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$E2$ transitions and quadrupole moments in the E(5) symmetry

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$E2$ transitions and quadrupole moments are studied in the recently proposed E(5) symmetry by using the intrinsic state formalism. It is shown that the values of these magnitudes can be obtained for the different bands to higher order in the boson number N by projecting the intrinsic state on γ and β variables. The formalism allows to find easily the dependence of those magnitudes on the structure parameter of the quadrupole operator, χ.

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I. INTRODUCTION

Recently a new class of dynamic symmetry has been proposed by Iachello [1]. This symmetry is expected to be of use when analyzing systems undergoing phase transitions between traditional dynamic symmetries. In particular, the example presented in Ref. [1] considers the Bohr Hamiltonian [2] and discusses the case in which the potential is γ independent and in addition the β dependence of the potential is modeled by a five-dimensional infinite well. This seems to be applicable in nuclear spectroscopy when nuclei are at the critical point in a transition from spherical to γ-unstable shape. The E(5) symmetry is discussed in [1] in connection with the interacting boson model (IBM) [3]. Energy levels are given and transition probabilities for selected states are calculated by using the following quadrupole operator depending linearly on β:

$$T_{\mu}^{(E2)}=t\beta\left[\mathcal{D}_{\mu 0}^{(2)}(\theta_i)\cos\gamma+\frac{1}{\sqrt{2}}(\mathcal{D}_{\mu 2}^{(2)}(\theta_i)+\mathcal{D}_{\mu -2}^{(2)}(\theta_i))\sin\gamma\right], \tag{1}$$

where t is a scale factor. Experimental examples of this new class of symmetry have already been proposed [4].

Within the geometrical model the case of γ independent potential surface was discussed some time ago by Wilets and Jean [5], while the equivalent situation within the IBM is known as the O(6) limit and was discussed first in Ref. [6]. In both cases the energy surface has a definite equilibrium value for β, being otherwise γ independent. On the other hand, the vibrational Bohr Hamiltonian [2] and the corresponding SU(5) limit in IBM [7] provide with a situation in which the energy surface has equilibrium value $\beta=0$ and is γ independent too. As mentioned above, the newly proposed E(5) symmetry seems to be appropriate when discussing potentials with flat behavior as a function of some coordinate, as it could be the case of the β coordinate in a SU(5)-O(6) transition.

First, in Sec. II a brief review of the example of E(5) symmetry presented in Ref. [1] is given. In Sec. III the formalism used is developed, including the intrinsic state description and the projection onto the laboratory frame. Results are presented in Sec. IV. Finally, Sec. V is devoted to a summary.

II. THE E(5) SYMMETRY

Consider the Bohr Hamiltonian

$$H=-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}-\frac{1}{4\beta^2}\sum_{\kappa}\frac{Q_{\kappa}^2}{\sin^2(\gamma-\frac{1}{3}\pi\kappa)}\right]+V(\beta,\gamma), \tag{2}$$

where β,γ are the shape variables and the Q_{κ}'s are the components of the angular momentum written in terms of Euler angles. In cases in which the potential depends only on β, $V(\beta,\gamma)=U(\beta)$, the wave function can be factorized as

$$\Psi(\beta,\gamma,\theta_i)=f(\beta)\Phi(\gamma,\theta_i), \tag{3}$$

where θ_i stands for the three Euler angles, and the Schrödinger equation can be split into two equations,

$$\left[-\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}+\frac{1}{4}\sum_{\kappa}\frac{Q_{\kappa}^2}{\sin^2(\gamma-\frac{2}{3}\pi\kappa)}\right]\Phi(\gamma,\theta_i)=\tau(\tau+3)\Phi(\gamma,\theta_i);\quad \tau=0,1,2,\ldots, \tag{4}$$

and

$$\left[-\frac{\hbar^2}{2B}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}-\frac{\tau(\tau+3)}{\beta^2}\right)+U(\beta)\right]f(\beta)=Ef(\beta). \tag{5}$$

If $U(\beta)$ can be modeled as a five dimensional infinite well, the problem is exactly solvable and the corresponding symmetry is called E(5). The solutions of the Schrödinger equations in β and (γ,θ_i) with the appropriate boundary conditions are known [1]. The wave functions on β are

$$f_{\xi,\tau}(\beta)=C_{\xi,\tau}\beta^{-3/2}J_{\tau+3/2}\left(\frac{x_{\xi,\tau}}{\beta_w}\beta\right), \tag{6}$$

where τ is the label associated to the O(5) algebra, ξ is a label that enumerates the zeros of the relevant Bessel function, $x_{\xi,\tau}$ is the ξth zero of the Bessel function $J_{\tau+3/2}(x)$, $C_{\xi,\tau}$ are normalization constants, and β_w is the range of the potential in the β variable.

The solutions of the (γ,θ_i) part were studied in Ref. [5] and tabulated in Ref. [12], where $\Phi_{\tau,L,M}(\gamma,\theta_i)$ are written in terms of $\mathcal{D}$ functions as

$$\Phi_{\tau,L,M}(\gamma,\theta_i)=\sum_{\rho} g_{\tau,L,\rho}(\gamma)\mathcal{D}^{(L)}_{M\rho}(\theta_i). \tag{7}$$

The intrinsic functions $g_{\tau,L,\rho}(\gamma)$ are explicitly given in [12] and only even values of ρ appear in the sum.

The E(5) states are labeled by $|N;\xi\tau LM\rangle$. N is the boson number, ξ is a label related to the solution of the Schrödinger equation in the β variable as mentioned above, τ is the label associated to the O(5) algebra, L is the total angular momentum, and M its projection on one axis.

III. THE FORMALISM

In this work it is shown that the values of the $E2$ transitions and quadrupole moments in the E(5) dynamic symmetry can be obtained by using the intrinsic state formalism in IBM [8–10] and projecting on the appropriate variables. The starting point is that the $E2$ transition operator in IBM is written as

$$T^{(E2)}_{\mu}=qQ^{(sd)}_{\mu}, \tag{8}$$

where q is a scale factor and $Q^{(sd)}$ is the IBM quadrupole operator,

$$Q^{(sd)}_{\mu}=(s^{\dagger}\tilde{d}+d^{\dagger}s)^{(2)}_{\mu}+\chi(d^{\dagger}\tilde{d})^{(2)}_{\mu}. \tag{9}$$

The parameter χ is its structure constant. The operators $\tilde{d}_m=(-1)^m d_{-m}$ are introduced so as to have tensors with the appropriate properties under spatial rotations.

The basic idea of the intrinsic frame formalism is to consider that the pure quadrupole states are globally described by a boson condensate of the form

$$|g\rangle=\frac{1}{\sqrt{N!}}(\Gamma^{\dagger}_{g})^{N}|0\rangle, \tag{10}$$

where the basic boson is given by

$$\Gamma^{\dagger}_{g}=\frac{1}{\sqrt{1+\beta^2}}\left[s^{\dagger}+\beta\cos\gamma d^{\dagger}_{0}+\frac{1}{\sqrt{2}}\beta\sin\gamma(d^{\dagger}_{2}+d^{\dagger}_{-2})\right], \tag{11}$$

which depends on the β and γ shape variables. The equilibrium values for β and γ are obtained by minimizing the energy surface in the boson condensate (10). In the SU(5) and O(6) limits of IBM this minimization leads to definite values of β ($\beta_0=0$ and $\beta_0=1$, respectively) and the energy surface is γ independent in both cases. At the critical point in the phase transition from SU(5) to O(6) the energy surface is expected to be rather flat in the β variable and the symmetry E(5) seems to be appropriate. When calculating $E2$ transitions and moments in the traditional [SU(5) or O(6)] IBM limits, the β variable is fixed in Eqs. (10) and (11) to its equilibrium value and integration on γ has to be performed. In the E(5) case, since the behavior of the energy surface is flat in both β and γ variables, integration on both variables has to be done.

Electromagnetic $E2$ transition rates and quadrupole moments are evaluated by taking matrix elements of the quadrupole operator (8) and (9). These matrix elements in the boson condensate $|g\rangle,\langle g|Q^{(sd)}_{m}|g\rangle\equiv Q^{(sd)}_{m}(\beta,\gamma)$, have already been calculated [11],

$$Q^{(sd)}_{0}(\beta,\gamma)=\frac{N}{1+\beta^2}\left[2\beta\cos\gamma-\sqrt{\frac{2}{7}}\chi\beta^2\cos2\gamma\right], \tag{12}$$

$$Q^{(sd)}_{2}(\beta,\gamma)=Q^{(sd)}_{-2}(\beta,\gamma)=\frac{1}{\sqrt{2}}\frac{N}{1+\beta^2}\left[2\beta\sin\gamma+\sqrt{\frac{2}{7}}\chi\beta^2\sin2\gamma\right]. \tag{13}$$

The matrix elements of $Q^{(sd)}$ not specified are zero. In the O(6) limit the IBM $E2$ transition operator is usually defined with $\chi=0$ since in that case it is a generator of O(6) and definite selection rules appear. This approximation has proved to be good for studying nuclei at the O(6) limit, but for transitional SU(5)-O(6) nuclei at the critical point the quadrupole operator could depend on χ. Thus, in the following the general form in Eqs. (12) and (13) will be kept.

With the help of Eqs. (6) and (7), states in the laboratory can be obtained from the boson condensate (10) and (11) as

$$|N;\xi\tau LM\rangle=f_{\xi,\tau}(\beta)\Phi_{\tau,L,M}(\gamma,\theta_i)|g\rangle. \tag{14}$$

Thus, the matrix elements of the quadrupole operator are given by

$$\begin{aligned}\langle N;\xi\tau LM|Q^{(sd)}_{\mu}(\mathrm{lab})|N;\xi'\tau'L'M'\rangle&=\langle N;\xi\tau LM|\sum_{m}Q^{(sd)}_{m}(\mathrm{int})\mathcal{D}^{(2)}_{\mu m}(\theta_i)|N;\xi'\tau'L'M'\rangle\\&=\int d\Omega\int\beta^4 d\beta\int|\sin3\gamma|d\gamma f^{*}_{\xi,\tau}(\beta)\Phi^{*}_{\tau,L,M}(\gamma,\theta_i)\\&\quad\times\sum_{m}\langle g|Q^{(sd)}_{m}(\mathrm{int})|g\rangle\mathcal{D}^{(2)}_{\mu m}(\theta_i)f_{\xi',\tau'}(\beta)\\&\quad\times\Phi_{\tau',L',M'}(\gamma,\theta_i),\end{aligned} \tag{15}$$

where the IBM quadrupole operator in the laboratory has been transformed to the intrinsic frame by using $\mathcal{D}$ functions. The intrinsic matrix elements $\langle g|Q^{(sd)}_{m}(\mathrm{int})|g\rangle$ are those given in Eqs. (12) and (13).

IV. RESULTS

Within the scheme presented in the preceding section, the calculation of matrix elements of the quadrupole operator implies integration on the variables β and γ, as well as on the Euler angles θ_i, in addition to the relevant matrix elements in the intrinsic frame, Eqs. (12) and (13).

Thus, the quadrupole matrix elements are given by

$$\langle N;\xi\tau LM|Q_{\mu}^{(sd)}(\mathrm{lab})|N;\xi'\tau'L'M'\rangle$$
$$=\frac{8\pi^2}{2L+1}\langle 2\mu L'M'|LM\rangle\sum_{m\rho\rho'}\langle 2mL'\rho'|L\rho\rangle$$
$$\times\int|\sin 3\gamma|d\gamma g_{\tau,L,\rho}(\gamma)$$
$$\times\left[\int\beta^4 d\beta f_{\xi,\tau}(\beta)Q_m^{(sd)}(\beta,\gamma)f_{\xi',\tau'}(\beta)\right]g_{\tau',L',\rho'}(\gamma), \tag{16}$$

where the integration on the Euler angles has already been done. Only even values of m, ρ, and ρ' appear in the sum. With this expression it is straightforward to calculate $E2$ transition and moments since the integrals in β and γ can be easily evaluated.

The quadrupole moments of the different states (ξ,τ,L) are given by [q is the scale factor in Eq. (8)]

$$Q(\xi,\tau,L)$$
$$=\sqrt{\frac{16\pi}{5}}\langle N;\xi\tau LM=L|qQ_0^{(sd)}(\mathrm{lab})|N;\xi\tau LM=L\rangle. \tag{17}$$

The corresponding quadrupole moments for some selected states are

$$Q(\xi=1,\tau=1,L=2)=0.1425N\chi q, \tag{18}$$

$$Q(\xi=1,\tau=2,L=2)=-0.0514N\chi q, \tag{19}$$

$$Q(\xi=1,\tau=2,L=4)=0.2400N\chi q, \tag{20}$$

$$Q(\xi=1,\tau=3,L=6)=0.3112N\chi q, \tag{21}$$

$$Q(\xi=1,\tau=3,L=4)=0.0751N\chi q, \tag{22}$$

$$Q(\xi=2,\tau=1,L=2)=0.1202N\chi q, \tag{23}$$

$$Q(\xi=2,\tau=2,L=2)=-0.0428N\chi q, \tag{24}$$

$$Q(\xi=2,\tau=2,L=4)=0.1998N\chi q. \tag{25}$$

The calculation of $E2$ transition probabilities can be done straightforward from

$$B(E2;\xi,\tau,L\rightarrow\xi',\tau',L')$$
$$=\frac{1}{2L+1}|\langle N;\xi\tau L||qQ^{(sd)}(\mathrm{lab})||N;\xi'\tau'L'\rangle|^2. \tag{26}$$

The corresponding $B(E2)$ transition rates for some selected transitions are

$$B(E2;\xi=1,\tau=1,L=2\rightarrow\xi=1,\tau=0,L=0)=0.1459N^2q^2, \tag{27}$$

$$B(E2;\xi=1,\tau=2,L=2\rightarrow\xi=1,\tau=0,L=0)=0.0044N^2\chi^2q^2, \tag{28}$$

$$B(E2;\xi=1,\tau=2,L=2\rightarrow\xi=1,\tau=1,L=2)=0.2282N^2q^2, \tag{29}$$

$$B(E2;\xi=1,\tau=2,L=4\rightarrow\xi=1,\tau=1,L=2)=0.2282N^2q^2, \tag{30}$$

$$B(E2;\xi=1,\tau=3,L=6\rightarrow\xi=1,\tau=2,L=4)=0.2806N^2q^2, \tag{31}$$

$$B(E2;\xi=2,\tau=0,L=0\rightarrow\xi=1,\tau=1,L=2)=0.0710N^2q^2, \tag{32}$$

$$B(E2;\xi=2,\tau=0,L=0\rightarrow\xi=1,\tau=2,L=2)=0.0082q^2N^2\chi^2, \tag{33}$$

$$B(E2;\xi=1,\tau=3,L=0\rightarrow\xi=1,\tau=1,L=2)=0.0090N^2\chi^2q^2, \tag{34}$$

$$B(E2;\xi=1,\tau=3,L=0\rightarrow\xi=1,\tau=2,L=2)=0.2806N^2q^2. \tag{35}$$

TABLE I. Comparison of some $B(E2)$ ratios in ^{134}Ba with the E(5) symmetry. Experimental data are from Ref. [14].

	$\frac{B(E2;4^+_{1,2}\rightarrow 2^+_{1,1})}{B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})}$	$\frac{B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})}{B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})}$	$\frac{B(E2;0^+_{2,0}\rightarrow 2^+_{1,2})}{B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})}$	$\frac{B(E2;0^+_{1,3}\rightarrow 2^+_{1,1})}{B(E2;0^+_{1,3}\rightarrow 2^+_{1,2})}$
E(5) [a]	1.68	0.86	0	0
E(5) [b]	1.56	0.49	0.12	0.032
Expt.	1.56(18)	0.42(12)	0.18(8)	0.037(3)

[a]E(5) as calculated in Ref. [1] with operator (1).

[b]E(5) from this work with the IBM quadrupole operator (8) and (9) and $\chi=1$.

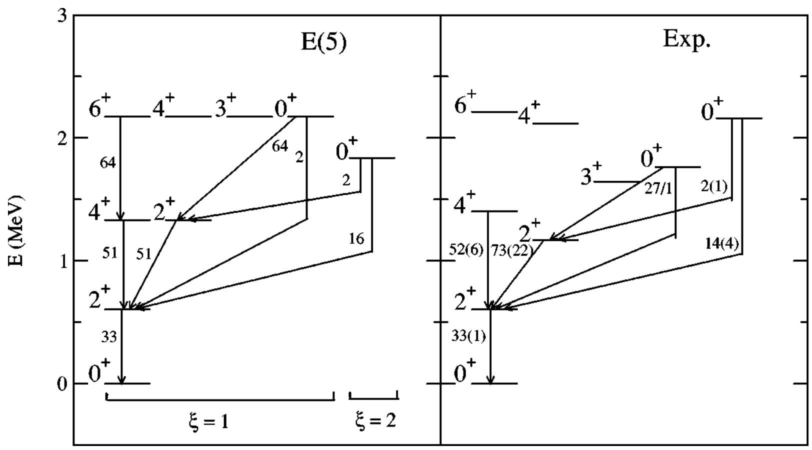

FIG. 1. Level schemes and $B(E2)$ values (in W.u.) for ^{134}Ba in the E(5) symmetry with $\chi=1$ (left) and corresponding experimental data [14] (right). The scale factor q in Eq. (8) has been adjusted to match the experimental $B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})$ value. For the decay from the state 0^+ at 1.761 MeV to the states 2^+_2 and 2^+_1 only the branching ratio (27/1) is known.

It has been checked that these results converge to known results in simpler situations. On the one hand, the O(6) values [13] are obtained if the integral in β, between brackets in Eq. (16), is substituted by $Q^{(sd)}_m(1,\gamma)$ $(\beta=1)$. On the other hand, these results reduce to those presented in Ref. [1], where the $E2$ operator (1) is used, if χ is taken as zero in the IBM quadrupole operator (8) and (9) and the normalization factor $2/(1+\beta^2)$ in Eqs. (12) and (13) is substituted by 1. With these changes the IBM quadrupole operator used here reduces to the $E2$ transition operator used in [1].

^{134}Ba has been proposed [4] as a first evidence in nuclear physics of the E(5) symmetry. In Table I some important $E2$ branching ratios for this nucleus are compared with the results obtained in the E(5) symmetry. Experimental data are taken from Ref. [14]. The notation used for denoting the states is $L^{\pi}_{\xi,\tau}$. Two kinds of E(5) results are shown. The results E(5) labeled with (a) are those taken from Ref. [1]. Due to the form of the transition quadrupole operator used there, Eq. (1), transitions $B(E2;0^+_{2,0}\rightarrow 2^+_{1,2})$ and $B(E2;0^+_{1,3}\rightarrow 2^+_{1,1})$ are forbidden. The results E(5) labeled with (b) are those obtained in this work in which the $E2$ transition operator is Eqs. (8) and (9) with $\chi=1$. It is observed in Table I that the formalism presented here allows for even a better description of the ^{134}Ba when comparing to the newly proposed E(5) symmetry. This improvement comes from the two differences the IBM quadrupole operator, Eqs. (8),(9) and Eqs. (12),(13), introduces with respect to the usually used quadrupole operator (1). On the one hand, the inclusion of the term depending on β^2 (χ) in Eqs. (8) and (9) is crucial to describe the ratios $B(E2;0^+_{2,0}\rightarrow 2^+_{1,2})/B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})$ and $B(E2;0^+_{1,3}\rightarrow 2^+_{1,1})/B(E2;0^+_{1,3}\rightarrow 2^+_{1,2})$. On the other hand, the normalization factor $2/(1+\beta^2)$ in Eqs. (12) and (13) improves the description of the ratios $B(E2;4^+_{1,2}\rightarrow 2^+_{1,1})/B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})$ and $B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})/B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})$. Therefore, the IBM transition operator (8) and (9), with intrinsic matrix elements (12) and (13), seems to provide a better description of the experimental data than the operator (1).

One important point as a signature of E(5) symmetry in comparison with the O(6) case is the transition $B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})$. This is forbidden in the O(6) limit even if one considers the general form of the quadrupole operator including the χ term, while it gives the correct ratio $B(E2;0^+_{2,0}\rightarrow 2^+_{1,1})/B(E2;2^+_{1,1}\rightarrow 0^+_{1,0})$ in the E(5) limit with $\chi=1$.

In Fig. 1 the observed $B(E2)$ transition rates in ^{134}Ba [14] are compared with the results obtained in this work assuming $\chi=1$. Units are given in W.u. For the decay from the state 0^+ at 1.761 MeV to the states 2^+_2 and 2^+_1 only the branching ratio is known 27/1. This branching ratio is nicely reproduced in the calculation.

V. SUMMARY

In this paper it has been presented how to use the intrinsic state formalism to evaluate electromagnetic transition rates and quadrupole moments in the recently proposed E(5) symmetry. It has been shown that dealing with β and γ dependent objects in the intrinsic frame can be done easily. The same technique can be used to calculate expectation values of other observables. In addition, it has been shown that the IBM $E2$ operator provides a better description of the data than the operator (1) used in Ref. [1].

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