# Simultaneous frequency and time-domain approximation method for discrete-time filters

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# A Simultaneous Frequency and Time-Domain Approximation Method for Discrete-Time Filters

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*Abstract*—A simultaneous frequency- and time-domain approximation method for discrete-time filters is proposed in this paper. In the proposed method, transfer function coefficients are divided into two subsets, $X_1$ and $X_2$, which are employed for optimizing a time response and a frequency response, respectively. Frequency and time responses are optimized through the iterative Chebyshev approximation method and a method of solving linear equations, respectively. At the $r$th iteration step, the maximum frequency response error, which appeared at the $(r-1)$th step, is minimized, and $X_2^{(r-1)}$ becomes $X_2^{(r)}$. $X_1^{(r)}$ is obtained from linear equations including $X_2^{(r)}$ as a constant. The frequency response at the $r$th step is evaluated using the above obtained $X_1^{(r)}$ and $X_2^{(r)}$. This means the optimum time response is always guaranteed in the frequency-response approximation procedure.

A design example of a symmetrical impulse response shows the new approach is more efficient than conventional methods from the filter order reduction viewpoint.

## I. Introduction

DISCRETE-TIME filters, such as digital, CCD, and switched-capacitor filters, have become very important for communication systems and other uses, due to the full integration possibility. Filters employed in communication systems, which transmit data or image signals, are usually required to satisfy specifications in both frequency and time domains. Therefore, simultaneous frequency- and time-domain approximation methods are inherently necessary design techniques.

Existing approaches to the above simultaneous approximation are mainly summarized as follows:

(1) Closed-form transfer functions are employed, which provide the optimum time response for arbitrary frequency response. A frequency response is approximated through iterative methods [1], [2]. The obtainable time responses are rather limited to, for instance, waveforms zero crossing at equally spaced sampling points.

(2) Specific transfer functions are employed, which can optimize one filter response and which do not affect the other filter response. These transfer functions include all-pass functions and linear phase finite impulse response (FIR) filters [3], [4]. Some constraints exist on pole–zero locations in these transfer functions and prevent sufficient reductions in filter orders.

(3) The coefficient subset approximating stopband attenuation is uniquely obtained from a closed-form transfer function having the rest coefficient subset as a constant [5], [6]. A time response is approximated through iterative methods. Attainable filter responses are, however, restricted to low-pass filters having equal-ripple stopband attenuation.

In the above approaches, the attainable filter responses are rather limited and some constraints on pole–zero locations exist. Furthermore, linear phase FIR filters and infinite impulse response (IIR) filters having a flat group delay response, in some sense, are practically employed for systems transmitting data and processing image signals [7]–[9]. In this case, however, it is difficult to determine the tolerance for group delay distortions, which guarantees the minimum time response deviation.

The approach proposed in this paper basically employs the first approach and extends the attainable filter responses by employing a method of solving linear equations in a time domain. As is well known, the transfer function coefficients are linearly related to the impulse response in discrete-time filters. Many approximation techniques in a time domain, based on the above linear relations, have been proposed [10]–[13]. They are, however, directed toward only time response approximation, and no simultaneous approximation methods have been reported in this direction.

In the proposed method, the transfer function coefficients are divided into two subsets $X_1$ and $X_2$ which are employed for approximating time and frequency responses, respectively. A frequency response is optimized through the iterative Chebyshev approximation [14] using the coefficient subset $X_2$. $X_1$ providing an optimum time response is obtained through solving linear equations which include $X_2$ as a constant. The frequency response is evaluated using the above obtained $X_2$ and $X_1$. This means that the optimum time response is always guaranteed in the frequency response approximation procedure. This method allows using two kinds of error criteria. They are exact interpolation, where no errors are caused, and the mean square error.

Section II describes the time response approximation through solving linear equations. A simultaneous frequency- and time-domain approximation algorithm is provided in

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Section III following a flowchart. Finally, a design example of a symmetrical impulse response is illustrated in Section IV. Comparison between the proposed and conventional methods are discussed, from the circuit complexity reduction viewpoint.

## II. Time Response Approximation by Solving Linear Equations

Time response targets are mainly classified into the following two categories.

1) Desired time response values are specified.

2) Desired time response figures are specified.

The first category includes, for instance, the Nyquist waveform zero crossing at equally spaced sampling points. Symmetrical impulse responses and minimum moment impulse responses are included in the second category.

In the proposed method, a transfer function $H(z)$ is expressed as

$$H(z)=\frac{P(z)}{Q(z)}G(z), \qquad z=\exp(j2\pi f/fs) \tag{1}$$

where $f_s$ is a sampling frequency. $P(z)$ and $Q(z)$ are polynomials in $z^{-1}$ and $G(z)$ is a rational function in $z^{-1}$. They are further expressed as

$$P(z)=\sum_{n=0}^{N_p-1} p_n z^{-n} \tag{2a}$$

$$Q(z)=\sum_{n=0}^{N_q-1} q_n z^{-n}, \qquad q_0=1 \tag{2b}$$

$$\frac{1}{Q(z)}=\sum_{n=0}^{\infty} \bar{q}_n z^{-n} \tag{2c}$$

$$G(z)=\frac{\sum_{n=0}^{N_a-1} a_n z^{-n}}{\sum_{n=0}^{N_b-1} b_n z^{-n}}=\sum_{n=0}^{\infty} g_n z^{-n}, \qquad b_0=1. \tag{2d}$$

$P(z)/Q(z)$ and $G(z)$ are used for approximating a time response and a frequency response, respectively. In other words, the coefficient subsets $X_1$ and $X_2$ consist of the coefficients of $P(z)/Q(z)$ and $G(z)$, respectively.

### A. Time Response Values Specified

Letting $d_n$, $0\leqslant n\leqslant N_d-1$ be a desired time response, time response approximation can be generally formulated so as to minimize

$$e_p=\left\{\sum_{n=0}^{N_d-1}|h_n-d_n|^p\right\}^{1/p} \tag{3}$$

where $h_n$ is an impulse response, that is the inverse $z$-transform of $H(z)$,

$$H(z)=\sum_{n=0}^{\infty} h_n z^{-n}. \tag{4}$$

By using $p_n$, $\bar{q}_n$, and $g_n$, $h_n$ is expressed as

$$h_n=p_n*\bar{q}_n*g_n \tag{5}$$

where the operation designated by the symbol $*$ means the convolution sum. From (3) and (5),

$$e_p=\left\{\sum_{n=0}^{N_d-1}|p_n*\bar{q}_n*g_n-d_n|^p\right\}^{1/p}. \tag{6}$$

In (6), $g_n$ is assumed to be fixed as will be described in the next section. The error evaluation $e_p$ is usually formulated as a high-order equation of $p_n$ and $q_n$. However, it is possible to express $e_p$ as a linear equation of $p_n$ and $q_n$ by selecting appropriate $N_p$, $N_q$, and $p$ values.

<u>$N_d=N_p+N_q-1$:</u>

In this case, the approximation error $e_p$ becomes exactly zero at specified sampling points by using the coefficient subset $X_1$ which is obtained through solving

$$p_n*\bar{q}_n*g_n-d_n=0, \qquad 0\leqslant n\leqslant N_d-1. \tag{7}$$

Equation (7) can be rewritten using $q_n$ instead of $\bar{q}_n$, as follows:

$$\sum_{m=0}^{n_1} p_m g_{n-m}-\sum_{m=1}^{n_2} q_m d_{n-m}=d_n, \qquad 0\leqslant n\leqslant N_d-1 \tag{8}$$

$$n_1=\min\{N_p-1,n\} \tag{9a}$$

$$n_2=\min\{N_q-1,n\}. \tag{9b}$$

Equation (8) means $N_d$-dimensional linear equations of $p_m$ and $q_m$. Thus when the number of the specified sampling points $N_d$ is equal to the degrees of freedom in $X_1$, that is $N_p+N_q-1$, time-response approximation can be carried out through solving linear equations and no approximation errors are caused.

<u>$N_d>N_p+N_q-1$:</u>

When $N_d$ is larger than the degrees of freedom in $X_1$, error evaluation $e_2$ is required to apply the method of solving linear equations. Since minimizing $e_2$ is equivalently carried out by the least mean square approximation, the error evaluation can be replaced by

$$E_1=\sum_{n=0}^{N_d-1}(p_n*\bar{q}_n*g_n-d_n)^2. \tag{10}$$

Equation (10) is rewritten as

$$E_1=\sum_{n=0}^{N_d-1}\{\bar{q}_n*(p_n*g_n-q_n*d_n)\}^2 \tag{11a}$$

$$=\sum_{n=0}^{N_d-1}\left\{\sum_{l=0}^{n-m}\bar{q}_l\left(\sum_{m=0}^{n_{1l}} p_m g_{n-l-m}-\sum_{m=0}^{n_{2l}} q_m d_{n-l-m}\right)\right\}^2 \tag{11b}$$

$$n_{1l}=\min\{N_p-1,n-l\} \tag{12a}$$

$$n_{2l}=\min\{N_q-1,n-l\}. \tag{12b}$$

Equation (11) is a set of high-order equations of $p_m$ and $q_m$. These equations, however, can be solved as linear equations of $p_m$ and $q_m$ through an iterative method [11]. In such a method, the values of $\bar{q}_n$ in (11) have the results determined in the previous iteration step, and are fixed in solving (11). On the other hand, the proposed simultaneous approximation method employs an iterative approach in a frequency domain, as described in the next section. Therefore, the iterative procedure required in solving (11) can be combined with the frequency-domain approximation.

Optimum $p_m$ and $q_m$ values, in the least mean square sense, are obtained by solving

$$\frac{\partial E_1}{\partial p_m}=0, \qquad m=0,1,\cdots,N_p-1 \tag{13a}$$

$$\frac{\partial E_1}{\partial q_m}=0, \qquad m=1,2,\cdots,N_q-1. \tag{13b}$$

From (11) and (13), the linear equations can be expressed as

$$\sum_{n=0}^{N_d-1}\left\{\sum_{l=0}^{n-m}\bar{q}_l\cdot g_{n-l-m}\left(\sum_{m=0}^{n_{1l}}p_m g_{n-l-m}-\sum_{m=0}^{n_{2l}}q_m d_{n-l-m}\right)\right\}=0 \tag{14a}$$

$$\sum_{n=0}^{N_d-1}\left\{\sum_{l=0}^{n-m}\bar{q}_l\cdot d_{n-l-m}\left(\sum_{m=0}^{n_{1l}}p_m g_{n-l-m}-\sum_{m=0}^{n_{2l}}q_m d_{n-l-m}\right)\right\}=0 \tag{14b}$$

where $\bar{q}_l$ have fixed values.

By letting $\bar{q}_l$ be

$$\bar{q}_0=1 \tag{15a}$$

$$\bar{q}_l=0, \qquad 1<l. \tag{15b}$$

Equation (14) become exact linear equations for $p_m$ and $q_m$, as shown in (16), and no iterative procedure is required [10].

$$\sum_{n=0}^{N_d-1} g_{n-m}\left(\sum_{m=0}^{n_1}p_m g_{n-m}-\sum_{m=0}^{n_2}q_m d_{n-m}\right)=0 \tag{16a}$$

$$\sum_{n=0}^{N_d-1} d_{n-m}\left(\sum_{m=0}^{n_1}p_m g_{n-m}-\sum_{m=0}^{n_2}q_m d_{n-m}\right)=0. \tag{16b}$$

In this case, however, $Q(z)$ becomes a weighting function for impulse response error $\Delta h_n$. The mean square error to be minimized through solving (16) can be expressed as

$$E_1^*=\sum_{n=0}^{N_d-1}\left(\sum_{m=0}^{n_2}q_m\Delta h_{n-m}\right)^2 \tag{17}$$

where

$$d_n=h_n+\Delta h_n. \tag{18}$$

A relation between $E_1$ and $E_1^*$ depends on desired responses in a time domain, and has been somewhat discussed [10].

In the above description, the desired time response $d_n$ is continuously given on the sampling points from $n=0$ to $N_d-1$. In the linear equations given by (8) and (14), the denominator coefficient $q_n$ formulates the convolution sum with $d_n$. Therefore, when $d_n$ is specified on discontinuous sampling points, (8) and (14) do not become linear equations for $q_n$.

### *B. Time Response Figure Specified*

In this category, desired time response values are not given. For simplicity, symmetrical impulse responses are taken as desired figures for description.

*Numerator Coefficients:*

When $X_1$ includes only the $P(z)$ coefficients, the impulse response $h_n$ can be expressed as

$$h_n=\sum_{m=0}^{n_1}p_m g_{n-m}. \tag{19}$$

Letting $K$ be the sampling point corresponding to the average delay time, that is the waveform center, the symmetrical impulse response condition is expressed as

$$h_{K+n}=h_{K-n}, \qquad n\in\Omega \tag{20}$$

where $\Omega$ is a set of sampling points at which the symmetrical condition must be satisfied. From (19), (20) becomes

$$\sum_{m=0}^{n_{1+}}p_m g_{K+n-m}=\sum_{m=0}^{n_{1-}}p_m g_{K-n-m}, \qquad n\in\Omega \tag{21}$$

$$n_{1+}=\min\{N_p-1,K+n\} \tag{22a}$$

$$n_{1-}=\min\{N_p-1,K-n\}. \tag{22b}$$

When $N_d$, which is the number of elements in the set $\Omega$, is equal to $N_p$, an exact symmetrical impulse response at the specified sampling points can be obtained by solving the linear equations given by (21) and (22).

On the other hand, when $N_d$ is larger than $N_p$, the least mean square approximation is required, as previously mentioned in the first category. The mean square error is expressed as

$$E_2=\sum_{n\in\Omega}\left(\sum_{m=0}^{n_{1+}}p_m g_{K+n-m}-\sum_{m=0}^{n_{1-}}p_m g_{K-n-m}\right)^2. \tag{23}$$

The optimum solution for $p_m$ is obtained by solving

$$\frac{\partial E_2}{\partial p_m}=0, \qquad m=0,1,\cdots,N_p-1. \tag{24}$$

Equation (24) is rewritten using (23), as follows:

$$\sum_{n\in\Omega}(g_{K+n-m}-g_{K-n-m})\left(\sum_{m=0}^{n_{1+}}p_m g_{K+n-m}-\sum_{m=0}^{n_{1-}}p_m g_{K-n-m}\right)=0, \qquad m=0,1,\cdots,N_p-1. \tag{25}$$

Apparently, (25) is a set of linear equations of $p_m$. Since the impulse response $h_n$ is expressed as the convolution sum of $p_m$ with $g_n$, as shown in (19), (23) has no weighting function for $\Delta h_n$ evaluation.

*Numerator and Denominator Coefficients:*

When the denominator $Q(z)$ is employed, the impulse response is not expressed as a linear combination of the denominator coefficients $q_n$. Furthermore, the desired time response values are not specified, and substituting $q_n$ for $\bar{q}_n$, as in (8) and (11), is impossible.

### C. Specified Sampling Point Range

In the proposed method, the time response is approximated on the specified $N_d$ sampling points. Time response samples on other sampling points cannot be controlled. Therefore, $N_d$ must be carefully determined so as to cover a sufficient time axis range. Detailed discussions on how to determine $N_d$ closely depend on actual applications.

### D. Stability

$G(z)$ can be optimized with arbitrary structure in the iterative approximation method [14]. Pole locations are easily observed and can be controlled. On the other hand, $Q(z)$ structure essentially has a direct form in the time response approximation, and the pole locations are not directly observed. Therefore, the stability of $Q(z)$ is not assured in the proposed method as in the conventional approaches [10]–[13].

## III. Simultaneous Approximation Algorithm

In the proposed algorithm, time and frequency responses are approximated through solving linear equations and iterative methods, respectively. This section describes how to combine both approximation processes.

A flowchart for the proposed algorithm is shown in Fig. 1. Details for each block are described in the following.

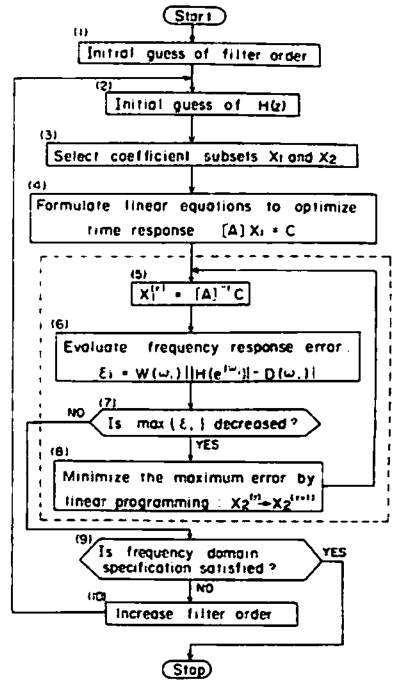


Fig. 1. Flowchart for simultaneous frequency- and time-domain approximation method.

### (1) Initial Guess of Filter Order

The optimum filter order allocation into a numerator and a denominator, by which the minimum filter order can be achieved as a whole, is a very important design problem. It is, however, generally difficult to obtain a unified method to give the optimum allocation for a wide range of filter responses. Therefore, design charts, mainly based on experience laws, must be prepared for actual use.

### (2) Initial Guess of H(z)

The optimum solution for any iterative method is highly dependent on the initial guess. Therefore, it is important to determine the initial guess as being near the optimum solution.

*Time Response Values Specified:*

In order to identify a transfer function using frequency responses, two kinds of responses are required, except for the minimum phase condition. Therefore, when an amplitude response and a time response are specified, phase calculation is required at first. On the contrary, a time response can uniquely identify a transfer function. For this reason, when desired time response values are given, it is computationally more efficient to determine the initial guess through time response approximation. The conventional approximation methods in a time domain [10]–[13] can be directly applied to this initial guess calculation.

*Time Response Figures Specified:*

Basically speaking, it is impossible to calculate the initial guess following the time response approximation in this case. However, there are several cases where the corresponding ideal time responses can be obtained. For example, a symmetrical time response is obtained assuming a linear phase response. The Gaussian or the raised cosine waveforms can be used as the ideal responses for the minimum moment waveform. Furthermore, in the case of Nyquist filters, the inverse $z$-transform of frequency responses with amplitude interpolated using the raised cosine function in the transition band and with a linear phase response can be utilized for the initial guess calculation.

### (3) Select Coefficient Subsets $X_1$ and $X_2$

Coefficient subsets $X_1$ and $X_2$ selections are dependent on the initial guess for the transfer function $H(z)$. When the conventional time response approximation methods [10]–[13] are employed, the resulting transfer function has a direct form. Therefore, it must be divided into the form $(P(z)/Q(z))G(z)$ as given by (1). In other words, it is necessary to select poles and zeros to be included in $X_1$ and $X_2$ from the initial direct form transfer function. The pole and zero selections are carried out, taking their contributions to filter responses into account. For example, zeros located in the stopband realize stopband attenuation. Therefore, they are selected for the $X_2$ elements. Zeros located in the passband mainly contribute to time response optimization, and are included in $X_1$. Poles are usually located in the passband, and are classified into two groups. One group mostly contributes to shaping an amplitude response, and is included in $X_2$. The other group contributes to time response optimization and becomes the $X_1$ elements.

### (4) Formulate Linear Equations to Optimize Time Response

After $X_1$ and $X_2$ have been selected, linear equations utilized for approximating a time response can be uniquely formulated. One example is presented here. Letting $Q(z)$ be unity, an impulse response $h_n$ is expressed as

$$h_n = \sum_{m=0}^{n_1} p_m g_{n-m}. \tag{26}$$

When desired time response values are given and $N_d$ is equal to $N_p$, the linear equations become

$$\begin{pmatrix} g_0 & & & 0 \\ g_1 & g_0 & & \\ g_2 & g_1 & g_0 & \\ \vdots & \vdots & \ddots & \\ g_{N_p-1} & g_{N_p-2} & \cdots & g_0 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_{N_p-1} \end{pmatrix} = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{N_d-1} \end{pmatrix}. \tag{27}$$

The matrix $[A]$, the vectors $X_1$ and $C$ in Steps (4) and (5) are expressed as

$$[A] = \begin{pmatrix} g_0 & & & 0 \\ g_1 & g_0 & & \\ g_2 & g_1 & g_0 & \\ \vdots & \vdots & \ddots & \\ g_{N_p-1} & g_{N_p-2} & \cdots & g_0 \end{pmatrix} \tag{28a}$$

$$X_1 = (p_0 \quad p_1 \quad p_2 \quad \cdots \quad p_{N_p-1})' \tag{28b}$$

$$C = (d_0 \quad d_1 \quad d_2 \quad \cdots \quad d_{N_d-1})'. \tag{28c}$$

The coefficient subset $X_2$ consists of the $G(z)$ coefficients $a_n$ and $b_n$. Since the time response $g_n$ can be obtained using $a_n$ and $b_n$, $X_2$ is equivalently included in $g_n$.

*(5)–(8) Iterative Chebyshev Approximation*

In Steps (5)–(8) enclosed with a dashed line, a frequency response is optimized through the iterative Chebyshev approximation method [14]. A time response is also approximated through solving linear equations in this procedure. In Step (5), the linear equations include $X_2^{(r)}$ optimized in Step (8) as a constant. $X_1^{(r)}$ is, therefore, uniquely determined for $X_2^{(r)}$. The amplitude response $|H(e^{j\omega_i})|$ is calculated using both $X_1^{(r)}$ and $X_2^{(r)}$ in Step (6). Therefore, frequency response evaluation automatically includes the optimum time response. $W(\omega_i)$ in Step (6) is a weighting function for error evaluation. The iteration procedure is finished when the maximum value for $\epsilon_i$ does not decrease from that obtained in the previous iteration step in Step (7). On the other hand, when the maximum value decreases, the above procedure is further repeated, based on the possibility of attaining maximum error reduction. The iterative Chebyshev approximation is carried out by employing a local linear programming technique at each iteration step. The nonlinear function of $X_2^{(r)}$ is approximately expressed with a linear function using the first-order differential coefficients for $X_2^{(r)}$. A further improved coefficient subset $X_2^{(r+1)}$ is obtained in Step (8). The same operations as those described above are repeated until the frequency response satisfies the given specification.

TABLE I
SPECIFICATIONS AND DESIGN PARAMETERS

| | |
|---|---|
| Sampling frequency | 400 Hz |
| Passband | 0~49 Hz |
| Stopband | 59~200 Hz |
| Desired time response | Partially symmetrical impulse response |
| Filter order allocations | 20/4, 16/8, 12/12 |
| Average delay time | 12T, 16T, 20T, 24T (T = 1/400 Sec) |

If the specification is not satisfied after the iterative Chebyshev approximation with the initial filter order, then the filter order is increased and the operations from Steps (2)–(9) are repeated.

## IV. Design Examples

### *A. Specifications and Design Parameters*

Table I shows specifications and design parameters. The frequency values do not make sense in actual applications, but ratios among them specify the frequency response. A symmetrical impulse response is taken as a desired time response. The number of specified sampling points is equal to the number of the $X_1$ elements. Degrees of freedom exist for selecting filter order allocations and average delay time. In this case, the average delay time means a sampling point at which an impulse response has the maximum value. Several values are assigned to these parameters, and the optimum result having good performances is selected.

### *B. Initial Guess of $X_2$*

The ideal frequency response having an amplitude response shown in Fig. 2(a) with linear phase is used for the initial guess calculation. In this figure, $f_p$ and $f_s$ are taken as 45 and 55 Hz, respectively. The initial guess is approximated through the Padé approximation in a time domain, taking the corresponding impulse response shown in Fig. 2(b) as a target. The exact symmetrical waveform condition is imposed on the samples designated by the symbol * in Fig. 2(b), which correspond to the peaks and valleys in the ringing.

*Select Coefficient Subsets $X_1$ and $X_2$:*

The numerator coefficients, corresponding to two zeros which appear in the passband in the initial guess, are selected as the $X_1$ elements. The remaining numerator and denominator coefficients are included in $X_2$.

### *C. Filter Responses Optimized*

Among the design parameters, 12/12th-order allocation and 20T average delay time provide the smallest passband ripple and the highest stopband attenuation. In the case of 12/12th-order, however, a pole and zero pair, which mostly cancelled each other, appeared in the passband at the iterative approximation procedure. Therefore, the approximation was continued after removing them. As a result, the filter order became 11/11th. Fig. 3(a) and (b) shows the optimized frequency and impulse responses, respectively. The resulting pole–zero locations are shown

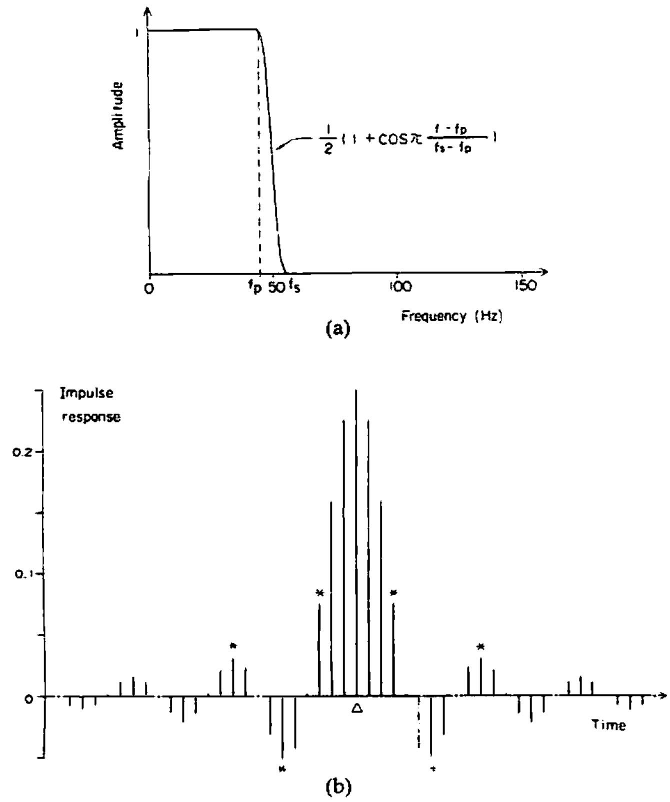


Fig. 2. Ideal filter responses for initial guess calculation. (a) Amplitude response. (b) Impulse response. Symbol Δ indicates average delay time. Symmetrical conditions are imposed on samples designated by symbol *. Sampling frequency is 400 Hz.

in Fig. 3(c). The numerator coefficients corresponding to the zeros enclosed with a dashed line are used for the time response approximation. As shown in Fig. 3(a), the group delay distortion is well decreased, except for the transition band, because the impulse response is approximated as a symmetrical waveform.

*D. Comparison between New and Conventional Approaches*

*Linear Phase FIR Filter:*

A 73 tap filter length is required to achieve the same frequency response shown in Fig. 3(a), using the Remez-exchange method [4]. When the input signals have some band limited spectra, an exact linear phase is not optimum, from the filter order reduction viewpoint [15].

*Elliptic Filter with All-Pass Function:*

It is possible to optimize a time response using an all-pass function without affecting an amplitude response through the iterative method [14]. This approach was tried during this study. A sixth-order elliptic filter and an eighth-order all-pass function are required to achieve the same results, as those shown in Fig. 3.

*Circuit Complexity Comparison:*

Numbers in operations such as adders, multipliers and delay elements are listed in Table II, where the FIR filter has a symmetrical direct form using 37 multipliers. Other approaches employ a cascade form of biquad sections. The elliptic filters and all-pass functions require three and two multipliers, respectively, for the biquad implementation.

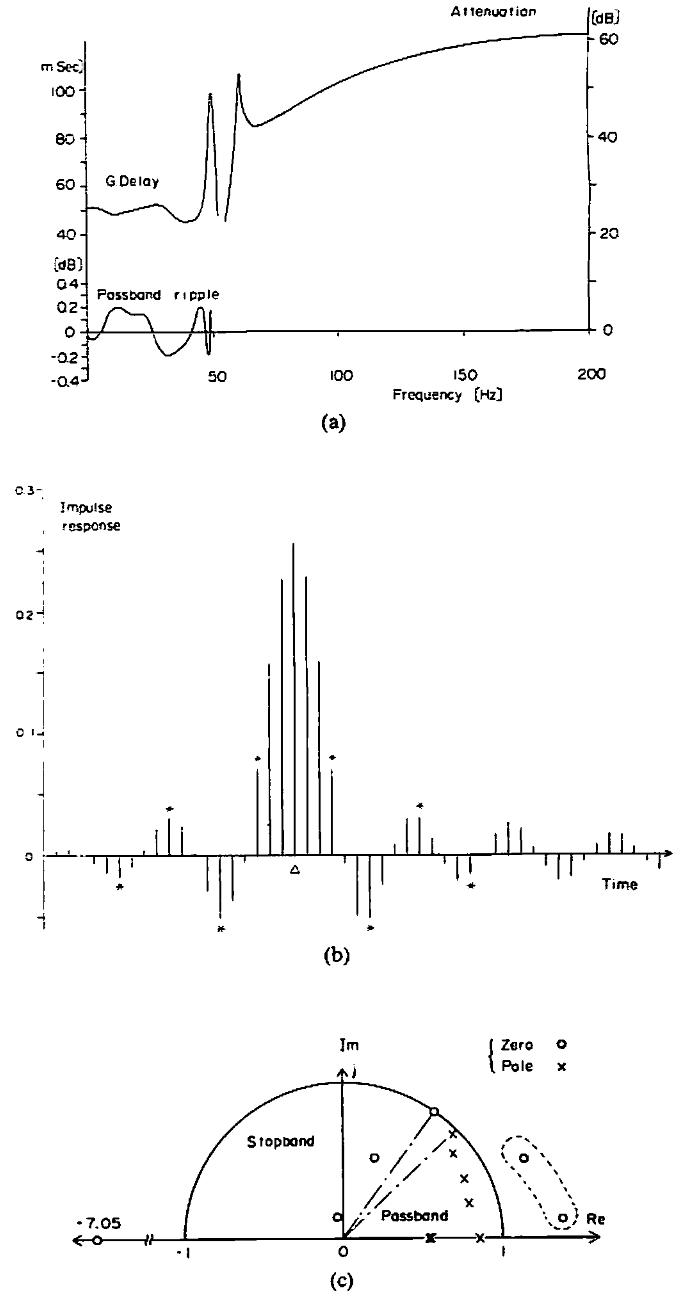


Fig. 3. Filter responses designed through proposed method with 11/11th-order transfer function, 20T average delay time and 400 Hz sampling frequency. (a) Amplitude response in decibels. (b) Impulse response. (c) Pole–zero locations.

The approach using elliptic filters and all-pass functions is superior to the proposed method, when circuit complexity is evaluated only based on the number of multipliers. On the other hand, when digital filters are implemented on high level functional LSI's for digital signal processing [16] or on signal processors including a hardware multiplier [17], the circuit complexity is mainly determined by the filter order. Furthermore, circuit complexities for other sampled data filters, such as CCD and switched capacitor filters [18], are mostly determined by the filter order. In these cases, the proposed method becomes a more efficient approach.

TABLE II
CIRCUIT COMPLEXITY COMPARISON BETWEEN CONVENTIONAL METHODS AND PROPOSED METHOD

| Methods \ Operations | Linear phase FIR filter | Elliptic filter with all-pass function | Proposed method |
|---|---|---|---|
| Filter order | 72nd | 14/14th | 11/11th |
| Multipliers | 37 | 17 | 22 |
| Adders | 72 | 28 | 22 |
| Delay elements | 72 | 14 | 11 |

## V. CONCLUSION

A simultaneous frequency- and time-domain approximation method for discrete-time filters is proposed. A time response is approximated through solving linear equations. The optimum solution is always guaranteed in a frequency response approximation procedure. The frequency response is optimized through the iterative Chebyshev approximation. This approach does not impose any constraints on pole–zero locations and filter responses. Hence, filter order reductions can be achieved for a wide range of filter responses.

Approximation error criteria in the proposed time response approximation are restricted to two categories, including exact interpolation and the mean square error. By extending the method of solving linear equations to linear programming techniques, a weighted mini-max error criterion can be employed.

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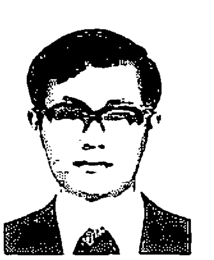

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