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A SIMULTANEOUS FREQUENCY AND TIME DOMAIN APPROXIMATION METHOD FOR DISCRETE-TIME FILTERS

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ABSTRACT

A simultaneous frequency and time domain approximation method for discrete-time filters is proposed in this paper. Filter coefficients are divided into two subsets X_1 and X_2 which are used to optimize a time response and a frequency response, respectively. The optimum solution for X_1 is always guaranteed through linear equations during the frequency response optimization. The frequency response is optimized by iterative methods. The proposed method does not impose any constraints on pole-zero locations and on filter responses. Hence, sufficient reductions in filter order and in final hardware size can be achieved.

INTRODUCTION

Simultaneous frequency and time response approximation methods are inherently necessary and become very important design techniques for filters employed in image signal transmitting and processing systems.

Existing approaches to the simultaneous approximation can be summarized as follows:
(1) A subset of filter coefficients which approximates a time response is independently determined from the rest of filter coefficients. A frequency response is optimized through iterative methods [1].
(2) Specific transfer functions are employed, which can optimize one filter response without the other response distortions. These transfer functions include all-pass functions and finite impulse response (FIR) filters [2].
(3) A subset of filter coefficients which approximates stopband attenuation is uniquely solved based on a closed form using the rest of filter coefficients employed for time response optimization [3]. Attainable filter responses by this approach are restricted to lowpass filters and an equal-ripple stopband attenuation.

The proposed method, in this paper, is based on the third approach, and extend the attainable filter responses by employing linear equations instead of the closed forms. As is well known, there exist linear relations between the filter coefficients and an impulse response in discrete-time filters. Many approximation techniques in a time domain based on the linear relations have been proposed [4] - [7]. They are, however, directed toward only time response approximation.

TIME RESPONSE OPTIMIZATION BY LINEAR EQUATIONS

Desired time responses are mainly classified into the following two categories.
(1) Desired time response values are given.
(2) Desired time response properties are given.
The first category includes, for instance, the Nyquist waveform zero crossing at equally spaced sampling points. Symmetrical impulse responses and minimum moment impulse responses are contained in the second category.

The transfer function $H(z)$ can be generally expressed as

$$H(z) = \frac{P(z)}{Q(z)}G(z)\ , \quad z = e^{j\omega T} \tag{1}$$

where $P(z)$ and $Q(z)$ are polynomials of z^{-1} and $G(z)$ is a rational function of z^{-1}. $P(z)$ and $Q(z)$ are used to optimize a time response, and a frequency response is approximated by $G(z)$.

Time Response Values Specified

Letting d_n be a desired time response, time response approximation is carried out based on

$$\frac{P(z)}{Q(z)}G(z) = \sum_{n=0}^{N_d-1} d_n z^{-n}, \tag{2}$$

where N_d is a number of specified impulse response samples. $G(z)$ can be assumed to be a fixed function as will be discussed in an approximation algorithm. From Eq. (2)

$$P(z)G(z) = \Big(\sum_{n=0}^{N_d-1} d_n z^{-n}\Big)Q(z) \tag{3}$$

Letting

$$P(z) = \sum_{n=0}^{N_p-1} p_n z^{-n} \tag{4a}$$

$$Q(z) = \sum_{n=0}^{N_q-1} q_n z^{-n} \tag{4b}$$

$$G(z) = \sum_{n=0}^{\infty} g_n z^{-n} \tag{4c}$$

Eq. (3) can be rewritten as

$$\sum_{m=0}^{n_1} p_m g_{n-m} = \sum_{m=0}^{n_2} q_m d_{n-m}, \quad 0 \leq n \leq N_d-1$$
$$n_1 = \min\{N_p-1,\ n\}$$
$$n_2 = \min\{N_q-1,\ n\}. \tag{5}$$

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When N_d is equal to or less than $N_p + N_q - 1$, exact interpolation can be obtained by solving linear equations Eq. (5). On the other hand, if N_d is larger than $N_p + N_q - 1$, a least square approximation technique is required to optimize a time response through linear equations. A squared error is evaluated by

$$E_1 = \sum_{n=0}^{N_d-1} \left(\sum_{m=0}^{n_1} p_m g_{n-m} - \sum_{m=0}^{n_2} q_m d_{n-m} \right)^2 \quad (6)$$

The optimum p_m and q_m in the least mean square sense are obtained by solving

$$\frac{\partial E_1}{\partial p_m} = 0, \; m = 0, 1, \ldots, N_p-1 \quad (7a)$$

$$\frac{\partial E_1}{\partial q_m} = 0, \; m = 0, 1, \ldots, N_q-1 \quad (7b)$$

When P(z) is only employed to optimize a time response, the error evaluation by Eq. (6) is directly related to the impulse response error. However, the employment of the Q(z) coefficients for time response approximation does not assure direct evaluation of the impulse response error. Letting the impulse response error be Δh_n, Eq. (6) can be rewritten as

$$\sum_{m=0}^{n_1} p_m g_{n-m} - \sum_{m=0}^{n_2} q_m (d_{n-m} - \Delta h_{n-m}) = 0, \; 0 \leq n \leq N_d-1 \quad (8)$$

From Eqs. (6) and (8), the following error is minimized by solving Eq. (7),

$$E_1' = \sum_{n=0}^{N_d-1} \left(\sum_{m=0}^{n_2} q_m \Delta h_{n-m} \right)^2 \quad (9)$$

Thus, the impulse response error is minimized with Q(z) as a weighting function.

Time Response Property Specified

A symmetrical impulse response is taken as a desired time response property in the following discussions.

(1) Numerator Coefficients

When P(z) is only used to optimize the time response, the impulse response can be expressed as

$$h_n = \sum_{m=0}^{n_1} p_m g_{n-m} \quad (10)$$

Letting K be the sampling point corresponding to the average delay time, the condition for a symmetrical impulse response is expressed as

$$h_{K+n} = h_{K-n}, \; 1 \leq n \leq K \quad (11)$$

From Eqs.(10) and (11),

$$\sum_{m=0}^{n_{1+}} p_m g_{K+n-m} = \sum_{m=0}^{n_{1-}} p_m g_{K-n-m}, \; 1 \leq n \leq K$$
$$n_{1+} = \min\{N_p-1, K+n\} \quad (12)$$
$$n_{1-} = \min\{N_p-1, K-n\}$$

When a number of the sampling points at which Eq. (11) must be satisfied is equal to or less than N_p, exact symmetrical impulse response at the specified sampling points can be obtained through linear equations by Eq. (12). When the number of the specified sampling points is larger than N_p, the least square approximation is required. A squared error is expressed as

$$E_2 = \sum_{n \in \Omega_2} \left(\sum_{m=0}^{n_{1+}} p_m g_{K+n-m} - \sum_{m=0}^{n_{1-}} p_m g_{K-n-m} \right)^2 \quad (13)$$

where Ω_2 is a set of sampling points at which the symmetrical conditions must be satisfied. The optimum solutions are obtained by

$$\frac{\partial E_2}{\partial p_m} = 0, \quad m = 0, 1, \ldots, N_p-1 \quad (14)$$

The error evaluation by Eq. (13) is directly related to the impulse response error.

(2) Numerator and Denominator Coefficients

When the denominator coefficients are employed, the time response is not directly optimized through linear equations. Modification is necessary in order to formulate linear equations of the filter coefficients. In this paper, a two stage approximation method is proposed. P(z)/Q(z) is expressed as

$$\frac{P(z)}{Q(z)} = \sum_{n=0}^{\infty} f_n z^{-n} \quad (15)$$

First stage approximation for the symmetrical impulse response is carried out through linear equations using the modified coefficients f_n, and p_n and q_n are determined by also linear equations so as to approximate the resulting f_n at the second stage. The first stage approximation employs the same number of the modified coefficients f_n as that of specified sampling points, and produces no approximation errors.

Error evaluation for the symmetry of the impulse response requires further discussions. Letting f_n^* be the result at the first stage approximation, and f_n be an impulse response of P(z)/Q(z) obtained at the second stage, then f_n and f_n^* can be related as

$$f_n = f_n^* + \Delta f_n^* \quad (16)$$

where Δf_n^* is an approximation error produced at the second stage. From Eq. (9), the following error function is minimized at the second stage.

$$E_3 = \sum_{n \in \Omega_2} \left(\sum_{m=0}^{n_3} q_m \Delta f_{n-m}^* \right), \; n_3 = \min\{N_q-1, n\} \quad (17)$$

As mentioned in Eq. (9), the approximation error for the impulse response of P(z)/Q(z) is not directly evaluated. It includes Q(z) as a weighting function. The whole impulse response using f_n becomes

$$h_n = \sum_{m=0}^{n_4} (f_m^* + \Delta f_m^*) g_{n-m}, \; n_4 = \min\{N_d-1, n\} \quad (18)$$

The difference between the impulse response samples located at symmetrical sampling points can be expressed as

$$(h_{K+n} - h_{K-n}) = \sum_{m=0}^{n_{4+}} f_m g_{K+n-m} - \sum_{m=0}^{n_{4-}} f_m g_{K-n-m},$$
$$n_{4+} = \min\{N_d-1, K+n\} \quad (19)$$
$$n_{4-} = \min\{N_d-1, K-n\}$$

The error Δf_n^* is minimized with Q(z) as a weighting function at the second stage, and the final impulse response error is evaluated with $G(z)$ as a weighting function.

APPROXIMATION ALGORITHM

Based on the discussions in the previous section, the following approximation algorithm is introduced.
(1) Time response optimization is carried out through linear equations, and its optimum solution is always guaranteed during a frequency response optimization procedure.
(2) A frequency response is optimized through iterative methods taking a weighting function $P(z)/Q(z)$ into account.

Design Flow Chart

Fig.1 shows a flow chart for the proposed simultaneous frequency and time domain approximation method. In Fig.1, X_1 and X_2 mean sets of the $P(z)/Q(z)$ coefficients and the $G(z)$ coefficients, respectively. A matrix [A] and a vector C include X_2 as constant coefficients. The iterative Chebyshev approximation method [8] is employed for frequency response optimization.
(4) Initial Guess for X_2
A transfer function is usually identified by two kinds of frequency responses. On the other hand, a time response can uniquely identify the transfer function. For this reason, the filter coefficient initialization based on a desired time response is effective.
(5) ~ (8) Iterative Chebyshev Approximation
The frequency response is optimized in blocks (5) ~ (8) using the $G(z)$ coefficients. The transfer function in these blocks can be expressed as

$$H(z,\ X_1,\ X_2) \tag{21}$$

X_1 is obtained as the solution of the linear equations in block (5), including X_2 optimized through the iterative method. By evaluating the frequency response error using X_1 obtained in block (5), the time response optimization is automatically achieved.

DESIGN EXAMPLE

Image signal transmitting filters are taken as design examples.

Specifications and Design Parameters

Table 1 shows filter response specifications and design parameters.
Initial Guess for Filter Coefficients: The initial guess for the filter coefficients is obtained through the Padé approximation using the ideal impulse response shown in Fig.2(b). The frequency response is also shown in Fig.2(a) where f_p and f_s are taken as 45Hz and 55Hz, respectively. The factors, consisting of two zeros appear in the passband, are taken as X_1. The remaining factors are used as X_2.
Specification for Time Response: The exact symmetrical condition is imposed on the impulse response samples designated by the symbol * in Fig.2(b).

Desired Time Response

Both passband ripple and stopband attenuation are specified in such filters. Time response distortion is caused by group delay distortion and an asymmetrical waveform results. Hence, a symmetrical waveform is taken as the time domain target.

Filter Responses Optimized

Among the design parameters, 12/12th-order allocation and 20T average delay time provide excellent frequency response. In the case of 12/12th-order filter approximation, a pole and zero pair which mostly cancel each other appear in the passband. Therefore, the optimization was continued after removing them, and the 11/11th-order filter results. Figs.3(a), (b) and (c) show the resulting amplitude response in dB, impulse response and pole-zero locations, respectively. The zeros shown by ⊙ in Fig.3(c) correspond to X_1 and are used for the time response optimization.
Comparison with Conventional Methods: Linear phase FIR filters designed through the Remez-exchange method require 73 tap filter lengths to meet the same frequency response in Fig.3(a). Thus, the proposed method can reduce the filter circuit complexity. As mentioned previously, all-pass functions can be used for time response optimization. This approach was tried in this paper, and a 6th-order elliptic filter with a 8th-order all-pass function results. The conventional method requires high filter order compared with the proposed approach.

CONCLUSION

A simultaneous frequency and time domain approximation method for discrete-time filters is proposed. Time response optimization is carried out through linear equations. The optimum solution is always guaranteed during the frequency response optimization. This approach does not impose any constraints on pole-zero locations and filter responses. Hence, sufficient reduction in filter circuit complexity can be achieved.

Approximation error criteria in the linear equation algorithm are restricted to two categories including exact interpolation and the least mean square error. By extending the linear equation method to linear programming methods, it will be possible to approximate a time response in the weighted Chebyshev sense.

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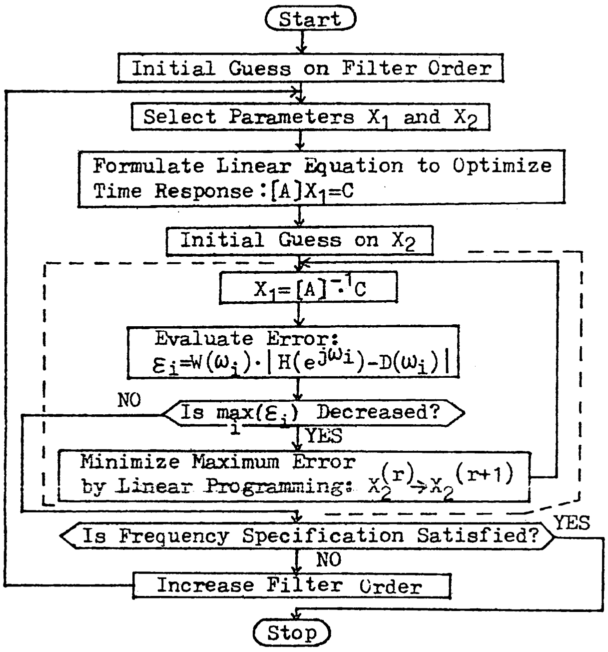
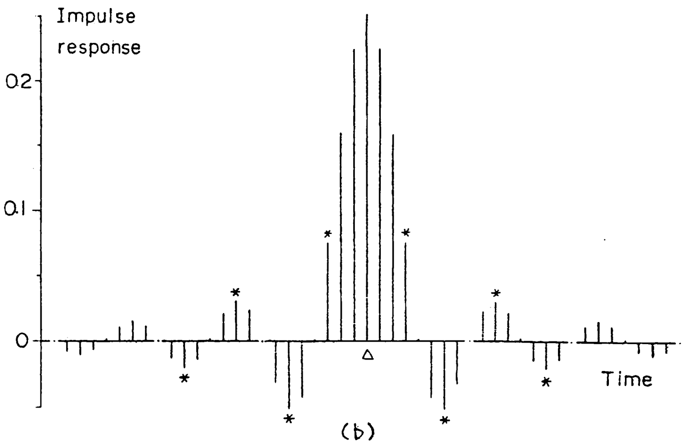


Fig.1 Design flow chart for proposed simultaneous frequency and time domain approximation method.

Table 1 Specifications and design parameters

Sampling Rate	400Hz
Passband	0 – 49Hz
Stopband	59 – 200Hz
Filter Order Allocations	20/4,16/8,12/12
Average Delay Time	12T,16T,20T,24T, T=1/400Sec.



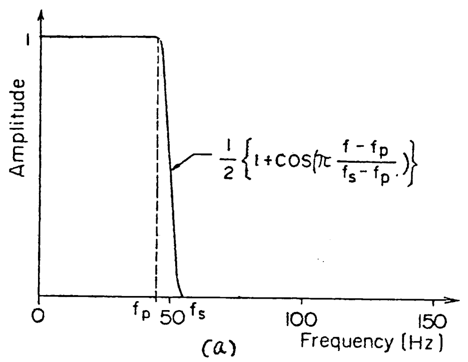


Fig.2 Ideal filter responses for initial guess calculation. (a)Amplitude response. (b)Impulse response. Symmetrical conditions are imposed on samples designated by *.

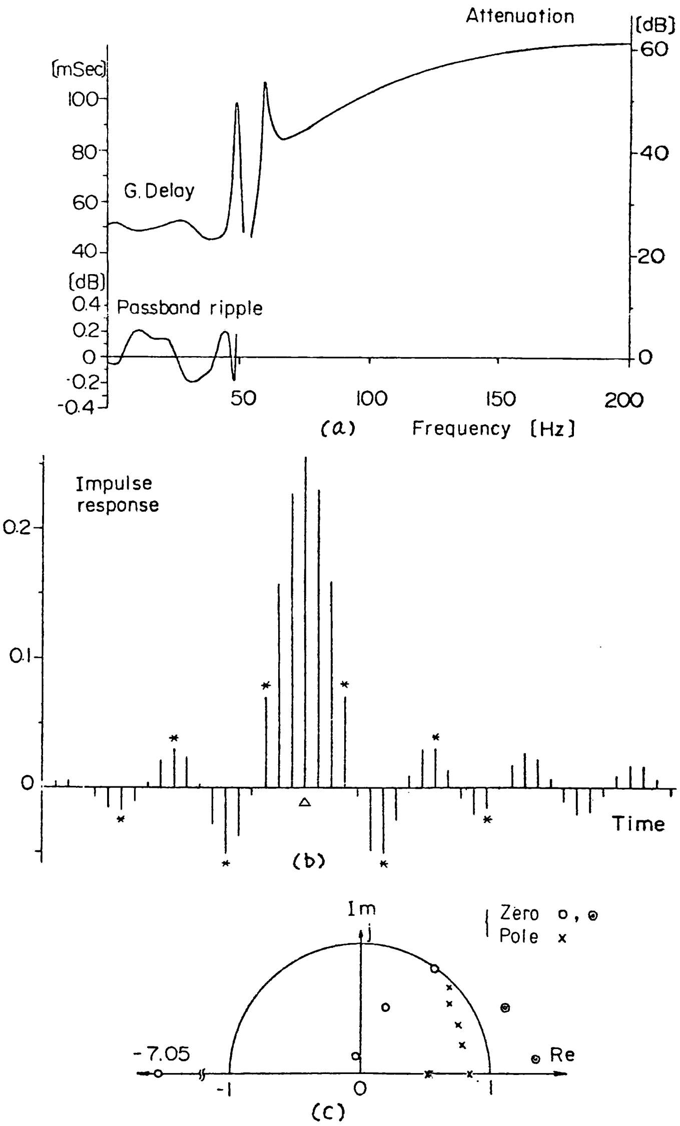


Fig.3 Optimized filter responses with 11/11th-order and 20T average delay. (a)Amplitude response in dB. (b)Impuse response. (c)Pole-zero locations. Zeros shown by ⊚ correspond to X_1.