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HARMONIC BALANCE FINITE ELEMENT METHOD TAKING ACCOUNT OF EXTERNAL CIRCUITS AND MOTION

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Abstract —The alternating-current electric machines are usually excited by voltage sources and the external elements are connected in the magnetizing circuit. Therefore the magnetic fields including nonlinear characteristics, voltage sources and external circuits should be considered together in the analysis model. Furthermore, the motion effects in field problem of electric machines must be sometimes taken into account.

This paper describes a new approach for analyzing the time-periodic nonlinear magnetic field problems considered external circuits and motion in the harmonic domain by using the harmonic balance finite element method. The system equation for governing magnetic field with motion and circuits is investigated. A few applications are discussed.

INTRODUCTION

There is sometimes the necessity to take account of both the nonlinear magnetic field and the magnetizing circuit in field problems of electric machines. Because the electric machines are usually excited by voltage power sources, then the corresponding magnetizing currents are unknown. The external circuits connected with magnetic field have some influences over magnetizing currents and magnetic field. The motion may change the distribution of flux and eddy currents. In this case, we should make a system matrix equation governing all of them in detail.

In order to solve such kind of time-periodic solutions, we propose the harmonic balance finite element method which assumes that the time-periodic solutions are expressed as the sum of fundamental and harmonic components. Eventually, the calculation with respect to time is carried out in the harmonic domain (the frequency domain). Therefore, the procedure of numerical analysis becomes easy to deal with like the analysis of static field. The system equation governing the magnetic field is reasonably combined with the equation of circuits in a single matrix equation.

In this paper, firstly we describe the formulation of 2-dimensional harmonic balance finite element method(HBFEM) which includes voltage power sources, circuit elements and motion. Secondly we apply this method to the magnetic frequency tripler with five-legged core. For the dynamic problem, the high-speed hybrid induction motor proposed by us is discussed and some numerical results are shown.

HBFEM TAKING ACCOUNT OF MOTION EFFECT

As the analysis model includes rotating or moving part, according to Maxwell's equations, the formulation of 2-dimensional magnetic field related to the vector potential A is given by

$$\frac{\partial}{\partial x}\left[\nu\frac{\partial A}{\partial x}\right]+\frac{\partial}{\partial y}\left[\nu\frac{\partial A}{\partial y}\right] = -J_o+\sigma\frac{\partial A}{\partial t}+\sigma V_x\frac{\partial A}{\partial x}+\sigma V_y\frac{\partial A}{\partial y} \qquad (1)$$

By using Galerkin's procedure, Eq.(1) becomes

$$\int\!\!\int_{\Omega}\left\{\frac{\partial N_i}{\partial x}\left[\nu\frac{\partial A}{\partial x}\right]+\frac{\partial N_i}{\partial y}\left[\nu\frac{\partial A}{\partial y}\right]\right\}dx\,dy - \int\!\!\int_{\Omega}\left[J_o-\sigma\frac{\partial A}{\partial t}-\sigma V_x\frac{\partial A}{\partial x}-\sigma V_y\frac{\partial A}{\partial y}\right]N_i\,dx\,dy=0 \qquad (2)$$

where N_i is the shape function of the first-order triangular element as a weighting function. The speeds of x- and y-direction (V_x, V_y) are assumed to be constant. ν and σ are the magnetic reluctivity and the conductivity respectively. The variables such as the vector potentials A, the magnetizing current density J_o and the magnetic reluctivity ν can be approximately expressed in harmonic solutions, then

$$A^i=\sum_{n=1,3,5...}\{A_{ns}{}^i\sin(n\omega t)+A_{nc}{}^i\cos(n\omega t)\} \qquad (3.a)$$

$$J_o=\sum_{n=1,3,5...}\{J_{ns}\sin(n\omega t)+J_{nc}\cos(n\omega t)\} \qquad (3.b)$$

$$\nu(t)=H\{B(t)\}/B(t) = \nu_0+\sum_{n=2,4,6.....}\{\nu_{ns}\sin(n\omega t)+\nu_{nc}\cos(n\omega t)\} \qquad (3.c)$$

where

$$\nu_0=\frac{1}{T}\int_0^T\nu(t)dt \qquad (4.a)$$

$$\nu_{ns}=\frac{2}{T}\int_0^T\nu(t)\sin(n\omega t)dt \qquad (4.b)$$

$$\nu_{nc}=\frac{2}{T}\int_0^T\nu(t)\cos(n\omega t)dt \qquad (4.c)$$

ω is the fundamental angular frequency. $H(B)$ is the magnetizing characteristic of the core which is expressed by an approximate function[1].

The matrix expression of HBFEM for a single element is obtained as follows:

$$[S^e]\{A^e\}+[N^e]\{A^e\}+[M^e]\{A^e\}-\{K^e\}=0 \qquad (5)$$

where the vector $\{A^e\}$ and $\{K^e\}$ are expressed as

$$\{A^e\}=\{A_{1s}{}^1\ A_{1c}{}^1\ A_{3s}{}^1\ A_{3c}{}^1,\cdots,\ A_{1s}{}^2\ A_{1c}{}^2\ A_{3s}{}^2\ A_{3c}{}^2,\cdots,\ A_{1s}{}^3\ A_{1c}{}^3\ A_{3s}{}^3\ A_{3c}{}^3,\cdots\}^T \qquad (6)$$

$$\{K^e\}=\Delta/3\{J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots\}^T \qquad (7)$$

where the superscript e means a single element. The matrix $[S^e]$ and $[N^e]$ have these forms:

$$[S^e]=\frac{1}{4\Delta}\begin{bmatrix}(b_1b_1+c_1c_1)D & (b_1b_2+c_1c_2)D & (b_1b_3+c_1c_3)D\\(b_2b_1+c_2c_1)D & (b_2b_2+c_2c_2)D & (b_2b_3+c_2c_3)D\\(b_3b_1+c_3c_1)D & (b_3b_2+c_3c_2)D & (b_3b_3+c_3c_3)D\end{bmatrix} \qquad (8)$$

$$[N^e]=\frac{\sigma\omega\Delta}{12}\begin{bmatrix}2N & N & N\\N & 2N & N\\N & N & 2N\end{bmatrix} \qquad (9)$$

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where D and N are the reluctivity and the harmonic matrices as shown in reference [1]. The matrix $[M^e]$ is related with motion and can be given by

$$[M^e] = \frac{\sigma V_x}{6}\begin{bmatrix} b_1 I & b_2 I & b_3 I \\ b_1 I & b_2 I & b_3 I \\ b_1 I & b_2 I & b_3 I \end{bmatrix} + \frac{\sigma V_y}{6}\begin{bmatrix} c_1 I & c_2 I & c_3 I \\ c_1 I & c_2 I & c_3 I \\ c_1 I & c_2 I & c_3 I \end{bmatrix} \tag{10}$$

where I is the unit matrix.

SYSTEM EQUATION FOR DYNAMICS MAGNETIC FIELD WITH EXTERNAL CIRCUITS

Usually the electric machines are excited by three-phase voltage sources and connected with external circuit as shown in Fig.1. In this case, the magnetic field and circuits should be considered in the analysis.

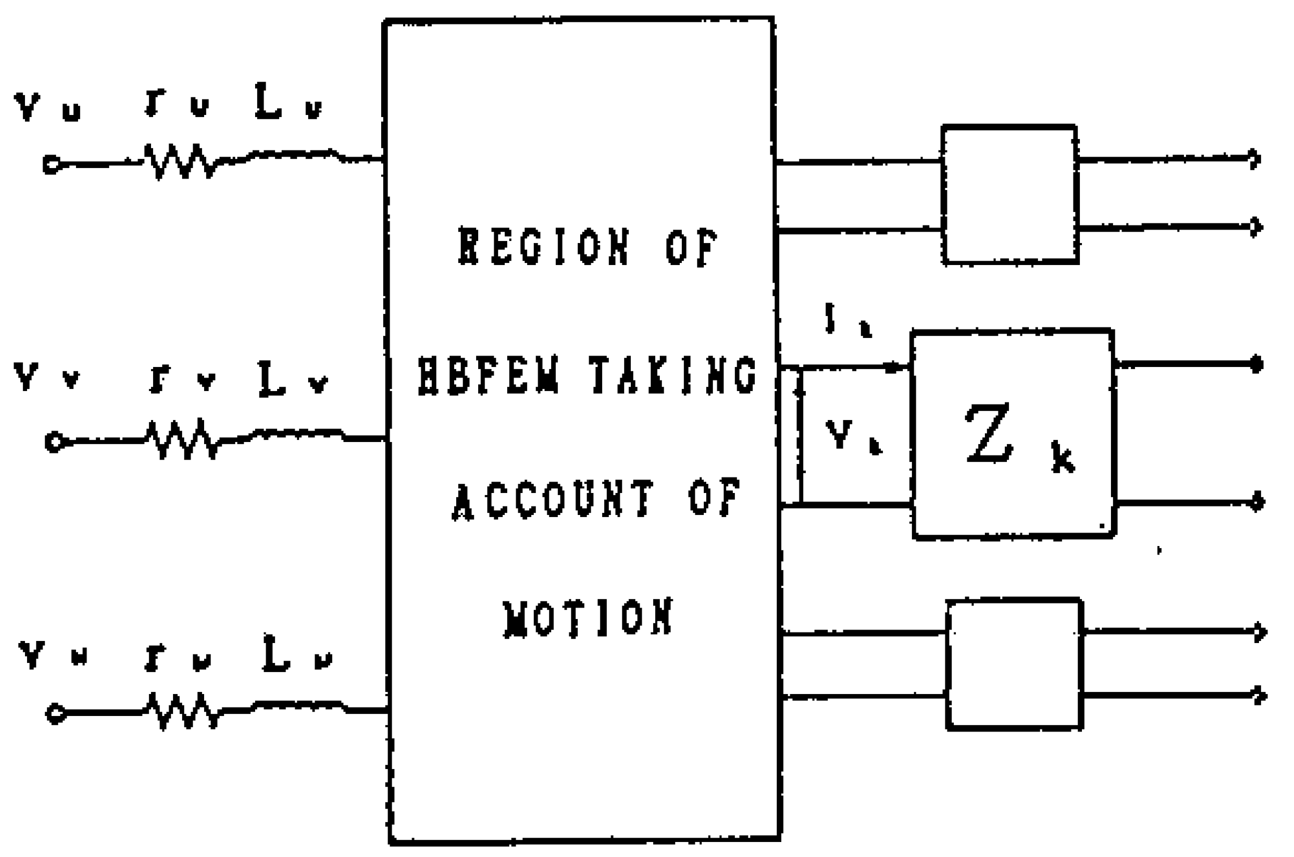


Fig.1 Analysis model taking account of external circuits and motion

According to Faraday's law, the relation between magnetic vector potential and terminal voltage can be given by the following equation, then

$$v = -d/dt\,(\oint A\,d\ell) \tag{11}$$

where the integration is done along magnetizing coil.

Substituting magnetic vector potential in Eq.(3.a) into Eq.(11), the terminal voltage related to magnetic field can be obtained by

$$V_k = \sum_{coil}\{-\sum_{n=1,3,5,\dots}(nA_{ns}{}^{l}\cos(n\omega t) + nA_{nc}{}^{l}\sin(n\omega t))\}\ \omega d_o\Delta/3S_{ck} \tag{12}$$

where k denotes the number of circuits. S_{c_k} and d_o are the area for a coil and the depth in z-direction respectively.

Based on the harmonic balance method, the terminal voltage $\{V_k\}$ for a single element in the k-th circuit is expressed as

$$\{V_k{}^e\} = \frac{\omega d_o\Delta}{3S_{ck}}\begin{bmatrix} N & N & N \end{bmatrix}\begin{Bmatrix} \{A^1\} \\ \{A^2\} \\ \{A^3\} \end{Bmatrix} = [C_k{}^e]\begin{Bmatrix} \{A^1\} \\ \{A^2\} \\ \{A^3\} \end{Bmatrix} \tag{13}$$

According to Kirchhoff's law, the matrix equations of magnetizing voltage sources and external circuits related to the field are obtained by

$$[C_u]\{A\} + [I]\{V_{NN'}\} + S_{cu}[Z_u]\{J_u\} = \{V_u\} \tag{14}$$

$$[C_v]\{A\} + [I]\{V_{NN'}\} + S_{cv}[Z_v]\{J_v\} = \{V_v\} \tag{15}$$

$$[C_w]\{A\} + [I]\{V_{NN'}\} + S_{cw}[Z_w]\{J_w\} = \{V_w\} \tag{16}$$

$$[C_k]\{A\} \qquad - S_{ck}[Z_k]\{J_k\} = \{0\} \tag{17}$$

$$S_{cu}[I]\{J_u\} + S_{cv}[I]\{J_v\} + S_{cw}[I]\{J_w\} = \{0\} \tag{18}$$

where the applied three-phase voltage sources $\{V_u\}, \{V_v\}$ and $\{V_w\}$ are expressed as

$$\{V_u\} = \{V_{u1s}\ \ V_{u1c}\ \ V_{u3s}\ \ V_{u3c}\ \cdots\}^T \tag{19a}$$

$$\{V_v\} = \{V_{v1s}\ \ V_{v1c}\ \ V_{v3s}\ \ V_{v3c}\ \cdots\}^T \tag{19b}$$

$$\{V_w\} = \{V_{w1s}\ \ V_{w1c}\ \ V_{w3s}\ \ V_{w3c}\ \cdots\}^T \tag{19c}$$

and $[C_k]$ is obtained as

$$[C_k] = \sum_e [C_k{}^e] \tag{20}$$

$\{J_u\}, \{J_v\}, \{J_w\}$ and $\{J_k\}$ are the magnetizing current densities and the current densities of external circuits. $\{V_{NN'}\}$ is the voltage at the neutral point. These variables have the same expressions as Eq.(19). $[Z_k]$ is the impedance of circuits for each harmonic component, hence

$$[Z_k] = \begin{bmatrix} [z_{1k}] & & 0 \\ & [z_{3k}] & \\ 0 & & \ddots \end{bmatrix} \tag{21}$$

The system equation of magnetic field for the whole region can be written as follows:

$$[H]\{A\} - [G_u]\{J_u\} - [G_v]\{J_v\} - [G_w]\{J_w\} \cdots - [G_k]\{J_k\} \cdots = \{0\} \tag{22}$$

the coefficient matrix $[H]$ of HBFEM has the form

$$[H] = \sum_e ([S^e] + [N^e] + [M^e]) \tag{23}$$

$[G_k]$ is the constant matrix related to the currents, that is

$$[G_k] = \sum_e [G_k{}^e] \tag{24}$$

where $[G_k^e]$ is defined as

$$\{K^e\} = [G_k{}^e]\ \{J_k\} \tag{25}$$

$\{A\}$ is vector potential for each harmonic component and can be expressed as

$$\{A\} = \{\{A^1\}^T\ \ \{A^2\}^T\ \cdots\ \{A^l\}^T \cdots\}^T \tag{26a}$$

$$\{A^l\} = \{A_{1s}{}^l\ \ A_{1c}{}^l\ \ A_{3s}{}^l\ \ A_{3c}{}^l\ \cdots\}^T \tag{26b}$$

Combining Eqs.(14)-(18) and (22), the system matrix equation for the whole magnetic field, voltage sources and external circuit are obtained as follows:

$$\begin{bmatrix} [H] & -[G_u] & -[G_v] & -[G_w] & \cdots & -[G_k] & \cdots & 0 \\ [C_u] & S_{cu}[Z_u] & & & 0 & & & [I] \\ [C_v] & & S_{cv}[Z_v] & & & & & [I] \\ [C_w] & & & S_{cw}[Z_w] & & & & [I] \\ \vdots & & & \vdots & & & & \vdots \\ [C_k] & & & & -S_{ck}[Z_k] & & & 0 \\ \vdots & & & & \vdots & & & \vdots \\ 0 & S_{cu}[I] & S_{cv}[I] & S_{cw}[I] & & 0 & & 0 \end{bmatrix}\begin{Bmatrix} \{A\} \\ \{J_u\} \\ \{J_v\} \\ \{J_w\} \\ \vdots \\ \{J_k\} \\ \vdots \\ \{V_{NN'}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{V_u\} \\ \{V_v\} \\ \{V_w\} \\ \vdots \\ \{0\} \\ \vdots \\ \{0\} \end{Bmatrix} \tag{27}$$

Since $[H]$ is a lare-sized band matrix, the system matrix Eq.(27) becomes a doubly-bordered band diagonal, and it will be solved by Gaussian elimination.

EXAMPLES OF APPLICATION FOR STATIC MACHINE

Since our purpose is to make an optimal design of high-speed hybrid induction motor which is combined by three-phase magnetic frequency tripler and induction motor[2], the analysis of magnetic frequency tripler as a static machine should be discussed first. We use magnetic frequency tripler with five-legged core as an analysis model. This is a typical analysis model

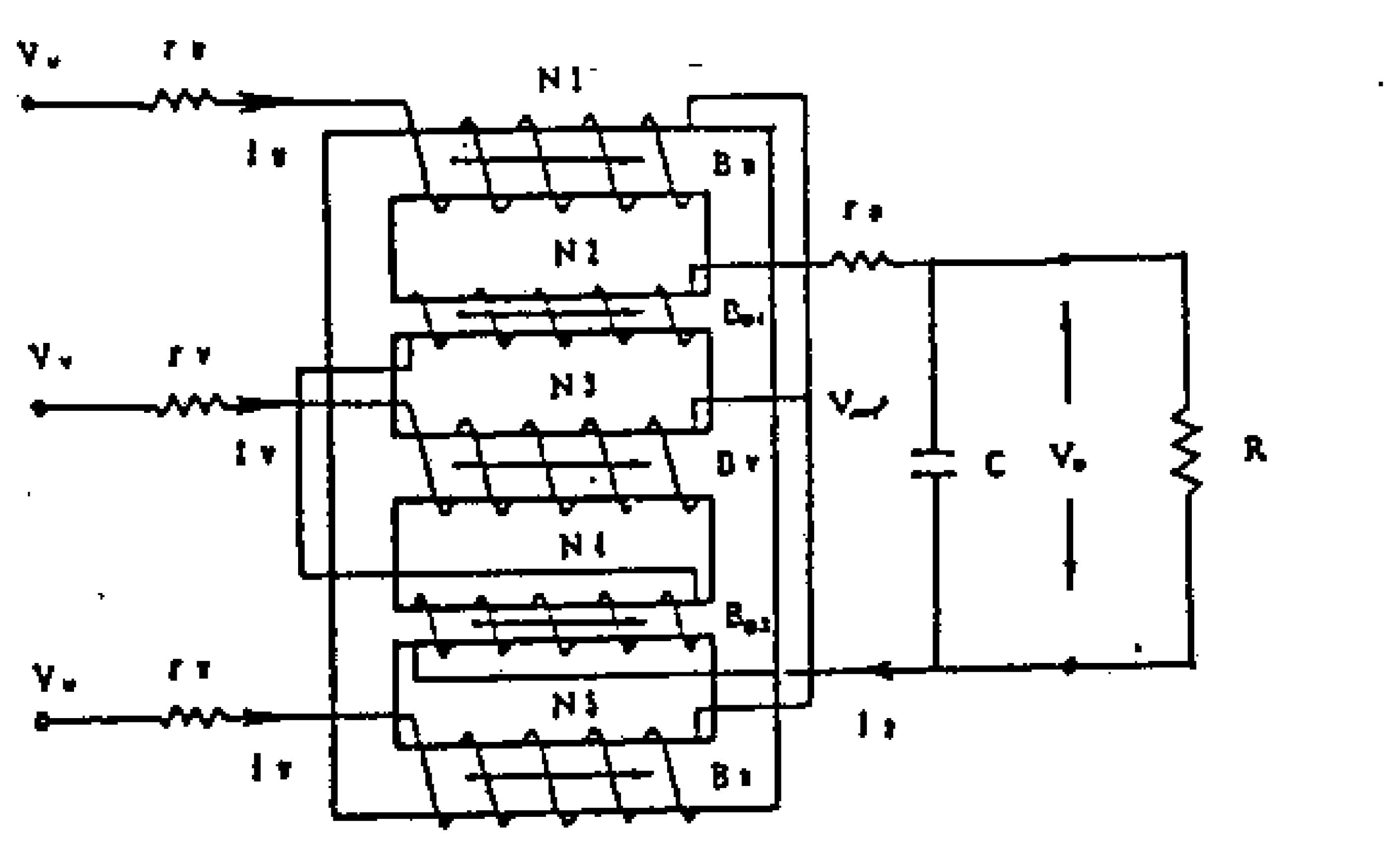


Fig.2 Magnetic frequency tripler

for the application of HBFEM taking account of voltage power sources and external circuits as shown in Fig.2. This device contains a five-legged core, three magnetizing coils and two secondary coils connected in series as an output power[3]. It can be described by Eq.(27) without motion term. Because magnetic frequency tripler usually works in the magnetic saturating state, the harmonic components will be generated in the magnetic core. According to the calculated results, the fundamental component and high-order harmonic components distribute in the different way as shown in Fig.3.

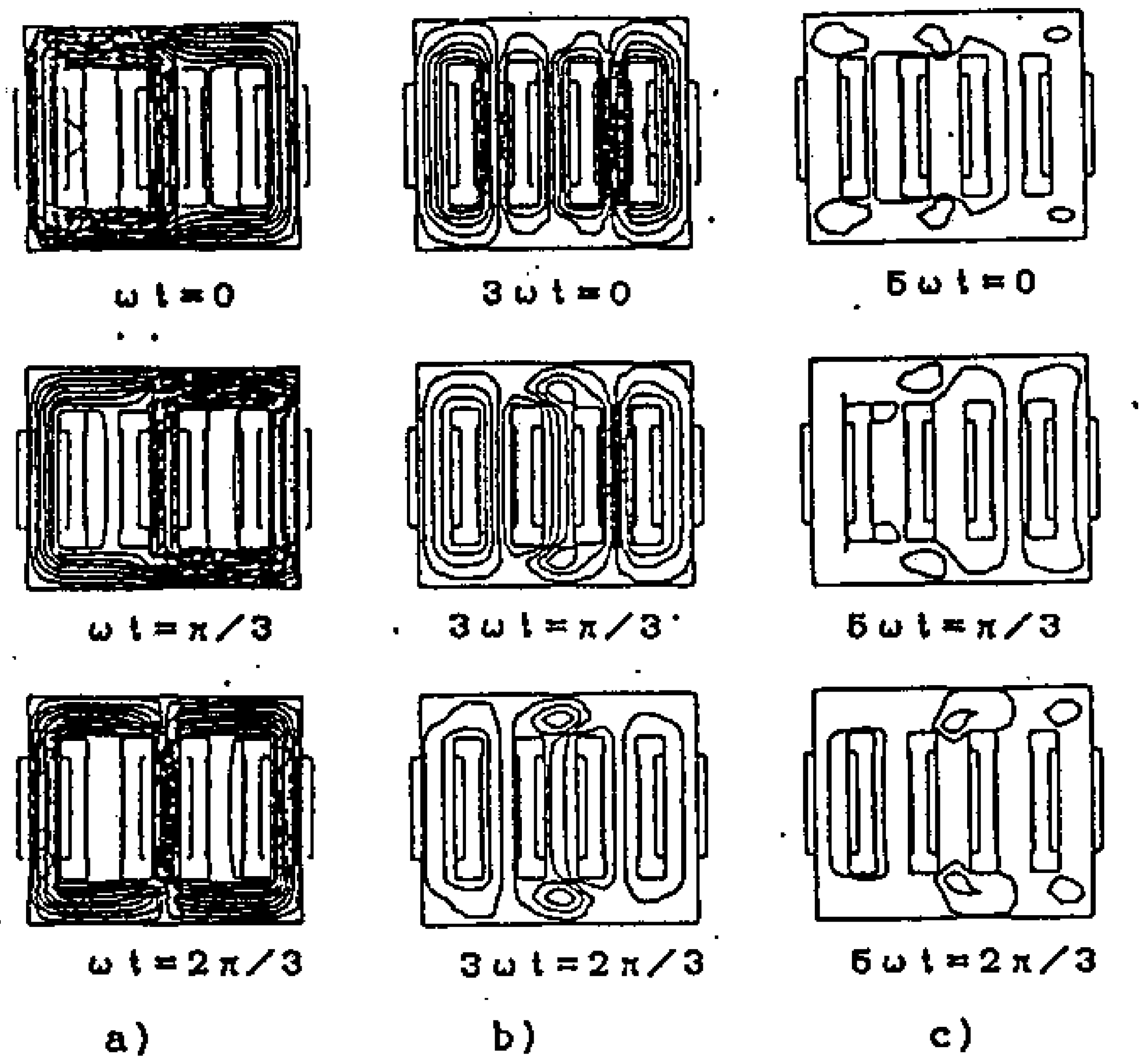


Fig.3 Distribution of magnetic flux.
a) is fundamental harmonic component.
b) is third harmonic component.
c) is fifth harmonic component.

As the secondary coils are connected in series, the fundamental component of opposite-direction currents will be canceled out in the secondary coils. Thus we can obtain high frequency power from the secondary coils. The analysis results of magnetizing currents and output current compared with experimental results are shown in Fig.4. We can find that there are some third and fifth harmonic components in the magnetizing currents and fundamental component in the output current.

ANALYSIS OF HIGH-SPEED HYBRID INDUCTION MOTOR

The high-speed hybrid induction motor consists of three-phase

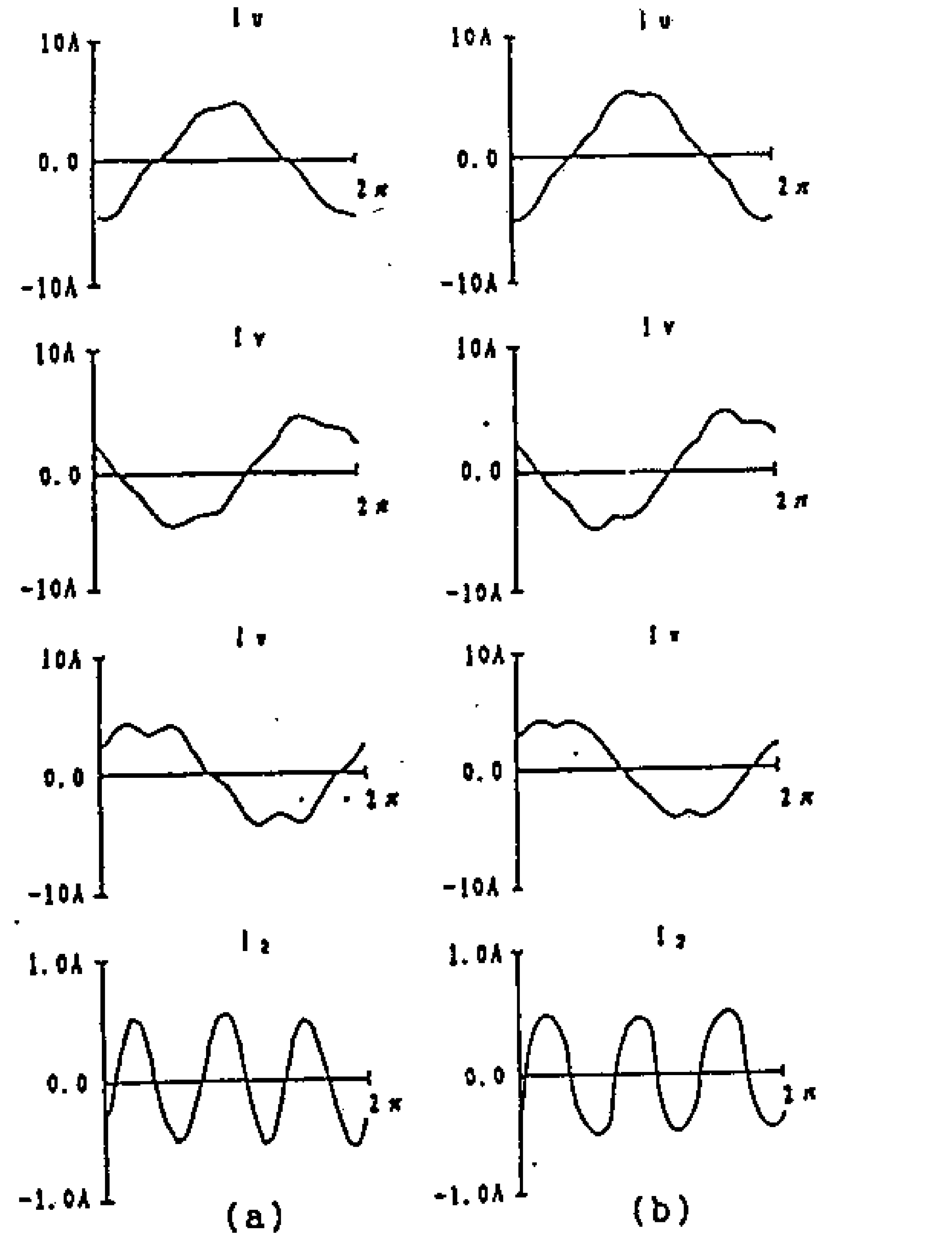
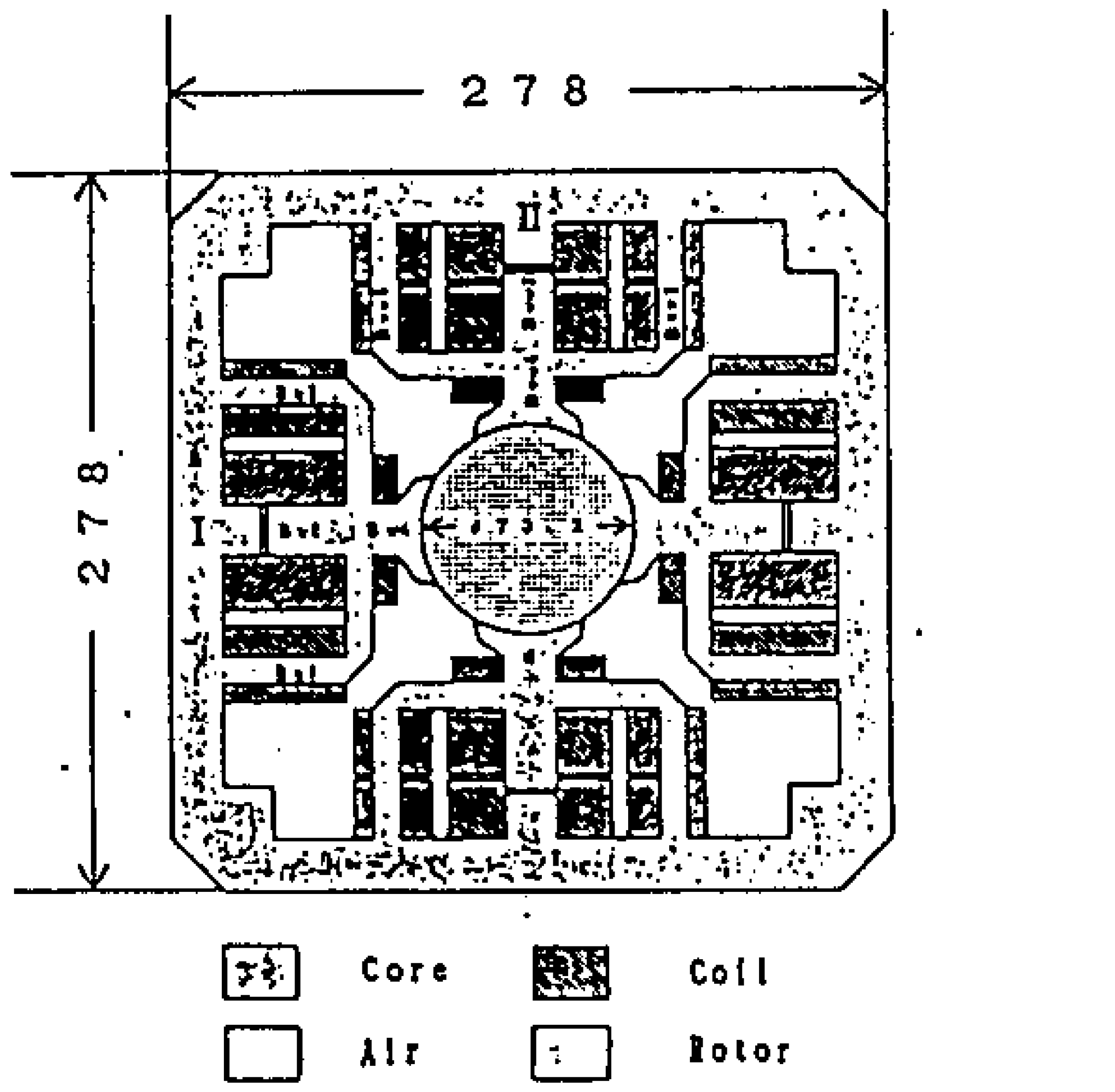


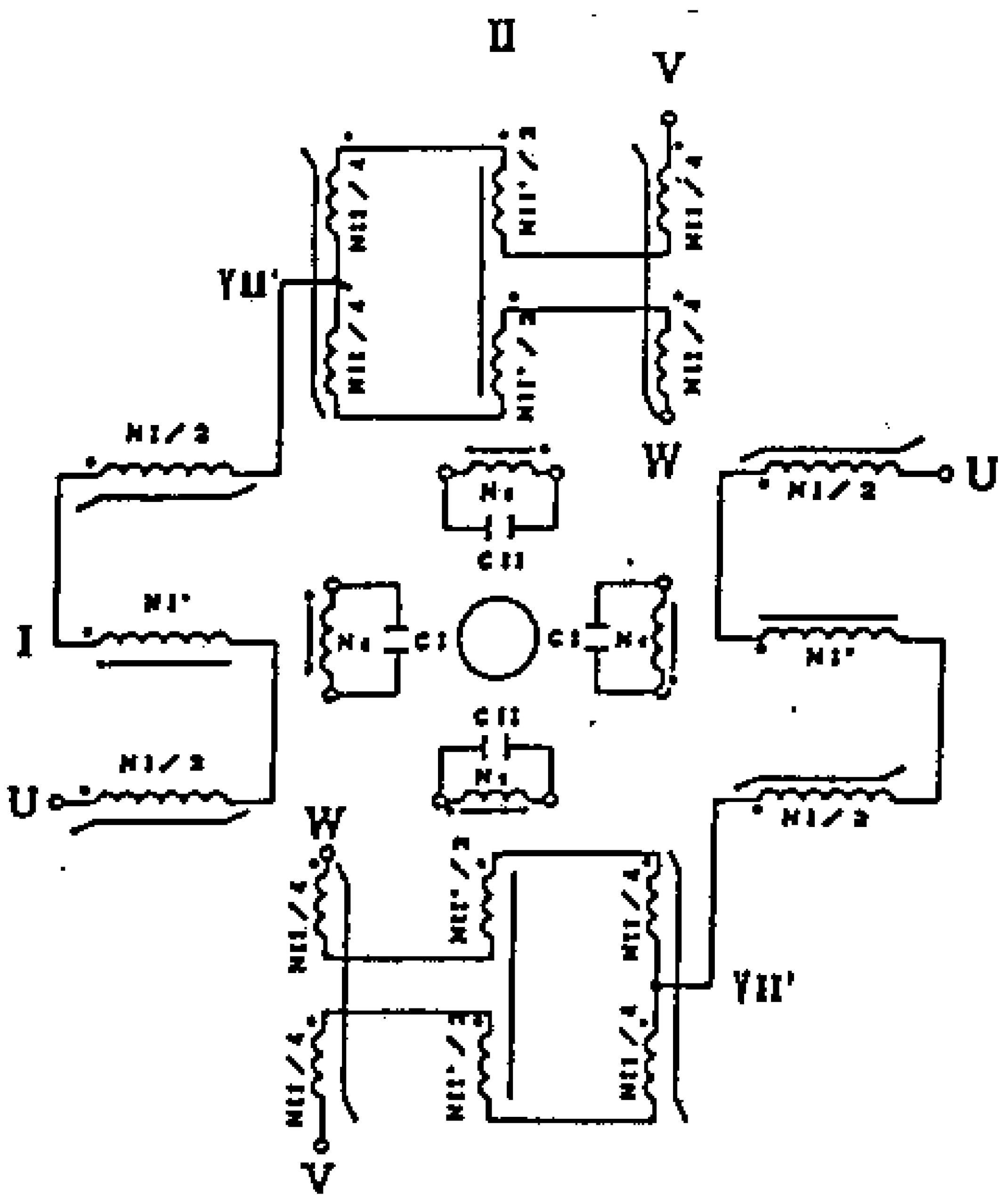
Fig.4 Waveform of currents
(a) Calculated results
(b) Experimental results

to two-phase magnetic frequency tripler and induction motor as shown in Fig.5. The induction motor has two pairs of magnetic frequency tripler and four magnetic poles with the air-gap in the middle leg of cores. The three-phase magnetizing windings are connected as Scott-connection and four additional coils connected with the capacitors for increasing output power are put in the poles. When a 60Hz commercial source is applied to hybrid induction motor, the rotation speed(10800rpm) will be gained between the poles.

The principle of three-phase to two-phase magnetic frequency tripler is that two single-phase triplers composed of 3-legged cores are connected in Scott-connection. Therefore, the input voltages shifted at 90 degree are applied to two triplers as shown in Fig.6.



(a) Configuration



(b) Connection of windings

Fig.5 High-speed hybrid two-phase induction motor

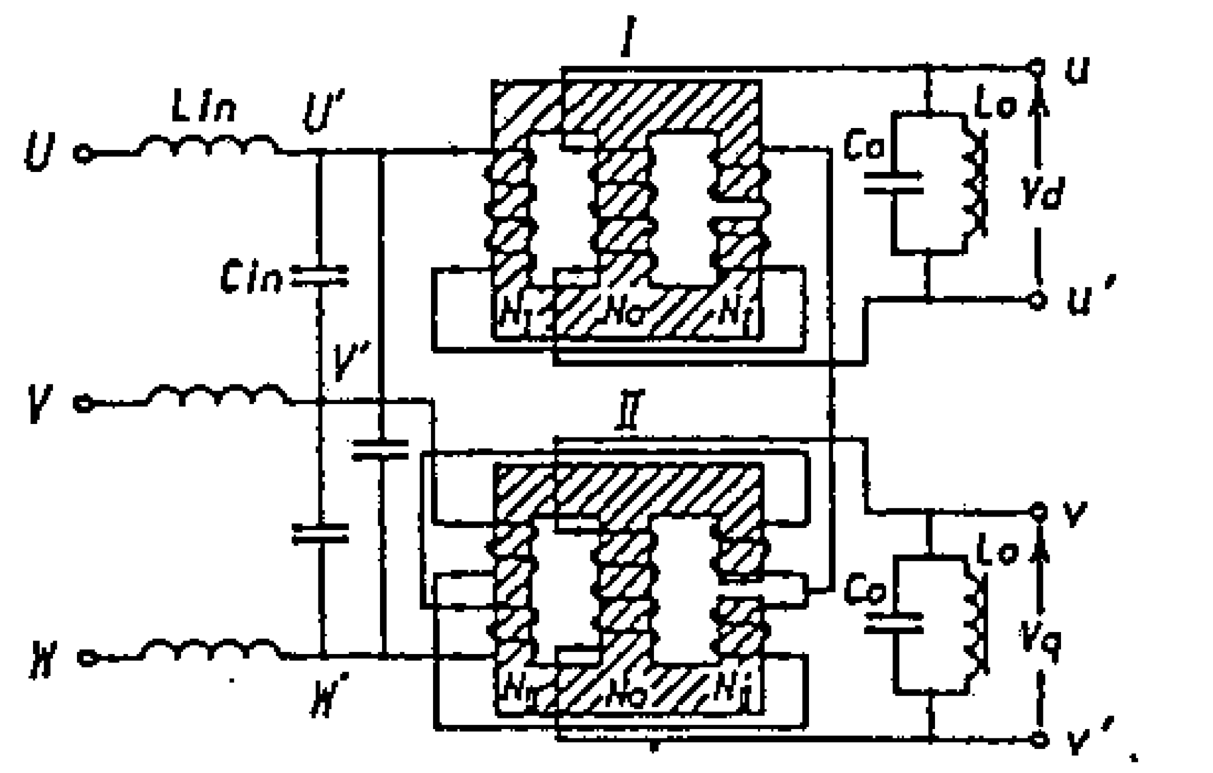


Fig.6 Three-phase to two-phase magnetic frequency tripler

These two triplers produce three-times frequency voltage due to the third harmonic component. The output voltage of tripler [I] is expressed as

$$V_d(t)=\sum_{n=1}^{\infty} V_{2n-1}\sin((2n-1)\omega t+\theta_{2n-1}) \quad (28)$$

where n, θ_{2n-1} and V_{2n-1} are integer, phase difference and peak value of harmonic respectively. As two triplers have the same structures, the voltage of the other tripler [II] has the form

$$V_q(t+(\pi/2\omega))=\sum_{n=1}^{\infty} V_{2n-1}\sin((2n-1)\omega t+(2n-1)\pi/2+\theta_{2n-1}) \quad (29)$$

Therefore, two output voltages (V_d, V_q) have the phase difference of 270 degree as a two-phase voltage source. The phase sequence of three-times frequency voltage rotates reversely against fundamental frequency voltage. It has a negative-phase-sequence.

Since the high-speed hybrid motor has a very complex configuration, the optimal design is requested in the design of electric machine. The good design leads the flux of fundamental harmonic component not to pass through the poles and rotor, and only the three-times frequency flux passes through the poles and moves the rotor. In our numerical analysis the half model is used as a analysis area. The magnetizing windings are applied with 60Hz commercial three-phase voltage source. The harmonic components will be generated in the core, when the core becomes saturated. Fig.7 shows the distributions of magnetic flux in the core. Since the comparisions are made between the different slips, we can find that the motion has made some influence on the distribution of magnetic flux.

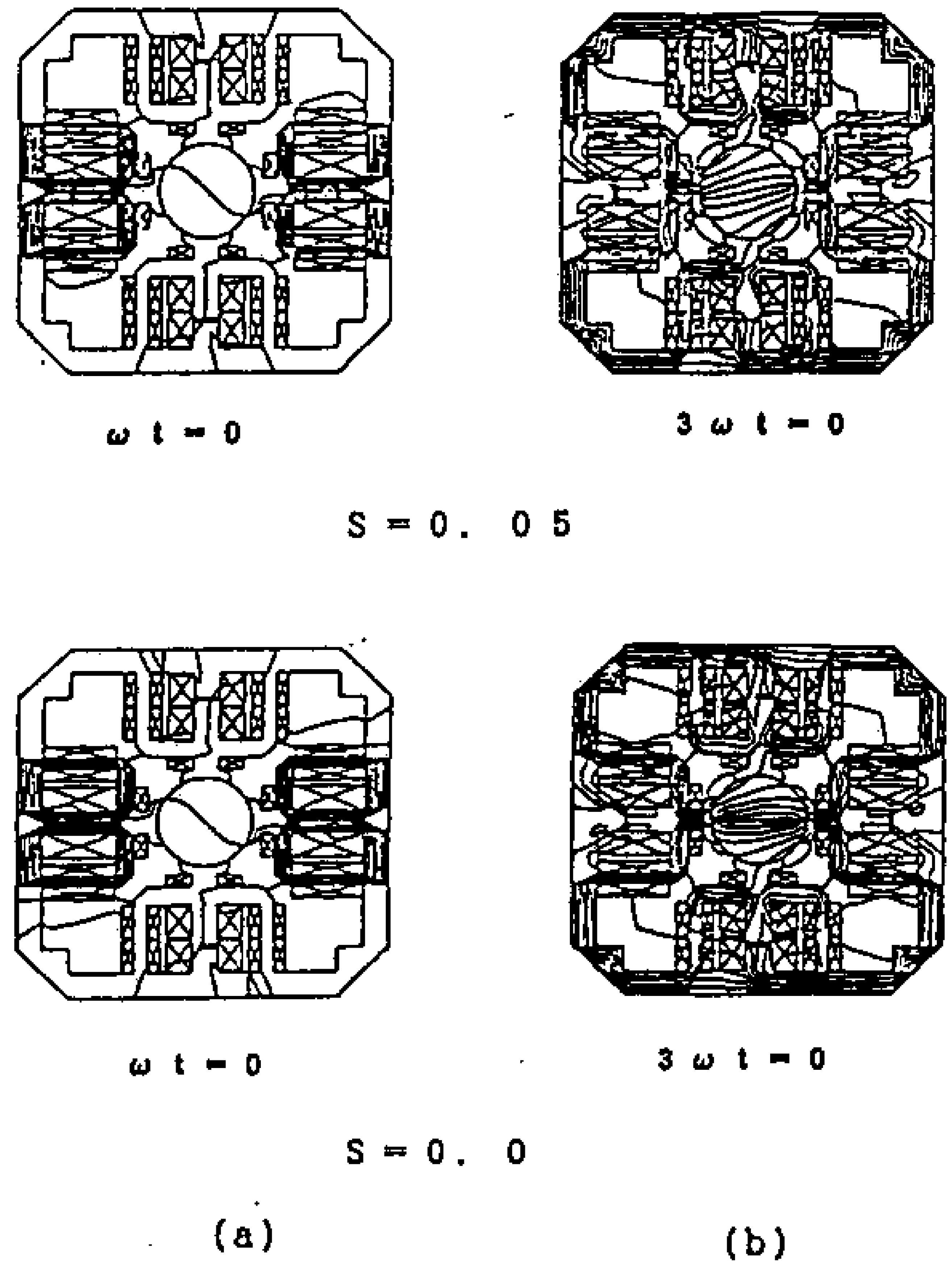


Fig.7 Distribution of magnetic flux
(a) Fundamental component
(b) Third harmonic component

CONCLUSIONS

The paper proposed a formulation of HBFEM taking account of external circuits and motion, and showed some applications. A comparison is made between experimental and numerical results for the static model, and shows a good agreement. It is suitable to analyze the high-speed hybrid induction motor by using HBFEM. The further investigation for analyzing the characteristic of the high-speed hybrid induction motor will be continued, and comparison between experimental result and numerical result will be made.

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