Optimization of Grasping an Object by Using Required Acceleration and Equilibrium-Force Sets

著者	Watanabe Tetsuyou, Yoshikawa Tsuneo
journal or publication title	Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)
number	2003
page range	338-343
year	2003-01-01
URL	http://hdl.handle.net/2297/35232

Optimization of Grasping an Object by Using Required Acceleration and Equilibrium-Force Sets

Tetsuyou Watanabe and Tsuneo Yoshikawa
Department of Mechanical Engineering, Kyoto University
Kyoto, 606-8501 Japan
te-watanabe@ieee.org, yoshi@mech.kyoto-u.ac.jp

Abstract

In this paper, we search optimal grasp points and configurations of fingers for not only resisting an external force applied to a grasped object but also generating a desirable acceleration of the object. Based on the concept of required external force set, we define required acceleration and equilibrium-force sets. By using the sets, we formulate an optimization problem from the viewpoint of decreasing the magnitudes of the joint torques required to generate the required acceleration and equilibrium-force. We also show that we can solve the problem by using a branch-and-bound method. The validity of our approach is shown by numerical examples.

1 INTRODUCTION

When we grasp an object by a robotic hand, force closure is one of the important properties of grasping [1]. Force closure is the concept which we can interpret in the following two ways; " any arbitrary acceleration and angular acceleration of a grasped object can be generated by joint actuating torques. " or " the motion of a grasped object can be completely constrained by virtue of contact forces, whatever external force and moment are applied to the object. " Yoshikawa [2] called the former concept active closure, and the latter concept passive closure when the motion of the object can be constrained without changing the pre-loaded joint torques. In the case where there exist multiple contact points between a finger and an object (for example, enveloping grasp), or the case where the number of joints of a finger is smaller than the number of the dimension of contact force applied by the finger, the contact force between the finger and the object can be generated not actively by the joint torques but passively by the mechanism of the geometric constraints. In this case, even if we can resist certain external force and moment in a certain direction and the motion of the object can be completely constrained, there is no guarantee that we can generate an acceleration of the object in the same direction. So far, there doesn't exist any researches about searching optimal grasp points on a grasped object, in which the above two concepts are distinguished [3]–[8].

In order to deal with this problem, we deal with searching optimal both configuration of fingers and contact positions for not only resisting an external force applied to a grasped object but also generating a desirable acceleration of the object.

This paper is organized as follows. At first, we set the problem to solve. Then, we formulate the problem, and show an algorithm to solve the problem. Lastly, numerical examples are presented to show the effectiveness of our approach.

2 PROBLEM DEFINITION

In this section, we define an optimization problem of grasping. At first, we describe the target system. Then, we define required acceleration and equilibrium-force sets. Lastly, we give the optimization problem of grasping.

2.1 Target System

The target system is shown in Fig.1. In this paper, we consider the case where an arbitrary shaped rigid object is grasped by N fingers of a robotic hand (Note that $N = 2$ in Fig.1). We make the following assumptions: 1) Each finger makes a frictional point contact with the object. 2) The unique normal direction at each contact point can be obtained. 3) There exists at most one contact point on each link of each finger. 4) Base doesn't make any contacts with the object. 5) In the manipulation of the object, both the number of contact points and the contact positions on the object/fingers don't change (We don't consider the manipulation in which a certain contact point removes from the object nor in which a certain point on a certain finger, which isn't a contact point, makes a new contact with the object). Under these assumptions, we consider the case where in a given workspace, we generate a desirable acceleration of the object, resisting external force and moment, such as gravitational force.

2.2 Required Acceleration and Equilibrium-Force Sets

In this section, we define required acceleration and equilibrium-force sets based on the concept of required external force set [9] [10].

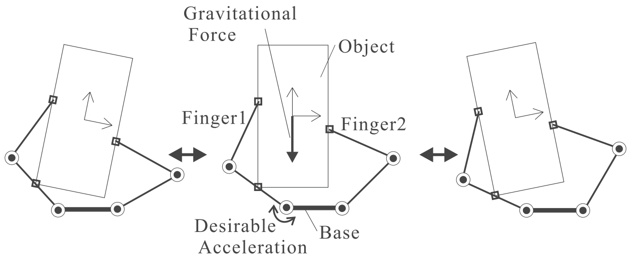

Figure 1: Target System

Required Acceleration Set (RAS) *We call a set, composed of desirable accelerations which the grasped object must be able to generate, required acceleration set* $\mathcal{A}_R \subset \mathcal{R}^D$*(D=* $3/6$ *in 2/3 dimensional space).*

Required Equilibrium-Force Set (REFS) *We call a set, composed of necessary resultant forces and moments to counteract external forces and moments and to maintain the current position and orientation of the object, required equilibrium-force set* $\mathcal{W}_R \subset \mathcal{R}^D$.

2.3 Problem to Solve

In general, there exist an infinite number of both configurations of fingers and contact positions, where we can generate the required acceleration and equilibrium-force. We think it is suitable to minimize the magnitude of the joint torques required to generate the required acceleration and equilibrium-force. Then, we consider the following problem.

Find the configuration of fingers and the contact positions, which minimize the magnitude of the necessary joint torques to generate both any arbitrary acceleration contained in the given RAS and any arbitrary equilibrium-force contained in the given REFS, at each position and orientation of the object in the given workspace.

3 FORMULATION OF THE PROBLEM

In this section, we formulate the problem described above. At first, we consider the case where the object is at certain position and orientation, and the fingers are in a certain configuration and grasp the object at certain contact points. The conditions for the joint torques to generate certain required acceleration and equilibrium-force in this case are described. Then, we formulate the optimization problem of grasping.

3.1 Conditions for Joint Torques

Let $\boldsymbol{q}_i \in \mathcal{R}^{M_i}$ $(i = 1, 2, \cdots, N)$ be the joint angles of the i th finger. Let $\boldsymbol{r} \in \mathcal{R}^D$ be the position of the origin and orientation of, Σ_O, the object coordinate frame fixed at the center of gravity of the object. Let $\boldsymbol{p}_{C_{ij}} \in \mathcal{R}^d (j = 1, 2, \cdots, L_i)$ be the position of, C_{ij}, the j th contact point between the object and the i th finger. Here M_i denotes the number of the joints of the i th finger, d= 2/3 in 2/3 dimensional space, and L_i denotes the number of the contact points on the i th finger. Note that we represent the combination of configuration of the N fingers and the L $(= \Sigma_{i=1}^{N} L_i)$ contact positions as $\mathcal{C}$.

3.1.1 Kinematic Constraints

The relation between the velocities of $\boldsymbol{p}_{C_{ij}}$ and $\boldsymbol{q}_i$, and the one between the velocities of $\boldsymbol{p}_{C_{ij}}$ and $\boldsymbol{r}$ are given as follows respectively.

$$\dot{\boldsymbol{p}}_{C_{ij}} = \boldsymbol{J}_{ij}\dot{\boldsymbol{q}}_i, \quad \dot{\boldsymbol{p}}_{C_{ij}} = \boldsymbol{G}_{ij}^T\dot{\boldsymbol{r}} \tag{1}$$

where $\boldsymbol{J}_{ij} \in \mathcal{R}^{d\times M_i}$ denotes the Jacobian matrix and $\boldsymbol{G}_{ij} = (\,\boldsymbol{I} \quad [(\boldsymbol{p}_{C_{ij}} - \boldsymbol{p}_o)\times]\,)^T$ in 3 dimensional space. Here, $\boldsymbol{I}$ represents an identify matrix, $\boldsymbol{p}_o$ represents the origin of Σ_O, and $[\boldsymbol{a}\times]$ represents a skew symmetric matrix equivalent to the cross product operation ($[\,\boldsymbol{a}\times]\boldsymbol{b} = \boldsymbol{a}\times\boldsymbol{b}$).

Let M be $\Sigma_{i=1}^{N} M_i$. By using the following vectors and matrices,

$$\dot{\boldsymbol{q}} = (\ \dot{\boldsymbol{q}}_1^T \quad \dot{\boldsymbol{q}}_2^T \quad \cdots \quad \dot{\boldsymbol{q}}_N^T\ )^T \in \mathcal{R}^M$$
$$\boldsymbol{G} = (\ \boldsymbol{G}_{11} \quad \boldsymbol{G}_{12} \quad \cdots \quad \boldsymbol{G}_{NL_N}\ ) \in \mathcal{R}^{D\times Ld}$$
$$\boldsymbol{J} = \mathrm{diag}\left(\begin{pmatrix}\boldsymbol{J}_{11}\\ \vdots \\ \boldsymbol{J}_{1L_1}\end{pmatrix} \begin{pmatrix}\boldsymbol{J}_{21}\\ \vdots \\ \boldsymbol{J}_{2L_2}\end{pmatrix} \cdots \begin{pmatrix}\boldsymbol{J}_{N1}\\ \vdots \\ \boldsymbol{J}_{NL_N}\end{pmatrix}\right) \in \mathcal{R}^{Ld\times M}$$

where diag means a block diagonal matrix, we get the following expression from (1)

$$\boldsymbol{G}^T\dot{\boldsymbol{r}} = \boldsymbol{J}\dot{\boldsymbol{q}} \tag{2}$$

Differentiating both sides of this equation with respect to time, we can get the following relation.

$$\boldsymbol{G}^T\ddot{\boldsymbol{r}} + \dot{\boldsymbol{G}}^T\dot{\boldsymbol{r}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} \tag{3}$$

3.1.2 Statics

Let $\boldsymbol{f} \in \mathcal{R}^{Ld}$ be the contact force vector which combines the contact forces at all contact points, $\boldsymbol{\tau}_c \in \mathcal{R}^M$ be the joint torque equivalent to $\boldsymbol{f}$, and $\boldsymbol{w} \in \mathcal{R}^D$ be the resultant force and moment applied to the object at Σ_O. From (2) and the principle of virtual work, we get the following relations.

$$\boldsymbol{\tau}_c = \boldsymbol{J}^T\boldsymbol{f}, \quad \boldsymbol{G}\boldsymbol{f} = \boldsymbol{w} \tag{4}$$

3.1.3 Frictional Constraint

The frictional constraint at the j th contact point on the i th finger is represented by

$$\mathcal{F}_{fij} = \{\boldsymbol{f}_{ij} | \sqrt{t_{f_{ij},1}^2 + t_{f_{ij},2}^2} \le \mu_{ij} n_{f_{ij}},\ n_{f_{ij}} \ge 0\} \tag{5}$$

in 3 dimensional space. Here, $n_{f_{ij}}$ denotes the normal component of the contact force $\boldsymbol{f}_{ij}$, $t_{f_{ij},1}$ and $t_{f_{ij},2}$ denote the tangential components of the contact force $\boldsymbol{f}_{ij}$, and μ_{ij} denotes the frictional coefficient at the contact point.

3.1.4 Equation of Motion

From (4), the equation of motion of the object and the fingers can be represented as follows respectively.

$$\boldsymbol{M}_B\ddot{\boldsymbol{r}} + \boldsymbol{h}_B - \boldsymbol{w}_e = \boldsymbol{G}\boldsymbol{f} \quad (6)$$

$$\boldsymbol{M}_r\ddot{\boldsymbol{q}} + \boldsymbol{h}_r + \boldsymbol{g}_r + \boldsymbol{J}^T\boldsymbol{f} = \boldsymbol{\tau} \quad (7)$$

where $\boldsymbol{\tau}$ denotes the joint torques, $\boldsymbol{M}_B$ and $\boldsymbol{M}_r$ are the inertia tensors of the object and the fingers respectively, $\boldsymbol{h}_B$ and $\boldsymbol{h}_r$ are the terms representing centrifugal and Coriolis forces of the object and the fingers respectively, $\boldsymbol{g}_r$ is the term representing gravitational force of the fingers, and $\boldsymbol{w}_e$ denotes external force and moment applied to the object, such as gravitational force. Note that the force, which counteracts $\boldsymbol{w}_e$, is the required equilibrium-force $\boldsymbol{w}_d$ ($\boldsymbol{w}_d + \boldsymbol{w}_e = \boldsymbol{o}$).

3.1.5 Conditions for Joint Torques

Based on the above discussion, we consider the conditions for the joint torques to generate both certain required acceleration $\ddot{\boldsymbol{r}}_d$ (which corresponds to $\ddot{\boldsymbol{r}}$ in (3) and (6)) and certain required equilibrium-force $\boldsymbol{w}_d$, when the position and orientation of the object are $\boldsymbol{r}$ and the combination of configuration of fingers and contact positions is $\mathcal{C}$. From (3), (5), (6), and (7), The condition is that there exist joint torques, $\boldsymbol{\tau}$, which satisfy the following constraints.

$$\boldsymbol{A}_1\boldsymbol{\tau} + \boldsymbol{A}_2\boldsymbol{f} = \boldsymbol{G}^T\ddot{\boldsymbol{r}}_d + \boldsymbol{a} \quad (8)$$

$$\boldsymbol{G}\boldsymbol{f} = \boldsymbol{M}_B\ddot{\boldsymbol{r}}_d + \boldsymbol{w}_d + \boldsymbol{h}_B \quad (9)$$

$$\boldsymbol{f}_{ij} \in \mathcal{F}_{fij}\ (j = 1, 2, \cdots, L_i)(i = 1, 2, \cdots, N) \quad (10)$$

where

$$\boldsymbol{A}_1 = \boldsymbol{J}\boldsymbol{M}_r^{-1},\ \boldsymbol{A}_2 = -\boldsymbol{J}\boldsymbol{M}_r^{-1}\boldsymbol{J}^T$$

$$\boldsymbol{a} = \boldsymbol{J}\boldsymbol{M}_r^{-1}(\boldsymbol{g}_r + \boldsymbol{h}_r) - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} + \dot{\boldsymbol{G}}^T\dot{\boldsymbol{r}}$$

3.2 Formulation of the Problem

In this subsection, we formulate the optimization problem of grasping. Let $\mathcal{B}$ be the position and orientation of the object in the given workspace. Let $\mathcal{S}_B$ be the set of all possible candidates of $\mathcal{B}$. In the given workspace, if the position and orientation of the object change, the configuration of fingers also changes. Then, in order to clarify $\mathcal{C}$ to be optimized, let $\mathcal{B}_r$ be the representative $\mathcal{B}$ in the given workspace and $\mathcal{C}_r$ be the $\mathcal{C}$ at $\mathcal{B}_r$. Now, we assume that if $\mathcal{C}$ at $\mathcal{B}_r$, namely $\mathcal{C}_r$ is given, we can get each $\mathcal{C}$ at each $\mathcal{B} \in \mathcal{S}_B$. Meaning, each $\mathcal{C}$ at each $\mathcal{B} \in \mathcal{S}_B$ can be determined by the relation between the $\mathcal{C}_r$ and the each $\mathcal{B}$. Then, $\mathcal{C}_r$ can represent each $\mathcal{C}$ at each $\mathcal{B} \in \mathcal{S}_B$. Hence, optimizing $\mathcal{C}_r$ means optimizing each $\mathcal{C}$ at each $\mathcal{B} \in \mathcal{S}_B$. Let $\mathcal{S}_C$ be the set of all possible candidates of $\mathcal{C}_r$. Let $\mathcal{W}_{cB}$ be the combinational required set given at $\mathcal{B}(\in \mathcal{S}_B)$:

$$\mathcal{W}_{cB} = \{\boldsymbol{x} = \left(\ddot{\boldsymbol{r}}_d^T\ \boldsymbol{w}_d^T\right)^T | \ddot{\boldsymbol{r}}_d \in \mathcal{A}_R,\ \boldsymbol{w}_d \in \mathcal{W}_R\} \quad (11)$$

Let ϕ be the largest magnitude of joint torque among all magnitudes of all joint torques:

$$\phi = \max_{i,j} |\tau_{ij}| \quad (12)$$

where τ_{ij} denotes the j th joint torque of the i th finger. Then, the problem can be formulated as follows.

Optimization Problem of grasping Find the combination of configuration of fingers and contact positions, $\mathcal{C}_r^*$ and ρ such that,

$$\rho = \min_{\mathcal{C}_r \in \mathcal{S}_C}\ \max_{\boldsymbol{x} \in \mathcal{W}_{cB}, \mathcal{B} \in \mathcal{S}_B}\ \min_{\boldsymbol{\tau}, \boldsymbol{f}\ \text{satisfy(8)}\sim\text{(10)}} \phi \quad (13)$$

4 ALGORITHM

In this section, we describe an algorithm to solve the optimization problem of grasping (13). In order to solve the problem, we use a branch-and-bound method [11]–[13], by representing the candidate configurations of fingers as discrete configurations, the candidate contact positions as discrete positions, and the candidate positions and orientations of object as discrete positions and orientations.

We make the following assumptions: 1) The number of the candidate $\mathcal{C}_r$'s is finite n_c. Let $\mathcal{C}_{rk}$ ($k = 1, 2, \cdots, n_c$) be the $\mathcal{C}_r$ contained in $\mathcal{S}_C$. 2) The number of the candidate $\mathcal{B}$'s is finite n_b. Let $\mathcal{B}_I$ ($I = 1, 2, \cdots, n_b$) be the $\mathcal{B}$ contained in $\mathcal{S}_B$. Let $\mathcal{C}_{kI}$ be the $\mathcal{C}$ at $\mathcal{B}_I$ in the case of $\mathcal{C}_r = \mathcal{C}_{rk}$. If certain $\mathcal{C}_{rk}$ is given, each $\mathcal{C}$ at each $\mathcal{B}_I$ ($\mathcal{C}_{kI}$'s ($I = 1, 2, \cdots, n_b$)) can be uniquely obtained. 3) We consider only the case where the system is stationary state at each $\mathcal{B}_I$. Namely, the velocities of the all variables in the constraints (8) and (9) are assumed to be zero. 4) The RAS and REFS ($\mathcal{A}_{RI}$ and $\mathcal{W}_{RI}$) given at each $\mathcal{B}_I$ can be expressed by convex polyhedrons composed of l_a and l_e vertices respectively. 5) When the object is at a certain $\mathcal{B}_I$ and $\mathcal{C}$ is a certain $\mathcal{C}_{kI}$ (k and I are fixed), $\boldsymbol{g}_r$ in (7) can be expressed by a convex polyhedron composed of l_e vertices, whose element corresponds one to one to the element contained in $\mathcal{W}_{RI}$. Note that here, we may suppose the case where the direction of gravitational force can change with respect to Σ_O, resulting from the manipulation of the robotic arm equipped with the robotic hand.

We remark $\boldsymbol{g}_r$ (and $\mathcal{W}_{RI}$). We consider the case where the object is at a certain $\mathcal{B}_I$ and $\mathcal{C}$ is a certain $\mathcal{C}_{kI}$. Let $\mathcal{W}_{RI_g}$ be the set composed of the forces and moments, which are contained in $\mathcal{W}_{RI}$ and counteract gravitational force. Let $\mathcal{W}_{RI_o}$ be the set composed of the forces and moments, which are contained in $\mathcal{W}_{RI}$ and counteract the other external forces and moments. We assume that $\mathcal{W}_{RI_\nu}$($\nu = g$ or o) can be expressed by a convex polyhedron composed of $l_{e_\nu}(\leq l_e)$ vertices:

$$\mathcal{W}_{RI_\nu} = \{\boldsymbol{w} = \Sigma_{i=1}^{l_{e_\nu}}\lambda_{e_\nu i}\boldsymbol{w}_{I\nu_i},\ \Sigma_{i=1}^{l_{e_\nu}}\lambda_{e_\nu i} = 1,\ \lambda_{e_\nu i} \geq 0\}$$

where $\boldsymbol{w}_{I\nu_i}$ denotes the i th vertex of $\mathcal{W}_{RI_\nu}$. Then, $\mathcal{W}_{RI}$

can be expressed by

$$\mathcal{W}_{RI} = \{\boldsymbol{w} = \Sigma_{i=1}^{l_{eg}}\Sigma_{j=1}^{l_{eo}}\lambda_{e_{i,j}}(\boldsymbol{w}_{Ig_i} + \boldsymbol{w}_{Io_j}), \Sigma_{i=1}^{l_{eg}}\Sigma_{j=1}^{l_{eo}}\lambda_{e_{i,j}} = 1,\ \lambda_{e_{i,j}} \geq 0\} \quad (14)$$

Let $\tilde{\boldsymbol{g}}$ be gravitational acceleration vector and $\boldsymbol{w}$ be the correspondent element of the $\mathcal{W}_{RI_g}$ to the $\tilde{\boldsymbol{g}}$. The relation between $\tilde{\boldsymbol{g}}$ and $\boldsymbol{w}$ can be expressed by $-m_b\tilde{\boldsymbol{g}}= \boldsymbol{w}$ where m_b denotes the mass of the object. Therefore, by using $\tilde{\boldsymbol{g}}_i(= -\boldsymbol{w}_{Ig_i}/m_b)$ which corresponds to the i th vertex of the $\mathcal{W}_{RI_g}$, $\tilde{\boldsymbol{g}}$ can be expressed by the convex combination $\tilde{\boldsymbol{g}} =\Sigma_{i=1}^{l_{eg}}\lambda_{e_{g_i}}\tilde{\boldsymbol{g}}_i$, $\Sigma_{i=1}^{l_{eg}}\lambda_{e_{g_i}} = 1$, $\lambda_{e_{g_i}} \geq 0$. If $\mathcal{C}$ ($\mathcal{C}_{kI}$) doesn't change, $\boldsymbol{g}_r$ can be expressed by $\boldsymbol{A}\tilde{\boldsymbol{g}}$ where $\boldsymbol{A}$ is a certain constant matrix [14]. Then, by using $\boldsymbol{g}_{r_i}$ which corresponds to $\tilde{\boldsymbol{g}}_i$ which corresponds to the i th vertex of the $\mathcal{W}_{RI_g}$, $\boldsymbol{g}_r$ can be expressed by the convex combination $\boldsymbol{g}_r = \Sigma_{i=1}^{l_{eg}}\lambda_{e_{g_i}}\boldsymbol{g}_{r_i} = \Sigma_{i=1}^{l_{eg}}\Sigma_{j=1}^{l_{eo}}\lambda_{e_{i,j}}\boldsymbol{g}_{r_i}$, $\Sigma_{i=1}^{l_{eg}}\lambda_{e_{g_i}} = \Sigma_{i=1}^{l_{eg}}\Sigma_{j=1}^{l_{eo}}\lambda_{e_{i,j}} = 1$, $\lambda_{e_{g_i}}, \lambda_{e_{i,j}} \geq 0$. Therefore, $\boldsymbol{g}_r$ can correspond one to one to the element contained in $\mathcal{W}_{RI}$ given in (14).

In this paper, Let **Subproblem** 1 be the subproblem of the problem (13). In Subproblem 1, candidate $\mathcal{C}_r$ is fixed to a certain $\mathcal{C}_{rk}$ ($\in \mathcal{S}_C$) (k is fixed). If we can solve this Subproblem 1 for all $\mathcal{C}_{rk}$'s ($k = 1, 2, \cdots, n_c$), we can get the optimal solution of the problem (13). Note that in Subproblem 1, each $\mathcal{C}$ at each $\mathcal{B}_I$ (namely $\mathcal{C}_{kI}$'s ($I = 1, 2, \cdots, n_b$)) can be uniquely obtained. Let $\boldsymbol{x}_{I_{j\xi}}$ $\left(= \begin{pmatrix}\ddot{\boldsymbol{r}}_{d_{Ij}}^T & \boldsymbol{w}_{d_{I\xi}}^T\end{pmatrix}^T\right)$ be the combination of the required acceleration and equilibrium-force which correspond to the j th vertex of $\mathcal{A}_{RI}$ and the ξ th vertex of $\mathcal{W}_{RI}$ respectively. Let **Subproblem** 2 be the subproblem of Subproblem 1. In Subproblem 2, candidate $\mathcal{B}$ is fixed to a certain $\mathcal{B}_I$ ($\in \mathcal{S}_B$) and the candidate required acceleration and equilibrium-force are also fixed to a certain $\boldsymbol{x}_{I_{j\xi}}$ ((k,) I, j, and ξ are fixed). Note that when we solve Subproblem 2 for $\mathcal{B}_I$ (and $\boldsymbol{x}_{I_{j\xi}}$) which is a subproblem of Subproblem 1 for $\mathcal{C}_{rk}$, $\mathcal{C}$ is fixed to $\mathcal{C}_{kI}$. Let $\rho_{\mathcal{C}_{rk}, I_{j\xi}}$ be the solution of the Subproblem 2. Now, we consider the magnitude of the necessary joint torque ϕ (see(12)) to generate both an arbitrary acceleration contained in $\mathcal{A}_{RI}$ and an arbitrary equilibrium-force contained in $\mathcal{W}_{RI}$, in the case where $\mathcal{C}_r$ is a certain $\mathcal{C}_{rk}$ and $\mathcal{B}$ is a certain $\mathcal{B}_I$, namely $\mathcal{C}$ is $\mathcal{C}_{kI}$. Meaning, we consider Subproblem 2 for the $\mathcal{B}_I$, at the $\mathcal{C}_{rk}$. The constraints of Subproblem 2 is composed of linear equations, linear inequalities, and frictional conditions, which are all convex. Then, in this case, the necessary joint torque ϕ can be represented by the convex combination of $\rho_{\mathcal{C}_{rk}, I_{j\xi}}$'s ($k$ and I are fixed, $j = 1, 2, \cdots, l_a$, $\xi = 1, 2, \cdots, l_e$). Therefore, among all $n_b l_a l_e$ solutions of Subproblem 2, $\rho_{\mathcal{C}_{rk}, I_{j\xi}}$'s ($I = 1, 2, \cdots, n_b$, $j = 1, 2, \cdots, l_a$, $\xi = 1, 2, \cdots, l_e$), the largest value is the solution of Subproblem 1 (for $\mathcal{C}_{rk}$). Meaning, we can solve Subproblem 1 by solving Subproblem 2 for all $\boldsymbol{x}_{I_{j\xi}}$'s and all $\mathcal{B}_I$'s. Note that Subproblem 2 can be solved by simplex method, by approximating the friction corn by a polyhedral convex corn [15]. Based on the above discussion, we can solve the problem (13) by a branch-and-bound method.

4.1 Procedure of the Algorithm

In this subsection, we describe the algorithm to search the optimal both configuration of fingers and contact positions. Let $\hat{\rho}$ be the tentative optimal solution, $\rho_{\mathcal{C}_{rk}}$ be the optimal solution of Subproblem 1 for $\mathcal{C}_{rk}$, and $\hat{\rho}_{\mathcal{C}_{rk}}$ be its tentative optimal solution. We represent a list of feasible $\mathcal{C}_{rk}$'s as the LIST.

step 1 We put all candidate $\mathcal{C}_{rk}$'s, whose corresponding each $\mathcal{C}$ at each $\mathcal{B}_I$ (namely $\mathcal{C}_{kI}$'s ($I = 1, 2, \cdots, n_b$)) satisfies some conditions, into the LIST. The some conditions are, for example, no-interference between the object and the fingers, no-interference between the fingers, and limitation of the joint angles of the fingers. Let $\hat{\rho}$ be an appropriate lower bound value. Let each $\rho_{\mathcal{C}_{rk}}$ and $\hat{\rho}_{\mathcal{C}_{rk}}$ be appropriate upper and lower bound values respectively.

step 2 We solve Subproblem 1 for certain $\mathcal{C}_{rk}$ contained in the LIST.

step 3 If we can get the solution of the Subproblem 1 in step2, let $\hat{\rho}$ $(= \rho_{\mathcal{C}_{rk}} = \hat{\rho}_{\mathcal{C}_{rk}})$ be the solution, and let $\hat{\mathcal{B}}_I$ and $\hat{\boldsymbol{x}}_{I_{j\xi}}$, respectively, be $\mathcal{B}_I$ and $\boldsymbol{x}_{I_{j\xi}}$, which give the solution. Otherwise, we eliminate the $\mathcal{C}_{rk}$ form the LIST and go back to step2.

step 4 We solve Subproblem 2 for $\hat{\mathcal{B}}_I$ and $\hat{\boldsymbol{x}}_{\hat{I}_{j\xi}}$, at every $\mathcal{C}_{rk}$ contained in the LIST. If we cannot get the solution at some $\mathcal{C}_{rk}$, we eliminate this $\mathcal{C}_{rk}$ form the LIST. If we can get the solution $\rho_{\mathcal{C}_{rk}, \hat{I}_{j\xi}}$ at some $\mathcal{C}_{rk}$, we compute $\hat{\rho}_{\mathcal{C}_{rk}} = \max\{\hat{\rho}_{\mathcal{C}_{rk}},\ \rho_{\mathcal{C}_{rk}, \hat{I}_{j\xi}}\}$. If $\hat{\rho} <\hat{\rho}_{\mathcal{C}_{rk}}$, we eliminate this $\mathcal{C}_{rk}$ form the LIST.

step 5 Let $\widehat{\mathcal{C}}_{rk}$ be the $\mathcal{C}_{rk}$ at which $\rho_{\mathcal{C}_{rk}, \hat{I}_{j\xi}}$ is the least among those at all $\mathcal{C}_{rk}$'s contained in the LIST.

step 6 We solve Subproblem 1 for $\widehat{\mathcal{C}}_{rk}$. If we can get the solution, let $\rho_{\hat{\mathcal{C}}_{rk}}$ $(= \hat{\rho}_{\hat{\mathcal{C}}_{rk}})$ be the solution, and let $\hat{\mathcal{B}}_I$ and $\hat{\boldsymbol{x}}_{I_{j\xi}}$, respectively, be $\mathcal{B}_I$ and $\boldsymbol{x}_{I_{j\xi}}$, which give the solution. If $\hat{\rho} >\rho_{\hat{\mathcal{C}}_{rk}}$, $\hat{\rho} =\rho_{\hat{\mathcal{C}}_{rk}}$. If we cannot get the solution or $\hat{\rho} <\rho_{\hat{\mathcal{C}}_{rk}}$, we eliminate this $\widehat{\mathcal{C}}_{rk}$ form the LIST and go back to step5.

step 7 If we can get the relation $|\hat{\rho}-\rho_{\mathcal{C}_{rk}}| < \epsilon$ (ϵ denotes an arbitrary small positive value) for all $\mathcal{C}_{rk}$'s contained in the LIST, we finish the loop. Otherwise, we go back to step4.

5 NUMERICAL EXAMPLES

In order to show the effectiveness of our approach, we show some numerical examples in this section. The target system is shown in Fig.2(a). We set Σ_O is placed at the geometric center of the object. Note that the scale markings shown in Fig.2(a) represent the ones of the reference coordinate frame (the hand coordinate frame). Note also that

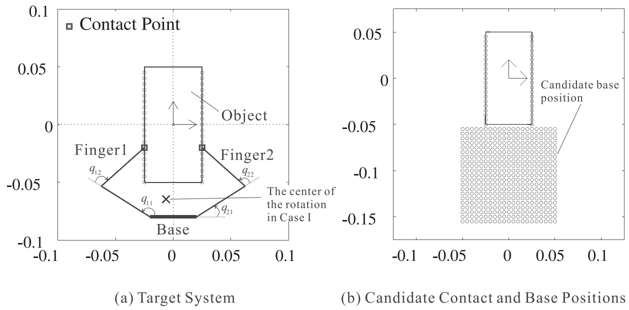

Figure 2: Target System in Numerical Examples

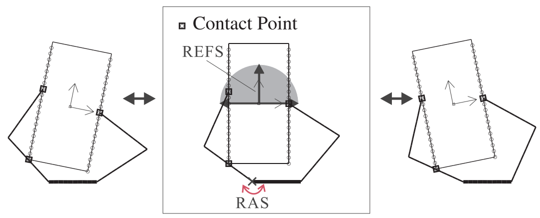

Figure 3: Target Operation with the Optimal Configuration in Case I

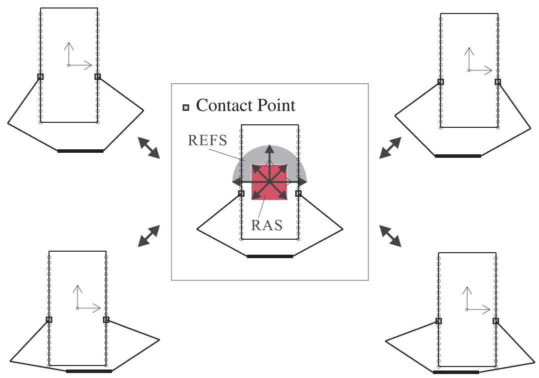

Figure 4: Target Operation with the Optimal Configuration in Case II

when the object is at the position and orientation shown in Fig.2(a), the position and orientation of the reference coordinate frame become the same as the ones of Σ_O. The robotic hand is composed of 2 fingers which are same form. We call the links of the fingers link 1 and link 2 in the order of closeness to the base side. The length of link i ($i = 1, 2$) is set to 0.05[m]. The mass of each link is set to 0.025[kg]. The limitations of the joint angles of the fingers are set as follows: $\pi/2 \leq q_{11} \leq \pi$, $-\pi \leq q_{12} \leq 0$, $0 \leq q_{21} \leq \pi/2$, $0 \leq q_{22} \leq \pi$, where q_{ij}([rad]) denotes the j th ($j = 1, 2$) joint angle of the i th ($i = 1, 2$) finger. In this numerical example, we represent the configuration of fingers as the position of base and the fingertip positions. Letting b_x([m]) and b_y([m]) be the x and y components of the position of the geometric center of the base respectively, we set the workspace of the base as follows (see Fig.2(b)): $-0.05 \leq b_x \leq 0.05$, $-0.155 \leq b_y \leq -0.055$. The number of the candidate positions of the base is set to 441 by giving the candidate positions with steps of 0.005 in the horizontal direction (x direction) and 0.005 in the vertical direction (y direction). The object is a 0.1([m])$\times$ 0.05([m]) quadrangle. The weight of the object is set to 4[N]. In Fig.2(b), the points on the object indicate the candidate contact points. The number of the candidate contact points is 40. The frictional coefficients at the contact points are set to 0.5. Note that we assume that the contact between link 1 of each finger and a vertex of the object can be represented by point contact if the intersection between the link and the vertex (object) is small.

At first, we consider Case I shown in Fig.3. This task is to rotate the object around the "X" mark, resisting gravitational force. Note that we suppose the case where the direction of gravitational force can be applied in multiple directions, resulting from the manipulation of the robotic arm equipped with the robotic hand. The "X" mark represents the point $(-0.005, -0.065)$ with respect to Σ_O (Note that it isn't easy to see the "X" mark. Then, we also show it in Fig.2(a)). The position and orientation of the object surrounded with a rectangle are $\mathcal{B}_r$ (This is the same in the following Case II). $\mathcal{B}_r$ is set to $(0, 0, 0)^T$. The candidate positions and orientations of the object are obtained as follows: the object, initially located at the $\mathcal{B}_r$, is rotated around the "X" mark over an interval of -0.2[rad] to 0.2[rad] with steps of 0.1[rad] ($n_b = 5$). We set the RAS is composed of angular accelerations around the X" mark $(-0.005, -0.065, 0)$, whose magnitude is 0.5[rad/s^2]. We set the REFS is a 33-side convex polyhedron which approximates the upward half circle whose radius is 4[N]. We show the RAS and the REFS at $\mathcal{B}_r$ as follows.

$$
\begin{aligned}
\mathcal{A}_{R1} &= \{\ddot{\boldsymbol{r}}_d = \lambda_1 \ddot{\boldsymbol{r}}_{d1} + \lambda_2 \ddot{\boldsymbol{r}}_{d2},\ \lambda_1 + \lambda_2 = 1,\ \lambda_i \geq 0, \\
&\qquad \ddot{\boldsymbol{r}}_{d1} = (-0.0325,\ 0.0025,\ 0.5)^T,\ \ddot{\boldsymbol{r}}_{d2} = -\ddot{\boldsymbol{r}}_{d1}\} \\
\mathcal{W}_{R1} &= \{\boldsymbol{w}_d | \boldsymbol{w}_d = \Sigma_{i=0}^{32} \lambda_i \boldsymbol{w}_{di},\ \Sigma_{i=0}^{32} \lambda_i = 1,\ \lambda_i \geq 0, \\
&\qquad \boldsymbol{w}_{di} = (4\cos(i\pi/32),\ 4\sin(i\pi/32),\ 0)^T\}
\end{aligned}
$$

The obtained optimal configuration of fingers and contact positions are shown in Fig.3. The obtained ρ was 0.270[Nm]. For someone's information, the necessary joint torque ϕ, in the case where $\mathcal{C}_r$ is the one shown in Fig.2(a), was 0.439[Nm]. At the optimal configuration, we can take advantage of the contact forces which can be generated without changing the magnitudes of the joint torques, in order to generate the required equilibrium-force. We think it is a reason why the configuration shown in Fig.3 is good compared with the configurations, at which each finger contacts with the object at only one point.

Next, we consider Case II shown in Fig.4. This task is to move the object in some translational directions, resisting gravitational force whose direction and magnitude are the same ones in Case I. $\mathcal{B}_r$ is set to $(0,0,0)^T$. The candidate positions and orientations of the object are obtained as follows: the object, initially located at the $\mathcal{B}_r$, is translated over an interval of -0.01[m] to 0.01[m] with steps of 0.005[m] in x direction and an interval of -0.01[m] to 0.01[m] with steps of 0.005[m] in y direction ($n_b = 25$). We set the RAS is composed of accelerations whose magnitude is 0.5[rad/s^2] in x and y directions. We show the RAS at $\mathcal{B}_r$ as follows (Note that the REFS is the same one in Case I).

$$\begin{aligned}\mathcal{A}_{R1} \;=\; &\{\ddot{\boldsymbol{r}}_d|\ddot{\boldsymbol{r}}_d = \Sigma_{i=1}^4 \lambda_i \ddot{\boldsymbol{r}}_{di},\ \Sigma_{i=1}^4 \lambda_i = 1,\ \lambda_i \geq 0,\\ &\ddot{\boldsymbol{r}}_{d1} = (0.5,\ -0.5,\ 0)^T,\ \ddot{\boldsymbol{r}}_{d2} = (-0.5,\ -0.5,\ 0)^T,\\ &\ddot{\boldsymbol{r}}_{d3} = (0.5,\ 0.5,\ 0)^T,\ \ddot{\boldsymbol{r}}_{d4} = (-0.5,\ 0.5,\ 0)^T\}\end{aligned}$$

The obtained optimal configuration of fingers and contact positions are shown in Fig.4. The obtained ρ was 0.393[Nm]. In order to generate the required accelerations in this case, the number of the contact points between the object and each finger must be one. We think it is a reason why we can get such a result shown in Fig.4.

In the above examples, we use SELECT PRO JP 3YEAR /OX KER(CPU : AMD ATHLON 1.2GHZ) made by GATEWAY. The running time is about 15 seconds for Case I and about 3 minutes for Case II.

6 CONCLUSION

In this paper, we have dealt with an optimization of grasping for not only resisting an external force applied to a grasped object but also generating a desirable acceleration of the object. In order to achieve an optimal grasp in which the two interpretations of force closure are distinguished, we have defined required acceleration and equilibrium-force sets. By using the sets, we have formulated an optimization problem from the viewpoint of decreasing the magnitudes of the joint torques required to generate the required acceleration and equilibrium-force, and showed that we can solve the problem by using a branch-and-bound method. We have also presented numerical examples in order to show the validity of our approach.

REFERENCES

[1] A. Bicchi and V. Kumar : "Robotic grasping and contact: A review," Proc. of IEEE International Conference on Robotics and Automation, pp.348–353, 2000.

[2] T. Yoshikawa : "Passive and active closures by constraining mechanisms," Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control, vol.121, pp.418–424, 1999.

[3] X. Markenscoff and C. H. Papadimitriou : "Optimum grip of a polygon," The International Journal of Robotics Research, vol.8, no.2, pp.17–29, 1989.

[4] Y. Chen, I. D. Walker and J. B. Cheatham : "Grasp synthesis for planar and solid objects," Journal of Robotic Systems, vol.10, no.2, pp.153–186, 1993.

[5] B. Mirtich and J. Canny : "Easily computable optimum grasps in 2-D and 3-D," Proc. of IEEE International Conference on Robotics and Automation, pp.739–747, 1994.

[6] L. Mangialardi, G. Mantriota and A. Trentadue : "A three-dimensional criterion for the determination of optimal grip points," Robotics & Computer-integrated Manufacturing, vol.12, no.2, pp.157–167, 1996.

[7] M. Y. Wang and D. M. Pelinescu : "Optimizing fixture layout in a point-set domain," IEEE Transactions on Robotics and Automation, vol.17, no.3, pp.312–323, 2001.

[8] D. Ding, Y. Liu and M. Y. Wang : "Automatic selection of fixturing surfaces and fixturing points for polyhedral workpieces," IEEE Transactions on Robotics and Automation, vol.17, no.6, pp.833–841, 2001.

[9] Y. Yu, K. Takeuchi and T. Yoshikawa : "Optimization of robot hand power grasps," Proc. of IEEE International Conference on Robotics and Automation, pp.3341–3347, 1998.

[10] T. Yoshikawa, T. Watanabe and M. Daito : "Optimization of power grasps for multiple objects," Proc. of IEEE International Conference on Robotics and Automation, pp.1786–1791, 2001.

[11] M. Fukushima : Introduction to Mathematical Programming, Asakura Book Store, 1996(in Japanese).

[12] C. H. Papadimitriou and K. Steiglitz : Combinatorial Optimization: Algorithms and Complexity, Prentice-Hall, Inc., Englewood Cliffs, New jersey, 1982.

[13] L. Stocco, S. E. Salcudean and F. Sassani : "Fast constrained global minimax optimization of robot parameters," Robotica, vol.16, pp.595–605, 1999.

[14] T. Yoshikawa : Foundations of Robotics, MIT Press, Cambridge, 1990.

[15] J. Kerr and B. Roth : "Analysis of multifingered hands," International Journal of Robotics Research, vol.4, no.4, pp.3–17, 1986.