NHDS: THE NEW HAMPSHIRE DISPERSION RELATION SOLVER

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Keywords: plasmas — methods: numerical — waves

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In collisionless astrophysical plasmas, waves and instabilities are well modeled by the linearized Vlasov–Maxwell equations, which have non-trivial solutions only when the complex frequency ω solves the hot-plasma dispersion relation. *NHDS* (New Hampshire Dispersion relation Solver) is a numerical tool written in Fortran 90 and first introduced by Verscharen et al. (2013) to solve this dispersion relation under the assumption that the plasma background distribution is a gyrotropic drifting bi-Maxwellian for each species j,

$$f_{0j}(v_{\perp}, v_{\parallel}) = \frac{n_j}{\pi^{3/2} w_{\perp j}^2 w_{\parallel}} \exp\left(-\frac{v_{\perp}^2}{w_{\perp j}^2} - \frac{\left(v_{\parallel} - U_j\right)^2}{w_{\parallel j}^2}\right),\tag{1}$$

in a cylindrical coordinate system aligned with the direction of the background magnetic field \mathbf{B}_0 , where n_j is the density, w_{\perp} (w_{\parallel}) is the perpendicular (parallel) thermal speed with respect to \mathbf{B}_0 , and U_j is the field-aligned drift speed. All floating-point quantities use double precision.

The NHDS code closely follows the formulation of the hot-plasma dispersion relation laid out by Stix (1992). It uses a Newton-secant method to identify those frequencies at which there are non-trivial solutions to the wave equation,

$$\begin{pmatrix}
\epsilon_{xx} - \frac{k_z^2 c^2}{\omega^2} & \epsilon_{xy} & \epsilon_{xz} + \frac{k_\perp k_z c^2}{\omega^2} \\
\epsilon_{yx} & \epsilon_{yy} - \frac{k^2 c^2}{\omega^2} & \epsilon_{yz} \\
\epsilon_{zx} + \frac{k_\perp k_z c^2}{\omega^2} & \epsilon_{zy} & \epsilon_{zz} - \frac{k_\perp^2 c^2}{\omega^2}
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix} = 0,$$
(2)

based on an initial guess for ω , where ϵ is the dielectric tensor, \boldsymbol{E} is the vector of the electric-field Fourier amplitudes, c is the speed of light, and $\boldsymbol{k}=(k_{\perp},0,k_z)$ is the wavevector. The initial guess defines the plasma mode that the code follows in \boldsymbol{k} . The Newton-secant method converges if the absolute value of the determinant of the matrix in Equation (2) is less than a user-defined value. All frequencies are given in units of the proton gyro-frequency $\Omega_{\rm p}$ and all length scales in units of the proton inertial length $d_{\rm p}$.

For each of the up to ten plasma components j, the user defines the temperature anisotropy $T_{\perp j}/T_{\parallel j}$ with respect to \boldsymbol{B}_0 , the value of $\beta_{\parallel j} \equiv 8\pi n_j k_{\rm B} T_{\parallel j}/B_0^2$, the relative charge $q_j/q_{\rm p}$, the relative mass $m_j/m_{\rm p}$, the relative density $n_j/n_{\rm p}$, and the normalized drift velocity $U_j/v_{\rm A}$, where $k_{\rm B}$ is the Boltzmann constant, $v_{\rm A}$ is the proton Alfvén speed, and $T_{\parallel j}$ is the temperature parallel to \boldsymbol{B}_0 . Furthermore, the ratio $v_{\rm A}/c$ and the angle of propagation θ are user-defined parameters.

The calculation of ϵ_{ik} entails the evaluation of the modified Bessel function $I_m(\lambda_j)$ of the first kind and the plasma dispersion function $Z(\zeta)$, where $\lambda_j \equiv k_\perp^2 w_{\perp j}^2/2\Omega_j^2$, and ζ is a dimensionless complex number. For the evaluation of I_m , NHDS applies the recursion method supplied by the Numath Library (Clenshaw 1962). It determines the maximum order m_{max} of I_m as the smaller of either a user-defined limit or as the number for which $I_{m_{\text{max}}}(\lambda_j)$ is less than a user-defined value. NHDS evaluates $Z(\zeta)$ following Poppe & Wijers (1990) by computing the complex error function $w(\zeta) = Z(\zeta)/i\sqrt{\pi}$ through one of the following methods, depending on the value of $|\zeta|$: a power series, the Laplace continued fraction method, or a truncated Taylor expansion. This combined method is faster than alternative approaches and calculates $w(\zeta)$ to an accuracy of 14 significant digits for almost all ζ .

NHDS determines the polarization of the wave solutions as the ratios E_y/E_x and E_z/E_x from Equation (2), which translate to ratios of the magnetic-field amplitudes through Faraday's law. In addition, as described by Verscharen & Chandran (2013) and Verscharen et al. (2016), NHDS calculates the relative wave energy W_k and the Fourier amplitudes of the fluctuations in density, bulk velocity, and pressure. The code also calculates the contribution γ_j to the total growth/damping rate $Im(\omega)$ from each species j as described by Quataert (1998).

For a given wave solution, NHDS can determine the value of the self-consistent fluctuating distribution function on a user-defined Cartesian grid in velocity space as described by Verscharen et al. (2016). NHDS saves the fluctuating distribution function in HDF5 files and creates an XDMF file for visualization with programs like ParaView. This calculation entails the calculation of the Bessel function $J_m(k_{\perp}v_{\perp}/\Omega_j)$ of order m, which NHDS performs through a polynomial Chebyshev approximation. The maximum order m_{\max} for J_m is determined in the same way as m_{\max} for I_m in the calculation of ϵ_{ik} , except that m_{\max} for J_m is evaluated for each v_{\perp} .

Figure 1 shows the dispersion relations of Alfvén/ion-cyclotron (A/IC) and fast-magnetosonic/whistler (FM/W) waves in parallel and perpendicular propagation as well as some of their polarization properties determined with NHDS.

The code is publicly available for download (Verscharen & Chandran 2018, Codebase: https://github.com/danielver02/NHDS).

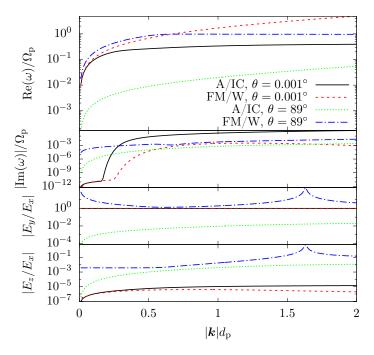


Figure 1. Dispersion relations for the A/IC and FM/W waves in parallel ($\theta = 0.001^{\circ}$) and perpendicular ($\theta = 89^{\circ}$) propagation. The panels show from the top to the bottom: the normalized real part of the frequency, the normalized damping rate, the ratio $|E_y/E_x|$, and the ratio $|E_z/E_x|$ as functions of $|\mathbf{k}|$.

We acknowledge support from NASA, NSF, and STFC. We appreciate discussions with Kris Klein.

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Verscharen, D. & Chandran, B. D. G. 2018, NHDS, v1.0,

Zenodo, doi:10.5281/zenodo.1227265