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Optimal Sensor Network Configuration based on Control Theory

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1 Introduction

Recently, much attention has been attracted to a wireless sensor network. It generally consists many sensor nodes with memory units, communications and calculation capabilities [1, 2]. In these researches, sensor nodes are connected wirelessly and some local estimates are merged into the common estimate via the wireless communication paths. It is well known that sensor networks are superior to an observation by a system with a single sensor in a fault tolerance, load reduction of operator, collection and application of information. Owing to some advantages, it is possible to apply various fields such as guidance control systems, traffic control systems, nano-medicines and disaster countermeasures. Meanwhile, each sensor node uses electric power for a communication and calculations, but the sensor nodes are generally powered and driven by built-in batteries [3, 4]. Moreover it is difficult to change batteries frequently or charge by power cable because of the increase in costs. Therefore, it is important to utilize the energy efficiently to achieve an energy-saving system and prolong sensor nodes life. For this objective, the sensor scheduling, the optimization of the communication rate or the buffer length and decreasing communication distances by the multi-hop communication have been studied [5–7]. Consequently, in this paper, we discuss a sensor scheduling problem considering the estimation error variance and the communication energy in the sensor networked feedback control system, one of the approach to this objective.

Firstly, we consider a optimal sensor network configuration via multi-hop communication for a feedback control system. The estimation problem in a sensor network system has been studied in [8–11]. A distributed Kalman filtering algorithm with a consensus strategy were proposed in [1, 12]. In these methods each sensor node communicates with its neighbors on a network. However, if the plant is applied control inputs from fusion center or one of sensor nodes, all sensor node have to obtain its information in real time and it is difficult to develop real system. In [13], a network configuration problem with a multi-hop communication and a feedback control system considering communication energy and estimation error variance. However amount of information transmitted from each sensor node increase with a number of sensor nodes.

Secondly, we consider a network configuration for a sensor network that sensor nodes have enought calculation ability. In beard,olf4,dwhyte,rant, it is difficult to apply to the guidance control that the plant receives arbitrary control inputs. Moreover, they do not consider the communication energy. Meanwhile, the network configuration and the sensor scheduling algorithm considering an estimation error variance and communication energy were proposed in [6, 7, 13]. However, each sensor node has only a observation and communication function and does not have a calculation function. The fusion center calculates the estimate and transmits the control input to the plant. In our framework, each sensor node has the calculation, communication and observation functions and the control input is applied to the plant. Thus we can not apply these previous methods.

In this paper, firstly, we deal with a optimal sensor network configuration via multi-hop communication for a feedback control system. We first define a sensor network with multi-hop communication. Then we assume that each sensor node transmit same amount of information for issue resolution of increasing amount of information transmitted. In this system, we discuss a estimation problem and a network configuration problem. Then we show that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update and a trade-off between the estimation error variance and a communication energy. Secondly, we propose a network configuration algorithm considering this trade-off. This network configuration algorithm achieves sub-optimal network topology with minimum energy and a desired error variance.

Secondly, we discuss a sensor scheduling problem considering the estimation error variance and communication energy in a feedback control system via a sensor network that sensor nodes have enought calculation ability. We first propose the estimation algorithm with the unknown input of the plant in the feedback control system via a sensor network. Each sensor node calculates the local estimate without information of the control input and transmits its information to the sensor node applying the control input to the plant. This sensor node calculates the common estimate and control input using received information. Then we show that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update. Secondly, we propose a sensor scheduling algorithm considering estimation error variance and communication energy. This scheduling algorithm achieves sub-optimal network topology with minimum energy and a desired error variance.

Finally, we verify effectiveness of a sensor scheduling algorithm by experiments.

This chapter is organized as follows. In section 2, we discuss a optimal sensor network configuration via multi-hop communication for a feedback control system. In section 3, we discuss a a sensor scheduling problem considering the estimation error variance and communication energy in a feedback control system via a sensor network that sensor nodes have enought calculation ability.

2 Optimal Sensor Network Configuration via Multi-Hop Communication

2.1 Problem formulation

Plant and Sensor Nodes

In this paper, we consider the feedback control system via a sensor network illustrated in Fig. 1. This system consists the plant and N sensor nodes S_i, $(i = 1, 2, ..., N)$. We assume all sensor nodes can take a measurement of the plant. The process dynamics of the plant and the measurement equation of a sensor node S_i are given by

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$
$$y_k^i = C_i x_k + v_k^i \tag{2}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k^i \in \mathbb{R}^{q_i}$ are the state, the control input and the measurement output of a sensor node S_i respectively. Additionally, $w_k \in \mathbb{R}^n$, $v_k^i \in \mathbb{R}^{q_i}$ are the process noise and measurement noise respectively. From (2), each sensor node take a different measurement. Moreover, (1) and (2) satisfy following *assumptions 1-3*.

Assumption 1 *w_k, $v_k = \left[(v_k^1)^{\mathrm{T}} \ (v_k^2)^{\mathrm{T}} \ \cdots \ (v_k^N)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^q$, $(q = \sum_i^N q_i)$ are zero mean white Gaussian noise and satisfy equations*

$$\mathrm{E}\left\{\begin{bmatrix} w_k \\ v_k \end{bmatrix}\begin{bmatrix} w_k^{\mathrm{T}} \ v_k^{\mathrm{T}} \end{bmatrix}\right\} = \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix}, \tag{3}$$
$$\mathrm{E}\left\{w_k x_0^{\mathrm{T}}\right\} = \mathrm{E}\left\{v_k x_0^{\mathrm{T}}\right\} = \mathbf{0}, \tag{4}$$

where Q, $R = \mathrm{diag}(R_1, R_2, ...)$ are the positive semidefinite and positive definite covariance matrix of noises w_k, v_k respectively.

Assumption 2 *The matrix pair $(A, Q^{\frac{1}{2}})$ is reachable.*

Assumption 3 *The matrix pair (C, A) is detectable, where*

$$C = \begin{bmatrix} C_1^{\mathrm{T}} \ C_2^{\mathrm{T}} \cdots C_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{5}$$

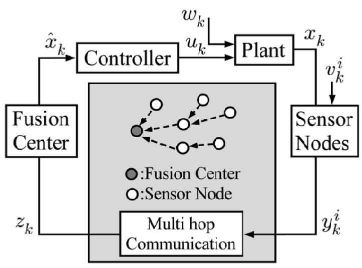


Fig. 1. Sensor network system.

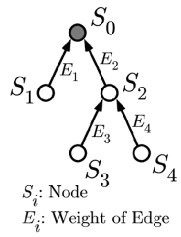


Fig. 2. An example of Network.

Network Topology

In this paper, we deal multi-hop communication. N sensor nodes and the fusion center S_0 are connected wirelessly and information transmitted from each sensor node are passed on to the fusion center via some relay nodes. The example of a network topology is illustrated in Fig. 2. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denoted a graph with the set of vertices $\mathcal{V}$ and the set of edges $\mathcal{E}$. Then sensor node S_i and network topology satisfy following Assumption 4, 5.

Assumption 4 *Sensor nodes S_i can transmit $z_k^i \in \mathbb{R}^r$ to the other sensor node once per time step with a time delay less than a sampling time. Additionally, when a sensor node S_i transmit information, this sensor node uses the energy E_i.*

Assumption 5 A network topology T is a directed spanning tree with root S_0.

From Assumption 5, a sensor node S_i transmits z_k^i containing information of a measurement of S_i to the other sensor node. the dimension of z_k^i is r in all sensor nodes. Moreover, each sensor node use a energy E_i for transmitting z_k^i to other sensor node. We assume $E = \sum_{i=1}^{N} E_i$ is the energy the whole system is using. The energy E_i is the weight of the edge of the network topology T. In general, the communication energy depend on a length of a communication pass between sensor nodes S_i and S_0. Consequently, if there are some relay node between S_i and S_0, the communication energy to pass to the sensor node S_0 from S_i will be reduced. But all sensor nodes transmit information once per one time step and the time delay between sensor nodes S_i and S_0 will increase. Consequently, there is a trade-off between an estimation accuracy and a communication energy.

Control Problems

In this paper, we discuss an estimation problem and a network configuration problem.

Problems can be formulated as following *problems 1, 2*.

Problem 1. We assume the plant and all sensor nodes satisfy *Assumption 1-5* and the network topology T is determined. Then compute the optimal state estimate $\hat{x}_k^-$ that minimizes following estimation error variance.

$$J = \mathrm{E}\left\{\left(x_k - \hat{x}_k^-\right)^{\mathrm{T}}\left(x_k - \hat{x}_k^-\right)\right\} \tag{6}$$

Problem 2. Find the optimal network topology T^* satisfying $J \leq \gamma$, Assumption 5 and following equation:

$$T^* = \arg\min_T E, \tag{7}$$

where $\gamma > 0$ is a design parameter.

2.2 Proposed method

Information merge method

In this paper, we define the sensor node receiving information from a sensor node S_i as the sensor node $\mathrm{Par}(S_i)$ and the set including sensor nodes transmitting information to a sensor nodeS_i as the set $\mathcal{N}_i = \{j|\mathrm{Par}(S_j) = S_i\}$. Moreover we define the depth h_i of a sensor node S_i, the hight $\bar{h} = \max_i h_i$ of the network topology T. For example, $\mathcal{N}_0 = \{1, 2\}$ and $\bar{h} = 2$ in Fig. 2.

A measurement output of each sensor node S_i have to merge via z_k^i with same dimension. Consequently, we propose following information fusion method for each sensor node.

$$z_k^i = C_i^{\mathrm{T}} R_i^{-1} y_{k-\bar{h}+h_i}^i + \sum_{j\in\mathcal{N}_i} z_{k-1}^j, \tag{8}$$

where $y_k^i = y_0^i, (k \leq 0)$. A dimension of $C_i^{\mathrm{T}} R_i^{-1} y_{k+h_i-\bar{h}}^i$ is n and all sensor nodes transmit information with same dimension. Moreover we propose a following information fusion method for fusion center.

$$z_k = \sum_{i\in\mathcal{N}_0} z_k^i, \tag{9}$$

where $z_k^0 = z_k$ is information merged in the fusion center. It follows from $y_{k-\bar{h}+h_i}^i$ and z_{k-1}^j, $(j \in \mathcal{N}_i)$ in (8) that z_k^i delays 1 time step per one relay node. Consequently, in a network topology with Assumption 5, information z_k merged in the fusion center is given by following equation.

$$z_k = \sum_{j \in \mathcal{N}_0} z_k^j = \sum_{j=1}^{N} C_j^{\mathrm{T}} R_j^{-1} y_{k-\bar{h}+1}^j \tag{10}$$

z_k is calculated in the fusion center at time step k and includes $C_i R_i^{-1} y_{k-\bar{h}+1}^i$ of all sensor nodes. The time step of measurements belonging to z_k depend on $\bar{h}$. The bigger $\bar{h}$ is, the bigger a time delay of measurement belonging to z_k.

State Estimation Algorithm

We showed fusion center calculate z_k including measurements with delay $y_{k-\bar{h}+1}^i$ at time step k. In this section, we propose a estimation algorithm using z_k. Then a estimation algorithm satisfies following Theorem 1 in a sensor network system (1) and (2).

Theorem 1 Consider the system (1), (2) and network topology T with *Assumption 1-5*. Then a estimation algorithm is given by following equations and the estimate $\hat{x}_k^j$ is minimum variance estimate based measurements of sensor node S_j:

$$\hat{x}_k^- = A^{\bar{h}-1} \hat{x}_{k-\bar{h}+1} + \bar{B}_{\bar{h}} \bar{u}_{k-\bar{h}+1}, \tag{11}$$

$$\hat{x}_{k-\bar{h}+1} = \hat{x}_{k-\bar{h}+1}^- + P_{k-\bar{h}+1} \left(z_k - C^{\mathrm{T}} R^{-1} C \hat{x}_{k-\bar{h}+1}^- \right), \tag{12}$$

$$P_k^- = A^{\bar{h}-1} P_{k-\bar{h}+1} \left(A^{\bar{h}-1} \right)^{\mathrm{T}} + G_{\bar{h}} \bar{Q} G_{\bar{h}}^{\mathrm{T}}, \tag{13}$$

$$P_{k-\bar{h}+1} = \left\{ \left(P_{k-\bar{h}+1}^- \right)^{-1} + C^{\mathrm{T}} R^{-1} C \right\}^{-1}, \tag{14}$$

where $\bar{B}_{\bar{h}}$, $\bar{G}_{\bar{h}}$, $\bar{Q} \in \mathbb{R}^{n(\bar{h}-1) \times n(\bar{h}-1)}$ is as follows

$$\bar{B}_{\bar{h}} = \left[B \ AB \cdots A^{\bar{h}-2} B \right], \tag{15}$$

$$G_{\bar{h}} = \left[I_n \ A \cdots A^{\bar{h}-2} \right], \tag{16}$$

$$\bar{Q} = \text{block diag}\{Q, Q, ..., Q\}. \tag{17}$$

Proof. we first define following fictitious measurement output $y_{k-\bar{h}+1}$.

$$\begin{aligned} y_{k-\bar{h}+1} &= \begin{bmatrix} y_{k-\bar{h}+1}^1 \\ y_{k-\bar{h}+1}^2 \\ \vdots \\ y_{k-\bar{h}+1}^N \end{bmatrix} \\ &= C x_{k-\bar{h}+1} + v_{k-\bar{h}+1}. \end{aligned} \tag{18}$$

$y_{k-\bar{h}+1}$ include measurements taken at time step $k - \bar{h} + 1$ of all sensor nodes. Then we consider the estimation algorithm using $y_{k-\bar{h}+1}$ taken at time step k. (1) can be rewritten as follow

$$x_k = A^{\bar{h}-1}x_{k-\bar{h}+1} + \bar{B}_{\bar{h}}\bar{u}_{k-\bar{h}+1} + G_{\bar{h}}\bar{w}_{k-\bar{h}+1}, \tag{19}$$

where $\bar{u}_{k-\bar{h}+1}$, $\bar{w}_{k-\bar{h}+1}$ is as follows

$$\bar{u}_{k-\bar{h}+1} = \begin{bmatrix} u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-\bar{h}+1} \end{bmatrix}, \ \bar{w}_{k-\bar{h}+1} = \begin{bmatrix} w_{k-1} \\ w_{k-2} \\ \vdots \\ w_{k-\bar{h}+1} \end{bmatrix}. \tag{20}$$

(19) is difference equation of time step k and $k - \bar{h} + 1$. Then we propose following estimation algorithm for (19) and (18).

$$\hat{x}_k^- = A^{\bar{h}-1}\hat{x}_{k-\bar{h}+1} + \bar{B}_{\bar{h}}\bar{u}_{k-\bar{h}+1} \tag{21}$$

$$\hat{x}_{k-\bar{h}+1} = \hat{x}_{k-\bar{h}+1}^- + K_{k-\bar{h}+1}\Big(y_{k-\bar{h}+1} - C\hat{x}_{k-\bar{h}+1}^-\Big) \tag{22}$$

where $\hat{x}_k^- = \mathrm{E}\{x_k | y_0, y_1, ..., y_{k-\bar{h}+1}\}$ and $\hat{x}_{k-\bar{h}+1} = \mathrm{E}\{x_{k-\bar{h}+1} | y_0, y_1, ..., y_{k-\bar{h}+1}\}$ are estimations of x_k and $x_{k-\bar{h}+1}$ based all measurements up to time step $k - \bar{h} + 1$. Now, the estimation error variance J is given by following equation.

$$J = E\{(x_k - \hat{x}_k^-)^{\mathrm{T}}(x_k - \hat{x}_k^-)\} = \mathrm{tr}P_k^- \tag{23}$$

The filter gain minimizing J satisfies following equations.

$$\frac{\partial}{\partial K_k}\mathrm{tr}P_k^- = \mathbf{0} \tag{24}$$

It follows from (19), (21) and (22) that the filter gain K_k and the error covariance matrix P_k satisfying (24) are as follows

$$\begin{aligned} K_{k-\bar{h}+1} &= P_{k-\bar{h}+1}^- C^{\mathrm{T}} \left(CP_{k-\bar{h}+1}^- C^{\mathrm{T}} + R\right)^{-1} \\ &= P_{k-\bar{h}+1}C^{\mathrm{T}}R^{-1} \end{aligned} \tag{25}$$

$$P_{k-\bar{h}+1} = \left\{\left(P_{k-\bar{h}+1}^-\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1} \tag{26}$$

Meanwhile, error covariance matrix P_k^- is as follow

$$P_k^- = A^{\bar{h}-1}\left\{\left(P_{k-\bar{h}+1}^-\right)^{-1} + C^{\mathrm{T}}R^{-1}C\right\}^{-1}\left(A^{\bar{h}-1}\right)^{\mathrm{T}} + G_{\bar{h}}\bar{Q}G_{\bar{h}}^{\mathrm{T}}, \tag{27}$$

where $\bar{Q}$ is covariance matrix of $\bar{w}_{k-\bar{h}+1}$. Consequently, a estimation algorithm using a measurement output (18).

Secondly, we show this algorithm is a estimation algorithm using z_k in (8).

It follows from (25), (18) and (22) that we can get following.

$$\hat{x}_{k-\bar{h}+1} = \hat{x}_{k-\bar{h}+1}^- + P_{k-\bar{h}+1}\Big(z_k - C^{\mathrm{T}}R^{-1}C\hat{x}_{k-\bar{h}+1}\Big). \tag{28}$$

(28) is a estimation algorithm using z_k merged in the fusion center. These equations complete the proof.

Relation between an estimation error variance and a network topology

In this section, we consider an estimation error variance $\mathrm{tr}P_k^-$ and a network topology. It follows from Assumptions 2, 3 that there is the unique positive definite solution $P_\infty^{\bar{h}}$ to algebraic Riccati equation (13) satisfying following equation:

$$P_\infty^{\bar{h}} = A^{\bar{h}-1} \left\{ \left(P_\infty^{\bar{h}} \right)^{-1} + C^{\mathrm{T}} R^{-1} C \right\}^{-1} \left(A^{\bar{h}-1} \right)^{\mathrm{T}} + G_{\bar{h}} \bar{Q} G_{\bar{h}}^{\mathrm{T}}. \tag{29}$$

From (29), the solution $P_\infty^{\bar{h}}$ depend on the depth $\bar{h}$. Now the solution $P_\infty^{\bar{h}}$ satisfies following Theorem 2.

Theorem 2 We assume if $\bar{h} = \alpha, \beta$, $(\alpha > \beta)$, there is the unique positive definite solutions P_∞^{α}, P_∞^{β} to algebraic Riccati equation (13) respectively. Then P_∞^{α} and P_∞^{β} satisfy following relation:

$$\mathrm{tr}P_\infty^{\alpha} \geq \mathrm{tr}P_\infty^{\beta}. \tag{30}$$

Proof. It follows from Assumptions 2 and 3 that the solution to (29) do not depend on initial value. Moreover (13) is different equation between k and $k - \bar{h} - 1$. Consequently it is apparent from these.

From Theorem 2, The smaller $\bar{h}$ is, the smaller priori estimation error is. Consequently, there is trade-off between an estimation error variance and communication energy.

2.3 Network Configuration algorithm

In this section, we discuss a network configuration algorithm. We have to configurate a network topology satisfying $J = \mathrm{tr}P_\infty^- \leq \gamma$ and Assumption 5. For this purpose, we first need to calculate $\bar{h}$ satisfying $J = \mathrm{tr}P_\infty^{\bar{h}} \leq \gamma$. secondly, we find rooted spanning tree where depths of all sensor node are less than $\bar{h}$ and a communication energy E is minimized. This tree is known as $\bar{h}$-HMST(the minimum-cost $\bar{h}$-hop spanning tree). In several researches, they showed approximation algorithm [14]. In this paper, we propose an algorithm minimizing in a subset of available network topology. We first consider following operation.

- Change destination of sensor nodes receiving information from sensor nodes belonging the set $\mathcal{V}_1$ into sensor nodes belonging the set $\mathcal{V}_2$,

where $\mathcal{V}_1 = \left\{ S_j | h_j > \bar{h} \right\}$ and $\mathcal{V}_2 = \left\{ S_j | h_j < \bar{h} \right\}$. It follows from this operation that all sensor nodes have depths with less than $\bar{h}$. We assume the set of all available network topology that we can get from this operation as $\mathcal{T}_s$. We rewrite Problem 2 to following problem.

Network Construction Algorithm

1: Compute of $\bar{h}$ satisfying

$$J = \mathrm{tr} P_\infty^{\bar{h}} \le \gamma.$$

2: Compute rooted minimum spanning tree T by Prim's algorithm and define

$$\mathcal{V}_1 = \left\{ S_j | h_j > \bar{h} \right\}, \\ \mathcal{V}_2 = \left\{ S_j | h_j < \bar{h} \right\}.$$

3: Change $\mathrm{Par}(S_i)$, $(S_i \in \mathcal{V}_1)$
if $\mathcal{V}_1$ is not an empty set

$$\begin{aligned} \mathrm{Par}(S_i) &:= \arg\min_{S_j \in \mathcal{V}_2} e\left(S_i, S_j\right) \\ E_i &:= e\left(S_i, S_j\right) \\ h_i &:= h_j + 1 \end{aligned}$$

end
4: **return** T

Problem 3. Find the optimal network topology T^* satisfying $J \le \gamma$, Assumption 5 and following equation:

$$T = \arg\min_{T \in \mathcal{T}_s} E, \tag{31}$$

where $\gamma > 0$ is a design parameter.

In Problem 2 we find the network topology minimizing a communication energy in all available network topology. However Problem 3 minimize in the subset of all available network topology.

We propose *Network Configuration Algorithm* and it is a solution of Problem 3. In this algorithm, we use Prim's Algorithm finding the minimum spanning tree. In *network configuration algorithm*, $e(S_i, S_j)$ is communication energy between sensor nodes S_i and S_j.

Network Construction Algorithm satisfying following theorem.

Theorem 3 *Network Construction Algorithm* minimize a communication energy E in subset $\mathcal{T}_s$ and it is the solution of problem 3.

Proof. In **3:** of *Network Construction Algorithm*, we select a sensor node with minimum communication energy belonging the set $\mathcal{V}_2$. because the operation are applied these sensor nodes, this algorithm is the solution of Problem 3.

Consequently, by designing γ, we can configurate a network topology what are superior to estimation accuracy or communication energy.

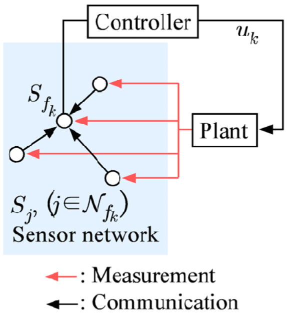


Fig. 3. Sensor network

3 Optimal Sensor Network Configuration Considering Estimation Error Variance and Communication Energy

4 PROBLEM FORMULATION

4.1 Plant and Sensor Nodes

In this paper, we consider the sensor networked feedback control system illustrated in Fig. 3. This system consists the plant and N sensor nodes S_i, $(i = 1, 2, ..., N)$. We assume all sensor nodes have enough computation capability and take a measurement of the plant. The process dynamics of the plant and the measurement equation of the sensor node S_i are given by

$$x_{k+1} = Ax_k + Bu_k + w_k, \tag{32}$$
$$y_k^i = C_i x_k + v_k^i, \tag{33}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k^i \in \mathbb{R}^{q_i}$ are the state, the control input and the measurement output of the sensor node S_i respectively. Additionally, $w_k \in \mathbb{R}^n$, $v_k^i \in \mathbb{R}^{q_i}$ are the process noise and the measurement noise respectively. We assume that the control input u_k is applied from the sensor node $S_{f_k}, (f_k = 1, 2, ..., N)$ to the plant and given by

$$u_k = L\hat{x}_k^{f_k}, \tag{34}$$

where $\hat{x}_k^{f_k} \in \mathbb{R}^n$ is the estimate of the sensor node S_{f_k} and L is the feedback gain. Now we assume we can arbitrarily determine which sensor node is the sensor node S_{f_k} at each time step. Thus, the task of the sensor node S_{f_k} is similar to the fusion center discussed in previous work, but it is not fixed. Moreover, (32) and (33) satisfy *assumptions 1-3*.

4.2 Network Topology

The sensor network consists N sensor nodes and one of them is the sensor node S_{f_k} applying the control input to the plant. We assume the sensor node S_{f_k} can communicate with other sensor nodes directory and define the set $\mathcal{N}_{f_k}$ containing sensor nodes communicating the sensor node S_{f_k}. Here there is no communication in between arbitrary two sensor nodes belonging to the set $\mathcal{N}_{f_k}$ at time step k. We assume we can arbitrary determine sensor nodes belonging to the set $\mathcal{N}_{f_k}$ as a case of the sensor node S_{f_k}.

Remark 1. The wireless communication between the sensor node $S_j, (j \in \mathcal{N}_{f_k})$ and S_{f_k} means that the sensor node $S_j, (j \in \mathcal{N}_{f_k})$ transmits information to the sensor node S_{f_k}. Thus, all communication paths are unidirectional.

In general, if there are the bidirectional communication paths, each sensor node can get and use a lot of information. But, in this paper, the network topology vary with time because we discuss a sensor scheduling problem determining the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ each time step. Due to different communication ranges of each sensor node or obstacles, it is difficult to keep bidirectional communication path at all times in real physical system. Moreover, it can cause high machinery costs. Thus, we deals with the unidirectional communication path. Consequently, all sensor nodes satisfy following *Assumption 6*.

Assumption 6 *The sensor node $S_j, (j \in \mathcal{N}_{f_k})$ can transmit to the sensor node S_{f_k} once while one time step with a time delay less than a sampling time. Additionally, when the sensor node S_{f_k} applies the control input u_k to the plant and sensor node S_j, $(j \in \mathcal{N}_{f_k})$ transmits information to the sensor node S_{f_k}, These sensor nodes use the communication energy $E_{f_k,p}$, $E_{j,f_k} \in \mathbb{R}_+$ respectively.*

We define the total communication energy E_k of the system. The energy E_k is described as follows

$$E_k = E_{f_k,x_k} + \sum_{j \in \mathcal{N}_{f_k}} E_{j,f_k}. \tag{35}$$

Remark 2. The communication energy $E_{i,j}$ generally can be $E_{i,j} = b_{i,j} + a_{i,j}(d_{i,j})^{c_{i,j}}$ and depend on a distance between sensor nodes S_i and S_j, where $b_{i,j}$ is a static part and $a_{i,j}$ is a dynamic part. $c_{i,j}$ is typically from 2 through 6 [13].

4.3 Control Problems

In this paper, we discuss the estimation problem with unknown input u_k and a sensor scheduling problem. Problems can be formulated as following *problems 4, 5.*

Problem 4. We assume the plant and all sensor nodes satisfy *Assumptions 1-3, 6* and the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ is determined. Then compute the optimal state estimate $\hat{x}_k^{f_k}$ that minimizes the following estimation error variance.

$$J = \mathrm{E}\left\{(x_k - \hat{x}_k^{f_k})^{\mathrm{T}}(x_k - \hat{x}_k^{f_k})\right\}. \tag{36}$$

Problem 5. At time step k, find the optimal network topology T_k^* satisfying $J \leq \gamma$ and the following equation.

$$T_k^* = \arg\min_{T_k} E_k, \tag{37}$$

where $\gamma > 0$ is a design parameter.

5 ESTIMATION ALGORITHM

In this section, we propose the estimation algorithm in the sensor networked feedback control system. The proposed algorithm based on extension of *Decentralized Kalman Filter* in [10]. Each sensor node S_j, $(j \in \mathcal{N}_{f_k})$ computes the local estimate $\hat{x}_k^j$. Here these sensor nodes can not know the control input because all communication paths are unidirectional. We can not apply an existing method to the feedback system via a sensor network. Consequently, we propose the novel estimation algorithm considering the unknown control input. In this algorithm, each sensor node S_j, $(j \in \mathcal{N}_{f_k})$ transmits $\hat{x}_k^{j-}$, $\hat{x}_k^j$, P_k^{j-}, P_k^j to the sensor node S_{f_k}. The sensor node S_{f_k} computes estimate $\hat{x}_k^{f_k}$ by information from sensor nodes S_j, $(j \in \mathcal{N}_{f_k})$.

5.1 Estimation Algorithm of sensor nodes S_j, $(j \in \mathcal{N}_{f_k})$

Firstly, we discuss an estimation algorithm of sensor nodes S_j, $(j \in \mathcal{N}_{f_k})$. Each sensor node S_j, $(j \in \mathcal{N}_{f_k})$ do not have information of the control input because all communication paths are unidirectional. Proposed algorithm satisfies the following *Theorem 4*.

Theorem 4 *Consider the system (32) and (33) with Assumption 1-3, 6. Then an estimation algorithm of each sensor node S_j, $(j \in \mathcal{N}_{f_k})$ is given by the following equations and the estimate $\hat{x}_k^j$ is the minimum variance estimate based measurements of sensor node S_j.*

$$\hat{x}_{k+1}^{j-} = A\hat{x}_k^j + \mathrm{B}\hat{u}_k^j, \tag{38}$$

$$\hat{x}_k^j = \hat{x}_k^{j-} + K_k^j(y_k^j - C_j\hat{x}_k^{j-}), \tag{39}$$

$$\hat{u}_k^j = L\hat{x}_k^j, \tag{40}$$

$$P_{k+1}^{j-} = (A + BL)\, P_k^j\, (A + BL)^{\mathrm{T}} + Q + BLP_k^{f_k} L^{\mathrm{T}} B^{\mathrm{T}}$$

$$
\begin{aligned}
&-(A+BL)\,M_k^j L^{\mathrm{T}} B^{\mathrm{T}} - BL(M_k^j)^{\mathrm{T}}\,(A+BL)^{\mathrm{T}}\,, && (41)\\
P_k^j &= \left\{(P_k^{j-})^{-1} + C_j^{\mathrm{T}} R_j^{-1} C_j\right\}^{-1}, && (42)\\
M_k^j &= (I-K_k^j C_j) M_k^{j-} (I - K_k^{f_k} C_{f_k})^{\mathrm{T}}, && (43)\\
M_{k+1}^{j-} &= (A+BL)\,M_k^j A^{\mathrm{T}} + Q - BLP_k^{f_k} A^{\mathrm{T}}, && (44)
\end{aligned}
$$

where definition of each variable is described as follows

$$
\begin{aligned}
\hat{x}_k^{j-} &= \mathrm{E}\left\{x_k | y_{k-1}^j, y_{k-2}^j, \cdots\right\},\\
\hat{x}_k^{j} &= \mathrm{E}\left\{x_k | y_{k}^j, y_{k-1}^j, \cdots\right\},\\
P_k^{j-} &= \mathrm{E}\left\{(x_k - \hat{x}_k^{j-})(x_k - \hat{x}_k^{j-})^{\mathrm{T}}\right\},\\
P_k^{j} &= \mathrm{E}\left\{(x_k - \hat{x}_k^{j})(x_k - \hat{x}_k^{j})^{\mathrm{T}}\right\},\\
M_k^{j} &= \mathrm{E}\left\{(x_k - \hat{x}_k^{j})(x_k - \hat{x}_k^{f_k})^{\mathrm{T}}\right\},\\
M_k^{j-} &= \mathrm{E}\left\{(x_k - \hat{x}_k^{j-})(x_k - \hat{x}_k^{f_k -})^{\mathrm{T}}\right\}.
\end{aligned}
$$

Proof. The filter equation for (32) and (33) are given by

$$
\begin{aligned}
\hat{x}_{k+1}^{j-} &= A\hat{x}_k^j + B\hat{u}_k^j, && (45)\\
\hat{x}_k^j &= \hat{x}_k^{j-} + K_k^j (y_k^j - C_j \hat{x}_k^{j-}), && (46)\\
\hat{u}_k^j &= L\hat{x}_k^j. && (47)
\end{aligned}
$$

From (32)-(34), (45), (46) and (47), errors $e_k^j = x_k - \hat{x}_k^j$, $e_k^{j-} = x_k - \hat{x}_k^{j-}$ can be described as follows

$$
\begin{aligned}
e_k^j &= (I - K_k^j C_j) e_k^{j-} - K_k^j v_k^j, && (48)\\
e_{k+1}^{j-} &= (A+BL)\,e_k^j + w_k - BLe_k^{f_k}. && (49)
\end{aligned}
$$

Thus estimation error covariance matrices P_k^j and P_{k+1}^{j-} are described as follows

$$
\begin{aligned}
P_k^j &= (I - K_k^j C_j) P_k^{j-} (I - K_k^j C_j)^{\mathrm{T}} + K_k^j R_j (K_k^j)^{\mathrm{T}}, && (50)\\
P_{k+1}^{j-} &= (A+BL)\,P_k^j\,(A+BL)^{\mathrm{T}} + Q + BLP_k^{f_k} L^{\mathrm{T}} B^{T}\\
&\quad - (A+BL)\,M_k^j L^{\mathrm{T}} B^{\mathrm{T}} - BL(M_k^j)^{\mathrm{T}}\,(A+BL)^{\mathrm{T}}\,, && (51)
\end{aligned}
$$

where M_k^j is the cross covariance matrix between the estimation errors of the estimate $\hat{x}_k^{f_k}$ and $\hat{x}_k^j$.

Firstly, we consider the covariance matrix (50). From the condition $\frac{\partial}{\partial K_k} \mathrm{tr} P_k^j = \mathbf{0}$ and (50), the filter gain K_k^j and error covariance P_k^j can be described as follows

$$K_k^j = P_k^j C_j^{\mathrm{T}} R_j^{-1}. \tag{52}$$

$$P_k^j = \left\{ (P_k^{j-})^{-1} + C_j^{\mathrm{T}} R_j^{-1} C_j \right\}^{-1}. \tag{53}$$

Secondly, we consider the cross covariance matrix M_k^j in (51). From its definition, M_k^j is described as follows

$$M_k^j = (I - K_k^j C_j) M_k^{j-} (I - K_k^{f_k} C_{f_k})^{\mathrm{T}}. \tag{54}$$

The sensor node S_{f_k} knows the value of the control input u_k because this sensor node applies the control input to the plant. Thus the estimation error $e_k^{f_k}$ is given as the following

$$e_{k+1}^{f_k -} = A e_k^{f_k} + w_k. \tag{55}$$

From (55) and its definition, the cross covariance matrix M_k^{j-} is given by

$$M_{k+1}^{j-} = (A + BL)\, M_k^j A + Q - BLP_k^{f_k} A^{\mathrm{T}}. \tag{56}$$

Next, we consider the estimation algorithm of the sensor node S_{f_k}. The estimation of the sensor node S_{f_k} is based on its measurement and the received information $\hat{x}_k^{j-}$, $\hat{x}_k^j$, P_k^{j-} and P_k^j from some sensor nodes $S_j, (j \in \mathcal{N}_{f_k})$. The sensor node S_{f_k} has information of the control input u_k. Thus, the estimation algorithm of the sensor node S_{f_k} is following *Decentralized Kalman Filter* proposed in [10].

$$\hat{x}_{k+1}^{f_k -} = (A + BL)\hat{x}_k^{f_k}, \tag{57}$$

$$\bar{x}_k^{f_k} = \hat{x}_k^{f_k -} + K_k^{f_k}(y_k^{f_k} - C_{f_k}\hat{x}_k^{f_k -}), \tag{58}$$

$$K_k^{f_k} = \bar{P}_k^{f_k} C_{f_k}^{\mathrm{T}} R_{f_k}^{-1}, \tag{59}$$

$$P_k^{f_k -} = A P_k^{f_k} A^{\mathrm{T}} + Q, \tag{60}$$

$$\bar{P}_k^{f_k} = \left\{ (P_k^{f_k -})^{-1} + C_{f_k}^{\mathrm{T}} R_{f_k}^{-1} C_{f_k} \right\}^{-1}, \tag{61}$$

$$P_k^{f_k} = \left[(\bar{P}_k^{f_k})^{-1} + \sum_{j \in \mathcal{N}_{f_k}} \left\{ (P_k^j)^{-1} - (P_k^{j-})^{-1} \right\} \right]^{-1}, \tag{62}$$

$$\hat{x}_k^{f_k} = P_k^{f_k} \left[(\bar{P}_k^{f_k})^{-1} \bar{x}_k^{f_k} + \sum_{j \in \mathcal{N}_{f_k}} \left\{ (P_k^j)^{-1} \bar{x}_k^j - (P_k^{j-})^{-1} \hat{x}_k^{j-} \right\} \right], \tag{63}$$

where the definition of variables is as follows

$$\bar{x}_k^{f_k} = \mathrm{E}\left\{ x_k | y_k^{f_k}, y_{k-1}^{f_k}, \ldots \right\},$$

$$\begin{aligned}
\hat{x}_k^{f_k} &= \mathrm{E}\left\{x_k | y_k^{f_k}, y_{k-1}^{f_k}, ..., y_k^j, y_{k-1}^j\right\}, j \in \mathcal{N}_{f_k}, \\
\bar{P}_k^{f_k} &= \mathrm{E}\left\{(x_k - \bar{x}_k^{f_k})(x_k - \bar{x}_k^{f_k})^{\mathrm{T}}\right\}, \\
P_k^{f_k} &= \mathrm{E}\left\{(x_k - \hat{x}_k^{f_k})(x_k - \hat{x}_k^{f_k})^{\mathrm{T}}\right\}.
\end{aligned}$$

The estimate $\bar{x}_k^{f_k}$ is only based on measurements of the sensor node S_{f_k}. But, the estimate $\hat{x}_k^{f_k}$ is based on measurements of the sensor node S_{f_k} and sensor nodes belong to the set $\mathcal{N}_{f_k}$. Then the covariance matrix $P_k^{f_k}$ satisfies the following *Theorem 5*.

Theorem 5 *Consider the system (32) and (33) with Assumptions 1-3, 6. If sensor nodes $S_{f_k} = S_f$, $S_{j_1}, S_{j_2}, ...,$ $(j_1, j_2 \in \mathcal{N}_f)$ are determined and the matrix pair (H_f, A), $H_f = [C_f^{\mathrm{T}} \quad C_{j_1}^{\mathrm{T}} \quad C_{j_2}^{\mathrm{T}} \cdots]^{\mathrm{T}}$ is detectable, then the estimate $\hat{x}_k^f$ is the solution of Problem 4 and there is a unique positive definite solution P_∞^f of the following algebraic Ricatti equation.*

$$P_\infty^f = AP_\infty^f A^{\mathrm{T}} + Q - AP_\infty^f H_f^{\mathrm{T}} \left(H_f P_\infty^f H_f^{\mathrm{T}} + V_f\right)^{-1} H_f P_\infty^f A^{\mathrm{T}}, \tag{64}$$

where $V_f = \mathrm{diag}\{R_f, R_{j_1}, R_{j2}, ...\}$.

Proof. Substituting (42) into (62), we can get

$$P_k^f = \left[\left(P_k^{f-}\right)^{-1} + H_f^{\mathrm{T}} V_f^{-1} H_f\right]^{-1} \tag{65}$$

From (60) and (65), this is the algebraic Ricatti equation. Consequently, From *Assumption 2* and detectability of the matrix pair (H_f, A), the covariance matrix P_k^f has the unique positive definite solution P_∞^f.

From *Theorem 5*, there is the unique positive definite solution of the algebraic Ricatti equation (60)-(62) while sensor nodes S_{f_k} and S_j, $(j \in \mathcal{N}_{f_k})$ are determined. Additionally, from *Assumption 3*, if we use $N-1$ sensor nodes as S_j, $(j \in \mathcal{N}_{f_k})$, there is the unique positive definite solution of the algebraic Ricatti equation. In next section, we propose a sensor scheduling algorithm considering the estimation error variance $J = \mathrm{tr} P_\infty^{f_k}$ and the communication energy. If we determine the set $\mathcal{N}_{f_k}$ including all sensor nodes, the estimation error variance of the common estimate is minimized. But the communication energy will increase because all sensor nodes have to transmit information to the sensor node S_{f_k}. On the contrary, if we determine the set $\mathcal{N}_{f_k}$ is empty set, the communication energy is zero because there are no communication paths. But the estimation error variance of the estimate will increase. Consequently, there is a trade-off between the estimation accuracy and the communication energy.

6 SENSOR SCHEDULING ALGORITHM

In previous section, we showed that the estimation error variance of the estimate $\hat{x}_k^{f_k}$ can be written as $J = \mathrm{tr}(P_\infty^{f_k})$. In this section, we propose a sensor scheduling algorithm minimizing communication energy in subset of all available network topology under the condition $J \leq \gamma$. The network topology can be fixed uniquely if and only if we determine the sensor nodes S_{f_k} and S_j, $j \in \mathcal{N}_{f_k}$. Here we can get that $N2^{N-1}$ network topologies are available. Consequently, we propose the following algorithm to reduce computation costs. In the proposed algorithm, $N(N-1)$ network topologies are available. Additionally, $E(S_i, \mathcal{N}_i)$ and $J(S_i, \mathcal{N}_i)$ are communication energy of the whole system and the estimation error variance respectively when sensor node $S_{f_k} = S_i$ and the set $\mathcal{N}_i$ are determined.

Sensor Scheduling Algorithm

1: **for** $\alpha = 1$ to N **do**

2: $\quad \mathcal{N}_\alpha = \mathcal{B} = \{1, ..., N\} \backslash \alpha$

3: $\quad$ **repeat** $N - 1$

4: $\quad\quad \beta = \arg\max_{j \in \mathcal{N}_\alpha \cap \mathcal{B}} E_{\alpha,j}$

5: $\quad\quad$ **if** $J(S_\alpha, \mathcal{N}_\alpha \backslash S_\beta) \leq \gamma$ **then**

$$\mathcal{N}_\alpha := \mathcal{N}_\alpha \backslash S_\beta$$

6: $\quad\quad \mathcal{B} := \mathcal{B} \backslash S_\beta$

7: **return** S_{i^*}, $\mathcal{N}_{i^*}$, $(i^* = \min_{i=1,...,N} E(S_i, \mathcal{N}_i))$

In this algorithm, firstly, we determine the sensor node $S_{f_k} = S_\alpha$, $(\alpha = 1)$. Secondly, we remove the sensor node S_β from the set $\mathcal{N}_\alpha$ in order of decreasing the communication energy $E_{\alpha,\beta}$ under the condition $J(S_\alpha, \mathcal{N}_\alpha \backslash S_\beta) \leq \gamma$. We calculate these subroutine N times $(\alpha = 1, 2, ..., N)$. Finally, the sensor node S_{f_k} and the set $\mathcal{N}_{f_k}$ minimizing communication energy in subset of all available network topology under the condition $J \leq \gamma$ are determined.

Example 1 is described as follows.

Example 1. Consider 3 sensor nodes $(N = 3)$ illustrated in Fig. 4. We assume the following conditions.

1) the Distances are $d_{1,2} = d_{2,3} = 1$, $d_{1,3} = 2$.

2) A communication energy is $E_{i,j} = \epsilon d_{i,j}^2$,$(\epsilon > 0)$.

3) The condition $J \leq \gamma$ is satisfied if and only if we use sensor nodes (S_1, S_2, S_3) or (S_1, S_3).

Now, we examine the proposed sensor scheduling algorithm in *Example 1.*

We first define $\alpha = 1$ and $\mathcal{N}_1 = \mathcal{B} = \{2, 3\}$. These mean that we first check the communication energy in a case of the sensor node S_{f_k} is S_1. Then 4:, 5:

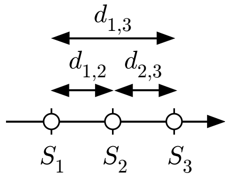
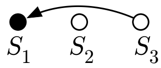
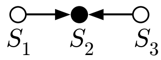


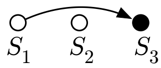
Fig. 4. Sensor nodes of *Example 1*



(a) A network topology ($S_{f_k} = S_1$).



(b) A network topology ($S_{f_k} = S_2$).



(c) A network topology ($S_{f_k} = S_3$).

Fig. 5. Network topologies of *Example 1*

and 6: in a sensor scheduling algorithm are calculated 2 times. we can chose $\beta = 3$ at the initial calculation. Then the sensor node S_3 would not be removed from $\mathcal{N}_\alpha$ because the condition $J(S_\alpha, \mathcal{N}_\alpha \backslash S_\beta = \{2\}) \le \gamma$ is not satisfied. Consequently, $\mathcal{N}_\alpha = \{2, 3\}$, $\mathcal{B} = \{2\}$. After the initial calculation, we can chose $\beta = 2$ at the second calculation. Because the condition $J(S_\alpha, \mathcal{N}_\alpha \backslash S_\beta = \{3\}) \le \gamma$ is satisfied, the sensor node S_2 is removed. Consequently, if we determine the sensor nodes S_{f_k} is S_1, the set $\mathcal{N}_1 = \{3\}$ (see Fig. 5(a)) and communication energy E_k is given by

$$E(S_1, \mathcal{N}_1) = E_{1,x_k} + \epsilon d_{1,3}^2 = E_{1,x_k} + 4\epsilon. \tag{66}$$

Next, we can define $\alpha = 2$ and $\mathcal{N}_\alpha = \mathcal{B} = \{1, 3\}$. We can calculate the communication energy E^2 and the set $\mathcal{N}_2$ by a method similar to above calculation. In this subroutine, because we can not remove sensor nodes from the set $\mathcal{N}_2$ under the condition $J_2 \le \gamma$, we can define $\mathcal{N}_2 = \{1, 3\}$ (see Fig. 5(b)) and the communication energy is given by the following equation when the sensor node S_{f_k} is S_2

$$E(S_2, \mathcal{N}_2) = E_{2,x_k} + \epsilon \left(d_{1,2}^2 + d_{2,3}^2\right) = E_{2,x_k} + 2\epsilon. \tag{67}$$

Finally we choose $\alpha = 3$ and $\mathcal{N}_3 = \{1, 2\}$. Then we can remove the sensor node S_2 from the set $\mathcal{N}_3$ under the condition. Consequently, $\mathcal{N}_\alpha = \{3\}$ (see Fig. 5(c)) and the communication energy is calculated as the following equation.

$$E(S_3, \mathcal{N}_3) = E_{3,x_k} + \epsilon d_{1,3}^2 = E_{3,x_k} + 4\epsilon \tag{68}$$

(66)-(68) are the communication energy when the sensor nodes S_{f_k} is S_1, S_2 or S_3 respectively. We consider the energy to transmit information from each sensor node to the plant is $E_{1,x_k} = \epsilon, E_{1,x_k} = 4\epsilon, E_{1,x_k} = 9\epsilon.$ at time step k. Then the communication energy are given as follows

$$E(S_1, \mathcal{N}_1) = 5\epsilon,\ E(S_2, \mathcal{N}_2) = 6\epsilon,\ E(S_3, \mathcal{N}_3) = 13\epsilon.$$

Consequently, we can determine $S_{i^*} = S_1$, $\mathcal{N}_{i^*} = \{3\}$ at time step k.

7 Experimental Evaluation

7.1 Experimental Setup

The experiment was carried out on a two-wheeled vehicle, a CCD camera and a computer as shown in Fig. 6.

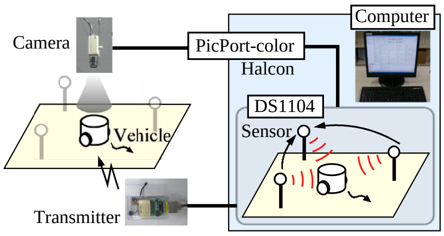


Fig. 6. Experimental setup.

Each measurement output is calculated from the image of a CCD camera mounted above the vehicle. The video signals are acquired by a frame grabber board PicPort-color and image processing software HALCON generate nine measurements. Consequently, nine sensor nodes, a network topology and measurement noises exist in the computer. We use DS1104 (dSPACE Inc.) as a real-time calculating for an estimation and sensor scheduling. Now Two-wheeled vehicle has the nonholonomic constraint. However two-wheeled vehicle can be defined following framework by virtual structure for feedback linearization [16].

$$A = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\delta^2}{2} & 0 \\ 0 & \frac{\delta^2}{2} \\ \delta & 0 \\ 0 & \delta \end{bmatrix},$$

where $\delta = 0.2$ and $x_0 = [\, 1.3 \;\; 0.7 \;\; 0 \;\; 0 \,]^{\mathrm{T}}$ are the sampling time and the initial state respectively. Additionally, we design the feedback gain L by LQG control.

7.2 Experimental Result of Network Configuration

In this subsection, an effectiveness of the network configuration algorithm proposed in section 2 by experiments. There are ten sensor nodes available and each sensor nodes has the following measurement equation and these position is shown in Fig. 7.

$$y_k^i = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_k + v_k^i, \quad (i = 1, 2)$$

$$
\begin{aligned}
y_k^i &= \begin{bmatrix} 0\,1\,0\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 3, 4) \\
y_k^i &= \begin{bmatrix} 0\,0\,1\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 5, 6) \\
y_k^i &= \begin{bmatrix} 0\,0\,0\,1 \end{bmatrix} x_k + v_k^i, \quad (i = 7, 8) \\
y_k^i &= \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 9, 10)
\end{aligned}
$$

Additionally, the covariance matrices of noises are $Q = 1\times10^{-4}I_4$, $R = 0.05I_{12}$ respectively.

Here we define the communication energy between arbitrarily two sensor nodes. We assume that the communication energy between sensor nodes S_i and S_j is $e_{i,j} = \epsilon d_{i,j}^2$. d_{i,f_k} is the distance between sensor nodes S_i and S_j and ϵ is the positive constant.

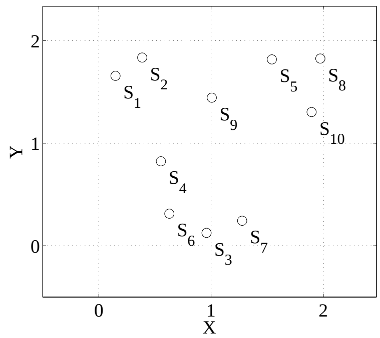


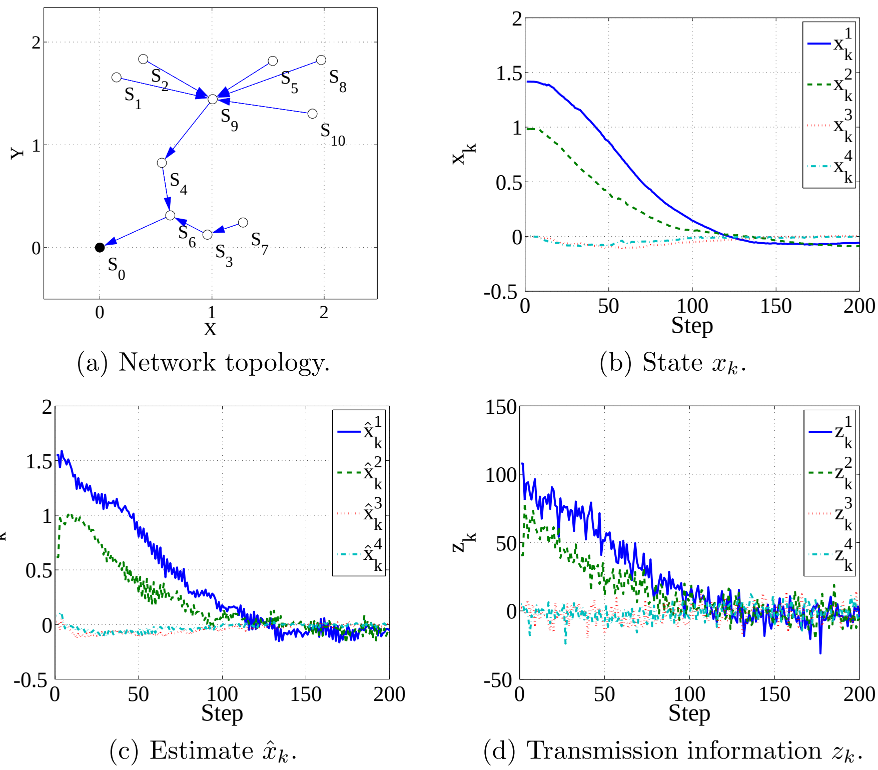
Fig. 7. Position of sensor nodes

Additionally, experiments were done following *Case 1* and *Case 2*.

Case 1 : The experiment designing $\gamma = 0.015$
Case 2 : The experiment designing $\gamma = 0.03$

The experimental results of *Case 1* and *Case 2* are shown in Fig. 8, 9. Fig. 8, 9(a), (b), (c) and (d) show a network topology, the state x_k, the estimate $\hat{x}_k$ and a information variable z_k respectively. As shown in Figs. 8(a), 9(a), network topologies satisfying the condition are $\bar{h} = 4, 6$ respectively. Additionally, error variances are J =0.0297, 0.0137 and communication energy are $E = 10.5\epsilon, 3.12\epsilon$ respectively. Consequently, there is a trade-off between an estimation accuracy and a communication energy. As shown in Figs. 8(c), 9(c), a vibration of the estimate in *case 1* is smaller than *Case 2*. As shown in Figs. 8(d), 9(d), z_k has information of weighted measurement. Fig. 10 shows the variance $J = \mathrm{tr}P_k^-$ in *Case 1* and *Case 2* respectively. As shown in Fig. 10, $\mathrm{tr}P_k^-$ converge on $\mathrm{tr}P_k^-$ =0.0297, 0.0137 and it is less than the design parameters respectively.

Consequently, we have showed that we can configurate a network topology what are superior to estimation accuracy or communication energy by designing γ.



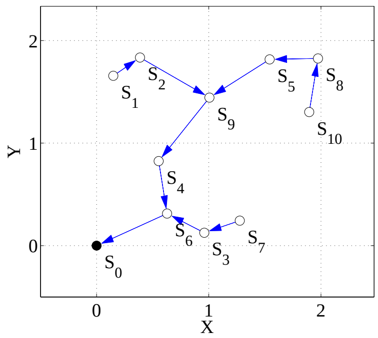
(a) Network topology.

(b) State x_k.

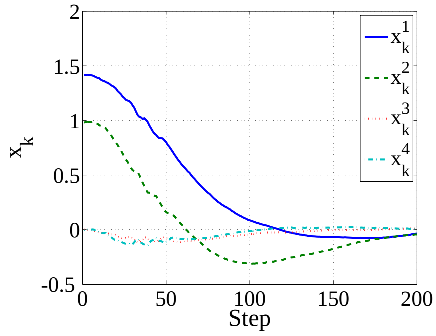
(c) Estimate $\hat{x}_k$.

(d) Transmission information z_k.

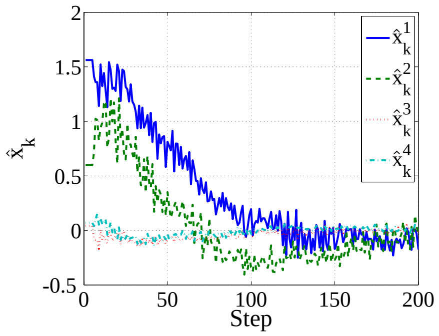
Fig. 8. Experimental results (Case 1).



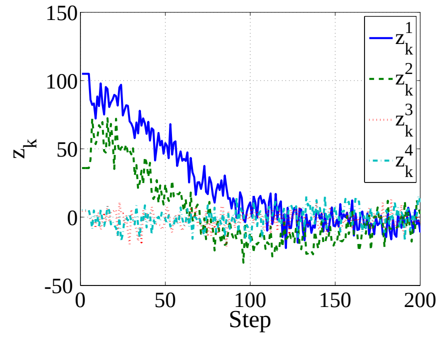
(a) Network topology.



(b) State x_k.



(c) Estimate $\hat{x}_k$.



(d) Transmission information z_k.

Fig. 9. Experimental results (Case 2).

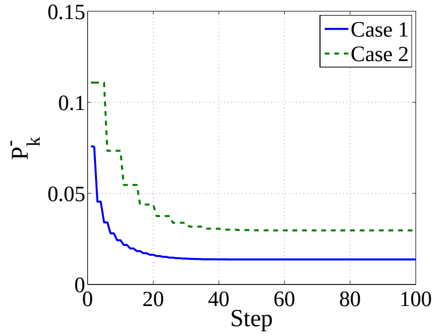


Fig. 10. Variance $\mathrm{tr}P_k^-$(Case 1, Case 2).

7.3 Experimental Result of Sensor Scheduling

In this subsection, effectiveness of a sensor scheduling algorithm proposed in section 3 by experiments.

There are nine sensor nodes available and each sensor nodes has the following measurement equation and these position is shown in Fig. 11.

$$\begin{aligned} y_k^i &= \begin{bmatrix} 1\,0\,0\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 1, 5, 9) \\ y_k^i &= \begin{bmatrix} 0\,1\,0\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 2, 6) \\ y_k^i &= \begin{bmatrix} 0\,0\,1\,0 \end{bmatrix} x_k + v_k^i, \quad (i = 3, 7) \\ y_k^i &= \begin{bmatrix} 0\,0\,0\,1 \end{bmatrix} x_k + v_k^i, \quad (i = 4, 8) \end{aligned}$$

Each measurement output is calculated from the image of a CCD camera mounted above the vehicle. The video signals are acquired by a frame grabber board PicPort-color and image processing software HALCON generate nine measurements. Consequently, nine sensor nodes, a network topology and measurement noises exist in the computer. We use DS1104 (dSPACE Inc.) as a real-time calculating for an estimation and sensor scheduling. Additionally, the covariance matrices of noises are $Q = 1 \times 10^{-4} I_4$, $R = 0.1 I_9$ respectively.

Here we define the communication energy between arbitrarily two sensor nodes. We assume that the communication energy between sensor nodes S_i and S_{f_k} is $E_{i,f_k} = \epsilon d_{i,f_k}^2$. d_{i,f_k} is the distance between sensor nodes S_i and S_{f_k} and ϵ is the positive constant.

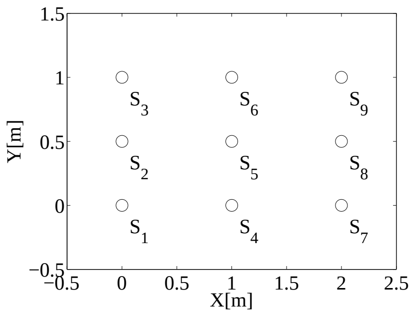


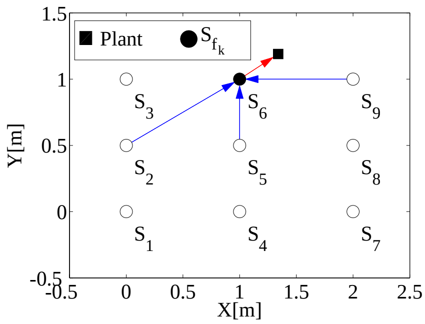
Fig. 11. Position of sensor nodes.

The experiment was done designing $\gamma = 0.02$. The experimental results are shown in Fig. 12. Fig. 12(a)-(c) show the trajectory of vehicle and network topology. As shown in Fig. 12(a)-(c), sensor nodes are switched while the vehicle is moving. Fig. 12(b) shows the estimation error. As shown in Fig. 12(b), the estimation error is zero mean. Fig. 12(e) shows the estimation error variance $P_k^{f_k}$. As shown in Fig. 12(e), the estimation error variance converge to the solution to algebraic Riccati equation and the solution is less than design parameter γ at all times. Finally, Fig. 12(e) is a comparison between following *Cases 1, 2*.

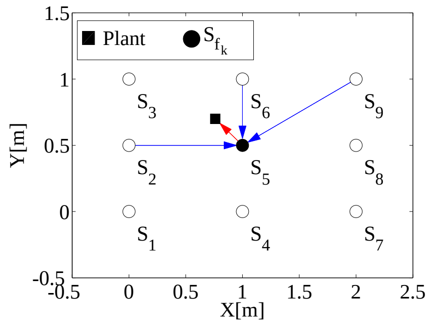
Case 1. A case that a sensor scheduling algorithm was applied.

Case 2. A case that the sensor node S_{f_k} was S_6 at all times.

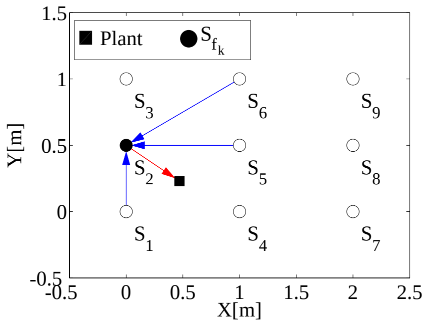
In these case, the error variance $\mathrm{tr}P_k^{f_k}$ is same. However from Fig. 12(f) the communication energy is different. This figure shows the energy of the whole system is reduced by a sensor scheduling algorithm. Consequently, by designing γ, a proposed algorithm reduce the communication energy under the condition that the estimation error is smaller than desired value.



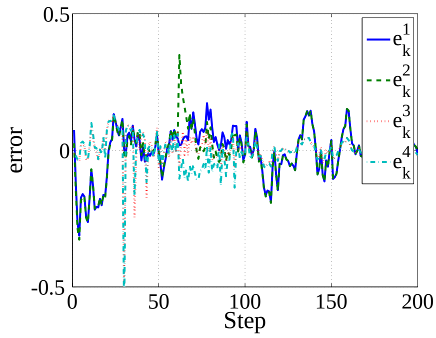
(a) Network structure at $k = 0$.



(b) Network structure at $k = 61$.

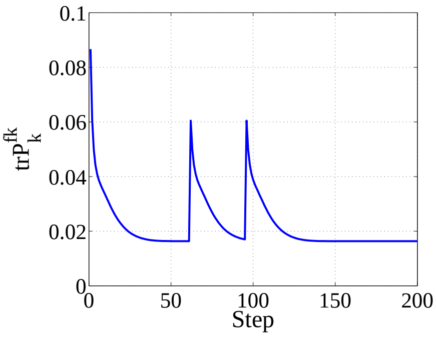


(c) Network structure at $k = 96$.

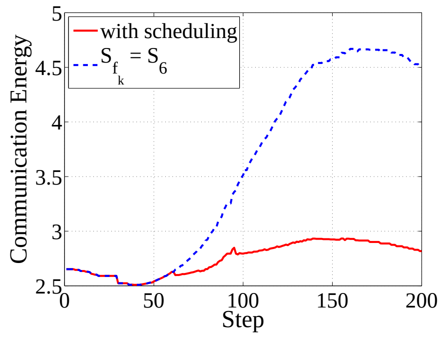


(d) Estimation error $e_k = x_k - \hat{x}_k$.

Fig. 12. Experimental results.



(e) Variance $J = \mathrm{tr} P_k^{f_k}$.



(f) Communication energy E_k.

Fig. 12. Experimental results.

8 Conclusions

In this paper, firstly, we discussed a network configuration problem considering the priori estimation error variance and communication energy in a feedback control system via a sensor network. We first have defined a sensor network with multi-hop communication. Then we have assumed that each sensor node transmit same amount of information for issue resolution of increasing amount of information transmitted. Then we showed that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update and a trade-off between the estimation error variance and a communication energy. Secondly, we have proposed a network configuration algorithm considering this trade-off.

Secondly, we discussed a sensor scheduling problem considering the estimation error variance and communication energy in a feedback control system via a sensor network. We first have proposed the estimation algorithm with the unknown input of the plant in the feedback control system via a sensor network. Each sensor node calculates the local estimate without information of the control input and transmits its information to the sensor node applying the control input to the plant. This sensor node calculates the common estimate and control input using received information. Then we showed that there is the unique positive definite solution to the discrete algebraic Riccati equation in the error covariance update. Secondly, we have proposed a sensor scheduling algorithm considering estimation error variance and communication energy. This scheduling algorithm achieved sub-optimal network topology with minimum energy and a desired error variance.

Finally, we have verified effectiveness of a proposed method by experiments.

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