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Consensus Control of Observer-based Multi-Agent System with Communication Delay

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Abstract: This paper proposes an observer-based consensus control strategy for multi-agent system (MAS) with communication time delay. The condition of stability for MIMO agents is derived by Lyapunov theorem. It gives systematic design procedure under assumed unidirectional network. Furthermore, new consensus control law using observers is proposed for the networked MAS with communication delays.
Experimental results show effectiveness of our proposed output consensus approaches.

Keywords: Multi-Agent System, Consensus Control, Output Feedback Communication Delay, Formation Control

1. INTRODUCTION

There have been rapidly progresses of new theories that create a fusion of graph theory and system control theory for cooperative control problem of distributed networked control systems[1],[2]. As one of these research works, a multi-agent system (MAS) is one of significant topics which each agent autonomously works by using information of other agents over the network [3]-[9].

Let consider an agent as a vehicle in MAS, then it is possible to apply formation control problem [3]-[4]. Formation control problems expect at various fields, e.g. satellites, airship, intelligent transport systems and load carriage. Therefore control problem of MAS is useful and practically problem.

In this paper, we proposes an observer-based consensus control strategy for multi-agent system with communication time delay. In the networked MAS, consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of of dynamical agents. Consensus algorithm with graph theory has been studied as a control problem of MAS in [5]-[8]. In [5] and [6], proposes a new control law to which consensus algorithm [7] is enhanced. It was shown in [5] that stability of MAS could be checked graphically using Nyquist-like criterion. Depending on the agent system, it is difficult to design controllers using Nyquist-like criterion. Also, [6] derive condition of stability of MAS using Lyapunov stability theorem under assumed unidirectional network (balanced graph) where an agent is second-order system. So the MAS is much less common than [5]. Therefore we consider that each agent is MIMO linear system and propose the condition of stability using Lyapunov stability theorem. Using Lyapunov theorem, controller design is easier than [5] under assumed unidirectional network.

In addition, there are some problem in network structure of MAS: communication delay and constrained communication. In this paper, new consensus control law using observers is proposed for constrained communication where communication delays are occurred in inter-agent communications. In [9], a Lyapunov-like criterion was derived for stability conditon in MAS with constant communication delay. The paper[4] considers parallel estimators for formation control of MAS where the agent is spacecraft. A spacecraft estimate states of other space that connected network. Hence order of the estimator under assumed large network is larger than other estimator under assumed small network. Therefore, the proposed observer-based output control strategy is much simpler than the methos in [4]. Finally, the experimental results show effectiveness of our proposed control law.

2. GRAPH THEORY

Graph is useful to represent network structures [11]. Let $G = (\mathscr{V}, \mathscr{A})$ be a graph of order N with the set of nodes $\mathscr{V} = \{v_1, \cdots, v_N\}$ and set of arcs $\mathscr{A} \subseteq \mathscr{V} \times \mathscr{V}$. An arc $(v_i,\ v_j)$ of G is shown by an arrow drawn from node v_j to node v_i. The set of neighbors of node v_i is denoted by $\mathscr{N}_i = \{v_j \in \mathscr{V} : (v_j, v_i) \in \mathscr{A}\}$.

In MAS, node v_i means ith agent and arc (v_i, v_j) is communication that jth agent send some information to ith agent.

In this paper, we use *Graph Laplacian* for graph structure expressed mathematically. Graph Laplacian $L = [l_{ij}]$ is matrix that $l_{ii} = \sum_{j \neq i} a_{ij},\ l_{ij} = -a_{ij}\ (i \neq j)$ where $a_{ij} = 1\ (v_j \in \mathscr{N}_i)$ and $a_{ij} = 0\ (v_j \notin \mathscr{N}_i)$. If $L = L^T$, Graph G is called *Undirected graph*, otherwise G is directed graph (*Digraph*). The undirected graph called *connected graph* if it is possible to reach any agent starting from any other agent by traversing network. In digraph case, the directed graph called strongly connected graph.

Assuming that the graph is (strongly) connected graph, Graph Laplacian L satisfies the following properties:

i) There is unique zero-eigenvalue of L
ii) All eigenvalues of L are nonnegative.

iii) An eigenvector of zero-eigenvalue of L is Nth column vector of all ones.

In this paper, we express eigenvalues of $N \times N$ symmetric matrix a as

$$\lambda_1(a) \leq \lambda_2(a) \leq \cdots \leq \lambda_N(a). \tag{1}$$

where $\lambda_k(a)$ is kth eigenvalue of a. It is well-known that eigenvalues of Graph Laplacian L in connected graph can be presented as

$$0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_N(L). \tag{2}$$

3. MULTI-AGENT SYSTEM

In this paper, the Networked MAS is treated. A plant is the MAS which consists N agents under following assumption.

[Assumption 1] The network structure of MAS is (strongly) connected graph.

Consider ith agent system as

$$\dot{x}_i = Ax_i + Bu_i, \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$. $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are ith state and input. Let $x = [x_1^T \ \cdots \ x_N^T]^T$ and $u = [u_1^T \ \cdots \ u_N^T]^T$, then MAS can be presented as

$$\dot{x} = (I_N \otimes A)x + (I_N \otimes B)u, \tag{4}$$

where I_N is Nth-order unit matrix and $\otimes$ is Kronecker product.

Throughout this paper, we assume that the agent (3) satisfies the following properties:

[Assumptions 2]
i) There is no positive real eigenvalue of A.
ii) (A,B) is controllable.

In this paper, stability of MAS means

$$\lim_{t\to\infty} x = \mathbf{1} \otimes \alpha \tag{5}$$

where $\mathbf{1}$ is Nth column vector of all ones and $\alpha \in \mathbb{R}^n$ is any vector and we define that α is consensus vector. We called *consensus* of MAS if system (4) achieve Eq. (5).

4. STABILITY OF MULTI-AGENT SYSTEM

First, we utilize a consensus algorithm[7] as

$$u_i = -K \sum_{j\in\mathcal{N}_i}^{N} (x_i - x_j), \tag{6}$$

where $K \in \mathbb{R}^{m\times n}$ is the controller gain. Consensus algorithm (6) can be written in matrix form as

$$u = -(L \otimes K)x. \tag{7}$$

By Eq. (7), MAS (4) is expressed as

$$\dot{x} = ((I_N \otimes A) - (L \otimes BK))x. \tag{8}$$

Following theorem is derived about consensus of MAS (8).

[Theorem 1] Assume that $\Lambda_e + \Lambda_e^T > 0$ and MAS (8) satisfy [**Assumption 1-2**]. Then MAS (8) achieve consensus

$$\alpha = N^{1/2} \exp(At)(l_1^T \otimes I_n)x(0) \tag{9}$$

if there is positive definite matrix P such that

$$A^T P + PA - \lambda_1(\Lambda_e \Lambda_e^T) PBB^T P < 0 \tag{10}$$

and controller gain K is
i) Digraph case

$$K = \frac{\lambda_{N-1}(\Lambda_e \Lambda_e^T)}{\lambda_1(\Lambda_e + \Lambda_e^T)} B^T P, \tag{11}$$

ii) Undirected graph case

$$K = \frac{1}{2}\lambda_N(L) B^T P, \tag{12}$$

where $\Lambda_e \in \mathbb{R}^{N-1\times N-1}$ is matrix that is gotten by Jordan form $\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_e \end{bmatrix} = SLS^{-1}$ of graph Laplacian L, where S is any regular matrix which first row vector is $N^{-1/2}\mathbf{1}$. l_1 is left-eigenvector corresponding to zero-eigenvalue of L where $l_1^T N^{-1/2}\mathbf{1} = 1$.

proof: Jordan form Λ of graph Laplacian L is expressed as

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_e \end{bmatrix} = S^{-1}LS. \tag{13}$$

Now, we consider coordinate transformation $z = (S^{-1} \otimes I_n)x$, then the system (8) is expressed as:

$$\begin{aligned} \dot{z} &= \big((S^{-1}S \otimes A) - (S^{-1}LS \otimes BK)\big) z \\ &= ((I_N \otimes A) - (\Lambda \otimes BK))\, z, \end{aligned} \tag{14}$$

where $z = [z_1^T \ z_e^T]^T \in \mathbb{R}^{Nn}$, $z_e = [z_2^T \ \cdots z_N^T]^T \in \mathbb{R}^{(N-1)n}$. Eq. (14) can be divided into two equations:

$$\dot{z}_1 = Az_1 \tag{15}$$

$$\dot{z}_e = ((I_{N-1} \otimes A) - (\Lambda_e \otimes BK))\, z_e, \tag{16}$$

If z_e converge to equilibrium ($z_e \to 0$ *as* $t \to \infty$), MAS (8) achieves consensus by Eqs. (15)-(16) and Assumption 2-i). Therefore, we get

$$\begin{aligned} \lim_{t\to\infty} x &= \lim_{t\to\infty} (S \otimes I_n) z = (N^{-1/2}\mathbf{1} \otimes I_n) z_1 \\ &= \mathbf{1} \otimes N^{-1/2} e^{At} (l_1^T \otimes I_n) x(0) \end{aligned} \tag{17}$$

This means each agent goes to a consensus. If we can prove an asymptotical stability of Eq. (16), then MAS achieves consensus. Consider a Lyapunov function V_1 as:

$$V_1 = z_e^T (I_{N-1} \otimes P) z_e. \tag{18}$$

Differentiate V_1 along the trajectories, we get

$$\dot{V}_1 = z_e^T \left(I_{N-1} \otimes (A^T P + PA) - \Lambda_e^T \otimes K^T B^T P - \Lambda_e \otimes PBK\right) z_e. \tag{19}$$

If there is positive definite matrix P such that

$$I_{N-1} \otimes (A^T P + PA) - (\Lambda_e^T \otimes K^T B^T P) - (\Lambda_e \otimes PBK) < 0. \tag{20}$$

Then the system (16) is stable.

Next, we consider stabilization condition of control gain K. By assumption 2-ii), $(A, \lambda_1(\Lambda_e\Lambda_e^T)^{-1/2}B)$ is stabilizable. Therefore, there is positive definite matrix P such that

$$A^TP + PA - \lambda_1(\Lambda_e\Lambda_e^T)PBB^TP < 0. \tag{21}$$

Because of $\Lambda_e\Lambda_e^T > 0$, Eq. (21) is expressed as:

$$\begin{aligned} 0 &> (I_{N-1} \otimes A^TP + PA) - (\lambda_1(\Lambda_e\Lambda_e^T) \otimes PBB^TP) \\ &\geq (I_{N-1} \otimes A^TP + PA) - (\Lambda_e\Lambda_e^T \otimes PBB^TP). \end{aligned} \tag{22}$$

First, let consider gain K in digraph case.

i) Digraph

(22) can be represent as

$$\begin{aligned} 0 &> (I_{N-1} \otimes A^TP + PA) - \left(\frac{\Lambda_e\Lambda_e^T}{\lambda_1(\Lambda_e + \Lambda_e^T)}\lambda_1(\Lambda_e + \Lambda_e^T) \otimes PBB^TP\right) \\ &\geq (I_{N-1} \otimes A^TP + PA) - \left(\frac{\Lambda_e\Lambda_e^T}{\lambda_1(\Lambda_e + \Lambda_e^T)}(\Lambda_e + \Lambda_e^T) \otimes PBB^TP\right) \\ &\geq (I_{N-1} \otimes A^TP + PA) - (\Lambda_e + \Lambda_e^T) \otimes \left(\frac{\lambda_{N-1}(\Lambda_e\Lambda_e^T)}{\lambda_1(\Lambda_e + \Lambda_e^T)}PBB^TP\right) \end{aligned} \tag{23}$$

If there is positive definite matrix P such that Eq. (21) and controller gain K define Eq. (11), MAS (8) satisfy Eq. (20). Therefore MAS (8) satisfy consensus.

It is clear that $\Lambda_e + \Lambda_e^T$ is positive definite matrix if there is no duplicate eigenvalue of Graph Laplacian L. And also $\Lambda_e + \Lambda_e^T$ is positive definite matrix if Graph is balanced graph by the property of Graph Laplacian. Therefore, there is $\Lambda_e + \Lambda_e^T > 0$ at least.

Next, we define controller gain K in undirected graph.

ii) Undirected graph

Graph Laplacian L of undirected graph is symmetric matrix. Therefore, $\Lambda_e > 0$ is diagonal matrix. Hence Eq. (22) can be written as

$$\begin{aligned} 0 &> (I_{N-1} \otimes A^TP + PA) - (\Lambda_e^2 \otimes PBB^TP) \\ &\geq (I_{N-1} \otimes A^TP + PA) - (\Lambda_e \otimes \lambda_{N-1}(\Lambda_e)PBB^TP). \end{aligned} \tag{24}$$

By Eq. (24), if there is positive definite matrix P such that Eq. (21) and control gain K define Eq. (12) when MAS (8) satisfies Eq. (20). Therefore MAS (8) satisfies a consensus. □

5. STABILITY OF MULTI-AGENT SYSTEM WITH COMMUNICATION DELAY

Consider the stability analysis of MAS with communication delay. Here a communication time delay is treated to be constant and a control law is proposed by using maximum tolerable communication delay τ.

The proposed control law of ith agent with communication delay can be represented as

$$u_i(t) = -K\sum_{j\in\mathcal{N}_i}^{N}(x_i(t-\tau) - x_j(t-\tau)). \tag{25}$$

Hence, the control law of MAS is expressed

$$u(t) = -(L \otimes K)x(t-\tau). \tag{26}$$

Therefore, MAS with communication delay can be represented as

$$\dot{x}(t) = (I_N \otimes A)x(t) - (L \otimes BK)x(t-\tau). \tag{27}$$

We analysis that MAS (27) achieve consensus.

[Theorem 2] Assume that MAS (27) satisfy [**Assumptions 1-2**]. Then MAS (27) achieve consensus if $A - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}BK$ is stable and there is positive definite matrix P such that

$$\begin{bmatrix} \Phi & PBK \\ K^TB^TP & -\frac{r_1r_2}{(r_1+r_2)\tau\lambda_{N-1}(\Lambda_e\Lambda_e^T)}I_n \end{bmatrix} < 0 \tag{28}$$

$$\begin{aligned} \Phi = {} & A^TP + PA - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}\left(K^TB^TP + PBK\right) \\ & + r_1\tau A^TA^T + r_2\tau\lambda_{N-1}(\Lambda_e^T\Lambda_e)K^TB^TBK \end{aligned}$$

where $r_1,\ r_2$ are positive scalar.

Proof: MAS (27) can be divided into two equation well as theorem 1 by coordinate transformation $z = (S^{-1} \otimes I_n)x$.

$$\dot{z}_1(t) = Az_1(t) \tag{29}$$

$$\dot{z}_e(t) = (I_{N-1} \otimes A)z_e(t) - (\Lambda_e \otimes BK)z_e(t-\tau) \tag{30}$$

Therefore it is clear that stability of Eq. (30) and MAS (27) achieve consensus are equivalent. Consequently, we prove stability of Eq. (30). Let

$$\begin{aligned} z_e(t-\tau) &= z_e(t) \\ &- \int_{t-\tau}^{t}(I_{N-1} \otimes A)z_e(\theta) - (\Lambda_e \otimes BK)z_e(\theta-\tau)d\theta. \end{aligned} \tag{31}$$

Then, Eq. (30) can be expressed as

$$\begin{aligned} \dot{z}_e(t) &= ((I_{N-1} \otimes A) - (\Lambda_e \otimes BK))z_e(t) \\ &+ (\Lambda_e \otimes BK)\int_{t-\tau}^{t}(I_{N-1} \otimes A)z_e(\theta) - (\Lambda_e \otimes BK)z_e(\theta-\tau)d\theta. \end{aligned} \tag{32}$$

Let us define a candidate of Lyapunov-like function as

$$\begin{aligned} V_2(z_e,t) = {} & z_e^T(t)(I_{N-1} \otimes P)z_ex(t) \\ & + r_1\int_{t-\tau}^{t}\int_{\theta}^{t}\|(I_{N-1} \otimes A)z_e(s)\|^2 ds d\theta \\ & + r_2\int_{t-\tau}^{t}\int_{\theta}^{t}\|(\Lambda_e \otimes BK)z_e(s)\|^2 ds d\theta \end{aligned} \tag{33}$$

where P is positive definite matrix. Then, the derivative V_2 along the trajectories as

$$\begin{aligned} \dot{V}_2(z_e,t) = {} & z_e^T\left(I_{N-1} \otimes (A^TP + PA)\right)z_e \\ & - z_e^T\left(\Lambda_e^T \otimes K^TB^TP\right)z_e + z_e^T(\Lambda_e \otimes PBK)z_e \\ & + 2z_e^T(I_{N-1} \otimes P)(\Lambda_e \otimes BK) \\ & \times \int_{t-\tau}^{t}(I_{N-1} \otimes A)z_e(\theta) - (\Lambda_e \otimes BK)z_e(t-\tau)d\theta \\ & + r_1\tau\|(I_{N-1} \otimes A)z_e(t)\|^2 + r_2\tau\|((\Lambda_e \otimes BK)z_e(\theta)\|^2. \end{aligned} \tag{34}$$

By Lyapunov krasovskii theorem, if there is positive definite matrix P such tha

$$\begin{aligned} & \left(I_{N-1} \otimes (A^TP + PA)\right) - \Lambda_e^T \otimes K^TB^TP - \Lambda_e \otimes PBK \\ & + \tau\left(\frac{1}{r_1} + \frac{1}{r_2}\right)(I_{N-1} \otimes P)(\Lambda_e \otimes BK)(\Lambda_e^T \otimes K^TB^T)(I_{N-1} \otimes P) \\ & + r_1\tau(I_{N-1} \otimes A^TA) + r_2\tau(\Lambda_e^T\Lambda_e \otimes K^TB^TBK) < 0, \end{aligned} \tag{35}$$

then MAS (27) satisfy consensus.

Next, assumed $A - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}$ is stable. Then the following inequality is derived by (35).

$$\begin{aligned} & A^TP + PA - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}(K^TB^TP + PBK) \\ & + \tau\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\lambda_{N-1}(\Lambda_e\Lambda_e^T)PBKK^TB^TP \\ & + r_1\tau A^TA + r_2\tau\lambda_{N-1}(\Lambda_e^T\Lambda_e)K^TB^TBK < 0 \end{aligned} \tag{36}$$

It is clear that (36) is satisfied then (35) is also satisfied. Hence, using schur complement, Theorem 2 is derived by (36). □

(36) is more conservative than (35). However, computational load of (36) according to increase of agents is lower than (35).

6. OBSERVER-BASED MULTI-AGENT SYSTEM

Let consider output feedback of consensus problem for reducing of communication traffic. Two control law in Section Ⅱ and Ⅲ are state feedback of consensus problem. In these case, an agent sends state and receives other agent state. Assumed MAS which consists N agents where an agent has n states and the network structure is complete graph, then an agent gets $n(N-1)$ information and send $n(N-1)$ information. If an agent sends m outputs where $m<n$ case, then an agent get $m(N-1)$ information and send $m(N-1)$ information. Hence using output information of agents, the communication traffic is reduced. Therefore, let consider ith information of output y_i and ith controlled information Z_i as

$$y_i = Cx_i,\ \ Z_i = \sum_{j\in\mathcal{N}_i}^{N}(y_i - y_j) = \sum_{j\in\mathcal{N}_i}^{N} C(x_i - x_j). \quad (37)$$

where $C \in \mathbb{R}^{m\times n}$. Then, the MAS can be expressed as

$$\begin{aligned} \dot{x} &= (I_N\otimes A)x + (I_N\otimes B)u \\ y &= (I_N\otimes C)x \\ Z &= (L\otimes I_n)y = (L\otimes C)x, \end{aligned} \quad (38)$$

where $y = [y_1^T\ y_2^T\ \cdots y_N^T]^T, Z = [Z_1^T\ Z_2^T \cdots Z_N^T]^T$ and this MAS satisfy under following assumption.

[Assumption 3] $(A,\ C)$ is observable.

ith observer-based control law is proposed as

$$\begin{aligned} \dot{\hat{x}}_{Fi} &= (A-HC)\hat{x}_{Fi} + H\sum_{j\in\mathcal{N}_i}^{N}(y_i - y_j) + Bu_i \\ u_i &= -K\hat{x}_{Fi}, \end{aligned} \quad (39)$$

where $\hat{x}_{Fi}$ is estimation vector that estimate $\sum_{j\in\mathcal{N}_i}^{N}(x_i - x_j)$ and $H\in\mathbb{R}^{n\times m}$ is observer gain. Then, the observer-based control law of MAS can be expressed as

$$\begin{aligned} \dot{\hat{x}}_F &= (I_N\otimes A - HC)\hat{x}_F + (L\otimes HC)x + (I_N\otimes B)u \\ u &= -(I_N\otimes K)\hat{x}_F, \end{aligned} \quad (40)$$

where $\hat{x}_F = [x_{F1}^T\ x_{F2}^T\ \cdots\ x_{FN}^T]^T$ is estimation vector that estimate $(L\otimes I_n)x$. Therefore, observer-based MAS is represented as

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}}_F \end{bmatrix} = \begin{bmatrix} I_N\otimes A & I_N\otimes BK \\ L\otimes HC & I_N\otimes(A-HC-BK) \end{bmatrix}\begin{bmatrix} x \\ \hat{x}_F \end{bmatrix}. \quad (41)$$

Let us consider stability of MAS (41).

[Theorem 3] MAS (38) satisfy [**Assumption 1-3**]. Then MAS (41) achieve consensus if $A - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}BK$ and $A-HC$ are stability.

Proof: Let $x_F = (L\otimes I_n)x$ and $u_F = (L\otimes I_m)u$, then the MAS (38) can be expressed as

$$\begin{aligned} \dot{x}_F &= (I_N\otimes A)x_F + (I_N\otimes B)u_F \\ Z &= (I_N\otimes C)x_F. \end{aligned} \quad (42)$$

The system (42) can be designed observers. Assumed the MAS (38) satisfy that $A-HC$ is stable. Hence, $\hat{x}_F$ achieve

$$\lim_{t\to\infty}\hat{x}_F = x_F = (L\otimes I_n)x. \quad (43)$$

Therefore, it is clear that MAS (41) achieves consensus by theorem 1. □

[Remark 1] It is defined that $x_F = (L\otimes I_n)x(t-\tau)$. Then a MAS with communication delay satisfy consensus. Then the control law of observer based MVS can be expressed as

$$\begin{aligned} \dot{\hat{x}}_F &= (I_N\otimes A - HC)\hat{x}_F + (L\otimes HC)x(t-\tau) + (I_N\otimes B)u \\ u &= -(I_N\otimes K)\hat{x}_F. \end{aligned} \quad (44)$$

Then, observer-based MAS with communication delay can be represented as

$$\begin{bmatrix} \dot{x}(t-\tau) \\ \dot{\hat{x}}_F \end{bmatrix} = \begin{bmatrix} I_N\otimes A & I_N\otimes BK \\ L\otimes HC & I_N\otimes(A-HC-BK) \end{bmatrix} \times \begin{bmatrix} x(t-\tau) \\ \hat{x}_F \end{bmatrix}. \quad (45)$$

[Theorem 4] MAS (38) satisfy [**Assumption 1-3**]. Then MAS (45) achieve consensus if $A - \lambda_1(\Lambda_e\Lambda_e^T)^{1/2}BK$, there is positive definite matrix P such that (28) and $A-HC$ are stability.

Proof: It is obvious that MAS (45) achieves consensus by Theorem 2 and 3. □

7. EXAMPLE AND ITS EXPERIMENTS

7.1 Multi-vehicle system

We consider a four-vehicle formation problem and a two-wheel vehicle is treated as an agent as in Fig. 1 (left below).

It is well-known that two-wheel vehicle has nonholonomic property. Assume that N vehicles can be represented same dynamics and there are no friction, then ith vehicle model can be expressed as

$$\begin{bmatrix} \dot{x}_{ri} \\ \dot{y}_{ri} \\ \dot{\theta}_{ri} \end{bmatrix} = \begin{bmatrix} \cos\theta_{ri} & 0 \\ \sin\theta_{ri} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \quad (46)$$

where (x_{ri}, y_{ri}) is center of gravity of ith vehicle, θ_{ri} its orientation, control input are its velocity v_i and its angular velocity ω_i.

Based on an idea of Virtual Structure, we consider Virtual Vehicle (VV) at each vehicles as Fig. 1 (upper right). If positional relationship between a real vehicle and a VV is given as Fig. 1, then position of center of gravity and angle of VV are expressed as

$$\begin{bmatrix} x_{vi} \\ y_{vi} \\ \theta_{vi} \end{bmatrix} = \begin{bmatrix} x_i + x_{di}\cos\theta_{ri} - y_{di}\sin\theta_{ri} \\ y_i + x_{di}\sin\theta_{ri} + y_{di}\cos\theta_{ri} \\ \theta_{ri} \end{bmatrix}. \quad (47)$$

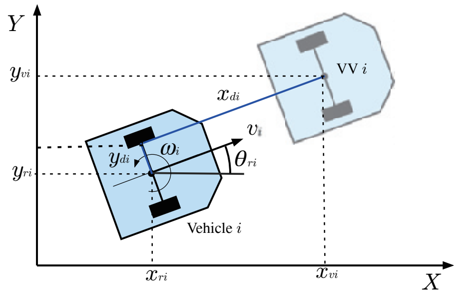


Fig. 1 ith Real Vehicle and Virtual Vehicle

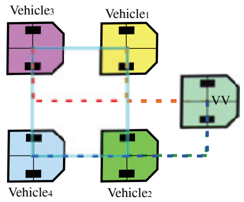


Fig. 2 Target $r_{ri} = 0$

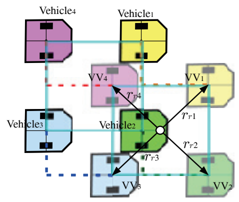


Fig. 3 Target $r_{ri} \neq 0$

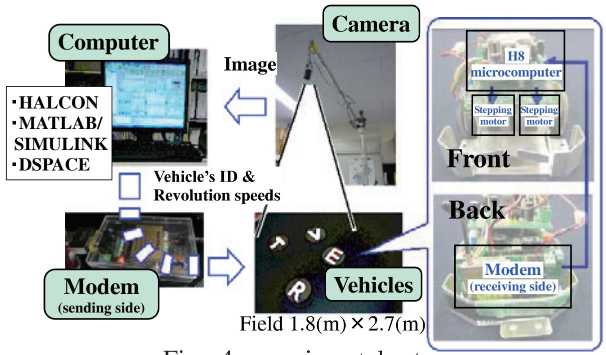
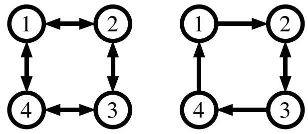


Fig. 4 experimental setup



(a) undirected graph (b) digraph

Fig. 5 Network structure

Derivatative of (47) gives

$$\begin{bmatrix} \dot{x}_{vi} \\ \dot{y}_{vi} \\ \dot{\theta}_{vi} \end{bmatrix} = \begin{bmatrix} B_{vi} \\ B_{\theta} \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{48}$$

$$B_{vi} = \begin{bmatrix} \cos\theta_{ri} & -x_{di}\sin\theta_{ri} - y_{di}\cos\theta_{ri} \\ \sin\theta_{ri} & x_{di}\cos\theta_{ri} - y_{di}\sin\theta_{ri} \end{bmatrix}$$

$$B_{\theta} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

If $x_{di} \neq 0$, B_{vi} is regular matrix. In this paper, consider formation problem at this VS system (48). Let center of gravity of VV $r_{vi} = (x_{vi},\ y_{vi})$, VV's velocity v_{vi} and

$$\dot{v}_{vi} = u_{vi},\ \begin{bmatrix} v_i & \omega_i \end{bmatrix}^T = B_{vi}^{-1} v_{vi}. \tag{49}$$

where u_{vi} is new input of VV. Then dynamics of center gravity of VV can be presented as

$$\dot{v}_{vi} = u_{vi},\ \dot{r}_{vi} = v_{vi}. \tag{50}$$

This dynamics (50) is a linear system.

For VV's uniform motion, we define u_{vi} as

$$u_{vi} = -k_{vr}(v_{vi} - v^*) + u_i, \tag{51}$$

where $k_{vr} > 0$ is design parameter, $v^* \neq 0$ is reference velocity and u_i is new input for consensus. Let $v_{ei} = v_{vi} - v^*, \tilde{r}_{vi} = r_{vi} - r_{ri}$, where r_{ri} is reference relative deviation for VV's system achieve any formation. Then the system (50) can be expressed as

$$\frac{d}{dt}\begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{vr} & 0 \end{bmatrix} \otimes I_2 \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_2 u_i$$
$$y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_2 \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix}. \tag{52}$$

If VVs achieve

$$\lim_{t\to\infty} \tilde{r}_{vi} = \tilde{r}_{vj},\ \lim_{t\to\infty} v_{ei} = v_{ej}\ , \tag{53}$$

then real vehicles make formation as Figs.2,3.

Because of the experimental setup can not be detected vehicles's velocities, we use observer based controller for achieve uniform motion of ith vehicle. Let be $x_i = [\tilde{r}_{vi}^T\ v_{ei}^T\ \hat{\tilde{r}}_{vi}^T \hat{v}_{ei}^T]^T$, then ith vehicle system can be expressed by Eq. (4), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -k_{vr} & 0 \\ h_1 & 0 & -h_1 & 1 \\ h_2 & 0 & -h_2 - k_{vr} & 0 \end{bmatrix} \otimes I_2,\ B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \otimes I_2,$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \otimes I_2, \tag{54}$$

$\hat{\tilde{r}}_{vi}$ is estimate vector of $\tilde{r}_{vi}$, $\hat{v}_{ei}$ is estimate vector of v_{ei}, $h_1,\ h_2 > 0$ are design parameters. In observer combined system case, (A, B) is not controllable but stabilizable if (A_l, B_l) is controllable where $A_l,\ B_l$ is system matrix before observer combined system. In this case, assumption 2-ii) can be replaced controllable to stabilizable. And also (A, C) is observable.

7.2 Experimental Evaluation

Fig.4 shows our experimental setup. This experiment shows the effectiveness of observer-based MAS with communication delay (Section VI). Network structures are considered two pattern: undirected graph and digraph as shown in Fig. 5. We consider virtual network and communication delay because it can not use inter-vehicle communication in our experimental setup. And inter-vehicle communication delay τ_{ij} to ith agent from jth agent is defined as

$$\tau_{ij} = 0.1\|r_{vi} - r_{vj}\| + 0.1. \tag{55}$$

For satisfy this conditions, maximum communication delay is set $\tau = 0.3$[s]. Let make set to parameters satisfy Theorem 4, these parameters are shown in Table I

Figs. 6-11 show our experimental result. "×"and "○" are first position and final position of real vehicles in Figs. 6, 9. In undirected graph case, real vehicles achieve formation. It is clear in Figs. 6, 7. In Fig. 7, VV achieve consensus. And also Fig. 8 shows calculated velocities and gives that VVs achieve reference velocity. In digraph case, Fig. 9 shows Trajectories, VV do not completely make formation in the experiment field. Fig. 10 shows

Table 1 Parameters

VV's parameter (x_{di}, y_{di})	(0.05,0) [m]
Reference VV's velocity v^*	$[0.05\ 0]^T$ [m/s]
Reference relative deviation r_{r1}	$[0.3\ 0.3]^T$ [m]
Reference relative deviation r_{r2}	$[0.3\ 0]^T$ [m]
Reference relative deviation r_{r3}	$[0\ 0.3]^T$ [m]
Reference relative deviation r_{r4}	$[0\ 0]^T$ [m]
Design parameter k_{vr}	0.23
Design parameter $[h_1 h_2]$	[2.0 2.0]
Controller gain K	$[0.0012\ 0.63\ 0.011\ \ -0.012] \otimes I_2$
Observer gain H	$[3.25\ 0.96\ 1.51\ 1.10]^T \otimes I_2$

consensus error $L\hat{\tilde{r}}(t-\tau)$ and gives VV toward consensus because $L\hat{\tilde{r}}(t-\tau)$ converge equilibrium. And also Fig. 11 shows calculated velocities and gives that VVs achieve reference velocity.

8. CONCLUSIONS AND FUTURE WORKS

In this paper, a new control law of observer-based MAS is proposed for consensus problem. An agent has MIMO system, stability problem of MAS is extended from undirected graph to digraph. And also condition of stability was derived using lyapunov stability theorem. We consider MAS with communication delay that present a problem in MAS. Consensus problem is extended from state feedback to output feedback case. The proposed algorithm was applied to Multi-vehicle formation control problem to demonstrate the effectiveness of our proposed strategies.

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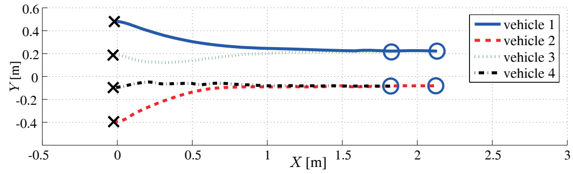

Fig. 6 Trajectories of four vehicles (Undirected graph)

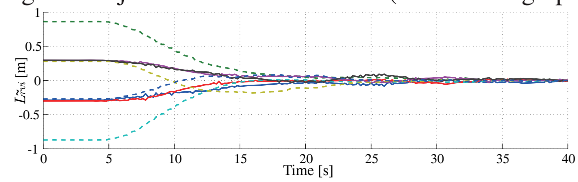

Fig. 7 Consensus error $L\tilde{r}_{vi}$ (Undirected graph)

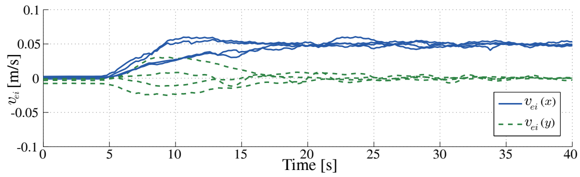

Fig. 8 Velocities of VVs v_{ei} (Undirected graph)

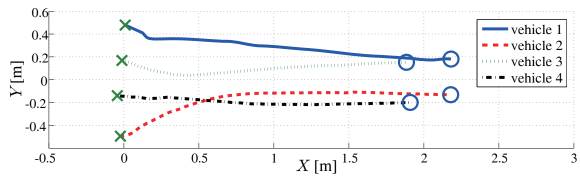

Fig. 9 Trajectories of four vehicles (Digraph)

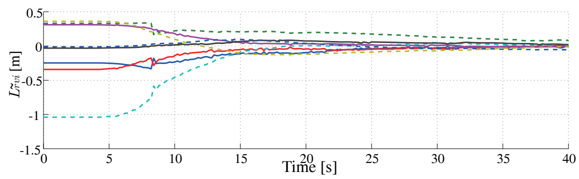

Fig. 10 Consensus error $L\tilde{r}_v(t-\tau)$ (Digraph)

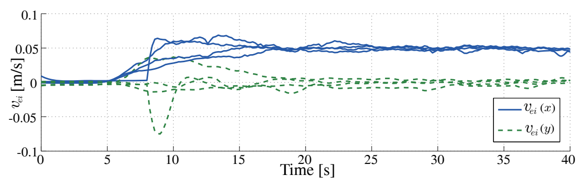

Fig. 11 Velocities of VVs v_{ei} (Digraph)

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