Joint Torque-velocity Pair Based Manipulability for Grasping System

著者	Watnabe Tetsuyou
journal or publication title	Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
volume	2008
number	4651119
page range	2264-2270
year	2008-01-01
URL	http://hdl.handle.net/2297/35225

doi: 10.1109/IROS.2008.4651119

Joint Torque-velocity Pair Based Manipulability for Grasping System

Tetsuyou Watanabe

***Abstract*— This paper provides a new approach of manipulability for general grasping system. While conventional manipulability is analysis in velocity domain and can not include force effect such as gravitational force, the proposing approach can include the force effect to keep grasping. For the purpose, an operation range is introduced. The operation range is for actuator attached with every joint of robot and provides generable joint torque and velocity and their relation (between generating torque/velocity and addable velocity/torque). Using the operation range, we derive manipulability set and measure in velocity domain, including force effect. The proposing method can evaluate not only the performance in velocity domain but also effects of friction, contact state, and external forces, which were not obtained in conventional studies.**

I. INTRODUCTION

Manipulability is a well known concept to evaluate the performance of robotic manipulator [1]. For a single-arm manipulator, it is defined as the set of generable endeffector velocity in the task space when the set of generable joint velocity is given. When the given set of joint velocity is a unit ball, the set of endeffector velocity becomes an ellipsoid. The ellipsoid is called manipulability ellipsoid. The volume of the ellipsoid is a quality measure to evaluate the performance in velocity domain. It is called manipulability measure. Based on the manipulability, many quality measures such as condition number are proposed [1].

This concept can be extended to the general constraining system such as a robotic hand [2]–[6]. In a general constraining system, object velocity is evaluated instead of endeffector velocity. For a dual-arm system, Chiacchio et al. [2] discussed manipulability. Bicchi et al. [3] analyzed manipulability for general grasping system including whole arm manipulation system. After that, Bicchi et al. [4], Wen et al. [5], and Park et al. [6] analyzed manipulability for general constraining system with underactuated joints.

In a grasping system, grasping itself is a key issue. But, the analysis for grasping is in force domain, while manipulability analysis is in velocity domain. In the above researches, force closure grasp was assumed, and force analysis was evaded (Force closure is defined that any force and moment in any direction can be applied to the object). However, for the assumption, the applicable classes of the analyses in the above researches are limited. It can be said that closed chained system was only considered. Therefore, exactly saying, a manipulability analysis in velocity domain for grasping system has not been done so far.

This work was not supported by any organization

T. Watanabe is with Graduate School of Natural Science & Technology, Kanazawa University, Kanazawa, 920-1192, Japan te-watanabe@ieee.org

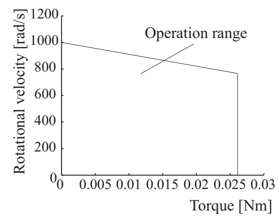


Fig. 1. Operation range for torque and velocity (maxon DC motor RE25 (20W))

In this paper, we propose a new approach to analyze manipulability in velocity domain, simultaneously including the effect of force/torque needed to keep grasping. For the purpose, we use an operation range (shown in Fig.1) of actuator attached with every joint of robot (operation range is originally used for motor selection). The operation range provides not only information about how magnitudes of torque and velocity the actuator can stably generate, but also the relation between generating velocity/torque and addable torque/velocity. Namely, we discuss manipulability from the view point of power, which have not been done in conventional studies and is first try.

First, we derive joint torques needed to keep grasping. We introduce required external force set (REFS) [7], [8] which is defined as a set of external wrenches (for example, gravitational force and corresponding forces to acceleration) required to be balanced. REFS can handle any desired grasp including force closure and equilibrium grasps. Using operation range and REFS, we compute necessary joint torques. Next, using the necessary joint torques and the operation range, we derive the set of generable object velocity. The set corresponds to conventional manipulability ellipsoid. The volume of the set corresponds to conventional manipulability measure. We call the set and its volume joint torque-velocity pair based manipulability set (TVMS) and measure (TVMM), respectively. The directions required to evaluate object velocity are limited in some tasks. For example, it is enough if considering unconstrained translational directions when lifting up an object on a table. To limit the directions to evaluate, we introduce required velocity direction set (RVDS). The set is defined as a set of object's twist direction in which it is required to evaluate how large velocity can be generated.

However, the computational load to derive the TVMS and TVMM is large since the set (polyhedron) used in the computation is high dimensional and constructed by many hyper planes and many vertices. The object can not always move in any arbitrary directions in grasping. In which direction the object can move depends on how to grasp the object. Therefore, motion planning is needed, and TVMS can be a criterion for the planning. For example, we can plan how to grasp to keep the object movable along a desired trajectory. Therefore, computational load should be lower. Then, we propose another evaluation method letting RVDS be a convex polyhedron (which is a set of scaled direction). We compute maximum α such that RVDS multiplied by α can be contained in the set of generable object velocity. For the linearity, only vertices of RVDS have to be considered, and then the computational load is reduced. Since the volume of RVDS is constant, the gauge parameter α can corresponds to manipulability measure. Then, we call the α joint torque-velocity pair based manipulability measure parameter (TVMMP). This approach has the following merits; 1) Friction effect can be included. 2) Contact state effect can be included. 3) Effect of external forces such as gravitational force can be included. These merits could not be obtained in the conventional studies.

As for dynamical analysis, Zheng et al. [9] and Fujiwara et al. [10] discussed a dynamic manipulability [1]. However, these analyses are not in velocity domain but in acceleration domain. Since power is not included/considered, the obtained generable accelerations are not associated with the generable velocities (obtained in this paper). They discussed only fingertip grasp. For the complexity, the computation load is very large. The other information such as inertia tensor is needed. In addition, in [10], internal force is fixed and then applicable class is limited.

II. Problem Definition

A. Target System

The target system is shown in Fig.2. In this paper, we consider a general grasping system where an arbitrary shaped rigid object is grasped by N fingers of a robotic hand. The nomenclatures are listed at appendix. We define that the contact state is any of the following four states: 1) F-point : the contact point with static friction, 2) N-point : the contact point without friction, 3) S-point : the contact point with kinetic friction, 4) D-point : the point about to detach.

We note that the contact force is zero at D-points. Also, we assume that every contact position, every contact state, every frictional coefficient are all given. For the convenient, the difference between static and kinetic frictional coefficients is not distinguished.

B. Problem Definition

After some definitions, we define the problem handled in this paper.

Joint Torque-velocity Pair Set (TVS): The set of generable joint torque and velocity at each joint, given by the operation range of the corresponding actuator, is named joint torque-velocity pair set (TVS).

TVS is usually given for absolute value of joint torque and velocity. Supposing that TVS is given as a convex polyheron

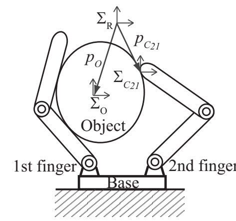


Fig. 2. Target System ($N = 2$)

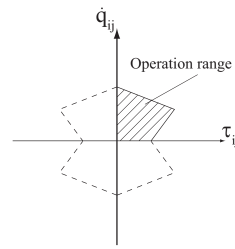


Fig. 3. Case when $\mathcal{S}_{tv_{ij}}$ does not constitute a convex polyhedron with respect to τ_{ij} and $\dot{q}_{ij}$

with respect to absolute value of joint torque and velocity, TVS is expressed by

$$\mathcal{S}_{tv_{ij}} = \{ \begin{bmatrix} |\tau_{ij}| \\ |\dot{q}_{ij}| \end{bmatrix} \mid \begin{bmatrix} |\tau_{ij}| \\ |\dot{q}_{ij}| \end{bmatrix} = \Sigma_{k=1}^{n_{tv_{ij}}} \nu_k \begin{bmatrix} \tau_{ij_{vk}} \\ \dot{q}_{ij_{vk}} \end{bmatrix},\ \nu_k \geq 0,$$
$$\Sigma_{k=1}^{n_{tv_{ij}}} \nu_k = 1\} (j = 1, 2, \cdots, M_i,\ i = 1, 2, \cdots, N). \quad (1)$$

When using the both plus and minus values of joint torque and velocity, the joint torque and velocity contained in $\mathcal{S}_{tv_{ij}}$ do not always constitute a convex polyhedron as shown in Fig.3. To resolve it, we introduce the variables $\tau_{ij_{max}}$, $\tau_{ij_{min}}$, $\dot{q}_{ij_{max}}$, and $\dot{q}_{ij_{min}}$, which, respectively, mean maximum and minimum values of joint torque, and maximum and minimum values of joint velocity. We consider to construct convex polyhedrons with respect to these four variables, instead of τ_{ij} and q_{ij}. Then, (1) becomes

$$\mathcal{S}_{tv_{ij}} = \{ \boldsymbol{y}_{ij} | \tau_{ij_{min}} \leq \tau_{ij} \leq \tau_{ij_{max}},\ \dot{q}_{ij_{min}} \leq \dot{q}_{ij} \leq \dot{q}_{ij_{max}},$$
$$\begin{bmatrix} \tau_{ij_{max}} \\ \dot{q}_{ij_{max}} \end{bmatrix} \in \mathcal{S}_{tv_{ij}}^{++}, \quad \begin{bmatrix} \tau_{ij_{max}} \\ \dot{q}_{ij_{min}} \end{bmatrix} \in \mathcal{S}_{tv_{ij}}^{+-}$$
$$\begin{bmatrix} \tau_{ij_{min}} \\ \dot{q}_{ij_{max}} \end{bmatrix} \in \mathcal{S}_{tv_{ij}}^{-+}, \quad \begin{bmatrix} \tau_{ij_{min}} \\ \dot{q}_{ij_{min}} \end{bmatrix} \in \mathcal{S}_{tv_{ij}}^{--} \}, \quad (2)$$

where

$$\mathcal{S}_{tv_{ij}}^{s_1 s_2} = \{ \boldsymbol{u}_{ij} | \boldsymbol{u}_{ij} = \Sigma_{k=1}^{n_{tv_{ij}}} \nu_k \begin{bmatrix} s_1 \tau_{ij_{vk}} \\ s_2 \dot{q}_{ij_{vk}} \end{bmatrix},\ \Sigma_{k=1}^{n_{tv_{ij}}} \nu_k = 1,$$
$$\nu_k \geq 0\} \qquad (s_1,\ s_2 \in \{+, -\}).$$

Aggregating (2) for all joints, we obtain

$$\mathcal{S}_{tv} = \{ \boldsymbol{y} | \boldsymbol{y}_{ij} \in \mathcal{S}_{tv_{ij}} {}^{\forall} i, j\}. \quad (3)$$

In some tasks, the directions required to evaluate how large object velocity can be generated are limited. For example, when lifting up an object on a table, the target directions required to evaluate are upward from the table and rotational directions do not have to be considered. Here, we consider only directions required to evaluate. For the purpose, we introduce RVDS.

Required Velocity Direction Set (RVDS): The set of object's twist direction in which it is required to evaluate how large velocity can be generated is named required velocity direction set (RVDS).

RVDS is assumed to be given by a convex polyhedral cone:

$$\mathcal{S}_{rvd} = \{\dot{\boldsymbol{r}} | \boldsymbol{A}_{rvd}\dot{\boldsymbol{r}} \leq \boldsymbol{o}\}, \tag{4}$$

where $\boldsymbol{A}_{rvd} \in \mathcal{R}^{n_{rvd} \times D}$.

In order to include the forces needed to be balanced to keep grasping, we introduce REFS.

Required External Force Set (REFS): The set of object's external wrench required to be balanced is named required external force set (REFS).

REFS is assumed to be given by convex polyhedron:

$$\mathcal{S}_{ref} = \{\boldsymbol{w}_{ex} | \boldsymbol{w}_{ex} = \Sigma_{i=1}^{n_{ref}} \kappa_i \boldsymbol{w}_{v_i},\ \Sigma_{i=1}^{n_{ref}} \kappa_i = 1,\ \kappa_i \geq 0\}, \tag{5}$$

Note the REFS indicates desired grasp. For example, if origin is in its interior, the desired grasp is force closure. Then, we define the following problem:

Problem 1: Suppose that $\mathcal{S}_{tv}$, $\mathcal{S}_{rvd}$ and $\mathcal{S}_{ref}$ are all given. In this case, find the set of generable object velocity, $\mathcal{S}_{ov}$ such that $\mathcal{S}_{ov} \subset \mathcal{S}_{rvd}$ and $\dot{\boldsymbol{r}} \in \mathcal{S}_{ov}$ is the velocity which can be generated even if any $\boldsymbol{w}_{ex} \in \mathcal{S}_{ref}$ is exerted on the object.

The obtained $\mathcal{S}_{ov}$ corresponds to conventional manipulability ellipsoid, and its volume corresponds to conventional manipulability measure. Therefore, we call $\mathcal{S}_{ov}$ and its volume joint torque-velocity pair based manipulability set (TVMS) and measure (TVMM), respectively.

III. Basic Formulation of the System

A. Kinematics of the system

The relation between velocities of $\boldsymbol{p}_{C_{F_{ij}}}$ and $\boldsymbol{q}_i$, and the relation between velocities of $\boldsymbol{p}_{C_{O_{ij}}}$ and $\boldsymbol{r}$, respectively, are given as follows:

$$\dot{\boldsymbol{p}}_{C_{F_{ij}}} = \boldsymbol{J}_{ij}\dot{\boldsymbol{q}}_i, \quad \dot{\boldsymbol{p}}_{C_{O_{ij}}} = \boldsymbol{G}_{ij}^T \dot{\boldsymbol{r}}, \tag{6}$$

where $\boldsymbol{J}_{ij} \in \mathcal{R}^{d \times M_i}$ denotes Jacobian matrix and

$$\boldsymbol{G}_{ij} = \begin{bmatrix} \boldsymbol{I} \\ [(\boldsymbol{p}_{C_{O_{ij}}} - \boldsymbol{p}_o)\times] \end{bmatrix}.$$

Here, $\boldsymbol{I}$ represents an identity matrix, $[\boldsymbol{a}\times]$ represents a skew symmetric matrix equivalent to the cross product operation ($[\ \boldsymbol{a} \times]\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$).

To derive the relation between $\dot{\boldsymbol{p}}_{C_{F_{ij}}}$ and $\dot{\boldsymbol{p}}_{C_{O_{ij}}}$ according to contact state, we introduce selection matrices. As for F and N-points and normal direction of S-point, the relation is

$$\begin{aligned} &\boldsymbol{H}_{c_{ij}}(\dot{\boldsymbol{p}}_{C_{F_{ij}}} - \dot{\boldsymbol{p}}_{C_{O_{ij}}}) = \boldsymbol{o}, \\ &\boldsymbol{H}_{c_{ij}} = \begin{cases} \boldsymbol{I} & \text{for F-point} \\ \boldsymbol{n}_{ij}^T & \text{for S-point and N-point} \end{cases}, \end{aligned} \tag{7}$$

where $\boldsymbol{o}$ denotes a zero vector. As for D-point, the relation is

$$\begin{aligned} &\boldsymbol{H}_{d_{ij}}(\dot{\boldsymbol{p}}_{C_{F_{ij}}} - \dot{\boldsymbol{p}}_{C_{O_{ij}}}) \leq \boldsymbol{o}, \\ &\boldsymbol{H}_{d_{ij}} = \ \boldsymbol{n}_{ij}^T \quad \text{for D-point}\ . \end{aligned} \tag{8}$$

As for tangential directions of S-point, the relation is

$$\begin{aligned} &\boldsymbol{H}_{s_{ij}}(\dot{\boldsymbol{p}}_{C_{F_{ij}}} - \dot{\boldsymbol{p}}_{C_{O_{ij}}}) = -\dot{\boldsymbol{p}}_{C_{sij}}, \\ &\boldsymbol{H}_{s_{ij}} = \ \boldsymbol{T}_{ij} \quad \text{for S-point}\ , \end{aligned} \tag{9}$$

aggregating (7), (9) and (8), we obtain

$$\boldsymbol{A}_c \begin{bmatrix} \dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T \end{bmatrix}^T = \boldsymbol{o}, \tag{10}$$

$$\boldsymbol{A}_d \begin{bmatrix} \dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T \end{bmatrix}^T \leq \boldsymbol{o}, \tag{11}$$

$$\boldsymbol{A}_s \begin{bmatrix} \dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T \end{bmatrix}^T = -\dot{\boldsymbol{p}}_{Cs}, \tag{12}$$

B. Statics of the system

First, we consider F and N-points and normal directions of S-points. From (10) and the principle of virtual work, the following relation is obtained:

$$\begin{bmatrix} \boldsymbol{\tau}_c \\ -\boldsymbol{w}_c \end{bmatrix} = \boldsymbol{A}_c^T \boldsymbol{f}_c = \begin{bmatrix} \boldsymbol{J}_c^T \\ -\boldsymbol{G}_c \end{bmatrix} \boldsymbol{f}_c. \tag{13}$$

Next, we consider tangential directions of S-points. From (12) and the principle of virtual work, the following relation is obtained:

$$\begin{bmatrix} \boldsymbol{\tau}_s \\ -\boldsymbol{w}_s \end{bmatrix} = \boldsymbol{A}_s^T \boldsymbol{f}_s = \begin{bmatrix} \boldsymbol{J}_s^T \\ -\boldsymbol{G}_s \end{bmatrix} \boldsymbol{f}_s. \tag{14}$$

Here, $\boldsymbol{f}_s$ is considered. When C_{ij} is a S-point, assuming friction obeys Coulomb's law, kinetic friction force $\boldsymbol{f}_{s_{ij}}$ can be expressed by

$$\boldsymbol{f}_{s_{ij}} = \boldsymbol{H}_{s_{ij}} \boldsymbol{f}_{ij} = -\mu_{ij} n_{f_{ij}} \hat{\dot{\boldsymbol{p}}}_{C_{sij}} = -\mu_{ij} \hat{\dot{\boldsymbol{p}}}_{C_{sij}} \boldsymbol{f}_{c_{ij}}, \tag{15}$$

where $\hat{\dot{\boldsymbol{p}}}_{C_{sij}} = \dot{\boldsymbol{p}}_{C_{sij}} / |\dot{\boldsymbol{p}}_{C_{sij}}|$. Aggregating for all S-points, (15) becomes

$$\begin{aligned} &\boldsymbol{f}_s = \mathrm{col}\left[-\mu_{ij}\hat{\dot{\boldsymbol{p}}}_{C_{sij}} \boldsymbol{f}_{c_{ij}}\right] = \boldsymbol{W}_s \boldsymbol{f}_c, \\ &\boldsymbol{W}_s = \mathrm{diag}\left[\boldsymbol{W}_{s_{ij}}\right], \\ &\boldsymbol{W}_{s_{ij}} = \begin{cases} -\mu_{ij}\hat{\dot{\boldsymbol{p}}}_{C_{sij}} & \text{for } \boldsymbol{f}_{c_{ij}} \text{ corresponding to S-point} \\ \text{nothing} & \text{for } \boldsymbol{f}_{c_{ij}} \text{ corresponding to F and N-points} \end{cases}. \end{aligned} \tag{16}$$

From (14) and (16), (14) becomes

$$\begin{bmatrix} \boldsymbol{\tau}_s \\ -\boldsymbol{w}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_s^T \\ -\boldsymbol{G}_s \end{bmatrix} \boldsymbol{W}_s \boldsymbol{f}_c. \tag{17}$$

Letting gravity term of fingers be $\boldsymbol{g}_f$, from (13) and (17), the total joint torque $\boldsymbol{\tau}$ is expressed by

$$\boldsymbol{\tau} = (\boldsymbol{J}_c^T + \boldsymbol{J}_s^T \boldsymbol{W}_s)\boldsymbol{f}_c + \boldsymbol{g}_f. \tag{18}$$

Also, total resultant force $\boldsymbol{w}$ is expressed by

$$\boldsymbol{w} = (\boldsymbol{G}_c + \boldsymbol{G}_s \boldsymbol{W}_s)\boldsymbol{f}_c. \tag{19}$$

C. Frictional constraints

Since kinetic friction is described above, static friction is only described here. The frictional constraint for F-point can be represented by

$$\mathcal{F}_{fij} = \{\boldsymbol{f}_{c_{ij}} \,||\boldsymbol{T}_{ij}\boldsymbol{f}_{ij}| \leq \mu_{ij} n_{f_{ij}},\, n_{f_{ij}} \geq 0\}. \quad (20)$$

The frictional constraint for N-point and normal direction of S-point can be represented by

$$\mathcal{F}_{nij} = \{\boldsymbol{f}_{c_{ij}} | n_{f_{ij}} \geq 0\}. \quad (21)$$

Aggregating (20) and (21) for all F-points, S-points, and N-points, we obtain

$$\begin{aligned}\mathcal{F} = \{\boldsymbol{f}_c | \boldsymbol{f}_{c_{ij}} \in \mathcal{F}_{fij},\ {}^{\forall}C_{ij}\text{which is F-point},\\ \boldsymbol{f}_{c_{ij}} \in \mathcal{F}_{nij},\ {}^{\forall}C_{ij}\text{which is N or S-point}\}. \quad (22)\end{aligned}$$

IV. Joint torque-velocity pair based manipulability

Firstly, we consider the case when there is no S-point. After that, we consider the case when S-points exist.

A. When there is no S-point

Firstly, we consider necessary joint torques to balance $\boldsymbol{w}_{vi} \in \mathcal{S}_{ref}$ and keep gasping.

We approximate the frictional constraint (20) by a n_{fric}-side convex polyhedral cone circumscribed in the friction cone [11]. Then, (22) becomes

$$\begin{aligned}\mathcal{F}_{lin} = \{\boldsymbol{f}_c | \boldsymbol{V}_{ij}\boldsymbol{f}_{c_{ij}} \leq \boldsymbol{o},\ {}^{\forall}C_{ij}\text{which is F-point},\\ \boldsymbol{f}_{c_{ij}} \geq 0,\ {}^{\forall}C_{ij}\text{which is N or S-point}\} \quad (23)\end{aligned}$$

where $\boldsymbol{V}_{ij} \in \mathcal{R}^{n_{fric}\times d}$. Then, from (19) and (23), in order to balance $\boldsymbol{w}_{v_i}$, the contact force has to satisfy the following constraints: $\boldsymbol{G}_c\boldsymbol{f}_{cv_i} + \boldsymbol{w}_{v_i} = \boldsymbol{o},\ \boldsymbol{f}_{cv_i} \in \mathcal{F}_{lin}$ where $\boldsymbol{f}_{cv_i}$ is $\boldsymbol{f}_c$ which balances $\boldsymbol{w}_{v_i}$.From (18), the joint torque ($\boldsymbol{\tau}_{v_i}$) corresponding to $\boldsymbol{f}_{cv_i}$ is expressed by $\boldsymbol{\tau}_{v_i} = \boldsymbol{J}_c^T\boldsymbol{f}_{cv_i} + \boldsymbol{g}_f$. The $\boldsymbol{\tau}_{v_i}$ has to be contained in TVS: $\boldsymbol{y}_{v_i} \in \mathcal{S}_{tv}$ where $\boldsymbol{y}_{v_i} = [\boldsymbol{\tau}_{v_i}^T\ \boldsymbol{\tau}_{max}^T\ \boldsymbol{\tau}_{min}^T\ \dot{\boldsymbol{q}}^T\ \dot{\boldsymbol{q}}_{max}^T\ \dot{\boldsymbol{q}}_{min}^T]^T$.

Then, the joint velocity has to satisfy the following constraints to balance $\boldsymbol{w}_{vi} \in \mathcal{S}_{ref}$:

$$\begin{aligned}&\boldsymbol{y}_{v_i} \in \mathcal{S}_{tv},\ \boldsymbol{\tau}_{v_i} = \boldsymbol{J}_c^T\boldsymbol{f}_{cv_i} + \boldsymbol{g}_f,\ \boldsymbol{G}_c\boldsymbol{f}_{cv_i} + \boldsymbol{w}_{v_i} = \boldsymbol{o},\\ &\boldsymbol{f}_{cv_i} \in \mathcal{F}_{lin}. \quad (24)\end{aligned}$$

Since $\mathcal{S}_{ref}$ and the constraints (24) are all linear, if satisfying the constraints (24) w.r.t. all vertices of REFS, the constraints becomes the set of joint velocity which can be generated even if any $\boldsymbol{w}_{ex} \in \mathcal{S}_{ref}$ is exerted on the object. Then, from (10), (11) and (24), $\mathcal{S}_{ov}$ is expressed by

$$\begin{aligned}\mathcal{S}_{ov} = \{\dot{\boldsymbol{r}} | &\boldsymbol{A}_c \begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T = \boldsymbol{o},\ \boldsymbol{A}_d \begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T \leq \boldsymbol{o},\\ &\boldsymbol{y}_{v_i} \in \mathcal{S}_{tv},\ \boldsymbol{\tau}_{v_i} = \boldsymbol{J}_c^T\boldsymbol{f}_{cv_i} + \boldsymbol{g}_f,\ \boldsymbol{G}_c\boldsymbol{f}_{cv_i} + \boldsymbol{w}_{v_i} = \boldsymbol{o},\\ &\boldsymbol{f}_{cv_i} \in \mathcal{F}_{lin},\ (i = 1, 2, \cdots, n_{ref})\}. \quad (25)\end{aligned}$$

Since both equations and inequalities constituting $\mathcal{S}_{ov}$ are linear and $\boldsymbol{\tau}$ and $\dot{\boldsymbol{q}}$ are bounded by $\mathcal{S}_{tv}$, $\mathcal{S}_{ov}$ is a convex polyhedron. Therefore, we call $\mathcal{S}_{ov}$ joint torque-velocity pair based manipulability polyhedron (TVMP). $\mathcal{S}_{ov}$ can be expressed using its vertices by a programming method such as a pivoting algorithm [13]:

$$\mathcal{S}_{ov} = \{\dot{\boldsymbol{r}} | \dot{\boldsymbol{r}} = \Sigma_{i=1}^{n_{s_{ov}}} \lambda_i \dot{\boldsymbol{r}}_{ovi},\ \Sigma_{i=1}^{n_{s_{ov}}} \lambda_i = 1,\ \lambda_i \geq 0\}, \quad (26)$$

where $\dot{\boldsymbol{r}}_{ovi}$ denotes the vertex of $\mathcal{S}_{ov}$ and $n_{s_{ov}}$ denotes the number of the vertices.

Here, we decompose $\mathcal{S}_{ov}$ into simplices by, for example, triangulation method using delaunay triangulation [12], [14] since the volume of simplex can be calculated. Letting $\int_i$ ($i = 1, \cdots, n_{s_{sov}}$) be the decomposed simplex of $\mathcal{S}_{ov}$ and its volume $V(\int_i)$, the volume of $\mathcal{S}_{ov}$ (TVMM), $V_{s_{ov}}$, is calculated by $V_{s_{ov}} = \Sigma_{i=1}^{n_{s_{sov}}} V(\int_i)$.

The derivation of both (26) and TVMM is sometimes time-consuming. The computational complexity of the pivoting algorithm (the derivation of (26)) is $\mathrm{O}(mn_{s_{ov}}D)$ where m denotes the number of inequalities constituting the convex polyhedron (note that equations in the constraints of (25) can be easily transformed into inequalities). As for delaunay triangulation, if the dimension of the polyhedron, l, is fixed, there is an incremental $\mathrm{O}(n_{s_{ov}}^{\frac{l}{2}})$ algorithm [12]. Therefore, whole calculation is time-consuming, especially in the case of high dimensional space. Also, the calculation of TVMM is available only when TVMP is full-dimensional.

Here, we consider to reduce the computational load. For the purpose, we consider using RVDS which is given by convex polyhedron:

$$\mathcal{P}_{rvd} = \{\hat{\dot{\boldsymbol{r}}} | \hat{\dot{\boldsymbol{r}}} = \Sigma_{i=1}^{n_{v_{rvd}}} \lambda_i \hat{\dot{\boldsymbol{r}}}_{v_i},\ \Sigma_{i=1}^{n_{v_{rvd}}} \lambda_i = 1,\ \lambda_i \geq 0\}. \quad (27)$$

where $\hat{\dot{\boldsymbol{r}}}$ denotes the direction of object velocity. Note that since a direction is determined by origin and a point, origin should be included in $\mathcal{P}_{rvd}$. This RVDS can be regarded as a set of scaled directions of interest object velocity. In consistent with this RVDS, we modify Problem 1 as follows:

Problem 2: Suppose that $\mathcal{S}_{tv}$, $\mathcal{P}_{rvd}$ and $\mathcal{S}_{ref}$ are all given. Consider the set of generable object velocity, $\mathcal{S}_{ov}$, such that $\dot{\boldsymbol{r}} \in \mathcal{S}_{ov}$ is the velocity which can be generated even if any $\boldsymbol{w}_{ex} \in \mathcal{S}_{ref}$ is exerted on the object. In this case, find the maximum $\alpha (> 0)$ such that $\alpha\mathcal{P}_{rvd} \subset \mathcal{S}_{ov}$.

Here, since the volume of $\mathcal{P}_{rvd}$ multiplied by α corresponds to manipulability measure and the volume of $\mathcal{P}_{rvd}$ is constant, α can be regarded as a criterion to evaluate TVMM. Therefore, we name α joint torque-velocity pair based manipulability measure parameter (TVMMP). Note that $\alpha\mathcal{P}_{rvd}$ can correspond to TVMP.

Here, we consider to generate object velocity in the direction of $\hat{\dot{\boldsymbol{r}}}_{v_\iota} \in \mathcal{P}_{rvd}$, balancing $\boldsymbol{w}_{v_k} \in \mathcal{S}_{ref}$. From (25), the maximum magnitude of generable object velocity in $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$ direction is derived by solving the following problem:

$$\begin{aligned}&\alpha_{k\iota} = \max_{\boldsymbol{x}}\ \alpha \quad \text{subject to} \quad \alpha\hat{\dot{\boldsymbol{r}}}_{v_\iota} = \dot{\boldsymbol{r}} \in \bar{\mathcal{S}}_{ov}, \quad (28)\\ &\bar{\mathcal{S}}_{ov} = \{\boldsymbol{A}_c \begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T = \boldsymbol{o},\ \boldsymbol{A}_d \begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T \leq \boldsymbol{o},\\ &\boldsymbol{y} \in \mathcal{S}_{tv},\ \boldsymbol{\tau} = \boldsymbol{J}_c^T\boldsymbol{f}_c + \boldsymbol{g}_f,\ \boldsymbol{G}_c\boldsymbol{f}_c + \boldsymbol{w}_{v_k} = \boldsymbol{o},\ \boldsymbol{f}_c \in \mathcal{F}_{lin}\}.\end{aligned}$$

Since the constraints of the above problem (28) are all linear and both RVDS and REFS are convex polyhedra, we only have to consider the problem (28) with respect to the vertices of RVDS and REFS. The minimum value among the $\alpha_{k\iota}$'s for all $\boldsymbol{w}_{v_k}$'s and $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$'s is the TVMMP. Therefore, the Problem 2 can be expressed as follows:

$$\min_{\boldsymbol{w}_{v_k}\in\mathcal{S}_{ref},\hat{\dot{\boldsymbol{r}}}_{v_\iota}\in\mathcal{P}_{rvd}} \max_{\boldsymbol{x}} \alpha \quad \text{subject to} \quad \alpha\hat{\dot{\boldsymbol{r}}}_{v_\iota}=\dot{\boldsymbol{r}}\in\bar{\mathcal{S}}_{ov} \tag{29}$$

This problem can be solved by the following way:

1) Solve the problem (28) for every $\boldsymbol{w}_{v_k}$ and $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$ by a linear programming ($k=1,2,\cdots,n_{ref}$, $\iota=1,2,\cdots,n_{v_{rvd}}$).

2) If obtaining the solution ($\alpha_{k\iota}$) for every combination of $\boldsymbol{w}_{v_k}$ and $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$, $\min_{k,\iota}\alpha_{k\iota}$ is the overall solution. Otherwise, there is no solution.

Remarks:

1. When obtaining the solution $\alpha_{k\iota}$ for a certain combination of $\boldsymbol{w}_{v_k}$ and $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$ in the problem (28), it means that the object velocity with the magnitude of from zero to $\alpha_{k\iota}$ can be generated in the direction $\hat{\dot{\boldsymbol{r}}}_{v_\iota}$ (balancing $\boldsymbol{w}_{v_k}$). Let $\boldsymbol{f}_c$, $\boldsymbol{\tau}_{max}$, $\boldsymbol{\tau}_{min}$, $\dot{\boldsymbol{q}}_{max}$ and $\dot{\boldsymbol{q}}_{min}$ giving $\alpha_{k\iota}$ be fixed among the elements of $\boldsymbol{x}$, while let the other elements $\dot{\boldsymbol{q}}$, $\dot{\boldsymbol{r}}$ and α ($=\alpha_{k\iota}$) giving $\alpha_{k\iota}$ be multiplied by ϵ ($0\le\epsilon\le1$):

$$\boldsymbol{x}\rightarrow\boldsymbol{x}_\epsilon=[\boldsymbol{f}_c^T\ \boldsymbol{\tau}_{max}^T\ \boldsymbol{\tau}_{min}^T\ \dot{\boldsymbol{q}}_{max}^T\ \dot{\boldsymbol{q}}_{min}^T\ \epsilon\dot{\boldsymbol{q}}^T\ \ \epsilon\dot{\boldsymbol{r}}^T\ \epsilon\alpha_{k\iota}]^T.$$

Obviously, the constraints in the problem (28) can be satisfied with respect to $\boldsymbol{x}_\epsilon$.

2. Let α^* be the overall solution. Then, object velocity with magnitude of $\epsilon\alpha^*$ can be generated in any arbitrary direction contained in $\mathcal{S}_{rvd}$.

3. We consider the case when one of the vertices of $\mathcal{P}_{rvd}$, $\hat{\dot{\boldsymbol{r}}}_{v_i}$, is $\boldsymbol{o}$. From the above remark, we can omit the calculation with respect to $\hat{\dot{\boldsymbol{r}}}_{v_i}=\boldsymbol{o}$ direction if $\mathcal{P}_{rvd}$ is not constructed by only $\boldsymbol{o}$.

4. We can make the distribution of generable object velocity, whose vertex is $\min_k\alpha_{k\iota}\ \hat{\dot{\boldsymbol{r}}}_{v_\iota}$ ($\iota=1,2,\cdots,n_{v_{rvd}}$). The obtained distribution corresponds to the conventional manipulability ellipsoid.

B. When there are some S-points

In this case, by actuating joint velocities, not only object velocity but also sliding velocity are generated. Then, if focusing only on deriving the set of generable object velocity, in which direction sliding should be generated to generate the derived object velocity becomes unclear. Here, we focus on in which direction object velocity can be generated and how large object velocity can be generated, with the finger sliding direction ($\hat{\dot{\boldsymbol{p}}}_{C_s}=\dot{\boldsymbol{p}}_{C_s}/|\dot{\boldsymbol{p}}_{C_s}|$) fixed.

Due to entity of kinetic friction, the terms and the constraints related with sliding and kinetic friction should be added. From (12), (18) and (19), $\mathcal{S}_{ov}$ (25) becomes

$$\begin{aligned}&\mathcal{S}_{ov}=\{\dot{\boldsymbol{r}}|\boldsymbol{A}_c\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T=\boldsymbol{o},\ \boldsymbol{A}_d\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T\le\boldsymbol{o},\\ &\boldsymbol{A}_s\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T=-\rho\hat{\dot{\boldsymbol{p}}}_{C_s},\ (\boldsymbol{G}_c+\boldsymbol{G}_s\boldsymbol{W}_s)\boldsymbol{f}_{cv_i}+\boldsymbol{w}_{v_i}=\boldsymbol{o},\\ &\boldsymbol{y}_{v_i}\in\mathcal{S}_{tv},\ \boldsymbol{\tau}_{v_i}=(\boldsymbol{J}_c^T+\boldsymbol{J}_s^T\boldsymbol{W}_s)\boldsymbol{f}_{cv_i}+\boldsymbol{g}_f,\\ &\boldsymbol{f}_{cv_i}\in\mathcal{F}_{lin}\ (i=1,2,\cdots,n_{ref})\}.\end{aligned} \tag{30}$$

Note that $\mathcal{S}_{ov}$ is also a convex polyhedron. Therefore, the volume of $\mathcal{S}_{ov}$ can be derived by the same way as the case described in section IV-A. Similarly, Problem 2 (29) becomes

$$\min_{\boldsymbol{w}_{v_k}\in\mathcal{S}_{ref},\hat{\dot{\boldsymbol{r}}}_{v_\iota}\in\mathcal{P}_{rvd}} \max_{\boldsymbol{x}_s} \alpha \quad \text{subject to} \quad \alpha\hat{\dot{\boldsymbol{r}}}_{v_\iota}=\dot{\boldsymbol{r}}\in\bar{\mathcal{S}}_{ov}$$

$$\begin{aligned}&\bar{\mathcal{S}}_{ov}=\{\dot{\boldsymbol{r}}|\boldsymbol{A}_c\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T=\boldsymbol{o},\ \boldsymbol{A}_d\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T\le\boldsymbol{o},\\ &\boldsymbol{A}_s\begin{bmatrix}\dot{\boldsymbol{q}}^T & \dot{\boldsymbol{r}}^T\end{bmatrix}^T=-\rho\hat{\dot{\boldsymbol{p}}}_{C_s},\ (\boldsymbol{G}_c+\boldsymbol{G}_s\boldsymbol{W}_s)\boldsymbol{f}_c+\boldsymbol{w}_{v_k}=\boldsymbol{o},\\ &\boldsymbol{y}\in\mathcal{S}_{tv},\ \boldsymbol{\tau}=(\boldsymbol{J}_c^T+\boldsymbol{J}_s^T\boldsymbol{W}_s)\boldsymbol{f}_c+\boldsymbol{g}_f,\ \boldsymbol{f}_c\in\mathcal{F}_{lin}\}.\end{aligned}$$

Using the same way as the case described in section IV-A, this problem can be solved.

V. Numerical Examples

In order to verify our approach, we show some numerical examples. Fig.4 shows the target system. Σ_R is placed at the contact point between the object and the base in the configuration shown in Fig.4. Σ_O is placed at geometric center of the object. The object is a ball with radius of 0.1[m]. The robotic hand is composed of 4 fingers which are same forms and have 4 joints. The length of every link is set to 0.1[m] and the gravity center of every link is set to geometric center of the link. Mass of every link is set to 0.051[kg]. The actuator attached with every joint is all same and attaches a gear with $1/40$ of reduction ratio. The base positions of fingers are set to $[-0.1\ -0.05\ 0]^T$, $[0.05\ 0.1\ 0]^T$, $[0.05\ 0\ 0]^T$ and $[0.05\ \ -0.1\ 0]$. We use the operation range shown in Fig.1. We set $n_{fric}=16$.

First, we consider Problem 1 for the case shown in Fig.5. Here, we consider to grasp an object by only Finger 1 and 3. Σ_O is placed at $[0.075\ 0\ 0.1]^T$. The contact position of Finger 1 is $[-0.022\ \ 0\ \ 0.074]^T$. The contact positions of Finger 3 are $[0.075\ 0\ 0]^T$ and $[0.171\ 0\ 0.072]^T$. μ_{ij} is set to 0.3. RVDS is not used and REFS is set as follows (Here, we set $mg=1$ [N]):

$$\mathcal{S}_{ref}=\{\boldsymbol{w}_{ex}|\boldsymbol{w}_{ex}=[0\ 0\ -mg\ \ 0\ 0\ 0]^T\}. \tag{31}$$

First, we consider the case when all contact points are F-points. The obtained TVMP is shown in Fig.6 (a). Denoting the components of $\dot{\boldsymbol{r}}$ by $[\dot{x}\ \ \dot{y}\ \ \dot{z}\ \ \dot{\phi}\ \ \dot{\psi}\ \ \dot{\theta}]^T$ and letting $\beta=[0.697\ 0\ \ -0.174\ 0\ 0.697\ 0]^T$, the TVMP is contained in the space expressed by $[\beta\ \dot{y}\ \dot{\phi}\ \dot{\theta}]^T$. The TVMP is not full dimensional. Since the overall set can not be shown, the TVMP shown in Fig.6 (a) is the set mapped to $\dot{\theta}=0$. From Fig.6 (a), it can be seen that the directions of generable velocities are limited for the geometrical constraints and large object velocity can be generated in the specific directions. Note that TVMM is zero but if calculating TVMM in the 4 D space, TVMM $=84.2$.

In the first case, the translational velocity can be generated only in y direction. Then, changing one of F-points to S-point, we try to generate translational velocity in another direction. We consider the case when the contact point on Finger 3 closer to the base is S-point. $\boldsymbol{H}_s^T\hat{\dot{\boldsymbol{p}}}_{C_s}$ is set to $[1\ \ 0\ \ 0\ \ ]^T$ (note that for easy to understand the direction of $\hat{\dot{\boldsymbol{p}}}_{C_s}$, $\boldsymbol{H}_s^T$ is added). Here, we only consider translational directions (we set RVDS is $\dot{\phi}=\dot{\psi}=\dot{\theta}=0$). The obtained TVMP is shown in Fig.6 (b). From Fig.6 (b), it can be seen that velocity in another translational direction can be

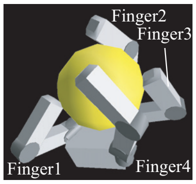


Fig. 4. Target system in numerical examples

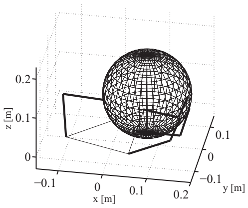
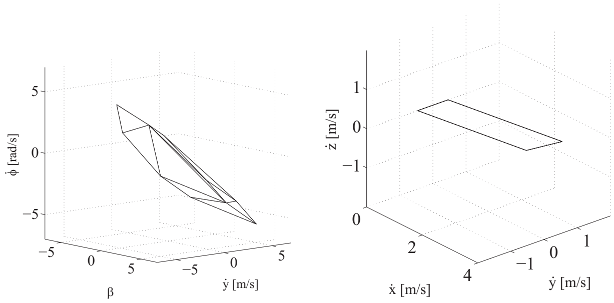
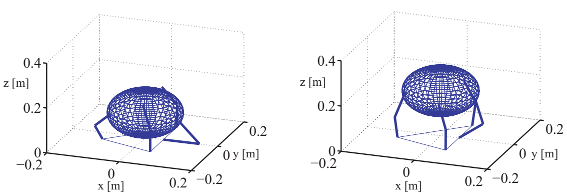


Fig. 5. Case 1



(a) (b)

Fig. 6. TVMP: (a) when all contact points are F-points, (b) when contact point on Finger 3 closer to the base is S-point



(a) (b)

Fig. 7. Case 2 : (a) initial state, (b) final state

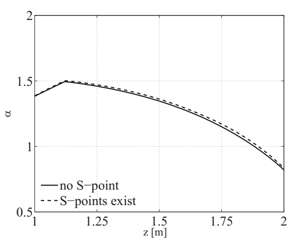
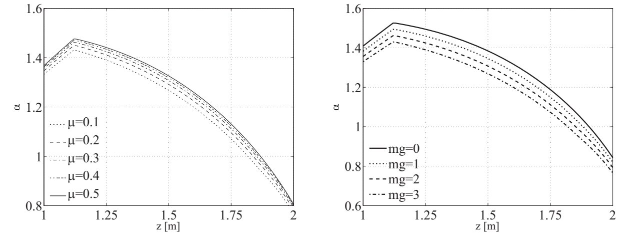


Fig. 8. TVMMP (α). The solid line denotes the results when the contact points on Finger 2 and 4 are N-points, and the dash line denotes the results when the contact points on Finger 2 and 4 are S-points.

generated. The object velocity can be generated only in $\dot{x} > 0$ direction due to the direction of $\hat{\dot{\boldsymbol{p}}}_{C_s}$.

Next, in order to see the effects of changes of configuration, friction and external force, we consider Problem 2 for the case shown in Fig.7 where the object moves in positive z direction. The contact points on Finger 1 and 3 are set to be F-point. The contact points on Finger 2 and 4 are set to be S or N-point. The contact point on the base is set to be D-point if it exists. The contact positions at the fingers and the base in the initial state are set to $[-0.077 \ \ -0.038 \ \ 0.086]^T$, $[0.033 \ \ 0.083 \ \ 0.14]^T$, $[0.097 \ 0 \ 0.074]^T$, $[0.033 \ -0.083 \ 0.14]^T$, and $[0 \ 0 \ 0]^T$. $\hat{\dot{\boldsymbol{p}}}_{C_s}$



(a) (b)

Fig. 9. TVMMP (α) for (a) various frictional coefficients and (b) various magnitudes of the external force.

is set such that $\dot{\boldsymbol{p}}_{C_s}$ is in parallel with $\boldsymbol{A}_s \ [(\boldsymbol{J}_c^+ \boldsymbol{G}_c^T)^T \boldsymbol{I}]^T$ $[0 \ 0 \ 1]^T$ according to the object motion. We use the same REFS (31) as the previous case. RVDS is set as follows:

$$\mathcal{P}_{rvd} = \{\hat{\dot{\boldsymbol{r}}} | \hat{\dot{\boldsymbol{r}}} = \Sigma_{i=1}^5 \lambda_i \hat{\dot{\boldsymbol{r}}}_{vi}, \ \Sigma_{i=1}^5 \lambda_i = 1, \ \lambda_i \geq 0\},$$
$$\hat{\dot{\boldsymbol{r}}}_{v1} = [1 \ 1 \ 1 \ \ 0 \ 0 \ 0]^T, \ \hat{\dot{\boldsymbol{r}}}_{v2} = [1 \ \ -1 \ 1 \ \ 0 \ 0 \ 0]^T,$$
$$\hat{\dot{\boldsymbol{r}}}_{v3} = [1 \ \ -1 \ 1 \ \ 0 \ 0 \ 0]^T, \ \hat{\dot{\boldsymbol{r}}}_{v4} = [-1 \ \ -1 \ 1 \ \ 0 \ 0 \ 0]^T, \hat{\dot{\boldsymbol{r}}}_{v5} = \boldsymbol{o}.$$

Note that $\dot{\boldsymbol{r}}_{v5}$ can be omitted in the computation.

We compute TVMMP when the object moves from the initial state to the final state shown in Fig.7. The results are shown in Fig.8 whose horizontal axis denotes the z coordinate of Σ_O, and whose vertical axis denotes TVMMP (α). From Fig.8, it can be seen that as the object moves in positive z direction, TVMMP becomes large at first and becomes small after that. It can be also seen that there is little difference between the cases when there is no S-point and when S-points exist. TVMMP is large since around the initial state, every finger (especially finger 1 and 3) is in the state where large contact velocity and force in any direction can be generated. Around the final state, the elbow (third joint) of every finger is extended, and then TVMMP is small.

In order to investigate the effects of friction and external forces, we compute TVMMP for various frictional coefficients and various magnitudes of external force. We compute in the case when the contact points on Finger 2 and 4 are N-points. We consider the following two cases: (1) μ_{ij} is changed to 0.1, 0.2, 0.3, 0.4, and 0.5 while $mg = 2$, (2) mg is changed to 0, 1, 2 and 3 while $\mu_{ij} = 0.3$. Fig.9 (a) shows the results when frictional coefficient changes, and (b) shows the results when mg changes. From Fig.9 (a), it can been seen that TVMMP becomes large with the increase of the frictional coefficient. When the frictional coefficient increases, the range of applicable contact forces becomes large. Therefore, we can balance external force by smaller joint torques, and then generable object velocity becomes large. From Fig.9 (b), it can been seen that TVMMP becomes small with the increase of magnitude of external force. When external force becomes large, necessary joint torques to keep grasping becomes large. Therefore, generable object velocity becomes small.

VI. Conclusion

In this paper, we have proposed a new approach to analyze manipulability for general grasping system. Using an operation range of every actuator attached with joint, we define a set of generable joint torque and velocity, TVS (joint torque-velocity pair set). Also, we define a set of external force required to be balanced (REFS (required external force set)) and a set of direction of object velocity required to evaluate ((RVDS) required velocity direction set). Using these sets, we have derived new manipulability set (joint torque-velocity pair based manipulability set (TVMS)) and measure (joint torque-velocity pair based manipulability measure (TVMM)) which can evaluate generable object velocities, including the effect of force/torque needed to keep grasping.

As shown in the numerical examples, this approach can evaluate the effects of friction, contact state and external force, which could not be included in the conventional approaches.

APPENDIX

NOMENCLATURES

col	A column vector or matrix formed by the following elements.
diag	A block diagonal matrix.
N	Number of fingers.
M_i	Number of joints of the ith finger ($i = 1, 2, \cdots, N$).
L_i	Number of contact points on the ith finger.
M	Number of total joints ($= \Sigma_{i=1}^{N} M_i$).
L	Number of total contact points ($= \Sigma_{i=1}^{N} L_i$) .
D	3/6 in 2/3 dimensional space.
d	2/3 in 2/3 dimensional space.
Σ_R	Reference coordinate frame.
Σ_O	Object coordinate frame fixed at the object.
$\mathcal{S}_{tv_{ij}}$	TVS at every joint ($j = 1, 2, \cdots, M_i$, $i = 1, 2, \cdots, N$).
τ_{ij}	The jth joint torque of the ith finger ($j = 1, \cdots, M_i$, $i = 1, \cdots, N$).
q_{ij}	The jth joint angle of the ith finger ($j = 1, \cdots, M_i$, $i = 1, \cdots, N$).
$\boldsymbol{u}_{ij}$	$= [\tau_{ij}\ \dot{q}_{ij}]^T$.
$\tau_{ij_{vk}}$	(≥ 0)Torque component of the kth vertex of $\mathcal{S}_{tv_{ij}}$.
$\dot{q}_{ij_{vk}}$	(≥ 0)Velocity component of the kth vertex of $\mathcal{S}_{tv_{ij}}$.
$n_{tv_{ij}}$	Number of the vertices of $\mathcal{S}_{tv_{ij}}$.
$\boldsymbol{y}_{ij}$	$= [\tau_{ij}\ \tau_{ij_{max}}\ \tau_{ij_{min}}\ \dot{q}_{ij}\ \dot{q}_{ij_{max}}\ \dot{q}_{ij_{min}}]^T$.
$\mathcal{S}_{tv}$	TVS aggregating TVS at every joint.
$\boldsymbol{y}$	=col[$\boldsymbol{y}_{ij}$].
$\mathcal{S}_{rvd}$	RVDS.
$\boldsymbol{r}$	Position and orientation of Σ_O ($\in \mathcal{R}^D$).
$\boldsymbol{A}_{rvd}$	Matrix constituting inequality constraints representing $\mathcal{S}_{rvd}$.
n_{rvd}	Number of the rows of $\boldsymbol{A}_{rvd}$.
$\mathcal{S}_{ref}$	REFS.
$\boldsymbol{w}$	Resultant force and moment applied to the object ($\in \mathcal{R}^D$).
$\boldsymbol{w}_{ex}$	External wrench ($\in \mathcal{R}^D$).
$\boldsymbol{w}_{vi}$	The ith vertex of $\mathcal{S}_{ref}$.
n_{ref}	Number of the vertices of $\mathcal{S}_{ref}$.
C_{ij}	The jth contact point of the ith finger ($j = 1, \cdots, L_i$, $i = 1, \cdots, N$).
$\Sigma_{C_{F_{ij}}}$	Coordinate frame fixed at the contact point on finger corresponding to C_{ij}.
$\Sigma_{C_{O_{ij}}}$	Coordinate frame fixed at the contact point on object corresponding to C_{ij}.
$\boldsymbol{q}_i$	$= [q_{i1}\ q_{i2}\ \cdots\ q_{iM_i}]^T$ ($\in \mathcal{R}^{M_i}$).
$\boldsymbol{p}_O$	Position of the origin of Σ_O ($\in \mathcal{R}^d$).
$\boldsymbol{p}_{I_{ij}}$	Position of the origin of $\Sigma_{I_{ij}}$ ($I \in \{C_F, C_O\}$) ($\in \mathcal{R}^d$).
$\boldsymbol{n}_{ij}$	Unit normal vector (directing to the inward of the object) at C_{ij}.
$\boldsymbol{t}_{k_{ij}}$	Unit tangential vector at C_{ij} ($k \in \{1, 2\}$).
$\boldsymbol{T}_{ij}$	$= [\boldsymbol{t}_{1_{ij}}\ \boldsymbol{t}_{2_{ij}}]^T$ ($\in \mathcal{R}^{d-1\times d}$). $[\boldsymbol{t}_{1_{ij}}]^T$ in 2 dimensional space.
$\dot{\boldsymbol{p}}_{C_{ij}}$	$= \dot{\boldsymbol{p}}_{C_{O_{ij}}} - \dot{\boldsymbol{p}}_{C_{F_{ij}}}$.
$\dot{\boldsymbol{p}}_{C_{sij}}$	$= \boldsymbol{H}_{s_{ij}} \dot{\boldsymbol{p}}_{C_{ij}}$.
$\boldsymbol{J}$	=diag[col[$\boldsymbol{J}_{F_{1j}}$] $\cdots$ col[$\boldsymbol{J}_{F_{Nj}}$]] $\in \mathcal{R}^{Ld\times M}$.
$\boldsymbol{G}$	$= [\boldsymbol{G}_{11}\ \boldsymbol{G}_{12}\ \cdots\ \boldsymbol{G}_{NL_N}] \in \mathcal{R}^{D\times Ld}$.
$\boldsymbol{A}$	$= [\boldsymbol{J}\ -\boldsymbol{G}^T] \in \mathcal{R}^{Ld\times(M+D)}$.
L_K	Number of K-points ($K \in \{\mathrm{F, N, S, D}\}$).
L_c	$= L_F d + L_N + L_S$.
L_s	$= L_S(d-1)$.
L_d	$= L_D$.
$\boldsymbol{H}_k$	=diag[$\boldsymbol{H}_{ij}$] $\in \mathcal{R}^{L_k\times Ld}$($k \in \{c, s, d\}$).
$\boldsymbol{H}_{ij}$	$= \begin{cases} \boldsymbol{H}_{k_{ij}} & \text{when } \boldsymbol{H}_{k_{ij}} \text{ exists} \\ \text{nothing} & \text{when } \boldsymbol{H}_{k_{ij}} \text{ doesn't exist} \end{cases}$.
$\boldsymbol{q}$	= col [$\boldsymbol{q}_i$] ($\in \mathcal{R}^M$).
$\boldsymbol{p}_I$	=col[$\boldsymbol{p}_{I_{ij}}$] ($I \in \{C_F, C_O\}$) ($\in \mathcal{R}^{Ld}$).
A_k	$= \boldsymbol{H}_k \boldsymbol{A}$ ($k \in \{c, s, d\}$).
$\dot{\boldsymbol{p}}_{Cs}$	= col[$\dot{\boldsymbol{p}}_{C_{sij}}$].
$\boldsymbol{f}_{ij}$	Contact force vector ($\in \mathcal{R}^d$).
$\boldsymbol{f}_{k_{ij}}$	$= \boldsymbol{H}_{k_{ij}} \boldsymbol{f}_{ij}$ ($k \in \{c, s\}$).
$\boldsymbol{f}$	=col[$\boldsymbol{f}_{ij}$] $\in \mathcal{R}^{Ld}$.
$\boldsymbol{f}_k$	$= \boldsymbol{H}_k \boldsymbol{f}$ = col[$\boldsymbol{f}_{k_{ij}}$] ($k \in \{c, s\}$).
$\boldsymbol{\tau}$	= col [τ_{ij}] ($\in \mathcal{R}^M$).
$\boldsymbol{\tau}_k$	$\boldsymbol{\tau}$ corresponding to $\boldsymbol{f}_k$ ($k \in \{c, s\}$).
$\boldsymbol{w}_k$	Resultant force and moment by $\boldsymbol{f}_k$($k \in \{c, s\}$).
$n_{f_{ij}}$	Normal force component of $\boldsymbol{f}_{ij}$ ($= \boldsymbol{n}_{ij}^T \boldsymbol{f}_{ij}$).
μ_{ij}	Frictional coefficient at C_{ij}.
$\hat{\dot{\boldsymbol{p}}}_{C_{sij}}$	$= \dot{\boldsymbol{p}}_{C_{sij}} / \lvert\dot{\boldsymbol{p}}_{C_{sij}}\rvert$.
$\mathcal{F}_{lin}$	Linearized frictional constraints.
$\boldsymbol{\tau}_{max}$	= col[$\tau_{ij_{max}}$].
$\boldsymbol{\tau}_{min}$	= col[$\tau_{ij_{min}}$].
$\dot{\boldsymbol{q}}_{max}$	= col[$\dot{q}_{ij_{max}}$].
$\dot{\boldsymbol{q}}_{min}$	= col[$\dot{q}_{ij_{min}}$].
$\mathcal{P}_{rvd}$	RVDS expressed by convex polyhedron.
$\hat{\dot{\boldsymbol{r}}}$	The direction of object velocity.
$\hat{\dot{\boldsymbol{r}}}_{vi}$	The ith vertex of $\mathcal{P}_{rvd}$.
$n_{v_{rvd}}$	Number of the vertices of $\mathcal{P}_{rvd}$.
$\boldsymbol{x}$	$= [\boldsymbol{f}_c^T\ \boldsymbol{\tau}_{max}^T\ \boldsymbol{\tau}_{min}^T\ \dot{\boldsymbol{q}}_{max}^T\ \dot{\boldsymbol{q}}_{min}^T\ \dot{\boldsymbol{q}}^T\ \dot{\boldsymbol{r}}^T\ \alpha]^T$.
$\hat{\dot{\boldsymbol{p}}}_{C_s}$	$= \dot{\boldsymbol{p}}_{C_s} / \lvert\dot{\boldsymbol{p}}_{C_s}\rvert$.
ρ	The magnitude of sliding velocity in $\hat{\dot{\boldsymbol{p}}}_{C_s}$ direction.
$\boldsymbol{x}_s$	$= [\boldsymbol{x}^T\ \rho]^T$.

REFERENCES

[1] T. Yoshikawa, *Foundations of Robotics*. MIT Press, Cambridge, 1990.

[2] P. Chiacchio, S. Chiaverini, L. Sciavicco, and B. Siciliano, "Global task space manipulability ellipsoids for multiple-arm systems," *IEEE Trans. on Robotics and Automation*, vol. 7, no. 5, 1991, pp. 678–685.

[3] A. Bicchi, C. Melchiorri, and D. Balluchi, "On the mobility and manipulability of general multiple limb robots," *IEEE Trans. on Robotics and Automation*, vol. 11, no. 2, 1995, pp. 215–228.

[4] A. Bicchi and D. Prattichizzo, "Manipulability of cooperating robots with unactuated joints and closed-chain mechanisms," *IEEE Trans. on Robotics and Automation*, vol. 16, no. 4, 2000, pp. 336–345.

[5] J. T. Wen and L. S. Wilfinger, "Kinematic manipulability of general constrained rigid multibody systems," *IEEE Trans. on Robotics and Automation*, vol. 15, no. 3, 1999, pp. 558–567.

[6] F. C. Park and J. W. Kim, "Manipulability and singularity analysis of multiple robot systems: A geometric approach," *Proc. of IEEE Int. Conf. on Robotics and Automation*, 1998, pp. 1032–1037.

[7] T. Watanabe and T. Yoshikawa, "Optimization of grasping by using a required external force set," *Proc. of IEEE Int. Conf. on Robotics and Automation*, 2003, pp. 1127–1132.

[8] ——, "Optimization of grasping an object by using required acceleration and equilibrium-force sets," *Proc. of IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics*, 2003, pp. 338–343.

[9] X. Z. Zheng, N. Tomochika, and T. Yoshikawa, "Dynamic manipulability of multiple robotic mechanisms in coordinated manipulation," *Proc. of IFToMM-jc Int. Symp. on Theory of Machines and Mechanisms*, 1992, pp. 147–152.

[10] M. Fujiwara, Y. Yokokohji, and T. Yoshikawa, "Guideline for designing haptic master hands based on dynamic muli-fingered manipulability," *Proc. of the Symp. on Haptic Interfaces for Virtual Environment and Teleoperator Systems*, 2003, pp. 77–84.

[11] J. Kerr and B. Roth, "Analysis of multifingered hands," *The Int. J. of Robotics Research*, vol. 4, no. 4, 1986, pp. 3–17.

[12] K. Fukuda, *Frequently Asked Questions in Polyhedral Computation*, 2004, [Online]. Available: http://www.ifor.math.ethz.ch/~fukuda/polyfaq/polyfaq.html.

[13] D. Avis and K. Fukuda, "A pivoting algorithm for convex hulls and vertex enumeration of arrangements and polyhedra," *Discrete & Computational Geometry*, vol. 8, 1992, pp. 295–313.

[14] K. Fukuda, *cdd/cdd+ Reference Manual*, 1999, [Online]. Available: http://www.ifor.math.ethz.ch/~fukuda/cdd_home/index.html.