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A Hybrid Information Approach to Predicting Corporate Credit Risk*

Di Bu¹, Simone Kelly³, Yin Liao^{†2}, and Qing Zhou^{1,4}

¹*Macquarie University, Australia*

^{2*}*Business School, Queensland University of Technology, 2 George St, Brisbane City QLD 4000,
Australia*

³*Department of Finance, Business School, Bond University, 14 University Dr, Robina QLD
4226, Australia*

⁴*School of Management, Xi'an Jiaotong University, China*

Abstract

This article proposes a hybrid information approach to predict corporate credit risk. In contrast to the previous literature that debates which credit risk model is the best, we pool information from a diverse set of structural and reduced-form models to produce a model combination based credit risk prediction. Compared with each single model, the pooled strategies yield consistently lower average risk prediction errors over time. We also find that while the reduced-form models contribute more in the pooled strategies for speculative grade names and longer maturities, the structural models have higher weights for shorter maturities and investment grade names.

Keywords: Corporate Credit Risk, Bond Spread, Structural Model, Reduced-form Model, Model Combination.

JEL classification: C22, G13

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[†]Corresponding author. Email: yin.liao@qut.edu.au, Fax: +61 7 3138 1500, Phone: +61 7 3138 2662

1 Introduction

The valuation and prediction of corporate credit risk is an important topic in both empirical and theoretical research. The structural and reduced-form models are two competing paradigms in this research field, and the literature (Jarrow and Protter, 2004) differentiates between the two modeling frameworks from the information perspective. In structural models, the modelers, as the firms' managers, are assumed to have complete knowledge of the firms' assets and liabilities. The corporate default, therefore, occurs when the firm's value hits a default barrier. In contrast, the reduced-form models show that the modelers have incomplete knowledge of the firm's conditions as normal market participants, in which the firm's default time is not accessible and can be simply specified by a hazard rate process. While early studies debate which modeling framework between the two better captures credit risk and conclude that all the credit risk models consistently underpredict corporate bond spreads (e.g., Eom, Helwege, and Huang, 2004, Collin-Dufresne and Goldstein, 2001), in this paper, we marry the two modeling frameworks together and propose a model combination approach to improve corporate credit risk prediction.

From the modeling framework, we choose two classic representatives to form the model pool. The first structural model we consider is the Merton model (Merton, 1974), which serves as the cornerstone for all the other structural models. The Merton model regards corporate liabilities as contingent claims on the assets of firms and applies option theory to derive the value of a firm's liabilities in the presence of default. In doing so, the firm's equity value can be viewed as a call option on the value of the firm's assets, and default will occur if the firm's asset value is not enough to cover the firm's liabilities. Despite the Merton model building up the theoretical foundation for the structural models, most assumptions in the model are not held in reality. Therefore, we consider the second structural model, the Black-Cox model (Black and Cox, 1976), in which the default occurs before the end of the debt maturity. Additionally, instead of considering only a single type of debt, the model allows for a tranche structure in the senior and subordinated bonds. The formulations of the two structural models are consistent with the manager's perspective that the firm's condition is observable and that default is an accessible stopping time.

The two classic reduced-form models we considered here include the Jarrow and Turnbull model (Jarrow and Turnbull, 1995) and the Duffie and Singleton model (Duffie and Singleton,

1999). The two reduced-form models treat the default as an unpredicted event given by a hazard process; hence, the firm will default when the exogenous random variable changes its level over a certain time interval. As a result, the default event is not dependent on the value of the firm's asset. Specifically, the Jarrow and Turnbull (JT) model assumes that the recovery rate is exogenous and that recovery can only be received at the time of maturity if the default occurs prior to maturity. The Duffie-Singleton (DS) model extends the JT model by allowing the recovery payment to be made at any time. Therefore, constructing reduced-form models presumes that the market does not have the same information set as the firm's management. The imperfect knowledge of the market is due to the fact that accounting reports and/or management press releases either purposefully or inadvertently add extraneous information that obscures the knowledge of the firm's asset value (Cetin, Jarrow, Protter, and Yildirim, 2004), leading to an inaccessible default time.

Given the distinct information foundation of these models, we propose combining the aforementioned four credit risk model representatives to construct a hybrid information-based forecast for corporate credit spread. Model combination has been widely used in econometric forecasting since the pioneer work by Bates and Granger (1969). The model combination method was later extended by Granger and Ramanathan (1984) and has spawned much literature. Some excellent reviews include Clemen (1989), Diebold and Lopez (1996), Clements, Hendry, and College (2002) and Timmermann (2006). Recently, forecast combinations have received renewed attention in the macroeconomic forecasting literature (e.g., Stock and Watson, 2003) and increasing attention in finance (e.g., Rapach, Strauss, and Zhou, 2010, O'Doherty, Savin, and Tiwari, 2012, Durham and Geweke, 2014). Because the underlying market condition changes over time, the firm asset returns and default events are generated from different data-generating processes (DGPs) over distinct economic states. Thus, there is no single model dominating all others in all the market conditions or economic states. The combination of different models with dynamically updated weights would allow for this model uncertainty. In addition, previous empirical studies (e.g., Gündüz and Homburg, 2014) suggest that the reduced-form approach outperforms the structural models for investment-grade names and longer maturities, and the structural approach performs better for shorter maturities and sub-investment grade names. Given the cross-sectional dispersion of the model performance in different types of corporate debts, the model combination would result in better performance on average across a wide range of corporate debts. To accommodate the above arguments, we implement a bias-variance trade-off framework to achieve

the combination. We first decompose the forecast errors of each individual model into bias and variance components. Then, we determine the optimal weights for individual models by achieving global minimum variance. Last, we correct the bias by assuming the prediction bias this period is the same as that of the last period. The pooled model, therefore, has the minimum variance and negligible bias.

We next gauge the empirical performance of our combined models in corporate bond spread prediction. Our dataset consists of 279,826 monthly corporate bond yield spreads to the swap rate of non-callable bonds issued by industrial firms over the period 1992-2016. We first explore the ability of both the combined model and all the individual models to explain the cross-sectional variation of bond spreads across different maturity ranges and credit ratings. We find that the performance of the combined model is constantly superior to other four individual models with 99% confidence level for all maturity/rating buckets in terms of root mean square error (RMSE). When looking at the performance of the individual models, we find that the reduced-form approaches outperform the structural for speculative-grade credit bonds and longer maturities, while the structural models do better for investment-grade credit and shorter term bond spreads. Structural models assume complete knowledge of a very detailed information set, akin to that held by the firm's managers, while reduced-form models assume knowledge of a less detailed information set, akin to that observed by the market (Jarrow and Protter, 2004). Taking this insight, we can interpret the results from information perspective by saying that the lower the credit rating and longer the term of the bonds are, the harder it is for bond holders to access the complete knowledge of the bond's condition. Therefore, the assumption of reduced-form models is more realistic than that of structural models, resulting in a better empirical performance for speculative grade and longer term bonds. Similar results for individual models are also reported by Gündüz and Homburg (2014). Second, using time-series regression, we test for whether the combined model can also better capture the time variations in corporate bond spread than the individual models and find that the combined model also significantly outperforms in both stable and volatile periods.

Our work makes three contributions to the corporate credit risk literature. First, it improves the performance of credit risk models for corporate credit spread forecasts by combining the two well-known competing model classes. As a barometer of financial health of corporations and sovereign entities, an accurate forecast for corporate credit spread is useful for corporate and government decision-making. The better risk prediction from our combined models improves the

pricing of credit derivatives for private traders; the measurement of corporate risk for regulatory agencies; and the assessment of systemic credit risk for macroeconomic policymakers. Second, model combination is an intuitive and easy-to-implement approach to integrate different sources of information, and our work is one of the few studies in the credit risk area to propose a model pooling approach, which acknowledges the advantage of utilizing hybrid corporate default-related information in credit risk prediction. Third, we contribute to the literature by examining the time-varying and cross-sectional performance of the individual popular credit risk models and investigate the economic rationale of pooling the models as well as an application of the pooled model in a real investment practice.

The remainder of this paper is organized as follows. Section 2 presents the two structural and reduced-form models and interprets their difference from the information perspective. Section 3 describes the procedure to construct the combined corporate bond spread forecasts from the three models. Section 4 provides an empirical analysis of the performance of combined model prediction using 2,436 corporate bonds, and Section 5 concludes.

2 Credit risk models and model calibration

In this section, we consider four classic credit risk models, including two structural models (Merton, 1974, Black and Cox, 1976), and two reduced form models (Jarrow and Turnbull, 1995, Duffie and Singleton, 1999). We describe each of the model's set-up and the methods we used to calibrate the model parameters.

2.1 Structural models

The Merton model: Merton (1974) laid the foundation on the structural approach to credit risk modeling. In this model, the asset value of a firm at time t , S_t , is assumed to follow a geometric Brownian motion, which is governed by the drift and volatility rate parameters μ and σ as follows:

$$\log S_t = \log S_{t-\tau} + (\mu - \frac{1}{2}\sigma^2)\tau + \sigma\sqrt{\tau}dW_t^S, \quad (1)$$

where both the drift μ and the volatility μ are constant.

Given that the firm has two types of outstanding claims, they are an equity and a zero-coupon debt maturing at time T with face value F , the following accounting identity holds for

every time t as

$$S_t = E_t + D_t, \quad (2)$$

where E_t and D_t are, respectively, the market value of equity and debt at time t . When debt matures, the default occurs in the event that the firm's assets are less than the face value of the debt, i.e., $S_T < F$. Otherwise, equity holders repay the debt and keep the balance. Therefore, the payout to the debt holders at the maturity time T is

$$D_T = \min(S_T, F), \quad (3)$$

and the equity holders, on the other hand, receive at time T

$$E_T = \max(S_T - F, 0). \quad (4)$$

Therefore, the firm's equity can be regarded as if it is a call option on the total asset value S of the firm with the strike price of F and the maturity date T . Assuming the risk-free interest rate is r , the equity claim in (4) can be priced at time $t < T$ by the standard Black-Scholes option pricing model to yield the following solution:

$$E_t = E(S_t; \sigma_t^2, F, r, T - t) = S\Phi(d_t) - Fe^{-r(T-t)}\Phi(d_t - \sigma\sqrt{T-t}), \quad (5)$$

where

$$d_t = \frac{\ln(\frac{S_t}{F}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \quad (6)$$

and Φ is the standard normal distribution function. Note that the equity pricing formula is not a function of the drift term μ and is invertible with respect to the asset value.

Once the parameter estimates are obtained from the Merton model, we can first generically compute the distance to default as the number of standard deviations between the expected asset value at maturity T and the face value of the debt:

$$DD = \frac{-\log L + (\mu - \sigma^2/2)T}{\sigma\sqrt{t}}, \quad (7)$$

where $L = (\frac{F}{S_0})$ is the firm leverage ratio. Second, the cumulative default probability of the

Merton model at time T can be calculated as:

$$\pi^P(T, Merton) = 1 - N(DD), \quad (8)$$

where $N(DD)$ is the cumulative distribution function of DD , and π^P is the cumulative probability that the asset value falls below the face value of the debt at the end of the time horizon T . Third, the credit spread of a risky corporate bond is defined as the premium required to compensate for the expected loss in the event of default, that is, $s_t = y_t - r$, where y_t is the yield of the risky corporate bond, and r is the risk-free interest rate. According to the payoff corporate debt holders receive, the risky debt can be priced by the difference between a default-free debt and a put option on the total asset value S_t of the firm with the strike price of F and the maturity date T . Therefore, we have

$$B_t = Fe^{-r(T-t)} - P_t, \quad (9)$$

where F is the face value of the zero coupon debt at the maturity time, and P_t is the price of a put option on the asset value S_t with the strike price F and the maturity date T . Then, the yield y_t of the risky corporate bond can be derived from

$$e^{-y_t(T-t)}F = B_t, \quad (10)$$

and the credit spread s_t can be computed as

$$s_t = y - r = -\frac{1}{T} \ln[1 - (1 - R)\pi^Q(T, Merton)], \quad (11)$$

where R is the recovery rate, T is the bond maturity and $\pi^Q(T, Merton)$ is the risk-neutral default probability, which is obtained by replacing μ with r in $\pi^P(T)$.

The Black-Cox model: Black and Cox (1976) extend the original Merton model by removing some unrealistic assumptions. First, while the Merton model allows for the firm to default only at the end of the maturity, Black and Cox (1976) add safety covenants that entitle debt holders to force the firm to reorganize when its value falls below a threshold and receive a discounted value of the debt's principal amount. Second, the original Merton model assumes the firm has only single-type debt, but the corporate debt has a tranche structure that causes the

subordinated bonds to receive no payments until all payments for the senior bonds have been made. Therefore, the tranche structure should be allowed in the firms' debt, and the Black-Cox model incorporates this debt characteristic.

Again, Black and Cox (1976) assume that a firm's asset value follows a geometric Brownian motion but allow for the payout rate to debt:

$$\log S_t = \log S_{t-\tau} + (\mu - \frac{1}{2}\sigma^2 - \delta)\tau + \sigma\sqrt{\tau}dW_t^S, \quad (12)$$

where δ is the payout rate to debt and equity holders. Different from the Merton model, the firm defaults the first time the asset value is below some fraction d of the face value of debt, rather than the end of the maturity period. The cumulative default probability of the Black-Cox model at time T is:

$$\pi^P(T, B - C) = N(-(\frac{-\log(dL) + (\mu - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}})) \quad (13)$$

$$+ \exp(\frac{2\log(dL)(\mu - \delta - \frac{\sigma^2}{2})}{\sigma^2})N(\frac{\log(dL) + (\mu - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}), \quad (14)$$

where $L = \frac{F}{S_0}$ is the leverage ratio. Once we obtain $\pi^P(T, B - C)$, the firm credit spread again can be calculated using equation 11.

2.2 Reduced-form models

The Jarrow and Turnbull model: the reduced-form models were originally introduced by Jarrow and Turnbull (1995). In contrast with structural models where the default time is endogenously determined and corresponds to the hitting time of the default barrier, both default timing and recovery rate are exogenously specified in the reduced-form model. The default time is a stopping time generated by a Cox process $N_t = 1_{\tau \leq t}$ with an intensity process λ_t depending on the vector of state variables X_t (often assumed to follow a diffusion process). In this formulation, the stopping time is totally inaccessible and not predictable. The probability of default prior to time T is therefore given by

$$Q(\tau \leq T) = E^Q(e^{-\int_0^T \lambda_s ds}). \quad (15)$$

To complete this formulation, Jarrow and Turnbull (1995) also give the payoff to the firm's debt in the event of default, called the recovery rate. This is usually given by a stochastic process δ_t . To be consistent with the structural model in the previous section, the recovery rate δ_τ is paid at time T .

The value of the firm's debt is therefore given by

$$B_t = E([1_{\tau \leq t} \delta_\tau + 1_{\tau > t} 1] e^{-\int_0^t r_s ds}). \quad (16)$$

For example, if the recovery rate (δ) and intensity processes (λ) are constants, then this expression can be evaluated explicitly, generating the model in Jarrow and Turnbull (1995) where the debt's value is given by

$$B_t = P_t(Q(\tau \leq T)\delta + (1 - Q(\tau \leq T))e^{\lambda T}), \quad (17)$$

where $P_t = E^Q(e^{-\int_0^t r_s ds})$.

The Duffie-Singleton model: similar to Jarrow and Turnbull (1995), Duffie and Singleton (1999) also assumes a Poisson process for defaults. Unlike the Jarrow-Turnbull model, the Duffie-Singleton model assumes that recovery is paid immediately upon default and equals a fraction of what the bond is worth immediately prior to default. In our formulation, it means:

$$w(t) = qB_t, \quad (18)$$

where q is the constant recovery ratio on the value of the bond prior to default. Substituting this result back into equation 15, we obtain

$$Q^*(\tau \leq T) = E^Q(e^{-(1-q)\int_0^T r_s ds}) = Q(\tau \leq T)^{1-q}. \quad (19)$$

It is clear that in the Duffie-Singleton model, recovery is blended into survival probabilities. In other words, recovery in the Duffie-Singleton model contains survival probabilities. Therefore, the debt's value is given by

$$B_t = P_t(Q^*(\tau \leq T)\delta + (1 - Q^*(\tau \leq T))e^{\lambda T}). \quad (20)$$

2.3 Model calibration

We calibrate the two structural model parameters using a recent approach by Feldhütter and Stephen (2017). First, we estimate the default boundary (that is L in the Merton model and dL in the Black-Cox model) by matching the model-implied default probability with the Moody’s reported default frequency. Different from previous studies (see Chen, Collin, and Goldstein (2009) for example) that estimate the default boundary separately for each maturity and rating, conditional on the other parameters, Feldhütter and Stephen (2017) assume that all the firms have the same default boundary and use a wide cross-section of default rates at different maturities and ratings to estimate the default boundary. Specifically, given the estimates of the issuing firms’ asset return (μ), asset volatility (σ), and payout ratio (δ) (the estimation procedure for these parameters is standard, in which we refer to Feldhütter and Stephen (2017) regarding the details). This approach fits the historical rates on all available ratings and maturities and estimates the default boundary by minimizing the sum of absolute deviations between annualized model-implied and historical default rates:

$$\min_d \sum_{a=AAA}^C \sum_{T=1}^{20} \frac{1}{T} |\pi^P(T, Merton) - \hat{\pi}_{aT}^P|, \quad (21)$$

where $\hat{\pi}_{aT}^P$ is the historical cumulative default rate for rating a and maturity T . Second, we calculate the credit spread using equation (11) above by obtaining $\pi^Q(T, \cdot)$ from $\pi^P(T, \cdot)$ and set the recovery rate to $R = 37.8\%$, which is Moody’s average recovery.

In reduced-form models, we need three pieces of information to complete the calibration: risk free zero yield curve, a set of risky bond prices, and a recovery assumption. In both the Jarrow-Turnbull and Duffie-Singleton models, we again assume a recovery rate of 37.8%. Given the price, face value and coupon collected for each specific firm bond, the default probability can be estimated by

$$\hat{\lambda}_t = \operatorname{argmin} \sum [B_i^{obs}(t) - B_i(t)]^2. \quad (22)$$

The firm credit spread can be calculated accordingly based on the model-implied bond price B_t and the risk-free rate.

3 Model combination framework

Using the corporate bond spreads obtained from the above four models, we can form the combined forecast for corporate bond spread in the following way:

$$\hat{s}_{t|t-1} = \alpha_0 + \sum_i^3 \omega_{i,t} \hat{s}_{i,t|t-1}, \quad (23)$$

where $\hat{s}_{t|t-1}$ is the weighted combination of the predicted bond spreads from the considered individual credit risk models; α_0 is the estimated bias correction term; and $\hat{s}_{i,t|t-1}$ and $\omega_{i,t}$ are the predicted bond spreads from each single model and the corresponding weights of the model in the model combination. Obviously, the key input in (23) is the model combination weights $\omega_{i,t}$. Several weighting schemes have been proposed in the literature, and here, we adopt the optimal weighting scheme, which obtains the model weights throughout, minimizing the mean squared forecast errors (MSFE).

It is well known that the MSFE loss function can be decomposed into forecasting bias and forecasting variance as follows:

$$E(\hat{s} - s_0)^2 = E[(s_0 - E(\hat{s})) + (E(\hat{s}) - \hat{s})]^2, \quad (24)$$

where \hat{s} is the bond spread forecast, and s_0 is the true bond spread. Assuming that biases are not correlated with random errors, we can rearrange the above expression as

$$E(\hat{s} - s_0)^2 = \underbrace{E[s_0 - E(\hat{s})]^2}_{Bias^2} + \underbrace{E(E(\hat{s}) - \hat{s})^2}_{Variance}, \quad (25)$$

which provides an explicit interpretation on how the MSFE loss is determined by both the forecasting bias and variance.

Due to the trade-off relationship between the bias and variance (Geman, Bienenstock, and Doursat, 1992), that is, the lower variance (bias) is necessarily associated with the greater bias (variance), we propose a bias-corrected optimal weighting scheme for the model combination, which obtains the model weights by minimizing the forecast variance at a given bias level that takes the value of forecasting bias in the last time period and subsequently corrects for the biases by simply removing it from the achieved combined forecasts. The resulting bias-corrected combined forecast will therefore have the smallest possible variance but also exhibit negligibly

small bias. Intuitively, the trade-off relationship between the model forecast bias and variance is analogous to the mean-variance (return-risk) trade-off in modern portfolio theory. We therefore rely on the global-minimum-variance (GMV) portfolio theory to find the model combination weights, which achieves the minimum forecasting variance at first. The corresponding model combination weights and the combined forecast bias are calculated as follows:

$$\omega = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \quad (26)$$

and

$$S = \frac{s'\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \quad (27)$$

In equations (26) and (27), the parameter to be estimated is the inverse variance-covariance matrix of forecasting errors from single models and is critical for the overall performance of the combined forecast. One of the most direct ways is to use the sample covariance estimator. The bias is removed accordingly to further reduce *MSFE*.

We further illustrate the rationality of the bias-corrected optimal weighting scheme in Figure 1, in which forecast bias (on the *y*-axis) is plotted against forecast variance (on the *x*-axis). The scattered internal dots represent the status of diverse individual forecasts that exhibit various levels of bias and variance. Building on the trade-off approach, we generate a hypothetical "estimation frontier". Point *G* represents a combination of single forecast that produces minimum variance, and point *U* represents an unbiased combination of single forecast. The consensus bond spreads forecast should feature negligible bias and minimum variance, which is represented by point *O* in Figure 1. Point *O* indicates the ideal situation of an unbiased forecast with minimum variance. This ideal condition cannot be feasibly achieved by any single individual forecast. In essence, our strategy for achieving outcome *O* is to use a global minimum variance weighting scheme to create an optimal combination of individual bias-corrected estimators.

[Insert Figure 1 here]

4 Data

Our bond data are extracted from REUTERS and supplemented by the Merrill Lynch corporate bond index database, which is also used in Schaefer and Strebulaev (2008) and Feldhütter and Stephen (2017). The data covers the period from December 1992 to March 2016. Follow-

ing previous studies, the data sample covers senior unsecured bonds issued by corporate firms without the following bond characteristics: floating rate coupons, issued by banks, government guaranteed, and with special clauses, and financial or government related firms and bonds with embedded options, such as convertible or callable bonds. Additionally, we use only bonds issued by industrial firms and ones with a maturity of less than 20 years, which is consistent with the maturities of the default rates used in Feldhütter and Stephen (2017). Applying these refinements, this paper obtains a sample of 2,436 corporate bonds, which leads to 279,826 monthly bond observations. The bond characteristics in the dataset contain yield spreads, issued amount, coupon rate, and issue date.

Moody’s credit rating and SWAP rates are collected from REUTERS, and the SWAP rates are used as the proxy of interest rates and to estimate the bond yield spreads. The firm characteristic variables, equity return volatility and leverage ratio, are obtained from the Center for Research in Security Prices (CRSPs) database, and the corporate financial statement information is collected from the COMPUSTAT database.

Table 1 reports the summary statistics of bonds spreads in basis points, denoted bps. As expected, the mean and standard deviation of bond spreads are greater for bonds with greater default risk. The monthly mean spread on AAA-rated bonds is 18 bps with a standard deviation of 33 bps, and for C bonds, the mean and standard deviation are, respectively, 1,217 bps and 1,864 bps. For some of our analysis, we rely on groupings into investment-grade (IG, BBB-rated and above) and speculative-grade (SG, below BBB-rated) bonds. For this grouping, we find that the IG and SG bonds spreads are, respectively, 68 and 452 bps, and the respective standard deviations are 87 and 759 bps.

[Insert Table 1 here]

5 Model performance evaluation

To assess the empirical performance, we compare the combined model with the four single models in terms of both forecasting bias and root mean square error (RMSE) of corporate bond spreads. The forecasting bias and RMSE are defined as $E(s - \hat{s})$ and $E(s - \hat{s})^2$, respectively, where \hat{s} is the model predicted bond spread and s is the actually observed bond spread. We further employ the Diebold and Mariano (1995) (DM) test to investigate whether the bond spreads prediction improvements from the combined model are statistically significant and use *, ** and *** to

indicate the significant superiority at 10%, 5% and 1% significance levels in the empirical results. More details of the DM test are provided in the Appendix.

5.1 Cross-sectional performance

First, we investigate the empirical performance of the model combination across different groups of bonds by classifying the bonds based on their term to maturity and credit rating.

5.1.1 Term structure of bond spreads

To understand the dependence of model performance on bond maturity, we follow Feldhütter and Stephen (2017) to classify the corporate bonds based on their term to maturity into three segments: 3-7 years, 7-13 years and 13-20 years, and compare the forecasting performance of the model combination with the four individual models for all the bonds and each segment of bonds.

Table 2 shows the bias and RMSE of bond spread forecasts from each model scheme, and Table 3 reports the model weights in the model combinations. There are several noteworthy findings summarized as follows. First, it is clear that although the structural and reduced-form models perform quite similarly when we focus on all the sample of bonds, the structural models outperform the reduced-form models for the bonds with maturities below 13 years, and the reduced-form models perform better than the structural models for bonds with longer maturities (13-20 years). Taking the view of Jarrow and Protter (2004), we can interpret the results from information perspective by saying that the longer the investors hold the bonds, the harder it is for them to access the complete knowledge of the firm's condition. In this case, the assumption of reduced-form models is more realistic than that of structural models, resulting in a better empirical performance. Second, the combined model outperforms all the individual models for all maturity ranges by providing smaller RMSE, and the DM test results confirm that the superiority is significant.

[Insert Table 2 here]

In addition, looking at Table 3 for the model optimal weights, we can see that more weights are given to the two structural models due to their superior performance for bond maturity ranging from 3-7 years, and the reduced-form models take higher weights for longer term maturity bonds. These observations further collaborate our previous arguments that the structural models outperform (underperform) the reduced-form models in forecasting the spread of bonds with

shorter (longer) maturity. The model pool outperforms each single model across all the ranges of bonds.

[Insert Table 3 here]

5.1.2 Model prediction breakdown to ratings

Next, we test the impact of credit rating on model performance by further dividing the bond sample based on credit rating. Due to the small number of bonds in AAA and B rating categories, we combine AAA and AA into one rating group and BB and B into one rating group to form AAA/AA and BB/B rating categories, which brings our total groups to five: AAA, AA/A, BBB, BB/B, and C. Table 4 reports the performance of all models in terms of bias and RMSE of bond spread forecasting for different credit rating categories, and Table 5 reports the model weights in the model combinations.

[Insert Table 4 here]

First, focusing on individual model performance, we find that the RMSE of bond spread forecasts from both the structural and reduced-form models increase as the credit rating worsens. In the bond group with longer terms to maturity, we observe that the reduced-form models outperform the structural models for speculative grade bonds (B and C rated). While turning to the bond group with short terms to maturity, the structural models perform better in pricing investment grade bonds (BBB rated and above). These findings are consistent with the results found in the section term structure above. Furthermore, the predicted spreads from the combined model track the actual bond spreads in all categories more precisely than the counterparts from each of the individual models, and the DM test shows that the superior performance is significant. Last, the bias-variance trade-off framework efficiently corrects the bias and successfully achieves the minimum variance in the model combination.

[Insert Table 5 here]

5.2 Time-series performance

The above sections examine the cross-sectional performance of the model combination across different groups of bonds, and in this section, we investigate the time-series performance of the model combination.

5.2.1 Time variation of bond spreads

We first employ a time-series regression to investigate whether the combined model also better captures time variations of bond spreads than the individual models. In each day, we calculate the average actual yield spread for a given rating along with the corresponding model-implied average spread and investigate the daily time series.

To reveal whether the above-documented prediction improvements are statistically significant, we regress the predicted bond spreads $\hat{s}_{i,t}$ from each model on the actual bond spreads $s_{i,t}$ for bonds in different rating categories:

$$s_{i,t} = \beta_0 + \beta_1 \hat{s}_{i,t} + \varepsilon_{i,t}, i = 1, \dots, 20 \quad (28)$$

where $s_{i,t}$ is the actual bond i spread at time t , and $\hat{s}_{i,t}$ is the model predicted spread of bond i at time t .

Both β_1 and R^2 of the regressions for different credit ratings are presented in Table 6. We report R^2 instead of the sum-of-squared errors of the fitted regression, as the two measures convey the same information, but the former better shows how much time variation of the actual spreads has been explained by the model's predicted ones. The results shown in Table 6 are consistent with what we find in the sections above that the ability of all the credit risk models to explain the time-series variation of bond spreads declines when moving from high rating category to the low rating category, as can be observed by the lower R^2 in speculative grade categories.

Next, we observe that the optimal forecast hypothesis (that is, $\beta_1 = 1$) is rejected in all the model predicted spreads, but β_1 is closer to one for the combined model in all five bond spreads categories. These findings provide supportive evidence that the biased and inefficient spread predictions are improved by the forecast combination. This is further corroborated by the increase of R^2 of the combined model across the individual models in all the cases. In general, the regression-based model comparison results suggest that in all the cases the combined model is able to better capture bond spreads' time-series variation than all the individual models.

These findings are further illustrated in Figures 2 and 3, which plot real daily average investment grade bond spreads and speculative bond spreads in conjunction with model estimated spreads for maturity from 3-20 years. The figures show that the estimated spreads for all models track the investment-grade bond spreads better than speculative-grade bond spreads and that

the combined model is able to better capture the spreads variation.

[Insert Table 6 here]

5.3 Model performance in different financial market conditions

We further investigate the model performance across different market conditions. We split the sample into two sub-periods. The period from March 1992 to March 2001 is defined as a stable period. We define the period from April 2001 to August 2009 as a volatile period, since it experiences both the dot-com bubble collapse and the Global Financial Crisis (GFC), in which the volatility of both stock return and bond spreads is relatively high. We then test the impact of credit rating on model performance by dividing the sample into both investment grade and speculative grade categories. We analyze the patterns of bond spread predictions from different models during both volatile and stable periods for different credit rating categories.

In Table 7, the DM tests show that the combined model significantly outperforms all the individual models in both stable and volatile periods for both investment and speculative grade categories with only one exception. The exception occurs only with the reduced-form model in volatile periods for the speculative grade category where the combined model does not significantly outperform the reduced form models. Table 7 also shows that using the bias and RMSE criteria, the Merton model performs the best among all individual models yielding the best results with RMSE of 4.17 bps for investment-grade bond spreads category in stable period. For the speculative-grade bond spreads category, the DS model outperforms other individual models in volatile period with RMSE of 82.29 bps.

[Insert Table 7 here]

In Table 8, we can see that the highest weight 33% is placed on the Merton model in the combined model during stable period for the investment grade category due to its superior performance, while during the volatile periods from 4/2001 to 08/2016, the reduced-form models outperform the structural models' speculative-grade categories. In the volatile period, more weight is given to the two reduced models.

[Insert Table 8 here]

[Insert Figure 2 here]

[Insert Figure 3 here]

6 Conclusion

This paper proposes a model combination approach to improve the corporate credit risk prediction from the conventional structural and reduced-form models. We implement the bias-variance trade-off framework to combine the forecasts from two structural and two reduced-form models, and we study the properties of the bond spread forecasts from the model combination via a set of empirical analyses. The empirical results verify the superior out-of-sample forecasting performance of the model combination compared with each individual model and provide empirical guidance about how to combine the structural and reduced-form models for different types of corporate bonds and under different economic scenarios.

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7 Appendix

The null hypothesis of the DM test is that the RMSE of two forecasting models is equivalent. In our empirical analysis, we are particularly interested in whether the combined model is able to significantly outperform the individual models in terms of RMSE.

Here, each single-model forecast is an obvious benchmark. To highlight the role of combination, the combined forecasts will be compared to each single-model forecast. To achieve this, the pairwise test for equal predictive accuracy (EPA) of Diebold and Mariano (1995) (DM) is employed. Let $\mathcal{L}(f_t^a)$ and $\mathcal{L}(f_t^b)$ represent a generic loss function defined on two competing bond spread forecasts f_t^a and f_t^b ; then, the relevant null and alternative hypotheses are

$$\begin{aligned} H_0 &: \mathbb{E}[\mathcal{L}(H_t^a)] = \mathbb{E}[\mathcal{L}(H_t^b)] \\ H_A &: \mathbb{E}[\mathcal{L}(H_t^a)] \neq \mathbb{E}[\mathcal{L}(H_t^b)]. \end{aligned} \quad (29)$$

The null hypothesis of the test is that the predictive ability of the two forecasting models is equivalent.

The test is based on the computation of

$$DM_T = \frac{\bar{d}_T}{\sqrt{\widehat{\text{var}}[\bar{d}_T]}}, \quad \bar{d}_T = \frac{1}{T} \sum_{t=1}^T d_t, \quad d_t = \mathcal{L}(f_t^a) - \mathcal{L}(f_t^b), \quad (30)$$

where $\widehat{\text{var}}[\bar{d}_T]$ is an estimate of the asymptotic variance of the average loss differential, \bar{d}_T .

To begin, forecast performance will be compared using the simple root mean squared forecast error of the $i - th$ forecast, defined as

$$RMSE^i = \sqrt{\frac{1}{T} (s_t - f_t^i)^2}, \quad (31)$$

where T is the total number of forecast periods, f_t^i is the forecast from the $i - th$ model and s_t is the target. To implement the DM test, given the $i - th$ forecast, the MSE loss function is chosen to represent $\mathcal{L}()$,

$$MSE_t^i = (s_t - f_t^i)^2. \quad (32)$$

Table 1: Bond Spread Summary Statistics

	Number of bonds	Average spread	Standard Deviation	Amount outstanding
AAA	42	18	33	2107
AA	239	29	52	1164
A	698	71	103	1398
BBB	769	105	176	1429
BB	262	319	571	1587
B	72	589	893	1079
C	23	1217	1864	1570
IG	1748	68	87	1079
SG	357	452	759	1570

Note: Table 1 provides summary statistics on bond spreads by credit rating classes (in basis points). IG stands for bonds rated BBB and above. SG stands for bonds rated BB and below. "Average spread" is the average actual spread to the swap rate. The average spread is calculated by first calculating the average spread of bonds in a given month and then calculating the average of these spreads over months. The bond yield spreads are from the period from December 1996 to December 2016

Table 2: Model performance sorted according to term to maturity

Time to Maturity	Merton Model		Black Cox Model		JT Model		DS Model		Combined Model	
	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
Full sample period	-14.52	35.62***	-16.87	37.59***	15.35	38.26***	14.28	36.35***	8.26	25.31
3-7 Year	-11.38	27.41**	-10.13	27.09**	14.45	36.16***	15.27	34.42***	7.49	19.51
7-13 Year	-16.25	38.07***	-14.51	34.46***	-17.78	39.42***	-18.47	42.98***	11.76	25.82
13-20 Year	-17.12	42.46***	-18.69	44.05***	-15.26	34.73*	-14.89	36.07**	12.74	28.49

Note: Table 2 shows the bias and root mean square errors (RMSEs) of bond spreads predictions from each model for different time to maturity ranges: 3-7 year, 7-13 years and 13-20 years. The bias and RMSE of the model predicted spread are defined as $E(s - \hat{s})$ and $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread. Diebold and Mariano (1995)(DM) test is employed to reveal whether the bond spread-s prediction improvements from the combined model are statistically significant against all the individual models in terms of RMSE. "*" implies that the combined model RMSE is significantly different from the individual model RMSE with 10% level, "***" at the 5% level and "****" at the 1% level.

Table 3: Optimal weights of each individual model for different maturity ranges

Credit Rating	Merton Model	Black Cox Model	JT Model	DS Model
Whole sample period	28%	24%	23%	25%
3-7 year	31%	33%	17%	19%
7-13 year	25%	28%	23%	21%
13-20 year	16%	14%	36%	34%

Note:This table reports the weights of each individual model in the combined model for different time to maturity ranges: 3-7 year, 7-13 years and 13-20 years. The weights are obtained through minimizing the root mean square errors (RMSEs) of the combined model predicted spread. The RMSE of the model predicted spread is defined as $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread.

Table 4: Model performance sorted according to both credit rating and term to maturity

Time to Maturity	Merton Model		Black Cox Model		JT Model		DS Model		Combined Model	
	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
Full sample period										
AAA	0.96	2.74	0.75	2.65	-1.67	5.12***	-1.35	4.84***	0.62	1.92
A	3.19	9.62*	2.81	8.62*	-4.73	13.15***	-4.53	12.81***	-2.42	6.51
BBB	12.56	34.02***	13.28	36.29***	-15.72	44.14***	14.97	41.12***	8.45	26.93
B	-29.51	62.59***	-27.29	62.62***	21.95	65.39***	-23.91	58.85*	-15.37	50.52
C	-43.37	110.35***	45.81	113.25***	35.52	92.53**	-36.43	95.26***	-21.61	68.72
3-7 Years										
AAA	0.56	1.64	0.65	1.82	-1.27	3.62	-1.75	3.84	0.42	1.43
A	-1.28	4.15*	-2.03	4.81*	3.19	8.62	3.31	7.42	-2.02	5.51
BBB	8.72	26.02**	8.28	28.29***	-9.72	29.14***	9.97	31.12***	6.05	21.93
B	23.31	62.39***	-23.91	63.85***	-20.51	55.59***	-21.29	58.62***	-17.51	49.57
C	23.19	59.53***	-24.43	56.26**	-16.36	49.35***	17.81	48.25***	-12.61	37.72
7-13 Years										
AAA	-1.23	4.29	-2.07	5.61*	0.91	4.02	1.16	4.29*	-0.72	3.06
A	-5.24	12.91***	6.43	14.42***	4.38	10.35*	4.75	11.12*	-3.16	7.72
BBB	9.62	29.45***	9.08	27.68**	11.45	27.52***	12.94	28.93***	6.17	22.27
B	19.31	59.14***	-17.92	57.72***	-24.41	68.93***	26.48	-72.45***	-15.25	40.81
C	-48.76	127.58***	-43.91	121.75***	-72.81	182.79***	-81.52	214.39***	33.24	98.51
13-20 Years										
AAA	-2.46	6.74*	-2.23	5.29	-4.42	11.87**	5.93	12.84**	-1.58	4.25
A	-7.85	16.39***	-7.41	18.76***	-4.49	12.46*	-4.83	12.13*	-3.73	9.52
BBB	-18.74	44.53***	19.82	46.04***	18.53	45.78***	17.93	46.87***	-11.86	33.43
B	-36.36	87.92***	-41.97	90.03***	28.37	73.97**	29.61	74.42***	14.83	49.36
C	40.83	107.39***	-46.36	116.41***	32.92	79.53**	-34.79	81.64***	22.24	56.89

Note: This table reports bias and RMSEs of bond spreads predictions from each model. Bias and RMSE are grouped according to both remaining bond maturity and credit rating. The bias and RMSE of the model predicted spread are defined as $E(s - \hat{s})$ and $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread. Diebold and Mariano (1995) (DM) test is employed to reveal whether the bond spread prediction improvements from the combined model are statistically significant against all the individual models in terms of RMSE. "*" implies that the combined model RMSE is significantly different from the individual model RMSE with 10% level, "***" at the 5% level and "****" at the 1% level.

Table 5: Optimal weights of individual models for different credit ratings

Credit Rating	Merton Model	Black Cox Model	JT Model	DS Model
AAA	32%	34%	16%	18%
A	30%	31%	18%	21%
BBB	28%	27%	21%	24%
B	26%	28%	22%	24%
C	18%	17%	31%	34%

Note:This table reports the weights of each individual model in the combined model for different credit ratings. The weights are obtained through minimizing the root mean square errors (RMSEs) of the combined model predicted spread. The RMSE of the model predicted spread is defined as $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread.

Table 6: Regression-based forecasting comparison

	Merton Model		Black Cox Model		JT Model		DS Model		Combined Model	
	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2	β_1	R^2
AAA	0.78*	0.72	0.74*	0.71	0.95***	0.87	0.92***	0.85	0.92***	0.88
A	1.23	0.83	1.48	0.81	0.87***	0.85	0.86***	0.82	0.93***	0.91
BBB	0.86***	0.71	0.94***	0.73	1.31*	0.62	0.76*	0.68	0.89***	0.84
B	0.63*	0.23	0.71*	0.29	0.48	0.18	0.54	0.22	1.22**	0.42
C	0.56**	0.11	0.51**	0.09	0.42*	0.07	0.48*	0.08	0.62**	0.26

Note:For a given rating and maturity group, we calculate a monthly average spread by computing the average yield spread for bonds with the corresponding rating and maturity observed in that month. We do this for both model predicted spreads and actual bond spreads (to the swap rate) resulting in a time series of monthly actual spreads for each bond s_1, s_2, \dots, s_t and model predicted spreads from each model $\hat{s}_1, \hat{s}_2, \dots, \hat{s}_t$. The table shows the regression coefficient and R^2 in the regression of the actual bond spread on the model predicted spread $s_{i,t} = \beta_0 + \beta_1 \hat{s}_{i,t} + \varepsilon_{i,t}$. "*" implies that β_1 is significantly different from one at the 10% level, "***" at the 5% level and "****" at the 1% level.

Table 7: Model performance in different sample periods

Time to Maturity	Merton Model		Black Cox Model		JT Model		DS Model		Combined Model	
	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
Full sample period										
Investment grade	7.06	21.52***	6.75	20.65*	-10.17	29.12***	-10.85	30.84***	5.35	15.52
Speculative grade	33.62	79.62***	32.81	78.62***	-26.73	71.15***	-27.53	73.81***	18.47	52.81
Stable period										
Investment grade	4.17	13.15***	4.63	14.81***	8.09	23.61*	9.39	24.14	-2.52	8.51
Speculative grade	23.25	61.62***	24.21	63.29***	21.72	56.14***	20.91	54.11***	14.02	42.56
Volatile period										
Investment grade	-14.28	32.73***	-12.51	36.81***	12.21	35.62***	13.36	36.42***	7.21	20.31
Speculative grade	-39.37	96.28***	41.78	103.25***	30.81	81.02***	32.28	82.29***	22.52	59.93

Note: This table shows bias and RMSEs of bond spreads predictions from each model. Bias and RMSE are grouped according to both sub-sample period and credit rating. The full sample is split into two sub-periods. The period from December 1993 to March 2001 is defined as a stable period. We define the period from April 2001 to August 2009 as a volatile period during which the volatility of both the stock return and bond spreads is relatively high. We divide credit rating into investment-grade (BBB-rated and above) and speculative-grade (below BBB-rated) bonds. The bias and RMSE of the model predicted spread are defined as $E(s - \hat{s})$ and $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread. Diebold and Mariano (1995) (DM) test is employed to reveal whether the bond spread prediction improvements from the combined model are statistically significant against all the individual models in terms of RMSE. "*" implies that the combined model RMSE is significantly different from the individual model RMSE with 90% confidence, "***" at the 95% confidence and "****" at the 99% confidence.

Table 8: Optimal weights of individual models in different sample periods

Credit Rating	Merton Model	Black Cox Model	JT Model	DS Model
Whole sample period				
Investment grade	29%	28%	21%	22%
Speculative grade	21%	23%	29%	27%
Stable period				
Investment grade	33%	31%	17%	19%
Speculative grade	24%	25%	26%	25%
Volatile period				
Investment grade	22%	25%	28%	25%
Speculative grade	19%	17%	31%	33%

Note: This table reports the weights of each individual model in the combined model for both Investment grade (BBB-rated and above) and speculative-grade (below BBB-rated) and different sample periods. The full sample is split into two sub-periods. The period from December 1993 to March 2001 is defined as a stable period. We define the period from April 2001 to August 2009 as a volatile period during which the volatility of both stock return and bond spreads is relatively high. The weights are obtained through minimizing the root mean square errors (RMSEs) of the combined model predicted spread. The RMSE of the model predicted spread is defined as $E(s - \hat{s})^2$, where \hat{s} is the model predicted bond spread and s is the actual observed bond spread.

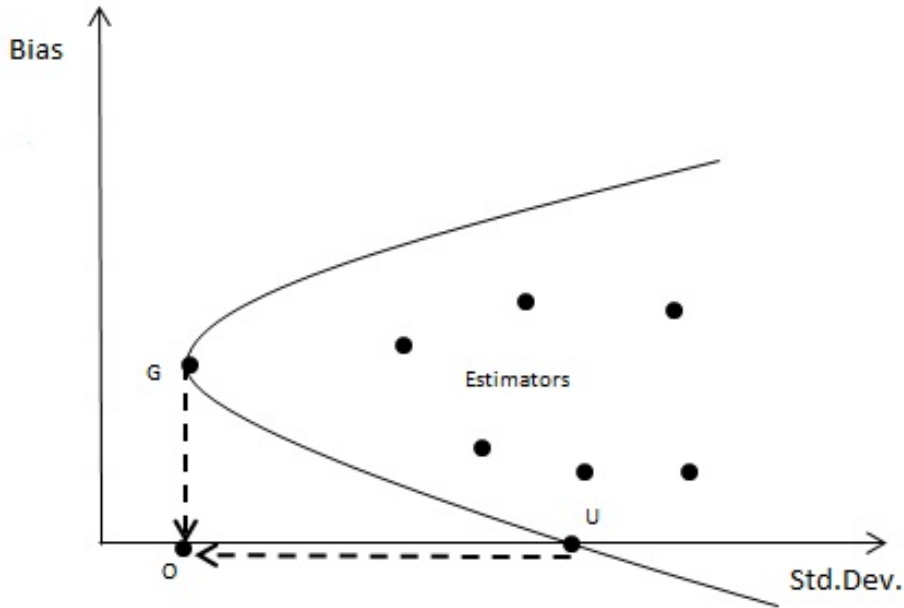


Figure 1: Bias/Variance Trade-off Framework

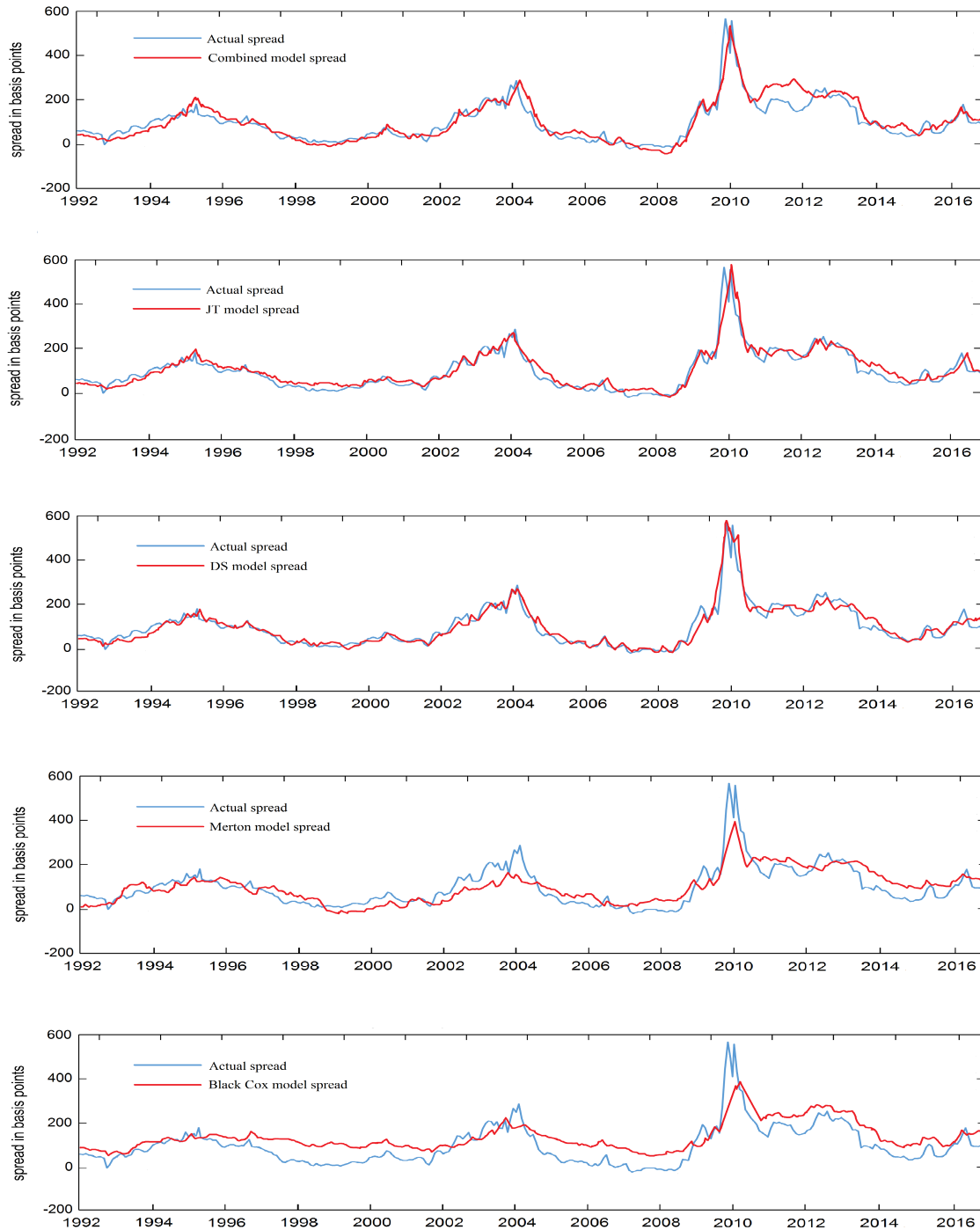


Figure 2: **Time-series variation in investment grade spreads.** This graph shows the time series of actual and model-implied speculative-grade corporate bond spreads. Each month, all yield observations in bonds with an investment grade rating and with a maturity between 3-30 years are collected and the average actual spread (to the swap rate) and the average model-implied spread in the combined model, the Jarrow and Turnbull model (JT model), the Duffie-Singleton model (DS model), the the Merton model and the Black-Cox model are computed. The graph shows the time series of monthly spreads

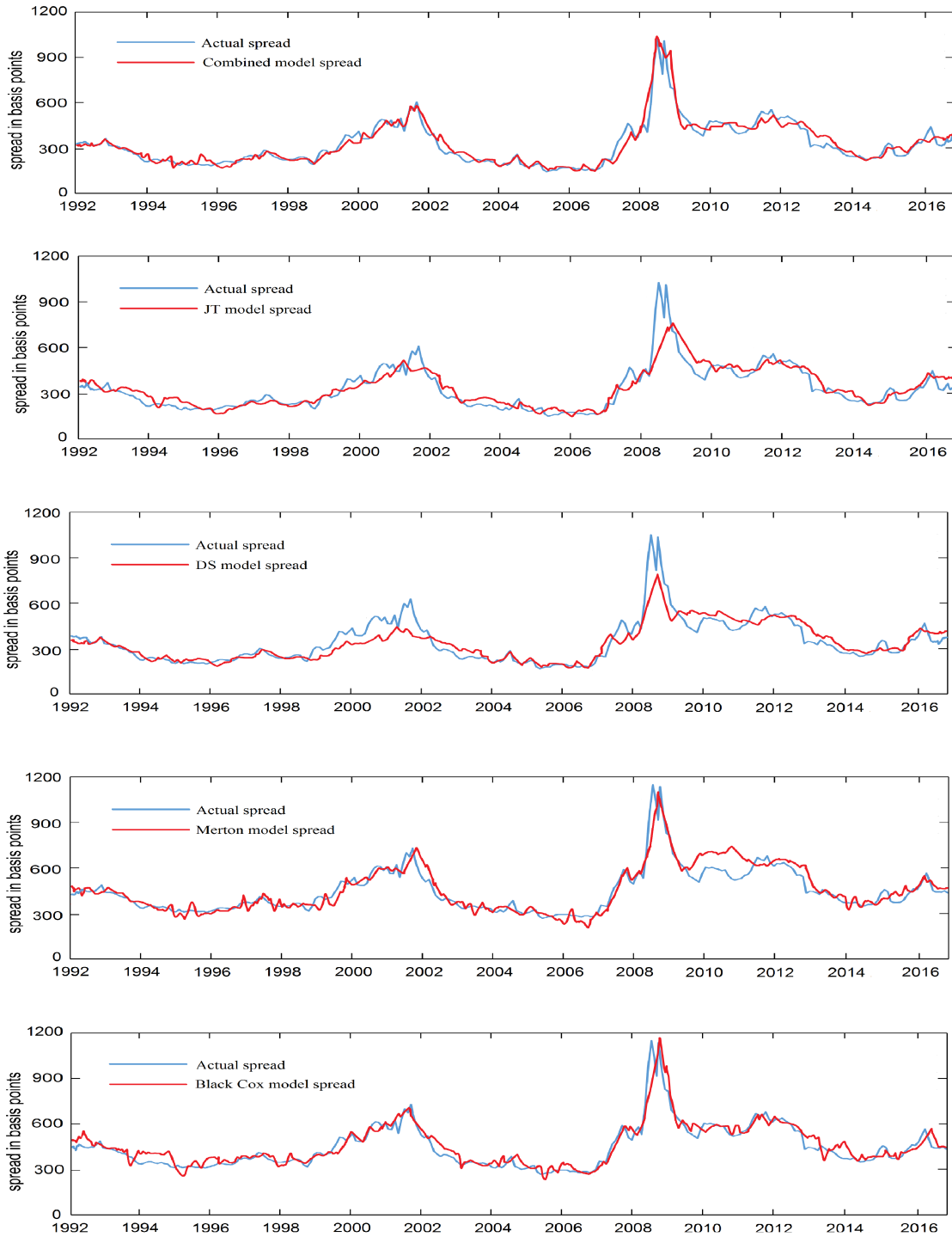


Figure 3: **Time-series variation in speculative grade spreads.** This graph shows the time series of actual and model-implied speculative-grade corporate bond spreads. Each month, all yield observations in bonds with an investment grade rating and with a maturity between 3-30 years are collected, and the average actual spread (to the swap rate) and the average model-implied spread in the combined model, the Jarrow and Turnbull model (JT model), the Duffie-Singleton model (DS model), the the Merton model and the Black-Cox model are computed. The graph shows the time series of monthly spreads.