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A Systematic Strategy to Incorporate Intensification and Diversification into Ant Colony Optimisation

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Abstract. Modern meta-heuristic search strategies are often used to solve discrete optimisation problems with little regard to varying the level of search intensity. Search intensity refers to whether the search concentrates in a particular region of space or is allowed to visit disparate regions. An intensification/diversification strategy for ant colony optimisation is developed based on the tabu search notion of the frequency of incorporation of elements into solutions. The tabu search meta-heuristic in particular has had a set of systematic intensification/diversification strategies developed for it. In this paper, these strategies are adapted for use in the relatively new ant colony optimisation techniques. The travelling salesman problem is used as the benchmark with encouraging results, particularly for larger problem instances.

Keywords: Search optimisation, intensification/diversification, ant colony optimisation, intelligent agents.

1 Introduction

The tabu search meta-heuristic [7] incorporates explicit strategies to control the level of search intensity. For instance, the search may be intensified in a promising region of space. This means that solution elements that have been associated with good solutions will be more favoured than other elements. As such, large variations to the solution state are not encouraged. However, if the search is in a region of space in which the solutions are poor, large changes to the solution will be necessary in order to find another pocket of high quality solutions (diversification).

A simple intensification strategy in tabu search is to record the best solutions as they are found. Periodically throughout the search process, the search is restarted from one of these solutions. This has the effect of "intensifying" the search around known good solutions that often form a cluster in state space. Another intensification strategy is to preserve a part of the solution by not allowing local search operators to change its elements. The preserved partial solution would contain elements that have traditionally been associated with good solutions. A popular method of achieving diversification in tabu search is to use a restarting method (in particular Battiti [1]). However, implementations of this method have tended to be non-systematic as they simply generate a random solution and restart the tabu search from this solution (with an empty tabu list). However, Glover and Laguna [7] suggest a method in which a set of diversifying moves (deliberately different from recent moves) are made and recorded on the tabu list to ensure that the search does not return to the previous region.

Ant colony optimisation (ACO) algorithms have some inbuilt intensification/diversification mechanisms. In terms of the ant colony system (ACS) algorithm in particular, intensification is achieved by a) reinforcing the pheromone levels of the elements that comprise the best found solution at each iteration of the search process and b) by often biasing the choice of the next element to be the one with the best combination of pheromone and heuristic value. Some measure of diversification is obtained by a) decreasing the pheromone on a selected solution element by use of a local updating rule, b) decreasing the pheromone on solution elements through evaporation at each iteration of the algorithm and c) occasionally allowing the probabilistic choice of the next element to be added to the ant's solution. For the $\mathcal{MAX}-\mathcal{MIN}$ Ant System meta-heuristic [12], some implicit diversification is achieved because pheromone values are bounded by a lower and upper value. Thus, as in the case with ACO techniques, a small set of very good solution elements will not dominate the other elements.

There have also been some attempts to explicitly incorporate intensification/diversification strategies into ACO techniques. Gambardella, Taillard and Dorigo [6] propose a hybrid ant system in which the usual constructive component is replaced by local search instead (thus it is an unusual ACO technique in this sense). It has been implemented for the quadratic assignment problem (QAP) and is subsequently known as HAS-QAP. It incorporates simple intensification and diversification processes into the algorithm. HAS-QAP begins each iteration with a complete solution (rather than constructing it). In an intensification phase, this initial solution is the best solution found to date. When diversification is activated, both the pheromone matrix and the initial solution are reinitialised. In the case of the latter, the solution is a random solution (i.e. a random permutation for the QAP). Blum [2] follows similar ideas to the above by allowing the search to intensify around a set of elite solutions (intensification phase) and diversifying by having a number of restart phases and resetting the pheromone values to random levels.

Randall and Tonkes [10] outline a scheme based on the ACO meta-heuristic ACS in which the characteristic element equations (see Equations 1 and 2) are modified so that the level of pheromone, in relation to the heuristic information, is varied. The premise is that elements having higher pheromone levels have shown in the past to be attractive and vice-versa. During an intensification phase, pheromone levels have a higher influence, while during diversification, elements with large amounts of pheromone are actively discouraged. Unfortunately there was no significant difference in solution quality between the intensification/diversification schemes and a control ACS strategy. Nakamichi and Arita [9] define a simple diversification strategy for the travelling salesman problem (TSP). At each step of the ant algorithm, each ant will have a probability of selecting a city at random, without regard to pheromone or heuristic (cost) information. This will allow the search to diversify, however, the results were far from conclusive as only one relatively small problem instance (eil51, see TSPLIB [11]) was used.

A common theme to the aforementioned works is that they rely on some form of randomness to achieve intensification and diversification. A more systematic approach to intensification/diversification, based on the frequency of incorporating elements into the solution, is outlined herein. The remainder of this paper is organised as follows. Section 2 outlines the standard mechanics of the ACS search paradigm in regards to the TSP while Section 3 outlines how intensification/diversification strategies can be applied to the ACO meta-heuristic, ACS. Section 4 gives the computational results and Section 5 has the conclusions of this work.

2 Ant Colony System for the Travelling Salesman Problem

ACO is modelled on the foraging behaviour of *Argentine* ants. The seminal work by Dorigo [3] showed that this behaviour could be used to solve discrete optimisation problems. ACO is in fact a collection of meta-heuristic techniques. This section gives a brief overview of one of these, ant colony system [4].

ACS can best be described with the TSP [4, 8] metaphor as it is a well understood optimisation problem. In addition, it will be used to test the strategies outlined in this paper. Consider a set of cities, with known distances between each pair of cities. The aim of the TSP is to find the shortest path to traverse all cities exactly once and return to the starting city. The ACS paradigm is applied to this problem in the following way. Consider a TSP with N cities. Cities i and j are separated by distance $d(i, j)$. Scatter m ants randomly on these cities ($m \ll N$). In discrete time steps, all ants select their next city then simultaneously move to their next city. Ants deposit a substance known as *pheromone* to communicate with the colony about the utility (goodness) of the edges. Denote the accumulated strength of pheromone on edge (i, j) by $\tau(i, j)$.

At the commencement of each time step, Equations 1 and 2 are used to select the next city s for ant k currently at city r. Equation 1 is a greedy selection technique that will choose the city that has the best combination of short distance and large pheromone levels. Using the first branch of Equation 1 exclusively will lead to sub-optimal solutions due to its greediness. Therefore, there is a probability that Equation 2 will be used to select the next city instead. This equation generates a probability and then roulette wheel selection is used to generate s.

$$s = \begin{cases} \arg\max_{u \in J_k(r)} \left\{ \tau(r,u)[d(r,u)]^{\beta} \right\} & \text{if} q \leq q_0 \\ \text{Equation 2} & \text{otherwise} \end{cases} \tag{1}$$

$$p_k(r,s) = \begin{cases} \frac{\tau(r,s)[d(r,s)]^{\beta}}{\sum_{u \in J_k(r)} \tau(r,u)[d(r,u)]^{\beta}} & \text{if} s \in J_k(r) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Note that $q \in [0, 1]$ is a uniform random number and q_0 is a parameter. To maintain the restriction of unique visitation, ant k is prohibited from selecting a city which it has already visited. The cities which have not yet been visited by ant k are indexed by $J_k(r)$. It is typical that the parameter β is negative so that shorter edges are favoured. Linear dependence on $\tau(r, s)$ ensures preference is given to links that are well traversed (i.e. have a high pheromone level). The pheromone level on the selected edge is updated according to the local updating rule in Equation 3.

$$\tau(r,s) \leftarrow (1-\rho) \cdot \tau(r,s) + \rho \cdot \tau_0 \tag{3}$$

Where:

ρ is the local pheromone decay parameter, $0 < \rho < 1$.
τ_0 is the initial amount of pheromone deposited on each of the edges.

Upon conclusion of an iteration (i.e. once all ants have constructed a tour), global updating of the pheromone takes place. Edges that compose the best solution (so far) are rewarded with an increase in their pheromone level while the pheromone on the other edges is evaporated (decreased). This is expressed in Equation 4.

$$\tau(r,s) \leftarrow (1-\gamma) \cdot \tau(r,s) + \gamma \cdot \Delta\tau(r,s) \tag{4}$$

$$\Delta\tau(r,s) = \begin{cases} \frac{Q}{L} & \text{if}(r,s) \in \text{best tour} \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Where:

$\Delta\tau(r,s)$ is used to reinforce the pheromone on the edges of the iteration best solution (see Equation 5).
L is the length of the best (shortest) tour to date while Q is a constant.
γ is the global pheromone decay parameter, $0 < \gamma < 1$.

It is typical that a local search phase is performed on the best solution in the current iteration, before the pheromone updating takes place. In the experimental work carried out in this paper, 200 inversion transitions are performed in a greedy fashion for the best solution. An inversion consists of randomly choosing two cities and inverting the sequence of cities between them.

An in-depth pseudocode description of the ACS algorith can be found in Dorigo and Gambardella [5].

3 Applying Intensification and Diversification Strategies to ACS

In this paper, ACS, with intensification/diversification extensions, is used to solve TSP problem instances. However, the techniques described in this section are adaptable to a range of discrete optimisation problems. There are two parts to defining an intensification/diversification strategy:

1. *how* intensification/diversification is to be achieved (i.e. the mechanics) and
2. *when* an intensification or a diversification phase is to be triggered and how long should it last.

In regards to the former, the frequency with which edges are incorporated into ant solutions is important for the operation of intensification and diversification. A matrix $u,\ (u(i,j),\ 1 \leq i,j \leq N)$ is used to store the number of times each edge has been incorporated into the ant solutions. This corresponds to a long term memory structure (as outlined in Glover and Laguna [7]).

During an intensification phase, edges that have been frequently used (as they have been found to be historically good) will receive a higher weighting by a new term w that is incorporated into the characteristic element selection equations (see 1 and 2). Equation 6 outlines how w is calculated.

$$w(i,j) = \frac{u_(i,j)}{\max_{k=1}^{N} u_(k,j)} \quad (6)$$

Thus the w values are bounded between 0 and 1. Diversification encourages the use of edges that have not been incorporated into solutions frequently. The weighting factor is given by Equation 7.

$$w(i,j) = 1 - \frac{u(i,j)}{\max_{k=1}^{N} u(k,j)} \quad (7)$$

If neither intensification nor diversification are required (i.e. these phases have not been invoked or are finished), $w(i,j) = 1,\ \forall i,j$ (referred to as the "normal" phase). The weighting function is incorporated into the standard element selection equations as shown in Equations 8 and 9.

$$s = \begin{cases} \arg\max_{u \in J_k(r)} \left\{ w(r,u)\tau(r,u)[d(r,u)]^{\beta} \right\} & \text{if} q \leq q_0 \\ \text{Equation 2} & \text{otherwise} \end{cases} \quad (8)$$

$$p_k(r,s) = \begin{cases} \frac{w(r,s)\tau(r,s)[d(r,s)]^{\beta}}{\sum_{u \in J_k(r)} w(r,u)\tau(r,u)[d(r,u)]^{\beta}} & \text{if} s \in J_k(r) \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

A method of determining if an intensification or diversification phase is required relies on the solution cost information generated by the ant colony. Some simple measures to determine if either intensification or diversification are necessary are outlined below:

– *Intensification* - Improved solution costs are received at a frequency greater than $intens$ iterations of ACS (where $intens$ is a parameter). This would indicate that the colony is in a promising region of the state space. Thus, highly used edges are encouraged while less used edges will receive little attention.
– *Diversification* - An improved solution cost has not been received for $divers$ iterations of ACS (where $divers$ is a parameter). This indicates that the search is stuck (or has prematurely converged) and requires new areas of the state space to be explored.

During the intensification and diversification phases, w will be calculated according to Equations 6-7 respectively. Each phase will last for $phase\_length$ iterations of ACS (where $phase\_length$ is a parameter).

4 Computational Experience

Twenty-one TSP problem instances are used to test both the control and the intensification/diversification settings of the solver[1]. The control strategy is ACS without the explicit intensification and diversification mechanism. These problems are from TSPLIB [11] and are given in Table 1. The computing platform used to perform the experiments is a Sun Ultra 5. Each problem instance is run across ten random seeds.

The ACS parameter settings are given in Table 2. The values of the parameters β, γ, ρ, m and q_0 have been found to be robust by Dorigo and Gambardella [5]. Three thousand iterations per run were used as this could be carried out in a reasonable amount of computational time while allowing for the interplay between the intensification and diversification mechanisms. In order to test the effect of the intensification/diversification parameters, three problems from the test set, eil51, kroA100 and d198, have been run with varying levels of $intens$, $divers$ and $phase\_length$. $intens$ and $divers$ were given the levels of $\{100, 200, 500\}$ while $phase\_length$ had $\{10, 20, 100\}$. Each combination of variable values were tested, giving 270 runs for each problem instance. A univariate analysis of variance procedure with log-based transform on the dependent variable and $\alpha = 0.05$ was used to test if any statistical differences arose. The analysis showed that no significant variations occurred for any of the variables or combinations thereof. This suggests that the intensification/diversification parameters are fairly robust in terms of choices of values.

The results of the control and intensification/diversification strategies are shown in Tables 3 and 4. In order to describe the range of costs gained by these experiments, the minimum (denoted "Min"), median (denoted "Med"), maximum (denoted "Max") and Inter Quartile Range (denoted "IQR") are given. Non-parametric descriptive statistics are used as the data are highly non-normally distributed.

Table 1. Problem instances used in this study.

Name	Size (cities)	Best-Known Cost
gr24	24	1272
fri26	26	937
swiss42	42	1273
hk48	48	11461
eil51	51	426
berlin52	52	7542
st70	70	675
eil76	76	538
kroA100	100	21282
bier127	127	118282
si175	175	21407
d198	198	15780
ts225	225	126643
gil262	262	2378
pr299	299	48919
lin318	318	42029
pcb442	442	50778
d493	493	35002
si535	535	48450
u574	574	36905
rat575	575	6773

Table 2. Parameter settings used in this study.

Parameter	Value
β	-2
γ	0.1
ρ	0.1
m	10
q_0	0.9
$intens$	20
$divers$	200
$phase\_length$	200
iterations	3000

It is evident from the result tables that the control strategy is fairly consistent within problem instances. For the smaller problems, it proves to be more effective than the intensification/diversification strategy. However, for the larger instances, the intensification/diversification strategy (in a few cases at least) seems to be able to break out of sub-optimal regions of space to find better solutions. To test this statistically, a one tailed t test with $\alpha = 0.05$ on normalised transformed data revealed that there

[1] This software is available upon request from the author.

Table 3. The results for the control strategy.

Problem	Best-known Cost	Min	Med	Max	IQR
gr24	1272	1272	1278	1336	5.25
fri26	937	937	955	961	0
swiss42	1273	1273	1307.5	1369	34
hk48	11461	11461	11509.5	11772	100
eil51	426	427	434.5	451	11.5
berlin52	7542	7542	7770	7961	207.75
st70	675	681	692.5	721	14.75
eil76	538	550	558	568	4.75
kroA100	21282	21292	21438	22659	839.25
bier127	118282	120637	123171	124039	903.5
si175	21407	23142	23322	23394	166
d198	15780	15972	16116.5	16311	93.25
ts225	126643	133055	134149.5	135523	784.75
gil262	2378	2430	2483.5	2532	56.25
pr299	48919	54441	55378	56332	825.5
lin318	42029	48172	48712.5	49383	645.25
pcb442	50778	61170	61658	62593	671.75
d493	35002	40989	41859	42250	509
si535	48450	54071	55375	56180	667
u574	36905	45801	46662.5	47457	752.75
rat575	6773	7965	8129	8242	138.75

was not a significant difference across the entire problem set. However, the larger problems (pr299 onwards) showed that intensification/diversification scheme produced significantly improved results over the control strategy.

5 Conclusions

In order to find and explore promising regions of the state space, it is often necessary to use intensification and diversification strategies. These strategies have been explicitly described in the tabu search literature [7], however, many practical implementations of meta-heuristic search techniques tend not to include these strategies, despite their potential benefits.

This paper described a method of incorporating intensification/diversification strategies into the ACO meta-heuristic ACS. The approach used herein is considered systematic because it explicitly utilises frequency information about solution elements to decide whether the search should concentrate on a particular region in state space or find another, more promising, region. In contrast to other techniques it does not rely on randomness to achieve intensification and diversification. This approach is consistent with tabu search long term memory techniques.

The results indicate that the intensification/diversification technique finds improved solutions (over the control strategy) to many of the large problems. In addition, it was found that it produced significantly better results for the largest problems. This is an encouraging initial result, however, the strategy must be improved so that it is more uniformly effective.

Table 4. The results of the intensification/diversification strategy.

Problem	Best-known Cost	Min	Med	Max	IQR
gr24	1272	1272	1278	1336	1
fri26	937	937	955	961	0
swiss42	1273	1273	1313.5	1369	35
hk48	11461	11461	11645.5	11876	207.75
eil51	426	427	435	455	12.25
berlin52	7542	7542	7801	8067	157.5
st70	675	682	698	721	12.75
eil76	538	551	558	570	8.5
kroA100	21282	21452	22107	23228	1041
bier127	118282	119721	121256.5	127380	4546.75
si175	21407	21757	23241	23394	940
d198	15780	16246	16400	16740	130.25
ts225	126643	129634	130802.5	138909	6647.75
gil262	2378	2564	2626.5	2715	64
pr299	48919	51751	53382	57295	3043
lin318	42029	45012	50260.5	51170	4325.5
pcb442	50778	53777	62305.5	63935	6212
d493	35002	36938	37695.5	43346	757.25
si535	48450	54071	55375	56180	667
u574	36905	39636	41556	47769	6795.25
rat575	6773	7509	7780	8322	631

In future, it will be interesting to study different mechanisms for triggering each of the intensification and diversification phases. In the current implementation, an intensification or diversification phase is forced onto the entire colony. It would be interesting to see the effect of allowing each ant to control its own search intensity. Also, at the present time, each phase lasts for a fixed number of iterations. It may be preferable to allow the phases to terminate if sufficient intensification/diversification progress has been made. How this condition is judged is an open question. Once these issues have been resolved, this technique will be empirically compared with other intensification/diversification approaches.

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