Probing ^{6}He structure from proton inelastic collisions

R. Crespo*, A.M. Moro†, I.J. Thompson**, M. Rodríguez-Gallardo†, J. Gómez-Camacho† and J.M. Arias†

*Departamento de Física, Instituto Superior Técnico, Av. Prof. Cavaco e Silva, Taguspark, 2780-990 Porto Salvo, Oeiras, Portugal
† Departamento de Física Atómica Molecular y Nuclear, Facultad de Física, Universidad de Sevilla, Apartado 1065, 41080 Sevilla, Spain
**Physics Department, University of Surrey, Guildford, Surrey GU2 7XH, U.K.

Abstract. We explore the Hyperspherical Harmonics pseudostate method to describe the ^{6}He continuum. The method is use it within the multiple scattering of the transition amplitude (MST) approach to study inelastic scattering of p-^{6}He at 700 MeV/u.

Keywords: ^{6}He continuum, Inelastic scattering, Halo nuclei.
PACS: 24.10.-i, 24.10.Ht, 25.40.Cm

INTRODUCTION

The structure of the low-lying continuum in ^{6}He has been subject to several theoretical studies using a variety of approaches: The Hyperspherical Harmonics scattering method (HHS) [1], the complex rotation method [2], the adiabatic HH approach [3] and, more recently, the Transformed Harmonic Oscillator (THO) method [4]. These methods predict a variety of different resonances in the continuum but they disagree with respect to the 1^- low-lying state which appears as a resonance only in the adiabatic HH framework.

By definition a resonance should not depend upon the excitation mechanism and we shall explore here the ^{6}He continuum through proton inelastic collisions at intermediate energies. In order to obtain information on the structure of the halo nucleus the reaction mechanism needs to be well understood. We shall be using here the multiple scattering expansion of the total transition amplitude (MST) [5] which has been subject to a very detailed analysis for the case of elastic scattering [6]. The HH pseudostate method has already been succesfully applied to describe the ^{11}Li continuum which, combined with the MST scattering framework, allowed the study of inelastic collisions from protons at intermediate energies [5].

It is the aim of the present work to explore the low-lying ^{6}He continuum using the HH pseudostate method. Particular attention will be given to the choice of the square integrable basis used in the expansion of the wave function for the excited states.

CP831, *Frontiers in Nuclear Structure, Astrophysics, and Reactions: FINUSTAR*
edited by S. V. Harissopulos, P. Demetriou, and R. Julin

SCATTERING FORMALISM

The ^{6}He borromean halo system is assumed here to be well described by a three-body model of two valence nucleons (labeled 2 and 3) and a core (labeled 4). Let us consider that the ^{6}He, in the initial ground state $|\Psi_{\varepsilon_0}^{J_0M_0}\rangle$, interacts with a proton with momentum and spin $|\vec{k}_1\chi_{S_1}^{\sigma}\rangle$ leading to the final state $|\vec{k}_1'\chi_{S_1}^{\sigma'};\Psi_{\varepsilon_i}^{JM}\rangle$ in the continuum. Within the single scattering approximation, the double cross section is

$$\frac{d\sigma_{JJ_0}}{d\Omega d\varepsilon_i} = \frac{1}{(\hat{S_1})^2}\sum_{\sigma\sigma'}|\frac{\hbar^2}{4\pi^2\mu_{1A}}\langle\vec{k}_1'\chi_{S_1}^{\sigma'};\Psi_{\varepsilon_i}^{JM}|\hat{t}_{12}+\hat{t}_{13}+\hat{t}_{14}|\vec{k}_1\chi_{S_1}^{\sigma};\Psi_{\varepsilon_0}^{J_0M_0}\rangle|^2 \quad . \tag{1}$$

In here, $\hat{t}_{1\mathscr{I}}$ are the transition amplitudes for the scattering from the valence neutrons and the core. The proton - $\mathscr{I}$ subsystem transition amplitudes satisfy

$$\hat{t}_{1\mathscr{I}} = v_{1\mathscr{I}} + v_{1\mathscr{I}}G_0\hat{t}_{1\mathscr{I}} \quad , \tag{2}$$

with $v_{1\mathscr{I}}$ is the interaction between the proton and the $\mathscr{I}$ subsystem. The propagator is $G_0 = (E^+ - K)^{-1}$, where E is the kinetic energy in the overall center of mass frame. Within the impulse approximation, K contains the kinetic energy operators of the projectile and all the target subsystems.

STRUCTURE

Assuming the alpha core is inert and spinless, the three-body bound-state and continuum wave functions can be written in the T-basis as

$$\Phi_{\varepsilon_i}^{JM}(\vec{x},\vec{y},\vec{\xi}_c) = \varphi_{\varepsilon_i}^{JM}(\vec{x},\vec{y})\otimes\varphi_{\text{core}}(\vec{\xi}_c) \quad , \tag{3}$$

where $\varphi_{\text{core}}(\vec{\xi}_c)$ is the core internal wave function and $\varphi_{\varepsilon_i}^{JM}(\vec{x},\vec{y})$ the three-body (3b) valence wave function relative to the core with scaled coordinates $\vec{x} = 2^{-1/2}\vec{r}$ from the relative coordinate between the valence nucleons, and $\vec{y} = (2A_{\text{core}}/(2+A_{\text{core}}))^{1/2}\vec{R}$ from the core position to the center of mass of the neutron pair. The 3b valence wave function is either bound $\varphi_{\varepsilon_0}^{JM}(\vec{x},\vec{y})$ or in the continuum $\varphi_{\varepsilon_i}^{JM}(\vec{x},\vec{y})$.

For the purpose of evaluating the wave functions, the nature of the Borromean system makes it convenient to introduce a set hyperradial coordinates: the hyperradius ρ and five hyperspherical polar angles $\Omega_5 = \{\alpha,\theta_x,\phi_x,\theta_y,\phi_y\}$. Expanding both the bound-state and continuum wave functions using HH basis functions, the 3b wave function has the general form

$$\varphi_{\varepsilon_i}^{JM}(\vec{x},\vec{y}) = \rho^{-5/2}\sum_{\mathscr{K}\ell_x\ell_yLS} F_{\mathscr{K}\gamma}^{J\varepsilon_i}(\rho)\Upsilon_{\mathscr{K}\gamma}^{JM}(\Omega_5) \quad . \tag{4}$$

The quantum number $\mathscr{K} = \ell_x + \ell_y + 2n$ (n=0,1,2, ...) is called hyperangular momentum and γ represents the set of quantum numbers $\gamma = \{\ell_x\ell_yLS\}$. The radial wave functions $F_{\mathscr{K}\gamma}^{J\varepsilon_i}(\rho)$ are found by solving a coupled channel Schrödinger problem with a three body

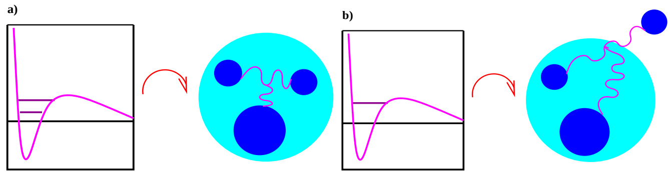

FIGURE 1. a) Schematic representation of a potential pocket for the sum of all binary interactions in the case of a true 3b resonance (T3bR). b) Schematic representation of a potential pocket for the case of a long lived binary resonace (LL2bR)

effective centrifugal barrier and coupling potentials. The coupling potentials include all pairwise interactions between the valence nucleons and the core. To simulate the Pauli principle between the interacting neutrons of the halo and of the core a repulsive part in the *s* component was included. In addition the coupling potentials include an effective 3b potential.

The ^{6}He ground state wave function, $\varphi_{\varepsilon_0}^{JM}(\vec{x},\vec{y})$, is the solution of the coupled equations for $\varepsilon_0 \sim -0.97$ MeV. The spatial density is highly correlated and is dominated by a "di-neutron" and a "cigar" peak [1].

As for the continuum within the HH expansion there are several methods for solving the coupled equations which differ essentialy in the asymptotic behaviour of the wave function: the scattering method (HHS) and the Pseudostates (HHPS). These methods are numerically very demanding specially if the potential barrier is very low and shallow. Within the the pseudo-states approach, the coupled equations are solved in a hyperradial box. This involves expanding the wave function with an square-integrable basis, and then diagonalising the Hamiltonian matrix. The set of these energy eigenfunctions will be a good representation of continuum states at least in the interior regions.

The (HHS) has been used to study the continuum of Borromean nuclei. A variety of type of excitations were founded for these nuclei [1]

- True 3b resonances (T3bR) which arise from pockets in *all* the diagonal components of the coupling potentials and are caused by the interaction of *all the 3 particles* in the interior domain as represented schematically in Fig. (1 a)
- Long-lived binary resonances in *one* of the constituent pairs (LL2bR) as represented schematically in Fig. (1 b)
- Resonances due to *some* of nondiagonal couplings between the potentials corresponding to dominant channels (DCC)

It was found in [1] that 2_1^+ and 1^+ are T3bR, 0_2^+, 0^- are no resonances and 1^- a LL2bR.

The HHPS method is applied in this work to ^{6}He considering two basis: The Gauss-Laguerre and the Transformed Harmonic Oscillator (THO) basis. Within the the former the hyperradial basis, $R_n(\rho)$, is

$$R_n(\rho) = {\rho_0}^{-3}[n!/(n+5)!]^{1/2} L_n^5(z) \exp(-z/2) \tag{5}$$

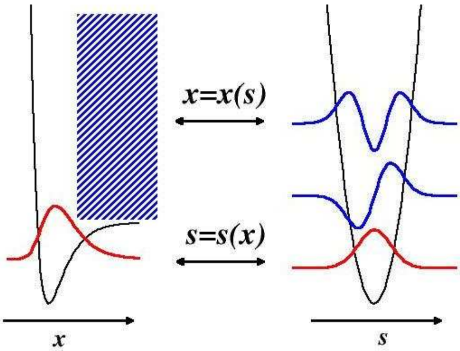

FIGURE 2. THO transformation

with $z = \rho/\rho_0$, ρ_0 a parameter to set the radial scale of the basis, orthonormalised such that

$$\int_0^\infty \rho^5 R_n(\rho) R_{n'}(\rho) = \delta_{nn'} \ , \tag{6}$$

The radial functions for positive energies are then the quasi-bound states, $\hat{\varphi}^{JM}_{\hat{\varepsilon}_i}(\vec{r},\vec{R})$

$$\hat{\varphi}^{JM}_{\hat{\varepsilon}_i}(\vec{x},\vec{y}) = \rho^{-5/2} \sum_{\mathscr{K}\gamma} \hat{F}^{J\hat{\varepsilon}_i}_{\mathscr{K}\gamma}(\rho) \Upsilon^{JM}_{\mathscr{K}\gamma}(\Omega_5) \tag{7}$$

with

$$\hat{F}^{J\kappa_i}_{\mathscr{K}\gamma}(\rho) = \sum_{n=1}^{N} c^{nJ\hat{\varepsilon}_i}_{\mathscr{K}\gamma} R_n(\rho) \tag{8}$$

Within the THO approach, the basic idea is to perform a local scale transformation (LST) for each channel $\{\mathscr{K}\gamma\}$ included in the ground state wave function $s_{\{\mathscr{K}\gamma\}}(\rho)$ such that

$$\int_0^\rho d\rho' |R^{\varepsilon_0 J}_{\mathscr{K}\gamma}(\rho')|^2 = \int_0^s ds' |R^{HO}_{0\mathscr{K}}(s')|^2 \tag{9}$$

Once the LST for each state is obtained, the THO basis is defined by applying a polynomial transformation

$$R^{THO}_{nJ\mathscr{K}\gamma}(\rho) = R^{\varepsilon_0 J}_{\mathscr{K}\gamma}(\rho) L^{\mathscr{K}+2}_n \left(s^2_{\mathscr{K}\gamma}(\rho) \right) \tag{10}$$

where $L^{\mathscr{K}+2}_n$ are generalized Laguerre polynomials with n the index of the polynomial. For J different from the ground state a slight different method is used [4]. The THO states are not eigenstates of the Hamiltonian, but provide a complete orthonormal set, where the Hamiltonian can be diagonalized. The procedure is represented schematically in Fig. 2. The advantages of the method are (i) the basis has the correct behaviour at the origin and, for bound states, the proper asymptotic behaviour, (ii) it is a more complete basis than the Gauss-Laguerre since it is a basis for each component of a state $\{nJ\mathscr{K}\gamma\}$.

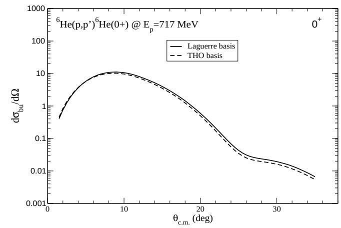

FIGURE 3. Calculated differential cross section for p-^{6}He at 700 MeV for the 0+ state using the Gauss-Laguerre basis (solid line) and THO basis (dashed line)

RESULTS AND CONCLUSIONS

Fig. 3 shows that the calculated differential cross section for p-^{6}He at 700 MeV for the 0^+ state using the Gauss-Laguerre basis (solid line) and THO basis (dashed line) are identical. We conclude that although the two basis have a different asymptotic behaviour the calculated cross section is the same and thus probes essentially the interior of the continuum wave function. Therefore the results suggest that the HH pseudo state method is a valid approach used to describe the continuum for the purpose of studying inelastic scattering of halo nuclei from protons.

Further work is underway to evaluate the contributions from the other states to the scattering.

ACKNOWLEDGMENTS

The financial support of FCT from grant POCTI/FNU/43421/2001 and Acção Integrada Luso-Espanhola E-75/04 is gratefully acknowledged.

REFERENCES

1. B.V. Danilin, I.J. Thompson, M.V. Zhukov and J.S. Vaagen, Nucl. Phys. **A632**, 383 (1998).
2. T. Myo, K. Kato, S. Aoyama, K. Ikeda, Phys. Rev. C **63**, 054313 (2001).
3. A. Cobis, D.V. Fedorov and A.S. Jensen, Phys. Rev. C **58**, 1403 (1998).
4. M. Rodríguez-Gallardo et al, Phys. Rev. C **72**, 024007 (2005).
5. R. Crespo, I.J. Thompson and A.A. Korsheninnikov, Phys. Rev. C **66**, 021002 (R) (2002).
6. R. Crespo, A.M. Moro and I.J. Thompson, *Four-body Multiple Scattering expansion of the total Transition amplitude - MST*,to be published in Proceedings of the LV National Conference, St. Petersbourg, Russia, (2005).