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Theoretical Proposal for a Unidirectional Optical Amplifier

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***Abstract*—The possibility of achieving a unidirectional optical amplifier is theoretically predicted with a model in the vacuum environment. The operation of this amplifier is based on the transfer of the kinetic energy of an electron beam to an optical one where both are propagating in the same direction. The optical beam propagates in a dielectric waveguide where it partly penetrates into the vacuum in the form of an evanescent wave. The electron beam is emitted from an electron gun and propagates along the surface of the dielectric waveguide, exciting the optical beam. The propagation speed of the optical beam is slowed down with the aid of the dielectric waveguide and is made to coincide with that of the electron beam. Quantum mechanical analysis of the interaction between the optical beam and the electron beam is given, based on the density matrix formalism. At the wavelength of 0.5 μm, the gain coefficient is calculated to be about 12 cm^{-1} under the excitation voltage of 64 kV and the electron beam current of several microamperes.**

***Index Terms*— Dielectric waveguides, electron beams, optical amplifiers, traveling-wave amplifiers, traveling-wave tube.**

I. Introduction

HISTORICAL developments in electronics are mostly caused by inventions of new active devices such as the vacuum tube, the transistor, and the integrated circuit. Fundamental features of these active devices are addressed by both their amplification and unidirectional properties. The former property not only gives amplification of a weak signal but also induces many other functions such as oscillation, modulation, detection, and switching. The latter property is essential in the cascade connection of these devices to perform complicated functions. The word unidirectional used in this paper indicates the characteristic of a signal to propagate in the forward direction but not in the counter direction.

Conventional vacuum tubes and transistors have the unidirectional property based on carrier transportation phenomena. Operational speeds of these devices are determined by the transport time of the carriers and are limited to frequencies below 10^{10} Hz. In these devices, the length of the interaction region between the signal and the carrier is required to be much shorter than the signal wavelength.

The most popular unidirectional active device realized at shorter wavelengths is the traveling wave tube [1]. The free-electron laser [2], [3] and the Cherenkov maser (or laser) [4]–[6], which have been developed for generation of high-power electromagnetic (EM) waves at frequencies above microwaves, also exhibit unidirectional amplification. In these devices, the kinetic energy of the emitted electron beam is utilized. Conditions for obtaining amplification of an EM wave from an electron beam are the following.

1) Energy must be conserved between the electron motion and the EM wave.
2) Momentum must be conserved between the electron motion and the EM wave. That is, the phase velocity of the EM wave should be the same as the velocity of the electron beam.
3) The electric polarization of the EM wave must have a component in the direction of the electron motion.

The traveling wave tube utilizes a helix waveguide to reduce the velocity of the EM wave as well as to give a polarization component along the electron beam [1]. Since the fabrication technology of the helix waveguide is limited to submillimeter size, traveling wave tubes cannot be used at optical frequencies.

In the case of the free-electron laser, electrons are accelerated to energies higher than typically 10 MeV to approach optical velocity and are forced to undertake periodic vibrations in the transverse direction with the help of a periodic magnetic field called the wiggler field [3]. Although the free-electron laser is able to operate at shorter wavelengths than the optical ones, this device may not be suitable for application in the field of electronics, because it requires high voltages and a strong magnetic field to generate high-power EM waves.

The Cherenkov maser utilizes a dielectric wall in a cylindrical waveguide to reduce the velocity of the EM waves [5], [6]. However, this device may not be suitable for direct applications in the field of electronics, because investigations of the device have been performed at voltages larger than several hundred volts to generate high-power EM waves at wavelengths longer than 100 μm.

Other ideas have also been proposed for achieving unidirectional optical amplifiers, but these ideas have not yet been realized [7]–[9].

The amplification mechanism of the proposed amplifier also utilizes the kinetic energy of the electron beam and is similar to those in the traveling wave tubes and the Cherenkov masers. The velocity of the optical beam is effectively reduced and a polarization component along the electron beam is generated. However, the remarkable difference between our amplifier and the traveling wave tube and the Cherenkov maser is the usage of the dielectric slab waveguide instead of the helix or the dielectric cylindrical waveguide which makes it possible to

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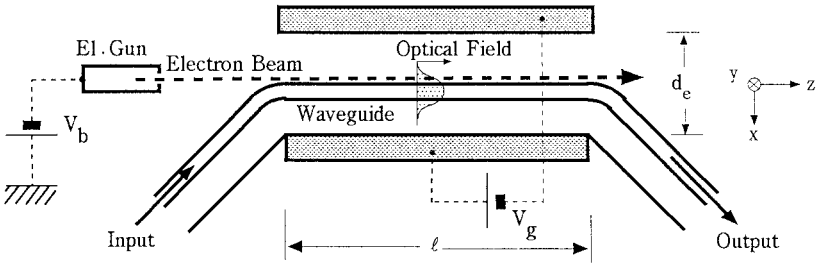

Fig. 1. Structure of a unidirectional optical amplifier.

fabricate devices in the nanometer size range. The electron beam is concentrated at the surface of the dielectric slab waveguide by the applied electric and not magnetic field.

Another feature of this paper is that the theoretical analysis is given in a quantum mechanical manner to provide a direct insight into both the amplification mechanism and the concept of the electron wave, which has not been given in the theoretical analyses of traveling wave tubes and Cherenkov masers.

Variation of the optical field is formulated in Section II based on classical field equations. Dynamics of the electron are formulated with the density matrix equation to examine the quantum statistical properties of the electron wave in Section III. Operating conditions for a given model are determined in Section IV. Numerical calculations of the amplifier gain are given in Section V. The gain coefficient based on the quantum mechanical treatment is compared in Section VI with that obtained in classical mechanics. Conclusions are given in Section VII.

II. Equation of the Optical Field

The structure of the proposed unidirectional optical amplifier is illustrated in Fig. 1. The device consists of a dielectric optical waveguide, an electron gun, and a pair of parallel electrodes, all in a vacuum. The optical beam propagates through the dielectric waveguide and partly penetrates the vacuum region in the form of an evanescent wave. The electron beam is emitted from the electron gun with an excitation voltage of V_b and propagates along the surface of the dielectric waveguide. An external dc voltage V_g is applied to the parallel electrodes to concentrate the electron beam on the surface of the waveguide. Then, the optical beam can be amplified by receiving energy from the electron beam.

Variations of the vector potential $\boldsymbol{A}$ of the optical field are given in terms of the current density $\boldsymbol{J}$ under the Lorentz condition as

$$\nabla^2 \boldsymbol{A} - \mu_o \varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu_o \boldsymbol{J}. \tag{1}$$

The current density is given in terms of velocity $\boldsymbol{v}$ of the electron, charge unit e, electron density N, electron mass m, and the vector potential $\boldsymbol{A}$ where

$$\boldsymbol{J} = eN\boldsymbol{v} = (e/m)N(\boldsymbol{P} - e\boldsymbol{A}) \tag{2}$$

and $\boldsymbol{J}, N$, and $\boldsymbol{P}$ should be evaluated in a quantum mechanical manner, as will be shown later.

We assume that the vector potential consists of forward and backward components, the amplitudes of which are denoted by $F(z)$ and $B(z)$ in the form of

$$\boldsymbol{A} = F(z)\boldsymbol{T}(x,y)e^{j(\omega t - \beta z)} + B(z)\boldsymbol{T}(x,y)e^{j(\omega t + \beta z)} + \text{c.c.} \tag{3}$$

where $\boldsymbol{T}(x,y)$ is the transverse field distribution function characterized by

$$(\nabla^2 + \mu_o \varepsilon \omega^2)\boldsymbol{T}(x,y)e^{\pm j\beta z} = 0 \tag{4}$$

and normalization in the form of

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\boldsymbol{T}(x,y)|^2 \, dx \, dy = 1. \tag{5}$$

Variations of amplitudes $F(z)$ and $B(z)$ are assumed to be slower than those of the optical phase $\exp(\pm j\beta z)$. Substitution of (3) into (1) and multiplication by $\boldsymbol{T}^*(x,y)e^{-j(\omega t - \beta z)}$ gives

$$\frac{\partial F}{\partial z}|\boldsymbol{T}(x,y)|^2 + \cdots = -j\frac{\mu_o e N}{2\beta m}\{\boldsymbol{P}\boldsymbol{T}^*(x,y)e^{-j(\omega t - \beta z)} - eF(z)|\boldsymbol{T}(x,y)|^2\}. \tag{6}$$

Variables related to the electron charge in the above equation should be substituted as expectation values obtained by the quantum mechanical treatment.

Multiplication of the electron density N and another quantum mechanical operator or a spatially varying function Q is given by taking the trace Tr together with the density matrix ρ [10], [11]

$$NQ \Rightarrow N_t \,\mathrm{Tr}\,\{\rho Q\} = N_t \sum_{\nu} \langle \nu | \rho Q | \nu \rangle = N_t \sum_{\nu} \int_V \varphi_\nu^*(\boldsymbol{r}) \rho Q \varphi_\nu(\boldsymbol{r}) \, d^3\boldsymbol{r} \tag{7}$$

where N_t shows the total electron density covering all possible energy levels, $|\nu\rangle$ denotes an energy level, and $\varphi_\nu(\boldsymbol{r})$ stands for the electron wave function given by

$$\varphi_\nu(\boldsymbol{r}) = \sqrt{\frac{1}{V}} e^{j\boldsymbol{k}_\nu \boldsymbol{r} + j\theta(\boldsymbol{r})} \tag{8}$$

where $\boldsymbol{k}_\nu$ is the electron wavenumber, $\theta(\boldsymbol{r})$ is the fluctuating component of the phase, and V is the volume of the electron given by spreading thickness h along x, width W along y (cross-sectional area $S = hW$), and length ℓ as

$$V = S\ell = hW\ell. \tag{9}$$

Here, ℓ is the length of the interaction region in Fig. 1.

Spatial integration in the transverse cross section changes (6) to

$$\frac{\partial F}{\partial z} = -j\frac{\mu_o e N_t}{2\beta m}\left[\int_S \mathrm{Tr}\{\rho \boldsymbol{P}\boldsymbol{T}^*(x,y)e^{j\beta z}\}\, d^2\boldsymbol{r}e^{-j\omega t} - e\int_S \mathrm{Tr}\{\rho\}|\boldsymbol{T}(x,y)|^2\, d^2\boldsymbol{r}F(z)\right] \quad (10)$$

where $\int_S d^2\boldsymbol{r}$ shows the two-dimensional integration over the cross-sectional area of the electron beam.

The trace operation Tr in this model is done only for excited energy levels of the electron beam. Therefore, the trace of the density matrix is not unity, $\mathrm{Tr}\{\rho\} \neq 1$.

III. Equation of the Density Matrix

The dynamic motion of the electrons is expressed by the next equation of the density matrix ρ

$$\frac{d\rho}{dt} = \frac{1}{j\hbar}[H,\rho] - \frac{1}{2}\{(\rho-\tilde{\rho})\Gamma + \Gamma(\rho-\tilde{\rho})\} + \Lambda. \quad (11)$$

The Hamiltonian H includes the interaction between the optical field and the electrons

$$H = H_o - \frac{e}{2m}(\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}\boldsymbol{P}) \quad (12)$$

where H_o is the principal Hamiltonian giving an eigen energy E_ν with an eigen state $|\nu\rangle$ of an electron in the interaction region

$$H_o|\nu\rangle = E_\nu|\nu\rangle. \quad (13)$$

$\tilde{\rho}, \Gamma$, and Λ in (11) are the thermal equilibrium distribution functions of the electron, the operator giving the electron relaxation, and the operator giving the electron flow, respectively [11]. These functions and operators are assumed to have only diagonal elements for the energy eigen state μ or ν

$$\langle\nu|\tilde{\rho}|\mu\rangle = \tilde{\rho}_\nu\delta_{\nu,\mu} \quad (14)$$
$$\langle\nu|\Lambda|\mu\rangle = \Lambda_\nu\delta_{\nu,\mu} \quad (15)$$
$$\langle\nu|\Gamma|\mu\rangle = 1/\tau_\nu\delta_{\nu,\mu}. \quad (16)$$

The electron wave function was defined in (8). When the phase fluctuations are not large ($|\nabla\theta| \ll |\boldsymbol{k}_\nu|$), the momentum operator has only diagonal elements in the form of

$$\langle\nu|\boldsymbol{P}|\mu\rangle = \int_V \varphi_\nu^*(\boldsymbol{r})(-j\hbar\nabla)\varphi_\mu(\boldsymbol{r})\, d^3\boldsymbol{r} = \hbar\boldsymbol{k}_\nu\delta_{\nu,\mu}. \quad (17)$$

The remarkable feature of this analysis is evaluation of the classical function $\boldsymbol{A}$ as an operator in quantum mechanical calculations because both $\boldsymbol{A}$ and the electron wave function $\varphi_b(\boldsymbol{r})$ are spatially varying functions. The following notations are introduced to indicate the relation between the optical field distribution and the electron wave function:

$$\langle\mu|\boldsymbol{T}(x,y)e^{-j\beta z}|\nu\rangle = \boldsymbol{T}_{\mu\nu}^{(-)} \quad (18)$$
$$\langle\mu|\boldsymbol{T}^*(x,y)e^{-j\beta z}|\nu\rangle = \boldsymbol{T}_{\mu\nu}^{*(-)} = (\boldsymbol{T}_{\nu\mu}^{(+)})^* \quad (19)$$
$$\langle\mu|\boldsymbol{T}(x,y)e^{j\beta z}|\nu\rangle = \boldsymbol{T}_{\mu\nu}^{(+)} \quad (20)$$
$$\langle\mu|\boldsymbol{T}^*(x,y)e^{j\beta z}|\nu\rangle = \boldsymbol{T}_{\mu\nu}^{*(+)} = (\boldsymbol{T}_{\nu\mu}^{(-)})^* \quad (21)$$

where the energy level ν should differ from μ because $\beta \neq 0$.

The off-diagonal elements of the vector potential $\boldsymbol{A}$ are written as

$$\langle\mu|\boldsymbol{A}|\nu\rangle = \boldsymbol{A}_{\mu\nu} = [F(z)\boldsymbol{T}_{\mu\nu}^{(-)} + B(z)\boldsymbol{T}_{\mu\nu}^{(+)}]e^{j\omega t} + \text{c.c.} \quad (22)$$

and

$$\boldsymbol{A}_{\nu\mu} = \boldsymbol{A}_{\mu\nu}^* \quad (23)$$

while its diagonal elements should vanish

$$\langle\nu|\boldsymbol{A}|\nu\rangle = 0. \quad (24)$$

The combined matrix elements in the interaction terms of (11) are

$$\langle\nu|(\boldsymbol{P}\boldsymbol{A}\rho + \boldsymbol{A}\boldsymbol{P}\rho - \rho\boldsymbol{P}\boldsymbol{A} - \rho\boldsymbol{A}\boldsymbol{P})|\nu\rangle = \hbar\sum_\mu(\boldsymbol{k}_\nu + \boldsymbol{k}_\mu)(\boldsymbol{A}_{\nu\mu}\rho_{\mu\nu} - \rho_{\nu\mu}\boldsymbol{A}_{\mu\nu}) \quad (25)$$

and

$$\langle\nu|(\boldsymbol{P}\boldsymbol{A}\rho + \boldsymbol{A}\boldsymbol{P}\rho - \rho\boldsymbol{P}\boldsymbol{A} - \rho\boldsymbol{A}\boldsymbol{P})|\mu\rangle = \hbar(\boldsymbol{k}_\mu + \boldsymbol{k}_\nu)\boldsymbol{A}_{\nu\mu}(\rho_{\mu\mu} - \rho_{\nu\nu}). \quad (26)$$

Equations for the diagonal and the off-diagonal elements of the density matrix are then given by

$$\frac{d\rho_{\nu\nu}}{dt} = j\frac{e}{2m}\sum_\mu(\boldsymbol{k}_\nu + \boldsymbol{k}_\nu)(\boldsymbol{A}_{\nu\mu}\rho_{\mu\nu} - \rho_{\nu\mu}\boldsymbol{A}_{\mu\nu}) - \frac{\rho_{\nu\nu} - \tilde{\rho}_\nu}{\tau_\nu} + \Lambda_\nu \quad (27)$$

$$\frac{d\rho_{\mu\nu}}{dt} = \left(j\omega_{\nu\mu} - \frac{1}{\tau}\right)\rho_{\mu\nu} + j\frac{e}{2m}(\boldsymbol{k}_\mu + \boldsymbol{k}_\nu)\boldsymbol{A}_{\mu\nu}(\rho_{\nu\nu} - \rho_{\mu\mu}) \quad (28)$$

where

$$\omega_{\nu\mu} = (E_\nu - E_\mu)/\hbar \quad (29)$$
$$1/\tau = (1/2)(1/\tau_\nu + 1/\tau_\mu). \quad (30)$$

These equations are quite similar in form to those in semiconductor lasers [11].

Assuming that $E_\nu > E_\mu$ and very slow variations of the diagonal elements $\rho_{\nu\nu}$ and $\rho_{\mu\mu}$, the off-diagonal elements of the density matrix are obtained as

$$\rho_{\mu\nu} = j\frac{e(\rho_{\nu\nu} - \rho_{\mu\mu})(\boldsymbol{k}_\nu + \boldsymbol{k}_\mu)}{2m[j(\omega - \omega_{\nu\mu}) + 1/\tau]} \times [\boldsymbol{T}_{\mu\nu}^{(-)}F(z) + \boldsymbol{T}_{\mu\nu}^{(+)}B(z)]e^{j\omega t} \quad (31)$$

and

$$\rho_{\nu\mu} = \rho_{\mu\nu}^*. \quad (32)$$

Meanwhile, the value of Tr{ } in the first term on the right-hand side of (10) becomes

$$\mathrm{Tr}\{\rho\boldsymbol{P}\boldsymbol{T}^*(x,y)e^{j\beta z}\} = \hbar\sum_{\nu,\mu}\rho_{\mu\nu}\boldsymbol{k}_\nu\boldsymbol{T}_{\nu\mu}^{*(+)} = \hbar\sum_{\nu,\mu}\rho_{\mu\nu}\boldsymbol{k}_\nu(\boldsymbol{T}_{\mu\nu}^{(-)})^*. \quad (33)$$

Since $\rho_{\mu\nu}$ is given as a complex function, the first term on the right-hand side of (10) is complex, while the second term is purely imaginary. Then, variations of the carried power are expressed as

$$\frac{d|F|^2}{dz} = g|F|^2 \quad (34)$$

with the gain coefficient g given by

$$g = \frac{\mu_o \hbar}{2\beta}\left(\frac{e}{m}\right)^2 N_t \,\mathrm{Re}\left\{\int_S \sum_{\nu>\mu}\sum_{\mu} \cdot \frac{(\rho_{\nu\nu}-\rho_{\mu\mu})\{(\boldsymbol{k}_\nu+\boldsymbol{k}_\mu)\boldsymbol{T}^{(-)}_{\mu\nu}\}\{\boldsymbol{k}_\nu(\boldsymbol{T}^{(-)}_{\mu\nu})^*\}}{j(\omega-\omega_{\nu\mu})+1/\tau}\, d^2\boldsymbol{r}\right\}. \tag{35}$$

The backward field component $B(z)$ is already dropped in (34), because the term $\boldsymbol{T}^{(+)}_{\mu\nu}$ should be 0 when $\boldsymbol{T}^{(-)}_{\mu\nu}$ is non zero. The condition of keeping the value of the term $\boldsymbol{T}^{(-)}_{\mu\nu} \neq 0$ corresponds to the wavenumber selection or the momentum conservation rule.

Gain is realized when the optical frequency ω coincides with the energy difference, $\omega \to \omega_{\nu\mu} = (E_\nu - E_\mu)/\hbar$. This relation expresses the energy conservation rule.

Variations of the diagonal elements of the density matrix become

$$\frac{d\rho_{\nu\nu}}{dt} = -\frac{1}{2}\left(\frac{e}{m}\right)^2 \cdot \sum_{\mu} \frac{(\rho_{\nu\nu}-\rho_{\mu\mu})|(\boldsymbol{k}_\nu+\boldsymbol{k}_\mu)\boldsymbol{T}^{(-)}_{\mu\nu}|^2|F(z)|^2}{\{(\omega-\omega_{\nu\mu})^2+1/\tau^2\}\tau} - \frac{\rho_{\nu\nu}-\tilde{\rho}_\nu}{\tau_\nu} + \Lambda_\nu. \tag{36}$$

A similar equation for $d\rho_{\mu\mu}/dt$ is also obtained.

IV. Operating Conditions

It is assumed, in this section, that the electrons propagate along z and the wavenumber components along x and y directions are negligibly small, such that

$$|\boldsymbol{k}_\nu| = k_\nu = k_{z\nu} \tag{37}$$

where k_ν is the absolute value of the wavenumber. Then only the z component of the optical field can contribute to amplification. The z component of $\boldsymbol{T}^{(-)}_{\mu\nu}$ becomes

$$T^{(-)}_{z\mu\nu} = \langle\mu|T_z(x,y)e^{-j\beta z}|\nu\rangle = \frac{1}{S}\int_S T_z(x,y)\, d^2\boldsymbol{r}\,\mathrm{sinc}\left[(k_\nu - k_\mu - \beta)\frac{\ell}{2}\right] \tag{38}$$

where $\mathrm{sinc}\,[\mathrm{X}] = \sin(X)/X$. The wavenumber selection rule is represented by this Sinc function with $X = 0$, that is,

$$k_\nu - k_\mu = \beta. \tag{39}$$

Now b is assigned as the energy level of the incident electron beam with an energy of

$$eV_b = E_b = \frac{\hbar^2 k_b^2}{2m} = \frac{mv_b^2}{2} \tag{40}$$

corresponding to the electron velocity v_b $(= \hbar k_b/m)$. Then, the current density of the electron beam is given by

$$J = e\langle N_b\rangle v_b = eN_t \sum_b v_b \rho_{bb}. \tag{41}$$

We need to take other energy levels c and a as final levels in the upper and the lower sides to account for optical absorption and emission, respectively, as shown in Fig. 2. Initial settings of these levels are $\rho_{aa} = \rho_{cc} = 0$. The electron can undergo

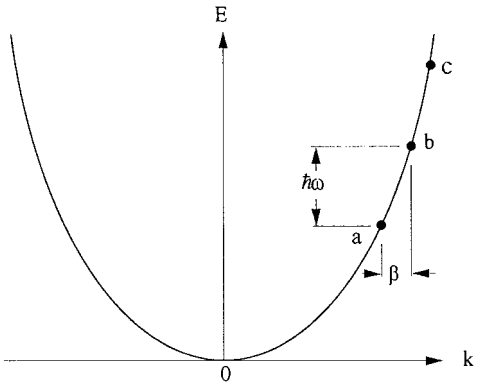

Fig. 2. Energy diagram of the electron transition. The electron can transit from upper level b to lower level a when the energy conservation rule of $E_b - E_a = \hbar\omega$ and the wavenumber selection rule of $k_b - k_a = \beta$ are obeyed, resulting in optical amplification. However, another transition from level b to upper level c is also possible as optical absorption. A large interaction length ℓ and precise adjustment of the exciting voltage V_b is required to tune to the optical amplification.

a transition from level b to the lower level a when the energy conservation rule $E_b - E_a = \hbar\omega$ and the wavenumber selection rule $k_b - k_a = \beta$ are obeyed, resulting in optical amplification. On the other hand, another transition from level b to the upper level c is also possible, which corresponds to optical absorption.

If electron scattering is small enough, the relaxation time τ could be counted as the time taken in passing through the interaction length ℓ as

$$\tau = \ell/v_b. \tag{42}$$

Based on the above assumptions and notations, the gain g of the optical beam is simply rewritten as

$$g = \sqrt{\frac{\mu_o}{\varepsilon_o}}\frac{eJ\ell}{n_{\mathrm{eff}}\hbar\omega}\xi D \tag{43}$$

where n_{eff} is the effective refractive index which indicates slowing down of the optical beam in the form of

$$\beta = n_{\mathrm{eff}}\omega/c = 2\pi n_{\mathrm{eff}}/\lambda \tag{44}$$

where λ is the optical wavelength, ξ in (43) is the coupling coefficient between the optical field and the electron beam, defined as

$$\xi = \xi_x\xi_y = \int_S \left|\frac{1}{S}\int_S T_z(x,y)\, d^2\boldsymbol{r}\right|^2 d^2\boldsymbol{r} \tag{45}$$

with ξ_x and ξ_y representing its components along x and y directions, respectively. D is a dispersion function determining the difference between the optical emission $(D>0)$ and the optical absorption $(D<0)$ as

$$D = \mathrm{sinc}^2[(k_b - k_a - \beta)\ell/2] - \mathrm{sinc}^2[(k_c - k_b - \beta)\ell/2] = \mathrm{sinc}^2\left[\left\{\frac{\sqrt{2m}}{\hbar}(\sqrt{E_b} - \sqrt{E_b - \hbar\omega}) - \frac{n_{\mathrm{eff}}\omega}{c}\right\}\frac{\ell}{2}\right] - \mathrm{Sinc}^2\left[\left\{\frac{\sqrt{2m}}{\hbar}(\sqrt{E_b + \hbar\omega} - \sqrt{E_b}) - \frac{n_{\mathrm{eff}}\omega}{c}\right\}\frac{\ell}{2}\right]. \tag{46}$$

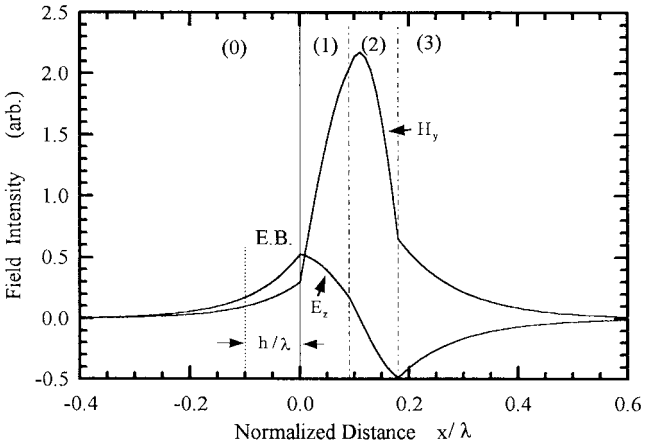

Fig. 3. Calculated example of the optical field distribution. Three-layer structure of the waveguide with $n_1 = 2.8, n_2 = 3.5$, and $n_3 = 1.4$ is assumed. E.B. is the region where the electron beam propagates, whose range is assumed as $h/\lambda = 0.1$. Thicknesses of the layers are examined to give the maximum value of ξ_x for the E_z component of TM mode under assumed setting of the refractive index $n_{\text{eff}} \approx 2.0$. The maximum value of $\xi_x = 0.13$ was obtained.

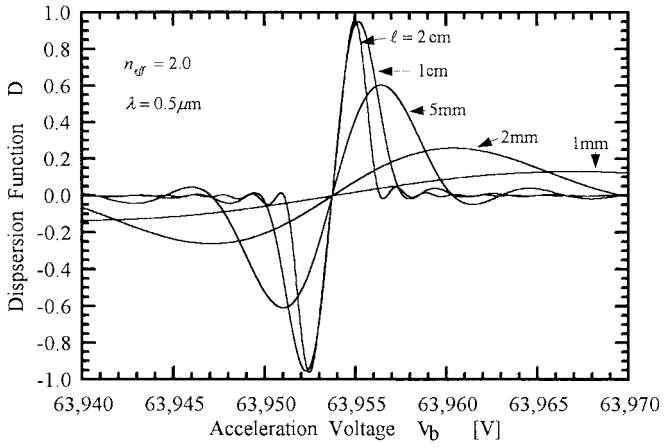

Fig. 4. Variation of the dispersion function with the excitation voltage. State of the optical absorption and the optical emission varies sensitively with V_b when the interacting length ℓ is long enough.

In achieving optical amplification, a long interaction length ℓ and precise adjustment of the exciting voltage V_b are required.

The accelerating voltage V_b and the electron velocity v_b are determined by the condition that the value inside the brackets in (46) becomes zero such that

$$V_b = \frac{E_b}{e} \approx \frac{m}{2}\left(\frac{c}{n_{\text{eff}}}\right)^2 \tag{47}$$

and

$$v_b = \frac{c}{n_{\text{eff}}}. \tag{48}$$

Equations (47) and (48) are obtained when $E_b \gg \hbar\omega$, which does not give the detailed difference between optical emission and absorption. An exact calculation of (46) is necessary in determination of the dispersion relations of both the emission and the absorption, as will be shown in the next section.

V. Numerical Example

The dielectric waveguide is assumed to consist of a three-layer structure with refractive indices $n_1 = 2.8, n_2 = 3.5$, and $n_3 = 1.4$. The optical field distribution is analyzed by the conventional F-matrix method. The spreading range of the electron beam is indicated by E.B. and is assumed to be $h/\lambda = 0.1$ in this example, where h is the spreading thickness of the electron beam along x. Thicknesses of the waveguide layers are adjusted to get the maximum value of the coupling coefficient ξ_x for the E_z component of the TM mode under an assumed setting of the refractive index $n_{\text{eff}} \approx 2.0$. Calculated field distributions H_z and E_z are shown in Fig. 3. The region denoted by (0) is vacuum space, while regions denoted by (1)–(3) are made of the dielectric materials. The resulting thicknesses of the layers are $d_1/\lambda = 0.09, d_2/\lambda = 0.09$, and $d_3/\lambda \geq 0.5$. The coupling coefficient is $\xi_x = 0.13$.

The voltage V_g applied to the parallel electrodes is determined to confine the electron beam in the narrow region of the E.B. in Fig. 3. The electrons may have the kinetic energy kT for transverse motion along x, where T is the cathode temperature in the electron gun. Then, the potential should be larger than the kinetic energy kT at the boundary $x = -h$. The kinetic energy is $kT = 0.17$ eV at $T = 2000$ K. When the thickness h of the E.B. region and the gap distance d_e of the parallel electrodes are assumed to be $h = 50$ nm with $\lambda = 0.5\,\mu$m and $d_e = 500\,\mu$m, respectively, $V_g \geq 1700$ V is required.

An example of the calculated dispersion function D is shown in Fig. 4 against the accelerating voltage V_b at $\lambda = 0.5\,\mu$m. As can be seen, the optical emission and the absorption are exchanged if the acceleration voltage is varied by only a few volts around its mean value of 63 954 V. A rather long interaction length of $\ell = 1$ cm is required to obtain $D \approx 1$. However, much longer distances are not suitable because V_b is required to be set very accurately.

By supposing $\lambda = 0.5\,\mu\text{m}, n_{\text{eff}} = 2.0, \ell = 5$ mm, $\xi_x = 0.13, \xi_y = 0.5, D = 0.607$, and $J = 10^5$ A/m^2, the gain coefficient becomes $g = 11.85\,\text{cm}^{-1}$. This value is sufficiently large, giving the amplification factor of $\exp[g\ell] = \exp[88.275] = 3925$. The emission current I is $I = JhW = 2.5\,\mu$A when the beam width along y is $W = 500\,\mu$m.

VI. Comparison with Classical Treatment

Amplification of the EM wave by the electron beam has been also analyzed by the classical treatment of coupled mode theory [7]. Two types of *fast* and *slow* electron beams are introduced with corresponding plasma frequencies of the electron motion. Both amplification and absorption are derived in terms of the dispersion relation of the EM wave on the plasma frequencies. The introduction of the fast and the slow electron beams may correspond to the energy levels b and a for optical emission and to the levels c and b for optical absorption.

One remarkable difference between the classical treatment and the quantum treatment is that the latter includes the Planck's constant $\hbar$ while the former does not. In this regard, (43) and (46) are rewritten to remove $\hbar$ under the condition

$E_b \gg \hbar\omega$. The first variable in (46) is rewritten as

$$\left\{\frac{\sqrt{2m}}{\hbar}(\sqrt{E_b}-\sqrt{E_b-\hbar\omega})-\beta\right\}\frac{\ell}{2} \approx \left\{\frac{\omega}{v_b}\left(1+\frac{\hbar\omega}{2E_b}\right)-\beta\right\}\frac{\ell}{2}. \quad (49)$$

Here, we put

$$\left(\beta-\frac{\omega}{v_b}\right)\ell \equiv U. \quad (50)$$

The square value of the sinc function is also approximated for small variations of $\delta = \hbar\omega^2\ell/4v_bE_b$ as

$$\frac{\sin^2(U/2-\delta)}{(U/2-\delta)^2} \approx \frac{\sin^2(U/2)}{(U/2)^2}+\frac{4}{U}\left\{\frac{\sin^2(U/2)}{(U/2)^2}-\frac{\sin U}{U}\right\}\delta. \quad (51)$$

The other sinc function with a variable of $(\sqrt{2m}/\hbar)(\sqrt{E_b+\hbar\omega}-\sqrt{E_b})-\beta\}\ell/2$ is similarly approximated. Then, the part including $\hbar\omega$ in the gain coefficient expressed by (43) becomes

$$\frac{D}{\hbar\omega} \approx \frac{2\omega\tau}{E_b}f(U) \quad (52)$$

where τ is counted to be the propagation time defined in (42), and $f(U)$ is the dispersion function obtained by Yariv and Shih as [7]

$$f(U) \equiv \frac{1}{U}\left\{\frac{\sin^2(U/2)}{(U/2)^2}-\frac{\sin U}{U}\right\}. \quad (53)$$

Then, the gain coefficient reduces to

$$g \approx \sqrt{\frac{\mu_o}{\varepsilon_o}}\frac{4eJ\omega\tau^3}{n_{\text{eff}}m\ell}\xi f(U). \quad (54)$$

The gain derived by Yariv and Shih in [7] corresponds to the term $g \times \ell$. Trivial differences between their equation and our results originated from parameters of different geometrical waveguides and the fact that they took into account relativistic effects.

Our density matrix formalism is an analytical method which includes quantum statistical properties and is advantageous to the analysis based on classical mechanics, as clarified in the following. Since the energy levels contributing to amplification are explicitly represented, we can choose suitable materials to fabricate the device. The analysis is applicable to the case where E_b is comparable to $\hbar\omega$ such as the electron emission in a solid material. Effects of electron scattering can be investigated by returning to the original equation of the gain coefficient g in (35) from (43) without the assumption of (42). Saturation and other nonlinear effects can be analyzed by applying the perturbation expansion to the dynamic equation of the density matrix of (11) similar to what has been done in the analysis of semiconductor lasers [11]. The amplifier noise can be also evaluated by adding quantum mechanical fluctuations to (11).

Detailed analysis of the operational characteristics mentioned above are expected after experimental realization of the proposed amplifier.

VII. Conclusions

The possibility of achieving a unidirectional optical amplifier was discussed. The density matrix formalism was applied to analyze the interaction between the electron beam and the optical field in a quantum mechanical manner. The amplification mechanism was shown to be the transfer of energy from the electron beam to the optical field, conserving both the energy and the momentum. The condition for achieving this amplification is to increase the momentum of the optical field, that is, to reduce the phase velocity. Utilization of the evanescent wave propagating along the dielectric waveguide was proposed in reduction of the phase velocity. Sufficient amplification of the optical field was confirmed through numerical calculations.

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