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# Mechanics of Hybrid Active/Passive-Closure Grasps

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**Abstract** - In this paper, we discuss the directions of active and passive force closures in hybrid active/passive-closure grasps. We show the directions are orthogonal to each other. We also discuss the magnitudes of the internal forces in the manipulation of the object. In hybrid active/passive-closure grasps, there exist two kinds of magnitudes of internal forces. One is the magnitude of internal forces, which changes if the object moves and the geometry of the fingers changes. The other is the one which don't change even when the object moves. We derive these two magnitudes.

**Index Terms** - Active/Passive Force Closure, Grasping, Internal Forces

## I. INTRODUCTION

When we grasp an object by a robotic hand, force closure is one of the important properties of grasping [1]. Force closure is the concept which we can interpret in the following two ways; "any arbitrary force and moment can be exerted on a grasped object." or "the motion of a grasped object can be completely constrained without changing the pre-loaded joint torques, whatever external force and moment are applied to the object." Yoshikawa [2] called the former concept active force closure, and the latter concept passive force closure. The former concept corresponds to that the fingers (limbs) can move the grasped object in arbitrary directions. Note that in the definition of active force closure [2], we can regard active force closure as included in passive force closure (active force closure is necessary but not sufficient for passive force closure [3]). But active force closure in itself corresponds to the motion of the grasped object. Then, in this paper, we put attention on this property of active force closure. On the other hand, passive force closure doesn't correspond to the motion of the grasped object. Consider the case where there exist multiple contact points between a finger and an object (for example, enveloping grasp). In this case, some contact forces between the finger and the object can be generated not actively by the joint torques but passively by the mechanism of the geometric constraints. Then, even if we can resist certain external force and moment in a certain direction and the motion of the object can be completely constrained, there is no guarantee that we can generate the motion of the object in the same direction.

There are some cases where certain directions of the grasped object correspond to active force closure and the other directions correspond to passive force closure. Yoshikawa [2] called such closures hybrid active/passive closure. In our previous paper [4], we show hybrid active/passive closure is optimal in a certain planning of grasping to manipulate a grasped object. But the relation between the directions of active force closure

and those of passive force closure is still not clear.

In this paper, we show the directions of active force closure are orthogonal to those of passive force closure in hybrid active/passive-closure grasps.

In hybrid active/passive-closure grasps, not only certain external forces and moments can be counteracted without changing the pre-loaded joint torques, but also the object can move in certain directions. When the object moves, the configurations of the fingers and the objects change. Then, the constant pre-loaded joint torques may or may not need to change for resisting external forces in the directions of passive force closure. This is related to the change of the internal forces. The internal forces are to satisfy the frictional constraints and to grasp the object stably. In the manipulation of the object, we have to control this internal force for the stable grasp. However, since hybrid active/passive-closure grasps have properties of passive force closure, there can exist some internal forces, which don't change even if the object moves and the configurations of the fingers change. Then, we also discuss this problem.

This paper is organized as follows. At first, the target system is shown and the directions of active and passive force closures are defined. Then, we show the directions of active force closure are orthogonal to those of passive force closure. We also discuss the internal forces in the manipulation of the object when the grasp is of hybrid active/passive closure.

### A. Related Works

The simplest case of active force closure is fingertip grasp. As for the manipulation of an object grasped by fingertips, Li et al. [5], Cole et al. [6], and Yokokohji et al. [7] presented control algorithms for the case of point contact, rolling contact, and soft-finger contact respectively. Cole et al. [8] extended the algorithm to the case of sliding contact. But the algorithm was limited in the 2 dimensional case. Zheng et al. [9] presented a control algorithm for the case of sliding contact in the 3 dimensional case. As for the manipulation of multiple objects grasped by fingertips or end links of fingers, Harada et al. [10] presented a control algorithm for the case of rolling contact. Harada et al. [11] also discuss active force closure for the case of grasping multiple objects.

On the other hand, passive force closure corresponds to power grasps. Power grasp is a grasp that can hold objects stably without changing the pre-loaded joint torques of the fingers. The researches about power grasps have been done from the following viewpoint [12]–[18]: the condition for power grasps, the formulation of the contact force distribution,

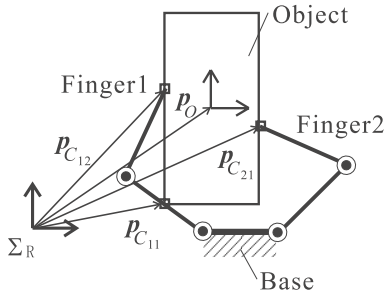


Fig. 1. Target System

the optimization of the pre-loaded joint torques, and so on.

It is hard to manipulate an object under hybrid active/passive closure. Trinkle et al. [19] discussed a grasp planning for manipulating an enveloped object with sliding contacts in 2 dimensional space. Bicchi et al. [20] analyzed the manipulability of the general grasping systems including enveloping grasp. Harada et al. [21] called the working style, where multiple contacts are allowed between chains (fingers or legs) and an object (or an environment), Envelope Family and presented a sufficient condition for the manipulation of Envelope Family. But, these researches didn't put attention on resisting external forces, which is another property of hybrid active/passive closure. Passive force closure in hybrid active/passive-closure grasps is discussed in [3], [22], [23]. But, these researches didn't discuss the manipulation of a grasped object. In this paper, we discuss not only active force closure (motion of a grasped object) but also passive force closure in the case where the grasped object is in hybrid active/passive closure.

## II. TARGET SYSTEM AND DEFINITION

### A. Target System

The target system is shown in Fig.1. In this paper, we consider the case where an arbitrary shaped rigid object is grasped by  $N$  fingers of a robotic hand. Note that we show the case where  $N = 2$  in Fig.1. We make the following assumptions: 1) Each finger makes a frictional point contact with the object, and the contacts are neither rolling nor sliding. 2) The unique normal direction at each contact point can be obtained. 3) There exists at most one contact point on each link of the fingers. 4) When the object moves, both the numbers of contact points and contact positions on the object/fingers don't change (we don't consider the manipulation in which a certain contact point removes from the object or in which a certain point on a certain finger, which isn't a contact point, makes a new contact with the object).

### B. Definition

**Space of Active Force Closure (SAFC);** Suppose that the grasped object is in force closure. Then, we call the direction, in which the grasped object can move by the corresponding motion of the fingers, the direction of active force closure (DAFC). We call the space, spanned by a set of the all DAFC's, the space of active force closure (SAFC).

**Space of Passive Force Closure (SPFC);** We call the direction, in which external force or moment can be counteracted without changing the joint torques, the direction of passive force closure (DPFC). We call the space, spanned by a set of the all DPFC's, the space of active force closure (SPFC).

## III. ORTHOGONALITY OF DIRECTIONS OF ACTIVE AND PASSIVE FORCE CLOSURES

In this section, we describe the orthogonality of SAFC and SPFC. At first, we describe the kinematics and statics of the system. Then, we show the orthogonality.

### A. Kinematics

Let  $\mathbf{q}_i \in \mathcal{R}^{M_i}$  ( $i = 1, 2, \dots, N$ ) be the joint angles of the  $i$ th finger of the robotic hand. Let  $\mathbf{r} \in \mathcal{R}^D$  be the position of the origin and the orientation of the object coordinate frame fixed at the object. Let  $\mathbf{p}_{C_{ij}} \in \mathcal{R}^d$  ( $j = 1, 2, \dots, L_i$ ) be the position of the  $j$ th contact point between the object and the  $i$ th finger. Here  $M_i$  denotes the number of the joints of the  $i$ th finger,  $D = 3$ ,  $d = 2$  in 2 dimensional space and  $D = 6$ ,  $d = 3$  in 3 dimensional space, and  $L_i$  denotes the number of the contact points on the  $i$ th finger. The relation between the displacements of  $\mathbf{p}_{C_{ij}}$  and  $\mathbf{q}_i$ , and the one between the displacements of  $\mathbf{p}_{C_{ij}}$  and  $\mathbf{r}$ , respectively, are given as follows;

$$\Delta \mathbf{p}_{C_{ij}} = \mathbf{J}_{ij} \Delta \mathbf{q}_i, \quad \Delta \mathbf{p}_{C_{ij}} = \mathbf{G}_{ij}^T \Delta \mathbf{r} \quad (1)$$

where  $\mathbf{J}_{ij} \in \mathcal{R}^{d \times M_i}$  denotes the Jacobian matrix and  $\mathbf{G}_{ij}$  denotes

$$\mathbf{G}_{ij} = \begin{pmatrix} \mathbf{I} \\ [(\mathbf{p}_{C_{ij}} - \mathbf{p}_o) \times] \end{pmatrix}$$

in 3 dimensional space. Here,  $\mathbf{I}$  represents an identify matrix,  $\mathbf{p}_o$  represents the position of the origin of the object coordinate frame,  $[\mathbf{a} \times]$  represents a skew symmetric matrix equivalent to the cross product operation ( $[\mathbf{a} \times] \mathbf{b} = \mathbf{a} \times \mathbf{b}$ ).

Let  $M = \sum_{i=1}^N M_i$  and  $L = \sum_{i=1}^N L_i$ . By using the following vectors and matrices,

$$\begin{aligned} \Delta \mathbf{q} &= (\Delta \mathbf{q}_1^T \quad \Delta \mathbf{q}_2^T \quad \cdots \quad \Delta \mathbf{q}_N^T)^T \in \mathcal{R}^M, \\ \mathbf{J} &= \text{diag} \left( \begin{pmatrix} \mathbf{J}_{11} \\ \vdots \\ \mathbf{J}_{1L_1} \end{pmatrix}, \begin{pmatrix} \mathbf{J}_{21} \\ \vdots \\ \mathbf{J}_{2L_2} \end{pmatrix}, \cdots, \begin{pmatrix} \mathbf{J}_{N1} \\ \vdots \\ \mathbf{J}_{NL_N} \end{pmatrix} \right) \in \mathcal{R}^{Ld \times M}, \\ \mathbf{G} &= (\mathbf{G}_{11} \quad \mathbf{G}_{12} \quad \cdots \quad \mathbf{G}_{NL_N}) \in \mathcal{R}^{D \times Ld}, \\ \mathbf{A} &= (\mathbf{J} \quad -\mathbf{G}^T) \in \mathcal{R}^{Ld \times (M+D)} \end{aligned}$$

where  $\text{diag}$  denotes a block diagonal matrix, we get the following expression from (1);

$$\mathbf{A} (\Delta \mathbf{q}^T \quad \Delta \mathbf{r}^T)^T = \mathbf{o}. \quad (2)$$

### B. Statics

Let  $\mathbf{f} \in \mathcal{R}^{Ld}$  be the contact force vector which combines the contact forces at all contact points,  $\boldsymbol{\tau} \in \mathcal{R}^M$  be the joint torques equivalent to  $\mathbf{f}$ , and  $\mathbf{w} \in \mathcal{R}^D$  be the resultant force and moment applied to the object at the object coordinate

frame. From (2) and the principle of virtual work, we get the following relation;

$$\begin{pmatrix} \boldsymbol{\tau}^T & -\boldsymbol{w}^T \end{pmatrix}^T = \boldsymbol{A}^T \boldsymbol{f} = \begin{pmatrix} \boldsymbol{J} & -\boldsymbol{G}^T \end{pmatrix}^T \boldsymbol{f}. \quad (3)$$

### C. Orthogonality

Now, we consider the case where the object is grasped and is in stationary state with non-zero contact force (internal force) at every contact point, which satisfies the frictional constraint. In addition, we assume that  $\boldsymbol{G}$  has full row rank. This means the grasp is of force closure. Namely, we consider SAFC and SPFC in force-closure grasps. Let  $\boldsymbol{\tau}_{pre} (\neq 0)$  be the pre-loaded joint torques corresponding to the internal forces. Then, from (3), we get

$$\begin{pmatrix} \boldsymbol{\tau}_{pre}^T & \boldsymbol{o}^T \end{pmatrix}^T = \boldsymbol{A}^T \boldsymbol{f}. \quad (4)$$

1) *SAFC*: In order to move the grasped object, the corresponding motion of the fingers is needed. The motions of the object and the fingers are constrained by the kinematic constraints (2). Then, the allowable motions of the object and the fingers are expressed by

$$\begin{pmatrix} \Delta \boldsymbol{q} \\ \Delta \boldsymbol{r} \end{pmatrix} = \boldsymbol{E}_P^T \Delta \boldsymbol{k}_1 = \begin{pmatrix} \boldsymbol{E}_{P1}^T \\ \boldsymbol{E}_{P2}^T \end{pmatrix} \Delta \boldsymbol{k}_1 \quad (5)$$

where  $\boldsymbol{E}_P \in \mathcal{R}^{a \times (M+D)}$  is an orthogonal matrix whose rows form bases of the null space of  $\boldsymbol{A}$ ,  $\Delta \boldsymbol{k}_1 \in \mathcal{R}^a$  is an arbitrary vector expressing the magnitude of the each column of  $\boldsymbol{E}_P^T$ ,  $a$  is the dimension of the null space of  $\boldsymbol{A}$ , and  $\boldsymbol{E}_{P1}$  and  $\boldsymbol{E}_{P2}$  are, respectively,  $M \times a$  and  $D \times a$  block matrices.

Since  $\boldsymbol{G}$  has full row rank, there don't exist the cases where in a certain column of  $\boldsymbol{E}_P^T$ , the corresponding component vector of  $\boldsymbol{E}_{P1}^T$  is  $\boldsymbol{o}$  and the corresponding component vector of  $\boldsymbol{E}_{P2}^T$  isn't  $\boldsymbol{o}$ . If in a certain column of  $\boldsymbol{E}_P^T$ , both the corresponding component vectors of  $\boldsymbol{E}_{P1}^T$  and  $\boldsymbol{E}_{P2}^T$  aren't  $\boldsymbol{0}$ , the motion of the object, in the direction corresponding to the vector of  $\boldsymbol{E}_{P2}^T$ , can be achieved by the motion of the finger in the direction corresponding to the vector of  $\boldsymbol{E}_{P1}^T$ . If in a certain column of  $\boldsymbol{E}_P^T$ , the corresponding component vector of  $\boldsymbol{E}_{P1}^T$  isn't  $\boldsymbol{0}$  and the corresponding component vector of  $\boldsymbol{E}_{P2}^T$  is  $\boldsymbol{0}$ , the motion of the finger, in the direction corresponding to the vector of  $\boldsymbol{E}_{P1}^T$ , makes no influence on the motion of the object. Then,  $\boldsymbol{E}_{P2}^T$  expresses SAFC.

2) *SPFC*: For the sake of simplicity, we consider the case where all fingers are not in a singular configuration and there is no kinematical redundancy with respect to the task in every finger. This means  $\boldsymbol{J}$  has full column rank. Now, we consider the contact forces which can generate without changing the pre-loaded joint torques. From (4), the contact forces can be expressed by

$$\boldsymbol{f} = (\boldsymbol{J}^T)^+ \boldsymbol{\tau}_{pre} + (\boldsymbol{I} - (\boldsymbol{J}^T)^+ \boldsymbol{J}^T) \tilde{\boldsymbol{k}}_2, \quad (6)$$

where  $(\boldsymbol{J}^T)^+$  denotes the pseudo-inverse matrix of  $\boldsymbol{J}^T$  and  $\tilde{\boldsymbol{k}}_2$  denotes an arbitrary vector. Then, by substituting (6) into

(3), we get

$$\begin{aligned} \begin{pmatrix} \boldsymbol{\tau} \\ -\boldsymbol{w} \end{pmatrix} &= \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{G}(\boldsymbol{J}^T)^+ \end{pmatrix} \boldsymbol{\tau}_{pre} + \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{G}(\boldsymbol{I} - (\boldsymbol{J}^T)^+ \boldsymbol{J}^T) \end{pmatrix} \tilde{\boldsymbol{k}}_2 \\ &= \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{O} \end{pmatrix} \boldsymbol{\tau}_{pre} + \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{\Xi} \end{pmatrix} \boldsymbol{k}_2 \quad (\text{from (4)}), \end{aligned} \quad (7)$$

where  $\boldsymbol{\Xi} \in \mathcal{R}^{D \times p}$  is a full column rank matrix whose columns form bases of the  $\boldsymbol{G}(\boldsymbol{I} - (\boldsymbol{J}^T)^+ \boldsymbol{J}^T)$ ,  $p$  is the rank of the  $\boldsymbol{G}(\boldsymbol{I} - (\boldsymbol{J}^T)^+ \boldsymbol{J}^T)$ , and  $\boldsymbol{k}_2 \in \mathcal{R}^p$  is an arbitrary vector expressing the magnitude of the each column of  $\boldsymbol{\Xi}$ . Note that  $\boldsymbol{\tau}_{pre}$  corresponds to the internal forces and then  $\boldsymbol{G}(\boldsymbol{J}^T)^+ \boldsymbol{\tau}_{pre} = \boldsymbol{o}$ .

DPFC is the direction where an external force can be counteracted without influence on the joint torques. Then,  $\boldsymbol{\Xi}$  in the second term of (7) expresses SPFC.

From the definition of  $\boldsymbol{E}_P$ ,  $\boldsymbol{A} \boldsymbol{E}_P^T = \boldsymbol{O}$ . Then, from (3), (6), and (7), we can get

$$\begin{aligned} \boldsymbol{E}_P \boldsymbol{A}^T \boldsymbol{f} &= \boldsymbol{o}, \\ \boldsymbol{E}_P \boldsymbol{A}^T (\boldsymbol{I} - (\boldsymbol{J}^T)^+ \boldsymbol{J}^T) \tilde{\boldsymbol{k}}_2 &= \boldsymbol{o}, \\ \boldsymbol{E}_P \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{\Xi} \end{pmatrix} \boldsymbol{k}_2 &= \boldsymbol{o}. \end{aligned}$$

$\boldsymbol{k}_2$  is an arbitrary vector in the last equation. Then, we can get

$$\begin{aligned} \boldsymbol{E}_P \begin{pmatrix} \boldsymbol{O} \\ -\boldsymbol{\Xi} \end{pmatrix} &= \boldsymbol{O}, \\ \boldsymbol{E}_{P2} \boldsymbol{\Xi} &= \boldsymbol{O}. \end{aligned} \quad (8)$$

(8) represents the orthogonality of SAFC (DAFC) and SPFC (DPFC) because  $\boldsymbol{E}_{P2}^T$  represents SAFC and  $\boldsymbol{\Xi}$  represents SPFC.

In the above discussion, the generalized velocity vector  $\begin{pmatrix} \Delta \boldsymbol{q}^T & \Delta \boldsymbol{r}^T \end{pmatrix}^T$  corresponds to the generalized force vector  $\begin{pmatrix} \boldsymbol{\tau}^T & -\boldsymbol{w}^T \end{pmatrix}^T$ . Then, equations (3) and (5) can correspond to the artificial constraints used in the context of hybrid force/position control [24]. Since the second term of (7) is included in the artificial constraint, we can regard the above deviation as based on the orthogonality of the artificial constraint, though the purpose and the result of the derivation are different from those of the hybrid force/position control (the purpose is to divide into the directions of position-control and the directions in which any external force can be counteracted without changing the joint torques (not the directions of force-control)).

### D. Examples

Consider the cases shown in Fig.2. In Fig.2 (a),  $\boldsymbol{A}$  is given by

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}. \quad (9)$$

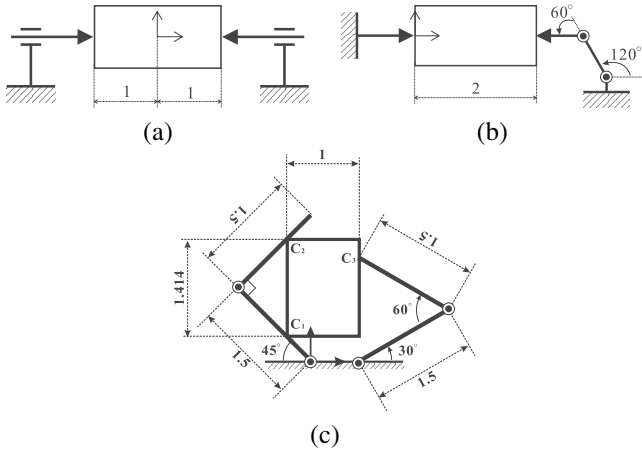


Fig. 2. Examples of Hybrid Active/Passive Closure

Then,  $E_p$  and  $\Xi$  are, respectively, expressed by

$$\begin{aligned} E_p &= \begin{pmatrix} -0.5774 & 0.5774 & -0.5774 & 0 & 0 \end{pmatrix}, \\ \Xi &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T. \end{aligned} \quad (10)$$

From (10), we can see that SAFC (DAFC) corresponds to the horizontal direction of this paper ( $x$  direction of the object coordinate frame) and that SPFC corresponds to  $y$  and the rotational directions of the object coordinate frame.

Next, consider the case shown in Fig.2 (b). The object is manipulated, contacting with the environment. Let the length of each link of the fingers be 1. Then,  $A$  is given by

$$A = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -\sqrt{3}/2 & 0 & -1 & 0 & 0 \\ -1.5 & -1 & 0 & -1 & -2 \end{pmatrix}. \quad (11)$$

where the position of the origin of the object coordinate frame is located at the contact point between the left finger and the object. In this case,  $E_p$  and  $\Xi$  are, respectively, expressed by

$$\begin{aligned} E_p &= \begin{pmatrix} 0 & -0.8944 & 0 & 0 & 0.4472 \end{pmatrix}, \\ \Xi &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^T. \end{aligned} \quad (12)$$

From (12), we can see that SAFC (DAFC) corresponds to the rotational direction of the object coordinate frame and that SPFC corresponds to the translational directions of the object coordinate frame.

Finally, consider the case shown in Fig.2 (c). The object is manipulated, contacting with the left finger at two points. In this case,  $A$  is given by

$$A = \begin{pmatrix} -0.3536 & 0 & 0 & 0 & -1 & 0 & 0.3536 \\ -0.3536 & 0 & 0 & 0 & 0 & -1 & 0.3536 \\ -1.7678 & -0.7071 & 0 & 0 & -1 & 0 & 1.7678 \\ -0.3536 & 0.7071 & 0 & 0 & 0 & -1 & 0.3536 \\ 0 & 0 & -1.5 & -0.75 & -1 & 0 & 1.5 \\ 0 & 0 & 0 & -1.299 & 0 & -1 & -0.6464 \end{pmatrix}. \quad (13)$$

where the position of the origin of the object coordinate frame is located at the first joint (the proximal end of the first link) of the left finger. In this case,  $E_p$  and  $\Xi$  are, respectively, expressed by

$$\begin{aligned} E_p &= \begin{pmatrix} 0.5125 & 0 & 0.64 & -0.255 & 0 & 0 & 0.5125 \end{pmatrix}, \\ \Xi &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^T. \end{aligned} \quad (14)$$

From (14), we can see that SAFC (DAFC) corresponds to the rotational direction of the object coordinate frame and that SPFC corresponds to the translational directions of the object coordinate frame. From the result that the second elements of  $E_p^T$  in (14) is zero, we can also see that the object cannot move with the change of the angle of the second joint of the left finger (note that the left finger is the first finger).

#### IV. INTERNAL FORCES IN THE MANIPULATION

In this section, we discuss the internal forces in the manipulation when the grasp is of hybrid active/passive closure.

For the sake of simplicity, suppose that the normal unit vector at every contact point is constant with respect to the object coordinate frame. Since the contact positions are all constant with respect to the object coordinate frame for the assumption given in section 2, the directions of the internal forces are also constant with respect to the object coordinate frame even if the object moves. Hence, whether the frictional constraints can be satisfied depends on only the magnitudes of the internal forces. In order to satisfy the frictional constraints in the manipulation of the object, we take the following simple way: at first find the appropriate constant magnitudes of the internal forces for the grasping and the manipulation of the object, and then assign the obtained constant appropriate magnitudes to the magnitudes of the internal forces and keep the magnitudes of the internal forces constant. Since there exists SPFC in hybrid active/passive closure-grasps, it is possible that there exist the magnitudes of the internal forces, which don't change despite the motion of the object. We don't have to change the joint torques corresponding to the magnitudes, while we have to change the joint torques corresponding to the magnitudes of the internal forces, which change due to the motion of the object. Namely, we have only to control the magnitudes of the internal forces, which change due to the motion of the object. In the following, we derive the relation between the joint torques and the magnitudes of the internal forces, which do or don't change due to the motion of the object.

If  $A$  doesn't have any null space, the object cannot move due to the geometric constraints. Then, in hybrid active/passive-closure grasps,  $A$  has null space. Here, we assume that  $A$  has full row rank. Then,  $f$ , which satisfies (4), can be uniquely obtained. Let such  $f$  be  $f_{int}$ .  $f_{int}$  is expressed by

$$f_{int} = (A^T)^+ \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \quad (15)$$

Since  $f_{int}$  is the internal forces,  $f_{int}$  satisfies  $Gf_{int} = \mathbf{o}$ . Then, the following another expression of  $f_{int}$  can be

obtained.

$$\mathbf{f}_{int} = (\mathbf{I} - \mathbf{G}^+ \mathbf{G}) \tilde{\mathbf{k}}_3 \triangleq \Phi \mathbf{k}_3 \quad (16)$$

where  $\tilde{\mathbf{k}}_3 \in \mathcal{R}^{Ld}$  denotes an arbitrary vector,  $\Phi \in \mathcal{R}^{(Ld) \times (Ld-D)}$  denotes an orthogonal matrix whose columns form bases of the null space of  $\mathbf{G}$ , and  $\mathbf{k}_3 \in \mathcal{R}^{Ld-D}$  denotes an arbitrary vector.  $\Phi$  expresses the directions of the internal forces and  $\mathbf{k}_3$  expresses their magnitudes.

From (15) and (16), we get

$$\mathbf{k}_3 = \Phi^T (\mathbf{A}^T)^+ \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \quad (17)$$

If the object and the fingers infinitesimally move, (17) becomes

$$\begin{aligned} \Delta \mathbf{k}_3 &= (\Delta(\Phi^T (\mathbf{A}^T)^+)) \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \\ &\quad + \Phi^T (\mathbf{A}^T)^+ \begin{pmatrix} \Delta \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \end{aligned} \quad (18)$$

Letting  $\mathbf{x} = (\mathbf{q}^T \mathbf{r}^T)^T \in \mathcal{R}^{M+D}$  and  $\mathbf{B} = \Phi^T (\mathbf{A}^T)^+ \in \mathcal{R}^{(LD-D) \times (M+D)}$ , (18) becomes

$$\Delta \mathbf{k}_3 = \left( \frac{\partial \mathbf{B}}{\partial \mathbf{x}} \Delta \mathbf{x} \right) \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} + \mathbf{B} \begin{pmatrix} \Delta \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \quad (19)$$

where  $\frac{\partial \mathbf{B}}{\partial \mathbf{x}}$  is a third-order tensor and  $\frac{\partial \mathbf{B}}{\partial \mathbf{x}} \Delta \mathbf{x}$  is a second-order tensor (matrix). By using the similar formulation of Chen and Kao [25], the first term of (19) becomes

$$\begin{aligned} \left( \frac{\partial \mathbf{B}}{\partial \mathbf{x}} \Delta \mathbf{x} \right) \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} &= \left[ \sum_{i=1}^{M+D} \left( \frac{\partial \mathbf{B}}{\partial x_i} \Delta x_i \right) \right] \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \\ &= \sum_{i=1}^{M+D} \left( \frac{\partial \mathbf{B}}{\partial x_i} \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \right) \Delta x_i \\ &= \left[ \left( \frac{\partial \mathbf{B}}{\partial x_1} \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \right) \left( \frac{\partial \mathbf{B}}{\partial x_2} \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \right) \right. \\ &\quad \left. \dots \left( \frac{\partial \mathbf{B}}{\partial x_{M+D}} \begin{pmatrix} \tau_{pre} \\ \mathbf{o} \end{pmatrix} \right) \right] \Delta \mathbf{x} \\ &\triangleq \Psi \Delta \mathbf{x}. \end{aligned} \quad (20)$$

Then, (19) becomes

$$\Delta \mathbf{k}_3 = \Psi \Delta \mathbf{x} + \mathbf{B} \begin{pmatrix} \Delta \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \quad (21)$$

We consider hybrid active/passive closure here. Then, arbitrary  $\Delta \mathbf{x}$  cannot be obtained. The allowable  $\Delta \mathbf{x}$  is expressed by (5). Then, (21) becomes

$$\Delta \mathbf{k}_3 = \Psi \mathbf{E}_P^T \Delta \mathbf{k}_1 + \mathbf{B} \begin{pmatrix} \Delta \tau_{pre} \\ \mathbf{o} \end{pmatrix}. \quad (22)$$

In order to keep the magnitudes of the internal forces constant in the manipulation, we have only to add  $\Delta \tau_{pre}$ , given by (22) with  $\Delta \mathbf{k}_3 = \mathbf{o}$ , to the joint torques. Then,  $\Psi \mathbf{E}_P^T \Delta \mathbf{k}_1$  is the magnitudes of the internal forces which we have to react. We call such magnitudes of the internal forces *reaction-needed magnitudes of the internal forces*. On the other

hand, we don't have to react the magnitudes of the internal forces, contained in the null space of  $\mathbf{E}_P \Psi^T$ , namely

$$\{\mathbf{k}_3 | \mathbf{k}_3 = (\mathbf{I} - \Psi \mathbf{E}_P^T (\Psi \mathbf{E}_P^T)^+) \mathbf{k}_5, \mathbf{k}_5 \in \mathcal{R}^{Ld-D}\} \quad (23)$$

where  $\mathbf{k}_5$  denotes an arbitrary vector. We call such magnitudes of the internal forces *reaction-not-needed magnitudes of the internal forces*.

In (22), letting  $\Delta \mathbf{k}_3 = \mathbf{o}$  and multiplying  $\mathbf{A}^T \Phi$  from the left, we can get

$$\begin{pmatrix} \Delta \tau_{pre} \\ \mathbf{o} \end{pmatrix} = -\mathbf{A}^T \Phi \Psi \mathbf{E}_P^T \Delta \mathbf{k}_1. \quad (24)$$

Then, if the joint torques change from  $\tau_{pre}$  to  $\tau_{pre} + \Delta \tau_{pre}$  (given by (24)) according to the change of the geometry, we can manipulate the object with the constant internal forces which satisfy the frictional constraints.

#### A. Examples

We consider the same cases as the previous examples. In the following examples, let the each magnitude of the internal forces be 1 (Let the every element of  $\mathbf{k}_3$  be 1). We compute  $\tau_{pre}$  form the magnitude of the internal forces.

At first, consider the case shown in Fig.2 (a). In this case, the direction of the internal force  $\Phi$  and  $\Psi$  in (20), respectively, are

$$\Phi = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}^T, \quad (25)$$

$$\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (26)$$

It is clear that the magnitude of the internal force is *reaction-not-needed magnitudes of the internal forces*. Namely, even if the object moves in the direction of active closure (DAFC), we don't have to change the pre-loaded joint torques in order to keep the magnitudes of the internal forces constant and to satisfy the frictional constraints.

Next, consider the case shown in Fig.2 (b). In this case, the direction of the internal force  $\Phi$  and  $\Psi$  in (20), respectively, are

$$\Phi = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}^T, \quad (27)$$

$$\Psi = \begin{pmatrix} 1.3856 & 0.8083 & 0 & 0 & -1.3856 \end{pmatrix}. \quad (28)$$

Then, from (12),  $\Psi \mathbf{E}_P^T$  is

$$\Psi \mathbf{E}_P^T = -1.3426. \quad (29)$$

It is clear that the magnitude of the internal force is *reaction-needed magnitudes of the internal forces*. Namely, when the object moves in the direction of active closure (DAFC), we have to change the pre-loaded joint torques in order to keep the magnitudes of the internal forces constant and to satisfy the frictional constraints.

Finally, consider the case shown in Fig.2 (c). In order to be easy to differentiate  $\Phi$  with respect to  $\mathbf{x}$ , we use the following another expression of  $\Phi$  given by [26];

$$\Phi = \begin{pmatrix} \mathbf{o} & e_{13} & e_{12} \\ e_{23} & \mathbf{o} & e_{21} \\ e_{32} & e_{31} & \mathbf{o} \end{pmatrix} \quad (30)$$

where  $e_{ij}$  is the vector directing from the contact point  $C_i$  to the contact point  $C_j$ . Then,  $\Phi$  can be expressed by

$$\Phi = \begin{pmatrix} 0 & 0 & 1 & -0.2678 & -1 & 0.2678 \\ 1 & 1.1464 & 0 & 0 & -1 & -1.1464 \\ 0 & 1.4142 & 0 & -1.4142 & 0 & 0 \end{pmatrix}^T.$$

In this case, we use  $\Phi^+$  in place of  $\Phi^T$  in (17) ~ (24). Then,  $\Psi$  in (20) is expressed by

$$\Psi = \begin{pmatrix} 1.2249 & 0.2129 & -0.9741 & -1.4809 & 0 & 0 & -0.2509 \\ -0.4992 & -0.0867 & 0.7782 & 0.6256 & 0 & 0 & -0.2790 \\ -0.6811 & 0.2801 & 0.8118 & 1.2092 & 0 & 0 & -0.1307 \end{pmatrix}.$$

Then, from (14),  $\Psi E_P^T$  is

$$\Psi E_P^T = \begin{pmatrix} 0.2535 & -0.0603 & -0.2049 \end{pmatrix}^T. \quad (31)$$

This is *reaction-needed magnitudes of the internal forces*. In this case, there also exist *reaction-not-needed magnitudes of the internal forces*. These magnitudes are expressed by

$$\{\mathbf{k}_3 | \mathbf{k}_3 = \begin{pmatrix} 0.1820 & 0.6181 \\ 0.9812 & -0.0637 \\ -0.0637 & 0.7835 \end{pmatrix} \mathbf{k}'_5, \mathbf{k}'_5 \in \mathcal{R}^2\} \quad (32)$$

where  $\mathbf{k}'_5$  denotes an arbitrary vector. From this result, we can see that in this case there exist both *reaction-needed magnitudes of the internal forces* and *reaction-not-needed magnitudes of the internal forces*.

## V. CONCLUSION

In hybrid active/passive-closure grasps, there are two kinds of essential directions. One is the directions of active force closure. The other is the directions of passive force closure. In this paper, we showed the directions of active force closure are orthogonal to those of passive force closure in hybrid active/passive-closure grasps. When the object moves, the geometries of the fingers and of the objects change. Then, we have to control the internal forces for satisfying the frictional constraints. However, hybrid active/passive-closure grasps have property of passive force closure. Then, it is possible that there exist the magnitudes of the internal forces, which we don't have to control even when the object moves. We derived the magnitudes of the internal forces. If in the initial state, the object is stably grasped with appropriate  $\tau_{pre}$ , we can manipulate the object by using the commands of joint velocities  $\dot{\mathbf{q}} = \mathbf{E}_{p1}^T (\mathbf{k}_{1d} - \mathbf{k}_1)$  ( $\mathbf{k}_{1d}$  is a desired value of  $\mathbf{k}_1$ ) given by (5) and the commands of joint torques  $\tau_{pre} + \Delta\tau_{pre}$  given by (24). But the development of more sophisticated control algorithm including feedback or dynamics is our future work.

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