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# HARMONIC BALANCE FINITE ELEMENT METHOD APPLIED TO NONLINEAR AC MAGNETIC ANALYSIS

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## ABSTRACT

The harmonic balance finite element method(HBFEM) we proposed is suited to analyzing the time-periodic magnetic field. An AC magnetic field calculation involving saturation characteristics requires many iterations and the intricate procedures for the time derivative. The procedure of the numerical calculation for a new FEM is intrinsically similar to that for the static nonlinear magnetic problem. The paper describes the formulation for the time-periodic magnetic field problems with any saturation characteristics. The calculations applied to a reactor and an electromagnet verifies the formulation of the HBFEM.

## INTRODUCTION

The harmonic balance finite element method (HBFEM) was developed for the purpose of analyzing the time-periodic problems with a saturated core. The HBFEM is the combination of the finite element method and the harmonic balance method [1]. It provides the field distributions for each harmonic.

The previous material reported the formulation for only the magnetizing characteristics expressed by third order or fifth order polynomial functions[2]. But as the magnetizing curve of the magnetic core is more rectangular, it is difficult to represent the B-H curve by only the low-order polynomial functions. Here, we formulate the HBFEM corresponding to any profile of the magnetizing characteristics.

## FORMULATION OF HBFEM

For simplicity of the formulation, the following assumptions are made:

(1) The field is two-dimensional in the (x,y) plane.
(2) The problem is quasi-stationary.
(3) The saturated core is isotropic.
(4) The hysteresis is not considered.

Therefore, the vector potential $A=(0,0,A)$ satisfies, in the region of interest surrounded with some boundary conditions, the following:

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J_o+\sigma\frac{\partial A}{\partial t} \quad (1)$$

where $\nu$ and $\sigma$ are the magnetic reluctivity and the conductivity.

Formulation is made by use of the Galerkin procedure and the weighting functions are the same as the shape functions $N_i(x,y)$. Its integral form is,

$$\iint\left\{\frac{\partial N_i}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial N_i}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)\right\}dx\,dy-\iint\left(J_o-\sigma\frac{\partial A}{\partial t}\right)N_i\,dx\,dy \quad (2)$$

We are only interested in the time-periodic solution (the harmonic problem) when an alternating current is applied. Therefore, all variables, i.e. vector potentials, flux densities and applied current, are approximated as harmonic solutions, that is,

$$A^i=\sum_{n=1,3,5,\ldots}\{A_{ns}{}^i\sin(n\omega t)+A_{nc}{}^i\cos(n\omega t)\}$$

$$B_x{}^e=\sum_{n=1,3,5,\ldots}\{B_{xns}{}^e\sin(n\omega t)+B_{xnc}{}^e\cos(n\omega t)\} \quad (3)$$

$$B_y{}^e=\sum_{n=1,3,5,\ldots}\{B_{yns}{}^e\sin(n\omega t)+B_{ync}{}^e\cos(n\omega t)\}$$

$$J_o{}^i=\sum_{n=1,3,5,\ldots}\{J_{ns}{}^i\sin(n\omega t)+J_{nc}{}^i\cos(n\omega t)\}$$

where $\omega$ is the fundamental angular frequency.

The magnetizing characteristic of a core can be expressed as any function of the flux density B, that is,

$$H=H\ (B) \quad (4)$$

where the hysteresis characteristic is neglected. The magnetic reluctivity is written as,

$$\nu(t)=H\{B(t)\}\ /\ B(t) \quad (5)$$

where $B=(B_x{}^2+B_y{}^2)^{1/2}$ . The reluctivity is expressed as Fourier expansion, that is,

$$\nu(t)=\nu_0+\sum_{n=2,4,6,\ldots}\{\nu_{ns}\sin(n\omega t)+\nu_{nc}\cos(n\omega t)\} \quad (6)$$

where,

$$\nu_0=\frac{1}{T}\int_0^T\nu(t)dt \quad (7.a)$$

$$\nu_{ns}=\frac{2}{T}\int_0^T\nu(t)\cdot\sin(n\omega t)\ dt \quad (7.b)$$

$$\nu_{nc}=\frac{2}{T}\int_0^T\nu(t)\cdot\cos(n\omega t)\ dt \quad (7.c)$$

We substitute Eqs.(3) and (6) into (2) and equate the coefficients of sin(nωt) and cos(nωt)(n=1,3,...) on both sides according to the harmonic balance method. As a result, the matrix for one element is expressed as:

$$\frac{1}{4\Delta}\begin{bmatrix}(b_1b_1+c_1c_1)D & (b_1b_2+c_1c_2)D & (b_1b_3+c_1c_3)D\\(b_2b_1+c_2c_1)D & (b_2b_2+c_2c_2)D & (b_2b_3+c_2c_3)D\\(b_3b_1+c_3c_1)D & (b_3b_2+c_3c_2)D & (b_3b_3+c_3c_3)D\end{bmatrix}\{A\}$$

$$+\frac{\sigma\omega\Delta}{12}\begin{bmatrix}2N & N & N\\N & 2N & N\\N & N & 2N\end{bmatrix}\{A\}-\{K\} \quad (8)$$

where,

$$\{A\}=\{A_{1s}{}^1\ A_{1c}{}^1\ A_{3s}{}^1\ A_{3c}{}^1\ A_{5s}{}^1\ A_{5c}{}^1,\cdots,\ A_{1s}{}^2\ A_{1c}{}^2\ A_{3s}{}^2\ A_{3c}{}^2\ A_{5s}{}^2\ A_{5c}{}^2,\cdots,\ A_{1s}{}^3\ A_{1c}{}^3\ A_{3s}{}^3\ A_{3c}{}^3\ A_{5s}{}^3\ A_{5c}{}^3,\cdots\}^T$$

$$\{K\}=\Delta/3\{J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ J_{5s}\ J_{5c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ J_{5s}\ J_{5c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ J_{5s}\ J_{5c},\cdots\}^T \quad (9)$$

$b_i=y_j-y_k$, $c_i=x_k-x_j$, $\Delta$: cross section.

The block matrices D and N are given in Eqs.(10) and (11). The coefficients of the matrix D are determined by only the Fourier coefficients in Eq.(6). The matrix D acts as a reluctivity and is called the *reluctivity matrix*. On the other hand, the matrix N is a constant concerned with harmonic orders and is called the *harmonic matrix*.

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$$
D=\frac{1}{2}\begin{bmatrix}
2\nu_0-\nu_{2c} & \nu_{2s} & \nu_{2c}-\nu_{4c} & -\nu_{2s}+\nu_{4s} & \nu_{4c}-\nu_{6c} & -\nu_{4s}+\nu_{6s} & \nu_{6c}-\nu_{8c} & -\nu_{6s}+\nu_{8s} & \cdots\cdots \\
 & 2\nu_0+\nu_{2c} & \nu_{2s}+\nu_{4s} & \nu_{2c}+\nu_{4c} & \nu_{4s}+\nu_{6s} & \nu_{4c}+\nu_{6c} & \nu_{6s}+\nu_{8s} & \nu_{6c}+\nu_{8c} & \cdots\cdots \\
 & & 2\nu_0-\nu_{6c} & \nu_{6s} & \nu_{2c}-\nu_{8c} & -\nu_{2s}+\nu_{8s} & \nu_{4c}-\nu_{10c} & -\nu_{4s}+\nu_{10s} & \cdots\cdots \\
 & & & 2\nu_0+\nu_{6c} & \nu_{2s}+\nu_{8s} & \nu_{2c}+\nu_{8c} & \nu_{4s}+\nu_{10s} & \nu_{4c}+\nu_{10c} & \cdots\cdots \\
 & & & & 2\nu_0-\nu_{10c} & \nu_{10s} & \nu_{2c}-\nu_{12c} & -\nu_{4s}+\nu_{12s} & \cdots\cdots \\
 & & \text{symmetry} & & & 2\nu_0+\nu_{10c} & \nu_{2s}+\nu_{12s} & \nu_{4c}+\nu_{12c} & \cdots\cdots \\
 & & & & & & 2\nu_0-\nu_{14c} & \nu_{14s} & \cdots\cdots \\
 & & & & & & & 2\nu_0+\nu_{14c} & \cdots\cdots \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots
\end{bmatrix} \quad (10)
$$

$$
N=\begin{bmatrix}
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix} \quad (11)
$$

The system equation for the entire region is obtained by the same procedure as the conventional FEM and is solved by the iteration procedure for a nonlinear static field. We applied the successive under-relaxation method to the calculations.

The main feature is that the calculation concerned with time is not included and the procedure of the calculation is the same as the nonlinear static FEM.

It is possible to obtain approximate solutions by the sum of a finite number of harmonics because the higher order components are reduced. When the harmonic components up to (2m-1) order are considered, the size of the block matrices D and N are 2m*2m respectively and the order of the system matrix which is the sparse and band matrix is 2m times bigger than the number of nodes.

## ANALYSIS BY THE HBFEM

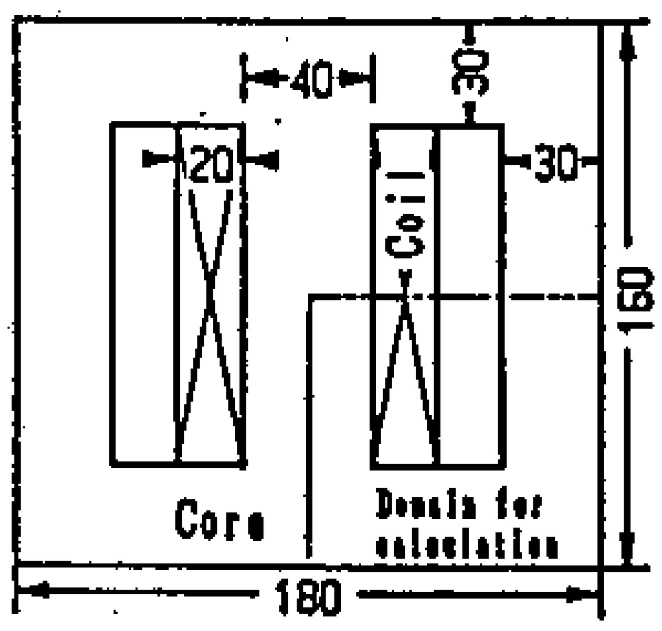
### Verification by simple problems

The procedure of the HBFEM is applied to a simple reactor with a saturated core as shown in Fig.1(a). For simplicity, eddy currents are not considered in this problem. The one quarter domain for calculation is subdivided as shown in Fig.1(b).

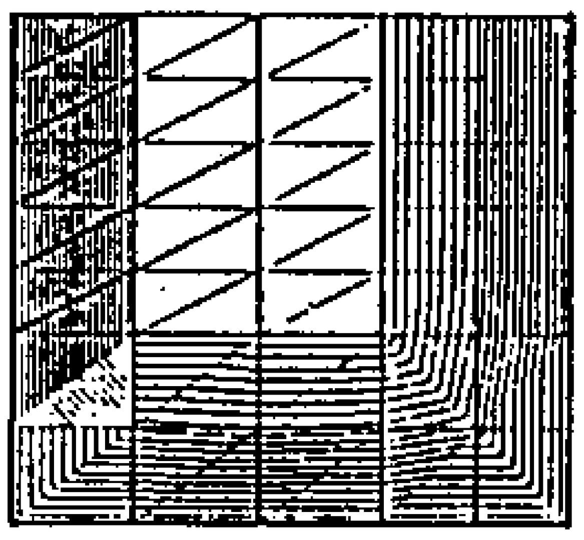
In the first problem, the magnetizing curve is expressed as a polygonal line as shown in Fig.2. The HBFEM including harmonics up to third order is applied to the reactor. The flux distribution of each harmonic is similar to that shown in Fig.1(b). The waveforms of the flux density in the middle leg are drawn in Fig.3. In this problem, it is possible to calculate the flux distribution by using the ordinary technique of the nonlinear static FEM as the current density is given at a particular instant.[3] The points illustrated as circles indicate the results. The agreement of the results verifies the procedure of the harmonic balance FEM.

In the second problem, the magnetizing curve of the core is expressed as the ninth order polynomial function as shown in Fig.4. We apply the HBFEM including harmonics up to seventh order to this problem. The waveform of the flux densities in the middle leg are illustrated in Fig.5. The analysis in this problem needs the HBFEM with the relative higher order harmonics because of the stronger magnetizing characteristics and no air gap.

Figure 6 show the waveform when the magnetizing current includes the components up to seventh order. In this calculation, the flux density is similar to a sinusoidal waveform. The results correspond to the analysis of the harmonic magnetizing currents on a transformer.




(a) Reactor



(b) Mesh and the distribution of the fundamental component

Fig.1 Reactor with a saturated core

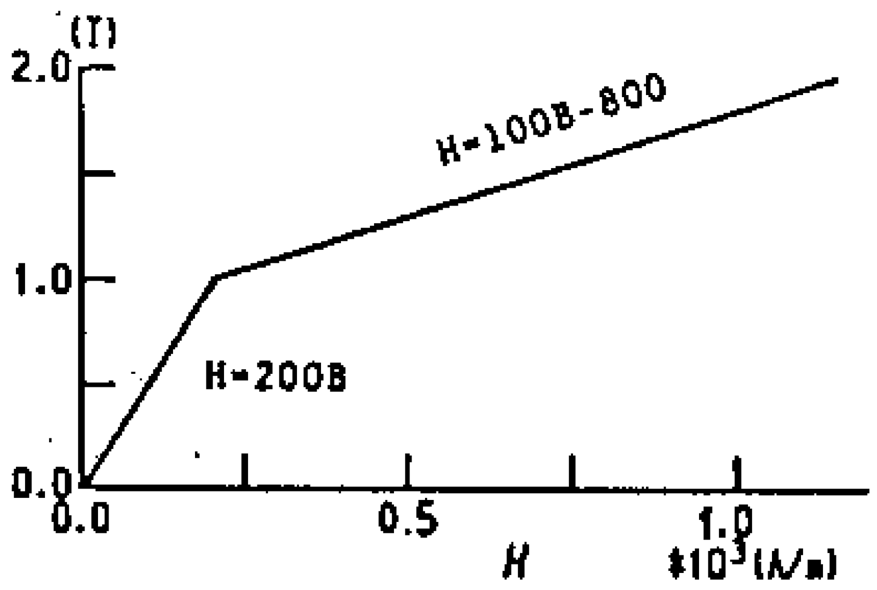



Fig.2 B-H curve of a saturated core expressed by the polygonal line

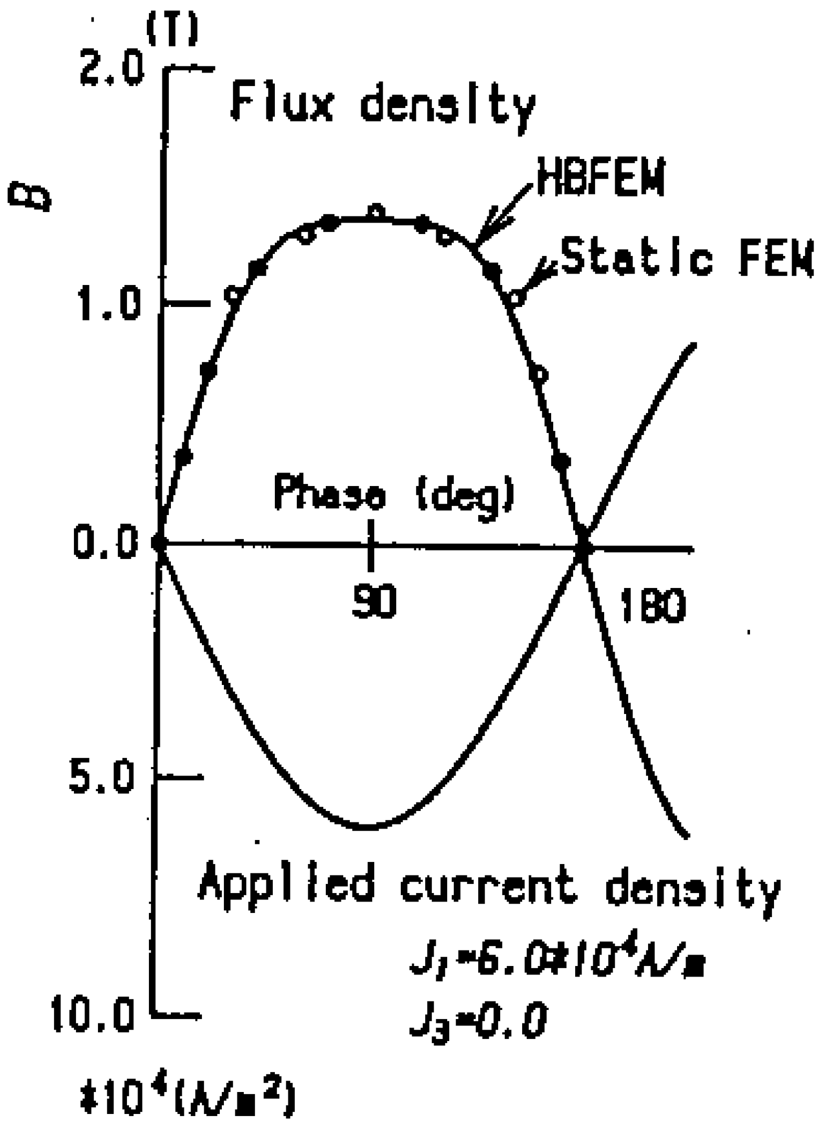



Fig.3 Comparison with results calculated by HBFEM and the nonlinear static FEM when the applied current is sinusoidal.

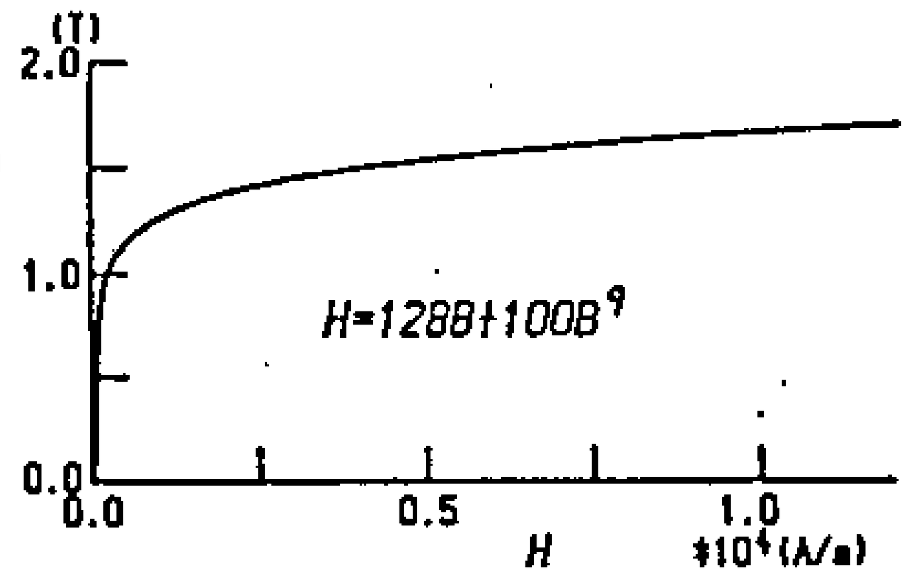



Fig.4 B-H curve of saturated core

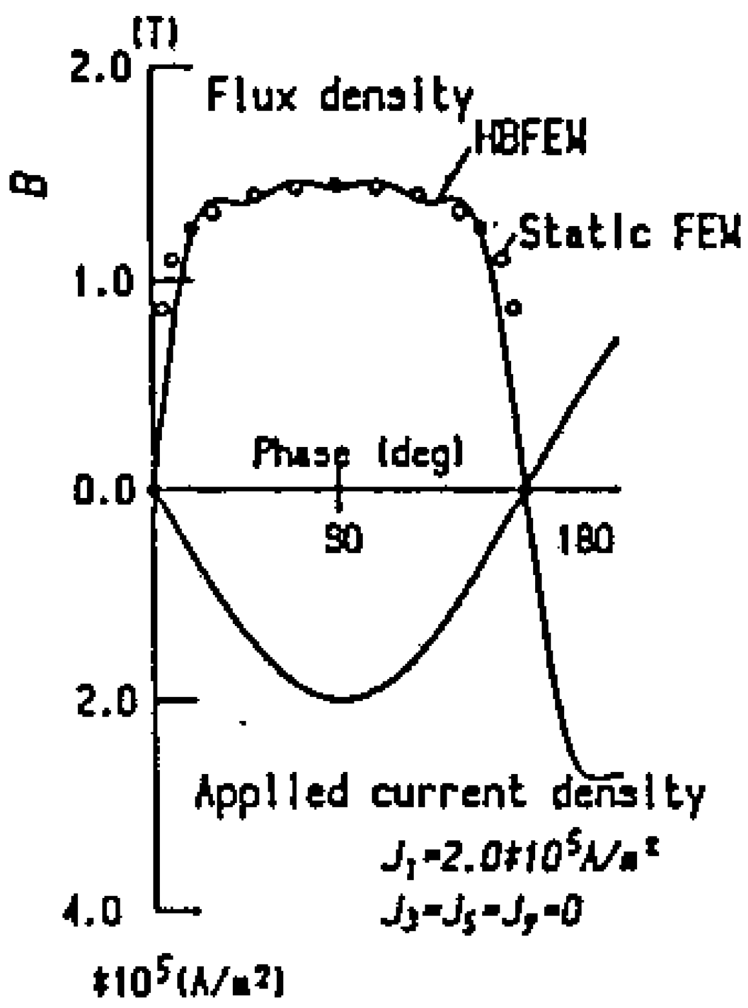



Fig.5 Comparison when the magnetizing current is sinusoidal.

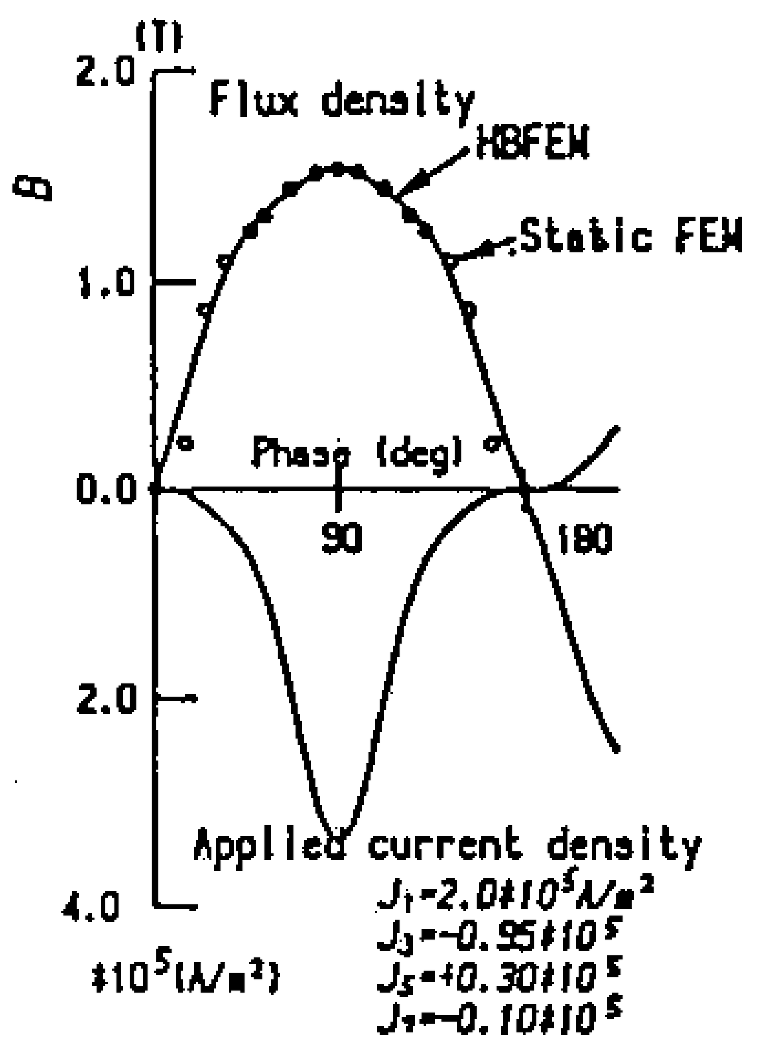



Fig.6 Comparison when the flux density is sinusoidal.

Magnet with Shading Coil

The HBFEM also is applied to the field calculation of the electromagnet in Fig.7. The middle leg has an air-gap and shading coils. The magnetization characteristic of the iron core is the same as in Fig 4. The parameters for the calculation are given as,

$$J_{1s}=5.0*10^6 \ A/m^2, \quad J_{1c}=J_{3s}=J_{3c}=0.0,$$
$$\sigma = 3.55*10^7 \ S/m.$$

The distributions of the fundamental and third harmonic component in Fig.7 are drawn at a particular phase. The HBFEM directly can give the component of each harmonic. The magnetic flux is delayed inside the shading coil and decreases with increasing the frequency. The third harmonic of the flux flows outside of the shading coil because the saturation generates the harmonics.

## CONCLUSIONS

The harmonic balance FEM for the time-periodic field distribution with magnetic saturation characteristics was described.

The features of the HBFEM are summaried as,

(1) The procedure of the calculation is the same as the static nonlinear FEM.

(2) The calculation of the time component is not included and the distribution of each harmonic component is directly obtained.

Comparison with corresponding results obtained from static FEM solutions confirmed the validity of this approach.

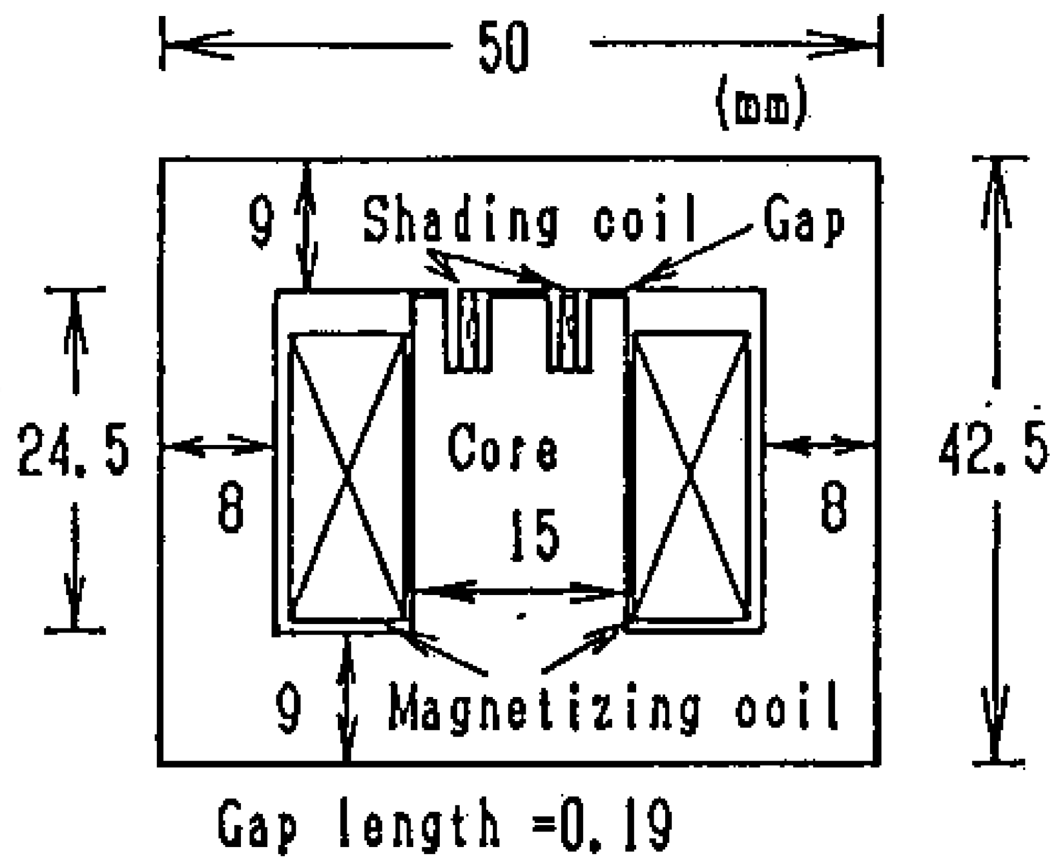
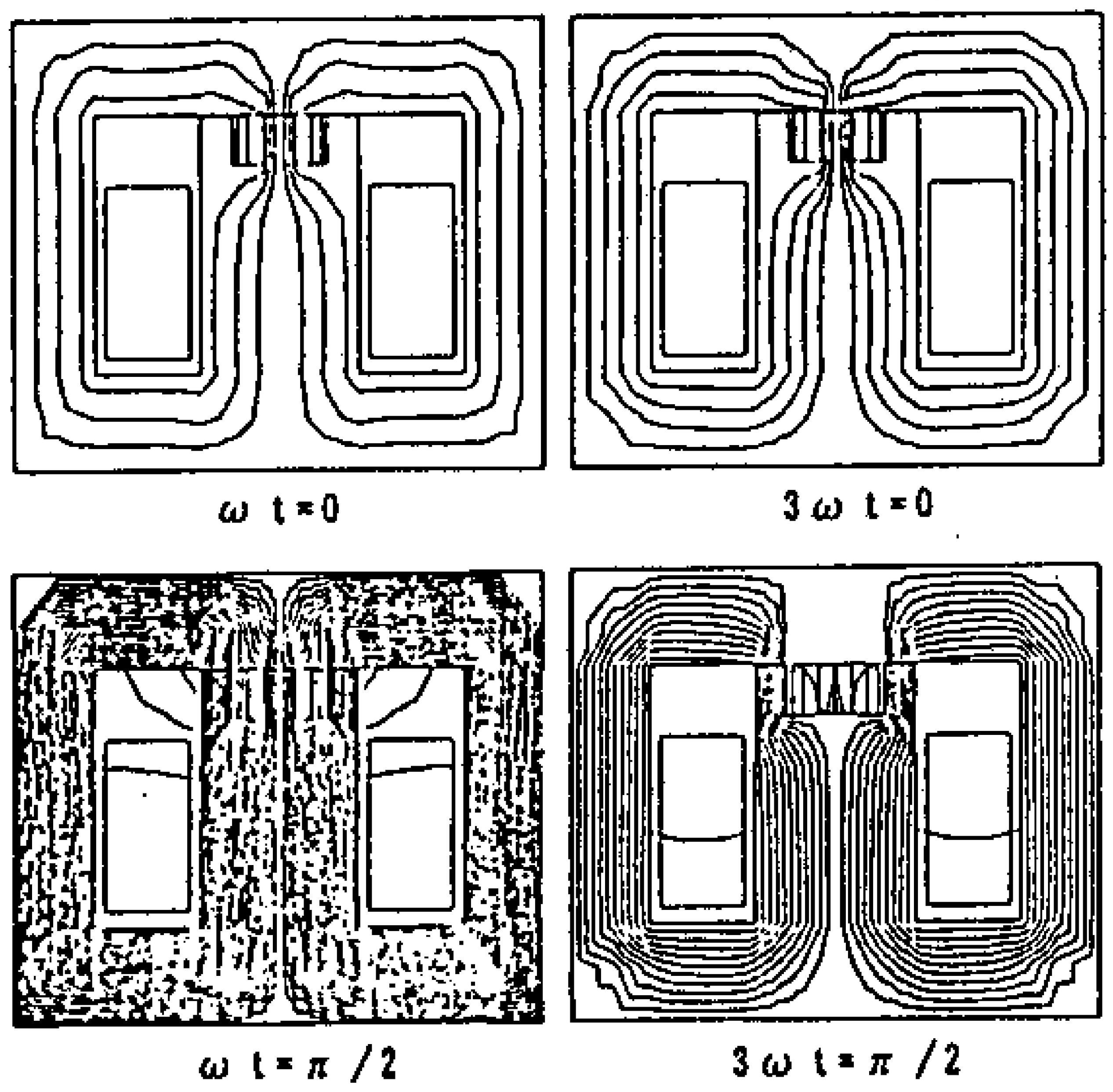
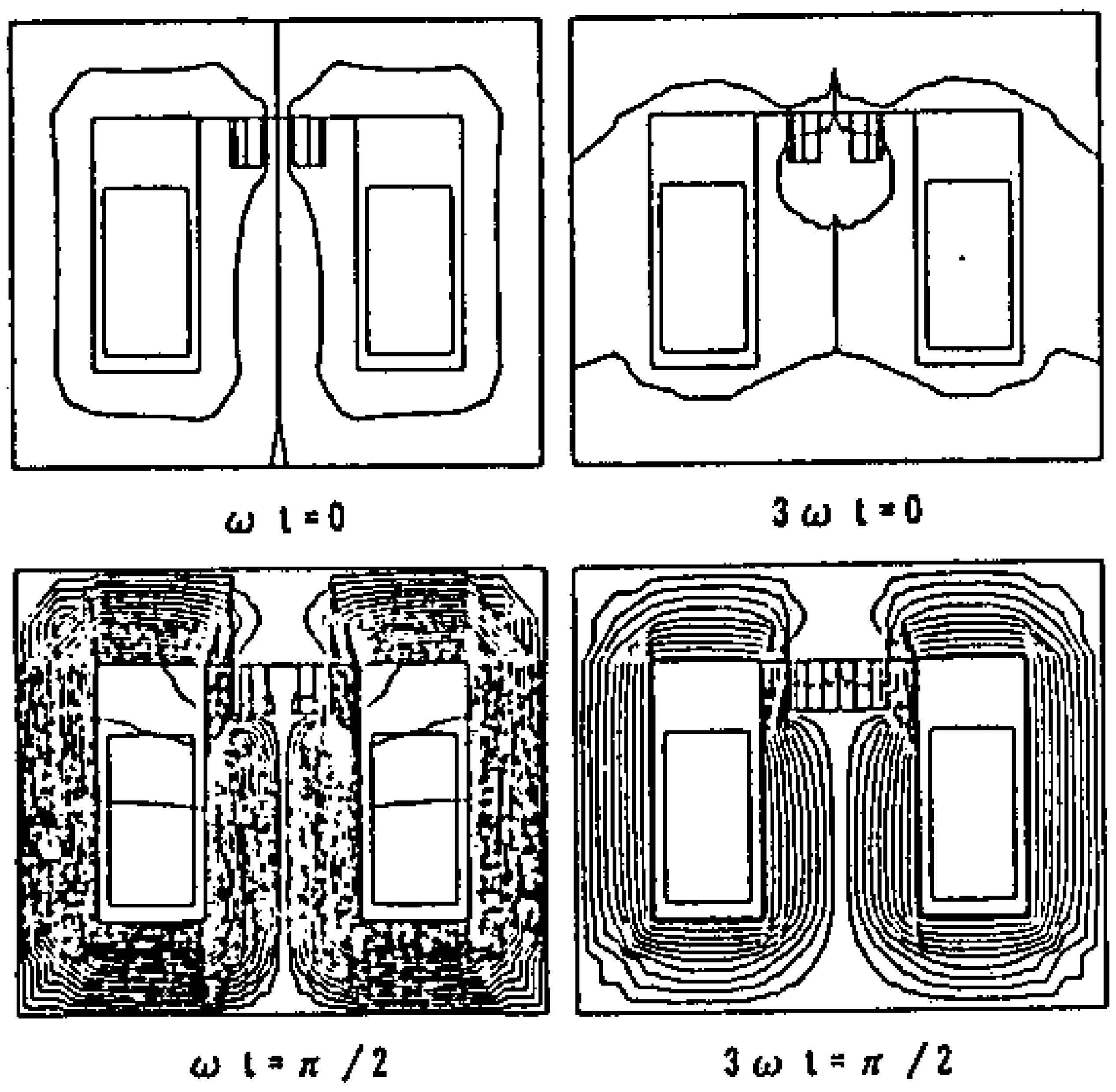



Fig.7 Electromagnet with a shading coil




(a) Fundamental component (b) Third harmonic component

Fig.8 Flux distributions when the frequency of a magnetizing current is 60 Hz.




(a) Fundamental component(b) Third harmonic component

Fig.9 Flux distributions when the frequency of a magnetizing current is 1000 Hz.

## REFERENCES

[1] C.Hayashi : Nonlinear Oscillations in Physical System, McGraw Hill, 1964, Ch. 1 & 5.

[2] S.Yamada, K.Bessho :"Harmonic Field Calculation by the Combination of Finite Element Analysis and Harmonic Balance Method", The 4th Joint MMM-INTERMAG Conf., ED-10, 1988

[3] P.Silvester, M.V.K.Chari :"Finite Element Solution of Saturable Magnetic Field Problem", IEEE Trans. PAS, Vol.PAS-89, No.7, pp1642-1649, 1970.