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Analysis of Intensity and Frequency Noises in Semiconductor Optical Amplifier

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***Abstract*—A theoretical analysis of the intensity and the frequency noise in semiconductor optical amplifiers (SOA) is given. Amplification of a traveling optical wave is formulated associating with fluctuations on the optical intensity, the optical phase and the electron numbers based on the classical wave-equation and quantum mechanical modification. Inclusion of the amplified spontaneous emission generated in the SOA is also taken into account. Amounts of noise are expressed in terms of the relative intensity noise (RIN), the spectrum line-width and the frequency noise (FM noise). Sensitive dependency of the RIN property on the optical input power is theoretically explained. The RIN increases after passing the SOA when the optical input power is small enough, but decreases when the optical input power is rather high. On the while, the spectrum line-width is found to be scarcely changed from the input light for conventional operation of the SOA.**

***Index Terms*— semiconductor optical amplifiers, noise, line-width, semiconductor lasers**

I. Introduction

THE Semiconductor optical amplifier (SOA) has excellent properties such as the high gain, the small size and making integration with the optical detector or the laser. Operating mechanism and properties of the SOA have been investigated by many authors [1]-[10]. However, there is still unclear subject to understand operating property of the SOA.

Important properties of an amplifier are the amplification rate, the operating power and the noise. The noise property of the optical amplifier has been evaluated in terms of the S/N (signal to noise) ratio and the NF (noise figure) by following the evaluating manor in the conventional electronic amplifiers. In the most cases of the optical amplifiers, the S/N ratio is counted with optical power of the coherent light to be the signal and that of the incoherent light to be the noise. However this definition is not suitable to evaluate real device and system, because the incoherent light also works as the signal in the intensity modulation systems which are the most popularly used. The noise should be defined for all fluctuating phenomena against to the signal with a solid definition of what is the signal.

Another difficulty to evaluate the SOA comes from the non-linear behavior not only on the amplification mechanism but also on the noise generating mechanism. It is widely believed that the S/N ratio must be degraded after passing an amplifier, because both the input signal and the input noise are amplified and another noise is added in the amplifier. This concept is effective when the amplification factors of the signal and the noise are identical and the generating mechanism of the additional noise is independent from the signal. The relative intensity noise (RIN) is one index to evaluate property of the noise in optical devices corresponding to an inverse value of the S/N ratio. Then the RIN level must increase after passing an SOA. However, the RIN level in the SOA varies with intensity of the input signal itself and is improved after amplification when the input power is rather high as shown in this paper.

Sensitive dependency of the RIN level in the SOA to the input optical power already reported experimentally by Shtaif and Eisenstein [5]. They also had theoretically postulated that dependency of the RIN level on the optical power is caused by strong nonlinear optical effect in the SOA [4],[7].

Theoretical analysis of the noise in the SOA including origin of the noise source in the quantum mechanical manner was started with the case having reflecting facets at the input and the output ports by Mukai and Yamamoto as an extending treatment of the semiconductor laser, where the optical mode is well defined with the resonating cavity [1]. However, theoretical analyses basing on the propagating wave to apply the SOA without reflecting mirrors are not unified because definition of "longitudinal mode" is ambiguous in an open waveguide [7],[12],[13].

A theoretical analysis of the intensity and the frequency noise in the SOA for traveling optical wave without reflecting facets is shown in this paper. The longitudinal mode is defined with a length in which one photon can be emitted as the spontaneous emission. Inclusions of spontaneously emitted lights into the signal light from both identical and not identical optical frequencies are taken into account. The sensitive dependency of the RIN level on the optical intensity is explained as different intrinsic properties between the signal of the continuous wave (CW) light and the noise which has temporally varying intensity, rather than the nonlinear effect on the optical intensity. On the other hand, less sensitivity of the frequency noise by optical amplification is theoretically shown.

In Section II, model and basic equations of this analysis are given. A wave equation is formulated with introduction of the spontaneous emission from the classical Maxwell's equation. Dynamic equations for the optical power, the optical phase and the electron density are derived by introducing the quantum mechanical properties of both the optical field and the electrons with generating terms of fluctuations. In Section III, fluctuations on the optical power, the optical phase and the electron density are expanded with angular frequency

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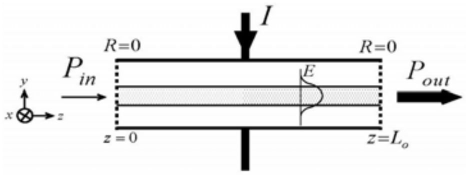


Fig.1 Structure of semiconductor optical amplifier (SOA). Facets of the amplifier are anti-reflection coated to prevent reflections.

components. Expressing equations of the RIN, the frequency noise and the spectral line-width are given. In Section IV, examples of numerically calculated data and discussions on the property and the noise generating mechanism are given. In Section V, reasons why the RIN level is improved after passing the SOA are more deeply discussed. In Section VI, conclusions are mentioned.

II. ANLYTICAL MODEL

A. Wave Equation

A model of SOA is shown in Fig.1. Here, an active region having width *w*, thickness *d* and length L_o are driven with driving current *I*. Both facets for the input and the output are anti-reflecting coated to prevent reflections. Spatial coordinates are noted with *x,y* and *z* as shown in the figure.

We form dynamic equations of the optical field from the classical Maxwell's equations. Electric polarization $\boldsymbol{P}$ for the optical emission consists of two components; one is for the stimulated emission which is characterized with a laser susceptibility χ and another is for the spontaneous emission $\boldsymbol{P}_{sp}$ as

$$\boldsymbol{P} = \varepsilon_o \chi \boldsymbol{E} + \boldsymbol{P}_{sp} \quad . \tag{1}$$

The guiding loss in the waveguide is given by

$$\boldsymbol{J} = \sigma \boldsymbol{E} \quad . \tag{2}$$

Then, the Maxwell's equations are

$$\nabla \times \boldsymbol{H} = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{P}}{\partial t} + \boldsymbol{J} = (\varepsilon + \varepsilon_o \chi)\frac{\partial \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{P}_{sp}}{\partial t} + \sigma \boldsymbol{E} \quad , \tag{3}$$

$$\nabla \times \boldsymbol{E} = -\mu_o \frac{\partial \boldsymbol{H}}{\partial t} \quad . \tag{4}$$

We suppose here that the waveguide is designed to guide only the fundamental TE mode consisted with components of E_x, H_y and H_z. From (3) and (4), a wave equation for the electric field E_x is given by

$$\nabla^2 E_x - \mu_o \varepsilon \frac{\partial^2 E_x}{\partial t^2} = \mu_o \sigma \frac{\partial E_x}{\partial t} + \mu \varepsilon_o \chi \frac{\partial^2 E_x}{\partial t^2} + \mu_o \frac{\partial^2 P_{sp}}{\partial t^2} \tag{5}$$

Solution of the equation is put to be

$$E_x = \sum_m A_m(t,z)\, \Phi(x,y)\, e^{-j\beta_m z + j\omega_m t} + c.c. \tag{6}$$

where m is the mode number covering not only the signal mode inputted into the SOA but also other modes grown up as the amplified spontaneous emission (ASE) as illustrated in Fig.2. Although the ASE shows continuous spectrum in experiment, we put discrete mode numbers. Continuous property of the spectrum will be expressed in terms of the spectrum broadening of the ASE modes as will be shown in (87) in later. As the beginning we deal these the signal and the ASE modes

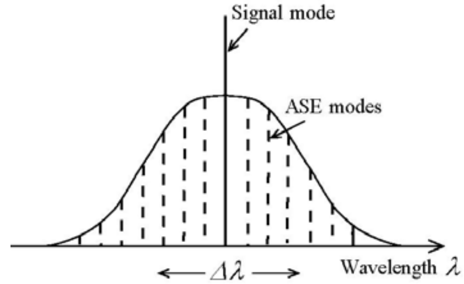


Fig.2 Modes in the SOA. Input light forms the signal mode and the spontaneous emission forms the ASE (amplified spontaneous emission) modes. The half width of the ASE profile is put be $\Delta\lambda$.

altogether with the mode number m . ω_m is the optical angular frequency, β_m is a propagation constant for z direction, $\Phi(x,y)$ is a transverse field distribution function which is obtained as a solution of an eigen equation of

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi(x,y) = \left(\beta_m^2 - \mu_o \varepsilon \omega_m^2\right)\Phi(x,y) \tag{7}$$

with a normalization condition of

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\Phi(x,y)\right|^2 dxdy = 1 \quad . \tag{8}$$

The term $A_m(t,z)$ is amplitude of the optical field slowly varying with both *t* and *z*.

By substituting (6) to (5) with (7) and by using condition of $\left|\partial A_m/\partial z\right| << \left|\beta_m A_m\right|$, $\left|\partial^2 A_m/\partial z^2\right| << \left|\beta_m \partial A_m/\partial z\right|$, $\left|\partial A_m/\partial t\right| << \left|\omega_m A_m\right|$ and $\left|\partial^2 A_m/\partial t^2\right| << \left|\omega_m \partial A_m/\partial t\right|$, we can drop several terms. Furthermore, we multiply $\Phi^*(x,y)\, e^{j\beta_m z - j\omega_m t}$ to the remaining equation, take spatial integrations in ranges $-\infty < x < \infty$ and $-\infty < y < \infty$, and take spatial and time averages over several wavelength along z and over several rotating time period along *t*, respectively. Then we get an equation for variation of the amplitude as

$$\begin{aligned} & -2j\left(\beta_m \frac{\partial A_m}{\partial z} + \omega \mu_o \varepsilon_{eff} \frac{\partial A_m}{\partial t}\right) \\ & = \left(j\omega_m \mu_o \sigma_{eff} - \mu_o \varepsilon_o \omega_m^2 \chi_{eff}\right) A_m \\ & + \mu_o \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial^2 P_{sp}}{\partial t^2}\, \Phi^*(x,y)\, e^{j\beta_m z - j\omega_m t}\, dxdy \end{aligned} \quad . \tag{9}$$

Here, the suffix *eff* means a value taken spatially average in the transverse cross-section such as

$$\varepsilon_{eff} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon \left|\Phi(x,y)\right|^2 dxdy \tag{10}$$

$$\sigma_{eff} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma \left|\Phi(x,y)\right|^2 dxdy \tag{11}$$

$$\chi_{eff} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi \left|\Phi(x,y)\right|^2 dxdy \equiv \xi \chi \tag{12}$$

where

$$\xi = \int_{-d/2}^{d/2}\int_{-w/2}^{w/2} \left|\Phi(x,y)\right|^2 dxdy \tag{13}$$

is called the confinement factor.

Equation (9) is rewritten to more simple form of

$$\frac{\partial A_m}{\partial z}+\frac{1}{v_m}\frac{\partial A_m}{\partial t}=\frac{g_m-\kappa_m+j\varphi_m}{2}A_m+U_m(t,z) \quad (14)$$

with the gain coefficient g_m

$$g_m=\frac{\mu_o\varepsilon_o\omega_m^2}{\beta_m}\operatorname{Im}\chi_{eff} \quad , \quad (15)$$

the guiding loss coefficient κ_m

$$\kappa_m=\frac{\omega_m\mu_o\sigma_{eff}}{\beta_m} \quad , \quad (16)$$

a term φ_m relating to the phase variation

$$\varphi_m=-\frac{\mu_o\varepsilon_o\omega_m^2}{\beta_m}\operatorname{Re}\chi_{eff} \quad . \quad (17)$$

and a velocity v_m of the field propagation defined as

$$v_m=\frac{\beta_m}{\omega_m\mu_o\varepsilon_{eff}}=\sqrt{\frac{\varepsilon_{eq}}{\mu_o}}\frac{1}{\varepsilon_{eff}} \quad (18)$$

with

$$\beta_m\equiv\omega\sqrt{\mu_o\varepsilon_{eq}} \quad . \quad (19)$$

The term $U_m(t,z)$ in (14) stands inclusion of the spontaneous emission. Evaluation of this term will be done after introducing the quantum mechanical property of the optical field as will be shown in later.

B. Dynamic Equations for Intensity and Phase Variations

The field amplitude is represented with an absolute value and a phase such as

$$A_m(t,z)=\left|A_m(t,z)\right|e^{j\theta_m(t,z)} \quad . \quad (20)$$

By substituting this equation to (14), we get following two equations :

$$\frac{\partial|A_m|}{\partial z}+\frac{1}{v_m}\frac{\partial|A_m|}{\partial t}=\frac{g_m-\kappa_m}{2}|A_m|+\operatorname{Re}\left\{U_m e^{-j\theta_m}\right\} , \quad (21)$$

$$\frac{\partial\theta_m}{\partial z}+\frac{1}{v_m}\frac{\partial\theta_m}{\partial t}=\frac{\varphi_m}{2}+\frac{1}{|A_m|}\operatorname{Im}\left\{U_m e^{-j\theta_m}\right\} . \quad (22)$$

As the first step, we exam variation of the optical intensity. An equation for squire value of the amplitude is given from (21) as

$$\frac{\partial|A_m|^2}{\partial z}+\frac{1}{v_m}\frac{\partial|A_m|^2}{\partial t}=\left(g_m-\kappa_m\right)|A_m|^2+2|A_m|\operatorname{Re}\left\{U_m e^{-j\theta_m}\right\} . \quad (23)$$

We now introduce a length L_f with which the longitudinal mode in the amplifier is defined. Effective optical energy $\Xi_m(L_f)$ in the space with length L_f is

$$\Xi_m(L_f)=\int_{z-L_f/2}^{z+L_f+2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varepsilon E_x^2\,dxdy\,dz=2\varepsilon_{eff}\left|A_m(t,z)\right|^2 L_f$$

$$=\begin{cases}\left[S_m(t,z)+1\right]\hbar\omega_m & \text{for optical emission}\\ S_m(t,z)\hbar\omega_m & \text{for optical absorption}\end{cases} \quad (24)$$

Difference of 1 between the optical emission and the optical absorption gives the spontaneous emission. Then, variation of the photon number is given as

$$\frac{dS_m}{dt}=\frac{\partial S_m}{\partial t}+v_m\frac{\partial S_m}{\partial z}=\frac{2\varepsilon_{eff}L_f}{\hbar\omega_m}\left\{\frac{\partial|A_m|^2}{\partial t}+v_m\frac{\partial|A_m|^2}{\partial z}\right\}$$

$$=\frac{2\varepsilon_{eff}L_f v_m}{\hbar\omega_m}\left[\left(g_m-\kappa_m\right)|A_m|^2+2|A_m|\operatorname{Re}\left\{U_m e^{-j\theta_m}\right\}\right] \quad (25)$$

$$=v_m(g_m-\kappa_m)S_m+v_m g_{em}+F_m(t,z)$$

Here, the gain coefficient g_m consists of two parts for the optical emission g_{em} and the optical absorption g_{am}, corresponding to the electron transition from the conduction band to the valence band and that from the valence band to the conduction band, respectively ;

$$g_m=g_{em}-g_{am} \quad . \quad (26)$$

The term $v_m g_{em}$ in (25) indicates the spontaneous emission going into the mode m. Then, $1/v_m g_{em}$ is a time period emitting one photon as the spontaneous emission. This emitted field propagates the length L_f with the velocity v_m during this time period of $1/v_m g_{em}$. Then, the length to define the longitudinal mode must be

$$L_f=1/g_{em} \quad . \quad (27)$$

The last term in (25)

$$F_m(t,z)=\frac{4\varepsilon_{eff}L_f v_m}{\hbar\omega_m}|A_m|\operatorname{Re}\left\{U_m e^{-j\theta_m}\right\} \quad (28)$$

gives generation of the intensity fluctuation called as the Langevin noise source.

Since the magnetic component H_y is related with E_x as

$$H_y=\frac{\beta_m}{\omega\mu_o}E_x=\sqrt{\frac{\varepsilon_{eq}}{\mu_o}}E_x \quad , \quad (29)$$

carrying power $P_m(t,z)$ of the mode m along the waveguide is

$$P_m(t,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_x\times H_y\,dxdy=2\sqrt{\frac{\varepsilon_{eq}}{\mu_o}}\left|A_m(t,z)\right|^2 , \quad (30)$$

and is related with the photon number by (24) as

$$S_m(t,z)=\frac{L_f}{\hbar\omega_m v_m}P_m(t,z) \quad . \quad (31)$$

Then, a dynamic equation for the carrying optical power is obtained as

$$\frac{\partial P_m}{\partial z}+\frac{1}{v_m}\frac{\partial P_m}{\partial t}=\left(g_m-\kappa_m\right)P_m+\hbar\omega_m v_m g_{em}^2+\hbar\omega_m g_{em}F_m(t,z) \quad . \quad (32)$$

As the second step, we exam dynamics the electron number or the electron density n. Here we need to define two types of volume. The first is full volume V_o of the active region in the amplifier

$$V_o=w\,d\,L_o \quad . \quad (33)$$

The second is the volume corresponding to the defined length L_f

$$V_f=w\,d\,L_f=w\,d/g_{em} \quad (34)$$

Then, variation of the electron number in the defined space is given as

$$\frac{d(nV_f)}{dt} = -\sum_m v_m g_m S_m - \frac{nV_f}{\tau} + \frac{V_f}{V_o}\frac{I}{e} + W(t,z) \quad (35)$$

where, τ is the electron life time, I is the driving current and $W(t,z)$ is another Langevin noise source. By dividing this equation with V_f and by help of (31), we get a dynamic equation for the electron density.

$$\frac{dn}{dt} = -\sum_m \frac{g_m P_m}{\hbar\omega_m wd} - \frac{n}{\tau} + \frac{I}{eV_o} + \frac{W(t,z)}{V_f} \quad (36)$$

As the electron life time, we count both the radiative and non-radiative recombinations as,

$$1/\tau = bn + 1/\tau_{nr} \quad . \quad (37)$$

The term bn means electron transition probability followed by the spontaneous emission for all guiding and radiation modes. Only 10^{-7} to 10^{-5} of the spontaneous emission enter to the signal mode which is given with $v_m g_{em}$ in (25) and (32) [14].

The third step is to get a dynamic equation of the phase variation. The refractive index changes with the change of the electron density n resulting in variation of the optical phase. This effect is introduced with the so called line enhancement factor α_m [15] such as

$$\alpha_m = -\frac{\partial \mathrm{Re}\,\chi / \partial n}{\partial \mathrm{Im}\,\chi / \partial n} \quad (38)$$

$$a_m \equiv \frac{d g_m}{dn} = \frac{d g_{em}}{dn} \quad (39)$$

$$\varphi_m = \alpha_m a_m \{n - \bar{n}(0)\} \quad (40)$$

where $\bar{n}(0)$ is a timely averaged value of the electron density at the input port (z=0) and a_m is a tangential coefficient of the gain to the electron density. Then equation (22) is rewritten as

$$\frac{d\theta_m}{dt} = \frac{\partial\theta_m}{\partial t} + v_m\frac{\partial\theta_m}{\partial z} = \frac{v_m \alpha_m a_m}{2}\{n - \bar{n}(0)\} + T_m(t,z) \quad (41)$$

where $T_m(t,z)$ is a generating term for the phase fluctuation.

$$T_m(t,z) = \frac{v_m}{|A_m|^2}\mathrm{Im}\{U_m e^{-j\theta_m}\} \quad (42)$$

The time averages of the intensity and the phase fluctuations are zero,

$$<\mathrm{Re}\{U_m e^{-j\theta_m}\}> = <\mathrm{Im}\{U_m e^{-j\theta_m}\}> = 0 \quad (43)$$

also the mutual correlation should be zero

$$<\mathrm{Re}\{U_m e^{-j\theta_m}\}\mathrm{Im}\{U_m e^{-j\theta_m}\}> = 0 \quad . \quad (44)$$

However, square values of these fluctuations must be equal.

$$<\mathrm{Re}^2\{U e^{-j\theta_m}\}> = <\mathrm{Im}^2\{U e^{-j\theta_m}\}> \quad (45)$$

Then we get an important relation between generating terms of the intensity and the phase fluctuations as.

$$<T_m^2(t,z)> = \frac{<F_m^2(t,z)>}{4(S_m+1)^2} = \frac{<F_m^2(t,z)>}{4(P_m/\hbar\omega_m v_m g_{em}+1)^2} \quad . \quad (46)$$

III. FLUCTUATING TERMS

A. *Intensity noise*

The generating terms of the fluctuation are expressed with angular frequency components as

$$F_m(t,z) = \int F_{m\Omega}(z) e^{j\Omega t} d\Omega \quad , \quad (47)$$

$$W(t,z) = \int W_\Omega(z) e^{j\Omega t} d\Omega \quad . \quad (48)$$

Then the optical power, the electron density and the gain coefficients are expanded with CW (continuous wave) terms and fluctuating terms such as

$$P_m(t,z) = \bar{P}_m(z) + \int P_{m\Omega}(z) e^{j\Omega t} d\Omega \quad , \quad (49)$$

$$n(t,z) = \bar{n}(z) + \int n_\Omega(z) e^{j\Omega t} d\Omega \quad , \quad (50)$$

$$\begin{aligned} g_m(t,z) &= \bar{g}_m(z) + \frac{d g_m}{dn}\int n_\Omega(z) e^{j\Omega t} d\Omega \\ &= \bar{g}_m(z) + a_m \int n_\Omega(z) e^{j\Omega t} d\Omega \end{aligned} \quad , \quad (51)$$

$$g_{em}(t,z) = \bar{g}_{em}(z) + a_m \int n_\Omega(z) e^{j\Omega t} d\Omega \quad . \quad (52)$$

By substituting these equations to (32), we get spatial changes of the CW and the fluctuating terms.

$$\frac{\partial \bar{P}_m}{\partial z} = (\bar{g}_m - \kappa_m)\bar{P}_m + v_m \hbar\omega_m \bar{g}_{em}^2 \quad (53)$$

$$\begin{aligned} \frac{\partial P_{m\Omega}}{\partial z} &= \left(-\frac{j\Omega}{v_m} + \bar{g}_m - \kappa_m\right)P_{m\Omega} \\ &+ a_m(\bar{P}_m + 2v_m\hbar\omega_m\bar{g}_{em})n_\Omega + \hbar\omega_m \bar{g}_{em} F_{m\Omega} \end{aligned} \quad (54)$$

Similarly by substitution to (36), we get two equations for the CW and the fluctuating terms for the carrier density.

$$\frac{I}{eV_o} = \sum_m \frac{\bar{g}_m \bar{P}_m}{\hbar\omega_m wd} + b\bar{n}^2 + \frac{\bar{n}}{\tau_{nr}} \quad (55)$$

$$n_\Omega = \frac{W_\Omega/V_f - \sum_m(\bar{g}_m/\hbar\omega_m wd)P_{m\Omega}}{j\Omega + \sum_m a_m \bar{P}_m/\hbar\omega_m wd + 2b\bar{n} + 1/\tau_{nr}} \quad (56)$$

By substituting (56) to (54), we get an equation to give spatial variation of $P_{m\Omega}$ as

$$\begin{aligned} \frac{\partial P_{m\Omega}}{\partial z} &= \left(-\frac{j\Omega}{v_m} + \bar{g}_m - \kappa_m\right)P_{m\Omega} \\ &+ a_m(\bar{P}_m + 2v_m\hbar\omega_m\bar{g}_{em})\frac{\dfrac{W_\Omega}{V_f} - \sum_p \dfrac{\bar{g}_p P_{p\Omega}}{\hbar\omega_p wd}}{j\Omega + \sum_p \dfrac{a_p \bar{P}_p}{\hbar\omega_p wd} + 2b\bar{n} + \dfrac{1}{\tau_{nr}}} \\ &+ \hbar\omega_m \bar{g}_{em} F_{m\Omega} \end{aligned} \quad (57)$$

Although power fluctuations among different modes should have mutual correlations through the fluctuation of n_Ω, we put in this paper that the mutual correlation is almost zero as the first order approximation ;

$$\langle P_{p\Omega} P_{m\Omega} \rangle \approx 0 \qquad \text{for } p \neq m \, . \quad (58)$$

Then power fluctuation for the total modes is given with summed value of the power fluctuation of each mode as

$$\left\langle \left(\sum_m P_{m\Omega} \right)^2 \right\rangle = \sum_m \left\langle P_{m\Omega}^2 \right\rangle \quad . \tag{59}$$

We may need to note here that the optical field components $\overline{P}_m(z)$ and $P_{m\Omega}(z)$ propagate along z direction, but other components such as $\overline{n}, n_\Omega \cdots$ do not propagate along z direction even they vary with the position z. The fluctuation $P_{m\Omega}(z)$ has very rapid spatial variation, but the auto-correlated term $< P_{m\Omega}^2 >$ has more smooth variation along z direction.

Then we form spatial variation of $< P_{m\Omega}^2 >$ from (57) as,

$$\frac{\partial < P_{m\Omega}^2 >}{\partial z} = \left\langle \frac{\partial P_{m\Omega}}{\partial z} P_{m\Omega}^* + P_{m\Omega} \frac{\partial P_{m\Omega}^*}{\partial z} \right\rangle$$
$$= 2\left[\overline{g}_m - k_m - \frac{a_m \overline{g}_m \left(\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em} \right) \left(\sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right)}{\left\{ \Omega^2 + \left(\sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right)^2 \right\} \hbar\omega_m wd} \right]$$
$$\times < P_{m\Omega}^2 >$$
$$+ \frac{2 a_m \left(\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em} \right)}{V_f} \mathrm{Re} \left\{ \frac{< P_\Omega W_\Omega >}{- j\Omega + \sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}}} \right\}$$
$$+ 2\hbar\omega_m \overline{g}_{em} \mathrm{Re} < P_{m\Omega} F_{m\Omega} > \tag{60}$$

The cross correlations $< P_{m\Omega} W_\Omega >$ and $< P_{m\Omega} F_{m\Omega} >$ are complex numbers and varied smoothly with the propagation of $P_{m\Omega}$. Since we can suppose relations of $\partial W_\Omega / \partial z = 0$ and $\partial F_\Omega / \partial z = 0$, changes of the cross-correlating terms are obtained by multiplying W_Ω^* and $F_{m\Omega}^*$ to (57) such as,

$$\frac{\partial < P_{m\Omega} W_\Omega >}{\partial z}$$
$$= \left\{ -\frac{j\Omega}{v} + \overline{g}_m - \kappa_m \right\} < P_{m\Omega} W_\Omega >$$
$$- \frac{a_m \overline{g}_m (\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em})}{\left\{ j\Omega + \sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right\} \hbar\omega_m wd} \sum_p < P_{p\Omega} W_\Omega >$$
$$+ \frac{a_m (\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em})}{\left\{ j\Omega + \sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right\} V_f} < W_\Omega^2 >$$
$$+ \hbar\omega_m \overline{g}_{em} < F_{m\Omega} W_\Omega > \tag{61}$$

$$\frac{\partial < P_{m\Omega} F_{m\Omega} >}{\partial z}$$
$$= \left[-\frac{j\Omega}{v_m} + \overline{g}_m - \kappa_m - \frac{a_m \overline{g}_m (\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em}) / \hbar\omega_m wd}{\left\{ j\Omega + \sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right\}} \right]$$
$$\times < P_{m\Omega} F_{m\Omega} >$$
$$+ \frac{a_m (\overline{P}_m + 2 v_m \hbar\omega_m \overline{g}_{em})}{\left\{ j\Omega + \sum_p \frac{a_p \overline{P}_p}{\hbar\omega_p wd} + 2b\overline{n} + \frac{1}{\tau_{nr}} \right\} V_f} < W_\Omega F_{m\Omega} >$$
$$+ \hbar\omega_m \overline{g}_{em} < F_\Omega^2 > \tag{62}$$

Amounts of the noise generating terms are evaluated with (32) and (35) by summing up all electron transition probabilities between the conduction and the valence bands and the electron injection into the active region with help of simple assumptions of the Poisson distributions for the statistical behavior ;

$$< F_{m\Omega}^2 > = \frac{\overline{g}_{em} + g_{am} + \kappa_m}{\hbar\omega_m \overline{g}_{em}} \overline{P}_m + v_m \overline{g}_{em} \tag{63}$$

$$< W_\Omega^2 > = \sum_m \frac{\overline{g}_{em} + g_{am}}{\hbar\omega_m \overline{g}_{em}} \overline{P}_m + \left(b\overline{n}^2 + \frac{\overline{n}}{\tau_{nr}} \right) V_f + \frac{V_f}{V_o} \frac{I}{e} \tag{64}$$

$$< F_{m\Omega} W_\Omega > = < W_\Omega F_{m\Omega} > = -\frac{\overline{g}_{em} + g_{am}}{\hbar\omega_m \overline{g}_{em}} \overline{P}_m - v_m \overline{g}_{em} \quad . \tag{65}$$

We trace terms of $\overline{P}_m(z)$, $< P_{m\Omega}^2(z) >$, $< P_{m\Omega}(z) W_\Omega(z) >$ and $< P_{m\Omega}(z) F_{m\Omega}(z) >$ from $z = 0$ to $z = L_o$ with numerical calculation.

We now put the mode number corresponding to the input signal to be s with initial conditions at $z = 0$ as

$$\overline{P}_s(0) = \overline{P}_{in} \quad \text{for } m = s , \tag{66}$$
$$\overline{P}_m(0) = 0 \quad \text{for } m \neq s , \tag{67}$$
$$< P_{s\Omega}^2(0) > = < P_\Omega^2 >_{in} \quad \text{for } m = s , \tag{68}$$
$$< P_{m\Omega}^2(0) > = 0 \quad \text{for } m \neq s \tag{69}$$

The initial conditions of the cross-correlations $z = 0$ are put to be zero for all modes :

$$< P_{m\Omega}(0) W_\Omega(0) > = < P_{m\Omega}(0) F_{m\Omega}(0) > = 0 \tag{70}$$

Values of the fluctuating power is expressed in tem of the relative intensity noise (RIN) for total modes as given by

$$\mathrm{RIN} = \frac{\sum_m < P_{m\Omega}^2 >}{\left(\sum_m \overline{P}_m \right)^2} \quad [\mathrm{Hz}^{-1}] \ . \tag{71}$$

B. Frequency Noise

Variation of the optical phase θ_m has been given in (41), where the partial time derivative of the phase is counted to be shift of the optical frequency f_m and is expressed with an averaged value $\bar{f}_m$ and fluctuated term $f_{m\Omega}$ such as

$$\frac{\partial \theta_m(t,z)}{\partial t} = 2\pi\, f_m = 2\pi \left\{ \bar{f}_m(z) + \int f_{m\Omega}(z) e^{j\Omega t}\, d\Omega \right\}. \quad (72)$$

Then, the optical phase is expressed with these terms as

$$\theta_m(t,z) = 2\pi \left\{ \bar{f}_m(z)\cdot t + \frac{1}{j\Omega}\int f_{m\Omega}(z) e^{j\Omega t}\, d\Omega \right\} \quad . \quad (73)$$

The generating term $T_m(t,z)$ for the phase fluctuation in (41) is given by

$$T_m(t,z) = \int T_{m\Omega}(z) e^{j\Omega t}\, d\Omega \quad . \quad (74)$$

By substituting these equations to (41), we get

$$2\pi \left\{ \bar{f}_m + \frac{\partial \bar{f}_m}{\partial z} v_m t \right\} = \frac{\alpha_m\, a_m\, v_m}{2} \{\bar{n}(z) - \bar{n}(0)\} \quad (75)$$

$$2\pi \left\{ f_{m\Omega} + \frac{v_m}{j\,\Omega}\frac{\partial f_{m\Omega}}{\partial z} \right\} = \frac{\alpha_m\, a_m\, v_m}{2} n_\Omega + T_{m\Omega} \quad . \quad (76)$$

Equation (76) is rewritten to an equation for spatial variation to be

$$\frac{\partial f_{m\Omega}}{\partial z} = j\Omega \left\{ \frac{\alpha_m a_m}{4\pi} n_\Omega + \frac{T_{m\Omega}}{2\pi v_m} - \frac{f_{m\Omega}}{v_m} \right\} \quad . \quad (77)$$

The frequency noise (FM noise) is $< f_{m\Omega}^2 >$, and whose spatial variation is given from (77) as

$$\frac{\partial < f_{m\Omega}^2 >}{\partial z} = \left\langle \frac{\partial f_{m\Omega}}{\partial z} f_{m\Omega}^* + f_{m\Omega}\frac{\partial f_{m\Omega}^*}{\partial z} \right\rangle$$
$$= \frac{\Omega}{\pi}\left\{ \frac{\alpha_m a_m}{2}\mathrm{Im} < f_{m\Omega}\, n_\Omega > + \frac{1}{v_m}\mathrm{Im} < f_{m\Omega}\, T_{m\Omega} > \right\} \quad (78)$$

Terms $< f_{m\Omega}\, n_\Omega >$ and $< f_{m\Omega}\, T_{m\Omega} >$ are complex numbers and whose varying equations are obtained from (77) by multiplying n_Ω^* and $T_{m\Omega}^*$ such as

$$\frac{\partial < f_{m\Omega} n_\Omega >}{\partial z} = j\,\Omega \left\{ \frac{\alpha_m\, a_m}{4\pi} < n_\Omega^2 > - \frac{< f_{m\Omega}\, n_\Omega >}{v_m} \right\} \quad (79)$$

$$\frac{\partial < f_{m\Omega} T_{m\Omega} >}{\partial z} = \frac{j\,\Omega}{v}\left\{ \frac{< T_{m\Omega}^2 >}{2\pi} - < f_{m\Omega}\, T_{m\Omega} > \right\} \quad (80)$$

where the relation of $< n_\Omega\, T_{m\Omega} > = 0$ is used. Terms $< n_\Omega^2 >$ and $< T_{m\Omega}^2 >$ are given from (56),(46) and (63) as

$$< n_\Omega^2 > = \left\{ < W_\Omega^2 > - \sum_m \frac{2\,\bar{g}_m}{\hbar\,\omega_m\,\bar{g}_{em}}\mathrm{Re} < P_{m\Omega} W_\Omega > + \sum_m \left(\frac{\bar{g}_m}{\hbar\,\omega_m\,\bar{g}_{em}} \right)^2 < P_{m\Omega}^2 > \right\}$$
$$\div \left[V_f^2 \left\{ \Omega^2 + \left(\sum_m \frac{a_m\,\bar{P}_m}{\hbar\omega_m\, wd} + 2b\bar{n} + \frac{1}{\tau_{nr}} \right)^2 \right\} \right] \quad (81)$$

$$< T_{m\Omega}^2 > = \frac{\left(\bar{g}_{em} + g_{am} + \kappa_m\right)\bar{P}_m / \hbar\omega_m\,\bar{g}_{em} + v_m\,\bar{g}_{em}}{4(\bar{P}_m / \hbar\omega_m\, v_m\,\bar{g}_{em} + 1)^2} \quad . \quad (82)$$

The frequency noise of the signal light $< f_{s\Omega}^2(z) >$ is calculated by tracing (78), (79) and (80) for $m = s$ with help of (81),(64),(61),(60) and (82) along $0 \le z \le L_o$ with the initial conditions of the incident signal ,

$$< f_{s\Omega}^2(0) > = < f_{s\Omega}^2 >_{in} \quad \text{for } m = s \ , \quad (83)$$

with

$$< f_{s\Omega}(0)\; n_\Omega(0) > = < f_{s\Omega}(0)\, T_{s\Omega}(0) > = 0 \quad \text{for } m = s\,. \ (84)$$

The spectral line-width Δf_s is derived from the frequency noise by putting $\Omega \to 0$ to be

$$\Delta f_s(z) = 4\pi < f_{s0}^2(z) > \qquad [\mathrm{Hz}] \quad . \quad (85)$$

We need to pay attention here that the increase of the frequency noise $< f_{s\Omega}^2(z) >$ is proportional to the noise angular frequency Ω. Then, the spectral line-width scarcely varies in the SOA.

The other modes in the ASE may have the frequency noise corresponding to the photon generating time of $1/v_m\, g_{em}$, that is

$$< f_{m\Omega}^2(z) > \approx v_m\,\bar{g}_{em} \qquad \text{for } m \ne s \quad . \quad (86)$$

This value is just same as the frequency separation of among the ASE modes obtained by relation of $(\beta_{m+1} - \beta_m)L_f = 2\pi$. Then, spectrum line-width

$$\Delta f_m(z) = 4\pi v_m\,\bar{g}_{em} \qquad \text{for } m \ne s \quad (87)$$

is 4π times wider than the frequency separation, resulting in a continuous profile of the ASE spectrum as experimentally observed.

IV. Caluculated Data

The SOA model for numerical calculation is made of InGaAsP. Although the gain coefficient g_m and the spontaneous emission g_{em} have specific wavelength dispersions, we take up M modes within the half wavelength $\Delta\lambda$ of the ASE profile indicated in Fig.2 and suppose the gain coefficients and the spontaneous emission rates are identical for all these modes for simple treatment such that

$$g_m = g \quad , \quad (88)$$

$$g_{em} = g_e \quad . \quad (89)$$

Since each mode has relation of

$$\beta_m\, L_f = 2\pi\, n_{eq}\, L_f / \lambda_m = 2m\pi \quad (90)$$

the number M of taken up modes giving the amplified spontaneous emission within $\Delta\lambda$ is

$$M = 2n_{eq}\, L_f\, \Delta\lambda / \lambda^2 - 1 = 2n_{eq}\, \Delta\lambda / \lambda^2\, \bar{g} - 1 \quad (91)$$

where two directions of the electric polarization, E_x and E_y, is counted in (91) and one mode is subtracted as the signal mode. Then the RIN of (71) is rewritten as

$$\mathrm{RIN} = \frac{< P_{s\Omega}^2 > + M < P_{m\Omega}^2 >}{\left(\bar{P}_s + M\,\bar{P}_m\right)^2} \qquad \text{where } m \ne s\,. \ (92)$$

Since the active region consists of the quantum well structure, the gain coefficient g shows a nonlinear relation for increase of the electron density n. We experimentally examined relation between the gain coefficient and the injected electron density in a device and fined a function by making best fitting to the experimental data as [16]

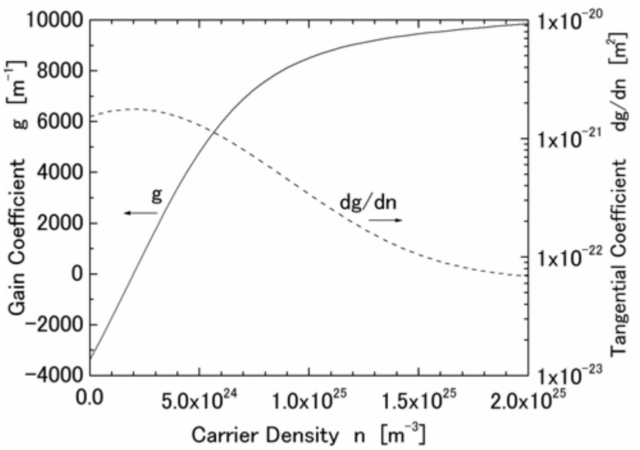


Fig.3 Variation of gain and tangential coefficients with electron density. The gain profile is obtained from experimentally measured data with (93) by making best fitting to the experimental data [16]. The dotted line is obtained from (93) as in (94)

$$g = 8.75\times10^{3}\times\left[1-\frac{2}{1+\exp\{3.9\times10^{-25}\times(n-n_g)\}}\right] + 6.25\times10^{-23}\times(n-n_g) \quad (93)$$

whose profile is given in Fig.3 with a solid line. The tangential coefficient of the gain to the electron density is obtained from this equation as

$$a=\frac{d\,g}{d\,n}=\frac{6.83\times10^{-21}\times\exp\{3.9\times10^{-25}\times(n-n_g)\}}{\left[1+\exp\{3.9\times10^{-25}\times(n-n_g)\}\right]^2}+6.25\times10^{-23} \quad (94)$$

and is also shown in Fig.3 with a broken line. The term for the electron transition from the valence to the conduction bands is given by putting $n=0$ in (93) as

$$g_a = -8.75\times10^{3}\times\left[1-\frac{2}{1+\exp\{-3.9\times10^{-25}\times n_g\}}\right] + 6.25\times10^{-23}\times n_g\ . \quad (95)$$

The term g_e for the electron transition from the conduction to the valence bands is evaluated in relation of $g_e = g + g_a$. Other parameters of used in the calculation are given in Table I.

Numerical examples for changes of several quantities along propagation in the SOA are shown in Fig.4. Fixed parameters are the length of $L_o = 1{,}000\mu\text{m}$, the driving current of $I = 100$ mA and the optical input signal of $\text{RIN}_{\text{in}} = \langle P_{s\Omega}^2(0)\rangle / \overline{P}_s^2(0) = 1\times10^{-15}\,\text{Hz}^{-1}$ and $\langle f_{s\Omega}^2(0)\rangle = 100\,\text{KHz}$ over angular frequency range of $0 \le \Omega/2\pi \le 100\,\text{GHz}$. The optical input power $P_{in} = \overline{P}_s(0)$ is selected in range of $1\mu\text{W} \le P_{in} \le 100\,\text{mW}$.

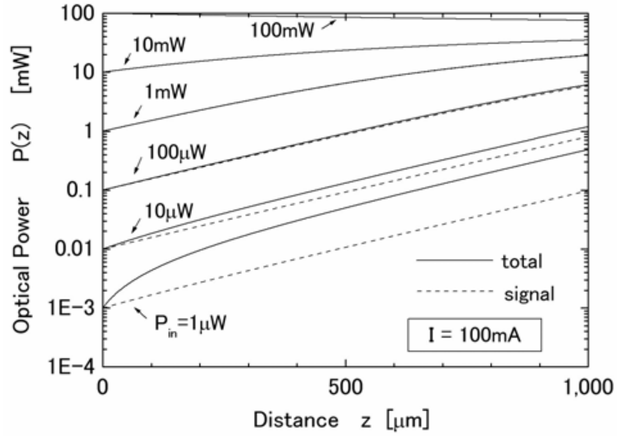
Variations of $\overline{P}(z)$, $\overline{P}(z)/P_{in}$ and $\overline{n}(z)$ along propagation direction z are shown in Fig.4(a),(b) and (c). In Fig.4(a), the signal powers are indicated with broken lines, while the total powers including the ASE are written with the solid lines. We find that the signal power is masked by the ASE when the input power is lower than $10\mu\text{W}$. When the input power is lager than several mW, the amplification suffers saturation as shown reduction of the electron density $\overline{n}(z)$ in Fig.4(c). When the input power exceeds 100 mW, we can not get the amplification any more, because supported electrical power, $\hbar\omega\times I = 0.8\text{eV}\times100\text{mA} = 80$ mW, into the SOA is lower than the optical input power.

TABLE I
Numerical values of used parameters

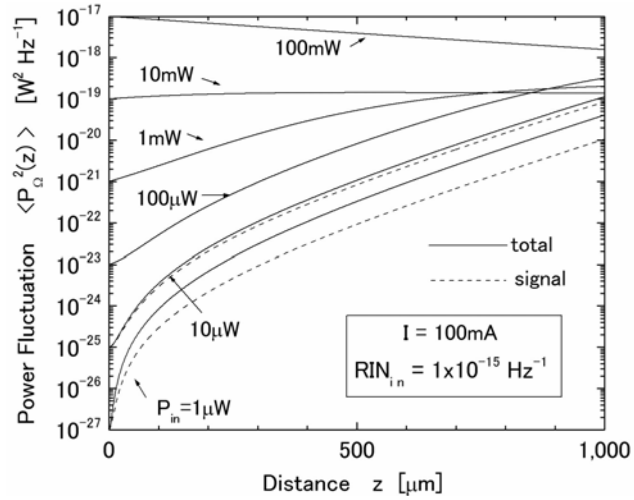
Symbol	Parameter	Value	Unit
W	activere region width	2.0	μm
d	active region thckness	40.0	nm
L_o	amplifier length	1.00	mm
V	volume of active region	8.0×10^{-17}	m^3
λ	optical wavelength	1.55	μm
$\hbar\omega$	photon energy	0.8	eV
$\Delta\lambda$	half width of ASE	80	nm
n_g	transparent electron density	2×10^{24}	m^{-3}
b	radiative recombination coefficient	2.2×10^{-16}	m^6/s
τ_{nr}	electron lifetime by non – radiative recombination	5.0×10^{-9}	s
c/v	equivalent refractive index	3.5	
κ	guiding loss coefficient	1000	m^{-1}
α	line - width enhancement factor	3	
g	gain coefficient	Eq.(93)	m^{-1}
a	tangential coefficient (= *dg/dn*)	Eq.(94)	m^2

Variations of the fluctuated optical power $\langle P_{\Omega}^2(z)\rangle$ at low frequency are shown in Fig.4(d) by supposing $\text{RIN}_{\text{in}} = 1\times10^{-15}\,\text{Hz}^{-1}$. The signal mode is again masked by the ASE when the input power is lower than $10\mu\text{W}$. The fluctuated optical power is amplified up to $P_{in} = 10\text{mW}$, but the RIN level reduces along propagation for larger optical input power than 1mW as shown in Fig.4(e). This situation comes from the fact that the fluctuation $\langle P_{\Omega}^2(z)\rangle$ is increasing but whose increasing rate is smaller than that of $\overline{P}^2(z)$. This relation is very similar to that in conventional laser oscillator, where RIN is reduced with increase of the lasing power. Therefore, application of a SOA as a pre-amplifier for very weak signal is not suitable because RIN increase after amplification, but application as a power amplifier connecting a laser oscillator to amplify the oscillated signal is profitable.

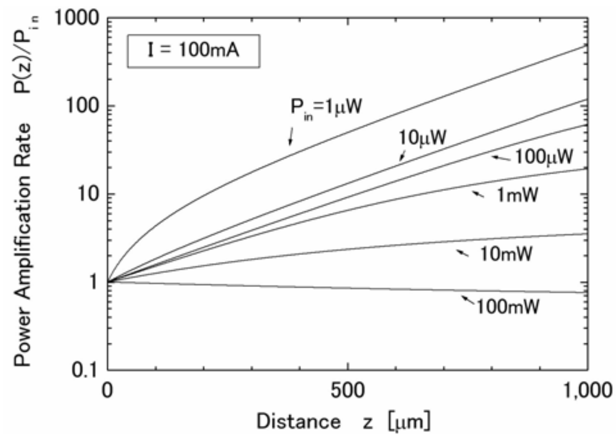
Property of the spectrum line-width is shown in Fig.4(f), where supposed line-width of the input light is $\Delta f_{in} = 4\pi\langle f_{s\Omega}^2(0)\rangle = 4\pi\times0.1\text{MHz} = 1.26\text{MHz}$. The spectrum line-width $\Delta f(z)$ hardly changes in the SOA. We understand this situation by putting $\Omega = 0$ in (77) to (80).



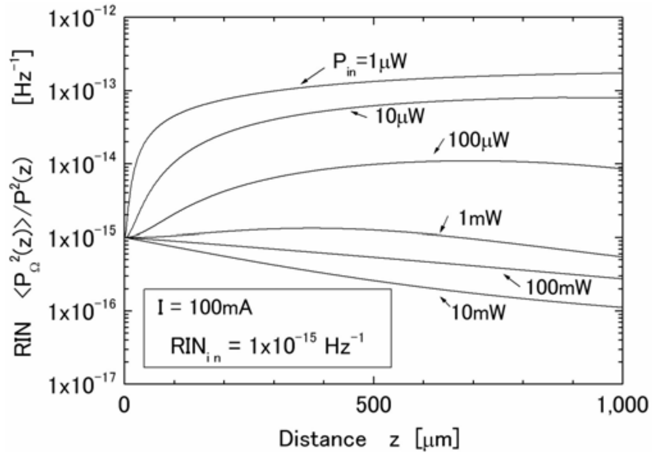
(a)



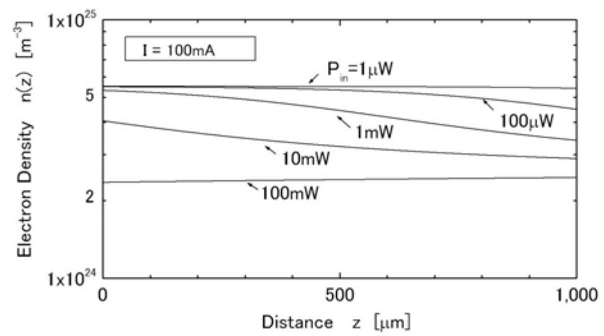
(d)



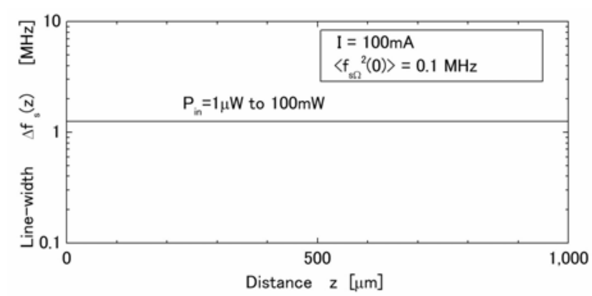
(b)



(e)



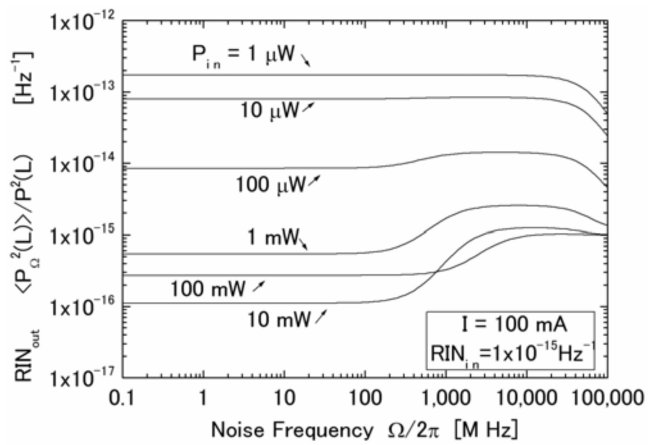
(c)



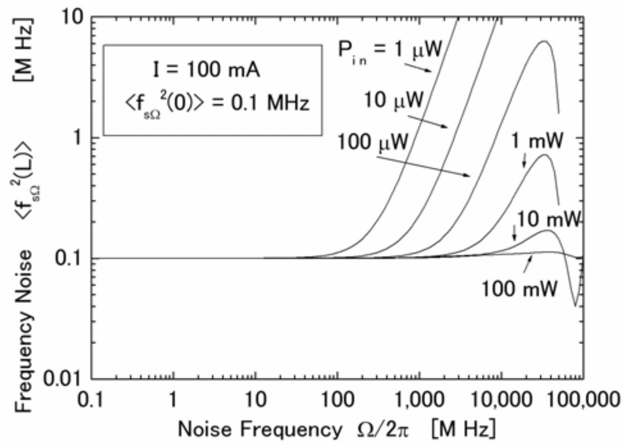
(f)

Fig.4 Variations of several quantities along propagation z in the SOA. (a) is optical power, (b) is the amplification rate, (c) is the electron density, (d) is the fluctuated power, (e) is the RIN and (f) is the line-width. The broken lines in (a) are signal powers, solid lines are total powers including the ASE. The signal power is masked by the ASE when the input power is lower than $10\mu\mathrm{W}$. When the input power is lager than several mW, the amplification suffers saturation as shown (b) with reduction of the electron density as shown in (c). The fluctuated optical power is amplified up to $P_{in} = 10\mathrm{mW}$ as shown in (d), but the RIN level reduces along propagation for larger optical input power than 1mW as shown in (e). The spectrum line-width hardly changes as shown in (f).

Frequency spectrum of the $\mathrm{RIN}_{out} =< P_{\Omega}^{2}(L) > / \overline{P}^{2}(L)$ and the frequency noise $< f_{\Omega}^{2}(L) >$ of the optical output are shown in Figs.5(a) and (b), where the noise frequency is defied be $\Omega / 2\pi$. Inputted spectrum of these noises are supposed to be uniformly spread over range of $0 \le \Omega / 2\pi < 1\times 10^{11}\,\mathrm{Hz}$. Both the intensity noise and the frequency noise have flat spectrum in the lower frequency region than 100 MHz. The RIN_{out} is increased or reduced in low frequency region giving $\mathrm{P}_{in} \approx 1\mathrm{mW}$ as a boundary as has been shown in Fig.4(e). The RIN_{out} can not be suppressed in the higher frequency range than 1,000MHz even P_{in} is larger than 10mW. Such frequency dependency for suppression of the RIN has been already reported experimentally by Shtaif and Eisenstein [5]. If $\mathrm{RIN}_{\mathrm{in}}$ has a the specific frequency profile as the input,

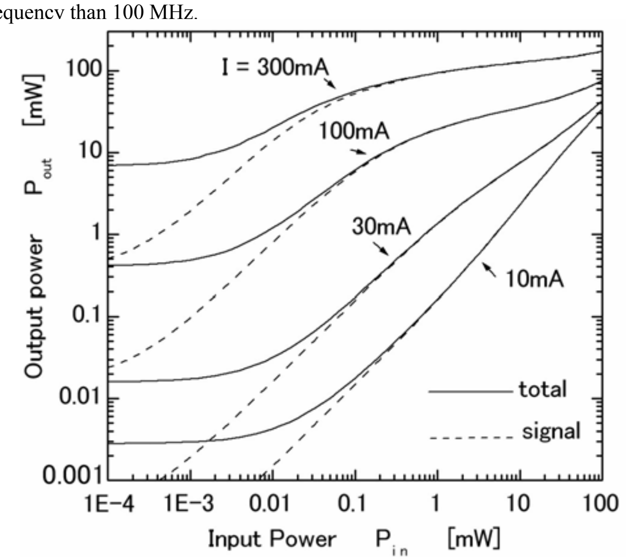


(a)

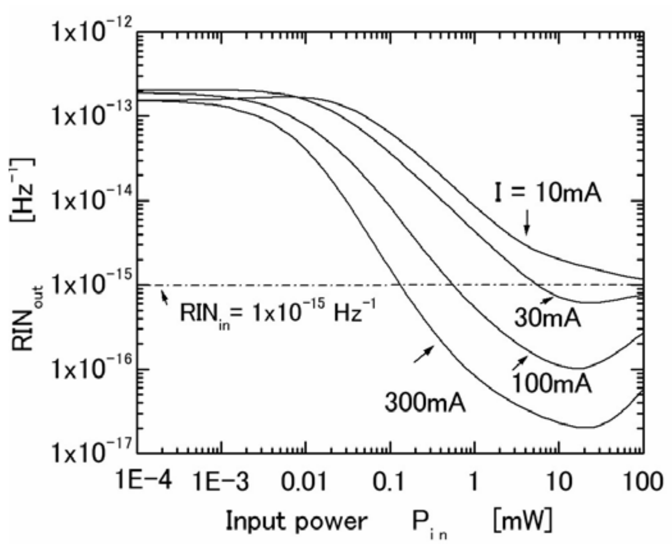


(b)

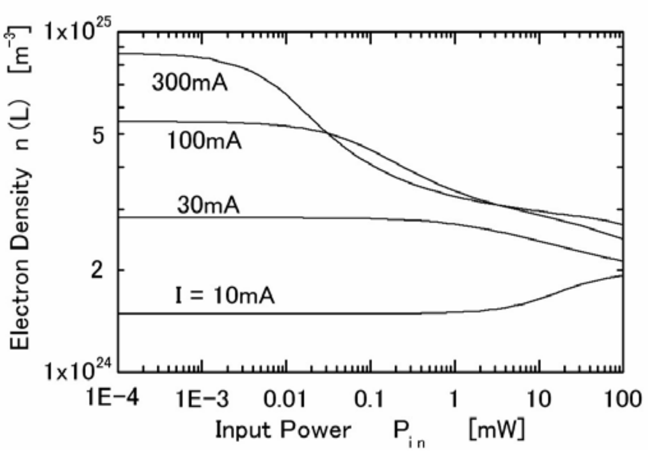
Fig.5 Frequency spectrum of the noise. (a) is the RIN and (b) is frequency noise. Both the intensity noise and the frequency noise have flat spectrum in the lower frequency than 100 MHz.



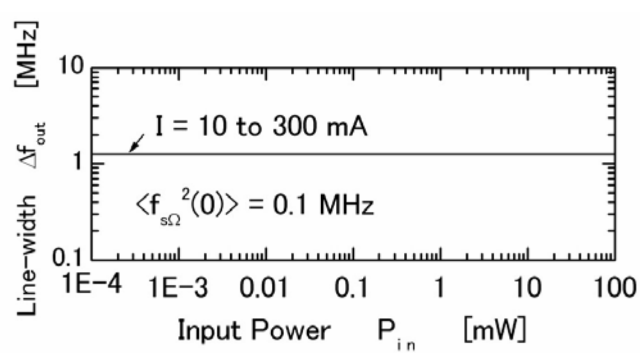
(a)



(c)



(b)



(d)

Fig.6 Variations of the output properties with the optical input power. (a) is the output power, (b) is the electron density, (c) is the RIN and (d) is the line-width. When the input powers are lower than 0.1mW, the input signal is buried in the ASE as shown in (a). Saturation of the output power and reduction of the electron density are induced for larger input power of several mW as shown in (a) and (b). The RIN can be reduced lower than that of the input signal when the input power and the driving electric current are large enough as shown in (c). The spectrum line-width of the signal hardly changes as shown in (d).

output profile RIN_{out} must also be affected by the input profile.

The frequency noise of the output signal $< f_{\Omega}^{2}(L) >$ show same value with the input signal $< f_{\Omega}^{2}(0) >$ in the lower frequency region but increase in the higher frequency region.

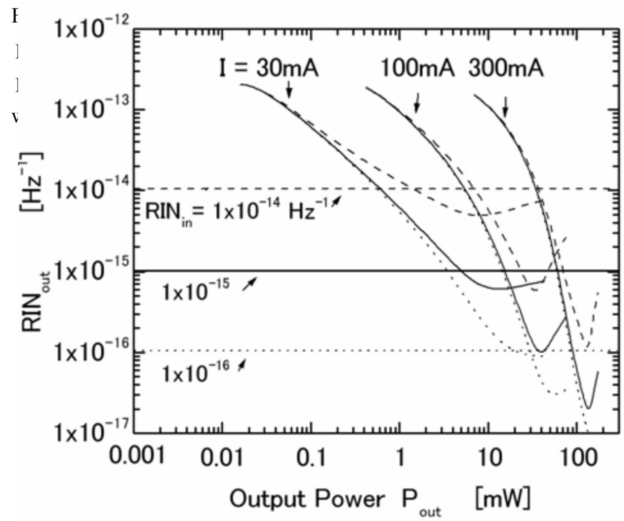
Variations of $P_{out} = \overline{P}(L)$, $\overline{n}(L)$, RIN_{out} and Δf_{out} with the input power P_{in} are shown in Figs.6(a),(b), (c) and (d) with parameters of the driving current I. In Fig.6(a), output powers of the signal light are drown with broken lines. It becomes clear that operation with lower input signal than $P_{in} = 100\mu\mathrm{W}$ is not suitable because of higher optical power of the ASE is generated in the amplifier. Saturation of the output power P_{out} and reduction of the electron density $\overline{n}(L)$ are induced for larger input power P_{in} of several mW. However, the RIN can be reduced lower than that of the input signal when the input power P_{in} and the supplied electric current I are large enough.

The spectrum line-width of the input signal hardly changes in the SOA as shown in Fig.6(d), but the optical input power P_{in} should be larger than 0.1mW to pick up the signal buried under the ASE power as shown in Fig.6(a).

When the input power P_{in} becomes larger, the output power P_{out} becomes larger. Relations between the relative intensity nose, RIN_{out}, of the optical output and the output power, P_{out}, are shown in Fig.7. Parameters in the figure are the operating driving I and the RIN_{in} of the input light. The broken, solid and dotted lines are cases of $\mathrm{RIN}_{in} = 1\times10^{-14}, 1\times10^{-15}$ and $1\times10^{-16}\ \mathrm{Hz}^{-1}$, respectively. Levels of RIN_{in} are indicated with strait lines. As found from this figure, the RIN_{out} of the output light is decided by the output optical power P_{out} from the SOA, independent from the RIN_{in} of the input light, when the driving current I is large enough.

V. DISCUSSION FOR THE REDUCTION OF RIN

Here we discuss the reason why the RIN is reduced by the amplification and whose value is decided mostly by the output optical power. Variations of the CW and the fluctuating components of the optical power are given in (53) and (54) for $\overline{P}_m$ and $P_{m\Omega}$. By comparing these equations, we find the term $a_m\left(\overline{P}_m + 2v_m\hbar\omega_m\overline{g}_{em}\right)n_{\Omega}$ is added only in the equation of fluctuating component $P_{m\Omega}$. Other terms have good correspondences between two equations. Also we find the fluctuation of n_{Ω} has an inverse phase vibration with the fluctuation of optical power $P_{m\Omega}$ as given in (56). Then the increasing rate of $< P_{m\Omega}^{2} >$ is reduced when $\overline{P}_m$ and $< P_{m\Omega}^{2} >$ become large as shown in (57) or (60). If we pick up only the



signal mode s and suppose $\partial P_{s\Omega}/\partial z \approx 0$ in (57), we get an approximated relation of

$$P_{s\Omega} \approx \frac{\hbar\omega_s\, g_e}{\kappa}\left(W_{\Omega} + F_{\Omega}\right) \qquad (93)$$

for sufficiently large $\overline{P}_s$. By help of (63) to (65), we find $< P_{s\Omega}^{2} >$ is given as a liner function of $\overline{P}_s$. Since RIN is defined be $< P_{s\Omega}^{2} > / \overline{P}_s^{\,2}$, the RIN must reduce with increase of $\overline{P}_s$, and its value is evaluated with the optical power $\overline{P}_s(z)$, the electron density $n(z)$ and the injection current density I/V_o at that position. Then the RIN of the output light can be decided by the output power P_{out} almost independent from the RIN of the input light, because of $\partial P_{s\Omega}/\partial z \approx 0$ at the output facet.

It is widely believed that the signal to the noise (S/N) ratio always degrades after passing an amplifier. However, the RIN of the optical light becomes better by passing the SOA as analyzed in this paper. This peculiar relation must be caused by the definition of the signal. The RIN is defined by supposing the CW light to be the signal. However the CW light is merely a carrier not the true "signal". "Signal" should be defined for the modulated light. If we apply an intensity modulation on the light with angular frequency Ω_M, a term $a_m\left(\overline{P}_m + 2v_m\hbar\omega_m\overline{g}_{em}\right)n_{\Omega_M}$ is added in the equation for the modulated light. Then amplification dynamics for the signal light becomes similar with that of the noise. The S/N ratio must be degraded for the modulated light by inclusion of the ASE. Our next job must be how define the S/N ratio for the modulated light.

Shtaif and Eisenstein introduced a non-linear effect generating the asymmetric gain saturation on the optical frequency to explain the sensitive dependency and frequency profile of the RIN [4][5]. When one optical field is applied in the SOA, the ASE must increase in the longer wave length side and decrease in the shorter wave length side based on the carrier beating effect as known as the asymmetric gain saturation in the semiconductor laser or the four wave mixing effect in the SOA [4][17]. However, this asymmetric property may not affect for total optical power and the intensity noise, during the optical intensity is not extremely large.

VI. CONCLUSIONS

The intensity and the frequency noises in semiconductor optical amplifier (SOA) are theoretically analyzed. Following results are obtained.

1) When the optical power of the signal light is very small, the signal light is buried in the amplified spontaneous emission from the SOA.
2) The RIN (relative intensity noise) level decrease after passing the SOA when the optical input power is rather high, although the RIN level increases when the input optical power is low.
3) When the driving injection current is large enough, the RIN level of the output light is decided by the output optical power without depending on the RIN level of the input light.
4) Reduction of the RIN is caused by the intrinsic properties between the continuous wave (CW) or DC like light as the signal and the temporally varying or AC like light as the noise.
5) The frequency noise increases in high frequency range, but hardly increases in low frequency range.
6) Then, the line-width of the inputted optical signal scarcely changes in the SOA.

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