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# ON THE OHSAWA–TAKEGOSHI EXTENSION THEOREM

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**Abstract.** Motivated by a recent work by B.-Y. Chen we prove a new estimate for the $\bar\partial$-operator, which easily implies the Ohsawa–Takegoshi extension theorem. We essentially only use the classical Hörmander estimate. This method gives the same constant as the one recently obtained by Guan–Zhou–Zhu.

**1. Introduction.** The Ohsawa–Takegoshi extension theorem [**14**] turned out to be one of the most important results in complex analysis and geometry. There have been various simplifications of its proof (see e.g. [**1**]) but the crucial one is due to B.-Y. Chen [**8**], who recently showed that it follows directly from Hörmander's estimate for the $\bar\partial$-equation. Using some of his ideas we obtain a generalization of an estimate due to Berndtsson [**1**] (see Theorem 1 below), from which the Ohsawa–Takegoshi theorem can be deduced directly.

We are also interested in the conjecture formulated by Suita [**15**]: for a bounded domain $D$ in $\mathbb{C}$ one has

$$c_D^2 \le \pi K_D.$$

Here

$$c_D(z) = \exp(\lim_{\zeta\to z}(G_D(\zeta,z) - \log|\zeta - z|)),$$

where $G_D(\cdot,z)$ is the (negative) Green function with pole at $z \in D$, and

$$K_D(z) = \sup\{|f(z)|^2 : f \text{ holomorphic in } D,\ \int_D |f|^2 d\lambda \le 1\}$$

is the Bergman kernel. Its relation to the extension theorem was found by Ohsawa [**13**] who, using methods of the $\bar\partial$-equation, showed the estimate

$$c_D^2 \le C\pi K_D \tag{1}$$

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with $C = 750$. It was improved in [**6**] to $C = 2$ and recently to $C = 1.95388...$ by Guan–Zhou–Zhu, as announced in [**11**]. We show, see Theorem 4 below, a result which covers both the Ohsawa–Takegoshi result and (1) with the same constant as in [**11**] (Theorem 4 below was originally shown in [**9**] with $C = 4$).

**2. The estimate for the $\bar\partial$-equation.** Let $\Omega$ be a pseudoconvex domain in $\mathbb{C}^n$ and assume that

$$\alpha = \sum_j \alpha_j d\bar z_j \in L^2_{loc,(0,1)}(\Omega)$$

satisfies $\bar\partial\alpha = 0$. We are looking for solutions of

$$\bar\partial u = \alpha \tag{2}$$

with $L^2$-estimates. The classical one is due to Hörmander [**12**]: for any plurisubharmonic $\varphi$ in $\Omega$ we can find $u$ with

$$\int_\Omega |u|^2 e^{-\varphi} d\lambda \le \int_\Omega |\alpha|^2_{i\partial\bar\partial\varphi} e^{-\varphi} d\lambda. \tag{3}$$

For $C^2$, strongly plurisubharmonic $\varphi$ we have

$$|\alpha|^2_{i\partial\bar\partial\varphi} = \sum_{j,k} \varphi^{j\bar k} \bar\alpha_j \alpha_k,$$

where $(\varphi^{j\bar k})$ is the inverse transposed of $(\partial^2\varphi/\partial z_j \partial\bar z_k)$, whereas for $\varphi$ which is only plurisubharmonic the right-hand side of (3) is a bit ambiguous. It makes sense however (and the estimate indeed holds – see [**4**] or [**5**]) if instead of $|\alpha|^2_{i\partial\bar\partial\varphi}$ we take any $h \in L^\infty_{loc}(\Omega)$ with

$$i\bar\alpha \wedge \alpha \le h\, i\partial\bar\partial\varphi.$$

Berndtsson [**1**] showed another estimate for (2): if in addition $\psi$ is a plurisubharmonic function in $\Omega$ satisfying

$$i\partial\psi \wedge \bar\partial\psi \le i\partial\bar\partial\psi \tag{4}$$

and $0 < \delta < 1$, then we can find $u$ with

$$\int_\Omega |u|^2 e^{\delta\psi-\varphi} d\lambda \le \frac{4}{\delta(1-\delta)^2} \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\delta\psi-\varphi} d\lambda. \tag{5}$$

The constant in (5) was improved in [**3**]: it was shown that the optimal $C(\delta)$ satisfies

$$\frac{4}{(1-\delta)(2-\delta)} \le C(\delta) \le \frac{4}{(1-\delta)^2}.$$

Then (5) makes sense also for $\delta = 0$: one obtains the following estimate due to Donnelly and Fefferman [**10**]:

$$\int_\Omega |u|^2 e^{-\varphi} d\lambda \le 4 \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{-\varphi} d\lambda.$$

It is also clear that we cannot have a finite constant in (5) $\delta = 1$.

Note that in our convention (4) can be written as $|\bar\partial\psi|^2_{i\partial\bar\partial\psi} \le 1$. Keeping this in mind we will formulate our main result which can be viewed as a variant of Berndtsson's estimate (5) for $\delta = 1$:

THEOREM 1. *Assume that $\Omega$ is a pseudoconvex domain in $\mathbb{C}^n$ and take $\alpha \in L^2_{loc,(0,1)}(\Omega)$ with $\bar\partial\alpha = 0$. Let $\varphi, \psi$ be plurisubharmonic functions in $\Omega$ such that $|\bar\partial\psi|^2_{i\partial\bar\partial\psi} \le 1$ in $\Omega$ and $|\bar\partial\psi|^2_{i\partial\bar\partial\psi} \le \delta < 1$ on $\operatorname{supp}\alpha$. Then there exists $u \in L^2_{loc}(\Omega)$ solving $\bar\partial u = \alpha$ and such that*

$$\int_\Omega (1 - |\bar\partial\psi|^2_{i\partial\bar\partial\psi})|u|^2 e^{\psi-\varphi} d\lambda \le \frac{1}{(1-\sqrt{\delta})^2} \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda. \tag{6}$$

PROOF. By standard approximation we may assume that $\varphi, \psi$ are smooth up to the boundary. We now use a trick from [**2**]. Let $u$ be the minimal solution to $\bar\partial u = \alpha$ in $L^2(\Omega, e^{-\varphi})$. This is equivalent to $u$ being perpendicular to $\ker\bar\partial$ in $L^2(\Omega, e^{-\varphi})$. Therefore, $v := ue^{\psi}$ is perpendicular to $\ker\bar\partial$ in $L^2(\Omega, e^{-\varphi-\psi})$, which means that $v$ is the minimal solution to $\bar\partial v = \beta$, where

$$\beta := (\alpha + u\bar\partial\psi)e^{\psi},$$

in $L^2(\Omega, e^{-\varphi-\psi})$. Therefore, by Hörmander's estimate (3)

$$\begin{aligned}\int_\Omega |u|^2 e^{\psi-\varphi} d\lambda &= \int_\Omega |v|^2 e^{-\varphi-\psi} d\lambda \\ &\le \int_\Omega |\beta|^2_{i\partial\bar\partial(\varphi+\psi)} e^{-\varphi-\psi} d\lambda \le \int_\Omega |\alpha + u\bar\partial\psi|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda.\end{aligned}$$

Denoting $h := |\bar\partial\psi|^2_{i\partial\bar\partial\psi}$, for any $t > 0$ we get

$$\begin{aligned}&\int_\Omega |\alpha + u\bar\partial\psi|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda \\ &\le (1+t^{-1}) \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda + t\int_{\operatorname{supp}\alpha} |u|^2 h e^{\psi-\varphi} d\lambda + \int_\Omega |u|^2 h e^{\psi-\varphi} d\lambda \\ &\le (1+t^{-1}) \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda + \delta(t+1)\int_{\operatorname{supp}\alpha} |u|^2 e^{\psi-\varphi} d\lambda \\ &\qquad + \int_{\Omega\setminus\operatorname{supp}\alpha} |u|^2 h e^{\psi-\varphi} d\lambda.\end{aligned}$$

Therefore

$$\begin{aligned}\int_{\Omega\setminus\operatorname{supp}\alpha} (1-h)|u|^2 e^{\psi-\varphi} d\lambda \;+\; \big(1-\delta(t+1)\big)\int_{\operatorname{supp}\alpha} |u|^2 e^{\psi-\varphi} d\lambda \\ \le (1+t^{-1}) \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\varphi} d\lambda.\end{aligned}$$

Since the left-hand side is bounded below by

$$\big(1-\delta(t+1)\big)\int_\Omega (1-h)|u|^2 e^{\psi-\varphi} d\lambda$$

for $t=\delta^{-1/2}-1$ we get (6). □

Note that, after replacing $\psi$ by $\delta\psi$, Theorem 1 gives the Berndtsson estimate (5) with the constant

$$\frac{1}{\delta(1-\delta)(1-\sqrt{\delta})^2}.$$

**3. The Ohsawa–Takegoshi extension theorem.** The following lemma is essentially contained in [**8**]:

Lemma 2. *For $\zeta\in\mathbb{C}$ with $|\zeta|\le(2e)^{-1/2}$ and $\varepsilon>0$ sufficiently small, set*

$$\psi(\zeta):=-\log\big[-\log(|\zeta|^2+\varepsilon^2)+\log\big(-\log(|\zeta|^2+\varepsilon^2)\big)\big].$$

*Then $\psi$ is subharmonic in $\{|\zeta|<(2e)^{-1/2}\}$ and there exist constants $C_1$, $C_2$, $C_3$ such that*

*i)* $\big(1-\dfrac{|\psi_\zeta|^2}{\psi_{\zeta\bar\zeta}}\big)e^\psi \ge \dfrac{1}{C_1\log^2(|\zeta|^2+\varepsilon^2)}$ *on* $\{|\zeta|\le(2e)^{-1/2}\}$;

*ii)* $\dfrac{|\psi_\zeta|^2}{\psi_{\zeta\bar\zeta}}\le\dfrac{C_2}{-\log\varepsilon}$ *on* $\{|\zeta|\le\varepsilon\}$;

*iii)* $\dfrac{e^\psi}{|\zeta|^2\psi_{\zeta\bar\zeta}}\le C_3$ *on* $\{\varepsilon/2\le|\zeta|\le\varepsilon\}$.

Using Theorem 1 and Lemma 2 similarly as in [**8**] we can easily prove an extended version of the Ohsawa–Takegoshi theorem:

Theorem 3. *Assume that $\Omega\subset\mathbb{C}^{n-1}\times\{|z_n|<(2e)^{-1/2}\}$ is pseudoconvex and let $\varphi$ be a plurisubharmonic function in $\Omega$. Then every holomorphic $f$ in $\Omega':=\Omega\cap\{z_n=0\}$ (we assume that $\Omega'$ is not empty) has a holomorphic extension $F$ in $\Omega$ satisfying*

$$\int_\Omega \frac{|F|^2e^{-\varphi}}{|z_n|^2\log^2|z_n|^2}\,d\lambda\le C\int_{\Omega'}|f|^2e^{-\varphi}d\lambda',$$

*where $C$ is a uniform constant.*

Proof. We follow the argument from [**8**]. By standard approximation we may assume that $\Omega$ is bounded with smooth boundary, $\varphi$ is smooth up to the boundary and $f$ is defined in a neighborhood of $\overline{\Omega'}$. Let $\chi\in C^\infty(\mathbb{R})$ be such that $\chi=1$ on $\{t\le 1/2\}$ and $\chi=0$ on $\{t\ge1\}$. For small $\varepsilon>0$ set

$$\alpha:=\bar\partial\big(f(z')\chi(|z_n|^2/\varepsilon^2)\big)=f(z')\chi'(|z_n|^2/\varepsilon^2)z_n\,d\bar z_n/\varepsilon^2.$$

We use Theorem 1 with $\psi$ given by Lemma 2 and $\widetilde{\varphi} = \varphi + 2\log|z_n|$. With $\delta := -C_2/\log\varepsilon$, we get a solution $u = u_\varepsilon$ to (2) with

$$\int_\Omega \frac{|u|^2 e^{-\varphi}}{|z_n|^2 \log^2(|z_n|^2+\varepsilon^2)}\,d\lambda \le \frac{C_1}{(1-\sqrt{\delta})^2}\int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\widetilde{\varphi}} d\lambda.$$

We have

$$\int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\widetilde{\varphi}} d\lambda \le \frac{(\sup\chi')^2}{\varepsilon^4}\int_{\{\frac{\varepsilon}{2}\le|z_n|\le\varepsilon\}} \frac{e^\psi}{\psi_{\zeta\bar\zeta}} d\lambda \sup_{\frac{\varepsilon}{2}\le|\zeta|\le\varepsilon}\int_{\Omega'_\zeta} |f|^2 e^{-\varphi} d\lambda',$$

where $\Omega'_\zeta = \{z' \in \mathbb{C}^{n-1} : (z',\zeta) \in \Omega\}$. It follows that $u = 0$ on $\{z_n = 0\}$ and

$$F_\varepsilon(z) := f(z')\chi(|z_n|^2/\varepsilon^2) - u_\varepsilon(z)$$

is a holomorphic extension of $f$ satisfying

$$\limsup_{\varepsilon\to 0}\int_\Omega \frac{|F_\varepsilon|^2 e^{-\varphi}}{|z_n|^2\log^2|z_n|^2}\,d\lambda \le C\int_{\Omega'} |f|^2 e^{-\varphi} d\lambda'.$$

The required $F$ is the weak accumulation point of $F_\varepsilon$. □

**4. The Suita conjecture and constants.** Similarly as in [**11**], we consider a decreasing convex $\eta : \mathbb{R}_+ \longrightarrow \mathbb{R}_-$ such that

$$\eta'' \ge \frac{(\eta')^2}{\eta + e^t} \tag{7}$$

and

$$C := \frac{1}{-\lim\limits_{t\to\infty}\eta'(t)} < \infty. \tag{8}$$

An example of such an $\eta$ is $-a(t+t^b)$, where $0 < b < 1$ and $a > 0$ is sufficiently small. The smallest $C$ that can be obtained this way (numerically with *Mathematica*) is for $\eta$ satisfying the equality in (7) and the initial condition $\eta(0) = 0$, $\eta'(0) = -2.216715...$; then $C = 1.95388...$

THEOREM 4. *Assume that $\Omega \subset \mathbb{C}^{n-1}\times D$ is pseudoconvex, where $D$ is a bounded domain in $\mathbb{C}$ containing the origin. Then for every plurisubharmonic $\varphi$ in $\Omega$ and $f$ holomorphic in $\Omega' := \Omega\cap\{z_n = 0\}$ (we assume that $\Omega'$ is not empty) there exists a holomorphic $F$ in $\Omega$ such that $F|_{\Omega'} = f$ and*

$$\int_\Omega |F|^2 e^{-\varphi}\,d\lambda \le \frac{C\,\pi}{(c_D(0))^2}\int_{\Omega'} |f|^2 e^{-\varphi} d\lambda',$$

*where $C$ is given by* (8).

Proof. First assume that $D = \{|\zeta| < 1\}$, so that in particular $c_D(0) = 1$. Let $0 < \varepsilon < 1$ and set

$$\alpha := \bar\partial\big(f(z')\chi(-2\log|z_n|)\big) = -f(z')\chi'(-2\log|z_n|)\frac{d\bar z_n}{\bar z_n},$$

where $\chi \in C^{0,1}(\mathbb{R}_+)$, such that $\chi(t) = 0$ for $t \le M := -2\log\varepsilon$ and $\lim_{t\to\infty}\chi(t) = 1$, will be determined later. Further, set $\widetilde\varphi := \varphi + 2\log|z_n|$ and $\psi := \gamma(-2\log|z_n|)$, where a convex decreasing $\gamma \in C^{1,1}(\mathbb{R}_+)$ will also be determined later. For $u$ given by Theorem 1 we have

$$\int_\Omega |u|^2 e^{-\varphi} d\lambda \le \int_\Omega (1 - |\bar\partial\psi|^2_{i\partial\bar\partial\psi})|u|^2 e^{\psi-\widetilde\varphi} d\lambda \le \frac{1}{(1-\sqrt{\delta})^2}\int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\widetilde\varphi} d\lambda,$$

provided that

$$\left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^{\gamma+t} \ge 1 \tag{9}$$

on $\mathbb{R}_+$ (then the first inequality holds), $(\gamma')^2/\gamma'' \le 1$ on $\mathbb{R}_+$ and $(\gamma')^2/\gamma'' \le \delta < 1$ on $\{t \ge M\}$ (then the second inequality follows from Theorem 1).

Similarly as in the proof of Theorem 3, we have (recall that $\alpha$, $\widetilde\varphi$ and $\psi$ depend on $\varepsilon$)

$$\limsup_{\varepsilon\to 0^+} \int_\Omega |\alpha|^2_{i\partial\bar\partial\psi} e^{\psi-\widetilde\varphi} d\lambda \le \limsup_{\varepsilon\to 0^+} \mathcal{A}(\varepsilon)\int_{\Omega'} |f|^2 e^{-\varphi} d\lambda',$$

where

$$\mathcal{A}(\varepsilon) = \int_{\{|\zeta|\le\varepsilon\}} \frac{(\chi'(-2\log|\zeta|))^2 e^{\gamma(-2\log|\zeta|)}}{|\zeta|^2\,\gamma''(-2\log|\zeta|)} d\lambda(\zeta) = \pi\int_M^\infty \frac{(\chi')^2 e^\gamma}{\gamma''} dt.$$

The optimal choice of $\chi$ with $\int_M^\infty \chi'(t)\,dt = 1$ is

$$\chi(t) := \begin{cases} 0, & t \le M \\ \dfrac{1}{c}\displaystyle\int_M^t w(s)\,ds, & t > M, \end{cases}$$

where $w = \gamma'' e^{-\gamma}$ and $c = \int_M^\infty w(s)\,ds$. Then

$$\mathcal{A}(\varepsilon) = \frac{\pi}{\displaystyle\int_M^\infty \gamma'' e^{-\gamma}\,dt}.$$

We now set

$$\gamma(t) := \begin{cases} -\log(-\eta(t)), & t \le M \\ -\delta\log(t - M + a) + b, & t > M, \end{cases}$$

where $a$, $b$ are chosen in such a way that $\gamma \in C^{1,1}(\mathbb{R}_+)$, that is

$$a = a(M) = \delta \frac{\eta(M)}{\eta'(M)},$$
$$b = b(M) = -\log(-\eta(M)) + \delta \log a.$$

We set $\delta = \delta(M) := M^{-1/2}$, so that in particular

$$\lim_{M\to\infty} a(M) = \infty. \tag{10}$$

One can easily check that on $\{t \le M\}$ by (7) $\gamma$ satisfies (9) and $(\gamma')^2/\gamma'' \le 1$. On $\{t > M\}$ we have $(\gamma')^2/\gamma'' = \delta$ and for sufficiently large $M$, since $-\delta \log(t - M + a) + t$ is increasing in $t$,

$$\left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^{\gamma+t} \ge (1-\delta) e^{-\log(-\eta(M))+M} \ge 1.$$

Moreover,

$$\int_M^\infty \gamma'' e^{-\gamma}\, dt = \delta e^{-b} \int_0^\infty (t+a)^{\delta-2} dt = \frac{-\eta'(M)}{1-\delta}$$

and this tends to 1/C as $M \to \infty$. Finally, we note that, if $0 < \widetilde{\varepsilon} \le \varepsilon$,

$$\begin{aligned}\int_{\{|\zeta|\le\widetilde{\varepsilon}\}} (1 - |\bar\partial\psi|^2_{i\partial\bar\partial\psi}) e^{\psi - 2\log|\zeta|} d\lambda &= \pi \int_{-2\log\widetilde{\varepsilon}}^\infty \left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^\gamma dt \\ &= \pi(1-\delta)e^b \int_{-2\log\widetilde{\varepsilon}-M}^\infty (t+a)^{-\delta} dt \\ &= \infty\end{aligned}$$

which ensures that $u = 0$ on $\{z_n = 0\}$. Defining the extension $F$ as in the proof of Theorem 3 gives the required result when $D = \{|\zeta| < 1\}$.

If $D$ is arbitrary we set $G := G_D(\cdot, 0)$, $\alpha$ is defined as before, and we modify the definitions of $\widetilde{\varphi}$, $\psi$ to $\widetilde{\varphi} := \varphi + 2G$, $\psi := \gamma(-2G)$. We have to show that

$$\limsup_{\varepsilon\to 0^+} \int_{\{|\zeta|\le\varepsilon\}} \frac{(\chi'(-2\log|\zeta|))^2 e^{\gamma(-2G)-2G}}{4|\zeta|^2\, |G_{\bar\zeta}|^2 \gamma''(-2G)}\, d\lambda(\zeta) \le \frac{C\,\pi}{(c_D(0))^2}. \tag{11}$$

We see that

$$\lim_{\varepsilon\to 0^+} \sup_{\{|\zeta|\le\varepsilon\}} |\zeta|^2 e^{-2G} = \frac{1}{(c_D(0))^2}. \tag{12}$$

We can write $G = \log|\zeta| + h$, where $h$ is a harmonic function. Then for some constant $A$ (independent of $\varepsilon$)

$$4|\zeta|^2\, |G_{\bar\zeta}|^2 = |1 + 2\bar\zeta h_{\bar\zeta}|^2 \ge (1 - A\varepsilon)^2 \tag{13}$$

if $\varepsilon$ is sufficiently small.

With the notation $t = -2\log|\zeta|$, we have $|2G + t| \le B$ for some constant $B$. We now modify the definition of $M$ to $M := -2\log\varepsilon - B$ and define $\gamma$ as before. Then for $t \ge -2\log\varepsilon$ we have $-2G \ge t - B \ge M$ and

$$\gamma(-2G) \le \gamma(t - B) \le \gamma(t) + \delta \log \frac{a + B}{a} \tag{14}$$

$$\gamma''(-2G) \ge \gamma''(t + B) \ge \left(\frac{a - B}{a}\right)^2 \gamma''(t). \tag{15}$$

We also have

$$\int_{M+B}^{\infty} \gamma'' e^{-\gamma}\, dt = \frac{-\eta'(M)}{1 - \delta} \left(\frac{a + B}{a}\right)^{\delta} \frac{a}{a - B/\eta'(M)}. \tag{16}$$

Combining (12)–(16) with (10) we now get (11). □

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**Remark.** After this paper was completed the optimal constant in the Suita conjecture and the Ohsawa–Takegoshi extension theorem was finally obtained in [**7**] building up on the methods developed here. However, Chen's proof from [**8**] of the extension theorem without optimal constant, presented here in a slightly different form, is probably the simplest one so far.

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