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著者	Nakayama Kenji, Hirano Akihiro, Sakaguchi Hiroaki
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A VIRTUAL BANDPASS FILTER IN SUB-BAND ADAPTIVE FILTERS AND CONVERGENCE ANALYSIS

Kenji Nakayama, Akihiro Hrano, Hiroaki Sakaguchi

Dept. of Information Systems Eng., Faculty of Eng., Kanazawa Univ.
2-40-20, Kodatsuno, Kanazawa, 920-8667, Japan
nakayama@t.kanazawa-u.ac.jp

ABSTRACT

In a sub-band adaptive filter, an i-th analysis filter $F_i(z)$ separates the i-th sub-band Ω_i. An unkown system $H(z)$ is identified as $H(z)F_i(z) = F_i(z)A_i(z^K), \omega \in \Omega_i$, where $A_i(z^K)$ is the i-th sub-band adaptive filter. $F_i(z)$ can be cancelled as $A_i(z^K) = H(z), \omega \in \Omega_i$. Furthermore, this can be expressed as $A_i(z^K) = B_i(z)H(z)$, where $B_i(z) = 1, \omega \in \Omega_i, = 0, \omega \in \Omega_j, i \neq j$. $B_i(z)$ is not really used, rather it appears equivalently in deriving the ideal sub-band adaptive filters. An impulse response $b_i(n)$ of $B_i(z)$ distributes over both negative and positive time domains. Since the impulse response $a_i(n)$ of $A_i(z^K)$ is a convolution sum of the impulse response $h(n)$ of $H(z)$ and $b_i(n)$, it also distributes in both time domains. Thus, **the ideal sub-band adaptive filters are noncausal**. Two methods for relaxing the noncausal condition are investigated. One of them is to insert time delay in serial to the unknown system in order to shift $a_i(n)$ toward the positive time domain. The other is to set the sampling rate in the sub-bands high so as to generate split bands and reduce the ratio of $a_i(n)$ in the negative time domain.

1. INTRODUCTION

Audio echo cancelers are important for TV conference systems. However, the impulse response of room acoustic characteristics is very long, for instance several thousand samples with a sampling rate of 8 kHz. This requires a very high-order adaptive filters. Therefore, sub-band adaptive filters become very atractive in order to save computational complexity and to make fast convergence possible [1]. One method is to use the sampling rate higher than twice of the sub-bandwidth. The other uses the lower sampling rate cancelling the cross terms [2],[3]. In this paper, the former approach is taken into account. The modulation sub-band adaptive filters [4],[5] can minimize the sampling frequency just twice of the sub-bandwidth. The Polyphase and FFT realization of a filter-bank is very efficient in order to reduce computational complexity [6],[7],[8].

The residual error in the sub-band adaptive filters is larger than that obtained by the full-band adaptive filters [5]. This point has not been well studied. In this paper, convergence property of the sub-band adaptive filters is theoretically analyzed. Two kinds of methods are investigated to improve the convergence property. Finally, several examples are shown to confirm the theoretical results.

2. MODULATION SUB-BAND ADAPTIVE FILTER

2.1. Block Diagram and Signal Spectra

Figure 1 shows a block diagram of the modulation sub-band adaptive filter [5],[7]. $F_i(z)$ and $G_i(z)$ are complex analysis and synthesis filter banks, respectively. The number of sub-bands is M and the sampling rate reduction is $1/K$. The carrier signals applied to both modulator (MOD) and demodulator (DEM) are complex. The input signal $x(n)$ is real. After the modulators, only the real part is transferred. The adaptive filters (AF) are real filters. After the synthesis filter bank, only the real part is transferred.

Figure 2 shows examples of the signal spectra at the analysis filter bank and the modulator. The number of sub-bands M is 6. The sampling rate f_s is 1. The output of $F_i(z)$, $i = 0 \sim 5$, are shown in Fig.2(a), and one of them, that is the $F_1(z)$ output, is shown in Fig.2(b). This spectrum is shifted to the origin by the modulator as shown in Fig.2(c). By taking the real part of the modulator output, the symmetry part appears as shown in Fig.2(d). Finally, this real signal is down sampled by $f_s/K, K = 4$, as shown in Fig.2(e). The spectrum is expanded over a whole band $0 \leq f \leq f_s/K$. However, aliasing does not occur. K is not the same as M. The signal spectra in the demodulator and the synthesis filter bank are the same in the reversed order.

2.2. Analysis Flters and Sampling Rates

The fundamental low-pass filter (LPF) is denoted

$$F(z), z = exp(j2\pi f/f_s) \tag{1}$$

$F_i(z)$ are obtained by shifting $F(z)$ as follows:

$$F_i(z) = F(z_i), \quad i = 0 \sim M-1 \tag{2}$$

$$z_i = exp\left[\frac{j2\pi(f - f_s/4M - if_s/2M)}{f_s}\right] \tag{3}$$

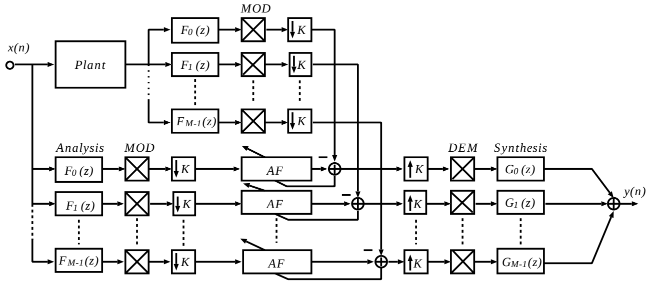


Fig. 1. Modulation sub-band adaptive filter.

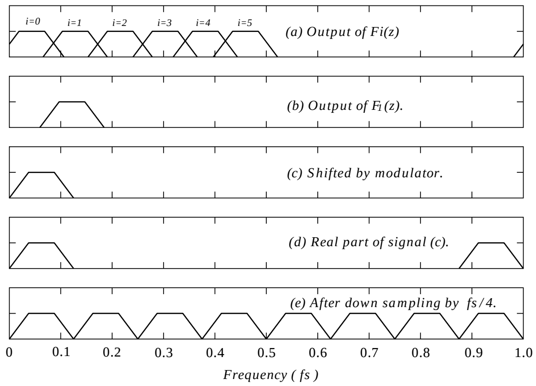


Fig. 2. Signal spectra at analysis filter bank and modulator outputs.

If the transient bandwidth is given by $2\Delta f$, then a single sub-band occupies $f_s/2M+2\Delta f$. Thus, the minimum sampling rate after down sampling is

$$f_{sl} = 2(f_s/2M + 2\Delta f). \tag{4}$$

Furthermore, the following relations are held.

$$f_{sl} = f_s/K \tag{5}$$

$$\Delta f = \frac{f_s}{4}(\frac{1}{K} - \frac{1}{M}) \tag{6}$$

Next, the carrier signal applied to the modulator is given by

$$c_i(n) = exp\left[\frac{j2\pi(-\Delta f + if_s/2M)n}{f_s}\right] \tag{7}$$

3. IDEAL CHARACTERESTICS OF SUB-BAND ADAPTIVE FILTERS

3.1. Virtual Bandpass Filters

Let sub-band width be B, and a sampling frequency in the sub-band satisfies $f_{sl} \geq 2B$. A system identification problem is taken into account as shown in Fig.1. Let a transfer function of the unknown system, "plant", be $H(z)$. The i-th analysis filter $F_i(z)$ separates the i-th sub-band Ω_i. The corresponding adaptive filter is denoted $A_i(z^K)$. We assume the separation is complete, and no aliasing occurs.

Suppose the unknown system is exactly identified in the sub-band Ω_i, then the following is held.

$$H(z)F_i(z) = F_i(z)\tilde{A}_i(z^K) \tag{8}$$

where $\tilde{A}_i(z^K)$ indicates the ideal sub-band adaptive filter. In the above equation, $F_i(z)$ can be cancelled as,

$$\tilde{A}_i(z^K) = H(z), \quad \omega \in \Omega_i \tag{9}$$

$\tilde{A}_i(z^K)$ is operated with f_{sl}, then the frequency band to be considered is limited to Ω_i.

Let $\tilde{A}_i(z)$ be a transfer function satisfies

$$\tilde{A}_i(z) = \begin{cases} \tilde{A}_i(z^K) & \omega \in \Omega_i \\ 0 & \omega \in \Omega_j, i \neq j \end{cases} \tag{10}$$

This means the impulse response $\tilde{a}_i(Kn)$ of $\tilde{A}_i(z^K)$ is obtained by down sampling the impulse response $\tilde{a}_i(n)$ of $\tilde{A}_i(z)$. Therefore, $\tilde{a}_i(n) is used instead \tilde{a}_i(Kn)$ in the following for convenience.

From Eq.(9), $\tilde{A}_i(z)$ is obtained by

$$\tilde{A}_i(z) = B_i(z)H(z) \quad \text{in a whole band} \tag{11}$$

$$B_i(z) = \begin{cases} 1 & \omega \in \Omega_i \\ 0 & \omega \in \Omega_j, i \neq j \end{cases} \tag{12}$$

$B_i(z)$ is an ideal filter with zero phase. The ideal adaptive filters in the sub-bands are obtained as a cascade form of the unknown system and the ideal bandpass filter. This ideal filter is not really used in the sub-band adaptive filter, rather it appears equivalently to relate the unknown system and the sub-band adaptive filters. So, it is called a **vertual bandpass filter** in this paper.

3.2. Noncausal Sub-band Adaptive Filters

Since $B_i(z)$ has zero phase, its impulse response $b_i(n)$ distributes over negative and positive time domain. Thus, $B_i(z)$ is a noncausal system. The impulse response $\tilde{a}_i(n)$ of the adaptive filter is a convolution sum of the impulse response $h(n)$ of the unknown system and $b_i(n)$, then it appears in the negative time domain. Since $H(z)$ is causal, that is $h(n) = 0, n < 0$, then $\tilde{a}_i(n)$ is given by

$$\tilde{a}_i(n) = \sum_{m=-\infty}^{n} b_i(m)h(n-m) \tag{13}$$

Even though $h(n)$ is causal, $\tilde{a}_i(n)$ can be noncausal due to $b_i(n)$, which occupy the negative time domain $(n < 0)$. This is a very important point, that is, **"the ideal characteristics of the sub-band adaptive filters are noncausal"**.

However, the noncausal filters cannot be actually realized. In other word, causal filters cannot approximate $\tilde{a}_i(n), n < 0$. This mismatch causes the residual error, which is larger than that of the full-band adaptive filters. In the latter case, the virtual bandpass filter does not appear.

4. RELAXING METHODS FOR NONCAUSAL CONDITION

4.1. Inserting Time Delay

In order to reduce the impulse response in the negative time domain, $h(n)$ is shifted toward the positive time domain by inserting time delay just before or after the unknown system, "plant" in Fig.1.

Let time delay be D samples, $h(n)$ is shifted as $h(n-D)$. $\tilde{a}_i(n)$ is changed as

$$\begin{aligned}\tilde{a}_i(n) &\rightarrow \sum_{m=-\infty}^{n-D} b_i(m)h(n-D-m) \\ &\rightarrow \tilde{a}_i(n-D) \qquad (14)\end{aligned}$$

Thus, $\tilde{a}_i(n)$ is also shifted toward the positive time axis, and the ratio of $\tilde{a}_i(n-D)$ in the negative time domain can be reduced.

The part of $\tilde{a}_i(n-D)$ in the negative time domain cannot be approximated by the causal filters. Thus, the error is estimated as follows:

$$E_{delay} = 10\log\frac{\sum_{n=-\infty}^{-1}\tilde{a}_i^2(n-D)}{\sum_{n=-\infty}^{\infty}\tilde{a}_i^2(n-D)} \quad dB \qquad (15)$$

Since $\tilde{a}_i(n)$ with high-Q poles spreads over in a wide range, the error in the sub-bands, which include high-Q poles, are dominant in the total error.

4.2. Oversampling Methods

The other method is to set f_{sl} higher than $2B$. In this case, there exist split bands between the sub-bands. Although $B_i(z)$ is required to have the unity amplitude and the zero phase responses in the sub-band Ω_i, it can take arbitrary amplitude and phase responses in the split bands.

$$B_i(z) = \begin{cases} 1 & \omega \in \Omega_i \\ \text{Arbitrary} & \omega \in \text{split bands} \\ 0 & \omega \in \Omega_j \quad i \neq j \end{cases} \qquad (16)$$

Let $b_i^{sub}(n)$ and $b_i^{split}(n)$ be impulse responses for $B_i(z)$, $\omega \in \Omega_i$ and $\omega \in$ split bands, respectively.

$$b_i(n) = b_i^{sub}(n) + b_i^{split}(n) \qquad (17)$$

$b_i^{sub}(n)$ distributes in the same range compared with using $f_{sl} = 2B$. $b_i^{split}(n)$ can be asymmetrical and its main part can be shifted toward the positive time domain by adjusting the amplitude and phase responses of $B_i(z)$ in a learning process. Totally, the component of $b_i(n)$ in the negative time domain can be reduced. Since the unknown system is causal, that is $h(n) = 0, n < 0$, the component of $\tilde{a}_i(n), n < 0$ is determined by that of $b_i(n)$ in the negative time domain.

5. SIMULATION

The following conditions are used. $M = 6$, $K = 4$, $f_{sl} = 2B$. Figure 3 shows amplitude responses of the unknown system and the analysis filter bank. One example of the optimum sub-band adaptive filters is shown in Fig.4. The number of taps of the full-band adaptive filter and sub-band adaptive filters are 160 taps and 40 taps×6bands =240 taps. Figure 5(a) shows an envelope of the impulse response $b_i(n)$. It symmetrically spreads over the negative and positive time axises. $h(n)$ is shown in Fig.5(b). Furthermore, $\tilde{a}_i(n)$ and its delayed versions with $D = 50$ and $D = 100$ are shown in Fig.6. From this figure, the original $\tilde{a}_i(n)$ spreads over the negative time axis. However, that of the delayed version is well reduced.

Figure 7 shows the learning curves for using no time delay ($MSE \simeq -25dB$) and 1200 sample time delay ($MSE \simeq -65dB$). The result using time delay is almost the same as that of the full-band adaptive filter. In this example, the time delay of the several hundred samples is enough.

Next, the oversampling method is examined. The down sampling rate is set to $K = 3$ and $K = 2$. Thus, f_{sl} is increased from $2B$ to $8B/3$ and $4B$, respectively. Figure 8 shows $\tilde{a}_i(n)$ with $K = 4$ and $K = 2$. It is confirmed that the component of $\tilde{a}_i(n)$ in the negaive time domain is well reduced for $K = 2$. The learning curves for $K = 3(MSE \simeq -40dB)$ and $K = 2(MSE \simeq -65dB)$ are shown in Fig.9. The convergence speed is a little slower, however, the final error for $K = 2$ is almost the same as using the time delay.

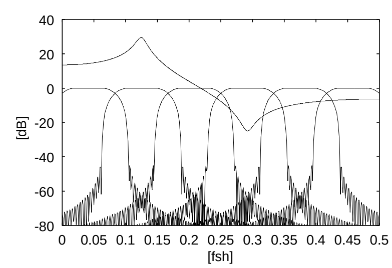


Fig. 3. Amplitude responses of $H(z)$ and $F_i(z)$.

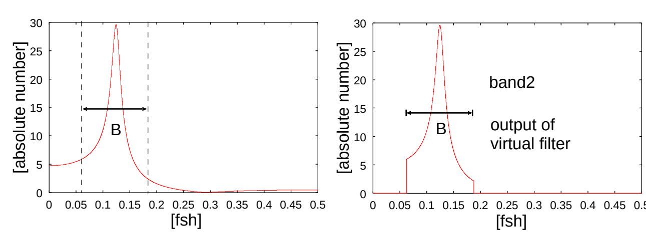
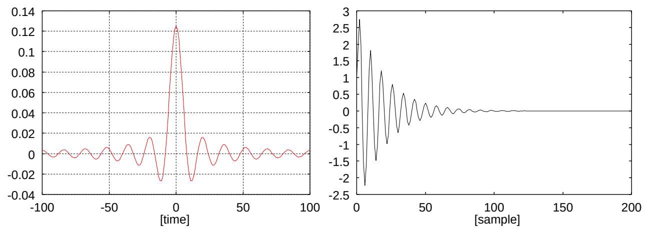


Fig. 4. Optimum sub-band adaptive filter.



(a) (b)

Fig. 5. (a)Envelope of $b_i(n)$ (b)$h(n)$ of unknown system.

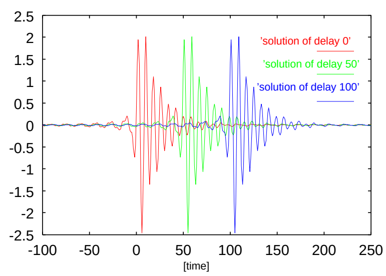


Fig. 6. $\tilde{a}_i(n)$ and its delayed versions with $D = 50$ and $D = 100$.

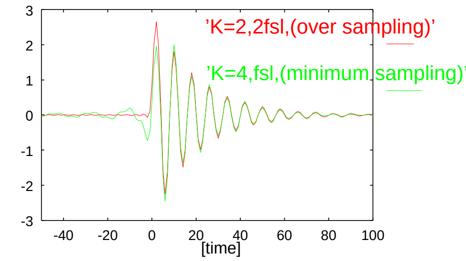


Fig. 8. $\tilde{a}_i(n)$ for minimum sampling rate $(K = 4, f_{sl} = 2B)$ and oversampling rate $(K = 2, f_{sl} = 4B)$.

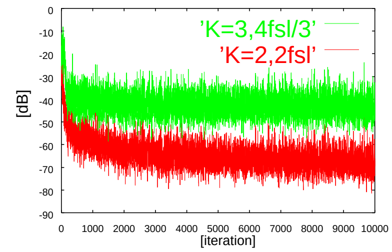


Fig. 9. Learning curves for oversampling rates with $K = 2, (f_{sl} = 8B/3)$ and $K = 3, (f_{sl} = 4B)$. MSE is approximately $-45dB$ and $-65dB$, respectively.

6. CONCLUSIONS

In this paper, it has been cleared that the ideal sub-band adaptive filters are noncausal. This property has been analyzed by introducing the virtual bandpass filter. Two kinds of methods to relax the noncausal condition have been investigated. One of them is to insert time delay in serial to the unknown system and the other is to set the sampling rate in the sub-bands high. The noncausal property and usefulness of the relaxation methods have been confirmed through the simulation.

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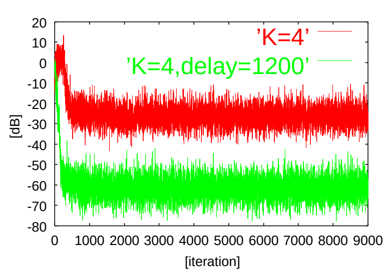

Fig. 7. Learning curves of sub-band adaptive filters with 1200 samples time delay ($MSE \simeq -65dB$) and no time delay ($MSE \simeq -25dB$).