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Automatic Optimum Order Assignment in IIR Adaptive Filters

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Abstract

When we use an IIR adaptive filter to identify a pole-zero unknown system, specially for a time varying unknown system, it is important to automatically estimate the order assignment on the numerator and the denominator. In our previous works, an algorithm for automatic order assignment in a separate form IIR adaptive filter has been proposed. The optimum order assignment is determined by minimizing the equation error of the separate form IIR adaptive filter. The equation error sometimes falls in local minima, specially for a colored input signal. The local minima interfere with the optimum order assignment. In this paper, an improved algorithm is proposed. A variable order increment ξ is used to avoid the effect of local minima. The efficiency of the improved algorithm is confirmed through computer simulations.

1 Introduction

For system identification problems, such as noise and echo cancellation, FIR adaptive filters are mainly used for their simple adaptation and numerical stability. When the unknown system is a high-Q resonant system, having a very long impulse response, IIR adaptive filters are more efficient for reduction in the order of a transfer function.

In the actual applications of an IIR adaptive filter, specially for a time varying unknown system, it is very important to automatically estimate the order assignment on the numerator and the denominator. When the total number of orders is limited by hardware, performance of the IIR adaptive filter is sensitive to the order assignment in numerator and denominator.

Some methods have been proposed to estimate the optimum order for time-invariant IIR filters. One of them is Akaike's Information Criterion (AIC) [1], which uses the final prediction error to calculate optimum order. Other methods, also have been proposed such as in speech processing problems [2][3][4]. All of this kind of methods can not be directly used in a online manner for a time-varying IIR filter.

In [5] [6], we have proposed an algorithm for automatically assigning the optimum order to the numerator and denominator of a separate form IIR adaptive filter by minimizing the equation error. However, the equation error sometimes falls in local minima, specially for a colored input signal. The local minima interfere with the optimum order assignment.

In order to overcome this shortage, we improved the proposed algorithm in this paper. In the improved algorithm, a variable order increment ξ is used to avoid the order assignments corresponding to the local minima of equation error. The efficiency of the improved algorithm is confirmed through computer simulations.

2 Separate form IIR adaptive filter

2.1 Structure

The structure of a separate IIR adaptive filter is shown in Fig.1. $X(z)$, $D(z)$ and $H(z)$ denote an input signal, a desired response and a transfer function of an unknown system, respectively. z^{-1} denotes one sample of delay. $Y_a(z)$ and $Y_b(z)$ are the outputs of FIR adaptive filters AF_a and AF_b. $Y(z)$ is the output of separate form IIR adaptive filter. An equation error is defined as the difference between $D(z)$ and $Y(z)$. The output error $E_0(z)$ is obtained by passing $E(z)$ through a filter whose transfer function is the dominator of the IIR adaptive filter.

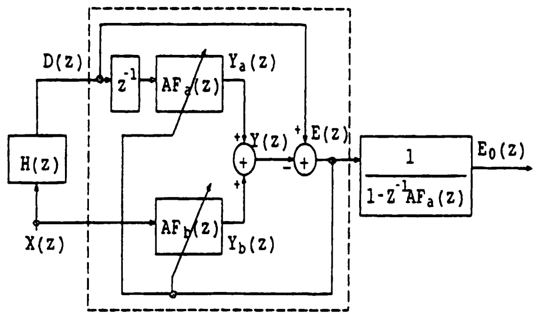

Figure 1: Separate realization of IIR adaptive filter

2.2 Equation Error Evaluation

From Fig.1, we obtain

$$D(z) = H(z)X(z) \quad (1a)$$
$$Y_a(z) = AF_a(z)z^{-1}D(z) \quad (1b)$$
$$Y_b(z) = AF_b(z)X(z) \quad (1c)$$
$$Y(z) = Y_a(z) + Y_b(z) \quad (1d)$$
$$E(z) = D(z) - Y(z) \quad (1e)$$
$$E_o(z) = \frac{E(z)}{1 - z^{-1}AFa(z)} \quad (1f)$$

By eliminating $D(z)$, $Y_a(z)$ and $Y_b(z)$, $E(z)$ can be expressed as

$$E(z) = [H(z) - H(z)z^{-1}AF_a(z) - AF_b(z)]X(z) \quad (2)$$

The ideal solution can be obtained by setting the inside of the bracket to be zero. From this condition, the following relation is obtained.

$$H(z) = \frac{AF_b(z)}{1 - z^{-1}AF_a(z)} \quad (3)$$

$E(z)$ equals zero in this case. On the other hand, if $E(z)$ not equal zero, $H(z)$ should include an error term $\Delta H(z)$,

$$H(z) + \Delta H(z) = \frac{AF_b(z) + E(z)/X(z)}{1 - z^{-1}AF_a(z)} \quad (4)$$

$\Delta H(z)$ can be denoted by

$$\Delta H(z) = \frac{E(z)}{X(z)\{1 - z^{-1}AFa(z)\}} = \frac{E_o(z)}{X(z)} \quad (5)$$

Even through $E(z)$ can not directly represent $\Delta H(z)$, but $\Delta H(z)$ will decrease along with the decrease of $E(z)$. Unlike the output error $E_0(z)$, $E(z)$ will not diverge during the adaptation. So that, it can be expect to find the optimum order assignment for the IIR adaptive filter by minimizing $E(z)$. The error evaluation is given by a mean squared equation error(MSEE)

$$MSEE = \frac{1}{K} \sum_{n=n_0-K+1}^{n_0} e^2(n) \quad (6)$$

where $e(i)$ denotes the equation error in time domain. It is assumed that at $n = n_0$ the adaptation already converges. K is the number of samples taken into account.

3 Automatic Order Assignment Algorithm

A block diagram for the proposed automatic order assignment is shown in Fig.2. AF_{main} and AF_{aux} denote main and auxiliary IIR adaptive filters, which have a separate form shown in Fig.1. $x(n)$, H and $d(n)$ denote an input signal, a unknown system and a desired response. e_{main} and e_{aux} are the equation errors of AF_{main} and AF_{aux}, respectively.

AF_{main} and AF_{aux} are adapted simultaneously using their respective equation errors. Both AF_{main} and AF_{aux} start adaptation with an initial order assignment guess. Initial order assignments of the numerator(N) and denominator(D) in AF_{main} and AF_{aux} are set to be $N_{main}(0) \approx D_{main}(0) \approx T_t/2$, $N_{aux}(0) = N_{main}(0) + \xi$, $D_{aux}(0) = D_{main}(0) - \xi$, Where T_t is the total number of taps which is limited by the hardware, and ξ is an order increment. For a given block of samples, the order assignment for both AF_{main} and AF_{aux} are kept constant. After convergence of both AF_{main} and AF_{aux}, MSEE of AF_{main} and AF_{aux}, denoted by $MSEE_{main}$ and $MSEE_{aux}$, are calculated.

If $MSEE_{main} > MSEE_{aux}$, then the order assignment of AF_{aux} is transferred to AF_{main}, and the order assignment of AF_{aux} is changed in the same direction, that is, N_{aux} increases by ξ, D_{aux} decreases by ξ. If $MSEE_{main} < MSEE_{aux}$, the order assignment of AF_{main} is held, and that of AF_{aux} is changed in the opposite direction, that is, N_{aux} decreases by ξ, D_{aux} increases by ξ. Therefore, the order assignment with smaller MSEE always held in AF_{main}. AF_{main}, AF_{aux} always adapt to track the time-varying unknown system.

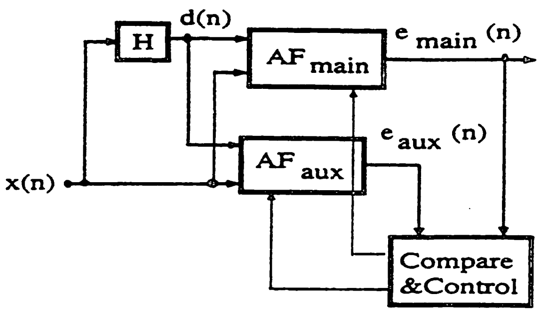

Figure 2: Block diagram of automatic order assignment

4 Improved Automatic Order Assignment Algorithm

In section 3, if within M intervals (n_0 iterations included in one interval), N_{main} and D_{main} keep unchanged and $MSEE_{main} < MSEE_{aux}$ is satisfied, the order assignment in AF_{main} is considered the optimal order assignment for present unknown system. However, sometimes $MSEE_{main}$ could be a local minimum. So that, the order assignment in AF_{main} is not the optimal order assignment for the unknown system in such case.

In order to avoid the effect of the local minima, instead of a fixed order increment $\xi = 1$ in [6], a variable order increment ξ is introduced. The improved automatic order assignment algorithm is described below.

1. Initial order assignment of numerator and denominator in AF_{main} and AF_{aux}. $N_{main}(0) \approx D_{main}(0) \approx \frac{T_t}{2}$, $N_{aux}(0) = N_{main}(0) + \xi$, $D_{aux}(0) = D_{main}(0) - \xi$. Index I=0.

2. Adapt AF_{main} and AF_{aux} simultaneously in the interval of n_0 iterations, n_0 is chosen so that after $(n_0 - K)$ iterations both AF_{main} and AF_{aux} almost converge.

3. Calculate $MSEE_{main}(k)$ and $MSEE_{aux}(k)$ defined by

$$MSEE_x(k) = \frac{1}{K} \sum_{n=kn_0-K+1}^{kn_0} e^2{}_x(n) \qquad (7)$$

where x=main or aux. If $MSEE_{main}(k) > MSEE_{aux}(k)$, then go to *4*, else I=I+1, go to *5*.

4. $N_{main}(k+1) = N_{aux}(k)$, $D_{main}(k+1) = D_{aux}(k)$, $\mathbf{W}_{main}(k+1) = \mathbf{W}^*_{aux}(k)$, $N_{aux}(k+1) = N_{aux}(k) + \xi$, $D_{aux}(k+1) = D_{aux}(k) - \xi$, $\mathbf{W}_{aux}(k+1) = \mathbf{W}^*_{aux}(k)$, go to *2*.

5. If $I < M$, go to *6*.
If $I = M$ and $|\xi| < \xi_{max}$ and $sign(\xi) = "+"$, then $\xi = -(\xi + 1)$, go to *6*.
If $I = M$ and $|\xi| < \xi_{max}$ and $sign(\xi) = "-"$, then $\xi = -(\xi - 1)$, go to *6*.
If $I = M$ and $|\xi| = \xi_{max}$, then $\xi = 1$, go to *6*.

6. $N_{main}(k+1) = N_{main}(k)$, $D_{main}(k+1) = D_{main}(k)$, $\mathbf{W}_{main}(k+1) = \mathbf{W}_{main}(k)$, $N_{aux}(k+1) = N_{main}(k) - \xi$, $D_{aux}(k+1) = D_{main}(k) + \xi$, $\mathbf{W}_{aux}(k+1) = \mathbf{W}^*_{main}(k)$, go to *2*.

W denotes the coefficient vector. Operator $*$ means removing ξ highest order terms, or adding ξ zero coefficients in the highest order terms, and keeping the remains of coefficients unchange.

5 Simulation and Discussion

Two unknown systems are used in the simulation. The numbers of coefficients of the denominator and numerator are 3 and 10 for System 1, and 5 and 8 for System 2. The location of zeros and poles of System 1 and 2 are shown in Table 1 and Table 2. Their amplitude characteristics are shown in Fig. 3 and 4.

Table 1: Location of zeros and poles of unknown system 1

Zeros		Poles	
r_z	θ_z	r_p	θ_p
0.30	0°	0.7	±142°
0.88	±20°		
0.88	±57°		
0.87	±96°		
0.85	±120°		

Table 2: Location of zeros and poles of unknown system 2

Zeros		Poles	
r_z	θ_z	r_p	θ_p
0.71	0°	0.90	±106°
0.93	±32°	0.90	±138°
0.81	±53°		
0.78	±86°		

The total number of coefficients is fixed to 13. The parameters used in the simulation are n_0 =

2000, $K = 400$, $M = 4$, $\xi = 1$, $\xi_{max} = 4$. The RLS algorithm was used, where $\lambda = 0.9$, $\delta = 0.1$ [7]. The input signal is a colored signal. It is obtained by passing a white noise through a low-pass filter. The location of the zeros and poles of the low-pass filter is shown in Table 3.

Table 3: Location of zeros and poles of the low-pass filter

Zeros		Poles	
r_z	θ_z	r_p	θ_p
1.0	180°	0.7	±30°

The relation between MSEE and the number of coefficients of the denominator(D_{main}) of System 1 is shown in Fig.5. For convenience, only D_{main} is shown here. There is a local minimum when D_{main} is 6.

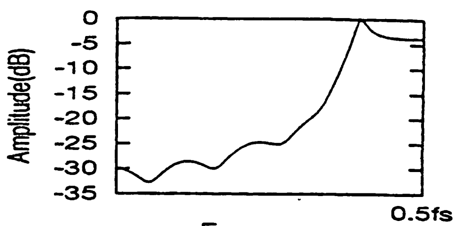

Figure 3: Amplitude characteristic of System 1

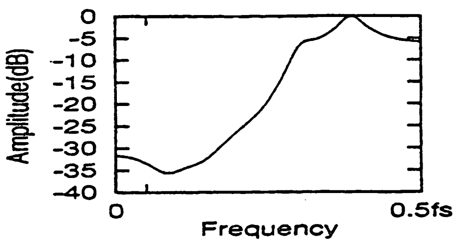

Figure 4: Amplitude characteristic of System 2

Table 4: Automatic optimal order assignment process

I	Iterations	N_{main}/D_{main}	$MSEE_{main}(dB)$	$IE_{main}(dB)$	N_{aux}/D_{aux}	$MSEE_{aux}(dB)$	$IE_{aux}(dB)$
1	2000	6/7	-30.20	-1.56	7/6	-39.17	-7.45
2	4000	7/6	-40.36	-15.49	8/5	-38.25	-15.97
3	6000	7/6	-39.69	-10.56	6/7	-29.72	-10.43
4	8000	7/6	-39.84	-15.43	8/5	-37.93	-15.81
5	10000	7/6	-40.22	-16.31	6/7	-29.66	-10.01
6	12000	7/6	-39.22	-14.31	9/4	-48.65	-23.07
7	14000	9/4	-52.64	-23.65	10/3	-98.62	-72.97
8	16000	10/3	-99.30	-83.42	11/2	-17.78	8.60
9	18000	10/3	-99.03	-72.19	9/4	-50.40	-25.22
10	20000	10/3	-99.17	-78.10	11/2	-18.01	5.04
11	22000	10/3	-99.20	-78.12	9/4	-48.97	-23.31
12	24000	10/3	-98.57	-78.36	12/1	-16.62	-2.14
13	26000	10/3	-100.53	-83.34	8/5	-37.42	-15.09
14	28000	10/3	-23.92	-8.64	1/12	-22.86	54.27
15	30000	10/3	-26.21	-9.08	7/6	-42.75	-21.81
16	32000	7/6	-53.01	-26.73	6/7	-39.24	-15.89
17	34000	7/6	-54.21	-31.12	8/5	-93.54	-86.72
18	36000	8/5	-93.51	-75.25	9/4	-25.99	0.77
19	38000	8/5	-95.05	-83.37	7/6	-42.28	-29.33
20	40000	8/5	-95.15	-75.78	9/4	-25.22	44.19
21	42000	8/5	-95.10	-75.72	7/6	-42.14	-29.27
22	44000	8/5	-94.01	-77.96	10/3	-24.70	-6.09
23	46000	8/5	-94.60	-81.60	6/7	-39.65	-21.70
24	48000	8/5	-95.46	-70.36	11/2	-20.45	2.84
25	25000	8/5	-93.94	-74.45	5/8	-40.73	-18.99
26	52000	8/5	-94.75	-74.43	12/1	-18.33	-2.30
27	54000	8/5	-93.59	-80.70	4/9	-34.98	-9.66
28	56000	8/5	-94.03	-85.57	9/4	-25.91	-1.20

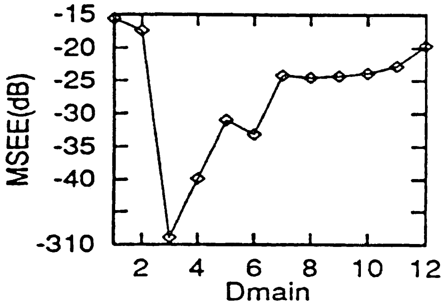

Figure 5: Relation between MSEE and D_{main} in System 1

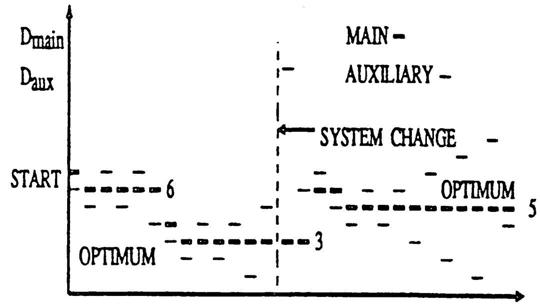

Figure 6: Process of automatic optimal order assignment

Figure 6 and Table 4 show the process of automatic optimal order assignment. In Fig.6 only D_{main} is shown instead of N_{main}/D_{main} for convenience. From 4000 iterations, D_{main} falls in a local minimum 6. The number of taps of AF_{aux} (N_{aux}/D_{aux}) vibrates between 8/5 and 6/7. After 4 times vibration, ξ increased to 2, D_{main} escapes from the local minimum 6. After 16000 iterations, the optimum $D_{main} = 3$ was found.

From 26001 iterations, System 1 changed to System 2. 8000 iterations needed to tracking this system change. At $I = 19$, the optimum $D_{main} = 5$ corresponding to System 2 was found. Then the number of taps of AF_{aux} (N_{aux}/D_{aux}) vibrates around $N_{mine}/D_{main} = 8/5$ by ± 1 in N_{aux} and D_{aux}. After 4 times vibration, $|\xi|$ increased to 2, N_{aux}/D_{aux} vibrates around $N_{mine}/D_{main} = 8/5$ by ± 2 in N_{aux} and D_{aux}. This process continues until $|\xi| = \xi_{max} = 4$. $MSEE_{main}$ is always smaller than $MSEE_{aux}$ during this period, it means that no local minimum happens. ξ was reset to 1, the process continued to track the unknown system.

IE_{main} and IE_{aux} in Table 4 denote the system identification errors. They are the differences between the impulse responses of the unknown system and that of the IIR adaptive filters AF_{main} and AF_{aux}.

6 Conclusions

An improved algorithm for automatic order assignment in a separate form IIR adaptive filter has been proposed. By using a variable order increment ξ, the order assignments corresponding to the local minima of the equation error can be avoided. When the unknown system changes, the improved algorithm can automatically track the change of the optimum order assignment. The efficiency has been confirmed through computer simulations.

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