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Predicting Quaking Aspen Stand Dynamics in Minnesota

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ABSTRACT. *This paper presents equations for predicting future basal area, number of trees, and total cubic-foot volume of aspen stands in Minnesota. The modeling methodology uses a fully-stocked yield table for quaking aspen as a density standard. A relative density change equation based on observed growth from permanent plots provides the basis for predicting the future relative density and therefore the future basal area, number of trees, and volume. The equations are easy to apply and require only site index, age, and beginning basal area, number of trees, and volume. North. J. Appl. For. 10(1):20–27.*

Quaking aspen is the most abundant commercial tree species in Minnesota (Jakes 1980). The aspen forest type covers approximately 34% of the commercial forestland in the state. This research focuses on predicting changes in volume, basal area, and number of trees of stands currently having a plurality of aspen. In this paper we describe the density standard models, the aspen permanent sample plot data available to us, and the relative density change models. We then test the different models, show how to use the models, and close with a discussion of the limitations of the aspen models developed.

Proposed Models of Quaking Aspen Stand Properties

The mathematical modeling method reported here is discussed in detail elsewhere (Leary and Smith 1990, Leary 1991). Briefly, this method combines the knowledge of quaking aspen stand property dynamics included in a classical yield table (printed in Brown and Gevorkiantz 1934) with information about current stand dynamics obtained from permanent growth plot remeasurement information. The yield table provides a density standard, and the growth plots capture the manner in which aspen stands approach the standard (providing data for relative density change equations). Some quaking aspen stand conditions observed by Brown and Gevorkiantz no longer exist in northern Minnesota, hence yield table information supplements stand dynamics information available from permanent growth plot remeasurements.

ACKNOWLEDGMENT: Our analyses would not have been possible without the foresight of people like Zehngraff (1936) and Heinselman (1953), who initiated long-term studies on the Chippewa National Forest, and the efforts of many scientists and technicians who helped establish the studies and expertly remeasured them in subsequent years.

Density Standard Models

Density standards for total cubic-foot stem volume, basal area, and number of trees, for site index (values >50) and age cells were taken from Brown and Gevorkiantz (1934). The equations given below represent the relations expressed in their quaking aspen Table 154 (temporary plots used in developing the table are from across the Lake States):

$$VBG = (-11908 + 416.5 \times s - 1.946 \times s^2)(1 - e^{-0.02556 \times age})^{1.970} \quad (1)$$

$$BBG = (-177.4 + 8.801 \times s - 0.05289 \times s^2)(1 - e^{-0.02425 \times age})^{0.8083} \quad (2)$$

$$TBG = 97.16 + 33773 \times e^{(-0.02889 \times s - 0.04826 \times age)} \quad (3)$$

where

s is site index (height in ft at base age 50),

age is mean tree age,

V_{BG} is total ft^3 vol/ac in trees 1 in. dbh and larger,

B_{BG} is total basal area (ba)/ac in trees 1 in. dbh and larger, and

T_{BG} is number of trees/ac 1 in. dbh and larger.

Statistics summarizing the calibration of these equations using nonlinear regression are listed in Table 1.

Real Growth Series to Calibrate Relative Density Change Equations

Remeasured permanent growth plots are required to calibrate relative density change equations. Two North Central Forest Experiment Station research studies, NC-96 and NC-52, sup-

Table 1. Calibration statistics for density standard equations fit to the quaking aspen yield table for trees 1 in. dbh and larger in Brown and Gevorkiantz (1934). Data consist of yields tabulated for 27 age-site cells with site index ranging from 50 to 80 ft.

Equation	Property	Measurement units	Adjusted R^2	S_e [a]	$\overline{Res}$ [b]	$\overline{RE}$ [c] (%)
(1)	Total volume	$ft^3\ ac^{-1}$	0.999	60.2	1.3	0.44
(2)	Total basal area	$ft^2\ ac^{-1}$	0.996	1.91	0.01	0.004
(3)	Number of trees	ac^{-1}	0.996	48.5	-0.17	-0.70

[a] S_e = Standard error of the estimate.

[b] $\overline{Res} = \text{Mean residual} = \frac{\sum_{i=1}^{n}(obs - pred)}{n}$

[c] $\overline{RE} = \text{Mean relative error} = \frac{\sum_{i=1}^{n}\frac{(obs_i - pred_i)}{obs_i}}{n}$

plied most of the long-term remeasurements. Both studies are on the Pike Bay Experimental Forest, Chippewa National Forest. Additional plot remeasurements came from Forest Inventory and Analysis permanent plots on the Chippewa National Forest, and from remeasurements of various studies located in north-central Minnesota. Figure 1 shows plot locations. Few plots are older than 50 yr (Table 2).

NC-96.—Data are from remeasurements of aspen thinning plots on the Pike Bay Experimental Forest. Quaking aspen site index ranged from 75 to 82 across the study area. The study was established in 1936 in a 13-yr-old stand. Treatments consisted of 6 different thinning intensities plus a control, each covering an area of 0.6 ac. There were no replications of thinning treatments. In the spring of 1946 a second thinning removed approximately 4.5 cords/ac of pulp and post material from each plot except the control plot. Stand summaries for the entire treated areas, based on a 10% sample of the area, were available for ages 13, 18, 23, 28, and 33. At age 33, four 1/7 ac permanent plots were established in each treatment and the control, each tree numbered, and its location mapped. Each of the four plots was summarized individually based on remeasurements at ages 33, 38, 43, 48, and 53. In summary, 9 measurements were available, extending from age 13 to 53, covering the period 1941–1981. The study area was harvested in the early 1980s.

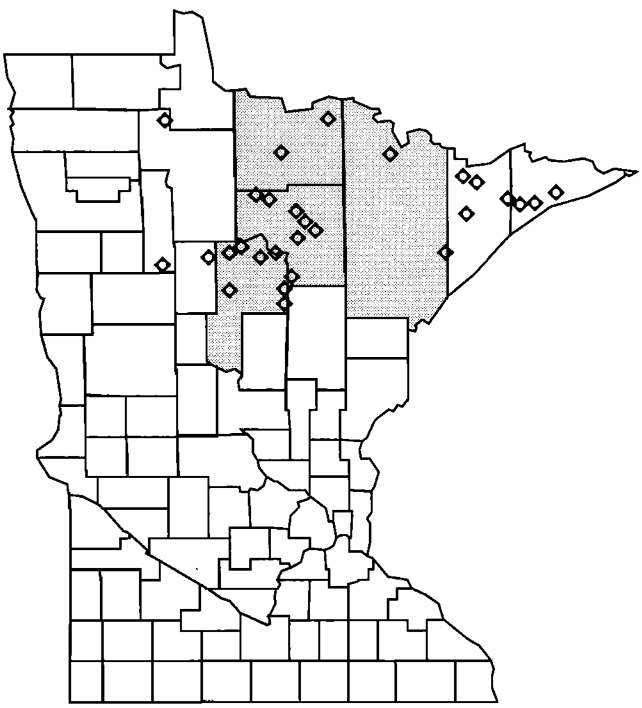

Figure 1. Location of long-term permanent growth plots in Northern Minnesota from which relative density change values were calculated (hatched area), and plots used to develop the stand volume Equation (4) (points).

NC-52.—The NC-52 remeasurements came from a thinning study established in a 10-yr-old aspen stand on the Pike Bay Experimental Forest in 1953. Half of the treatment area was thinned to about 750 trees/ac, 1/4 was left untreated, and 1/4 had hardwoods other than quaking aspen removed. Diameter at breast height was recorded for trees on forty 1/10-ac plots at stand ages 10, 19, 24, 29, 34, 39, and 46. Before measurement at age 34, individual tree identity was not recorded, so stand tables with 1 in. diameter classes formed the basis of the plot totals. Plot totals for the last three measurements were from diameters measured to the nearest 0.1 in. on numbered trees. The study is still active.

North-Central Minnesota Study Plots.—Additional data were obtained from a variety of studies established in north-central Minnesota (Table 3). The first study listed evaluated the effect of prescribed burning on a clearcut aspen stand. All other studies examined the effect of thinning. When available, we included control plots receiving no treatment as well as the thinned plots.

Chippewa NF plots.—Forest inventory plots located throughout the Chippewa National Forest were analyzed looking for plots with 50% or more quaking aspen (by basal area) which had no partial cutting during the measurement period 1970 to 1980. We found 13 plots that met the criteria. Each plot is a standard FIA design: 10 points, each point sampled with a 37.5 factor prism for trees 5 in. dbh and larger. Three 1/300-ac plots were sampled for trees smaller than 5 in. dbh. Per acre estimates of basal area and number of trees were obtained for each plot using standard FIA data analysis procedures.

Because data came from several sources, there was not a consistent method of calculating total cubic-foot volume per tree. To use a consistent method for each data source, we calculated total cubic-foot volume per acre for the plot using

Table 2. Frequency of available permanent plot observations by site and age class for all data sources.[a]

Age class	Site index class 46-55	56-65	66-75	76-85	86-95	96-105
0–9			7	9	7	
10–19		1	31	71	9	14
20–29	(5)	(6)	28 (7)	68	5	2
30–39	(4)	(17)	62 (5)	97 (1)	4	2
40–49	(4)	1 (8)	37 (4)	47		1
50–59	(4)	(12)	18 (2)	(1)		
60–69	(3)	(13)	(3)			
70–79			(3)			
80–89						

[a] Boxed cells are combinations of site and age classes for which Brown and Gevorkiantz had temporary plot information in constructing their normal yield table. In parentheses are number of plots used by Brown and Gevorkiantz. Note the absence of permanent plot measurements in older stands on higher sites.

Equation (4) (Schlaegel 1971). Figure 1 shows the geographical distribution of plots used by Schlaegel to develop Equation (4).

$$V = 0.41898 \times B \times h \quad (4)$$

where
B = total stand basal area in ft^2/ac
h = average total height in feet of dominants and codominants

Average height of dominants and codominants (h) was computed using age and site index in the Lundgren and Dolid (1970) equation:

$$h = s \times (1.46 - 1.4337e^{-0.02274 \times age}) \quad (5)$$

Relative Density Change Equation Calibration

Leary and Smith (1990) and Leary (1991) detail a method to calibrate relative density change equations. Briefly, the method requires calculations of relative density for each stand property (volume, basal area, and number of trees) on each permanent plot. Relative density is the ratio of the plot's stand property to the standard [in our case calculated with Equations (1), (2), or (3) for the same age and site index]. The difference between relative density at the end of the measurement interval and relative density at the beginning of the measurement interval gives an estimate of relative density change over the period. Due to the differing measurement intervals, relative density changes were standardized to the most common interval of 5 yr in length (71% of the intervals) by linear interpolation (23% of the intervals) or extrapolation (6% of the intervals).

Several relative density change equations were tested, starting from experiences in Leary (1991) and Leary and Smith (1990). Models selected produced predictor variables with greater statistical significance than the alternatives and did not show serious departures from the assumptions for linear regression. The calibration data consisted of 387 observed density changes. One number of trees relative density change was clearly deviant and was not used in the calibration of number of the trees equation. The equations are:

$$\frac{\Delta RDv}{\Delta t} = a_1 \times s + a_2 \times RD_v \quad (6)$$

$$\frac{\Delta RD_{ba}}{\Delta t} = \frac{b_1}{age} RD_{ba} + \frac{b_2}{age} RD_{ba}^2 + b_3 \times s + b_4 \times age \quad (7)$$

$$\frac{\Delta RDt}{\Delta t} = c_1 \times s + c_2 \times RD_t \quad (8)$$

where
a_1, a_2, b_1, b_2, b_3, b_4, c_1, and c_2 are numerical contents
s is site index,
age is mean tree age,
RD_v is volume relative density (observed volume divided by

Table 3. Selected information for aspen data from north-central Minnesota studies. The first column lists a reference that further describes the study.

Author	County	Measurement intervals	Site index	Age
Perala 1974	Cass	7	70	2–10
Hubbard 1972	Koochiching	6	90	7–24
Noreen 1968	Koochiching	6	80	4–20
Perala & Laidly 1989	St. Louis & Koochiching	24	83–103	5–21
Perala 1978 (H-70)	Cass	54	73	30–62
Perala 1978 (M-371)	Cass	8	85	37–47
Perala 1978 (NC-52i)	Cass	4	80	10–34
Perala 1978 (NC-93)	Cass	4	60–75	16–21
Unpub.	Koochiching	19	80	15–39

Table 4. Calibration results for relative density change equations. Units for volume and basal area are cubic feet and square feet.

Overall regression	Vol./ac			Basal area/ac			No. of trees/ac	
Growth intervals	387			387			386	
F-ratio	181			121			44.5	
Adjusted R^2	0.482			0.554			0.184	
S_e	0.0975			0.0882			0.1048	
$\overline{Res}$	-0.0013			-0.0009			-0.0008	
Mean sq. residual	0.0094			0.0077			0.0109	
Mean abs. residual	0.0697			0.0587			0.0680	
Numerical constants	a_1	a_2	b_1	b_2	b_3	b_4	c_1	c_2
Estimate	0.00354	-0.2912	5.953	-7.622	0.00153	-0.00191	0.001275	-0.1453
Standard error	0.0002	0.0196	1.056	0.819	0.00004	0.0005	0.0001	0.0153
t-ratio	17.9	14.9	5.64	-9.30	4.18	-3.59	9.09	-9.18

volume from the standard for the same age and site index),

RD_{ba} is basal area relative density (observed basal area divided by basal area from the standard for the same age and site index), and

RD_t is number of trees relative density (observed trees per acre divided by trees per acre from the standard for the same age and site index).

Table 4 summarizes how well the models fit the observations. The relatively poor fit for number of trees demonstrates the large variability in mortality found on small plots.

Model Tests

Both the density standard equation and relative density change equation are needed to predict stand dynamics. We tested the combined effect of the model for cubic volume, basal area, and number of trees in three ways:

1. comparing *accuracy* of predicted and observed values for periodic relative density change and final stand property (volume, basal area, number of trees),
2. calculating, i.e., *deducing,* selected *individual future stand properties* and checking the resulting numbers against published values and our collective experience, and
3. *deducing* selected *stand property interrelations,* and comparing with theory.

Accuracy tests were made using methods outlined in Rauscher (1986). Tests were done on the calibrated combinations of relative density change and density standard for each property (volume, basal area, and number of trees). One hundred thirty-four observations had been randomly drawn from the database of permanent growth plot observations for accuracy testing. One hundred nineteen were 10 yr of age or older and are used in tests described below. These observations were not used to calibrate the models.

Future individual stand properties of special interest were: mean annual property change by 5 yr increments, the maximum mean annual property change value, and the age at which mean annual property change is a maximum. Thus, our tests related primarily to the first column of the modified Bakuzis matrix (Leary 1988).

Tougher tests are possible by examining *stand property interdependence* as shown in the interior cells of the modified Bakuzis matrix. Two relations are examined: whether the models violate the -3/2 power law of self-thinning, and whether the models violate assumptions regarding the A and B lines in the stocking guide framework.

Our equations estimate 5 yr change. In tests described below we used the equations as they would be applied in practice. For example, if a plot remeasurement interval was 9 yr, the equations were iterated twice, but only 4/5 of the last increment used.

Accuracy Tests

To test models for volume, basal area, and number of trees we:

1. divided the initial and final stand property for the 119 remeasurement intervals by appropriate values from the density standard to obtain initial and final observed relative density,
2. predicted relative density change for the measurement interval,
3. added predicted relative density change to initial relative density to obtain final predicted relative density, and
4. multiplied final predicted relative density by the standard to estimate final predicted stand property.

In most cases, the measurement interval was 5 yr. Results are given in Table 5. The models for volume, basal area, and number of trees produce unbiased estimates of relative density change and standing crop (because each 95% confidence interval about the bias contains zero). As is usual, percentage errors are larger for growth (relative density change) than yield (standing crop).

Deduction Tests

Our deduction test for volume was simply to select sets of initial conditions, make projections for a number of 5 yr periods, and calculate mean annual volume increment at each period until MAI reached a maximum and started to decrease. We are particularly interested in the age at which MAI peaked, the value of MAI at that age, and how close the projections came to the 80,80,80 (80 cords/ac on site index 80 at age 80) condition reported in Brown and Gevorkiantz (1934). Table 6 presents the results from the test deductions. Numbers in parentheses under maximum MAI and age 80 columns are cord-equivalents, at 79 ft^3/cord.

The qualitative directions shown in Table 6 seem correct; age at maximum MAI is later if starting relative density is small and

Table 5. **Test of models showing accuracy measures of observed and predicted relative density change and final stand properties for volume, basal area, and number of trees (dbh ≥ 1 in.). The confidence interval is for a two-sided probability of 0.05.**

	Relative density change error		Final standing crop error	
	Absolute	Percent	Absolute	Percent
Total volume				
bias (obs. - pred.)	0.0117	47.2	21.6 ft^3 ac^{-1}	0.91
confidence interval ±	0.0167	56.2	48.8 ft^3 ac^{-1}	2.30
Total basal area				
bias (obs. - pred.)	0.0094	46.0	1.06 ft^2 ac^{-1}	0.89
confidence interval ±	0.0146	406.0	1.60 ft^2 ac^{-1}	2.06
Number of trees				
bias (obs. - pred.)	0.0104	53.7	6.5 ac^{-1}	0.61
confidence interval ±	0.0173	928.0	16.0 ac^{-1}	2.32

maximum MAI is higher on better sites. Qualitative behavior differs from expected in that the age of maximum MAI is greater on good than poor sites. Ages at which mean annual increment are maximum are reasonable. The model comes close to meeting the (80,80,80) condition (Table 6).

The deduction test for basal area dynamics was similar to that for volume. We calculated cubic volume mean annual increment [by calculating basal area and using Equations (4) and (5)], and determined the age at which it is a maximum and MAI at maximum (Table 7). Ages of maximum mean annual increment are very similar to those obtained from projected volume for stands fully stocked at age 10. When initial density is lower, the basal area equation predicts an earlier peak than shown in Table 6. The decline in basal area at later ages is consistent with Zehngraff (1947), who indicates basal area decline after age 60.

An important criterion for number of trees per acre deduction test is the Sukatchev effect (Harper 1977): trees are lost to mortality more quickly on good sites than on poor sites, if both start with the same number of trees. The model does not violate the effect at age 40 or 60, within the range of our calibration data, but does by age 80 (Table 8).

Stand Property Interrelations

The -3/2 Power Law of Self-Thinning.—The -3/2 power law of self-thinning asserts that tree frequency and volume per tree are closely linked in evenaged monocultures as follows:

$$volume\,/\,tree = k_0\,(number\ of\ trees)^{k_1} \qquad (9)$$

where k_1 is approximately -3/2.

Because quaking aspen often grows in evenaged monocultures, one might expect that it behaves according to Equation (9). Indeed, Perala and Cieszewski (in review) showed quaking aspen stands do self-thin according to the -3/2 rule. In our test we used reasonable initial conditions, applied the relative density change equations for volume, Equation (6), and number of trees per acre, Equation (9), to estimate change in each property, and checked if the resulting trajectory gives evidence of self-thinning. Because actual stands obey the rule, our models should produce it. If the combined use of the models [Equations (6) and (8)] do not produce self-thinning, then one or the other is incorrect, even if each performed reasonably well when tested separately. Results are shown in Figure 2. Clearly, the equation combinations show our models predict a period of self-thinning.

Trajectories in the Stocking Guide Framework.—A second test of stand property interrelations is to use the number of trees per acre and basal area prediction equations to examine stand trajectories in

< basal area - number of trees >

space, the framework in which evenaged stand stocking guides is expressed. Figure 3 shows trajectories for NC-52 plots used in validation testing. Upper limits of basal area range from 130 to 159 ft^2/ac. Perala's aspen stocking guide (Perala 1986) shows that upper management level peaks at about 170 ft^2/ac and that maximum density peaks at about 220 ft^2/ac. Figure 3 shows the basal area increase, as the number of trees decreases, found for other stocking guides (Gingrich 1967, Benzie 1977, Roach 1977). We conclude that the model produces reasonable shapes for the trajectories in the stocking guide framework, but the maximum basal areas attained may be low. Although not shown in Figure 3, regeneration of only, say, 400 trees/ac at age 10, will

Table 6. Summary of mean annual volume increment deduction test. All initial conditions were applied at age 10. Site index designates height at age 50, and rd designates volume relative density.

Initial conditions	Age at MAI maximum	Maximum MAI ft^3/ac (cds/ac)	Volume in ft^3/ac at age (cd/ac) 40	60	80
Site index 60					
rd 1.0	40	48.2 (0.61)	1926	2775	3380 (43)
rd 0.6	50	46.0 (0.58)	1798	2727	3365 (43)
rd 0.2	57	44.7(0.56)	1670	2679	3350 (42)
Site index 80					
rd 1.0	47	91.6 (1.16)	3629	5396	6624 (84)
rd 0.6	55	89.3 (1.13)	3440	5325	6602 (84)
rd 0.2	57	87.6 (1.11)	3251	5254	6580 (83)

Table 7. Summary of mean annual volume increment predicted by the relative density change equation for basal area. All initial conditions were applied at age 10. Site index designates height at age 50, and rd designates basal area relative density.

Initial conditions	Age at MAI maximum	Maximum MAI ft^3/ac (cds/ac)	Basal area in ft^2/ac at age 40	60	80
Site index 60					
rd 1.0	40	55.8 (0.71)	100	110	98
rd 0.6	40	54.5 (0.69)	98	109	98
rd 0.2	50	47.9 (0.61)	84	100	90
Site index 80					
rd 1.0	45	96.4 (1.22)	130	150	147
rd 0.6	45	95.2 (1.21)	135	149	147
rd 0.2	50	88.8 (1.12)	114	142	142

give an increase in number of trees to about 450 from age 10 to 15, but then a monotonic decrease in number of trees.

Implementation

For each stand property, the equations for the density standard and relative density change operate as a linked pair—a different density standard would require a different equation for relative density change. To apply our equations requires temporary inventory plot measurements of age, site index, and any one, or all three, stand properties (total cubic-foot volume, basal area, and number of trees) for trees 1 in. dbh and larger. The current relative density is the ratio of the value of the current stand property to the value of the density standard [Equations (1), (2), or (3)] with the same age and site index. Calculate relative density change [Equations (6), (7), or (8)] and add it to the current relative density to predict a new relative density. The product of the new relative density and the density standard [Equations (1), (2), or (3) using the original age + 5] is the predicted stand property. The procedure is repeated for additional 5-yr increments. When the interval is less than 5 yr, proportionately reduce the calculated relative density change.

For example, Table 9 shows, first in spreadsheet notation and then with numeric values, the prediction of number of trees at age 35 starting from a 28-yr-old, site index 78 aspen stand with 1000 trees/ac. The predicted number of trees is 697 trees/ac.

Discussion and Conclusions

Quaking aspen occurs in mixed stands with a number of other species. While most of the permanent growth plots used in calibration and validation had over 90% quaking aspen by basal area, the range extended to as little as 50%. We plotted residuals from fitting relative density change equations for each stand property against percentage quaking aspen for remeasurement intervals and found no trend. We also plotted residuals in the validation data set against percentage quaking aspen and found no trend. This strongly suggests that the models developed are robust relative to species composition, and may be used in stands having, say, 60% (by basal area) or more quaking aspen.

Table 8. Deduction test for number of trees per acre relative density change equation to simulate tree frequency dynamics from hypothetical initial stand conditions. All starting conditions were applied at age 10.

Initial conditions	No. of trees/ac at age 10	No. of trees/ac at age 40	60	80
Site index 60				
rd 1.00	3780	685	267	129
rd 0.57	2166	524	229	118
rd 0.20	756	384	196	109
Site index 80				
rd 1.75	3780	647	259	137
rd 1.00	2164	477	216	123
rd 0.35	755	329	177	111

Based on our analysis of total volume as a function of stand age, the models appear to be overestimating stand volume somewhat (more than 80 cords/ac at age 80 and site index 80 in Table 6). Some sources of possible overprediction are:

1. we ignored explicit inclusion of defect in volume computations,
2. our permanent plot remeasurements came from an area of Minnesota having good growing conditions, and
3. study areas were subjectively selected and then given special treatment during the study period.

Each possibility is addressed briefly.

1. Our predictions assume all trees are solid wood. Cull and defect were ignored in the volumes predicted. Hahn (1984) estimates that Lake States quaking aspen trees classified by Forest Inventory and Analysis as growing stock have about

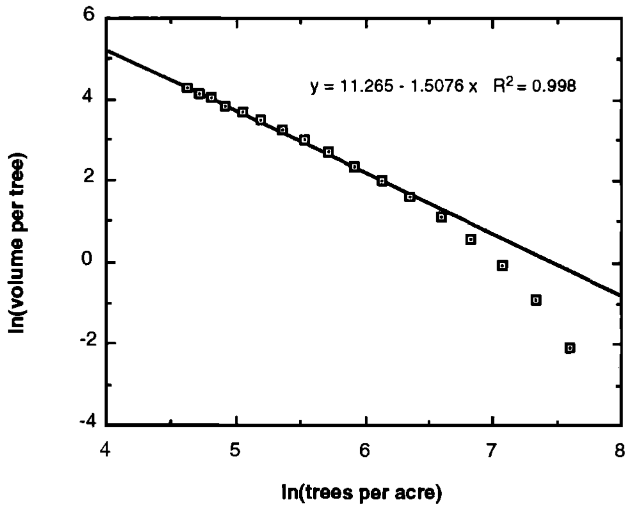

Figure 2. Check of predicted number of trees/ac and volume per tree stand property interdependence. Initial conditions were 10 yr, 600 trees/ac, and 250 ft^3. The regression was fit to observations after omitting predictions for the first 5 time steps. First plotted point corresponds to age 10, with later points having 5-yr age increments.

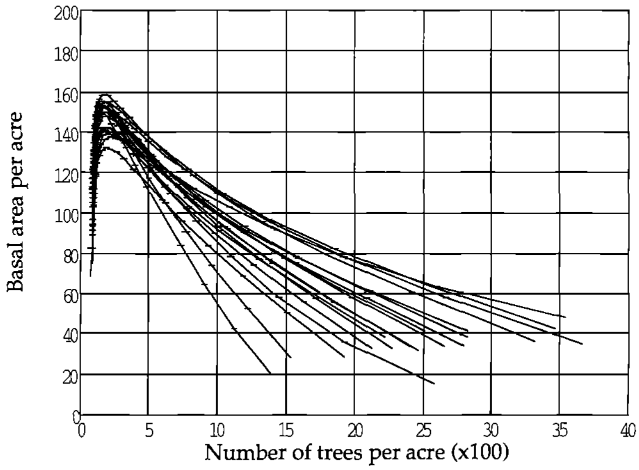

Figure 3. Trajectories in the stocking guide framework for 1ì combinations of number of trees per acre and basal area at age 10 from NC-52 plots not used in calibrating relative density change equations. Basal area is projected using Equation (7) and number of trees/ac with Equation (8). Tick marks on the trajectories signal 5 yr elapsed time.

5% cull. Cull percentage in other tree classes (rough, rotten) are much larger. W.B. Smith (personal communication) estimates about 90% of trees in aspen stands are growing stock. To estimate cubic volume of sound wood, our predictions must be decreased by about 15%.

2. There is an inadequate geographical distribution of the permanent growth plots used in calibrating the relative density change equation. The omission is not purposeful; all known sources of permanent growth plots having tree measurements to 1.0 in. dbh were used in our calibration. A result is that a "complete" range of the soils and climate of northern Minnesota is not included. Most plots were located in the portion of north central Minnesota, known as a generally good, although not the best, aspen growing area in Minnesota.
3. The method of selecting the physical location of studies 96 and 52 is unknown to the authors, and probably subjective. In similar cases, subjective location of scientific studies results in above average site qualities being selected. The use of subjective criteria in locating research study plots, as well as subsequent plot treatment, leads to what Bruce (1977) calls the "research plot effect" bias in growth studies. In at least one study we used, plots received special treatment dunng the study and should be ıncluded ın the research plot effect; i.e., historical records for NC-52 indicate it was aerially sprayed, along with recreational and administrative sites, during a severe outbreak of forest tent caterpillar in 1952 (letter to the NC-52 study files by M.L Heinselman 1952). The amount of the enhanced growth, sprayed over unsprayed, is not known although significant slowing of quaking aspen height growth is evident for the early 1950s in unpublished stem analysis information collected in the Pike Bay area.

When possible sources of overestimation are added—they would seem to be conformable for addition—estimates given by our projection models may approach 25% too high (say 15% for cull and 10% for "research plot effect"). Users should check our models against their own permanent growth plot remeasurement information to calculate their own reduction factors or to calıbrate their own relative density change equations. When no permanent growth plots are available, we suggest a 15–25% reduction to give a conservative yield figure.

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Table 9. An example showing how to predict number of trees at age 35 for an aspen stand currently 28 yr old, with 1000 trees/ac and site index 78. Equation form and parameters are described in the paper. The first part of the table is in spreadsheet format and the last part shows the results of the calculations.

	A	B	C	D	E	F
	Age	Density standard	Starting density	Starting relative density	5-yr relative density change	Projected relative density
1	28	Eq. (3)	Observed	C1/B1	Eq. (8)	D1+E1*(A2-A1)/5
2	33	Eq. (3)	F1*B2	C2/B2	Eq. (8)	D2+E2*(A3-A2)/5
3	35	Eq. (3)	F2*B3			
	28	1016	1000	0.9846	-0.0436	0.9410
	33	818.7	770	0.9410	-0.0373	0.9261
	35	752.3	**697**			

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