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著者	Komatsuzaki Toshihiko, Iwata Yoshio
journal or publication title	Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)
volume	4173
page range	357-366
year	2006-01-01
URL	http://hdl.handle.net/2297/3039

Modeling of Sound Absorption by Porous Materials using Cellular Automata

Toshihiko Komatsuzaki and Yoshio Iwata

Graduate School of Natural Science and Technology, Kanazawa University,
Kakuma-machi, Kanazawa, 920-1192, Japan
{toshi, iwata}@t.kanazawa-u.ac.jp

Abstract. In the present study, acoustic wave propagation in acoustic tube in-corporating sound absorbing material is simulated using Cellular Automata (CA). CA is a discrete system which consists of finite state variables, arranged on a uniform grid (cell). CA dynamics is described by a local interaction rule, which is used for computation of new state of each cell from the present state at every time step. In this study an acoustic tube model is introduced in which ab-sorbing material is characterized by direct modeling of porosity and flow resis-tance. Direct numerical simulation CA model is performed and evaluated by absorption coefficient using standing wave ratio measure. The results showed good correspondence with analytical solutions.

1 Introduction

The vibrating structures and various kinds of machineries often cause serious noise problems to humans within an environment. The passive sound attenuation method is generally employed using resonators, isolation walls and sound absorbing treatment. Among various kinds of sound absorption materials, porous materials such as glass wool quilting and polyurethane foams are the most common and significant technique which are widely used for room acoustics and various electric devices. However, the recent designing of compact and lightweight devices put limits on the application of such dissipative materials in conjunction with saving costs. Hence the material itself, amount and placement must be determined carefully that can realize high performance damping and low cost. The development of numerical model which can predict sound propagation and attenuation effect of those materials is then important for realizing efficient and suitable engineering design.

Before predicting desired sound absorption effect in a practical environment, material properties such as acoustic propagation constant and the absorption coefficient must be determined either numerically or experimentally. The more precise measurement system has been developed for the latter approach. On the other hand, theoretical prediction of sound absorbing mechanism of porous materials has long been investigated which coincides with basic experimental results[1]. The finite element and also the boundary element methods may be reliable and useful approach for exploring more realistic situations. However, on setting properties and shapes of porous materials with these models certain approximation must be incorporated which may lead to the lack of micro structure and the essential mechanism of sound absorption of materials itself. Also, obtaining transient response of the system with these models require elaborate modeling procedure.

In this paper, the acoustic wave model is developed using Cellular Automata. CA is a kind of discrete computations which has been developed for modeling wide range of phenomena including many physical processes described generally by partial differential equations[7]. Specifically the wave propagation models have been studied by researchers based on Cellular Automata[3]-[8]. The works include Chopard et al.[8] who had modeled wave propagation by Lattice Boltzmann approach applicable for practical situations such as the radio wave transmission in complex urban environments. The authors have also developed an acoustic wave propagation model for two dimensional acoustic problems for simulating sound source movement, sound diffraction by the presence of barriers and reflection due to inhomogeneity of acoustic media[9]. Due to its easiness and simplicity of modeling procedure, the modeling approach also seems suitable for the problems concerned. However, the preceding work does not include energy dissipating mechanisms which is nessesary for producing sound absorption effect. In the present study, the modified version of the acoustic wave propagation model is numerically developed using CA for understanding fundamental sound absorption mechanism of porous materials and evaluating sound absorption performance, where the details of porous material structure is considered in the model. The acoustic waveguide incorporating sound absorbing porous material is constituted and the sound absorption effect is predicted. The theoretical approach for obtaining absorption coefficient is also presented for comparison.

2 Theoretical Description of One-dimensional Acoustic Field

In this section, theoretical description of one-dimensional acoustic field is shown, and the material property related to acoustic characteristics which is commonly known as the sound absorption coefficient is also derived. Moreover, the parameter known as standing wave ratio (SWR) and used for determining absorption coefficient by numerically measured sound pressure amplitude is presented.

2.1 The Wave Equation

The generated pressure oscillation in an acoustic medium is observed as sound, which is described by a set of linear equations for one dimensional field under the presence of absorbing material[1]:

$$\frac{\rho_0}{\sigma}\frac{\partial \dot{u}(x,t)}{\partial t} = -\frac{\partial p(x,t)}{\partial x} - R_f \dot{u}(x,t) \tag{1}$$

$$\frac{\sigma}{\kappa}\frac{\partial p(x,t)}{\partial t} = -\frac{\partial \dot{u}(x,t)}{\partial x} \tag{2}$$

where $p(x,t)$ is a sound pressure and $\dot{u}(x,t)$ a particle velocity, ρ_0 density, κ volume elasticity, σ porosity of porous material and R_f flow resistance constant respectively. Equation (1) corresponds to equation of motion of the continuum per unit volume, and also (2) satisfies continuity of the medium. The solution to (1) and (2) without porous material is given by setting $\sigma = 1.0$ and $R_f = 0.0$, on the assumption that the wave is harmonic:

$$p(x,t) = j\omega\rho A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)} \tag{3}$$

where A and B are constants determined by boundary conditions, ω the sound source frequency, k the wave number respectively. The first term of (3) expresses a progressive wave, and the second a regressive wave.

If we employ acoustic tube model which has a sound source on one edge, the pressure distribution inside tube is then calculated by giving boundary conditions $\dot{u}(0,t) = \dot{u}_0 e^{j\omega t}$, and also $\dot{u}(l,t) = 0$ for the another edge closed:

$$p(x,t) = -j\rho c \dot{u}_0 \frac{\cos k(l-x)}{\sin kl} e^{j\omega t} \tag{4}$$

In the above (4), l stands for the tube length, $\dot{u}_0$ the driving source velocity.

2.2 Definition of propagation constant and characteristic impedance

Sound absorbing materials are usually characterized by acoustic properties known as propagation constant and characteristic impedance. The absorption coefficient is then determined by those constants. The characteristic impedance is defined by the ratio between acoustic pressure and particle velocity while the wave travels along the media, described as:

$$Z_c = \frac{p}{u} = \rho c_m \tag{5}$$

In (5), p and u denotes sound pressure and particle velocity, c_m sound speed along material and ρ the density of material, respectively. The propagation constant γ is defined by the damping the phase chance along the unit length of material axis, which is given by a complex form:

$$\gamma = \alpha + \beta, \quad \beta = \omega / c_m \tag{6}$$

In the above (6), α and β signifies damping and phase constant.

Sound propagation model inside acoustic waveguide incorporating absorbing materials is shown in Fig. 1. The sound wave propagating through the material 1 with thickness d is described by the following (7) with respect to the incoming sound pressure P_{i0} traveling through the air,

$$p_{1d} = p_{1i} e^{-\gamma d} \tag{7}$$

In the case material 1 is backed by another material 2, the inhomogenous boundary between these two materials is characterized by acoustic impedance Z_2. In

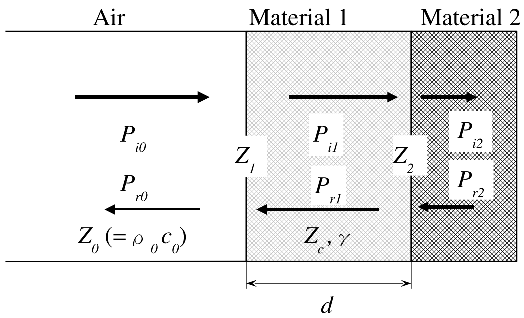

Fig. 1. Sound propagation model inside acoustic waveguide incorporating absorbing material. In this figure, three kinds of acoustic media exist. Hence two boundaries between air and material 1, and also between materials 1 and 2 are present. P denotes sound pressure, and Z acoustic impedance.

the same way the acoustic impedance Z_1 with respect to the boundary between air and material 1 is given by the following equation using Z_2.

$$Z_1 = Z_c \frac{Z_2 \cosh(\gamma d) + Z_c \sinh(\gamma d)}{Z_2 \sinh(\gamma d) + Z_c \cosh(\gamma d)} \tag{8}$$

Before calculating sound absorption coefficient α, the reflection constant r_p must be determined using acoustic impedance Z_1. The constant r_p is defined as follows.

$$r_p = \frac{Z_1 - \rho_0 c_0}{Z_1 + \rho_0 c_0} \tag{9}$$

In (9), ρ_0 and c_0 denotes density and sound speed of air, respectively. The absorbing coefficient α is then calculated using (9), according to the following (10).

$$\alpha = 1 - |r_p|^2 \tag{10}$$

As already described above, in order to obtain absorption coefficient the acoustic impedance Z_1 must be determined, however, Z_1 also depends on another impedance Z_2. Therefore, Z_2 must be first determined by setting the layer behind the target material become air, or directly backed by the rigid wall before calculating Z_1. (In the latter case Z_2 become zero.) The rest of the unknown parameter, propagation constant γ and characteristic constant Z_c, are usually determined by measurements. They are also derived analytically by solving (1) and (2), for the case the porous material is backed directly by the wall described as follows.

$$\gamma = \frac{\omega}{c_0}\sqrt{1 - j\frac{\sigma R_f}{\omega \rho_0}} \tag{11}$$

$$Z_c = \frac{\rho_0 c_0}{\sigma}\sqrt{1 - j\frac{\sigma R_f}{\omega \rho_0}} \tag{12}$$

Equations (11) and (12) are used for the comparison with results obtained by the Cellular Automata acoustic model in subsequent section. γ, Z_c and Z_1 are the important parameters for characterizing the property of porous materials. However, the measurement process as well as parameter calculation seem rather complex.

2.3 Determining Absorption Coefficient by Standing Wave Ratio Method

One of the most fundamental approaches for determining absorption coefficient experimentally is known as the standing wave ratio (SWR) method. Due to its simple idea and constitution, and also the needless for complex calculation, the method is suitable for the direct numerical approach such as the CA model dealt in the present study. As illustrated in Fig. 2, the progressive wave propagates into the material and a wave reflected at the face of material interferes and forms standing wave distribution. The standing wave ratio (SWR) is defined by the ratio between the maximum and the minimum peaks of standing wave. Practically, in an experimental situation, these peaks are explored by scanning microphone along acoustic tube axis. The SWR, n, is defined as follows.

$$n = \frac{|P_i| + |P_r|}{|P_i| - |P_r|} \tag{13}$$

The reflection constant r_p is then determined by the following equation and further absorption coefficient by (10), as well.

$$r_p = \frac{|P_r|}{|P_i|} = \frac{n-1}{n+1} \tag{14}$$

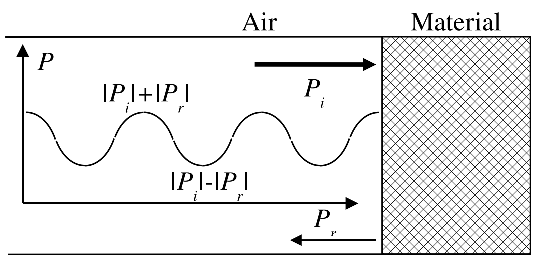

Fig. 2. Standing wave distribution inside an acoustic tube. The progressive wave propagates into the material and a wave reflected at the face of material interferes and forms standing wave distribution. The standing wave ratio (SWR) is defined by the ratio between the maximum peak and the minimum peak of the standing wave.

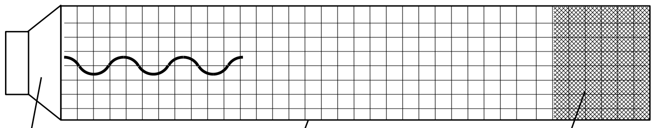

Fig. 3. Simulation model of acoustic tube incorporating absorbing material. A sound source is located at left hand side of the tube, whereas the porous material located on the other side.

3 The Cellular Automata Model

In this section, Cellular Automata model is developed for simulation of acoustic wave propagation in a media incorporating porous material. The simple finite difference scheme obtained by linear wave equation is referenced for developing local interaction rule, in a sense that discretized wave equation yields to an expression of local relationship of wave amplitudes. The rule is then extended to a more practical case, yet time and space are treated as discrete integers. The Cellular Automata approach to such a wave propagation problem was discussed for two dimensional models comparing with analytical solutions. Definitions for state variables and local interaction rules are presented in the following subsections.

3.1 Space Partitioning and State Definition

Figure 3 shows two dimensional space discretized into rectangular cells. Each one of the cell is distinguished for its state by three numbers; i) zero for acoustic media (air), ii) 1 for rigid wall , and iii) 2 for portion of absorbing material. Additionally, two variables which express the sound pressure and particle velocity are defined for the first acoustic medium state. These variables are updated at each simulation step according to the local interaction rules explained in the next subsection. In advance to composition of local rules, the definition of neighbor is specified as shown in Fig. 4. For the two dimensional model, cross-located four cells are neighbors which is conventionally called Neumann Style neighbors. In each medium state cell the sound pressure variable is assigned as well as particle velocities in four neighboring directions. Following Cellular Automata convention, time and space are treated as integers. In order for the model to be comparable with analytical solution, we assign unit cell length $dx = 0.001$[m], and also the sound speed $c = 344$[m/s]. Table 1 shows comparative listing between CA space and physical parameter.

3.2 Foundation of Local Rules

State parameters given in each one of the cells is updated every discrete time step according to a local interaction rule which is described in this section.

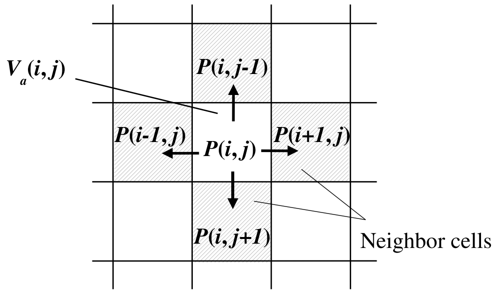

Fig. 4. Definition of neighbor in two dimensional acoustic model. Two state variables, sound pressure P and particle velocity V, are placed in each cell.

Table 1. Table 1. Equivalent system parameters. Parameters defined in the CA model are com-pared with those in physical system.

	Sound speed	Unit time step	Unit space size
Physical system	c = 344 [m/s]	$dt = 1/344$ [sec]	$dx = 0.001$ [m]
2-dim CA model	$c = 1/\sqrt{2}$ [cell/step]	$dt = 1$ [step]	$dx = 1$ [cell]

First, the particle velocities in four directions are updated in time with respect the difference of sound pressure between adjacent cells, whose update rule is described explicitly as,

$$V_a(\mathbf{x}, t+1) = V_a(\mathbf{x}, t) - \{P(\mathbf{x} + \mathbf{dx}_a, t) - P(\mathbf{x}, t)\} \tag{15}$$

V_a represents particle velocity of media and P the sound pressure. Two dimensional cell position is expressed as a vector $\mathbf{x}$ and discrete time step as t. A suffix a in (15) signifies index of four neighbors. The particle velocity further obeys (16), which expresses energy dissipation by the flow resistance due to presence of porous material.

$$V_a(\mathbf{x}, t) = (1 - n \cdot d)V_a(\mathbf{x}, t) \tag{16}$$

In the above (16), n represents number of porous material cells in neighbor, d a damping constant per unit cell. The pressure is then updated according to the rule described by (17),

$$P(\mathbf{x}, t+1) = P(\mathbf{x}, t) - c_a^2 \sum_a V_a(\mathbf{x}, t+1) \tag{17}$$

where c_a denotes the wave traveling speed in CA space. Sound pressure and particle velocities are updated according to the local rule described by above three equations.

Since calculation will be carried out between nearby cells that are separated only a unit length at every single step, any physical quantities cannot have the transport speed exceed to this calculation limit. This applies directly to one dimensional CA model with maximum speed condition $c_a \leq 1$, whereas not for two dimensional case. Since wave is assumed to propagate isotropically despite the square compartment of space and cross-style definition of neighbors, an effective traveling speed must be considered. It is known that the maximum wave speed becomes $c_a = 1/\sqrt{2}$ for two dimensional case, therefore the wave front travels $1/\sqrt{2}$ of unit cell length per calculation step. These conditions can also be obtained by the CFL condition, which provides requirement for numerical stability of finite difference scheme expressed explicitly as,

$$c = \frac{\Delta t}{\Delta x} \leq \begin{cases} 1 & \text{for one dimension} \\ 1/\sqrt{2} & \text{for two dimension} \end{cases} \tag{18}$$

where Δt and Δx are unit time step and unit length in difference scheme, respectively. By setting Δt and Δx be unity, we get $c \leq 1$ as one dimensional stability condition, and also $1/\sqrt{2}$ for another. It is straightforward to say that upper limit condition of propagating speed can be derived not numerically, but physically in the CA model.

4 Simulation of Wave Propagation

In this section, simulation of acoustic wave propagation is performed for the acoustic tube model incorporating porous material as shown in Fig. 3, which is described by the Cellular Automata. Analytical solution is also calculated using set of equations explained in section 2.

In the CA model, the space inside acoustic tube is divided into 100 x 1000 cells, where the unit size of a cell is assumed to be 1 [mm] for the comparison with physical system. Hence the size of acoustic tube corresponds to 100 [mm] in diameter and 1000 [mm] in length respectively. The sound source is provided by giving forced particle velocity to cells which are located on the left edge of the tube, whereas the sound absorbing material with certain thickness is located on the other side by assigning cell state as porous material.

Two cases of simulation are performed in the following subsections. The first case calculates acoustic field inside sound tube without porous material, where the resonance characteristic is investigated comparing with analytical solution. In the second case the CA model is tested for the presence of absorbing material, where the result is compared with analytically calculated absorbing coefficient.

4.1 Acoustic Tube Model Without Porous Material

The acoustic field inside sound tube model without porous material is calculated. Analytical pressure distribution caused by pulse excitation at the sound source can be obtained by (4). The resonance characteristic of the acoustic tube with length 1 [m] is shown in Fig. 5. The first and the second resonant frequencies for the tube are 172 and 344 [Hz], respectively. From Fig. 4, it is known that frequency response obtained by CA model well corresponds to analytical one.

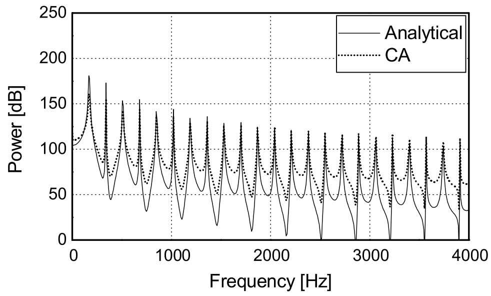

Fig. 5. Frequency response of acoustic tube. The CA model well coincides with analytically calculated response.

4.2 Acoustic Tube Incorporating Porous Material

The second case deals with an acoustic field inside sound tube under the presence of porous material. Two cases of porous material with thickness 50 and 100 [cells] are considered in the present simulation. Hence the thickness of material becomes 50 [mm] and 100 [mm] in the actual physical system, respectively. The damping parameter with respect to the (16) in the CA model is set to $d = 0.2$, and the inner pores of the material is expressed by randomly locating cell states by the mixture of medium and material state, so that the porosity becomes 0.8 apparently. Sinusoidal excitation at the sound source whose frequency varies from 10 to 4000 [Hz] is generated at the left end of the tube. The absorption coefficient is processed according to the SWR method depicted in section 2.3 by the measured standing wave amplitudes.

The absorption coefficient is also calculated analytically by using set of equations mentioned in section 2. In calculating propagation constant and characteristic impedance using (11) and (12), the flow resistance R_f is set 5000 [Ns/m^4], and the porosity $\sigma = 0.8$, respectively.

Calculation results obtained by both CA and analytical model are illustrated in fig-ure 6. The results calculated by the CA model well coincides with analytical one for two cases of material thickness except for considerable difference in relatively low and high frequency regions, which is due to the inadequate formation of standing wave for extremely low frequency in such an short distance of the present acoustic tube model, and also insufficient partition of space compared to the wave length in higher frequency.

5 Conclusions

In the present paper, the two dimensional acoustic wave propagation model is devel-oped using Cellular Automata. Moreover, the sound absorbing model

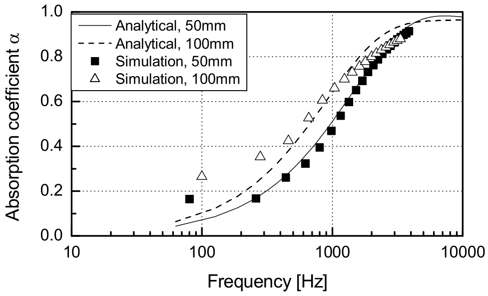

Fig. 6. Absorption coefficient obtained by the CA model. The absorption coefficient is determined according to the SWR method. The solid and dashed curve signifies analytically calculated absorption coefficient for the respective material thickness 50mm and 100mm.

incorporating porous material is investigated. It is shown that the CA model well illustrated results which are consistent with analytical solutions.

Acknowledgements

We thank Professor Shin Morishita at Yokohama National University for his advice and support to the present work.

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