Software of teaching material developed by graphic function

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グラフィック機能を利用した教材ソフトの開発

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Introduction

The software of the teaching material made by us enables the learners to change abstract expressions into concrete figures by using the graphic function of the personal computer, so that they can understand basic matters efficiently with great interest.

As for the expressive power of the graphic function, the following three points should be especially mentioned.

① Experssion of movement

We adopted the maneuvers by which we can answer to various data by using the function of changing the position of one figure immediately. For instance, when the learners make the software that the personal computer itself maneuvers without doing for themselves, they cannot get the sense that they maneuver together with the personal computers. Therefore, we tried to make the position of the figure move by the learners' key operation, so that they can deepen the understanding of basic matters in mathematics.

② Expression of the changes of shape and size.

If we try to make abstract expression which were made more concrete by the graphic function more visible, there will be a method of partial magnification. If this method is adoptsd by the personal computer, we can get the result more easily and quickly, though so far it has taken us so much time to get the result.

③ Application of color

We can draw the learners' attention by coloring the emphasized letters, sentences, and formulae etc., and dividing the colors of the figures and graphs for the explanation.

I. Subroutine

1. Input of a cubic function

This signifies to input the function not by BASIC expression, but by algebra expression.

algebra expression

$2x^2+x-1$

BASIC expression

$2*x^2+x-1$

In addition, the numbers and the variables x are inputted by the corresponding key, e.g. the index 2 is inputted by the function key 2, the index 3 is by the key 3.

2. Automatic establishment of the range of drawings

This signifies to calculate the range by which the function can be characterized, that is, the maximum and minimum values of both x axis and y axis, given an optional cubic or quadratic function (except a linear function, fixed number). This method is divided into the following three cases and for each of them the value of the range is calculated.

① A cubic function with extreme values

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When two values of x which are extreme values of x which are extreme values are x_1 and x_2 ($x_1 < x_2$), and absolute values of the difference between them is p, the minimum and maximum values of x axis are assumed to be $x_1 - \rho$, $x_2 + \rho$ respectively.

As for y axis, when two values are y_1 and y_2 ($y_1 < y_2$), and the absolute value of the difference is ρ, the minimum and maximum values of y axis are assumed to be $y_1 - \rho$, $y_2 + \rho$ respectively. (fig. 1)

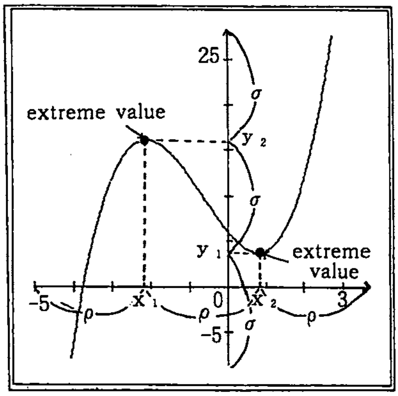


fig. 1

②A cubic function with no extreme values

When the values of x which is an inflection point is x_0, and the absolute value of the inclination of the function at $x_0 + 1$ is larger than 1, the minimum and maximum values are assumed to be $x_0 - 1$, $x_0 + 1$ respectively, and for y axis the minimum and maximum values are assumed to be

min $(f(x_0 - 1), (f(x_0 + 1))$

max $(f(x_0 - 1), (f(x_0 + 1))$

respectively. (fig. 2−a)

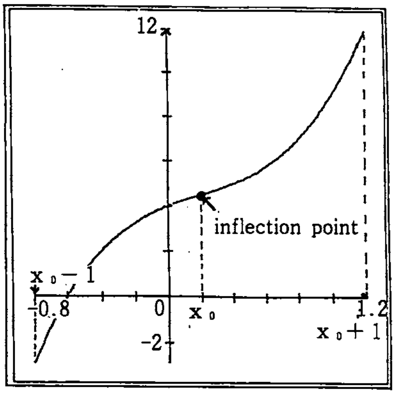


fig. 2−a

On the contrary, when the absolute value is less than or equal to 1, the minimum and maximum values of x axis are assumed to be $x_0 - 5$, $x_0 + 5$ respectively, and the minimum and maximum values of y axis are assumed to be

$\min(f(x_0 - 5), f(x_0 + 5))$

$\max(f(x_0 - 5), f(x_0 + 5))$

respectively. (fig.2−b)

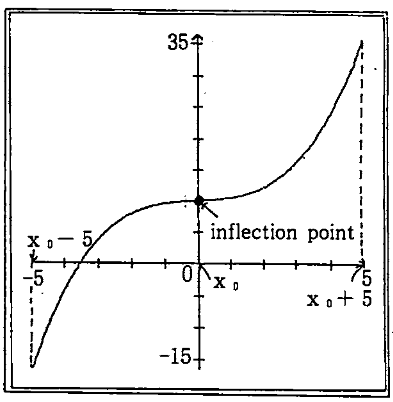


fig. 2−b

③A quadratic function

When the value of x which is an extreme value is x_0, for x axis it is the same with the case of ②.

Here, when the minimum value of x axis is X, and the one−second(1/2) of the absolute value of the difference between $f(x_0)$ and $f(X)$ is σ, if it is a convex function, the minimum and maximum values of y axis are assumed to be

$f(x_0) - \sigma$

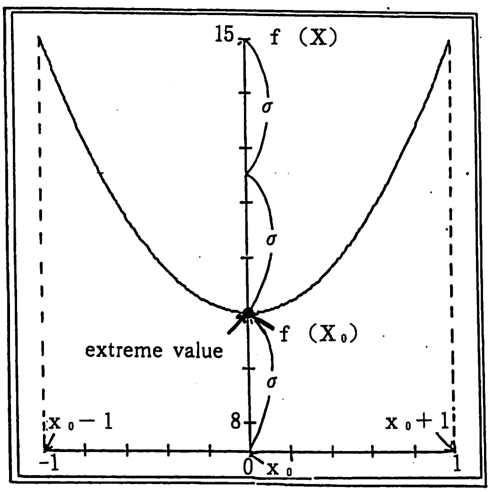


fig. 3—a

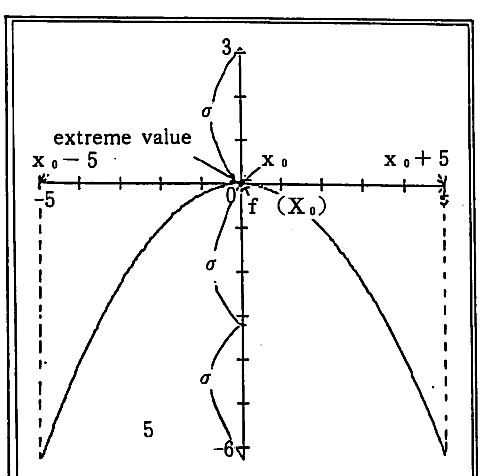


fig. 3—b

$f(X)$

respectively (fig.3—a), and if it is a dent function, the minimum and maximum values of y axis are assumed to be

$f(X)$

$f(x_0)+\sigma$

respectively (fig.3—b)

3. Automatic establishment of the unit of graduation

This signifies to calculate the unit of graduation which is drawn on the axis on the basis of the data of the maximum and minimum values of x and y axes in the range of drawings.

As the unit of graduation, we adopted either of 1×10^e, 2×10^e, and 5×10^e (e:integer).

Then we explain here only in case of x axis how the unit of graduation will be calculated. First of all, when the minimum and maximum values in the range of drawings of x axis is w_1 snd w_2 respectively, $\rho=w_2-w_1$ is assumed. Here, to calculate e which satisfies $5\times10^{e-1}\leqq\rho<5\times10^e$ we process as follows:

$$e=\min(E:\rho<5\times10^E)$$

The minimum of all values of M which satisfy $\rho/(M\times10^{e-1})\leqq10$ in these p, e, and which are equal to 1, 2, and 5 is assumed to be m.

By using this m, the unit of graduation of x axis is assumed to be $m\times10^{e-1}$.

If we arrange the unit of graduation like this, the unit of graduation taken, for instance, in case of $5\leqq\rho<50$ is either of 1, 2, and 5. Here, it is $5\leqq\rho<11$, $11\leqq\rho<22$, $22\leqq\rho<50$ that makes the unit 1, 2, and 5 respectively, and the number of intervals between graduations is from 5 to 10.

4. Parallel movement of segments

This is used when an optional segment is moved in the parallel in the software of the teaching material, "Mean value theorem"

The segments are moved here by the repetition of the instructions, GET and PUT.The larger the array variables used by this instruction becomes, the more time it takes us to process it. In other words, the longer the segment which we try to move becomes, the more time it takes us to move it. Therefore, wearranged that if the length of the segment is more than a certain one, only a part of it should be moved.

The direction of movement is inputted by the cursor key, and whenver the key ⊞ or ⊟ is pushed one time, the width which is moved when the cursor key is pushed one time becomes twice or one－second respectively.

Moreover, when the corresponding point of contact is found by this operation, we arranged to draw a tangent there, and when it is not found, we arranged to finish the operation automatically. (fig.4)

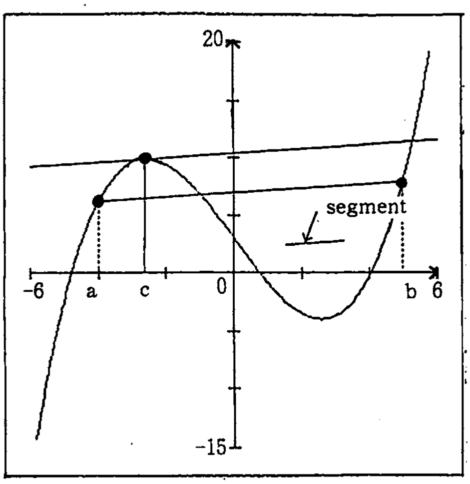


fig. 4

5. Partial magnification of graphs

This is used when the part of magnification is appointed in the scene of the magnification of graphs in the software of the teaching material, "Approximation expressions and errors".

First, a red point is displayed on the graph and is moved by the input data given by the cursor key (fig.5－a). If the return key is pushed, a point there becomes a point in the corner within the range of magnification (fig.5－b). Next, when the input data is given by the cursor key, the red point is further moved, and a rectangular frame is drawn which has the segment as a diagonal that connects the point and one point fixed first in the corner. The range enclosed here becomes a candidate of the part of magnification. If the return key is pushed, the part is made the position of magnification. (fig.5－c)

6. Graph of errors

This signifies to calculate the absolute value of errors in two approximation expressions in the software of the teaching material, "approximation expressions and errors", and to draw it on the graph. It is the most effective when it is difficult to distinguish with our eyes the difference between these two errors on the graph of approximation expresstions (hereafter it is simply called the graph) on the same screen. (fig.6－a, fig.6－b)

Here, the range of x axis is arranged to be the same as that of the graph and to be compared between on graph and on the graph of errors. Moreover, the range of y axis is arranged to take twice as many as the maximum value of errors in the value approximated from 0, and this value is in our experience taken from the most visible case of all on the screen of the software of this

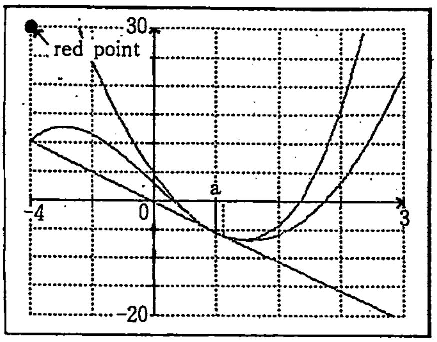


fig. 5—a

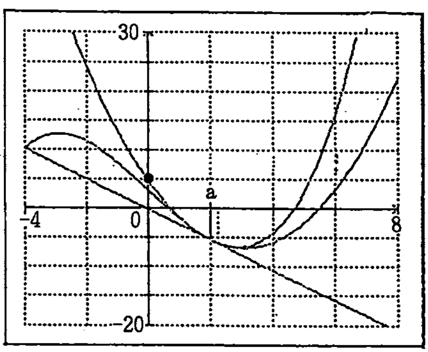


fig. 5—b

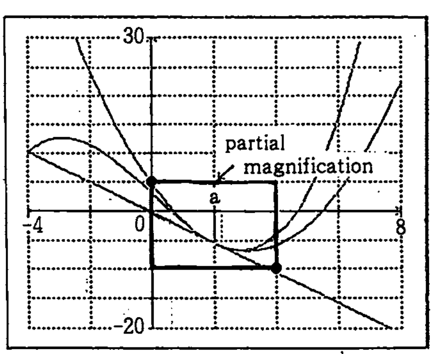


fig. 5—c

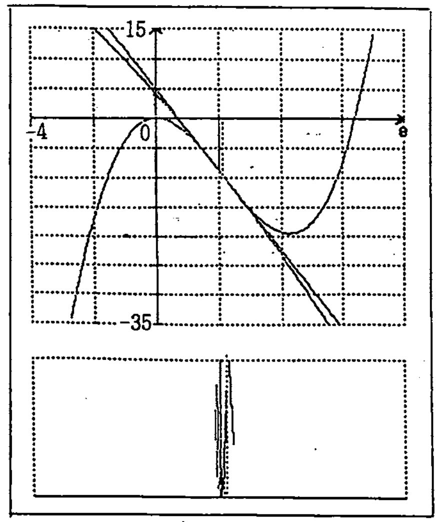


fig. 6—a

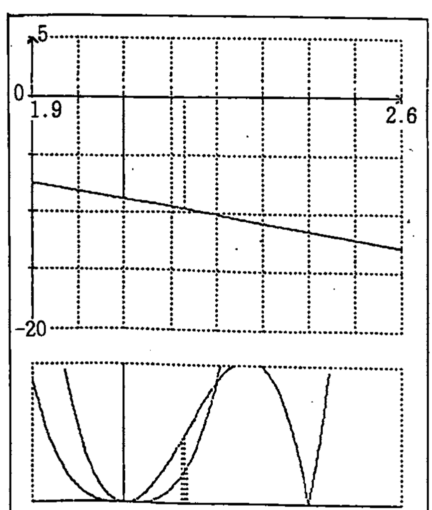


fig.6—b

teaching material.

Ⅲ Software of the teaching material

The software of the teaching material explained here consists of the following six items.

1 Explanation of the use of mean value theorem
2 Mean value theorem
3 Mean value theorem, Practice
4 Explanation of the use of approximation expressions and errors
5 Approximation expressions and errors
6 approximation expressions and errors, Practice

These are displayed on the "menu screen" of the personal computer and are selected at learners' choice. Each of them is explained in the following.

1. Explanation of the use of mean value theorem

The display of letters should be as minimum as possible to make the graphic the subject on the limited screen of the personal computer. Therefore, the method operated by the keyboard in each program is arranged as another program. It is this program and explains the method of operation to learn the programs of 2 and 3 about Mean value theorem.

2. Mean value theorem

This is the program which makes us understand what Mean value theorem means, but it is difficult to understand what it means from only the abstract formula in this theorem of average values. Here we want to impress the meaning of this theorem upon the learners strongly as an image. In this program, we arranged to get the value of c which satisfies Mean value theorem, that is,

$$\frac{f(b)-f(a)}{b-a}=f'(c),\ a \langle c \langle b$$

by the key operation of the learners.

Moreover, we adopted the scene in which learners define the cubic function f(x), and the value of a and b that are used here. We also made these repetitions possible, and tried to make learners fix the understanding.

Besides, when learners learn this, they are assumed to have finished learning Rolle's theoerm. (flowchart.1)

3. Mean value theorem, Practice

We want learners to maneuver by various functions after making them understand the contents of mean value theorem.

Here, learners are arranged to be able to confirm the value of f'(c) calculated by hand in the graph.

Well, errors are often included in the value calculated by the computer. If such a value is shown as it is, learners will be confused. Therefore, only a rough position is arranged to be confirmed not by value but by the position on the graph.

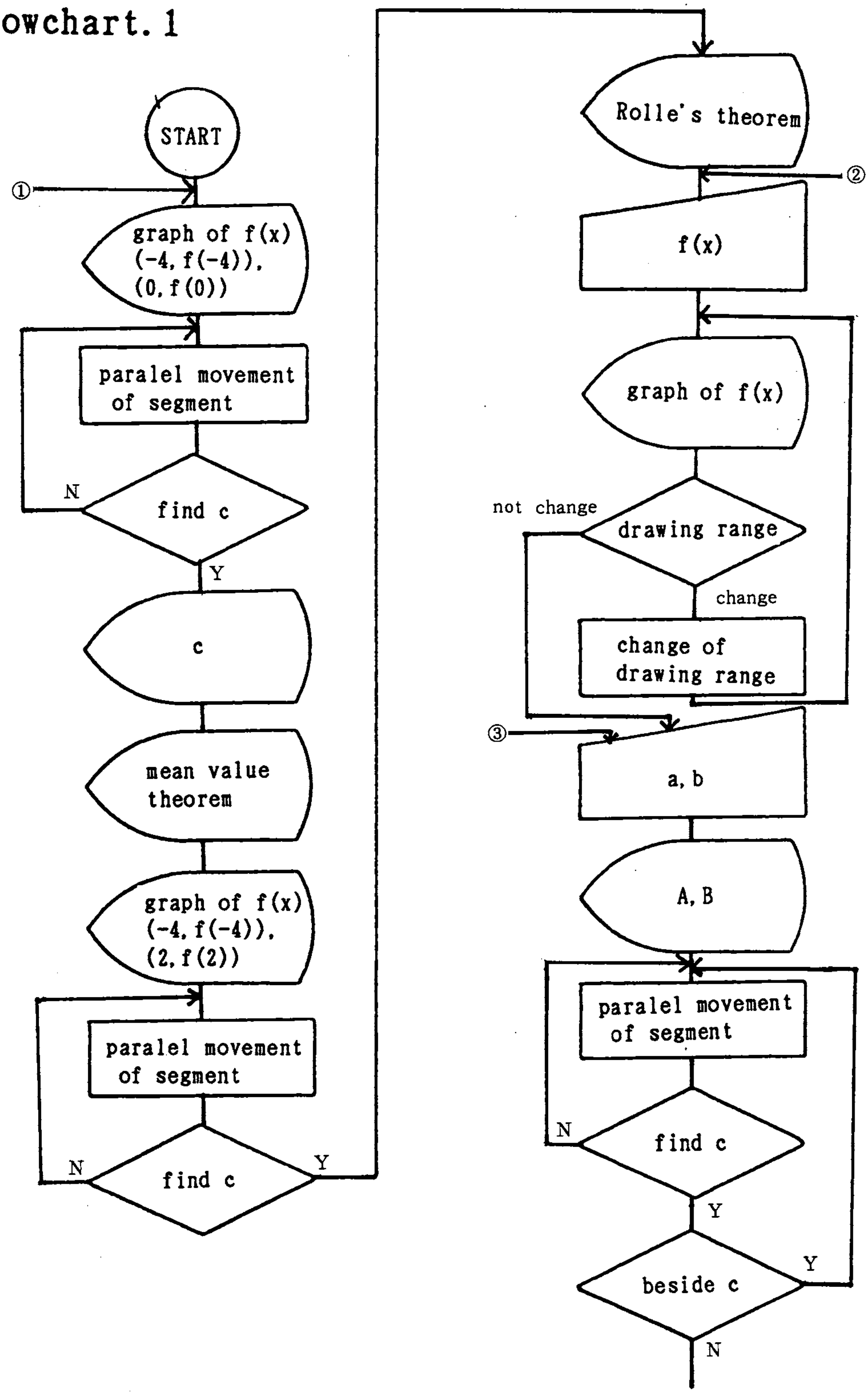
4. Explanation of the use of approximation expressions and errors

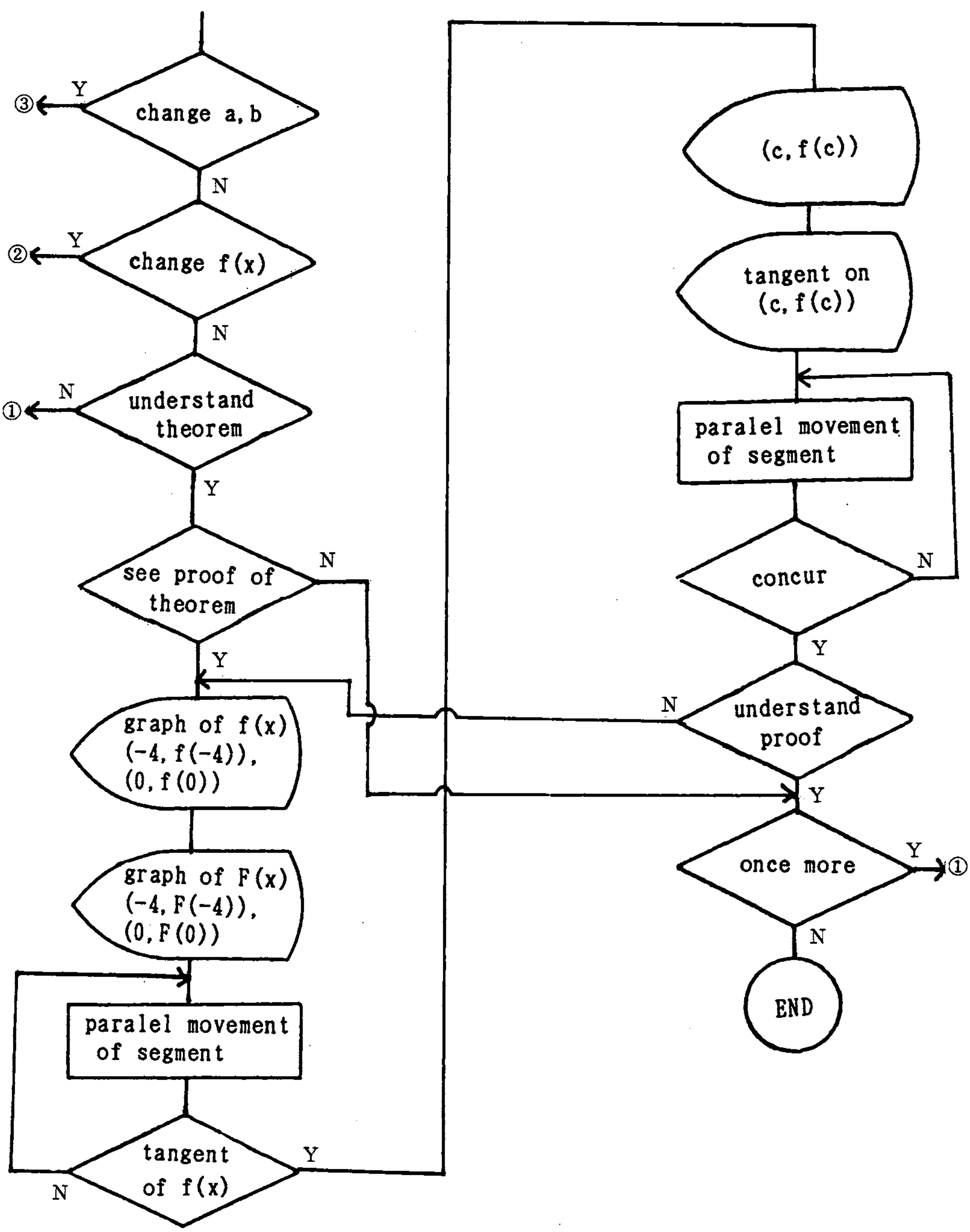
This program explains the method of operation to learn the 5 and 6 about approximation expressions and errors.

5. Approximation expressions and errors

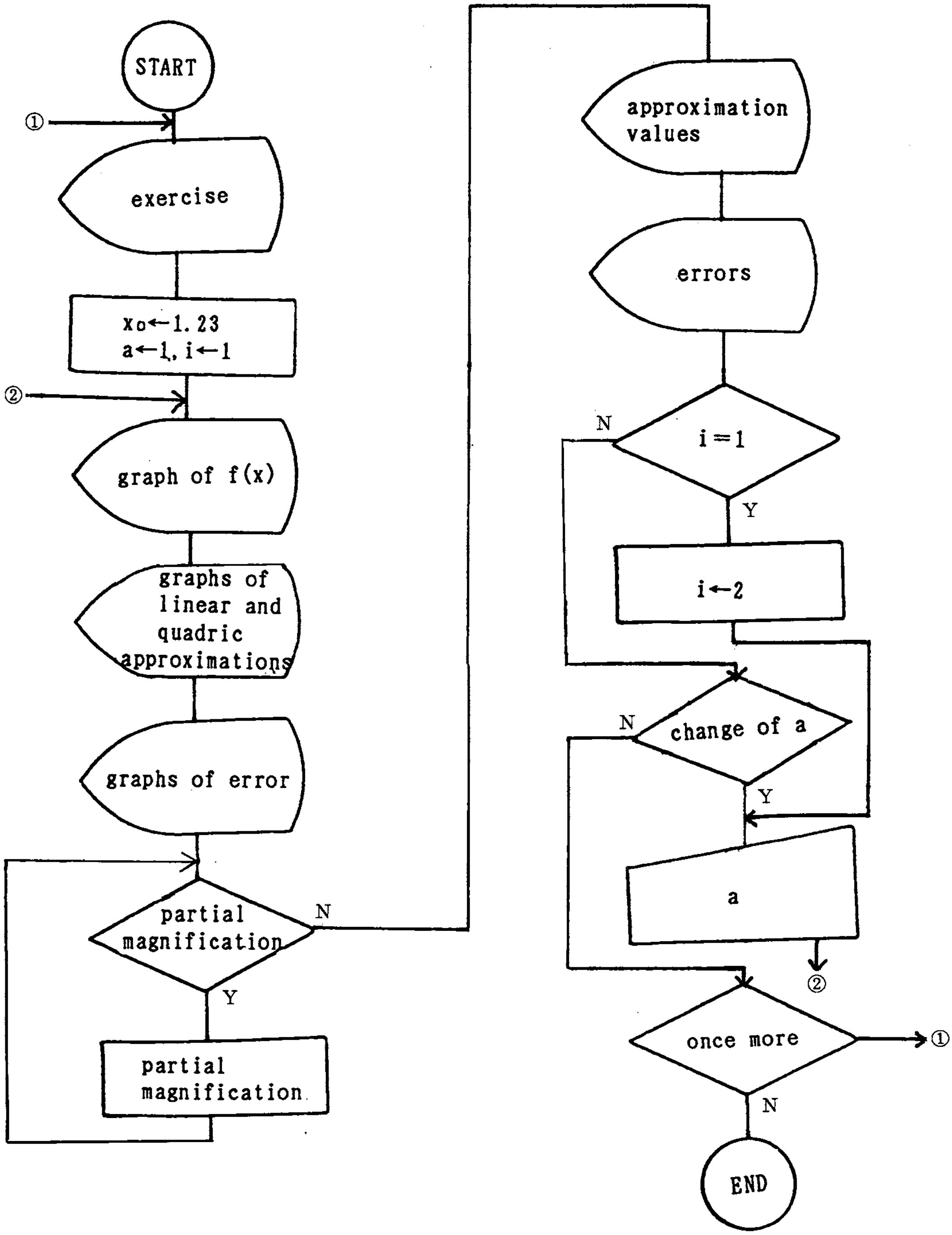
This program makes learners recognize visually the relation between the error of linear approximation and that of quadric approximation, and the relation between the center of the devrlopment and the error. It is devised to make learners understand the relation between the original function and two approximation function better.

flowchart. 1


START
①
graph of f(x)
(-4, f(-4)),
(0, f(0))
paralel movement of segment
N
find c
Y
c
mean value theorem
graph of f(x)
(-4, f(-4)),
(2, f(2))
paralel movement of segment
N
find c
Y
Rolle's theorem
②
f(x)
graph of f(x)
not change
drawing range
change
change of drawing range
③
a, b
A, B
paralel movement of segment
N
find c
Y
beside c
Y
N


change a, b
Y
③
N
change f(x)
Y
②
N
understand theorem
N
①
Y
see proof of theorem
N
Y
graph of f(x) (-4, f(-4)), (0, f(0))
graph of F(x) (-4, F(-4)), (0, F(0))
paralel movement of segment
tangent of f(x)
Y
N
(c, f(c))
tangent on (c, f(c))
paralel movement of segment
concur
N
Y
understand proof
N
Y
once more
Y
①
N
END

flowchart. 2


START
①
exercise
xo←1. 23
a←1, i←1
②
graph of f(x)
graphs of
linear and
quadric
approximations
graphs of error
partial
magnification
N
Y
partial
magnification
approximation
values
errors
i=1
N
Y
i←2
change of a
N
Y
a
②
once more
①
N
END

(flowchart.2)

6. Approximation expressions and errors, Practice

In this program learneres can deepen the understanding by giving an optional cubic function, the point which they want to approximate, and the center of the development, and getting the appeoximation and the error, after understanding the contents of appeoximation expressions and errors.