Water evaporation rate from a radiation-heated porous material considering a local variation of matric potential

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Abstract

In a previous study (Ichikawa *et al*., 2008), we found experimentally and confirmed theoretically that a porous column, the primary part of a solar pump, enhanced water evaporation. The evaporation rate from the surface of a sand column was higher than that from the water surface. Our previous study also showed that there was an optimum height of the sand column at which the rate of evaporation was the maximum. Experimental results for the water evaporation rate changed continuously with increasing water table depth and had a maximum at a particular depth value. The evaporation rates calculated from a mathematical model based on Philip and de Vries (1957), however, changed discontinuously at a particular value of water table depth at which maximum evaporation occurred. Assuming that the local evaporation rate on the column surface varies, following a statistical normal distribution, we computed its average over the column surface. The rates thus computed changed continuously with increasing water table depth, which agreed well with the experimental results. We therefore conclude that the discrepancy in the previous study arose because the experimental results are averages over the surface, while those theoretically calculated without considering the statistical distribution are merely local values on the surface.

Key words: Matric potential, Porous materials, Solar pump, Stochastic model, Water evaporation rate.

1. Introduction

A solar pump is a device that distills water while it is being transported. When a column composed of a porous material is immersed in a water reservoir, it absorbs and transmits water. Water thus transported along the water potential gradient evaporates at the end of the column where solar radiation is absorbed. This water is then recovered using a condensing cover. In our previous study (Ichikawa *et al*., 2008), we found experimentally that a sand column, a primary component of a solar pump, enhanced evaporation. In order to consider practical applications for this system, it is necessary to construct a quantitatively explainable model. Based on the above viewpoint, we theoretically confirmed the above experimental results by applying the mathematical model of simultaneous heat and moisture transfer in porous material proposed by Philip and de Vries (1957).

In our previous study, we also found that there was an optimum height of the sand column at which the rate of evaporation attained its maximum. Quantitatively, however, the experimentally determined relationship between the evaporation rate and the height of the column differed from that theoretically computed. The height of the sand column, denoted by l, is the distance between the column surface and the water level of the reservoir, *i.e.*, water table depth. The theoretical results showed that the water evaporation rate increased discontinuously when the water table depth l was around 0.1 m. In contrast, the experimental results changed continuously with increasing l.

The following reasons for the abovementioned discrepancy were hypothesized in Ichikawa *et al*. (2008). Because of the highly nonlinear relationship between the matric potential and the water content

of sand, a slight change in the matric potential may cause a drastic change in *l*. Such a change would result in a large change in the evaporation rate. The theoretical results in our previous study were based on these highly sensitive relationships. On the other hand, the evaporation rates in the experiments were measured as the average over the column surface. They should therefore be less sensitive to change in the matric potential than the theoretical values would be, thus showing no discontinuity in the relationship between the evaporation rate and *l*.

To date, no satisfactory theory exists for predicting the moisture characteristic curve, which is generally measured experimentally (*e.g.*, Hillel, 1971; Nakano, 1991). The moisture characteristic curve is obtained macroscopically by using a soil sample of which volume is greater than the representative elementary volume (Bear, 1991); however, from a microscopic viewpoint, it seems appropriate to manipulate it as a stochastic process. This stochastic behavior will be remarkable, particularly in the low-moisture region, where the matric potential varies drastically with change in moisture content.

The necessity of stochastic interpretation for the moisture characteristic curve of soil on a farmland scale was pointed out by Berge (1990), and it arises because of horizontal difference in soil texture. A stochastic treatment was reported by Dagan and Bresler (1979), who studied solute transport in infiltration processes under the spatial variation of soil texture. Haraguchi (1999) discussed the nonuniformity of soil physical properties in paddy fields. As far as the present authors know, however, there is no report describing a stochastic effect on the evaporation rate from a porous medium with a macroscopically uniform texture.

In this paper, we examine the abovementioned difference, in the relationship of evaporation rate and *l*, between experimentally measured results and the theoretically calculated results. Assuming that the local evaporation rate varies on the column surface and follows a statistical normal distribution of matric potential, we computed its average over the column surface. The rates thus computed were then compared with the experimental results.

2. Theoretical Consideration

2.1 Interpretation of noncontinuous change in the theoretical water evaporation rate with water table depth *l*

The simulation results in the earlier paper (Ichikawa *et al.*, 2008) showed a drastic change in volumetric moisture content θ_l at the surface of the sand column when *l* was about 0.1 m. The mechanism underlying this result is interpreted as follows. In the simultaneous transfer of heat and moisture in porous media, such as sand, because transport properties such as hydraulic and thermal conductivities are nonlinear functions of temperature and volumetric moisture content, both of these themselves also change nonlinearly with *l*. Therefore, when *l* is substantially small, the volumetric moisture content at the surface of the sand column remains sufficiently high to satisfy the conditions for potential evaporation. As *l* increases above a threshold, however, volumetric moisture content near the surface drops drastically. Under the same irradiation conditions, the height under which the driving force for moisture transfer becomes maximum owing to surface drying is believed theoretically to be 0.1 m for this sand column. Considering additional experimental results (provided in a later section), this is reasoned to be 0.07-0.08 m. In other words, the sand column is sufficiently moist, and only a wet region exists until *l* increases to 0.07-0.08 m; however, a dried-out region coexists with the wet region because drying near the surface proceeds as *l* increases. In the dried-out region, radiant heat is absorbed and accumulated, so that the temperature rises greatly because of a low heat capacity. Because the dried-out region resists vapor transfer, its development inhibits water evaporation and rapidly lowers the evaporation rate.

Formation of the dried-out layer depends on whether water reaches the surface of the sand column. The sand column sucks water up to a certain height (0.1 m for the experiments in the previous paper; however, it seemed to be 0.07-0.08 m upon later consideration), above which the sand cannot draw water to the surface. This means that a principal factor of this phenomenon is the water suction ability of the sand column, *i.e.*, its matric potential. If moisture transfer is controlled by liquid water transfer, hydraulic conductivity is also related to moisture transfer. In fact, moisture transfer is controlled by water vapor transfer near the sand surface; however, the influence of hydraulic conductiv-

ity on the water evaporation rate may be small.

2.2 Difference between the experimental results and those calculated from the model in the earlier paper

The somewhat different tendency of the experimental results for the relationship between l and the water evaporation rate, compared to the theoretically calculated results, may be explained as follows:

The moisture characteristic curve is shown in Fig. 1, which is the same as Fig. 3 in the earlier paper. Measurement of matric potential (Φ_p) was performed using a soil column method for $-2<\log_{10}(-\Phi_p)<0$, a centrifugation method for $0<\log_{10}(-\Phi_p)<2$, and a vapor pressure method for $2<\log_{10}(-\Phi_p)$. A corresponding equation based on Matsukawa and Sohma (1983) is also plotted in this figure.

According to Fig. 1, experimental results for the matric potential of the sample sand at an identical volumetric moisture content have an error of ±20% around the value estimated from the corresponding equation, particularly in the range $0<\theta_l<0.2$. This range of volumetric moisture content corresponds to the extent of change in the volumetric moisture content near the sand surface. According to the simulation results of simultaneous heat and moisture transfer obtained via Philip and de Vries's model, the value of l at which the moisture evaporation rate shows a maximum is 0.08 m in the case of $0.8\Phi_p$ (corresponding to a 20% smaller value of the matric potential than the criterion Φ_p), and 0.115 m in the case of $1.2\Phi_p$ (corresponding to a 20% larger value of the matric potential than the criterion Φ_p). The values 0.115 m
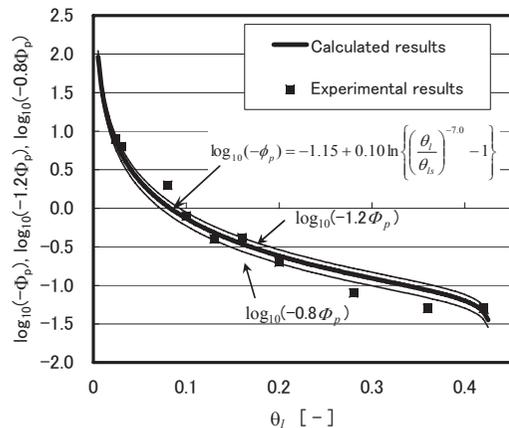
and 0.08 m are different from the standard value of 0.1 m. From the above, it is plausible to reason that local variation in Φ_p, due to the microscopic difference in the height of the evaporation front for individual pores, greatly affects the water evaporation rate. The experimentally observed water evaporation rates in the earlier paper are the averages of the local values obtained via Philip and de Vries's model over the entire surface of the sand column. In other words, the actual evaporation rate is assumed to be the average of the local values, the value of expectation of the water evaporation rate, supposing that the matric potential Φ_p varies stochastically. Standard deviation of the matric potential is determined by trial and error so as to find the best fit to the experimental results.

2.3 Proposed model

In quantifying the above supposition, the following assumptions are set forth:

1) Water evaporation occurs nonuniformly over the sand column surface; that is, the evaporation rate varies locally.
2) Local variation of the water evaporation rate is due to local differences in the water retention characteristics of the column; that is, the relation between matric potential and moisture content.
3) The value of the matric potential over the sand surface follows a normal distribution around the average (expectation), which has a particular standard deviation.

We define the nondimensional matric potential Ψ as

$$\Psi\left(=\frac{\Phi_p}{\Phi_{p0}}\right) \tag{1}$$

where Φ_{p0} is a criterion measure, the value of the matric potential obtained from an experimental equation from Matsukawa and Sohma's model (1983):

$$\Phi_{p0}=-10^{-1.31+0.10\ln\left\{\left(\frac{\theta_l}{\theta_{ls}}\right)^{-7.0}-1\right\}} \tag{2}$$

where θ_{ls} is the saturated volumetric moisture content. The value of Φ_{p0} in Fig. 1, -1.15, is replaced by -1.31 in Eq.(2), considering that the value of l corresponding to the maximum water evaporation rate is 0.07-0.08 m as was pointed out in 2.2.

Based on the experimental results in the earlier paper, supposing Ψ is a stochastic variable with an average of 1.0 and standard deviation σ, $f(\Psi)$, the probability density function of Ψ, is as follows:

Fig. 1. Moisture characteristic curve for the sample sand.

$$f(\Psi) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(\Psi-1)^2}{2\sigma^2}} \qquad (3)$$

Setting the value of the local evaporation rate at the sand column surface estimated from Philip and de Vries's model to $Evp(\Psi, l)$, its expectation $[Evp]_{av}$, the average over the column surface, is given by

$$[Evp]_{av} = \int_0^\infty Evp(\Psi, l)\, f(\Psi)\, d\Psi \qquad (4)$$

In practice, when calculating Eq.(4), we divide $Evp(\Psi, l)$ into a finite number N of discrete parts for every small increment $\Delta\Psi$, and we approximate the right-hand side of Eq.(4) by the following summation form equation:

$$[Evp]_{ave} = \sum_{i=1}^{11} Evp(\Psi_i, l)\, f(\Psi_i)\, \Delta\Psi \qquad (5)$$

Changing the values of parameters in the equation of matric potential, simulations of simultaneous heat and moisture transfer processes using Philip and de Vries's model were performed, and steady-state water evaporation rates were estimated for each case of l. Applying these water evaporation rates to Eq.(5), its expectation, the average over the column surface, was obtained for all values of l. In this paper, we set $N=$ 11 and $\Delta\Psi=0.143$.

3. Experiment

The apparatus and procedure used in the experiment on water evaporation from the sand column surface are the same as those used in the previous study. An acrylic resin column of 50 mm diameter, 150 mm length, and 5 mm thickness into which 0.5 kg of sample sand had been packed was immersed in a water bath. A 100 W incandescent lamp as a radiation heat source was set at an appropriate height above the column so that the radiative heat flux at the column surface was controlled at a prescribed value, 4860 kJ m^{-2} h^{-1}. The period of each experiment was 72

h, by which time the moisture and heat transfer had reached steady state. Because there were no previous experimental data for l=0.06-0.09 m and the value of l showing a maximum evaporation rate could not be clearly known, supplementary experiments for water evaporation were performed in this study for l=0.06, 0.07, 0.08, and 0.09 m. Table 1 shows the experimental conditions and the measured values of the steady-state water evaporation rate.
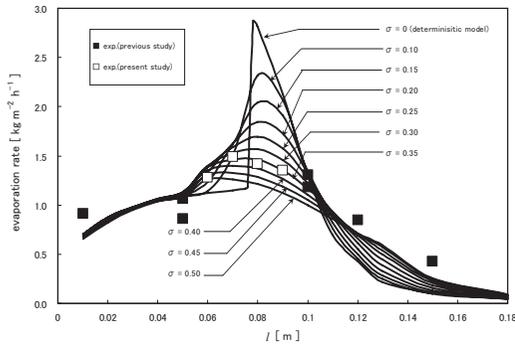
4. Results and Discussion

Fig. 2 shows the experimental results of the relationship between l and the evaporation rate obtained in the previous and present studies, along with the calculated results using the present model. Calculations were made under several values of σ, the standard deviation of the nondimensional matric potential, from 0.1[-] to 0.5[-], varying with an increment of 0.05[-], and the expectation of the water evaporation rate $[Evp]_{av}$ for each σ value was plotted in this figure. For comparison, the local evaporation rates calculated, as in the previous study, for the case of $\Phi_p = \Phi_{p0}$ are also shown in this figure. Experimental data are those from the previous study along with data from additional experiments.

Numerical simulation was performed under an increment of distance $\Delta z=5.0\times10^{-3}$ m and a time increment $\Delta t=1.0\times10^{-5}$ h. Validity of the calculated results was confirmed by fact that these results differed at most by 0.1% from the results obtained using a time increment of half the above value.

Transport properties used in the present theoretical calculations were the same as in the previous study (Table 2 in Ichikawa *et al.*, 2008), except for the mass transfer coefficient over the sand surface k_m. By trial and error, this was increased from 40 m h^{-1} to 60 m h^{-1} to bring the water evaporation rate as close to the measured rates as possible. The original value of k_m was estimated from an experimentally obtained equation for heat and moisture transfer (Seki and Komori, 1985). Because this equation has an error of ±50%, we deemed

Table 1. Experimental conditions and measured evaporation rate.

Run	Packed materials	Outcrop height above water level in a water bath l [m]	Temperature in a water bath T_l [K]	Room temperature (daily mean) T_a [K]	Water evaporation rate at the steady state [kg m^{-2} h^{-1}]
1	sand	0.06	295.0	295.0	1.27
2	sand	0.07	294.5	295.6	1.49
3	sand	0.08	295.0	295.0	1.42
4	sand	0.09	294.7	295.6	1.35

Fig. 2. Relationship between the steady-state evaporation rate and the distance *l* between the column surface and the water level in the reservoir.

the above modification of k_m to be justifiable.

According to the present experimental results, water evaporation rate shows a maximum of 1.49 kg m^{-2} h^{-1} when *l*=0.07 m. This maximum value is a half of a peak evaporation rate 2.8 kg m^{-2} h^{-1}, obtained when *l*=0.08 m by the deterministic model described in the earlier paper. On the other hand, according to the calculated results from Eq.(4), the relationship of evaporation rate *vs. l* is influenced by σ when 0.05 m < *l* < 0.15 m, but hardly influenced at all when *l* < 0.05 m. When *l* > 0.15 m, the evaporation rate is influenced by σ, but not substantially, because the evaporation rate is small. When the value of *l* is in the range of 0.05 m to 0.1 m, the maximum evaporation rate decreases gradually with increasing σ; in contrast, the rate gradually increases with increasing *l* from 0.1 m to 0.15 m. In other words, the probability of matric potential diffuses extensively as σ increases, and therefore, any influence of the value of *l* on the average water evaporation rate becomes alleviated.

According to the experimental results, the most suitable value of σ is 0.35[-], at which the calculated water evaporation rates conform most to the experimental results in the range of 0.06 m < *l* < 0.08 m, within which the evaporation rate is relatively large. In the range of *l* > 0.12 m, the calculated results are slightly smaller than the measured results, although both results show a similar tendency of decrease with increasing *l*. On the other hand, at *l*=0.01 m, the measured result is about 20% greater than the calculated result, and σ seems to be larger than 0.5[-]. The reason for the variation of σ with *l* cannot be adequately clarified here; however, the assumption of the probability density

function of matric potential as a normal distribution may be one cause. Calculation of the evaporation rate under σ=0.35[-] appears to be the best approximation for all the measured results obtained in the previous and present studies.

The value of σ was determined by a curve-fitting technique between the experimental results and theoretical calculation. Although it should be related to the degree of scattering of matric potential, the value of σ is not assured to be the same for any other sand of different particle-size distribution from the sand used in this study.

As shown in the earlier paper, the difference in the size of apparatus, that is, the container diameter, has no influence on the evaporation rate *vs. l* relationship. This means that the value of *l* under which the evaporation rate shows a maximum will remain the same even for cases of different container diameters, as long as the probability density function of matric potential is the same. The selection of porous material as a water transfer medium is a key to this system; the value of σ as a material characteristic must be examined in advance for various types of porous materials.

5. Conclusions

Assuming that the matric potential of the sand column is distributed normally, we computed the expectation (average) of the evaporation rate over the column surface. Expected values thus computed changed continuously with water table depth *l*, agreeing closely with the experimental results. Hence, we conclude that the water content at, and thus the evaporation rate from, the sand column surface may generally distribute unevenly. Given an appropriate estimate for the mass transfer coefficient and the water retention characteristics, our stochastic model proposed in this study is capable of dealing with such uneven distribution.

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