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Suppressing flavor-changing neutral currents and CP -violating phases by extra dimensions

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In extra dimensions the infrared attractive force of gauge interactions is amplified. We find that this force can align in the infrared limit the soft-supersymmetry breaking terms out of their anarchical disorder at a fundamental scale in such a way that flavor-changing neutral currents as well as dangerous CP -violating phases are sufficiently suppressed at the unification scale. The main assumption is that the matter and Higgs supermultiplets and the flavor-dependent interactions such as Yukawa interactions are stuck at the four-dimensional boundary. As a concrete example we consider the minimal model based on $SU(5)$ in six dimensions.

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I. INTRODUCTION

Low-energy softly broken supersymmetry (SUSY) has been the most promising idea in solving the gauge hierarchy problem [1]. However, the introduction of the superpartners of the known particles induces large flavor-changing neutral current (FCNC) processes and CP -violating phases, which are severely constrained by precision experiments [2–6]. Therefore, the huge degrees of freedom involved in the soft-supersymmetry breaking (SSB) parameters have to be highly constrained in all viable supersymmetric models. This has been called the supersymmetric flavor problem.

To overcome this problem, several ideas of SUSY breaking and its mediation mechanisms have been proposed: gauge mediation [7], anomaly mediation [8], gaugino mediation [9] and so on. The common feature behind these ideas is that the leading parts of the SSB parameters are given by flavor-blind radiative corrections. It is noted that the anomaly mediation and the gaugino mediation work on the assumption that the tree-level contributions for the SSB parameters at a fundamental scale M_{PL} are sufficiently suppressed, e.g., by sequestering of branes for the visible sector and the hidden SSB sector, since there is no reason for these terms to be flavor universal. However, it has been argued recently [10] that such a sequestering mechanism cannot be simply realized in generic supergravity or superstring inspired models. An interesting way out of this problem is to suppress the tree-level contributions by certain field theoretical dynamics. There have been indeed several attempts along this line of thought in which use has been made [11–13] that the SSB parameters are suppressed in the infrared limit in approximate superconformal field theories [14]. In this paper, we propose another possibility in more than four dimensions that flavor-blind radiative corrections are much more dominant than any other flavor nonuniversal contributions.

In Sec. II we will show that such a mechanism can be realized by implementing the power-law running of couplings [15,16] in supersymmetric field theories with δ extra compactified dimensions and at the same time by using the infrared attractiveness of the SSB parameters [17]. Here we consider the simplest case in which only the non-Abelian gauge supermultiplet propagates in the $(4 + \delta)$ -dimensional bulk and the supermultiplets containing the matter and Higgs fields are localized at our 3-brane [16,18,19]. In this mecha-

nism the gaugino mass M , which is assumed to be generated at the fundamental scale M_{PL} by some SUSY breaking mechanism, receives a correction proportional to $(M_{\text{PL}}/M_{\text{GUT}})^\delta$ at the grand unification scale M_{GUT} , and more importantly induces dominant flavor-blind corrections to other SSB parameters. The most interesting finding is that the squared soft-scalar masses $(m^2)_j^i$ and the soft-trilinear couplings h^{ijk} become so aligned at M_{GUT} that flavor changing neutral current processes and dangerous CP -violating phases are sufficiently suppressed. It will be seen that in this class of models, all the A -parameter h 's, B parameter B_H and soft-scalar masses m^2 's in the minimal supersymmetric standard model (MSSM) are basically fixed as functions of the unified gaugino mass M and the μ -parameter μ_H , up to corrections coming from Yukawa interactions. Therefore, this class of models cannot only overcome the supersymmetric flavor problem, but also have a large predictive power. Moreover, no charged sparticles become tachyonic in these models.

We shall consider in Sec. III the minimal supersymmetric $SU(5)$ grand unified theory (GUT) model in six dimensions as an explicit example, and take into account the logarithmic corrections, too. To simplify the model, we however neglect the neutrino masses and mixings. We find that the model can predict a set of the SSB parameters that are consistent with the radiative electroweak symmetry breaking and with other experimental constraints. Section IV is devoted to conclusion.

II. BULK GAUGE INTERACTIONS ALIGN THE SSB TERMS

As we have explained our basic idea in the Introduction, we assume that only the supersymmetric gauge interactions exist in the $(4 + \delta)$ -dimensional bulk while all the other interactions are confined at the four-dimensional boundary. Accordingly, the $(4 + \delta)$ -dimensional gauge supermultiplet propagates in the bulk, and all the $N=1$ chiral supermultiplets $\Phi_i = (\phi_i, \psi_i)$ containing matters and Higgs bosons propagate only in four dimensions. The gauge supermultiplet contains a chiral supermultiplet Γ in the adjoint representation, where we assume that δ is equal to one or two. We assign an odd parity to Γ so that it does not contain zero modes [16,19], and does not have any interactions with Φ 's.

To simplify the situation we further assume that each extra dimension is compactified on a circle with the same radius R . With these assumptions, the boundary superpotential has a generic form

$$W(\Phi) = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j, \quad (1)$$

and the SSB Lagrangian L_{SSB} can be written as

$$-L_{\text{SSB}} = \left(\frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B^{ij} \phi_i \phi_j + \frac{1}{2} \sum_{n=0} M \lambda_n \lambda_n + \text{H.c.} \right) + \phi^{*j} (m^2)_j^i \phi_i, \quad (2)$$

where λ_n 's are the Kaluza-Klein modes of the gaugino, and we have assumed a unique gaugino mass M for all λ 's.

The size of R is model dependent and is not related to the GUT scale M_{GUT} *a priori*, where we mean by M_{GUT} the energy scale at which the gauge coupling constants of the MSSM are unified. If $M_C = 1/R < M_{\text{GUT}} < 10^{16}$ GeV, we obtain $M_{\text{GUT}} \approx 10M_C$ [16,20]. Therefore, since we will consider GUTs, we may have a problem of the fast proton decay, if M_{GUT} is much smaller than $\sim 10^{16}$ GeV. We will consider the renormalization group (RG) running of the parameters between the fundamental scale $M_{\text{PL}} = M_{\text{Planck}} / \sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV and M_{GUT} . If $M_C > M_{\text{GUT}}$, the parameters evolve according to the power law [15,16] between M_{PL} and M_C , and to the logarithmic-law below M_C . So, if $M_C < M_{\text{GUT}}$, the parameters obey the power law between M_{PL} and M_{GUT} , so that the effect of the infrared attractiveness can be maximized in this case. Below M_{GUT} , the effective gauge symmetry is supposed to be $SU(3)_C \times SU(2)_L \times U(1)_Y$, and between M_{GUT} and M_C the parameters of the effective theory obey the power law. The power-law running of the parameters in this range has no influence on our purpose in this paper, because we are interested in the infrared attractiveness of the SSB parameters in GUTs with extra dimensions. Therefore, we simply assume that $M_{\text{GUT}} = M_C = 1/R$.

To see the gross behavior of the RG running, we first consider the contributions coming from only the gauge supermultiplet, because it is the only source responsible for the power-law running [15,16] of the parameters under the assumptions specified above. In the flavor bases in which couplings of the gauginos are diagonal, only diagonal elements of the anomalous dimensions can contribute. We find the following set of the one-loop β functions in this approximation [16,20]:

$$\Lambda \frac{dg}{d\Lambda} = -\frac{2}{16\pi^2} C(G) G_\delta^2 g, \quad (3)$$

$$\Lambda \frac{dM}{d\Lambda} = -\frac{4}{16\pi^2} C(G) G_\delta^2 M, \quad (4)$$

$$\Lambda \frac{dY^{ijk}}{d\Lambda} = -\frac{2}{16\pi^2} (C(i) + C(j) + C(k)) G_\delta^2 Y^{ijk}, \quad (5)$$

$$\Lambda \frac{d\mu^{ij}}{d\Lambda} = -\frac{2}{16\pi^2} (C(i) + C(j)) G_\delta^2 \mu^{ij}, \quad (6)$$

$$\Lambda \frac{dB^{ij}}{d\Lambda} = \frac{2}{16\pi^2} (C(i) + C(j)) G_\delta^2 (2M\mu^{ij} - B^{ij}), \quad (7)$$

$$\Lambda \frac{dh^{ijk}}{d\Lambda} = \frac{2}{16\pi^2} (C(i) + C(j) + C(k)) G_\delta^2 (2MY^{ijk} - h^{ijk}), \quad (8)$$

$$\Lambda \frac{d(m^2)_j^i}{d\Lambda} = -\frac{8}{16\pi^2} C(i) \delta_j^i G_\delta^2 |M|^2, \quad (9)$$

where $G_\delta = g X_\delta^{1/2} (R\Lambda)^{\delta/2}$, and [16]¹

$$X_\delta = \pi^{\delta/2} \Gamma^{-1}(1 + \delta/2) = \begin{cases} 2 & \text{for } \delta=1, \\ \pi & \text{for } \delta=2. \end{cases} \quad (10)$$

The gauge coupling is denoted by g , and $C(G)$ stands for the quadratic Casimir of the adjoint representation of the gauge group G , and $C(i)$ for that of the representation R_i . It is easy to show that the evolution of Y^{ijk} , μ^{ij} and M are related to that of g as

$$M(M_{\text{GUT}}) = \left(\frac{g(M_{\text{GUT}})}{g(M_{\text{PL}})} \right)^2 M(M_{\text{PL}}), \quad (11)$$

$$Y^{ijk}(M_{\text{GUT}}) = \left(\frac{g(M_{\text{GUT}})}{g(M_{\text{PL}})} \right)^{\eta_Y^{ijk}} Y^{ijk}(M_{\text{PL}}), \quad (12)$$

$$\mu^{ij}(M_{\text{GUT}}) = \left(\frac{g(M_{\text{GUT}})}{g(M_{\text{PL}})} \right)^{\eta_\mu^{ij}} \mu^{ij}(M_{\text{PL}}), \quad (13)$$

where

$$\eta_Y^{ijk} = \frac{C(i) + C(j) + C(k)}{C(G)},$$

$$\eta_\mu^{ij} = \frac{C(i) + C(j)}{C(G)}. \quad (14)$$

Therefore, these parameters can become very large if $g(M_{\text{PL}})/g(M_{\text{GUT}})$ is large. A rough estimate shows that

¹ X_δ is regularization scheme dependent. See [21] for a detailed analysis on the regularization dependence.

$$g(M_{\text{GUT}})/g(M_{\text{PL}}) \simeq \left[\frac{C(G)X_\delta \alpha_{\text{GUT}}}{\pi \delta} \right]^{1/2} \left(\frac{M_{\text{PL}}}{M_{\text{GUT}}} \right)^{\delta/2} \left(\frac{g(M_{\text{PL}})}{g(M_{\text{GUT}})} \right)^4 \left[\frac{(m^2)_j^i}{|M|^2} (M_{\text{PL}}) - \frac{C(i)}{C(G)} \delta_j^i \right]. \quad (17)$$

$$\simeq \begin{cases} 3.5 & \text{for } \delta=1, \\ 32 & \text{for } \delta=2, \end{cases} \quad (15)$$

where we have used $\alpha_{\text{GUT}}=0.04$, $M_{\text{PL}}/M_{\text{GUT}}=10^2$, $G=SU(5)$ to obtain the concrete numbers. These numbers should be compared with 1.3 in the corresponding four-dimensional case [17]. In the class of models we will be considering, we assume that the supersymmetric Higgs boson mass parameter μ of the MSSM, μ_H , is given appropriately at the fundamental scale M_{PL} and we may take it as a free parameter. Indeed μ_H is also enhanced according to the power law (13). However the Giudice-Masiero mechanism [22] will lead to small μ_H compared with the B parameter, which turns out to be of the order of the gaugino mass at M_{GUT} , unless η_μ is larger than or equal to 2.

In contrast to g , Y^{ijk} , μ^{ij} , M , the SSB parameters B^{ij} , h^{ijk} and $(m^2)_j^i$ have a completely different behavior. We find that the ratios of the SSB parameters to the gaugino mass M approach their infrared attractive fixed points:

$$B^{ij}/M\mu^{ij} \rightarrow -\eta_\mu^{ij},$$

$$h^{ijk}/MY^{ijk} \rightarrow -\eta_Y^{ijk},$$

$$(m^2)_j^i/|M|^2 \rightarrow \frac{C(i)}{C(G)} \delta_j^i, \quad (16)$$

where η 's are defined in Eq. (14). Note that so far no assumption on the reality of the SSB parameters has been made, and we recall that the phase of M and μ^{ij} can always be rotated away by a phase rotation that corresponds to the R symmetry and an appropriate rotation of the chiral superfields Φ , respectively. So, after these rotations, all the phases of M and μ^{ij} are transferred to those of Y^{ijk} , h^{ijk} , B^{ij} and $(m^2)_j^i$. Therefore, we may assume without loss of generality that M and μ^{ij} are real. We see from Eq. (16) that the low-energy structure is completely fixed by the group theoretic structure of the model. Furthermore, since h^{ijk} and $(m^2)_j^i$ become aligned in the infrared limit, i.e., $h^{ijk} \propto Y^{ijk}$ and $(m^2)_j^i \propto \delta_j^i$, the infrared forms (16) give desired initial values of the parameters at M_{GUT} to suppress FCNC processes in the MSSM, and they predict that the only CP -violating phase is the usual CKM phase.²

One can easily estimate how much of a disorder in the initial values at M_{PL} can survive at M_{GUT} . Suppose that there exists an $O(1)$ disorder in $(m^2)_j^i/|M|^2$. Using the β functions (4) and (9), we find the deviation from Eq. (16) to be

²Equation (16) means that the phases of (h/MY) and $(B/M\mu)$ that cannot be rotated away approach zero in the exact infrared limit.

Then inserting the value of $g(M_{\text{PL}})/g(M_{\text{GUT}})$ given in Eq. (15), we find that an $O(1)$ disorder at M_{PL} becomes a disorder of $O(10^{-2})$ and $O(10^{-6})$ at M_{GUT} for $\delta=1$ and 2, respectively. Note that the off-diagonal elements of $(m^2)_j^i$ as well as the differences among the diagonal elements $\Delta m^2(i,j) = (m^2)_i^i - (m^2)_j^j$ [if $C(i) = C(j)$] belong to the disorder. However, their contributions to $(\delta_{ij})_{LL,RR}$ of [6] are less than $O(10^{-6})$ for $\delta=2$, and therefore the most stringent constraints coming from the $K_S - K_L$ mass difference Δm_K and the decay $\mu \rightarrow e\gamma$ are satisfied [6]. In the case of five dimensions ($\delta=1$) the suppression of the disorder will be sufficient, if the gauginos are much heavier than the sfermions [6]. (If we use $M_{\text{PL}}/M_{\text{GUT}} \sim 10^3$, then the suppression is much improved.)

Similarly, using Eqs. (4) and (8), we obtain the deviation for the trilinear couplings from Eq. (16) as

$$\left(\frac{g(M_{\text{PL}})}{g(M_{\text{GUT}})} \right)^2 \left[\frac{h^{ijk}}{MY^{ijk}} (M_{\text{PL}}) + \eta_Y^{ijk} (M_{\text{PL}}) \right], \quad (18)$$

where use has been made of Eq. (12). Suppose the trilinear couplings to be of the order of MY^{ijk} at M_{PL} . Then we find that

$$\left| \frac{h^{ijk}}{MY^{ijk}} (M_{\text{GUT}}) + \eta_Y^{ijk} \right| \leq \left(\frac{g(M_{\text{PL}})}{g(M_{\text{GUT}})} \right)^{2+\eta_Y^{ijk}}. \quad (19)$$

Note that the phases of h^{ijk}/MY^{ijk} are also suppressed. In the case of $G=SU(5)$, $\eta_Y^{ijk} = 48/25(42/25)$ for the up (down) type Yukawa couplings. Using Eq. (15) again, we find that the right-hand side of Eq. (19) is $\sim 10^{-2(6)}$ for $\delta=1(2)$. This disorder contributes, for instance, to $\text{Im}(\delta_{ii})_{LR}$ as well as $\text{Re}(\delta_{ij})_{LR}$ of [6]. Therefore our suppression mechanism can satisfy the most stringent constraints coming from the electric dipole moments (EDM) of the neutron and the electron and also from ϵ'/ϵ in the $K^0 - \bar{K}^0$ mixing [6]. Similarly the phases of the B parameters, $B^{ij}/M\mu^{ij}$, are also suppressed.

In concrete examples, there will be logarithmic corrections to Eq. (16) to which the Yukawa couplings Y^{ijk} non-trivially contribute. How much the logarithmic corrections can amplify the disorder will be model-dependent. It is certainly worthwhile to note that the logarithmic interactions will be non-negligible only for Λ close to M_{GUT} , thereby overcoming the problem found in [23] that the GUT effects may destroy the universality of the SSB terms. In the next section we consider a concrete model based on $G=SU(5)$, and take into account the logarithmic corrections.

III. AN APPLICATION

A. The minimal $SU(5)$ model

To be more specific we consider the minimal GUT model based on $G=SU(5)$ in six dimensions. To simplify the situ-

ation we neglect the neutrino masses and their mixings.³ According to the previous section, we assume that only the $SU(5)$ gauge supermultiplet has the towers of Kaluza-Klein states. In the sense of four-dimensional supersymmetry, the multiplet contains an $N=1$ gauge supermultiplet and an $N=1$ chiral supermultiplet Γ in the adjoint representation. We assign an odd parity to this chiral supermultiplet so that it does not contain zero modes. Three generations of quarks and leptons are accommodated by three chiral superfields in $\Psi^i(\mathbf{10})$ and $\Phi^i(\bar{\mathbf{5}})$, where i runs over the three generations. A $\Sigma(\mathbf{24})$ is used to break $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, and $H(\mathbf{5})$ and $\bar{H}(\bar{\mathbf{5}})$ to describe the two Higgs superfields appropriate for the electroweak symmetry breaking. They are boundary superfield, and do not have any interaction with Γ which is a part of the gauge supermultiplet. The superpotential of the model is given by

$$W = \frac{Y_U^{ij}}{4} \epsilon^{\alpha\beta\gamma\delta\tau} \Psi_{\alpha\beta}^{(i)} \Psi_{\gamma\delta}^{(j)} H_\tau + \sqrt{2} Y_D^{ij} \Phi^{(i)\alpha} \Psi_{\alpha\beta}^{(j)} \bar{H}^\beta + \frac{Y_\lambda}{3} \Sigma_\alpha^\beta \Sigma_\beta^\gamma \Sigma_\gamma^\alpha + Y_f \bar{H}^\alpha \Sigma_\alpha^\beta H_\beta + \frac{\mu_\Sigma}{2} \Sigma_\alpha^\gamma \Sigma_\gamma^\alpha + \mu_H \bar{H}^\alpha H_\alpha, \quad (20)$$

where α, β, \dots are the $SU(5)$ indices, and Y_U^{ij} and Y_D^{ij} are the Yukawa couplings. The SSB Lagrangian is

$$-\mathcal{L}_{\text{SSB}} = m_{H_u}^2 \hat{H}_u^* \hat{H}_u + m_{H_d}^2 \hat{H}_d^* \hat{H}_d + m_\Sigma^2 \hat{\Sigma}_\alpha^\dagger \hat{\Sigma}_\alpha^\beta + \sum_{i,j}^3 [(m_\Phi^2)^{ij} \hat{\Phi}_\alpha^* \hat{\Phi}^{(j)\alpha} + (m_\Psi^2)^{ij} \hat{\Psi}^\dagger \hat{\Psi}^{(j)\alpha} \hat{\Phi}_{\beta\alpha}^{(j)}] + \left\{ \frac{1}{2} M \lambda \lambda + B_H \hat{H}^\alpha \hat{H}_\alpha + B_\Sigma \hat{\Sigma}_\alpha^\dagger \hat{\Sigma}_\alpha^\beta + h_f \hat{H}^\alpha \hat{\Sigma}_\alpha^\beta \hat{H}_\beta + \frac{h_\lambda}{3} \hat{\Sigma}_\alpha^\beta \hat{\Sigma}_\beta^\gamma \hat{\Sigma}_\gamma^\alpha + \frac{h_U^{ij}}{4} \epsilon^{\alpha\beta\gamma\delta\tau} \hat{\Psi}_{\alpha\beta}^{(i)} \hat{\Psi}_{\gamma\delta}^{(j)} \hat{H}_\tau + \sqrt{2} h_D^{ij} \hat{\Phi}^{(i)\alpha} \hat{\Psi}_{\alpha\beta}^{(j)} \hat{H}^\beta + \text{H.c.} \right\}, \quad (21)$$

where a hat is used to denote the scalar component of each chiral superfield. Then the gross infrared attractive form of the SSB parameters (16) becomes

$$B_\Sigma \rightarrow -2M\mu_\Sigma, B_H \rightarrow -\frac{24}{25} M\mu_H, \quad (22)$$

$$h_U \rightarrow -\frac{48}{25} M Y_U, h_D \rightarrow -\frac{42}{25} M Y_D, \quad (23)$$

³In a more realistic case, we should take into account the neutrino masses and their mixings, but they will not change the results we will find below, because we assume that the neutrino supermultiplets, too, are boundary multiplets.

$$h_f \rightarrow -\frac{49}{25} M Y_f, h_\lambda \rightarrow -3M Y_\lambda, \quad (24)$$

$$m_\Sigma^2 \rightarrow 2|M|^2, m_{H_d}^2, m_{H_u}^2,$$

$$m_\Phi^2 \rightarrow \frac{12}{25} |M|^2, m_\Psi^2 \rightarrow \frac{18}{25} |M|^2.$$

The unified gaugino mass M and μ are free parameters, but B_H is no longer a free parameter. We therefore have to check that the electroweak symmetry is correctly broken at low energies. All the scalars that belong to $\mathbf{5}$ or $\bar{\mathbf{5}}$ have the same positive squared soft mass [$\approx (0.69M)^2$], which does not differ very much from [$\approx (0.85M)^2$] for the scalars belonging to $\mathbf{10}$. So, the infrared attractive form in the present model is similar to the SSB terms of the constrained MSSM (CMSSM), implying that the model predicts a similar spectrum as in the CMSSM.

B. Logarithmic corrections

Next we are interested in how much the logarithmic corrections coming from the Yukawa interactions modify the infrared attractive values (22)–(24). In the following analyses we would like to neglect the mixings of the matter multiplets, because their effects will be very small as seen later. One of the pleasant features of the infrared attractive form of the SSB terms (16) is that the trilinear couplings, too, may be assumed to be small if the corresponding Yukawa couplings are small, as we have seen in Eq. (18). Therefore, we may also neglect the mixings among the scalar components of the matter multiplets. Consequently, we will work with

$$Y_{U,D}^{ij} \simeq Y_{t,b} \delta^{i3} \delta^{j3}, h_{U,D}^{ij} \simeq h_{t,b} \delta^{i3} \delta^{j3}. \quad (25)$$

We first write down the one-loop β functions of this model [$dA/d\ln\Lambda = \beta(A)/16\pi^2$] [24]:

$$\beta(g) = [-10G_2^2 + 7g^2]g, \quad (26)$$

$$\beta(M) = [-20G_2^2 + 14g^2]M, \quad (27)$$

$$\beta(Y_t) = \left[-\frac{96}{5} G_2^2 + 9|Y_t|^2 + \frac{24}{5} |Y_f|^2 + 4|Y_b|^2 \right] Y_t, \quad (28)$$

$$\beta(Y_b) = \left[-\frac{84}{5} G_2^2 + 3|Y_t|^2 + \frac{24}{5} |Y_f|^2 + 10|Y_b|^2 \right] Y_b, \quad (29)$$

$$\beta(Y_\lambda) = \left[-30G_2^2 + \frac{63}{5} |Y_\lambda|^2 + 3|Y_f|^2 \right] Y_\lambda, \quad (30)$$

$$\beta(Y_f) = \left[-\frac{98}{5} G_2^2 + 3|Y_t|^2 + 4|Y_b|^2 + \frac{53}{5} |Y_f|^2 + \frac{21}{5} |Y_\lambda|^2 \right] Y_f, \quad (31)$$

$$\beta(\mu_\Sigma) = \left[-20G_2^2 + 2|Y_f|^2 + \frac{42}{5}|Y_\lambda|^2 \right] \mu_\Sigma, \quad (32)$$

$$\beta(\mu_H) = \left[-\frac{48}{5}G_2^2 + \frac{48}{5}|Y_f|^2 + 4|Y_b|^2 + 3|Y_t|^2 \right] \mu_H, \quad (33)$$

$$\begin{aligned} \beta(B_H) = & \left[-\frac{48}{5}G_2^2 + \frac{48}{5}|Y_f|^2 + 4|Y_b|^2 + 3|Y_t|^2 \right] B_H \\ & + \left[\frac{96}{5}G_2^2M + \frac{96}{5}h_f Y_f^* + 8Y_b^* h_b + 6Y_t^* h_t \right] \mu_H, \end{aligned} \quad (34)$$

$$\begin{aligned} \beta(B_\Sigma) = & \left[-20G_2^2 + 2|Y_f|^2 + \frac{42}{5}|Y_\lambda|^2 \right] B_\Sigma \\ & + \left[40G_2^2M + 4h_f Y_f^* + \frac{84}{5}Y_\lambda^* h_\lambda \right] \mu_\Sigma, \end{aligned} \quad (35)$$

$$\begin{aligned} \beta(h_t) = & \left[-\frac{96}{5}G_2^2 + 9|Y_t|^2 + \frac{24}{5}|Y_f|^2 + 4|Y_b|^2 \right] h_t \\ & + \left[\frac{192}{5}MG_2^2 + 18h_t Y_t^* + 8h_b Y_b^* + \frac{48}{5}h_f Y_f^* \right] Y_t, \end{aligned} \quad (36)$$

$$\begin{aligned} \beta(h_b) = & \left[-\frac{84}{5}G_2^2 + 3|Y_t|^2 + \frac{24}{5}|Y_f|^2 + 10|Y_b|^2 \right] h_b \\ & + \left[\frac{168}{5}MG_2^2 + 6h_t Y_t^* + 20h_b Y_b^* + \frac{48}{5}h_f Y_f^* \right] Y_b, \end{aligned} \quad (37)$$

$$\begin{aligned} \beta(h_\lambda) = & \left[-30G_2^2 + \frac{63}{5}|Y_\lambda|^2 + 3|Y_f|^2 \right] h_\lambda \\ & + \left[60MG_2^2 + \frac{126}{5}h_\lambda Y_\lambda^* + 6h_f Y_f^* \right] Y_\lambda, \end{aligned} \quad (38)$$

$$\begin{aligned} \beta(h_f) = & \left[-\frac{98}{5}G_2^2 + 3|Y_t|^2 + 4|Y_b|^2 + \frac{53}{5}|Y_f|^2 \right. \\ & + \left. \frac{21}{5}|Y_\lambda|^2 \right] h_f + \left[\frac{196}{5}MG_2^2 + 6h_t Y_t^* + 8h_b Y_b^* \right. \\ & + \left. \frac{42}{5}h_\lambda Y_\lambda^* + \frac{106}{5}h_f Y_f^* \right] Y_f, \end{aligned} \quad (39)$$

$$\begin{aligned} \beta(m_{H_d}^2) = & -\frac{96}{5}G_2^2|M|^2 + \frac{48}{5}|Y_f|^2(m_{H_u}^2 + m_{H_d}^2 + m_\Sigma^2) \\ & + 8|Y_b|^2(m_{H_d}^2 + m_{\Psi_3}^2 + m_{\Phi_3}^2) + \frac{48}{5}|h_f|^2 + 8|h_b|^2, \end{aligned} \quad (40)$$

$$\begin{aligned} \beta(m_{H_u}^2) = & -\frac{96}{5}G_2^2|M|^2 + \frac{48}{5}|Y_f|^2(m_{H_u}^2 + m_{H_d}^2 + m_\Sigma^2) \\ & + 6|Y_t|^2(m_{H_u}^2 + 2m_{\Psi_3}^2) + \frac{48}{5}|h_f|^2 + 6|h_t|^2, \end{aligned} \quad (41)$$

$$\begin{aligned} \beta(m_\Sigma^2) = & -40G_2^2|M|^2 + 2|Y_f|^2(m_{H_u}^2 + m_{H_d}^2 + m_\Sigma^2) \\ & + \frac{126}{5}|Y_\lambda|^2 m_\Sigma^2 + 2|h_f|^2 + \frac{42}{5}|h_\lambda|^2, \end{aligned} \quad (42)$$

$$\begin{aligned} \beta(m_{\Phi_3}^2) = & -\frac{96}{5}G_2^2|M|^2 + 8|Y_b|^2(m_{H_d}^2 + m_{\Psi_3}^2 + m_{\Phi_3}^2) \\ & + 8|h_b|^2, \end{aligned} \quad (43)$$

$$\begin{aligned} \beta(m_{\Psi_3}^2) = & -\frac{144}{5}G_2^2|M|^2 + 6|Y_t|^2(m_{H_u}^2 + 2m_{\Psi_3}^2) \\ & + 4|Y_b|^2(m_{H_d}^2 + m_{\Psi_3}^2 + m_{\Phi_3}^2) + 6|h_t|^2 + 4|h_b|^2, \end{aligned} \quad (44)$$

$$\beta(m_{\Phi_{1,2}}^2) = -\frac{96}{5}G_2^2|M|^2,$$

$$\beta(m_{\Psi_{1,2}}^2) = -\frac{144}{5}G_2^2|M|^2, \quad (45)$$

where $G_2^2 = \pi(R\Lambda)g^2$ [see Eq. (10)].

Note that we identified $1/R$ with M_{GUT} ($\sim 2 \times 10^{16}$ GeV), so that the renormalization group flow above M_{GUT} is six-dimensional. We then require that the MSSM is the effective theory below M_{GUT} , and, as before, we denote the fundamental scale by M_{PL} , which we assume to be $10^2 \times M_{\text{GUT}}$. To compute explicitly the logarithmic corrections coming from the Yukawa couplings Y_t, Y_b, Y_f, Y_l , we have to choose their initial values at M_{GUT} . But they cannot be chosen arbitrarily, because Y_f and Y_λ have to satisfy the proton decay constraint [25], and Y_t and Y_b are related to the top quark mass M_t and $\tan\beta = \langle \hat{H} \rangle / \langle \hat{\bar{H}} \rangle$. So we impose that the mass of the colored Higgs boson is larger than 8×10^{16} GeV [25], and use $M_t = 174$ GeV. We also use M_τ (mass of the tau lepton) = 1.77 GeV, and impose the $b - \tau$ unification at M_{GUT} .⁴

As we see from Eq. (22) again, the soft parameter B_H is not an independent parameter. Furthermore, μ_H cannot assume an arbitrary value, because it is related to the electroweak symmetry breaking. They should be determined through the minimization of the scalar potential of the MSSM. For simplicity, we assume that the potential of the MSSM at $\Lambda = M_{\text{SUSY}}$ takes the tree-level form, so that the minimization conditions are given by

⁴But we will not take the mass of the bottom quark very seriously. It becomes larger than its experimental value.

$$0 = m_{H_d}^2 - m_{H_u}^2 + M_Z^2 \frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} - B_H \frac{\tan^2 \beta - 1}{\tan \beta}, \quad (46)$$

$$0 = 2\mu_H^2 + m_{H_d}^2 + m_{H_u}^2 - B_H \frac{\tan^2 \beta + 1}{\tan \beta}, \quad (47)$$

where M_Z is the mass of the Z boson, and all the parameters including M_Z are defined at M_{SUSY} , which we assume to be the unified gaugino mass M . Once the gaugino mass M is given, the other parameters $\tan \beta$ and μ_H are fixed by these equations. Therefore the viable scenario allows only a very restrictive set determined by the gaugino mass for the low energy parameters in MSSM. As we explain below, however, it is by no means trivial that these two conditions are simultaneously satisfied. Note first that $m_{H_d}^2$ and $m_{H_u}^2$ are indeed a unique function of the gaugino mass M in the zeroth order approximation (24), but their logarithmically corrected values nontrivially depend on $\tan \beta$: Not only their infrared attractive values at M_{GUT} , but also their RG evolution below M_{GUT} depends on Y_t and Y_b , and consequently on $\tan \beta$. Therefore, the minimization conditions define a highly non-linear problem, in which the RG flows of the couplings below and above M_{GUT} influence on each other in a non-trivial way. To explore the complete low energy parameters with respect to the gaugino mass, therefore, would go beyond the scope of this paper, and we leave this problem to future work.

In what follows we consider only one case: $M = 500 \text{ GeV}$, $g = (0.0406 \times 4\pi)^{1/2}$, $M_{\text{GUT}} = 1.83 \times 10^{16} \text{ GeV}$, and

$$\begin{aligned} \mu_H &= 926 \text{ GeV}, \quad Y_t = 0.767g, \quad Y_b = 0.201g, \\ Y_f &= 1.0g, \quad Y_\lambda = 0.01g, \quad \tan \beta = 19.5. \end{aligned} \quad (48)$$

In this case the infrared attractive values of the SSB terms are found to be

$$\begin{aligned} (m_{\Phi_{1,2}}^2, m_{\Phi_3}^2) &= (0.510[0.48], 0.507[0.48])|M|^2, \\ (m_{\Psi_{1,2}}^2, m_{\Psi_3}^2) &= (0.766[0.72], 0.726[0.72])|M|^2, \\ (m_{H_u}^2, m_{H_d}^2) &= (0.367[0.48], 0.402[0.48])|M|^2, \end{aligned} \quad (49)$$

$$h_t = -1.90[1.92]MY_t,$$

$$h_b = -1.68[1.68]MY_b,$$

$$B_H = -0.896[0.96]M\mu_H.$$

Here $m_{\Phi_{1,2}}^2$ and $m_{\Psi_{1,2}}^2$ are, respectively, the squared masses of the scalar components of $\Phi(\mathbf{10})$ and $\Psi(\bar{\mathbf{5}})$ of the first two generations, while $m_{\Phi_3}^2$ and $m_{\Psi_3}^2$ are those of the third generation. The numbers in [] are those without the logarithmic corrections. It should be noted also that no charged sparticle becomes a LSP. In Figs. 1, 2 and 3, we present the infrared convergence of the SSB parameters.

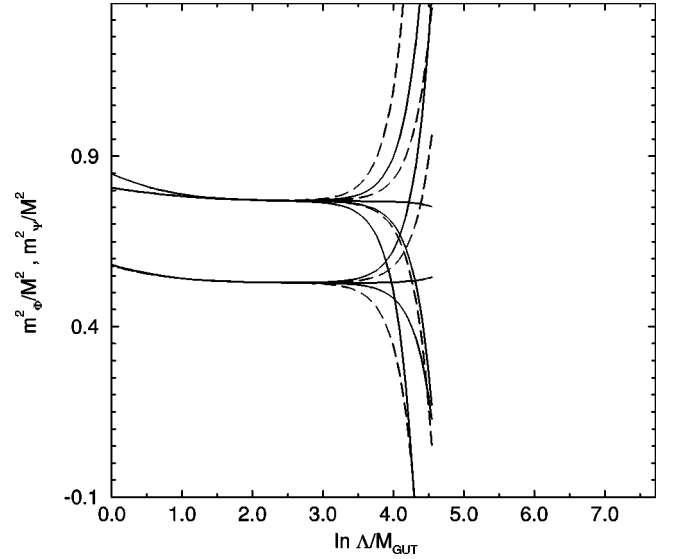


FIG. 1. Infrared attractiveness of $m_{\Phi}^2/|M|^2$ and $m_{\Psi}^2/|M|^2$. The dashed (solid) lines correspond to the third (first two) generation(s). $m_{\Psi_{1,2}}^2/|M|^2 > m_{\Psi_3}^2/|M|^2 > m_{\Phi_{1,2}}^2/|M|^2 \approx m_{\Phi_3}^2/|M|^2$ at $\Lambda = M_{\text{GUT}}$.

The β functions for $m_{\Phi_{1,2}}^2$ and $m_{\Psi_{1,2}}^2$ do not depend on Y_i and h_i ($i = t, b, f, \lambda$) in our approximation. Therefore, the infrared attractive values (24) are not modified by them. There exist of course logarithmic corrections coming from the gauge interaction, but they are flavor-blind. This is very pleasant, because the most stringent constraint from FCNC processes is the almost degeneracy of the squared soft masses of the first two generations. We have found that for the initial values of Y 's and g given in Eq. (48) the off-diagonal components $(m_{\Phi}^2)^{ij}/|M|^2$ with $i, j = 1, 2$ and the difference of diagonal elements, $\Delta m_{\Phi}^2(1,2)/|M|^2 = |m_{\Phi_{1,2}}^2 - m_{\Phi_2}^2|/|M|^2$ (and similarly for m_{Ψ}^2) are less than $O(10^{-4})$, which has been estimated to be $O(10^{-6})$ without the logarithmic corrections in Eq. (17). This order of disorder at M_{GUT} is still sufficient to satisfy the stringent constraints coming from Δm_K as well as $\mu \rightarrow e\gamma$ [6].

In contrast to the case of the first two generations, the β functions for $m_{\Phi_3}^2$ and $m_{\Psi_3}^2$ depend on Y_i and h_i . Therefore, they change their infrared attractive values. Figure 1 shows the evolution of $m_{\Phi}^2/|M|^2$ and $m_{\Psi}^2/|M|^2$, respectively. The dashed lines correspond to the third generation. The differences $\Delta m_{\Psi}^2(i,3)/|M|^2 = |m_{\Psi_i}^2 - m_{\Psi_3}^2|/|M|^2$ with $i = 1, 2$ directly contribute to Δm_B as well as to $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. We find that $\Delta m_{\Psi}^2(i,3)/|M|^2 \leq 0.04$ at M_{GUT} , which means that $|\langle \delta_{13,23}^{d,u} \rangle_{RR}|, |\langle \delta_{13,23}^{d,u} \rangle_{LL}| \leq \times 10^{-2}$ at M_{GUT} . Therefore, Δm_B in the $B - \bar{B}$ mixing and $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are sufficiently suppressed. The differences $\Delta m_{\Psi}^2(i,3)/|M|^2$ also contribute through the mixing between the first two generations and the third generation to Δm_K and $\mu \rightarrow e\gamma$. Assuming that the mass matrix of the up-type quarks is diagonal, and using the known values of Cabibbo-Kobayashi-Maskawa matrix V_{CKM} , we find that $\Delta m_{\Psi}^2(i,3)/|M|^2 \leq 0.04$ does not cause any problems with the FCNC processes mentioned above. The difference of -0.04 in $m_{\Psi}^2/|M|^2$ also causes no problem for $b \rightarrow s\gamma$ [6].

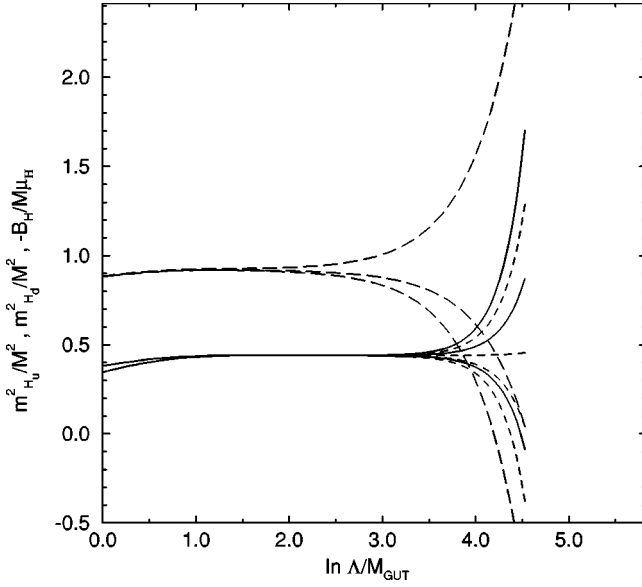


FIG. 2. Infrared attractiveness of $m_{H_d}^2/|M|^2$, $m_{H_u}^2/|M|^2$ and $-B_H/M\mu_H$. $-B_H/M\mu_H > m_{H_d}^2/|M|^2 > m_{H_u}^2/|M|^2$ at $\Lambda = M_{\text{GUT}}$.

The universality between $m_{H_d}^2$ and $m_{H_u}^2$ is also destroyed, as we see in Eq. (49). The main origins are the top Yukawa coupling Y_t and Y_f . This does not conflict with the FCNC problems and CP -violating processes. In Fig. 2 we show the infrared attractiveness of $m_{H_d}^2/|M|^2$, $m_{H_u}^2/|M|^2$ and $-B_H/M\mu_H$. In Fig. 3 the converging behavior for $-h_t/MY_t$ and $-h_b/MY_b$ is presented. There is also no universality between h_U and h_D from the beginning. In Eq. (18) we have found that the nonaligned part of h^{ijk} is suppressed by a factor of 10^{-6} in six dimensions, if the Yukawa couplings are neglected. Let us estimate how much of this suppression can survive if Y 's are taken into account. We find that the corrections can be written as

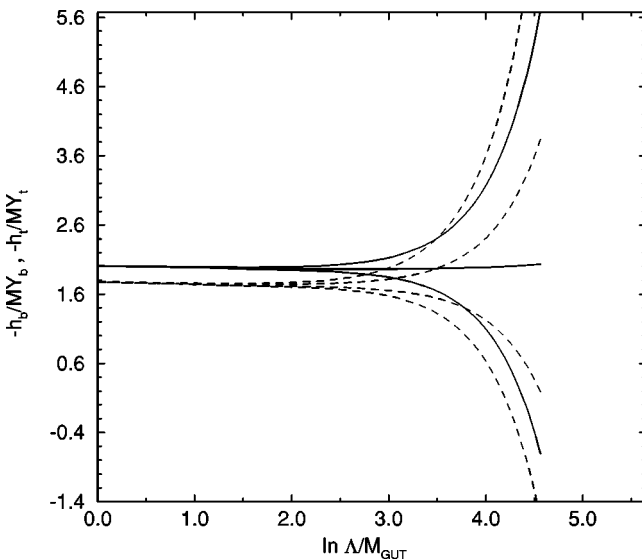


FIG. 3. Infrared attractiveness of $-h_t/M$ (solid lines) and $-h_b/M$ (dashed lines).

$$\frac{\Delta h_U^{ij}}{M} \sim \frac{1}{16\pi^2} (a_t Y_U^{i3} Y_U^{3j} Y_t + a_b Y_U^{i3} Y_D^{3j} Y_b) \ln \left(\frac{\Lambda_{\text{eff}}}{M_{\text{GUT}}} \right), \quad (50)$$

and similarly for $\Delta h_D^{ij}/M$, where a_t and a_b are $O(1)$ constants, and we have assumed that $h_{U,D}$ are proportional to $MY_{U,D}$ at a scale Λ_{eff} , at which Y 's become non-negligible. Further considerations in the basis where Y_U is diagonal yield that nonzero contributions (that are relevant to us) are

$$\begin{aligned} |\Delta h_U^{3j}(j \neq 3)| &\sim Y_t Y_b^2 L, \\ |\Delta h_D^{i3}(i \neq 3)| &\sim V_{CKM}^{ib} Y_t^2 Y_b L, \\ |\Delta h_D^{ij}(i, j \neq 3)| &\sim V_{CKM}^{ib} Y_b^3 L, \end{aligned} \quad (51)$$

where $L = M \ln(\Lambda_{\text{eff}}/M_{\text{GUT}})/16\pi^2$. Assuming that $\Lambda_{\text{eff}} \sim 50M_{\text{GUT}}$, we find that $|\Delta h_U^{3j}(j \neq 3)|/M \sim O(10^{-4})$ for the values given in Eq. (48), and the other Δh 's receive a further suppression from V_{CKM} . $\text{Im} \Delta(h_D^{11}/M)$, for instance, contributes to the EDM of the neutron, and can be estimated to be $O(10^{-7})$ which is small enough. $\text{Im} \Delta(h_D^{12}/M)$, which is of $O(10^{-7})$, too, is also small enough to satisfy the constraint from ϵ'/ϵ in the $K^0 - \bar{K}^0$ system. Therefore, we may conclude that the disorder of the trilinear couplings caused by the Yukawa couplings are sufficiently suppressed to satisfy even the most stringent constraints from the electric dipole moments EDMs [6].

IV. CONCLUSION

We conclude that gauge interactions in extra dimensions can be used to suppress the disorder of the SSB terms at the fundamental scale so that the FCNC processes and dangerous CP -violating phases become tiny at lower energy scales. Moreover, no charged sparticles become tachyonic in this scenario of the SSB parameters. As an explicit example we considered the minimal supersymmetric $SU(5)$ GUT model in six dimensions, and took into account the logarithmic corrections, too, where, to simplify the model, we neglected the neutrino masses and mixings. We found that the model can predict a set of the SSB parameters that are consistent with the radiative electroweak symmetry breaking and with other experimental constraints. We also found that the logarithmic corrections are not negligible. However, the infrared attractiveness of the SSB parameters does not change in the presence of the logarithmic corrections. The suppression mechanism of the FCNC and CP phases presented in this paper does not properly work in four dimensions. Therefore, the smallness of FCNC as well as of EDM is a possible hint of the existence of extra dimensions.

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