tau,\rho} h^{\bar{\tau}\rho} \left(\sum_{\alpha,\lambda} \frac{\partial h_{\lambda\bar{\tau}}}{\partial z^\alpha} \xi^\lambda dz^\alpha + \sum_{\lambda} h_{\lambda\bar{\tau}} d\xi^\lambda \right)$$
$$\cdot \left(\sum_{\beta,\mu} \frac{\partial h_{\rho\bar{\mu}}}{\partial \bar{z}^\beta} \bar{\xi}^\mu d\bar{z}^\beta + \sum_{\mu} h_{\rho\bar{\mu}} d\bar{\xi}^\mu \right)$$

The last term is non-negative since h^{-1} is a positive definite Hermitian matrix, needless to say of the first term. Therefore $L(\psi)$ is positive semi-definite, if only H_i is negative semi-definite matrix. Thus, we obtain

THEOREM 4. *A holomorphic vector bundle over a weakly* 1*-complete manifold is weakly* 1 *-complete, if it is weakly negative in the sense of S. Nakano.*

Suppose that $\pi : E \longrightarrow M$ be a holomorphic vector bundle over a Stein manifold M. Using a strongly pseudoconvex function on M diverging to ∞ at the boundary of M, we can introduce on the fibres of E a Hermitian metric, such that E is negative with respect to this metric. Therefore we can construct on E, in the same way as the above, strongly pseudoconvex function diverging to ∞ at the boundary.

We can prove similarly that every holomorphic vector bundle over a strongly q-complete manifold is strongly q-complete.

References

[1] K. Iwasawa, On some types of topological groups, Ann. of Math., **50** (1949), 507–558.

[2] H. Kazama, q-complete complex Lie groups and holomorphic fibre bundles over a Stein manifold, Mem. Fac. Sci. Kyushu Univ., **26** (1971), 102–118.

[3] H. Kazama, On pseudoconvexity of complex abelian Lie groups, J. Math. Soc. Japan, **25** (1973), 329–333.

[4] H. Kazama, On pseudoconvexity of complex Lie groups, Mem. Fac. Sci. Kyushu Univ., **27** (1973), 241–247.

[5] Y. Matsushima-A. Morimoto, Sur certains espaces fibrés holomorphes sur une variétés de Stein, Bull. Soc. Math. France, 88 (1960), 137–155.

[6] A. Morimoto, Non-compact complex Lie groups without non-constant holomorphic functions, Proc. Conf. on Complex Analysis at Univ. of Minneapolis, 1965, 256–272.

[7] S. Nakano, On complex analytic vector bundles, J. Math. Soc. Japan, **7** (1955), 1–12.