# DISTRIBUTED OPTIMIZATION WITH ITS APPLICATIONS TO POWER SYSTEMS

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# **Symbols**

- $\lambda(\cdot)$  The eigenvalue of a marix
- $\lambda_2(\cdot)$  The smallest non-zero eigenvalue of  $(\cdot)$
- $[\cdot]^+$  The projection operator on the non-negative real number
- $\mathcal{A}$  The adjacency matrix of the graph
- $\mathcal{E}$  The edge set of a graph
- $\mathcal{G}$  A graph
- $\mathcal{L}$  The Laplacian matrix
- $\mathcal{V}$  The node set of a graph
- $\|\cdot\|$  The  $l_2$  Euclidean norm of a matrix or a vector
- $\partial f(\cdot)$  The partial derivative of a function  $f(\cdot)$  with respect to a variable x
- $\nabla f(\cdot)$  The gradient of a function  $f(\cdot)$
- $A^T$  The transpose of a matrix or a vector A
- $a_i$  agent *i* chosen control action
- $a_{ij}$  The entry in the *i*th row and *j*th column of a matrix A
- $C(\cdot)$  Cost function
- $diag(\cdot)$  diagonal matrix
- *L* The operation of Lie derivative

- $P_i$  Active power from *i*th agent
- $U(\cdot)$  Utility function
- $v_i$  *i*th node

# Abbreviations

- BESSs Battery Energy Storage Systems.
- CG Conventional Generators.
- **DER** Distributed Energy Resource.
- **DG** Distributed Generator.
- **DGA** Distributed Gradient Algorithm.
- **DoD** Depth of Discharging.
- EGD Economic Generation Dispatch.
- EMS Energy Management Systems.
- ESS Energy storage system.
- EV Electric Vehicle.
- ICE Internal Combustion Engine.
- **ISS** Input-to-State Stability.
- KKT KarushKuhnTucker.
- LGD Laplacian-Gradient Dynamics.
- MAS Multi-agent Systems.
- MPPT Maximum Peak Power Tracking.

- PCC Point of Common Coupling.
- **PEV** Plug-in Electrical Vehicle.
- PMS Power Management System.
- **PV** Photovoltaic.
- **RG** Renewable Generator.
- **RTP** Real-time Price.
- SG Synchronous Generator.
- SoC State of Charge.
- SSC Storage System Controller.
- ToU Time-of-Use.
- V2G Vehicle-to-Grid.
- WT Wind Turbine.

# The University of Manchester

#### **Tianqiao Zhao**

#### **Doctor of Philosophy**

#### Distributed Optimization with Its Applications to Power Systems

### March 13, 2019

The traditional electric power grid is in the process of being transformed from the conventional power grid to an intelligent grid, a so-called smart grid. The operation of smart grids no longer has one-directional energy and information flow, and enables communication between utilities and consumers, which is a potential solution to motivating end users to participate in the decision-making of a demand response. The traditional solution for energy management problems is a centralized algorithm, which requires a control centre to collect and process the algorithm. However, it could lose its effectiveness due to the increasing level of distributed energy resources (DERs). The distributed method is a potential solution to the problem caused by the centralized method.

The research in this thesis mainly focuses on the feasibility of improving the energy management system (EMS) of smart grids from the demand side to the supply side. First, a coordination strategy of the plug-in electric vehicle (PEV) charging process is studied by maximizing the welfare and satisfaction of PEV owners. It is a distributed algorithm and analysis shows that it solves the optimal charging problem in an initialization-free approach. Second, the resource management of renewable generators (RGs) and energy storage systems (ESSs) is investigated. The objective is to minimize the curtailment of renewable energy, and at the same time, to minimize the power losses of ESSs. An optimal solution is proposed to the management problem by enhancing the communication and coordination under a multi-agent system (MAS) framework. Third, from the above research, it is found that the operation of microgrids may change frequently and without warning. A distributed finite-time algorithm is designed for EMSs in a microgrid under different operation modes. Fourth, a novel fixed-time distributed solution is introduced that both achieves a fast convergence speed and is robust to dealing with uncertain information. Finally, a novel distributed strategy for multiple BESSs is proposed for optimally coordinating them under uncertainties of wind power generation while considering the privacy of users.

# Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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# **Publications**

#### Journal papers:

- Tianqiao Zhao and Zhengtao Ding, 'Distributed initialization-free cost-optimal charging control of plug-in electric vehicles for demand management' IEEE Transactions on Industrial Informatics, vol. 13, no. 6, pp. 2791-2801, Dec. 2017.
- Tianqiao Zhao and Zhengtao Ding, 'Distributed Agent Consensus-Based Optimal Resource Management for Microgrids' IEEE Transactions on Sustainable Energy, vol. 9, no. 1, pp. 443-452, Jan. 2018.
- 3). Tianqiao Zhao and Zhengtao Ding, 'Distributed Finite-Time Optimal Resource Management for Microgrids Based on Multi-Agent Framework' IEEE Transactions on Industrial Electronics, vol. 65, no. 8, pp. 6571-6580, Aug. 2018.
- 4). Tianqiao Zhao and Zhengtao Ding, 'Cooperative Optimal Control of Battery Energy Storage System under Wind Uncertainties in a Microgrid' IEEE Transactions on Power Systems, vol. 33, no. 2, pp. 2292-2300, March 2018.
- 5). Tianqiao Zhao, Zhenhong Li and Zhengtao Ding, 'Consensus-based Distributed Optimal Energy Management with Less Communication in a Microgrid' IEEE Transactions on Industrial Informatics (in press), September 2018.
- 6). Tianqiao Zhao and Zhengtao Ding, 'Consensus-Based Distributed Fixed-time Economic Dispatch under Uncertain Information in Microgrids' submitted to IEEE Transactions on Control System Technology.

7). Tianqiao Zhao and Zhengtao Ding, 'A Distributed Optimal Management System for Coordinating the Generation Side and Demand Side Simultaneously' submitted to IET Generation, Transmission & Distribution

#### **Conference papers**

- Tianqiao Zhao, Zongyu Zuo, and Zhengtao Ding, 2017, 'Cooperative control of distributed battery energy storage systems in Microgrids ' in preceding of 2016 35th Chinese Control Conference (CCC), Chengdu, 2016, pp. 10019-10024.
- 2). Tianqiao Zhao and Zhengtao Ding, 2017, 'Optimal Energy Management of Microgrid via Distributed Primal-Dual Dynamics for Fast Frequency Recovery' in preceding of 2017 IEEE Power & Energy Society General Meeting, Chicago, IL, 2017, pp. 1-5.

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To My Loving Parents and Wife

# Chapter 1

# Introduction

### **1.1** Overview of Modern Power Systems

The traditional electric power grid is in the process of being transformed from the conventional power grid to an intelligent grid due to the growing demand for electrical energy from consumers, the integrated need for renewable energy resources, and the transformation of ageing power grid infrastructures. The future grid [4] is a so-called smart grid that is an aggregation of distributed generation units, responsive demands, and storage units each equipped with controllable power electronics devices connected to each other through communication networks [5,6].

The traditional power grid is centralized and has one-directional energy and information flow. In contrast, as shown in Fig. 1.1, a smart grid is characterized by the bi-directional flow of energy and information, and enables communication between utilities and consumers, which is a potential solution to motivating end users to participate in the decision-making of demand response. The following goals and characteristics are defined by the Energy Independence and Security Act of 2007 (EISA), TitleXIII and National Institute of Technology (NIST) [7]. First, a smart grid requires the dynamic optimization of grid operations and resources with full cyber-security; second, it should deploy and integrate distributed resources



Figure 1.1: The conceptual model of a smart grid [1]

and generation, including renewable resources; third, demand response, demand-side resources, and energy-efficiency resources should be incorporated and developed; lastly, advanced electricity storage and peak-shaving technologies, including plug-in electric and hybrid electric vehicles, and thermal-storage air conditioning should be deployed and integrated into smart grids.

These goals cannot be achieved without advanced control and optimization technologies. Considering the number and variety of distributed controllable devices, managing a smart grid requires a new paradigm shift in centralized EMS, which needs to consider the local intelligence of the distributed units and their communication capability [8]. Such units in Fig. 1.1 are able to sense their local environment, perform local computations, and coordinate with their neighbours through local communications. These elements play an important role in improving the flexibility and energy efficiency of smart grids.

#### 1.1.1 Microgrids

The conception of the microgrid is a key component of smart grids. Generally, variously distributed generators (DGs) comprising wind turbines (WTs) and photovoltaic (PV) generators, energy storage systems (ESSs), loads and other devices are integrated into a microgrid to support a flexible and efficient electric network [9]. It can help to integrate these low-carbon devices into the traditional power grid to enhance the reliability and reduce the carbon emission. Furthermore, its structure is fundamental in supporting increasing penetration of PEVs. On the one hand, the microgrid is a small-scale power system that can achieve the power dispatch and supply-demand balance economically within the microgrid because of the function of advanced EMSs. The EMS play a significant role in the control of a microgrid, which is able to optimally dispatch the active power of controllable DERs and control the consumption of controllable loads, subject to certain economic criteria. On the other hand, a microgrid can be treated as a 'virtual' power source or load. Through the coordinated control of the output power of distributed generation within the microgrid, the peak-shifting or load shaving can be realized, thus reducing the impact of intermittent renewable generation on main grids or users. A microgrid is usually operated in the gridconnected mode, which exchanges power with the main grid through the point of common coupling (PCC) [10, 11]. Due to planned or unexpected reasons [12], the microgrid will be operated in the islanded mode that needs to maintain its own active power supply-demand balance. However, because of the intermittency of WTs and PVs, a microgrid has to face new operational and control challenges of resource management, especially in the islanded operation mode.

### **1.1.2 Distributed Generation**

Generally, distributed generation includes combined heat and power (CHP), fuel cells, microturbines, photovoltaic systems, wind turbines and hydroelectric energy, etc., and is intended to be the main backup supplier of energy for improving the reliability in a microgrid. Due to the flexibility and low-carbon feature of DERs, they can be installed sufficiently close to energy consumers. As a result, the transmitting electricity loss could be reduced compared to conventional power systems. However, due to their non-dispatchable feature, they are complemented by controllable generators such as diesel generators, and ESSs.

### **1.1.3 Energy Storage System**

Distributed ESSs are referred to as devices that can absorb excessive power and compensate for insufficient power during peak generation and load periods. Furthermore, the system frequency in the low-inertia microgrid may change rapidly due to the increasing high penetration of intermittent renewable generation. Distributed ESSs are power electronic-based components and thus have a faster response time. Therefore, they are an ideal supplement for intermittent distributed renewable generation, such as solar power and wind power generation, since they can store excess energy and rapidly feed it back when needed to minimize the active power imbalance of an islanded microgrid.

### **1.1.4 Plug-in Electric Vehicles**

Driven by electricity stored in its rechargeable battery, a PEV can be recharged by power sources. It is widely recognized that electric vehicles (EVs) provide a promising alternative to the traditional internal combustion engine (ICE) vehicles, as they can reduce both environmental pollution and greenhouse effect. Enabling this transformation to PEVs will bring potential benefits to the smart grid [13, 14].

### **1.2 Research Scope**

The high penetration of renewable energy resources on the supply side or a large-scale integration of PEVs on the demand side might cause a more complicated design of EMSs in smart grids. As already known, the intermittent nature of renewable energy resources means that it is difficult to predict the output of a wind turbine or PV generation for the day-ahead or even the hourly-ahead markets. In addition, improperly controlled and non-coordinated PEVs could bring about an increase in peak demand that could destabilize the grid.



Figure 1.2: Outlines of research scope

In this thesis, the planning and operational problems of energy management in a smart grid environment are considered, and the thesis is divided into two major parts: namely supply side management and demand side management. As shown in Fig.1.2, at the demand side, the main concern is PEV charging management while providing ancillary services. On the other side, the problems of the supply side in this thesis are mainly: 1) power generation management that consists of conventional generators (CGs) and ESSs; 2) distributed energy resource management that focuses on the coordination of renewable energy resources and ESSs.

As a result, the first part of this research aims to provide a charging strategy for PEVs that avoids overload problems and maximizes user satisfaction. The second part is to coordinate distributed energy resources and ESSs that respect certain operational requirements. The third part investigates the energy management problem of a microgrid in different operation modes. The fourth part studies the optimal dispatch problem of islanded microgrids under uncertain information. The fifth part is the study of the cooperative control of ESSs.

## **1.3 Contributions**

This research investigates the energy management problem in the conception of smart grids. The major contributions of this project involved in the thesis are summarised as follows:

- The PEVs charging problem is investigated for demand management:
  - The optimal strategy maximizes the welfare and satisfaction of PEV's customers, i.e., the charging cost and the rate of change of State of Charge (SoC), by minimizing deviations between the charging current and the desired current value under the conditions of individual PEV charging limit.
  - Each PEV exchanges information with its own neighbours under a directed communication graph rather than under undirected graphs. The computational and communication burdens are shared among individual agents (PEVs) based on this distributed scheme, and thus the proposed algorithm can be flexible and scalable.
  - 3. In the real application, initialized errors may exist in load satisfaction; the initializationfree is desirable for a practical PEV charging control. To this end, by characterizing the omega-limit set of trajectories in our strategy, it is shown that the proposed algorithm does not require any specific initializing procedure and therefore PEVs can start from any charging power allocation.
- Energy management problem is studied for minimizing the operational cost of renewable generators (RGs) and their parallel-connected battery energy storage systems (BESSss) in an islanded microgrid:
  - By properly designing an objective function for RGs, the minimization of curtailment cost of RGs is achieved while guaranteeing the equal power sharing among RGs.
  - Implementing on a distributed manner through a communication network, the proposed algorithm is robust to the single point failure as long as the communication network remains connected.

- 3. To deal with the intermittency of renewable generation and load demand, an analysis is provided to show that the proposed algorithm can work even under the time-varying supply-demand imbalance.
- For microgrids that operate under various operating conditions, it is necessary to develop an optimization algorithm that can converge at a fast speed and a convergence time that can be estimated and tuned before implementation. To this end, a novel distributed algorithm is proposed to solve optimal resource management for a microgrids different operation modes:
  - Novel objective functions are formulated for different microgrid modes to minimize overall operation cost.
  - Instead of using projection methods [15, 16] to deal with generation constraints, we apply a smooth ε-exact penalty function [17] in problem formulation to handle the local constraints for the convenience of implementation.
  - 3. A finite-time algorithm is adopted to achieve a higher convergence speed, which is beneficial in maintaining the power balance of a low-inertial microgrid in the presence of unknown changes and agent trips. Furthermore, the convergence time can be estimated before implementation so that it can be tuned according to real requirements.
- A novel fixed-time distributed EMS under uncertain information is proposed for the economical and accurate operation of an islanded microgrid:
  - 1. A new objective function is designed for BESSs considering their DoD cost.
  - 2. In addition to the plug-and-play capability and scalability for the flexible operation of microgrids, the algorithm designed in this work guarantees the finite setting time that can be estimated before implementation, as well as the robust convergence of the optimal solution for the EMS, which achieves the objectives of fast convergence speed and robustness against uncertain information simultaneously.
  - It is beneficial in maintaining the power balance of the low-inertial islanded microgrid in the presence of unknown changes and uncertainties.

- A novel distributed algorithm is proposed to coordinate multiple BESSs under wind uncertainties by maximizing the total welfare of BESSs:
  - By considering the energy efficiency and ToU pricing, an objective function is formulated to maximize the total welfare of multiple BESSs that can encourage BESSs to participate in grid regulation.
  - 2. A coordination scheme of BESSs is proposed for multiple BESSs to maintain active power balance under uncertainties of wind power generation .
  - 3. The information sharing of the cost function may mean the participants have in privacy concerns. The proposed algorithm is able solve the formulated problem and respect the privacy of users at the same time, and is achieved by introducing a mismatch estimator to update the local power output and by removing the requirement of gradient information sharing.

### 1.4 Thesis Outline

In this research, the demand management of the PEV charging process is first investigated to enable the plug-and-play operation of PEVs. This algorithm is extended to the supply side in a microgrid, such as the energy management of RGs and ESSs. After starting from the EMS design, we found that the operation of microgrids may change frequent and unpredicted, it is desired to design an algorithm that can converge in a pre-determined time. From this perspective, distributed finite-time algorithms are designed for microgrids under different operating modes. In addition, a distributed robust finite-time algorithm is further proposed to deal with the uncertain information during the management process. However, the previous studies require the sharing of private information, e.g, gradients of cost functions. A novel distributed optimization algorithm is designed to consider the user privacy.

To understand the whole thesis structure, a general overview is provided in this section.

**Chapter 1** gives brief overviews of the modern power system and the essence of the smart grid. Then, the motivations and scopes of this project are proposed. Afterwards, the contributions that thesis makes to this research area, are discussed.

Chapter 2 contains notation and some preliminaries used throughout this thesis.

**Chapter 3** provides a literature review on the economic operation in microgrids. The problems of PEV charging management, resource management and optimal coordination of BESSs are studied. Recent advances in different methods can be found in this Chapter. Moreover, the gaps between the existing results and future needs are identified.

**Chapter 4** studies the management problem during the charging procedure of PEVs for maximizing user satisfaction and welfare, while respecting each PEV's physical constraints. An objective function is formulated to represent the interests of users. Then, a distributed cooperative scheme of PEVs is proposed to solve the problem in an initialization-free way.

**Chapter 5** investigates the resource management problem of RGs and parallel-connected BESSs. The objective functions for RGs and BESSs are formulated to minimize the system operational cost. The power sharing among RGs and the minimization of BESS power loss are achieved when the optimization problem is solved. Then, a distributed solution is introduced to solve this problem that is further robust to the single point/link failure.

**Chapter 6** proposes a two-level optimization system for optimal resource management for both grid-connected and islanded microgrid modes. For the islanded mode, the objective is defined to minimize overall system cost while maintaining supply-demand balance. In contrast, a grid-connected microgrid should follow the command generated by the main grid. Therefore, an objective function is defined to make the marginal cost to each participant equal to the price set by the main grid. Then, different distributed algorithms are proposed to solve the formulated problems

**Chapter 7** proposes a fixed-time distributed EMS under uncertain information in an islanded microgrid. The proposed EMS can achieve optimal dispatches in the fixed-time that can be estimated or designed before implementation and is simultaneously robust against uncertain information.

**Chapter 8** investigates the coordination problem of BESSs under uncertainties of wind power generation. A coordination scheme is introduced to enhance the BESS's performance under uncertain power output from wind turbines while increasing their profits and energy efficiency.

**Chapter 9** provides suggestions for future work. This chapter includes the possible extensions and interesting topics are also suggested for future research.

## 1.5 Summary

This chapter has introduced the background of a modern power system and the essential elements of smart grids. The motivation and research scope of this thesis have also been outlined. In addition, the contributions are discussed. Finally, the organization of each chapter has been summarized.

# Chapter 2

# **Preliminaries**

### 2.1 Notation

In this section, we recall some preliminaries about graph theory, nonsmooth analysis and differential inclusions that are used. For  $l \in \mathbb{R}$ , we denote  $\mathcal{H}_l = \{x \in \mathbb{R}^n \mid \mathbf{1}_n^T x = l\}$ , where  $\mathbf{1}_n = [1, 1...] \in \mathbb{R}^n$ . Let  $B(x, \varepsilon) = \{y \in \mathbb{R}^n \mid ||y - x|| < \varepsilon\}$ . A set-value map  $\mathcal{X} : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$  each value in  $\mathbb{R}^n$  associates with a set in  $\mathbb{R}^m$ . For  $u \in \mathbb{R}$ ,  $[u]^+$  denotes max $\{0, u\}$ . Also, for  $B_0 \in \mathbb{R}^n$ ,  $B_0 = [b_{10}, \ldots, b_{i0}]^T$ , where  $b_{i0} \neq 0$  denotes the ability of *i*th agent to receive the information of the total supply/demand power;  $b_{i0} = 0$  otherwise; and  $\mathbf{1}_n^T B_0 = 1$ .

### 2.1.1 Graph Theory

Following [18, 19], we present some basic notions of a directed graph. A directed graph is defined by  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  denotes the node set and  $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$  is the edge set. If  $(\mathbf{v}_i, \mathbf{v}_j) \in \mathcal{E}$  means node  $\mathbf{v}_i$  is a neighbour of node  $\mathbf{v}_j$ . A directed graph contains a directed spanning tree if there exists a root node that has directed paths to all other nodes. A directed graph is strongly connected if there exists a directed path that connects any pair of vertices. For a directed graph  $\mathcal{G}$ , its adjacency matrix  $\mathcal{A} = [a_{ij}]$  in  $\mathbb{R}^{n \times n}$  is defined by  $a_{ii} = 0, a_{ij} = 1$  if  $(\mathbf{v}_j, \mathbf{v}_i) \in \mathcal{E}$  and  $a_{ij} = 0$  otherwise. A weighted graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ consists of a digraph  $(\mathcal{V}, \mathcal{E})$  and an adjacency matrix  $\mathcal{A} \in \mathbb{R}_{\geq 0}^{n \times n}$  with  $a_{ij} > 0$  if and only if  $(i, j) \in \mathcal{E}$ . The weighted in-degree and out-degree of *i* are defined as  $d_{in}(i) = \sum_{j=1}^{n} a_{ij}$ and  $d_{out}(i) = \sum_{j=1}^{n} a_{ji}$ , respectively. The Laplacian matrix  $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{n \times n}$  associates with  $\mathcal{G}$  is defined as  $\mathcal{L}_{ii} = \sum_{j \neq i} a_{ij}$  and  $\mathcal{L}_{ij} = -a_{ij}, i \neq j$ .  $\mathcal{G}$  is defined as weight-balanced if  $d_{out}(\mathbf{v}) = d_{in}(\mathbf{v})$ , for all  $\mathbf{v} \in \mathcal{V}$  iff  $\mathbf{1}_{n}^{T} \mathcal{L} = 0$  iff  $\mathcal{L} + \mathcal{L}^{T}$  is positive semi-definite. If  $\mathcal{G}$  is strongly connected and weight-balanced, zero is a simple eigenvalue zero of  $\mathcal{L} + \mathcal{L}^{T}$ . In this case, there is

$$x^{T}(\mathcal{L}+\mathcal{L}^{T})x \geq \lambda_{2}(\mathcal{L}+\mathcal{L}^{T})\left\|x-\frac{1}{n}\mathbf{1}_{n}^{T}\mathbf{1}_{n}x\right\|^{2},$$
(2.1)

where  $\lambda_2(\mathcal{L} + \mathcal{L}^T)$  is the smallest non-zero eigenvalue of  $\mathcal{L} + \mathcal{L}^T$ .

### 2.1.2 Nonsmooth Analysis and Differential Inclusions

Following [20, 21], we present some basic notions of nonsmooth analysis and differential inclusions, respectively. A function  $f : \mathbb{R}^n \to \mathbb{R}^m$  is locally Lipschitz at  $x \in \mathbb{R}^n$  if for y,  $y' \in B(x, \varepsilon)$ ,  $||f(y) - f(y')|| \le L_x ||y - y'||$ , where  $L_x, \varepsilon \in (0, \infty)$ . A function  $f : \mathbb{R}^n \to \mathbb{R}^m$  is said as regular at  $x \in \mathbb{R}^n$  if, for all  $v \in \mathbb{R}^n$ , the right and generalized directional derivatives of f at x in the direction of v coincide [21]. A function is regular at x, if it is continuously differentiable at x. A convex function is regular.

Considering a set-valued map  $\mathcal{H}: \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ , a differential inclusion on  $\mathbb{R}^n$  is defined by

$$\dot{x} \in \mathcal{H}(x). \tag{2.2}$$

The set of equilibria of (2.2) is denoted by  $\text{Eq}(\mathcal{H}) = \{x \in \mathbb{R}^n | 0 \in \mathcal{H}(x)\}$ . A local Lipschitz function  $f: \mathbb{R}^n \rightrightarrows \mathbb{R}$ , the set-valued Lie derivative  $L_{\mathcal{H}}f, \mathbb{R}^n \rightrightarrows \mathbb{R}$ , of f with respect to (2.2) is defined as

$$L_{\mathcal{H}}f = \{ b \in \mathbb{R} \mid \exists v \in \mathcal{H}(x) \text{ s.t. } \alpha^T v = b \text{ for all } \alpha \in \partial f(x) \}.$$
(2.3)

For any  $\varepsilon \in (0,\infty)$ , a set-valued map  $\mathcal{H} : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$  is upper semi-continuous at  $x \in \mathbb{R}^n$ if there exists  $\delta \in (0,\infty)$  such that  $\mathcal{H}(y) \subset \mathcal{H}(x) + B(0,\varepsilon)$  for all  $y \in B(x,\delta)$ . Also,  $\mathcal{H}$  is locally bounded at  $x \in \mathfrak{R}^n$  if there exist  $\varepsilon, \delta \in (0,\infty)$  such that  $||z|| \le \varepsilon$  for all  $z \in \mathcal{H}(y)$  and  $y \in B(x,\delta)$ . Let  $\Omega_f$  be the set of points where f is not differentiable, and the generalized gradient of f is defined as

$$\partial f(x) = \operatorname{co} \{ \lim_{k \to \infty} \nabla f(x_k) \mid x_k \to x, x_k \notin \Omega_f \cup S \}$$

where co denotes convex hull and S is a set of measure zero.

A solution of  $\dot{x} \in \mathcal{H}(x)$  on  $[0,T] \subset \mathbb{R}$  is defined as an absolutely continuous map  $x : [0,T] \rightarrow \mathbb{R}^n$  that satisfies (2.2) for almost all  $t \in [0,T]$ . Also, if  $\mathcal{H}$  is locally bounded, upper semicontinuous, and takes non-empty, compact, and convex values, then the existence of solutions is guaranteed.

**Theorem 2.1.1.** (LaSalle Invariance Principle for differential inclusions) [22] Let  $\mathcal{H} : \mathbb{R}^n \Rightarrow \mathbb{R}^n$  be locally bounded, upper semicontinuous, with non-empty, compact, and convex values. let  $f : \mathbb{R}^n \to \mathbb{R}$  be locally Lipschitz and regular. If  $S \subset \mathbb{R}^n$  is compact and strongly invariant under (2.2) and  $\max \mathcal{L}_{\mathcal{H}} f(x) \leq 0$  for all  $x \subset S$ , then the solutions of (2.2) starting at *S* converge to the largest weakly invariant set *M* contained in  $S \cap \{x \in \mathbb{R}^n \mid 0 \in \mathcal{L}_{\mathcal{H}} f(x)\}$ . Moreover, if the set *M* is finite, then the limit of each solution exists and is an element of *M*.

### **2.2 Saddle Points**

The set of saddle points of a  $\mathcal{C}^1$  function  $F : X \times \mathcal{Y} \to \mathbb{R}$  is denoted by Saddle(*F*). If there exist open neighbourhoods  $\mathcal{U}_x \subset X$  of  $x_*$  and  $\mathcal{U}_y \subset \mathcal{Y}$  of  $y_*$  such that

$$F(x_*, y) \le F(x_*, y_*) \le F(x, y_*)$$
 (2.4)

for all  $y \in \mathcal{U}_y$  and  $x \in \mathcal{U}_x$ , and for  $(x_*, y_*) \in \text{Saddle}(F)$ , the points  $x_* \in X$  and  $y_* \in \mathcal{Y}$  are the local minimizer and local maximizer of the map  $x \mapsto F(x, y_*)$  and  $y \mapsto F(x_*, y)$  respectively. The point  $(x_*, y_*)$  are a global min-max saddle point of F are  $\mathcal{U}_{x_*} = X$  and  $\mathcal{U}_{y_*} = \mathcal{Y}$ . Also, the set of saddle points of a convex-concave function F is convex.
# Chapter 3

# Literature Review: Economic Operation in Microgrids

## 3.1 Introduction

This chapter concludes two sections. In **Section 3.2**, it provides a review of existing results on applications of MAS used for power systems and distributed energy management. Then, a literature review of typical economic operation problem in microgrids and the corresponding key issues are given in **Section 3.3**.

## **3.2** Overview of Multi-agent Systems

The evolution of modern power systems towards decentralized and scalable architectures brings new challenges to traditional system operators in terms of the control and management of modern power systems. MAS framework is a potential solution to meet these challenges [23]. Designing a MAS in the context of power systems, power system elements are evolved in a given environment, and modelled as distributed intelligent agents with characteristics of reactivity, pro-activeness and social ability. It is applicable to any level of

partitioning according to certain tasks; an individual agent could potentially represent a distributed energy (DER), a consumer or even a large region of the network.

#### 3.2.1 Multi-agent Systems in Modern Power Systems

MAS-based strategies [24] have been applied to a variety of power system applications, including load restoration [25], frequency regulation [26] and reactive power control [27]. How- ever, the MAS-based solutions in those research studies are based on set rules, and lack a rigorous stability analysis. The existing distributed solution is proposed under the undirected communication network, which requires a bi-directional communication, rather than a possible single-directional communication. In contrast, the scheme with directed information flow would have lower communication costs [28]. Since the development of the smart grid is becoming highly scalable due to the integration of smart meters and control-lable devices, the scalability of distributed resource management will be strengthened by introducing directed communication [29].

# **3.3** Economic Operation in Microgrids

The economic operation of microgrids involves optimal energy management on both supply and demand sides. The supply side, which includes resource management and optimal management of BESSs, aims to minimize the total operating cost on the generation side while meeting some equality and inequality constraints. On the demand side, demand management, i.e. PEV charging management, is designed to encourage consumers to participate in the power system and market operations to increase grid sustainability.

The following subsections provide a literature review for the economic operation of microgrids.

Firstly, in respect of smart grids, PEV charging management is introduced in Section Section 3.3.1. Secondly, the energy management problems regarding microgrids, under different operation modes, are discussed in Section 3.3.2. Thirdly, the problems regarding the cooperative optimal control of multiple BESSs are discussed in Section 3.3.3. In addition, the corresponding key issues of these problems are also discussed in each corresponding each subsection. Finally, a brief summary is given in Section 3.4.

#### **3.3.1** PEVs Charging Management

In this section, the PEV charging problem for demand management is considered, which aims to coordinate the charging process of a large number of PEVs in a distributed approach. The large-scale integration of PEVs may induce both adverse effects and incentives simultaneously on future grids. One of the apparent impacts is that the grid would be destabilized with a higher peak demand due to PEV integration [30]. On the other hand, the grid can benefit from the integration in load profile levelling and frequency regulation since PEVs can be treated as a flexible load due to their charging property [31]. Additionally, the satisfaction of PEV customers should be improved by designing a proper coordination strategy, which can achieve a high acceptance rate of EV utilization. The main concerns of PEV users are the total charging time, the total charging cost and the SoC at the end of the charging process. Therefore, with the development of vehicle-to-grid (V2G) technology [32, 33], it is important to design efficient energy-management policies to control and optimize the charging process of EVs for smart grid development [34]. The purpose of this work is to design a proper cooperative control strategy to maximize the benefits and satisfaction of PEVs customers while satisfying the constraints of PEV operations.

The coordination control and demand management problem for PEVs can be formulated as an optimization problem in a centralized manner [35–39]. Such kinds of control strategies usually require a control centre, i.e., an aggregator, to receive the charging status of each PEV from the charging station and a powerful computation centre to process the substantial information collected from PEVs [40]. However, with a large number of EVs introduced as controllable units, traditional centralized approaches may lose their efficiency due to the intractable computation burden, and they are sensitive to single-point failures. To overcome such problems, the decentralized PEVs coordination control strategies are presented in [41–45]. In [41], a decentralized approach is proposed for each charging station to regulate its charging power by responding to an external signal. The decentralized approach only collects a PEV's own information, which can relieve computation burdens on the control. However, a central external signal, i.e., a coordination control signal [42] or a real-time pricing signal [43], must be sent to an aggregator. A fully decentralized strategy only needs local information that is presented in [44, 45]. However, it is difficult to adjust droop coefficients to instantaneous operating conditions in real time when there is a lack of broadly available information in practice.

#### **Key Issues of PEVs Charging Management**

Concerning the charging management process of a large number of PEVs, there are existing distributed optimal strategies in literature that consider the charging control of EVs [46–48]. ?]. Indeed, these strategies could be the alternative solutions to dealing with the large-scale charging problems of PEVs. However, despite the practical implementation of charging strategies, the concerns of PEV users have not been fully addressed, and it cannot be guaranteed that the strategy can work due to the unpredictable nature of users charging behaviour and time-varying charging power available from the grid. Unfortunately, little attention has been paid to these issues.

#### **3.3.2 Energy Management System for Microgrids**

The task of energy management involves coordinating the dispatch of energy resources, in an economical way, which both meets the demands of consumers and minimizes generation costs. For this task, a proper algorithm is required to schedule power outputs of energy resources to meet the load demand while maintaining the system's constraints in a costeffective way by minimizing generation costs to all participants.

The energy management problem has been widely researched in demand response, economic generation dispatch (EGD), and loss minimization by different methods such as analytical methods [49, 50], hierarchical schemes [51–53], heuristic methods [54, 55] or centralised control schemes [56, 57]. These methods are effective for conventional power systems. How- ever, they may lose control efficiency in a microgrid due to the high penetration of RGs. The reason is that a powerful computation centre is indispensable for processing



Figure 3.1: A diagram of the RG and parallel-connected ESS

substantial collected data from RGs and ESSs in a microgrid [58]. As a result, it brings intractable computation burden, and is sensitive to single-point failures. In addition, the participants in the microgrid may be unwilling to release their local information globally such as local cost/utility function and power consumption [59]. The above problems can be avoided by solving the resource management problem in a distributed manner that only utilizes local information through a local private communication network [60]. In addition, an algorithm is proposed to solve the optimal reconfiguration problem in a distributed system [61]. Therefore, it is important to design a distributed solution that promotes the most flexible, reliable, and cost-effective development of a microgrid.

#### Key issues in Energy Management System in an Islanded Microgrid

Due to the intermittency of WTs and PVs, an islanded microgrid will face new operational and control challenges with regard to resource management. A solution is to install fast response ESSs for these non-conventional and intermittent renewable sources [62,63]. Such a hybrid system consists of load demands, distributed RGs and parallel-connected ESSs as depicted in Fig. 3.1 [64]. However, the integration of RGs and parallel-connected ESSs is limited by some adverse constraints [65–67], i.e. regulation problem [65]. Besides, integrating ESSs may require additional investment and increase in operating costs with strong physical restrictions [68]. Since subventions may be restricted in the short term, the objective is to reduce generation costs by adopting a proper resource management approach.

Results in existing literature have considered distributed solutions to optimal schedule generation supply and load demand [16, 69–75]. In [69], the power allocation problem of distributed ESSs was solved by a consensus-based control strategy. An external leader is needed to collect and broadcast the total supply-demand power mismatch, which means the control strategy is not fully distributed, and is sensitive to the measurement errors of actual power deviations. In [70], a distributed economic operation strategy for a microgrid was proposed to minimize the economic cost by jointly scheduling various participants without considering the single link/node failure. In [71] and [72], the authors presented a consensus+innovation framework and a consensus method for the economic dispatch problem in power systems, which employs a projection scheme to handle inequality constraints. As reported in [74], by modelling the distributed generators (DGs) and loads as agents, the microgrid can be operated economically through the management of DGs and price-sensitive loads in a MAS. Also, in [75], the active and reactive power of the islanded microgrid are dispatched optimally by a two-layer networked and distributed method in a MAS framework. A fully distributed control strategy is proposed in [16]. However, this control strategy relies on a specific initialization procedure during each update step.

#### Key issues in Energy Management System for Different Microgrid Modes

In addition to the above issues, in microgrids under different modes, due to the unpredictable nature of a microgrid, changes in their operation modes may be unpredicted and frequent [76,77]. Therefore, fast convergence algorithms are required to effectively coordinate both dispatchable and non-dispatchable energy resources to maintain microgrid stability [12,78], which is necessary for facilitating the development of a microgrid. To this end, a proper algorithm is required to schedule power outputs of energy resources to meet the load demand effectively at a fast and estimable convergence speed.

#### Key issues in Energy Management System under uncertain information in Microgrids

The microgrid may face not only frequent and unpredicted changes, but also uncertain information in EMSs. The existing EMS solutions may not be able to act fast enough to effectively maintain microgrid stability. Recently, literature [79,80] proposed distributed algorithms with fast response time for EMS problems, which do not consider uncertain information such as the inaccurate prediction of renewable generation [81] and communicational and computational uncertainties during the power management [82]. Although these algorithms are actually fast enough, the lack of robust design for uncertainties may result in an unreliable EMS and even the instability of islanded microgrids. Therefore, the distributed EMS that has both a fast convergence speed and is robust against uncertain information should be further considered.

# 3.3.3 Energy Management System for Multiple Battery Energy Storage Systems

With a properly designed pitch angle and rotor speed control strategy [83,84], the effectiveness of the wind power generation for maintaining grid frequency stability has been verified by several existing research studies in different areas, such as inertial, primary and second frequency control [85, 86]. However, due to the intermittency of wind power generation supply, a microgrid faces new challenges in terms of its operation and control, especially under high penetration levels. As a result, integrating high penetration wind power generation would affect the stability of a microgrid, which may cause a mismatch between supply and demand when the available wind power generation is not equal to total load demand.

As mentioned in **Section 3.3.2**, installing BESSs for the intermittent renewable sources [87–89] is a promising solution, since they can provide a faster response in terms of absorbing excessive power and compensating for insufficient power during peak generation and load periods respectively. Thus, the active power imbalance caused by integrating high penetration wind power generation can be addressed by properly installing and coordinating the BESSs.

The cooperative approaches in the existing studies are mainly clarified into three categories, namely centralized schemes [56, 90], decentralized approaches [91] and distributed control strategies. The smart grid will consist of more distributed controllable BESSs with the ability to exchange information through a communication network. Therefore, the emerging

management solution for multiple BESSs should be efficient for an economically.

#### Key Issues in Energy Management for Multiple BESSs

Recently, the control and optimization of BESSs have drawn the attention of researchers [92–95]. In [92], the size of operation BESSs is optimized based on adjusting SoC limits. In [93], group BESSs are coordinated by a distributed control algorithm for voltage and frequency deviation regulation. To achieve the SoC equalization, authors in [94] improved the conventional droop control by modifying a virtual droop resistance according to the SoC imbalance. A cooperative control method is presented in [95] for BESSs based on time-of-use (ToU) pricing. However, the relevant results in existing literature are designed by assuming that the energy efficiency of multiple BESSs remains a constant value. It is indicated that the variation of charging/discharging efficiency of multiple BESSs is indispensable [96]. Additionally, the experiment in [97] shows that energy efficiency fluctuates drastically according to the charging/discharging rate and SoC. In this case, energy efficiency should be taken into consideration in the optimization and control design of multiple BESSs. Furthermore, the owners of BESSs may not willing to share their private information, such as information about cost/utility functions. It is therefore desirable to design a novel algorithm that considers the privacy of users.

#### **3.4** Summary

In this chapter, an overview of applications of MAS used for power systems and typical optimization problems in microgrids, including PEV charging management, EMSs in microgrids and cooperative optimal control of BESSs, has been provided. Furthermore, the corresponding key issues of these problems have been identified. The remaining chapters will focus on designing proper solutions to these issues through different optimization models and algorithms.

# Chapter 4

# Distributed Initialization-Free Cost-Optimal Charging Control of Plug-in Electric Vehicles

# 4.1 Introduction

PEVs provide a promising alternative solution to the reduction of environmental pollution and fuel emissions. As the key issues indicated in **Section 3.3.1**, a well-designed charging coordination approach is needed to minimize the impact on the power system when a large number of PEVs is connected to the grid. The desired strategy should solve the problem in a distributed manner while both meeting system interests, i.e., the welfare and satisfaction of PEV customers, and respecting the charging constraint of each PEV.

To this end, this chapter focuses on designing a novel distributed optimal control strategy to solve the optimal charging problems of PEVs. The analysis in **Section 4.3** shows that the proposed strategy solves the key issues raised in **Section 3.3.1** successfully.

# 4.2 Problem Formulation for the Battery Charging Problem of PEVs

We assume that multiple PEVs are plugged into a charging station under a specific optimal charging control to schedule their charging profiles during the total charging duration time T. The charging station knows the total charging power capacity over a PEVs charging period. Each PEV is charged with a constant charging current to reach its desired SoC.

The objective is to design an optimal control method in terms of the economic factors of the PEV charging process, such as the total charging time, the total charging cost, etc. To address this objective, the modelling of the PEV battery and its charging property is first investigated. The existing results on the PEV battery modelling mainly consider two aspects: a) equivalent circuit models [98, 99] and b) electrochemical models [100]. For the equivalent circuit models, they are mainly used for online estimation and power management, and the electrochemical models are usually adopted for battery design optimization, health characterization, and health-conscious control.

#### 4.2.1 Battery Modelling

For the purpose of optimal charging control design, an equivalent circuit model is adopted in [101], which considers a Li-ion battery as an ideal energy storage unit. This model has been validated based on test data [102]. All the analyses in this section are based on the approximate battery model in which the battery parameters are independent of the depth of discharge (DoD), SoC and temperature. Such assumptions are widely applied in [48, 103, 104] for the optimization and control design.

The model is described as a constant voltage source in series with a constant resistance that considers resistive energy losses. This model can be represented as:

$$V_i = V_{o,i} + R_i I_i, \tag{4.1a}$$

$$\dot{\mu}_i = \frac{I_i}{Q_i},\tag{4.1b}$$

where  $V_i$  is the terminal voltage,  $V_{o,i}$  denotes the open circuit voltage,  $R_i$  represents the



Figure 4.1: The simple equivalent circuit model

equivalent battery internal resistance,  $I_i$  is the charging current,  $Q_i$  represents the battery charge capacity, and  $\mu_i$  is the battery state of charge SoC, of *i*th PEV, respectively. Note that  $V_{o,i}$  can vary with SoC. We treat it as a constant voltage source since the variation of  $V_{o,i}$  is very small within 25% – 90% SoC for lithium-ion batteries [104].

When a PEV is being charged after it is plugged in, the power consumed by the *i*th PEV can be represented by multiplying the terminal voltage and its charging current,

$$P_{EV,i} = V_{o,i}I_i + R_i I_i^2, (4.2)$$

where  $P_{EV,i}$  is the instantaneous charging power. Hence, the battery charging current can be expressed as

$$I_i = \frac{1}{2R_i} (\sqrt{4R_i P_{EV,i} + V_{o,i}^2} - V_{o,i}).$$
(4.3)

The battery charging current and power are positive during the charging process and negative for the discharging process.

#### 4.2.2 Constraints

The physical limits are represented by the following constraints.

#### **Global Constraint of the Charging Power Allocation**

There is a limit to the total amount of power that a charging station can provide; therefore the total amount of charging power of all PEVs should not exceed the stations total available power  $P_{\text{total}}$ , which is modelled as an upper bound of the utility's power delivery

$$\sum_{i=1}^{n} P_{EV,i} \le P_{\text{total}}.$$
(4.4)

#### **Local Constraint of Each PEV**

The allocated charging power of each PEV is locally bounded by different physical factors, such as the upper bound of the outlet's power output, the charging current's tolerance and the charging level [47]. One local constraint is proposed to map the physical above constraints, i.e.,

$$0 \le P_{EV,i} \le P_{EV,i}^M \tag{4.5}$$

where  $P_{EV,i}^{M}$  is the maximum charging power of *i*th PEV that considers the above limitations.

#### 4.2.3 Optimization Problem Formulation of PEVs

The charging time length over the charging duration  $T_i$  of *i*th PEV is denoted by  $\Delta T$ , and the time slots is expressed as  $K_i = \frac{T_i}{\Delta T}$ ,  $k \in K_i := \{1, ..., K_i\}$ .

In [105], a real-time price (RTP) model is formulated as the derivative of the generation sup- ply cost, which represents the marginal cost of the generation supply. Therefore, based on a similar concept in [106], the generation supply cost of *i*th PEV is supposed to  $C_{g,i} = \frac{1}{2}a(\sum_{k=1}^{K_i} P_{EV,i}(k))^2 + b(\sum_{k=1}^{K_i} P_{EV,i}(k)) + c$  with proper parameters *a*, *b*, and *c*. Therefore, a RTP model for *i*th PEV is adopted linearly with respect to its total demand during the charging duration, such that

$$P_{re,i} = a \sum_{k=1}^{K_i} P_{EV,i}(k) + b.$$
(4.6)

Note that the RTP model is widely applied for the coordination of PEV's charging process [107, 108].

Furthermore, the *i*th PEV needs to reach its desired SoC,  $\mu_i^*$  by its deadline that is determined by

$$\sum_{k=1}^{K_i} P_{EV,i}(k) \Delta T_i = (\mu_i^* - \mu_i(0)) Q_i.$$
(4.7)

By substituting (4.7) into (4.6), the price model is expressed as

$$P_{re,i} = a \frac{(\mu_i^* - \mu_i(0))Q_i}{T_i} + b.$$
(4.8)

The satisfaction of PEV customers with the charging service is usually dependent upon the SoC of their vehicles when the charging process is finished. Therefore, a concave utility function is defined to represent the satisfaction of the *i*th PEV customer, which relates to changing rate of SoC, such that

$$U_{EV,i}(\dot{\mu}_i) = -\left(P_{re,i}\frac{K_i\Delta T^2}{2}\right)\dot{\mu}_i^2 + \left(P_{re,i}(\mu_i^* - \mu_i(0))\Delta T\right)\dot{\mu}_i.$$
(4.9)

It should note that the utility function has the following properties:

- The satisfaction of PEV's customers is increasing according to the changing rate of SoC.
- The utility function has the decreasing marginal utility.

The utility function has been widely applied in [107, 109–111] for PEV charging coordination problems. From the users' perspective, the utility function of PEV users should be maximized when PEVs are connected to the charging station through a Smart Charger. The Smart Charger can regulate the charging current to maximize consumer satisfaction. It is desired to discover an optimal charging current for each plugged PEV according its own utility function. Therefore, the optimal charging current reference for each PEV is formulated as follows. The utility function is rewritten as:

$$U_{EV,i} = -\left(P_{re,i}\frac{K_i \Delta T^2}{2Q_i^2}\right)I_i^2 + \left(\frac{P_{re,i}(\mu_i^* - \mu_i(0))\Delta T}{Q_i}\right)I_i$$
(4.10)

Then, by taking the derivative of (4.10) with respect to  $I_i$  and equating to zero, the bliss point of the charging current of the utility function is

$$I_i^{ref} = \arg\max_{I_i} (U_i(\dot{\mu_i})) = \frac{\mu_i^* - \mu_i(0)}{T_i} Q_i,$$
(4.11)

where  $T_i = K_i \Delta T$ .

To maximize the satisfaction of each PEV user, the PEV should be ideally charged according to the desired charging current obtained in (4.11). However, due to the total available charging power constraints, it is almost impossible to realize the desired current charging for all PEVs. Therefore, inspired by a similar formulation process in [48], the objective function for PEVs is formulated by minimizing the difference between the charging current and the desired reference, i.e.,  $f_i(P_{EV,i}) = \varepsilon_i (I_i^{ref} - I_i)^2$ . The total deviations are denoted by  $f(P_{EV})$  with  $P_{EV} = [P_{EV,1}, \dots, P_{EV,n}]^T \in \mathbb{R}^n$ . With (4.3), the objective function is written as

$$\operatorname{Min} f(P_{EV}) = \operatorname{Min} \sum_{i=1}^{n} \varepsilon_{i} (I_{i}^{ref} - I_{i})^{2}$$
  
$$= \operatorname{Min} \sum_{i=1}^{n} \varepsilon_{i} \{ (I_{i}^{ref})^{2} + \frac{V_{o,i}^{2}}{2R_{i}^{2}} + \frac{2I_{i}^{ref}V_{o,i}}{R_{i}}$$
  
$$- \frac{V_{o,i} + 2R_{i}I_{i}^{ref}}{2R_{i}^{2}} \sqrt{4R_{i}P_{EV,i} + V_{o,i}^{2}} + \frac{P_{EV,i}}{R_{i}} \}, \qquad (4.12)$$

where  $\varepsilon_i$  is a non-negative weight given by  $\varepsilon_i = \frac{1}{\mu_i T_i + \kappa}$ .

This prior weight is introduced in terms of the current SoC and the total charging time of the *i*th PEV. A small positive value,  $\kappa$ , is set to avoid singularity. The weight can prioritize PEVs based on the time of charge and the remaining SoC to be charged.

The first three terms in (4.12) are independent of the decision variable  $P_{EV,i}$ , which can be neglected from the objective function. Hence (4.12) can be further simplified as

$$\operatorname{Min} \sum_{i=1}^{n} \varepsilon_{i} \left( \frac{P_{EV,i}}{R_{i}} - \frac{V_{o,i} + 2R_{i}I_{i}^{ref}}{2R_{i}^{2}} \sqrt{4R_{i}P_{EV,i} + V_{o,i}^{2}} \right)$$
  
s.t.  $0 \leq P_{EV,i} \leq P_{EV,i}^{M},$  (4.13)

Furthermore, for the purpose of fully using the available power, we assume that  $\sum_{i=1}^{n} P_{EV,i} = P_{\text{total}}$ .

The objective function (4.13) is convex, and the set of charging power allocations satisfying the box constrain is  $\mathcal{F}_{box} = \{P_{EV,i} \in \mathbb{R} \mid 0 \le P_{EV,i} \le P_{EV,i}^M\}$ . We denote the feasibility set of the above optimal problem as  $\mathcal{F}_{EV} = \{P_{EV,i} \in \mathbb{R} \mid 0 \le P_{EV,i} \le P_{EV,i}^M$  and  $\mathbf{1}_n^T P_{EV} = P_{total}\}$ . Besides, the solution is denoted by  $\mathcal{F}_{EV}^*$ .

A centralized strategy can solve the above problem, but it requires a powerful control centre

to receive all the information of each PEV for data management, communication and processing. The objective is to design a distributed cooperative control algorithm such that it can optimally allocate the charging power of all PEVs based on their priorities. Furthermore, from any initial conditions, this novel charging control algorithm can accommodate plugand-play operations and perform well under the time-varying supply-demand condition in an isolated power system.

#### 4.3 Distributed Optimal Solution

In this section, a distributed control algorithm is proposed to solve the optimal charging control problem.

#### 4.3.1 **Problem Reformulation**

The inequality constraint may cause difficulties in the optimal control design. Thus, the exact penalty function method is utilized to tackle this problem. The optimization problem (4.13) is reformulated by rewriting the objective function of *i*th PEV, i.e.

$$g_i(P_{EV,i}) = f_i(P_{EV,i}) + \frac{1}{\varepsilon} ([P_{EV,i} - P_{EV,i}^M]^+).$$
(4.14)

and  $g(P_{EV}) = \sum_{i=1}^{n} g_i(P_{EV,i})$ , which subjects to the available charging power constraint  $\mathbf{1}_n^T P_{EV}^* = P_{\text{total}}$ .

Note that  $g(P_{EV})$  is convex, locally Lipschitz, and continuously differentiable on  $\mathbb{R}$  except at  $P_{EV,i} = P_{EV,i}^M$ . According to the Proposition 5.2 in [112], the original optimization problem (4.13) and the reformulated optimal charging problem coincide if there exists  $\varepsilon \in \mathbb{R}_{>0}$  such that

$$\varepsilon < \frac{1}{2\max_{P_{EV} \in \mathcal{F}_{EV}} \left\| \bigtriangledown f(P_{EV}) \right\|_{\infty}}.$$
(4.15)

In our control design, we assume that (4.15) holds this condition.

A useful Lemma based on [113] is introduced as follows

**Lemma 4.3.1.** Since  $g(P_{EV,i})$  is convex, locally Lipschitz, and continuously differentiable except at  $P_{EV,i} = P_{EV,i}^M$ , the charging optimal problem has a solution  $P_{EV}^* \in \mathbb{R}^n$  if and only if, there exists  $\sigma \in \mathbb{R}$  such that

$$\sigma \mathbf{1}_n \in \partial g(P_{EV}^*)$$
 and  $\mathbf{1}_n^T P_{EV}^* = P_{\text{total}}.$  (4.16)

#### 4.3.2 Distributed Algorithmic Design

In [112], the author first proposed an algorithm to solve an ED problem in a distributed manner, i.e.,

$$\dot{P}_{EV,i} \in -\sum_{j \in \mathbb{N}(i)} a_{ij} (\partial g_i(P_{EV,i}) - \partial g_j(P_{EV,j})).$$
(4.17)

However, it requires that the initial condition starts from the feasible set. To further solve the problem in an initialization-free way, a distributed optimal solution is presented to the optimal charging problem as below in which a novel feedback term is introduced to maintain supply-demand balance. It allows the power allocation of each PEV to start from any initial conditions.

$$\dot{P}_{EV,i} \in -\sum_{j \in \mathbb{N}(i)} a_{ij} (\partial g_i(P_{EV,i}) - \partial g_j(P_{EV,j})) + \gamma x_i, \qquad (4.18a)$$
$$\dot{x}_i = -\beta (x_i - (b_{i0}P_{\text{total}} - P_{EV,i}))$$

$$-\alpha \sum_{j \in \mathbb{N}(i)} a_{ij}(x_i - x_j) - \sum_{j \in \mathbb{N}(i)} a_{ij}(\eta_i - \eta_j), \qquad (4.18b)$$

$$\dot{\eta}_i = \alpha \beta x_i, \tag{4.18c}$$

where  $\alpha, \beta, \gamma \in \mathbb{R}_{>0}$  are the parameters to be designed,  $\mathbb{N}(i)$  denotes the neighbour set of *i*th PEV. In (4.18a), the first term explores the minimization of the total cost and a feedback element,  $\gamma x_i$ , enforces *i*th PEV to satisfy the supply-demand equality condition. Furthermore,  $x_i$  is designed to track the average signal  $\frac{1}{n}(P_{total} - \mathbf{1}_n^T P_{EV}(t))$  for each PEV  $i \in \{1, ..., n\}$ . The proposed algorithm solves the optimal problem regardless of initial values of  $(P_0, x_0, \eta_0)$ . As proved by [114], the dynamic estimation method (4.18b) - (4.18c) is a low-pass filter that ensure robust average consensus estimation in a sensor network.

#### 4.3.3 Convergence Analysis

For convergence analysis, the algorithm is rewritten in a compact form represented by the set-valued map  $X_{EV}$ 

$$\dot{P}_{EV} \in -\mathcal{L}\partial g(P_{EV}) + \gamma x,$$
(4.19a)

$$\dot{x} = -\alpha \mathcal{L}x - \beta (x - (B_0 P_{\text{total}} - P_{EV})) - \mathcal{L}\eta, \qquad (4.19b)$$

$$\dot{\eta} = \alpha \beta x, \tag{4.19c}$$

where x,  $\eta$  are the column vectors of  $x_i$ , and  $\eta_i$  respectively.

We characterize the  $\omega$ -limit set of the trajectories of (4.19) with any initial conditions in  $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$ .

**Lemma 4.3.2.** The  $\omega$ -limit set of the trajectories of (4.19) with any initial conditions in  $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$  is contained in  $\mathcal{H}_{P_{total}} \times \mathcal{H}_0 \times \mathcal{H}_0$ .

*Proof.* Defining  $\psi(t) = \mathbf{1}_n^T P_{EV}(t) - P_{\text{total}}$ , one has

$$\dot{\boldsymbol{\psi}}(t) = \mathbf{1}_{n}^{T} \dot{\boldsymbol{P}}_{EV}(t) = \mathbf{1}_{n}^{T} \boldsymbol{\gamma} \boldsymbol{x}(t), \qquad (4.20)$$

and

$$\ddot{\psi}(t) = -\mathbf{1}_{n}^{T}\beta\gamma x(t) + \mathbf{1}_{n}^{T}\beta\gamma(B_{0}P_{\text{total}} - P_{EV}(t))$$
  
=  $-\beta\gamma\psi(t) - \beta\dot{\psi}(t).$  (4.21)

The system can be rewritten as

$$\dot{z} = Az, \tag{4.22}$$

where  $z = [z_1, z_2]^T$ , and  $z_1 = \psi, z_2 = \psi$ . The system matrix, *A*, is obtained as

$$A = \begin{bmatrix} 0 & 1 \\ -\beta\gamma & -\beta \end{bmatrix}.$$

Let  $M \in \mathbb{R}^{2 \times 2}$  be

$$M = \frac{1}{2\gamma\beta^2} \begin{bmatrix} \beta^2 + \beta\gamma + (\beta\gamma)^2 & \beta \\ \beta & 1 + \beta\gamma \end{bmatrix},$$
 (4.23)

which satisfies  $A^T M + MA + I = 0$ . Define  $V_z = z^T M z$  as a Lyapunov function candidate for (4.22), and the derivative of  $V_z$  is

$$\dot{V}_z = -z^T z. \tag{4.24}$$

Therefore, we can deduce that  $\lim_{t\to\infty} z_i(t) = 0$  for i = 1, 2, and the convergence rate is exponential. Furthermore,  $z_i(t) = 0$  implies that  $\mathbf{1}_n^T P_{EV}(t) \to P_{total}$  and  $\mathbf{1}_n^T x(t) \to 0$ . Note that  $\mathbf{1}_n^T \dot{\eta} = 0$ , as  $\mathbf{1}_n^T x(t) \to 0$ .

Based on Lemma 5.3.1 and the Proposition A.1 in [22], we now ready to establish that, with (4.19), the trajectories of the charging power allocations of PEVs converge to the solution of the optimal charging problem.

**Theorem 4.3.1.** The trajectories of (4.19) converge to the solution of the optimal charging problem if  $\alpha, \beta, \gamma \in \mathbb{R}_{>0}$  satisfy the condition that

$$\frac{\gamma}{\alpha\beta\lambda_2(\mathcal{L}+\mathcal{L}^T)} + \frac{\beta\lambda_{max}(\mathcal{L}^T\mathcal{L})}{2} < \lambda_2(\mathcal{L}+\mathcal{L}^T).$$
(4.25)

*Proof.* A change of coordinates is introduced to shift the equilibrium point of (4.19) to the origin. With  $\bar{\eta} = \mathcal{L}\eta - \beta(B_0 P_{total} - P_{EV})$ , the set-valued map  $\chi_{EV}$  is transformed as

$$\mathcal{X}_{EV}(P_{EV}, x, \eta) = \{ [-\mathcal{L}\xi + \gamma x, -(\alpha \mathcal{L} + \beta I)x - \bar{\eta}, (\alpha \beta \mathcal{L} + \gamma \beta I)x - \beta \mathcal{L}\xi]^T \in \mathbb{R}^{3n} \mid \xi \in \partial_g(P_{EV}) \}.$$
(4.26)

Consider a candidate Lyapunov function  $V_2 : \mathbb{R}^{3n} \to \mathbb{R}_{\geq 0}$ ,

$$V_2 = g(P_{EV}) + \frac{1}{2}\gamma\beta ||x||^2 + \frac{1}{2}||\beta x + \bar{\eta}||^2, \qquad (4.27)$$

and let  $\varphi_1 = x, \varphi_2 = \beta x + \overline{\eta}$ , then

$$V_2 = g(P_{EV}) + \frac{1}{2}\gamma\beta \|\varphi_1\|^2 + \frac{1}{2}\|\varphi_2\|^2.$$
(4.28)

Define the overall coordinate transformation  $\mathcal{T}: \mathbb{R}^{3n} \to \mathbb{R}^{3n}$  as

$$[P_{EV}, \varphi_1, \varphi_2]^T = \mathcal{T}(P_{EV}, x, \eta)$$
  
= 
$$[P_{EV}, x, \beta x + \mathcal{L}\eta - \beta (B_0 P_{total} - P_{EV})]^T.$$
(4.29)

Next step is to prove that, in the new coordinate, the trajectories of (5.1) converge to the set

$$\bar{\mathcal{F}}_{op} = \mathcal{T}(\mathcal{F}_{op}^*) = \mathcal{F}_{EV}^* \times \{0\} \times \{0\}.$$
(4.30)

Note that *g* is locally Lipschitz and regular, while the set-valued map  $X_{EV}$  is locally bounded, upper semi-continuous, and takes non-empty, compact and convex values. Take the setvalued Lie derivative  $L_{X_{EV}}V_2 : \mathbb{R}^n \rightrightarrows \mathbb{R}$  of  $V_2(P_{EV}, \varphi_1, \varphi_2)$  along the  $X_{EV}$ ,

$$L_{\chi_{EV}}V_{2} = \{-\xi^{T}\mathcal{L}\xi + \gamma\xi^{T}\varphi_{1} - \gamma\alpha\beta\varphi_{1}^{T}\mathcal{L}\varphi_{1} - \beta\|\varphi_{2}\|^{2} - \beta\varphi_{2}^{T}\mathcal{L}\xi \mid \xi \in \partial g(P_{EV})\}.$$

$$(4.31)$$

Denote  $\delta = [\delta_1, \delta_2, \delta_3]^T$ , where  $\delta_1 = \xi \in \partial g(P_{EV})$ ,  $\delta_2 = \beta \gamma \phi_1$  and  $\delta_3 = \phi_2$  respectively. A continuous function is defined as w:  $\mathbb{R}^{3n} \times \mathbb{R}^{3n} \to \mathbb{R}^{3n}$ ,

$$w(P_{EV}, \varphi_1, \varphi_2, \delta) = [-\mathcal{L}\delta_1 + \gamma\varphi_1, -\alpha\mathcal{L}\varphi_1 - \varphi_2, \beta\gamma\varphi_1 - \beta\varphi_2 - \beta\mathcal{L}\delta_1]^T, \quad (4.32)$$

and hence dynamics (4.26) can be expressed as

$$\mathcal{X}_{EV}(P_{EV}, \varphi_1, \varphi_2) = \{ w(P_{EV}, \varphi_1, \varphi_2, \delta) \mid \delta \in \partial V_2(P_{EV}, \varphi_1, \varphi_2) \}.$$
(4.33)

Since the directed graph  $\mathcal{G}$  is strongly connected and weight-balanced with the fact that  $\mathbf{1}_{n}^{T} \mathbf{\varphi}_{1} = 0$  for  $(P_{EV}, \mathbf{\varphi}_{1}, \mathbf{\varphi}_{2}) \in \mathcal{H}_{P_{total}} \times \mathcal{H}_{0} \times \mathcal{H}_{0}$ ,

$$\delta^{T} w(P_{EV}, \varphi_{1}, \varphi_{2}, \delta) = -\frac{1}{2} \xi^{T} (\mathcal{L} + \mathcal{L}^{T}) \xi + \gamma \xi^{T} \varphi_{1}$$
  

$$-\frac{1}{2} \alpha \beta \gamma \varphi_{1}^{T} (\mathcal{L} + \mathcal{L}^{T}) \varphi_{1} - \beta \|\varphi_{2}\|^{2} - \beta \varphi_{2}^{T} \mathcal{L} \xi$$
  

$$\leq -\frac{1}{2} \lambda_{2} (\mathcal{L} + \mathcal{L}^{T}) \left\| \xi - \frac{1}{n} \mathbf{1}_{n}^{T} \mathbf{1}_{n} \xi \right\|^{2}$$
  

$$+ \gamma (\xi - \frac{1}{n} \mathbf{1}_{n}^{T} \mathbf{1}_{n} \xi)^{T} \varphi_{1} - \frac{1}{2} \alpha \beta \gamma \lambda_{2} (\mathcal{L} + \mathcal{L}^{T}) \|\varphi_{1}\|^{2}$$
  

$$- \beta \|\varphi_{2}\|^{2} - \beta \varphi_{2}^{T} \mathcal{L} (\xi - \frac{1}{n} \mathbf{1}_{n}^{T} \mathbf{1}_{n} \xi) \qquad (4.34)$$

Defining  $\vartheta = \xi - \frac{1}{n} (\mathbf{1}_n^T \mathbf{1}_n \xi)$ , and  $\phi^T = [\vartheta^T, \phi_1^T, \phi_2^T]$ , we have

$$\delta^T w(P_{EV}, \varphi_1, \varphi_2, \delta) \le \phi^T R \phi \tag{4.35}$$

where

$$R = \begin{bmatrix} -\frac{1}{2}\lambda_2(\mathcal{L} + \mathcal{L}^T)I & \frac{1}{2}\gamma I & -\frac{1}{2}\beta\mathcal{L}^T \\ \frac{1}{2}\gamma I & -\frac{1}{2}\alpha\beta\gamma\lambda_2(\mathcal{L} + \mathcal{L}^T)I & 0 \\ -\frac{1}{2}\beta\mathcal{L}^T & 0 & -\beta I \end{bmatrix}$$

Applying the Schur complement,  $R \in \mathbb{R}^{3n \times 3n}$  is negative definite if

$$\begin{aligned} &-\frac{1}{2}\lambda_{2}(\mathcal{L}+\mathcal{L}^{T})I\\ &-\left[\frac{1}{2}\gamma I - \frac{1}{2}\beta\mathcal{L}^{T}\right] \begin{bmatrix} -\frac{1}{2}\alpha\beta\gamma\lambda_{2}(\mathcal{L}+\mathcal{L}^{T})I & 0\\ 0 & -\beta I \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{2}\gamma I\\ -\frac{1}{2}\beta\mathcal{L} \end{bmatrix}\\ &= -\frac{1}{2}\lambda_{2}(\mathcal{L}+\mathcal{L}^{T})I + \frac{\gamma}{2\alpha\beta\lambda_{2}(\mathcal{L}+\mathcal{L}^{T})}I + \frac{\beta}{4}\mathcal{L}^{T}\mathcal{L}, \end{aligned}$$

is negative definite, which is guaranteed by (4.25). Hence,  $\delta^T w(P_{EV}, \varphi_1, \varphi_2, \delta) \leq 0$ , and  $\delta^T w(P_{EV}, \varphi_1, \varphi_2, \delta) = 0$  if and only if  $\phi = \varphi_1 = \varphi_2 = 0$ . Reasoning with Lemma A.1 [22], we conclude that  $0 \in L_{X_{EV}}V_2$  if and only if there exists  $\sigma \in \mathbb{R}$  such that  $\sigma \mathbf{1}_n \in \partial g(P_{EV}^*)$ . With Lemma 4.3.1,  $P_{EV}^* \in \mathcal{F}_{EV}$  being a solution of the optimal charging problem.

The last step is to show the trajectories of  $\mathcal{T}$  are bounded. This follows similar lines in [22], and therefore we omit it.

**Remark 4.3.1.** The convergence of the proposed algorithm (4.18) does not rely on a specific graph. With this property, PEVs can easily implement plug-and-play operations, when a PEV arrives in or departs from the charging station. Furthermore, the proposed algorithm does not require any specific initializing procedures. Hence, PEVs can start from any charging power allocations.

#### 4.4 Simulation Results and Analysis

In the simulation studies, several cases are used to validate the effectiveness of the proposed distributed optimal strategy. The algorithm is tested for a 5-PEV system on a PC with Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 4GB RAM in MATLAB/Simulink. The parameters of the PEV battery are listed in Table 4.1, which are taken from the typical battery charging profiles provided in [115]. It is assumed that the PEVs are able to interact with their adjacent neighbours in the communication network. The total available charging power can be accessed by several PEVs with  $\mathbf{1}_n^T B_0 = 1$ .. Without loss of generality, we assume that PEV1 knows the total available charging power.

	Table 4.1: Parameters of PEV Batteries						
	$V_{o,i}(V)$	$R_i(Ohm)$	$Q_i(A.h)$	SoC(0)	$SoC^*$	$P^M_{EV,i}(kW)$	
Vehicle 1	303	1.13	25	0.20	0.90	3.3	
Vehicle 2	292	1.08	30	0.24	0.85	3.3	
Vehicle 3	289	1.17	28	0.30	0.80	3.3	
Vehicle 4	301	1.12	29	0.18	0.85	3.3	
Vehicle 5	298	1.07	32	0.25	0.90	3.3	
Vehicle 6	306	1.14	35	0.21	0.90	3.3	



Figure 4.2: Distributed demand management for PEVs charging

#### 4.4.1 Algorithm Implementation

The proposed optimal solution can be implemented in a MAS framework as shown in Fig. 4.2. The step-by-step algorithm for the *i*th PEV agent is shown in Algorithm. 1, and Fig. 4.3 gives the general operation structure of *i*th agent. The communication topology for PEVs charging is shown as Fig. 4.4. Each PEV is plugged into the charging station through a recharging socket, which is assigned as an agent. The recharging socket only interacts with its neighbouring agents to exchange the information, i.e.,  $(P_{EV,i}, x_i)$ . Each agent deploys the proposed algorithm (5.1) in Section IV which will provide an optimal power charging reference for all PEVs.

**Remark 4.4.1.** The applicability of the proposed algorithm can be investigated by calculating the minimum amount of data exchanged by each node. To this end, a simulation study is

		Num of Variable	bits (Single-precision)	Sample Rate	Data	
	$P_1$	1	32bit	100Hz	3200bps	
	<i>z</i> <sub>1</sub> 1		32bit	100Hz	3200bps	
$\eta_1$ 1		1	32bit	100Hz	3200bps	
Total amount of data						

Table 4.2: Total amount of data of the communication line between PEV1 and PEV2

provided to show the minimum amount of data exchanged by each node on the communication network. In this simulation study, the communication network is designed as a directed graph, and the communication channel between each PEV has the same tolerance as for the data transmission. Without loss of generality, the data exchanged in the communication channel between PEV1 and PEV2 was calculated. The sample time is set at 0.01s (100Hz). The algorithm in this study has three states for each agent that requires three real numbers. The amount of data of one directed communication channel from PEV1 to PEV2 can be calculated as Table 4.2. As the result show, the minimum amount of data of the communication channel from PEV1 to PEV2 is 9.6Kbps.

**Remark 4.4.2.** The results of these simulation studies are obtained based on rigorous parameter selection, which is taken from a standard charging profile, and the constraint of the charging rate is set based on the Level 2 charging profile. As a result, the proposed algorithm is well-adapted to a real PEV charging problem. Furthermore, as Remark 4.4.1 indicates, the minimal amount of data exchanged by each PEV may be compatible with the throughput of modern communication systems. With the above analysis, it is shown that the proposed algorithm can be potentially implemented in a testbed.

#### 4.4.2 Simulation Studies

In the case studies, the designed parameters are chosen as  $\alpha = 12$ ,  $\beta = 0.4$ ,  $\gamma = 2$ ,  $\varepsilon = 0.0085$ , which satisfy the condition in (4.15), and (4.25) specified in Theorem 4.3.1. In *Case* 4.4.1, investigates the optimal charging problem with the constant total available charging power while the plug-and-play operation is considered. *Case* 4.4.2 investigates the performance of the proposed strategy under the time-varying available charging power supply. Finally, the scalability analysis is validated in *Case* 4.4.3.

Algorithm 1 DISTRIBUTED OPTIMAL CHARGING CONTROL ALGORITHM

**Initialization:** For  $i \in \{1, ..., n\}$   $P_{EV,i} = P_{EV,i}(0), x_i = x_i(0), \eta_i = \eta_i(0)$  **Consensus Algorithm: Select**  $\alpha, \beta, \gamma \in \mathbb{R}_{>0}$ **Check If** variables the inequality (4.25),

$$\begin{cases} \text{Yes, Flag} = 1 \rightarrow \text{Continue} \\ \text{No, Flag} = 0 \rightarrow \text{Go back to Select} \end{cases}$$
(4.36)

#### Coordination

Each PEV *i* communicates with its adjacent PEV agent, and updates  $(P_{EV,i}, x_i, \eta_i)$  according to (4.18a)-(4.18c) in Section IV. End if Each PEV achieves the optimal operation



Figure 4.3: The general operation structure of *i*th agent



Figure 4.4: The communication topology for PEVs charging



Figure 4.5: The charging power updates for PEVs



Figure 4.6: Total allocated charging power updates for PEVs



Figure 4.7: Allocated charging power updates for PEVs



Figure 4.8: Supply-demand mismatch updates for PEVs



Figure 4.9: The SoC updates for PEVs

**Case 4.4.1.** In this case, the total available charging power is assumed as 12kW. As shown in Figs. 4.5 - 4.6, the charging rates of all PEVs quickly converge to their optimal values, while the total allocated charging power converges to 12kW, i.e., the total available charging power.

The plug-and-play adaptability of our strategy is investigated, e.g., 1) an EV arrives in the charging station at an arbitrary time; 2) an EV leaves the charging station when its SoC is charged to its desired value. The total available charging power is still 12kW. The communication network is weight-balanced, when PEVs are moving in and out. Here an imbalancecorrecting algorithm [116] is applied to this communication network design. Supposing that each agent can correct its weight by sending and receiving information from its neighbours, the digraph adapts and becomes weight-balanced in a finite time. Figs. 4.7 - 4.9 show the charging power allocations of PEVs during their charging process, the demand-supply mismatch and the SoC of PEVs. As shown in Fig. 4.7, the charging power allocations of each PEV converge to their optimal values. When one PEV, e.g., PEV6, arrives at the charging station, the charging power allocation can converge to the optimal values with the proposed strategy. After PEV3 leaves the charging station, the proposed strategy guarantees that the remaining PEVs can still reach their optimal charging rates by sharing the total available charging power. As shown in Fig. 4.8, during the whole PEV charging process, the deviations between the total available charging power and the total allocated charging power are very small. Fig. 4.9 gives the SoC update of the PEV charging process when a PEV leaves or arrives at the charging station. When PEV6 arrives at the charging station at a random time, its SoC starts from 20% and increases during the charging process. The SoC of PEV3 drops to zero because it has been charged to the desired SoC, and it is ready to depart from the station. Therefore, the proposed strategy will have little effect on the frequency disturbance of the system, which may be applied to isolated systems such as an autonomous microgrid.

**Case 4.4.2.** The effectiveness of the proposed strategy under a time-varying supply-demand condition is validated in this case. An isolated microgrid consisting of DGs and loads is considered here. Due to the intermittent nature of renewable energy sources, particularly wind power generation, the generated power may fluctuate and cause a frequency fluctuation problem in the microgrid. PEVs installed on the customer side are employed as a flexible



Figure 4.10: Allocated charging power updates for PEVs



Figure 4.11: Supply-demand mismatch updates for PEVs



Figure 4.12: Allocated charging power updates for 30-PEVs

load for alleviating frequency fluctuation [117]. To this end, we consider a time-varying non-PEV load condition with the total available power given by  $P_{total} = 12000 + 700 \sin(0.005t)$ . The communication network and the other operating conditions are the same as those in Case 4.4.1. As shown in Figs. 4.10 - 4.11, the allocated charging powers converge to their op- timal values under the time-varying supply-demand condition, while the mismatch between supply and demand power converges to zero. In addition, Fig. 4.11 shows that the proposed strategy effectively offsets the supply-demand mismatch, which will help with frequency fluctuation (caused by the intermittent nature of renewable sources) regulation.

**Case 4.4.3.** The scalability of the proposed strategy is validated. To do so, we implement the optimal control strategy to both 30-PEV system and 60-PEV system, and The total available charging power is supposed to be 60kW and 120kW, respectively. The communication network is weight-balanced and strongly connected.

As shown in Fig. 4.12 and Fig. 4.14, the proposed strategy can guarantee the allocated charging powers to converge to their optimal values within 25s. Figs. 4.13 - 4.15 illustrate the deviations of demand and supply power can converge to zero. It is worth noting that the convergence of the proposed algorithm is mainly determined by parameter selection and the knowledge of the communication network. The convergence can be ensured for a large number fleet by maintaining the inequality (4.25).



Figure 4.13: Supply-demand mismatch updates for 30-PEVs



Figure 4.14: Allocated charging power updates for 60-PEVs



Figure 4.15: Supply-demand mismatch updates for 60-PEVs

It should be noted that the proposed algorithm is applicable in a real scenario since the required communication network for data transmission is acceptable for a real communication network and the computational cost is acceptable for running an embedded system.

## 4.5 Conclusion

In this chapter, a cooperative distributed control strategy is proposed for PEV optimal charging by maximizing the welfare and satisfaction of PEV customers while considering the charging constraints of PEVs. The proposed distributed algorithm is implemented based on MAS-framework under a directed communication graph, which is robust to dealing with single-link failures compared with centralized methods. The initializing procedures are no longer needed in this control design. Thus, the PEVs can start from any charging power allocations. In addition, since the convergence of the proposed algorithm does not rely on a specific graph, it allows plug-and- play operation during the PEV charging process. Furthermore, the proposed distributed strategy can handle the time-varying supply-demand mismatch problem in isolated systems.

# Chapter 5

# Distributed Agent Consensus-Based Optimal Resource Management for Microgrids

# 5.1 Introduction

This chapter considers the optimal resource management problem for an islanded microgrid. Microgrids provide a promising approach to deal with challenges regarding the integration of distributed renewable generation and ESSs. However, as indicated in **Section 3.3.2**, resource management in a microgrid encounters a new difficulty, i.e. supply-demand imbalance, caused by the intermittence of renewable sources. Therefore, an optimal solution is proposed to the resource management by enhancing communication and coordination under a multi-agent system framework. An agent is a participant, for instance, the distributed RGs/ESSs of the microgrid. With this MAS, the distributed optimal solution only utilizes local information, and interacts with neighbouring agents. Thus, single-node congestion is avoided since the requirement for a central control centre is eliminated, and it is robust against single-link/node failures. The analysis will show that the proposed solution can solve the resource management problem in an initialization-free manner. Additionally, the

proposed strategy can maintain the supply-demand balance under a time-varying supplydemand deviation.

#### 5.2 **Problem Formulation**

The resource management of RGs is formulated to minimize the generation cost while satisfying the supply-demand constraint and the RG constraints. In this section, the objective function, while considering both RGs and parallel-connected ESSs costs, is constructed as:

$$\operatorname{Min} C(P_{R}, P_{E}) = \sum_{i \in \mathcal{N}_{RG}} f_{i}(P_{R,i}(t)) + \sum_{j \in \mathcal{N}_{ESS}} g_{j}(P_{E,j}(t))$$
(5.1)

where  $\mathcal{N}_{RG}$  and  $\mathcal{N}_{ESS}$  are the sets of RGs and ESSs, respectively.  $f_i(P_{R,i}(t))$  and  $g_j(P_{E,j}(t))$ are the cost function for *i*th RG and *j*th ESS, for  $i \in \mathcal{N}_{RG}$  and  $j \in \mathcal{N}_{ESS}$ .  $P_{R,i}(t)$  and  $P_{E,j}(t)$ are the output power of *i*th RG and *j*th ESS. Moreover, define  $C_k(P_{R,i}, P_{E,j}) = f_i(P_{R,i}(t)) + g_j(P_{E,j}(t))$  and  $C(P_R, P_E) = \sum_{k \in \mathcal{N}} C_k(P_{R,i}, P_{E,j})$  with  $P_R = [P_{R,i}(t), \dots, P_{R,n}(t)]^T \in \mathbb{R}^n$  and  $P_E = [P_{E,j}(t), \dots, P_{R,n}(t)]^T \in \mathbb{R}^n$ .

Then, the RG's objective of economic dispatch is to minimize the curtailment of renewable energy in a microgrid. To this end, the cost function of the *i*th RG is expressed as

$$f_i(P_{R,i}(t)) = a_i P_{R,i}(t)^2 + b_i P_{R,i}(t) + c_i$$
(5.2)

where  $P_{R,i}^{\max}$  is the predicted maximum power generation capacity of *i*th RG, and  $a_i = \frac{\varepsilon_i}{2P_{\max}}$ ,  $b_i = -\varepsilon_i$ , and  $c_i = \frac{\varepsilon_i P_{\max}}{2}$ , respectively. A trade-off factor  $\varepsilon_i$  difference between the capacity and generation cost, which can be selected according to the capacity required, the installation and other costs known as 'balance of system cost' for each type of renewable source [70, 118]. The formulated objective function (5.2) has a similar format to the cost function in [119, 120] with different the parameter setting. In addition, (5.2) indicates that a lower power generation cost is realized when the deviation of actual output power  $P_{R,i}(t)$  between  $P_{R,i}^{\max}$  is minimized. Hence, the minimization of renewable energy curtailment can be derived by minimizing the generation cost of RGs [121].

Following [122], a convex quadratic cost function is adopted to represent the power loss of ESSs during the charging/discharging process, i.e.,

$$g_j(P_{E,j}(t)) = a_j P_{E,j}(t)^2 + b_j P_{E,j}(t) + c_j$$
(5.3)

where  $a_j$ ,  $b_j$  and  $c_j$  are the non-negative parameters of *j*th ESS, which can be selected in terms of the Amoroso and Cappuccino's experimental results [123].

#### 5.2.1 Constraints

The physical limitations on the operation of the studied system are presented as follows:

#### **Global Constraint**

Since the change of frequency within a microgrid is mainly affected by the supply-demand mismatch, the supply-demand balance should be maintained to ensure the stability of the microgrid. To this end, the active power balance in a microgird can be expressed as:

$$P_D(t) = \sum_{i \in \mathcal{N}_{RG}} P_{R,i}(t) + \sum_{j \in \mathcal{N}_{ESS}} P_{E,j}(t)$$
(5.4)

where  $P_D(t)$  is the total load demand.

#### **Local Constraints**

The actual output power of each RG and ESS should belong to a feasible range, i.e.,

$$P_{R,i}^{\min} \le P_{R,i} \le P_{R,i}^{\max}$$

$$P_{E,j}^{\min} \le P_{E,j} \le P_{E,j}^{\max}$$
(5.5)

where  $P_{R,i}^{\min}$  and  $P_{R,i}^{\max}$  are the lower and upper bound for the *i*th RG;  $P_{E,j}^{\min}$  and  $P_{E,j}^{\max}$  are the lower and upper bound for the *j*th ESS.

#### 5.2.2 Objective Function

Considering both the global constraints (5.4) and local constraints (5.5), the resource management problem is formulated as

Min 
$$C(P_R, P_E)$$

subject to

$$P_{D}(t) = \sum_{i \in \mathcal{N}_{RG}} P_{R,i}(t) + \sum_{j \in \mathcal{N}_{ESS}} P_{E,j}(t),$$

$$P_{R,i}^{\min} < P_{R,i} < P_{R,i}^{\max}, \quad \text{for } i \in \mathcal{N}_{RG}$$

$$P_{E,j}^{\min} < P_{E,j} < P_{E,j}^{\max}, \quad \text{for } j \in \mathcal{N}_{ESS}.$$
(5.6)

**Remark 5.2.1.** In the formulation of the objective function, the cost function is used to reflect the operating cost of the islanded microgrid. The formulated cost function consists of a generation cost of each RG and a charging/discharging cost of each BESS. The generation cost of RGs is formulated to minimize the curtailment of renewable energy when the minimized generation cost is achieved. In the meantime, the cost function of BESSs rep- resents the power losses during the charging/discharging process as indicated in [122], which maximizes the actual power output of BESSs while minimizing the power losses. For this reason, the total operating cost will be minimized by the proposed cost function.

The feasibility set and the solution set of the resource management problem is denoted by  $\mathcal{F}_{RM}$  and  $\mathcal{F}_{RM}^*$ , respectively. Denote  $P_E = [P_{E,1}, \dots, P_{E,n}] \in \mathbb{R}^n$  and  $P_R = [P_{R,1}, \dots, P_{R,n}] \in \mathbb{R}^n$ , respectively. A useful Lemma in [113] is introduced as follows

**Lemma 5.2.1.** Since  $C_k(P_{R,i}, C_{E,j})$  is convex, locally Lipschitz, and continuously differentiable, the optimization problem has a solution  $(P_R^*, P_E^*) \in \mathbb{R}^n$  if and only if  $\exists \sigma \in \mathbb{R}$  such that  $\sigma \mathbf{1}_n \in \partial_{P_R} C(P_R, P_E)$ , and  $\sigma \mathbf{1}_n \in \partial_{P_E} C(P_R, P_E)$ , and  $\mathbf{1}_n P_R^* + \mathbf{1}_n P_E^* = P_D$ .

#### 5.3 Distributed solution of dynamic economic dispatch

The formulated problem (5.6) is a convex optimization problem with both equality and inequality constraints. Traditional centralized strategies may have some challenges, such as computation burden and non-timely response. In this section, in order to overcome these challenges, we develop a distributed cooperative strategy to deal with the resource management problem is developed, which only utilizes locally available information and interacts with its adjacent agents.

#### 5.3.1 Distributed Algorithm Design

Inspired by the dynamic average consensus estimation method proposed in [124], a distributed solution is developed for resource management, which allows the power allocation of RGs and ESSs to start from any initial condition. The distributed algorithm is formulated as:

$$\begin{split} \dot{P}_{R,i} &= -\sum_{h \in \mathbb{N}_{RG}} a_{ih} (\partial_{P_{R,i}} C_i(P_{R,i}, P_{E,j})) \\ &- \partial_{P_{R,h}} C_h(P_{R,h}, P_{E,j})), \end{split}$$
(5.7a)  
$$\dot{P}_{E,j} &= -\sum_{k \in \mathbb{N}_{ESS}} a_{jk} (\partial_{P_{E,j}} C_i(P_{R,i}, P_{E,j})) \\ &- \partial_{P_{E,k}} C_k(P_{R,i}, P_{E,k})) + \gamma x_j, \qquad (5.7b) \\ \dot{x}_j &= -\beta (x_j - (b_{i0} P_D - P_{E,j} - P_{R,i})) \end{split}$$

$$-\alpha \sum_{k \in \mathbb{N}_{\text{ESS}}} a_{jk}(x_j - x_k) - \sum_{k \in \mathbb{N}_{\text{ESS}}} a_{jk}(\eta_j - \eta_k), \qquad (5.7c)$$

$$\dot{\eta}_j = \alpha \beta x_j, \tag{5.7d}$$

where  $\alpha$ ,  $\beta$ ,  $\gamma \in \mathbb{R}_{>0}$  are the parameters to be designed.  $\mathbb{N}_{RG}$  and  $\mathbb{N}_{ESS}$  denotes the neighbour set of *i*th RG and *j*th ESS, respectively. In addition,  $x_j$  is designed to track the difference between supply and demand,  $P_D - \mathbf{1}_n^T P_E - \mathbf{1}_n^T P_R$ . Both RGs and ESSs are deployed through a Laplacian-gradient algorithm to explore the minimization of the generation cost. Meanwhile, we introduce a feedback element,  $\gamma x_j$  for *j*th ESS to meet the supply-demand equality condition.

**Remark 5.3.1.** Note that the renewable generation may not be considered as dispatchable if it is controlled in maximum peak power tracking (MPPT) mode because of their intermittent and irregular nature. because of its intermittent and irregular nature. As a result, RGs are not used to compensate the supply-demand mismatch. Thus, only ESSs are deployed
to maintain the supply-demand balance in the control design. The feedback term is only introduced for the *j*th ESS to satisfy the supply-demand equality condition. Furthermore, the *i*th RG only implements (5.7b) to explore the minimization of its own generation cost.

#### 5.3.2 Convergence Analysis

The algorithm is rewritten in a compact form for convergence analysis as

$$\dot{P}_E = -\mathcal{L}_E \partial_{P_E} C(P_R, P_E) + \gamma x, \qquad (5.8a)$$

$$\dot{P}_R = -\mathcal{L}_R \partial_{P_R} C(P_R, P_E), \tag{5.8b}$$

$$\dot{x} = -\alpha \mathcal{L}_E x - \beta (x - (B_0 P_D - P_E - P_R)) - \mathcal{L}_E \eta, \qquad (5.8c)$$

$$\dot{\eta} = \alpha \beta x, \tag{5.8d}$$

where x,  $\eta$  are the column vectors containing  $x_i$ , and  $\eta_i$ , respectively.  $\mathcal{L}_E$  and  $\mathcal{L}_R$  denote the corresponding Laplacian matrix of network connections of ESSs and RGs, respectively.

Let  $P_k = P_{R,i} + P_{E,j}$ , and  $P = [P_1, \dots, P_n] \in \mathbb{R}^n$ . The  $\omega$ -limit set of the trajectories of (5.7) under any initial condition in  $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$  is first characterized. Then, with  $\mathbb{T} = \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$  and  $\mathbb{T}_0 = \mathcal{H}_{P_D} \times \mathcal{H}_0 \times \mathcal{H}_0$ , the following lemma is introduced.

**Lemma 5.3.1.** The  $\omega$ -limit set of any trajectory of (5.8) with any initial condition in  $(P_{k,0}, x_0, \eta_0) \in \mathbb{T}$  is contained in  $\mathbb{T}_0$ .

*Proof.* Defining  $\phi(t) = \mathbf{1}_n^T P - P_D$ , one has

$$\dot{\boldsymbol{\phi}}(t) = \mathbf{1}_n^T \boldsymbol{\gamma} \boldsymbol{x}(t), \tag{5.9}$$

and

$$\ddot{\boldsymbol{\phi}}(t) = \mathbf{1}_{n}^{T} \boldsymbol{\gamma} \dot{\boldsymbol{x}}(t)$$

$$= -\mathbf{1}_{n}^{T} \boldsymbol{\beta} \boldsymbol{\gamma} \boldsymbol{x}(t) + \mathbf{1}_{n}^{T} \boldsymbol{\beta} \boldsymbol{\gamma} (\boldsymbol{x} - (B_{0} P_{D} - P))$$

$$= -\boldsymbol{\beta} \boldsymbol{\gamma} \boldsymbol{\phi}(t) - \boldsymbol{\beta} \dot{\boldsymbol{\phi}}(t).$$
(5.10)

The system can be rewritten as a linear system  $\dot{z} = Az$  with  $z = [z_1, z_2]^T$ , and  $z_1 = \phi(t), z_2 =$ 

 $\dot{\phi}(t)$ . The system matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -\beta\gamma & -\beta \end{bmatrix}.$$

Let  $R \in \mathbb{R}^{2 \times 2}$  be

$$R = \frac{1}{2\gamma\beta^2} \begin{bmatrix} \beta^2 + \beta\gamma + (\beta\gamma)^2 & \beta \\ \beta & 1 + \beta\gamma \end{bmatrix},$$
 (5.11)

which satisfies  $A^T R + RA + I = 0$ . Define  $V_z = z^T Rz$  as a Lyapunov function candidate, and the derivative of  $V_z$  is

$$\dot{V}_z = -z^T z. \tag{5.12}$$

Therefore, it can be deduced that  $(\phi(t); \dot{\phi}) \to 0$ . Furthermore,  $\phi = 0$  implies that  $\mathbf{1}_n^T P_E(t) + \mathbf{1}_n^T P_R(t) \to P_D$  and  $\mathbf{1}_n^T x(t) \to 0$ . Note that  $\mathbf{1}_n^T \dot{\eta} = 0$ , as  $\mathbf{1}_n^T x(t) \to 0$ .

**Remark 5.3.2.** The proof of Lemma 5.3.1 establishes the exponential stability of the demand mismatch dynamics. With the Theorem 5.4 in [125], it can ensure the input-to-state stability(ISS). Thus it is robust to arbitrary bounded perturbations. Additionally, from the Lyapunov equation  $V_z = z^T R z$ , it follows Theorem 4.6 in [125] that gives the convergence rate as

$$\left\|x(t)\right\| \le \sqrt{\frac{\lambda_{\max}(R)}{\lambda_{\min}(R)}} e^{\frac{-1}{\lambda_{\max}(R)}} \left\|x(0)\right\|.$$
(5.13)

The demand mismatch dynamics depends on the topology of the communication network. However, the convergence rate does not directly depend on the knowledge of the Laplacian matrix.

We are now ready to complete the convergence analysis. By applying (5.8), the trajectories of the actual output power of RGs and their parallel-connected ESSs converge to the solution of the optimization problem.

**Theorem 5.3.1.** The trajectories of (5.8) converge to the solution of the optimal charging problem if  $\alpha, \beta, \gamma \in \mathbb{R}_{>0}$  satisfy the following conditions, i.e.,

$$\frac{\beta \lambda_{\max}(\mathcal{L}_R^T \mathcal{L}_R)}{2\lambda_2(\mathcal{L}_R + \mathcal{L}_R^T)} \ge 0, \tag{5.14}$$

and

$$\frac{\gamma}{\alpha\beta\lambda_2(\mathcal{L}_E + \mathcal{L}_E^T)} + \frac{\beta\lambda_{\max}(\mathcal{L}_E^T\mathcal{L}_E)}{2} < \lambda_2(\mathcal{L}_E + \mathcal{L}_E^T).$$
(5.15)

*Proof.* A change of coordinates is performed to shift the equilibrium point of (5.8) to the origin, i.e.,  $\hat{\eta} = \mathcal{L}_E - \beta (B_0 P_D - P_E - P_R)$ . Then, the dynamics (5.8) can be transformed as

$$\dot{P}_{E} = -\mathcal{L}_{E}\xi_{1} + \gamma x,$$
  

$$\dot{P}_{R} = -\mathcal{L}_{R}\xi_{2},$$
  

$$\dot{x} = -\alpha\mathcal{L}_{E}x - \beta(x - (B_{0}P_{D} - P_{E} - P_{R})) - \mathcal{L}_{E}\eta,$$
  

$$\dot{\hat{\eta}} = \alpha\beta\mathcal{L}_{E}x + \gamma\beta x - \beta\mathcal{L}_{E}\xi_{1} - \beta\mathcal{L}_{R}\xi_{2},$$
(5.16)

with  $\xi_1 \in \partial_{P_E} C(P_R, P_E)$  and  $\xi_2 \in \partial_{P_R} C(P_R, P_E)$ .

Next step is to prove that the trajectories of (5.8) converge to the optimal solution of the formulated problem in the new coordinates.

Consider a candidate Lyapunov function

$$V = C(P_R, P_E) + \frac{1}{2}\gamma\beta \|\theta_1\|^2 + \frac{1}{2}\|\theta_2\|^2.$$
(5.17)

where an additional transformation is introduced to more easily identify the candidate Lyapunov function, with  $\theta_1 = x$  and  $\theta_2 = \beta x + \hat{\eta}$ .

Then the time derivative of (5.17) along the trajectories of (5.8) is given as

$$\dot{V} = -\xi_1^T \mathcal{L}_E \xi_1 + \gamma \xi_1^T \theta_1 - \xi_2^T \mathcal{L}_R \xi_2 - \gamma \alpha \beta \theta_1^T \mathcal{L}_E \theta_1 - \beta \theta_2^T \theta_2 - \beta \theta_2^T \mathcal{L}_E \xi_1 - \beta \theta_2^T \mathcal{L}_R \xi_2 = -\frac{1}{2} \xi_1^T (\mathcal{L}_E + \mathcal{L}_E^T) \xi_1 + \gamma \xi_1^T \theta_1 - \frac{1}{2} \xi_2^T (\mathcal{L}_R + \mathcal{L}_R^T) \xi_2 - \frac{1}{2} \gamma \alpha \beta \theta_1^T (\mathcal{L}_E + \mathcal{L}_E^T) \theta_1 - \beta \|\theta_2\|^2 - \beta \theta_2^T \mathcal{L}_E \xi_1 - \beta \theta_2^T \mathcal{L}_R \xi_2$$
(5.18)

Since the directed graph G is strongly connected and weight-balanced with the fact that

 $\mathbf{1}_{n}^{T}\mathbf{\theta}_{1} = 0$  for  $(P_{E}, P_{R}, \mathbf{\theta}_{1}, \mathbf{\theta}_{2}) \in \mathbb{T}_{0}$ , one has

$$\begin{split} \dot{V} &\leq -\frac{1}{2}\lambda_{2}(\mathcal{L}_{E} + \mathcal{L}_{E}^{T}) \left\| \boldsymbol{\xi}_{1} - \frac{1}{n} \boldsymbol{1}_{n}^{T} \boldsymbol{1}_{n} \boldsymbol{\xi}_{1} \right\|^{2} \\ &+ \gamma(\boldsymbol{\xi}_{1} - \frac{1}{n} \boldsymbol{1}_{n}^{T} \boldsymbol{1}_{n} \boldsymbol{\xi}_{1})^{T} \boldsymbol{\theta}_{1} \\ &- \frac{1}{2}\lambda_{2}(\mathcal{L}_{R} + \mathcal{L}_{R}^{T}) \left\| \boldsymbol{\xi}_{2} - \frac{1}{n} \boldsymbol{1}_{n}^{T} \boldsymbol{1}_{n} \boldsymbol{\xi}_{2} \right\|^{2} - \beta \|\boldsymbol{\theta}_{2}\|^{2} \\ &- \frac{1}{2}\gamma\alpha\beta\lambda_{2}(\mathcal{L}_{E} + \mathcal{L}_{E}^{T}) \|\boldsymbol{\theta}_{1}\|^{2} - \beta \boldsymbol{\theta}_{2}^{T} \mathcal{L}_{E}(\boldsymbol{\xi}_{1} - \frac{1}{n} \boldsymbol{1}_{n}^{T} \boldsymbol{1}_{n} \boldsymbol{\xi}_{1}) \\ &- \beta \boldsymbol{\theta}_{2}^{T} \mathcal{L}_{R}(\boldsymbol{\xi}_{2} - \frac{1}{n} \boldsymbol{1}_{n}^{T} \boldsymbol{1}_{n} \boldsymbol{\xi}_{2}) \end{split}$$
(5.19)

Defining  $\psi_1 = \xi_1 - \frac{1}{n} (\mathbf{1}_n^T \mathbf{1}_n \xi_1), \psi_2 = \xi_2 - \frac{1}{n} (\mathbf{1}_n^T \mathbf{1}_n \xi_2)$  and  $\upsilon_1^T = [\psi_1^T, \theta_1^T, \theta_2^T], \upsilon_2^T = [\psi_2^T, \theta_2^T]$ , we have

$$\dot{V} \le \mathbf{v}_1^T J_1 \mathbf{v}_1 - \mathbf{v}_2^T J_2 \mathbf{v}_2, \tag{5.20}$$

where

where 
$$J_{1} = \begin{bmatrix} -\frac{1}{2}\lambda_{2}(\mathcal{L}_{E} + \mathcal{L}_{E}^{T})I_{n} & M_{12}^{T} \\ M_{12} & M_{22} \end{bmatrix},$$
  
with  $M_{12}^{T} = \begin{bmatrix} \frac{1}{2}\gamma I_{n} & -\frac{1}{2}\beta\mathcal{L}_{E}^{T} \end{bmatrix}.$  and furthermore,  $M_{22} = \begin{bmatrix} -\frac{1}{2}\alpha\beta\gamma\lambda_{2}(\mathcal{L}_{E} + \mathcal{L}_{E}^{T})I_{n} & 0 \\ 0 & -\beta I_{n} \end{bmatrix}$  and  $J_{2} = \begin{bmatrix} -\frac{1}{2}\lambda_{2}(\mathcal{L}_{R} + \mathcal{L}_{R}^{T})I_{n} & -\frac{1}{2}\beta\mathcal{L}_{R}^{T} \\ -\frac{1}{2}\beta\mathcal{L}_{R} & 0 \end{bmatrix}.$ 

Resorting to the Schur complement,  $J_1$  is negative definite if

$$-\frac{1}{2}\lambda_2(\mathcal{L}_E + \mathcal{L}_E^T)I_n + \frac{\gamma}{2\alpha\beta\lambda_2(\mathcal{L}_E + \mathcal{L}_E^T)}I_n + \frac{\beta}{4}\mathcal{L}_E^T\mathcal{L}_E$$

is negative definite, and  $J_2$  is positive definite if

$$\frac{\beta \lambda_{\max}(\mathcal{L}_{R}^{T} \mathcal{L}_{R})}{2\lambda_{2}(\mathcal{L}_{R} + \mathcal{L}_{R}^{T})}$$
(5.21)

is positive definite, respectively. Furthermore, we can conclude that  $\dot{V} \leq 0$  by applying (5.15) and (5.14). Hence, with the application of LaSalle Invariance principle, we deduce that  $\dot{V} = 0$  iff  $\psi_1 = \psi_2 = \theta_1 = \theta_2 = 0$ , which implies that  $\partial_{P_E} C(P_R, P_E) \in \text{span}\{\mathbf{1}_n\}$  and  $\partial_{P_R} C(P_R, P_E) \in \text{span}\{\mathbf{1}_n\}$ . Recalling the Lemma 5.3.1 and the characterization of optimizers in Lemma 5.2.1, it indicates  $(P_E^*, P_R^*)$  is a solution of the optimization problem.



Figure 5.1: General operation diagram of an agent.

**Remark 5.3.3.** The local inequality constraints are taken into account by applying additional projection operations to each RG and ESSs. As shown in the literature [126, 127], the projection operation does not affect the convergence analysis of the distributed optimal strategy.

**Remark 5.3.4.** It might be possible to include the operating limits of the power network, such as voltage and line flow limits in the proposed algorithm by applying the projection method, which may lead to different optimization results. Additionally, the proposed method can also deal with the model of the transmission loss in [128], which assumes each agent could estimate the power loss of the line adjacent to it. With the available estimation value, each agent could add this value to the load quantity, which would result in the network discovering a new optimization result that considers power loss.

### 5.4 Simulation Results and Analysis

In the following case studies, the designed parameters are chosen as  $\alpha = 14$ ,  $\beta = 0.6$ ,  $\gamma = 3$ , which satisfies the condition specified in Theorem 5.3.1. The parameters of RGs and ESSs are summarized in Table 5.1. Each participant in the microgrid is regarded as an agent that equips with two level controls, i.e. top level control and bottom level control as shown in Fig. 5.3.2.



Figure 5.2: IEEE 14-bus system.

The first two cases test the proposed control strategy based on the IEEE 14-bus system with the base power  $P_{\text{base}}$ =100kW, and the communication topology is shown as Fig. 5.2. In *Case* 5.1, the optimal resource management problem is studied under both constant supply-demand deviation and time-varying deviation conditions. *Case* 5.2 investigates the performance of the proposed strategy under the link/node failures. To accomplish the tractability of the demonstration, it is assumed that if the single-link/node failure occurs in the communication topology, the rest of the weighted-balanced digraph should still remain connected, and thus the remaining nodes would be able to operate with their neighbouring nodes continuously [47]. Finally, the scalability analysis is validated in *Case* 5.3 in the IEEE 162-bus system with various RGs and ESSs, such as WTs, PVs, Solar thermal, BESSs and fuel cells. Without loss of generality, it is assumed that ESS1 knows the total supply-demand mismatch.

	Table 5.1: Simulation Parameters						
			$a_j$	$b_j$	$P_{E,j}^{\min}$ (p.u.)	$P_{E,j}^{\max}(p.u.)$	
	ESSs	ESS1	0.83	0.007	0	50	
		ESS2	0.79	0.006	0	50	
		ESS3	0.90	0.009	0	50	
		ESS4	0.93	0.012	0	50	
		ESS5	0.78	0.008	0	50	
			Predicated Maximum Capacity				
					$P_{R,i}^{\min}(p.u.)$	$P_{R,i}^{\max}(p.u.)$	$\epsilon_i$
					,	,	
	PVs	PV1	60		0	60	0.1
		PV2	62		0	65	0.2
		PV3	65		0	65	0.1
_	WTs	WT1	(	57	0	67	0.1
		WT2	(	58	0	68	0.2

#### 5.4.1 Case 5.1

In this case, the performance of the proposed distributed strategy for resource management is investigated in the IEEE 14-bus system. The designed communication network of agents can be independent of the physical bus connections, which is a directed network in this simulation study.

- Constant Supply-demand Mismatch Condition: The total supply-demand mismatch is assumed as 500 p.u.. As shown in Figs. 5.3 and 5.4, the power allocations of RGs and parallel-connected ESSs quickly converge to their optimal values, while the supply-demand balance converges to zero, i.e., the optimization objective is achieved.
- 2) Time-varying Supply-demand Mismatch Condition: if renewable energy resources are controlled by MPPT algorithms, this may cause a time-varying supply-demand imbalance when the available renewable generation cannot meet load demands. As a result, in this subcase, the time-varying imbalance is modelled by a time-varying function, i.e.,  $P_D = 500 + 10\sin(0.05t)$  p.u.. The simulation results are shown in Figs. 5.5 and 5.6. The allocated actual powers converge to their optimal values, and the deviation between supply and demand power is close to zero.



Figure 5.3: The actual output power of RGs and ESSs

In this case study, it is shown that the proposed strategy effectively offsets the supplydemand mismatch, which will help with frequency regulation in an islanded microgrid.

#### 5.4.2 Case 5.2

The robustness of single link/node failure is investigated in this case. In the first subcase, it is assumed that a breakdown of the transmission line for ESS3 is at 10s, and it can still communicate with its neighbours. Because RGs are not used for tackling demand mismatch, the remaining ESSs have to undertake the supply-demand balance in a fast response time. At 30s, the emergency line is employed, and hence ESS3 is plugged in to support the load demand. The results are shown in Figs. 5.7 - 5.9. After ESS3 is unplugged, the power allocations of all participants converge to the new optimal values. Furthermore, the supply-demand mismatch can be handled by the remaining ESSs. When ESS3 is connected again, all the results converge to those of the previous ones.

Next, the second subcase is considered when ESS3 fails to operate at 6s and recovers at 40s. Upon failure, ESS3 loses communication with its neighbours, and the remaining ESSs remain connected to support the load demand. The results in Fig. 5.10 and Fig. 5.11 show that the other participants can converge to a new optimal operation condition, and the supply-demand mismatch is eliminated when ESS3 fails. Moreover, the results converge to



Figure 5.4: The supply-demand mismatch update



Figure 5.5: The actual output power of RGs and ESSs



Figure 5.6: The supply-demand mismatch update



Figure 5.7: The actual output power of RGs and ESSs during the single link failure



Figure 5.8: The supply-demand mismatch update



Figure 5.9: Total allocated output power



Figure 5.10: The actual output power of RGs and ESSs during the single node failure

those of the previous ones after ESS3 is repaired.

It should be noted that during single link/node failure, the supply-demand deviations are small. Therefore, the proposed strategy will have little effect on the frequency disturbance of the system, which may be applied to isolated systems such as an autonomous microgrid.

**Remark 5.4.1.** One of the necessary conditions for the convergence of the proposed algorithm is the connectivity of the communication network. Therefore, when single- link/node failure occurs, the optimal solution is guaranteed if the communication network remains connected. Distributed information is also used in the proposed algorithm rather than global information, which is robust to dealing with link failures if the failure does not affect the connectivity of the communication network. Also, when one node fails to communicate with the other neighbours, the remaining groups of nodes can continue their operations if the network is connected.

#### 5.4.3 Case 5.3

The scalability of the proposed strategy is validated. This is achieved by implementing the optimal control strategy to the modified IEEE 162-bus system. There are three types of RGs in this system, i.e., eight PVs, eight WTs and one solar thermal, and two types of ESSs, i.e.,



Figure 5.11: The supply-demand mismatch update

battery storage system and fuel cells. The cost functions of the fuel cell and solar thermal are based on the cost model in [129, 130], respectively. The communication network is weight-balanced and strongly connected. The initial condition is given by [131] with a total load demand of 18422 MW.

As shown in Fig. 5.12 and Fig. 5.13, the proposed strategy can guarantee the allocated charging powers to converge to their optimal values within 12s while the deviations of demand and supply power converge to zero, demonstrating that the proposed algorithm is scalable.

## 5.5 Conclusion

In this chapter, a MAS-based distributed optimal strategy has been proposed to the optimal resource management in an islanded microgrid. The proposed strategy not only minimizes the generation cost to all participants, but also maintains the supply-demand equality condition within the microgrid system. The effectiveness of the proposed distributed strategy is demonstrated by the simulation that uses the IEEE test systems. To tackle the more practical constraints of resource management, voltage limits, line capacities, and network reconfiguration will be considered in future work.



Figure 5.12: The actual power output of RGs and ESSs



Figure 5.13: The supply-demand mismatch update

# **Chapter 6**

# Distributed Finite-Time Optimal Resource Management for Microgrids Based on Multi-Agent Framework

# 6.1 Introduction

The key issues proposed in **Section 3.3.2** mean that a fast convergence algorithm is required to coordinate both dispatchable and non-dispatchable energy resources to effectively maintain microgrid stability under different operation modes.

In this chapter, a two-level optimization system is proposed for optimal resource management with a fast convergence speed. At the top level, the proposed algorithm generates a reference point of optimal power output through local communication. The algorithm only requires information among neighbouring participants without a central control coordination, and simultaneously accomplishes resource optimization in a finite-time while respecting system constraints. The bottom-level control is responsible for the reference tracking of each corresponding participant in a microgrid. The convergent rate of the proposed algorithm is compared with other consensus-based algorithms through simulation studies. Furthermore, an actual islanded system is presented to demonstrate the overall effectiveness of the proposed strategy.

### 6.2 Multi-agent System Architecture of a Microgrid

In this section, the topology of the communication network for a microgrid is first introduced. Next, agents are defined in a microgrid context, and the cost function of each type of agents is formulated for distributed optimization.

#### 6.2.1 Multi-agent System Framework

In the MAS framework, a microgrid consists of a utility grid, non-dispatchable DGs including WTs and PVs, conventional dispatchable synchronous generators (SGs), ESSs and noncontrollable load demands. A Point of Common Coupling (PCC) of the utility grid is used to measure the power delivered/withdrawn and decide the operation mode of the micro- grid. In the MAS framework, each participant in the microgrid is assigned to an intelligent agent that is able to interconnect with its adjacent agents to accomplish established objec- tives. A two-level control model, namely the top-level control and bottom-level control, is deployed to each agent as depicted in Fig. 6.1. The top-level control is a communication network for each agent to transfer information that is generated by an optimal strategy. Both the control mode and the physical agent platform are located at the bottom-level control. According to the control schemes in [73],the control mode is used to adjust the power output of the agent to the reference signal generated by the top-level, and the settings of different control modes can be decided by the agents local operation conditions. Furthermore, the physical agent platform is the electrical components that transmit the power generated/consumed.

#### 6.2.2 The Agent Description under MAS Framework

In this section, agents are defined in the microgrid context under the proposed MAS framework, and the corresponding cost functions are defined for the algorithm design.



Figure 6.1: The communication network of agents in a microgrid

#### **Dispatchable Agents**

**Conventional Generator Agents** Conventional generator agents include fuel generators and gas plants. The generation cost of this type of agent is usually expressed as a typical quadratic function of active power output, i.e.,

$$C_k(P_{G,k}) = \frac{1}{2}a_k P_{G,k}^2 + b_k P_{G,k} + c_k, \ P_{G,k}^{\min} \le P_{G,k} \le P_{G,k}^{\max}$$
(6.1)

where non-negative  $a_k$ ,  $b_k$ , and  $c_k$  are the cost coefficients for *k*th generator;  $P_{G,k}^{\min}$  and  $P_{G,k}^{\max}$  are the lower and upper bounds of the power output, respectively. The marginal cost function is the derivative of the cost function with respect to  $P_{G,k}$ , which is used to later algorithm design.

**Storage Agents** Agents The storage agents can be treated as a dispatchable agent since they have the capacity to transfer power to absorb excessive power and compensate for insufficient power bidirectionally. Integrating storage agents to a microgrid may require additional costs, and thus, it is ideal to charge the storage agents when the marginal cost is low, and discharge them when the electricity rate is high otherwise [132]. Furthermore, battery life directly depends on different DoD scenarios in each cycle time [133], and the degradation cost is also an essential factor that may affect the economic decision for batteries [134]. To this end, inspired by [132], a general cost function for the *j*th battery is proposed

by considering DoD:

$$C_{j}(P_{B,j}) = \frac{1}{2}a_{j}(P_{B,j} + 3P_{B,j}^{\max}(\text{DoD}))^{2} + b_{j}((P_{B,j} + 3P_{B,j}^{\max}(\text{DoD})) + c_{j}, P_{B,j}^{\min} \le P_{B,j} \le P_{B,j}^{\max}$$
(6.2)

where  $a_j$ ,  $b_j$ , and  $c_j$  are the non-negative quadratic coefficients;  $P_{B,j}^{\min}$  and  $P_{B,j}^{\max}$  are the minimal and maximum charging rate, respectively. The DoD is the current depth of charge, and the marginal cost is defined as the derivative of the cost function with respect to  $P_{B,j}$ . From the cost function design, it is worth noting that the marginal cost is proportional to the DoD and the power withdrawn.

**Utility Agent** When the microgrid is operated under the grid-connected mode, the utility agent will monitor the power transfer between the utility grid and the microgrid through the PCC. Accordingly, the microgrid updates its current electricity rate based on the broadcast-ing of the utility grid. The cost function can be assumed as a function with constant marginal cost [132],

$$C_U = P_r \times P_U \tag{6.3}$$

where  $P_r$  is the current electricity rate and  $P_U$  is the exchanged power between the utility and the microgrid. During the grid-connected mode operation, the marginal cost of each dispatchable agent should be equal to the marginal cost given by the utility grid, which is equal to the electric rate.

#### Nondispatchable Agents

**Renewable Agents** The RGs, including WTs and PVs, are considered as non-dispatchable since they are usually controlled in MPPT mode. Therefore, the forecast of their generation cannot be accurate because of the stochastic property and intermittency.

**Load Demand Agents** The load demands, consisting of industrial and residential loads, have the stochastic property of the consumer behaviours, so they cannot be treated as dispatchable due to the unpredictable nature of power requirements.

Due to the intermittency of renewable generation, the operating condition of a microgrid may change frequently and unexpectedly. In order to ensure the performance and stability of the microgrid, the system should converge to the required target in a timely manner. As a result, a faster convergence rate of a distributed optimal solution is required to meet the challenges of microgrid development.

# 6.3 Finite-time Distributed Optimal Solution for the Islanded Microgrid

In this section, a finite-time distributed optimal solution is developed for the islanded microgrid to address the problems in the centralized strategies in the microgrid application.

The islanded microgrid should ensure supply-demand balance in a cost-effective way. To this end, the objective for agents in an islanded microgrid is to balance supply and demand, and minimize generation/operation costs in the meantime. The objective function can be formulated as:

$$\begin{array}{ll}
\operatorname{Min} & \sum_{k \subseteq G} C_k(P_{G,k}) + \sum_{j \subseteq B} C_j(P_{B,j}) \\
\text{s.t} & P_D = P_L - P_R = \sum_{k \subseteq G} P_{G,k} + \sum_{j \subseteq B} P_{B,j} \\
& P_{G,k}^{\min} \leq P_{G,k} \leq P_{G,k}^{\max} \\
& P_{B,j}^{\min} \leq P_{B,j} \leq P_{B,j}^{\max} \\
\end{array} \tag{6.4}$$

where  $P_D$  is the net power demand;  $P_L$  and  $P_R$  are all demands of load agents and all nondispatchable generation, respectively.

For convenience, we denote  $P_i$  as the output power, and  $P_i^{\min}$  and  $P_i^{\max}$  as the lower and upper bound of *i*th dispatchable agent, respectively. We further define  $\omega_i$ ,  $\sigma_i$  and  $c_i$  as the

coefficients of ith cost function. Then (6.4) can be written as

$$\begin{array}{ll}
\text{Min} & \sum_{i \in n} C_i(P_i) \\
\text{s.t} & \sum_{i \in n} P_i = P_L - P_R \\
& h_i(P_i) = (P_i^{\min} - P_i)(P_i^{\max} - P_i) \le 0.
\end{array}$$
(6.5)

The solution for optimal resource management can be obtained using a centralized approach, which requires a control centre and bi-directional communication lines between the controller and the connected agents. The central controller collects all data first, e.g., the cost functions, the local constraints, and the power generated/consumed. Then, it solves the optimal problem and broadcasts the solution to all agents. However, due to the high penetration of RGs, more frequent updates and a higher convergence speed are required. The centralized strategy may lose its control efficiency if operating conditions change frequently and unpredictably. To address this problem, the following sections will introduce two distributed solutions for optimal resource management.

#### 6.3.1 Alternative Formulation

Inequality constraints may give rise to difficulties in optimal control design. Before presenting the optimal solution, an alternative formulation of the optimization problem is developed for dealing with inequality constraints by using  $\varepsilon$ -exact penalty function proposed in [17], i.e.,

$$p_{\mathbf{\epsilon},i}(h_i(P_i)) = egin{cases} 0, & h_i(P_i) \leq 0 \ rac{h_i(P_i)^2}{2\mathbf{\epsilon}}, & 0 \leq h_i(P_i) \leq \mathbf{\epsilon} \ h_i(P_i) - rac{\mathbf{\epsilon}}{2}, & h_i(P_i) \geq \mathbf{\epsilon} \end{cases}$$

where  $\varepsilon$  is a positive coefficient. Thus the objective function is further written as:

Min 
$$C_{\varepsilon}(P) = \sum_{i \in n} C_{\varepsilon,i}(P_i)$$
  
s.t  $\sum_{i \in n} P_i = P_L - P_R$  (6.6)

where  $C_{\varepsilon,i}(P_i) = C_i(P_i) + \mu p_{\varepsilon,i}(h_i(P_i))$ , and  $P = [P_1, \dots, P_n]^T$ .  $\mu$  is a positive weight of the penalty function.

Let  $P^* = [P_1^*, \dots, P_n^*]^T$  be the optimal solution for (6.5), and  $\hat{P^*} = [\hat{P}_1^*, \dots, \hat{P}_n^*]^T$  be the solution for (6.6). Then, following Proposition 6 in [17], for  $\mu = \frac{1-n}{1-\sqrt{n}}\mu^*$ , the relationship between (6.5) and (6.6) is

$$0 \le C(P^*) - C_{\varepsilon}(\hat{P}^*) \le \varepsilon \mu n, \tag{6.7}$$

where  $\mu^* > \max{\{\eta^*\}}$  with  $\eta^* = {\{\eta_1^*, \dots, \eta_n^*\}}$  is the Lagrange multiplier vector satisfying the KarushKuhnTucker (KKT) condition [135]. According to the Proposition 4 in [136], the upper bound of  $\eta^*$  is given as

$$\max\{\eta_i^*\}_{i=1}^n \le \frac{2\max\{\max_{P_i \in P_{\text{fea},i}} |\nabla C_i(P_i)|\}_{i=1}^n}{\min\{P_i^M - P_i^m\}_{i=1}^n},\tag{6.8}$$

where  $\nabla C_i(P_i)$  is the gradient of  $C_i(P_i)$ , and  $P_{\text{fea},i} = \{P_i \in \mathcal{R} \mid \sum_{i \in n} P_i = P_L - P_R \text{ and } (P_i^{\min} - P_i)(P_i^{\max} - P_i) \le 0\}$ , for  $i \in n$ .

**Remark 6.3.1.** To ensure the  $\varepsilon$ -solution is equal to the solution of the original optimization problem exactly, the local bound of each dispatchable agent is further modified as  $h_i(P_i) = (P_i^{\min} - P_i)(P_i^{\max} - P_i) + \varepsilon \le 0$  that can meet the accurate requirement of the system. Note that s can be chosen to be significantly small so that it does not affect the convergence of the original problem. After the change of local bounds, the  $\varepsilon$ -feasible set is the same as the feasible set of the original optimization problem. The proof can be obtained by adopting a similar approach in [17].

#### 6.3.2 Finite-time Distributed Optimal Energy Management

By invoking the  $\varepsilon$ -penalty function, a finite-time distributed optimal solution is proposed to solve the optimization problem (6.6). The update dynamics for *i*th dispatchable agent is

$$\dot{P}_{i} = \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{\frac{p}{q}}$$

$$(6.9)$$

where p and q are the positive odd integers satisfying p < q. The dynamics is distributed in the sense that each dispatchable agent only requires local available information and the information from its adjacent agents through a local communication network. **Theorem 6.3.1.** Consider the optimization problem in (6.6) with the dynamics (9), the feasible set  $P_{\text{fea}}$  is time-invariant, and any trajectory starting from  $P_{\text{fea}}$  converges to the solution set of (6.6) in a finite-time.

*Proof.* A candidate function  $V_1 = |\sum_{i=1}^n P_i|$  is considered since the active power of ESSs would be negative/positive when they are charged/discharged. Also it should be noted that  $P_D \neq 0$ , we only consider the cases for  $P_D > 0$  or  $P_D < 0$  accordingly. Firstly, if  $\sum_{i=1}^n P_i > 0$ ,  $V_1 = \sum_{i=1}^n P_i$  and the derivative of  $V_1$  is

$$\dot{V}_{1} = \sum_{i=1}^{n} \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{2 - \frac{p}{q}} + \sum_{i=1}^{n} \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{\frac{p}{q}} = 0.$$
(6.10)

Then, if  $\sum_{i=1}^{n} P_i < 0$ ,  $V_1 = -\sum_{i=1}^{n} P_i$  and the derivative is

$$\dot{V_1} = -\left(\sum_{i=1}^n \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_j) - \nabla C_{\varepsilon,i}(P_i))^{2-\frac{p}{q}} + \sum_{i=1}^n \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_j) - \nabla C_{\varepsilon,i}(P_i))^{\frac{p}{q}}\right) = 0.$$
(6.11)

From the above results, it concludes that the total power output of network is conserved and the feasible set  $P_{\text{fea}}$  is time-invariant.

Next, we show that the trajectories starting from  $P_{\text{fea}}$  will converge to the solution set in a finite-time manner. Note that the solution to (6.6) is unique so that  $P^*$  will be the unique solution to the minimization problem. Denote  $V_2 = \sum_{i=1}^n C_{\varepsilon,i}(P_i) - \sum_{i=1}^n C_{\varepsilon,i}(P_i^*)$  as a candidate function. It is worth noting that  $V_2 \ge 0$ , and  $V_2 = 0$  when  $P_i = P^*$ . From (6.9), and let  $\xi_i \in \nabla C_{\varepsilon,i}(P_i)$ ,

$$\dot{V}_{2} = \sum_{i=1}^{n} \xi_{i} \left[ \sum_{j \in n} a_{ij} (\xi_{j} - \xi_{i})^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (\xi_{j} - \xi_{i})^{\frac{p}{q}} \right] \\ = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (\xi_{i} - \xi_{j}) (\xi_{j} - \xi_{i})^{2 - \frac{p}{q}} + \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (\xi_{i} - \xi_{j}) (\xi_{j} - \xi_{i})^{\frac{p}{q}} \\ = -\frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (\xi_{j} - \xi_{i})^{\frac{3q-p}{q}} - \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (\xi_{j} - \xi_{i})^{\frac{q+p}{q}} \\ = -\frac{1}{2} \sum_{i,j=1}^{n} \left[ a_{ij}^{\frac{2q}{3q-p}} (\xi_{j} - \xi_{i})^{2} \right]^{\frac{3q-p}{2q}} - \frac{1}{2} \sum_{i,j=1}^{n} \left[ a_{ij}^{\frac{2q}{q+p}} (\xi_{j} - \xi_{i})^{2} \right]^{\frac{q+p}{2q}}.$$

$$(6.12)$$

Following the Corollary 2 in [137] and Lemma 3.4 in [138], it can be obtained that

$$\dot{V}_{2} \leq -\frac{1}{2} n^{\frac{p-q}{q}} \sum_{i,j=1}^{n} \left[ a_{ij}^{\frac{2q}{3q-p}} (\xi_{j} - \xi_{i})^{2} \right]^{\frac{3q-p}{2q}} - \frac{1}{2} \left[ \sum_{i,j=1}^{n} a_{ij}^{\frac{2q}{q+p}} (\xi_{j} - \xi_{i})^{2} \right]^{\frac{q+p}{2q}} \\ = -\frac{1}{2} n^{\frac{p-q}{q}} \left[ 2\xi^{T} \mathcal{L}_{B} \xi \right]^{\frac{3q-p}{2q}} - \frac{1}{2} \left[ 2\xi^{T} \mathcal{L}_{C} \xi \right]^{\frac{q+p}{2q}}$$
(6.13)

where  $\xi = [\xi_1, \dots, \xi_n]^T$ . Consider the graphs  $\mathcal{G}_B$  and  $\mathcal{G}_C$  with the adjacency matrices  $\mathcal{A}_B = [a_{ij}^B]_{n \times n}$  and  $\mathcal{A}_C = [a_{ij}^C]_{n \times n}$ , respectively. Then  $\mathcal{L}_B$  and  $\mathcal{L}_C$  are the corresponding Laplacian matrices of  $\mathcal{G}_B$  and  $\mathcal{G}_C$  respectively. Define

$$\Gamma_B = 2\xi^T \mathcal{L}_B \xi \ge 2\lambda_2(\mathcal{L}_B) \left\| \xi - \frac{1}{n} (\mathbf{1}_n^T \xi) \mathbf{1}_n^T \right\|_2^2$$
(6.14)

$$\Gamma_C = 2\xi^T \mathcal{L}_C \xi \ge 2\lambda_2(\mathcal{L}_C) \left\| \xi - \frac{1}{n} (\mathbf{1}_n^T \xi) \mathbf{1}_n^T \right\|_2^2.$$
(6.15)

Since  $C_{\varepsilon}(P)$  is strongly convex, for  $\hat{P} = [\hat{P}_1, \dots, \hat{P}_n]$ , it follows that

$$C_{\varepsilon}(\hat{P}) \ge C_{\varepsilon}(P) + (\xi - \frac{1}{n}(\mathbf{1}_{n}^{T}\xi)\mathbf{1}_{n}^{T})(\hat{P} - P) + \frac{k}{2}\left\|\hat{P} - P\right\|^{2}.$$
(6.16)

It is worth noting that the minimum of (6.16) is  $C_{\varepsilon}(P) - \frac{1}{2k} \left\| \xi - \frac{1}{n} (\mathbf{1}_n^T \xi) \mathbf{1}_n^T \right\|$  for constant *P*. For  $\hat{P} = P^*$ , we have  $V_2 \leq -\frac{1}{2k} \left\| \xi - \frac{1}{n} (\mathbf{1}_n^T \xi) \mathbf{1}_n^T \right\|$ . Therefore,

$$\begin{split} \dot{V}_{2} &\leq -\frac{1}{2} n^{\frac{p-q}{q}} [4k\lambda_{2}(\mathcal{L}_{B})V_{2}]^{\frac{3q-p}{2q}} - \frac{1}{2} [4k\lambda_{2}(\mathcal{L}_{C})V_{2}]^{\frac{q+p}{2q}} \\ &\leq -\frac{1}{2} n^{\frac{p-q}{q}} [4k\overline{\lambda}V_{2}]^{\frac{3q-p}{2q}} - \frac{1}{2} [4k\overline{\lambda}V_{2}]^{\frac{q+p}{2q}} \\ &= -\frac{1}{2} \left[ n^{\frac{p-q}{q}} [4k\overline{\lambda}V_{2}]^{\frac{q-p}{q}} + 1 \right] ([4k\overline{\lambda}V_{2}]^{\frac{q+p}{2q}}), \end{split}$$
(6.17)

where  $\overline{\lambda} = \min \{\lambda_2(\mathcal{L}_B), \lambda_2(\mathcal{L}_C)\} > 0$ . Let  $\delta = \sqrt{4k\overline{\lambda}V_2}$ , then

$$\dot{\delta} = -n^{\frac{p-q}{q}} \overline{\lambda} \delta^{\frac{2q-p}{q}} - \overline{\lambda} \delta^{\frac{p}{q}}.$$
(6.18)

With Lemma 4.1 in [138] and Comparison Lemma in [125], we conclude that

$$\lim_{t \to T_1} V_2 = 0, \tag{6.19}$$

where  $T_1$  is given by

$$T_1 \le \frac{\pi q n^{\frac{q-p}{2q}}}{2\overline{\lambda}(q-p)}.$$
(6.20)

Note that  $V_2 \to 0$  implies  $\sum_{i=1}^{n} C_{\varepsilon,i}(P_i) - \sum_{i=1}^{n} C_{\varepsilon,i}(P_i^*) = 0$ , which can deduce that  $\xi \in \text{span}\{\mathbf{1}_n\}$ . Since  $\mathbf{1}_n^T P_0 = P_L - P_R$  with  $P_0 = [P_{1,0}, \dots, P_{n,0}]^T$ , it concludes that the trajectories starting from  $P_{\text{fea}}$  converge to the set of solutions of the optimal problem.

# 6.4 Finite-time Distributed Solution for Grid-connected Microgrid

Once the operation mode of a microgrid is switched to the grid-connected mode, the participants in the microgrid should follow the marginal cost set by the utility agent, which can be treated as a tracking problem, i.e.,

$$\nabla C_{\varepsilon,1}(P_1) = \nabla C_{\varepsilon,2}(P_2) = \dots = \nabla C_{\varepsilon,n}(P_n) = \nabla C_U.$$
(6.21)

To address this problem, a finite-time distributed solution is proposed as:

$$\dot{P}_{i} = \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (\nabla C_{\varepsilon,j}(P_{j}) - \nabla C_{\varepsilon,i}(P_{i}))^{\frac{p}{q}} + g_{i} (P_{r} - \nabla C_{\varepsilon,i}(P_{i}))^{2 - \frac{p}{q}} + g_{i} (P_{r} - \nabla C_{\varepsilon,i}(P_{i}))^{\frac{p}{q}}$$

$$(6.22)$$

where  $P_r = \nabla C_U$ , and  $g_i$  is the pinning gain of *i*th dispatchable agent. Define  $e_i = P_r - \xi_i$ , where  $\xi_i \in \nabla C_{\varepsilon,i}(P_i)$ , and then the derivative of  $e_i$  is given as

$$\dot{e}_{i} = \dot{\xi}_{i} \cdot \dot{P}_{i} = \omega_{i} \left[ \left( \sum_{j \in n} a_{ij} (\xi_{j} - \xi_{i})^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (\xi_{j} - \xi_{i})^{\frac{p}{q}} \right) + g_{i} (P_{r} - \xi_{i}^{2 - \frac{p}{q}}) + g_{i} (P_{r} - \xi_{i})^{\frac{p}{q}} \right) \right]$$
$$= \omega_{i} \left[ \left( \sum_{j \in n} a_{ij} (e_{j} - e_{i})^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (e_{j} - e_{i})^{\frac{p}{q}} \right) - g_{i} (e_{i})^{2 - \frac{p}{q}} - g_{i} (e_{i})^{\frac{p}{q}} \right]$$
(6.23)

where  $\nabla^2 C_i(P_i) = \omega_i > 0$ . Suppose  $\omega_i$  is known by *i*th dispatchable agent, and the communication network satisfies the condition in weights that  $\omega_i a_{ij} = \omega_j a_{ji}$ .

**Theorem 6.4.1.** Suppose the graph is connected. Then, by applying (6.22), the distributed tracking problem (6.21) is solved in a finite-time.

*Proof.* Consider a candidate Lyapunov function,  $V_3 = \frac{1}{2} \sum_{i=1}^{n} e_i \cdot e_i$ , and then differentiating  $V_3$  yields

$$\begin{split} \dot{V}_{3} &= \sum_{i=1}^{n} e_{i} \omega_{i} \left[ \left( \sum_{j \in n} a_{ij} (e_{j} - e_{i})^{2 - \frac{p}{q}} + \sum_{j \in n} a_{ij} (e_{j} - e_{i})^{\frac{p}{q}} \right) - g_{i} (e_{i})^{2 - \frac{p}{q}} - g_{i} (e_{i})^{\frac{p}{q}} \right] \\ &= \frac{1}{2} \sum_{i,j=1}^{n} \left[ (\omega_{i} a_{ij}) (e_{i} - e_{j}) (e_{j} - e_{i})^{2 - \frac{p}{q}} \right] + \frac{1}{2} \sum_{i,j=1}^{n} \left[ (\omega_{i} a_{ij}) (e_{i} - e_{j}) (e_{j} - e_{i})^{\frac{p}{q}} - \sum_{i=1}^{n} \left[ (\omega_{i} g_{i}) (e_{i})^{\frac{3q-p}{q}} - \sum_{i=1}^{n} \left[ (\omega_{i} g_{i}) (e_{i})^{\frac{2q}{q}} - \sum_{i=1}^{n} \left[ (\omega_{i} g_{i}) (e_{i})^{\frac{2q}{q}} - \frac{1}{2} \sum_{i,j=1}^{n} \left[ (\omega_{i} a_{ij})^{\frac{2q}{q+p}} (e_{j} - e_{i})^{2} \right]^{\frac{q+p}{2q}} - \sum_{i=1}^{n} \left[ (\omega_{i} g_{i})^{\frac{2q}{2q-p}} (e_{i})^{2} \right]^{\frac{3q-p}{2q}} - \sum_{i=1}^{n} \left[ (\omega_{i} g_{i})^{\frac{2q}{q+p}} (e_{i})^{2} \right]^{\frac{q+p}{2q}}. \end{split}$$

As a result,

$$\dot{V}_{3} \leq -\frac{1}{2} \left( \sum_{i,j=1}^{n} (\omega_{i}a_{ij})^{\frac{2q}{3q-p}} (e_{j}-e_{i})^{2} + 2\sum_{i=1}^{n} (\omega_{i}g_{i})^{\frac{2q}{3q-p}} (e_{i})^{2} \right)^{\frac{3q-p}{2q}} -\frac{1}{2} \left( \sum_{i,j=1}^{n} (\omega_{i}a_{ij})^{\frac{2q}{q+p}} (e_{j}-e_{i})^{2} + 2\sum_{i=1}^{n} (\omega_{i}g_{i})^{\frac{2q}{q+p}} (e_{i})^{2} \right)^{\frac{q+p}{2q}}.$$
(6.24)

Define

$$\Theta = \sum_{i,j=1}^{n} (\omega_{i}a_{ij})^{\frac{2q}{3q-p}} (e_{j} - e_{i})^{2} + 2\sum_{i=1}^{n} (\omega_{i}g_{i})^{\frac{2q}{3q-p}} (e_{i})^{2}$$
$$\Phi = \sum_{i,j=1}^{n} (\omega_{i}a_{ij})^{\frac{2q}{q+p}} (e_{j} - e_{i})^{2} + 2\sum_{i=1}^{n} (\omega_{i}g_{i})^{\frac{2q}{q+p}} (e_{i})^{2}.$$
(6.25)

Let  $\mathcal{G}_{\alpha}$  and  $\mathcal{G}_{\beta}$  be the graphs with  $\alpha_{ij} = (\omega_i a_{ij})^{\frac{2q}{3q-p}}$  and  $\beta_{ij} = (\omega_i a_{ij})^{\frac{2q}{q+p}}$ , respectively. The Laplacian matrix of each graph is  $\mathcal{L}_{\alpha}$  and  $\mathcal{L}_{\beta}$ .  $G_b = \text{diag}(g_{b,i})$  with  $g_{b,i} = (\omega_i g_i)^{\frac{2q}{3q-p}}$ , and  $G_c = \text{diag}(g_{c,i})$  with  $g_{b,i} = (\omega_i g_i)^{\frac{2q}{q+p}}$  are the diagonal matrix of pinning gains of each graph. Then, following Lemma 4 in [139], one has

$$\Theta = 2e(\mathcal{L}_{\alpha} + G_b)e \ge 2\lambda_1(\mathcal{L}_{\alpha} + G_b)e^Te > 0$$
  
$$\Phi = 2e(\mathcal{L}_{\beta} + G_c)e \ge 2\lambda_1(\mathcal{L}_{\beta} + G_c)e^Te > 0$$
(6.26)

which can be rewritten as  $4\lambda_1(\mathcal{L}_{\alpha} + G_b) \leq \frac{\Theta}{V_3}$  and  $4\lambda_1(\mathcal{L}_{\beta} + G_c) \leq \frac{\Phi}{V_3}$ . Then, following the similar lines in the proof of Theorem 6.3.1, (6.24) can be reformulated as

$$\dot{V}_{3} \leq -\frac{1}{2}n^{\frac{p-q}{q}} [4\lambda_{1}(\mathcal{L}_{\alpha}+G_{b})]^{\frac{3q-p}{2q}} - \frac{1}{2} [4\lambda_{1}(\mathcal{L}_{\beta}+G_{c})]^{\frac{q+p}{2q}} \leq -\frac{1}{2}n^{\frac{p-q}{q}} [4\overline{\lambda_{1}}]^{\frac{3q-p}{2q}} - \frac{1}{2} [4\overline{\lambda_{1}}]^{\frac{q+p}{2q}}$$
(6.27)

where  $\overline{\lambda_1} = \min\{\lambda_1(\mathcal{L}_{\alpha} + G_b), \lambda_1(\mathcal{L}_{\beta} + G_c)\}$ . Similarly, with Lemma 4.1 in [138] and Comparison Lemma in [125], there is

$$\lim_{t \to T_2} V_3 = 0, \tag{6.28}$$

where  $T_2$  is given by

$$T_2 \le \frac{\pi q n^{\frac{q-p}{2q}}}{2\overline{\lambda_1}(q-p)}.$$
(6.29)

Note that  $V_3 \to 0$  implies  $e_i \to 0$ , i.e.,  $\forall C_{\varepsilon,i}(P_i) \to P_r$ . Thus the tracking problem (6.21) is solved in a finite-time.

### 6.5 Simulation Results and Analysis

In this section, three case studies are provided to verify the effectiveness of the proposed finite-time algorithm. The simulation studies are carried out in IEEE 14-bus system, where Fig. 6.2 depicts the topology of the test system that includes three conventional generators, two storage systems, one renewable generator and nine loads. Table. 6.1 gives the parameters of each type of agents.

In *Case* 6.1, to verify the faster convergence property, the proposed finite-time algorithm is compared with other two optimization solutions for the resource management in microgrids. *Case* 6.2 investigates the performance of the proposed strategy under different conditions of microgrid operation. In *Case* 6.3, an actual islanded system is built to test the proposed algorithm. Finally, a modified IEEE 162-bus system [131] is adopted for verifying the scalability of the proposed algorithm.

**Remark 6.5.1.** Note that the upper bounds of the settling-time  $T_1$  and  $T_2$  only depend on the designed parameters p, q, the communication network structure and the volume n of the given system. As a result, the convergence time can be estimated based on the algorithm and network design.



Figure 6.2: IEEE 14-bus system

Table 6.1: Parameters of simulation studies									
		$\omega_i$	$\sigma_i$	$P_i^{\min}$	$P_i^{\max}$				
	G1	0.082	3.25	20	65				
	G2	0.068	4.2	20	65				
	G3	0.071	5.08	20	70				
	ESS1	0.3	0.07	-45	45				
	ESS2	0.4	0.062	-50	50				

#### 6.5.1 Case 6.1

The resource management problem has been solved in many studies by using the MAS framework. In [3], the authors proposed a distributed gradient algorithm (DGA) for the maximum social welfare in a microgrid by considering the actual situation of renewable energy. Also, a Laplacian-gradient dynamics (LGD) in [112] is proposed for the economic dispatch problem, which achieves supply-demand balance while minimizing the total generation cost. However, [3] solves the problem in the undirected graph and the fast convergence requirement of a microgrid is not considered in [112]. In contrast, we solve the optimal resource management problem by adopting a finite-time approach. To demonstrate the fast convergence property of the proposed algorithm, the proposed finite-time algorithm is first compared with the DGA and the LGD in this case. The communication network keeps the same configuration and is assumed to be connected during the simulation study. Fig. 6.3 shows the performance of the proposed finite-time algorithm. The results indicate that the marginal costs of each dispatchable agent converges to the optimal value in a finite-time. To give a marked comparison, Fig. 6.4 and 6.5 depict the comparative evaluation of the marginal cost update. Furthermore, as shown in Table. 6.2, the setting time of the proposed algorithm (1.2s) is shorter compared with the DGA (4.3s) and LGD (5.9s). As the results show, the proposed algorithm leads to a faster convergence compared with the algorithm proposed in [3] and [112], which is an indispensable part of microgrid operation because of the intermittent and unpredictable nature of non-dispatchable agents.

Table 6.2: The setting time of different algorithms

	Finite-time	DGA	LGD
Т	1.2s	4.3s	5.9s

It is worth noting that the setting time of the proposed finite-time algorithm depends on the parameter selection and the communication network structure. Once the size of the microgrid is increased, the network information may change and the upper bounds on the setting time will change accordingly. However, the proposed algorithm solves the optimal resource management problem using a distributed and sparse approach, and hence, it is scalable for a large-scale network, which is verified in *Case* 5.4. Also, real islanded microgrids are usually small-scale due to the mission-critical application so that they usually have the limited



Figure 6.3: The marginal cost update with the proposed finite-time algorithm

inverters. The applicability of the proposed algorithm for an actual islanded system is tested in Case *Case* 5.3.

#### 6.5.2 Case 6.2

As the results in *Case* 6.1 show, the proposed algorithm is applicable for microgrids under the unpredictable change of the operation mode. Then, in this case, the proposed algorithm is applied to a microgrid under both islanded and grid-tied modes to verify the effectiveness of the proposed algorithm during different operation modes of the microgrid. The microgrid is supposed to be operated in islanded mode at 0s, and be connected to the grid at 5s. During the simulation study, we assume a spanning tree exists in the communication network, which is used to ensure the convergence of the proposed algorithm. Fig. 6.6 and 6.7 give the updates of the marginal cost and actual output power during the microgrid operation. As Fig. 6.6 shows, the marginal cost of each dispatchable agent converges to the optimal value initially. After the microgrid is connected to the grid at 5s, the marginal cost converges to a new value because it is controlled to follow the reference set by the utility. The results show that both of them converge to the optimal value in a finite-time, and the setting time is bounded.



Figure 6.4: The marginal cost update with the distributed gradient algorithm



Figure 6.5: The marginal cost update with the Laplacian-gradient dynamics



Figure 6.6: The marginal cost update with the proposed finite-time algorithm



Figure 6.7: The actual output power

#### 6.5.3 Case 6.3

With the above results, *Case* 6.3 is then adopted to demonstrate the potentiality of the practical application of the algorithm. In this case, an actual islanded system located in Inner Mongolia of China [2] is developed to demonstrate the effectiveness of the proposed algorithm. The single line diagram of the islanded system is shown as Fig. 6.8, which consists of eight SGs, two wind power plants and three aluminium smelting loads. The relevant configurations of each participant can be found in [140]. In this islanded system, the penetration of wind power generation is up to 30% and therefore the stability of the islanded system is crucial. Due to stochastic wind power generation, there is a requirement

In this simulation study, the coefficients for SGs are analytically selected to proper practical conditions [140]. Three aluminium smelting loads are regarded as constant loads. Furthermore, renewable resources should be used as much as possible. As a result, it is assumed that the two wind power plants are operating properly and that the maximum available power of these plants should be equal to their capacity.

By supposing the communication network is connected, Fig. 6.9 and 6.10 show that the marginal cost of each dispatchable agent converges to the optimal value first, and the actual power outputs converge to the optimal values accordingly. However, the islanded system may encounter the tripping problem of SGs. As a result, the algorithm is further adopted to verify the effectiveness of the algorithm when the SG is tripped. In the simulation, the SG4 is supposed to be suddenly tripped at 5s, and the rest of SGs then share the mismatch to maintain the supply-demand balance. Therefore, as shown in Fig. 6.9 and 6.10, the marginal cost converges to a new optimal value and actual power outputs changes to new optimal values in a faster convergence manner consequently after 5s. It is worth noting that the SG3 and SG5 respond to the tripping of SG4 first as they are adjacent to SG4.

**Remark 6.5.2.** The intermittency of the wind power generation would cause the supplydemand mismatch, which could be treated as a constant variation in a short time. In the *Case* 6.3, since the SG tripping may cause load variation, by considering the SG tripping during simulation, the results could verify the effectiveness of the algorithm for the load demand variation from another side. At the same time, the storage system can be integrated into the system by deploying the formulated cost function and the proposed algorithm.



Figure 6.8: An actual islanded system in China [2]

#### 6.5.4 Case 6.4

In this case, the IEEE 162-bus system is adopted to further evaluate the performance of the proposed strategy. The adopted system is modified with nine generators and eight ESSs and the initial condition is given by [131] with a total load demand of 18422 MW. The communication network is assumed weight-balanced and strongly connected.

Deploying the algorithm at 0s, the results in Fig. 6.11 and Fig. 6.12 show the proposed strategy can guarantee the marginal costs to converge to the optimal value while the allocated active powers to optimal values correspondingly. Thus the scalability of the proposed algorithm is demonstrated.

# 6.6 Conclusion

This chapter introduces a MAS-based finite-time distributed optimization strategy to the optimal resource management in a microgrid under both islanded and grid-connected modes.



Figure 6.9: The marginal cost update with the proposed finite-time algorithm for an actual islanded microgrid



Figure 6.10: The actual output power



Figure 6.11: The marginal cost update



Figure 6.12: The actual output power

The proposed strategy can minimize the generation cost of each participant while maintaining the supply-demand balance. Achieving a fast convergence is necessary for facilitating the development of a microgrid because the changes in a microgrid can be frequent and unpredicted. A finite-time algorithm is adopted to achieve a higher convergence speed, which is beneficial to the low-inertial microgrid to maintain its power balance in the presence of unknown changes and agent trips.
# Chapter 7

# **Consensus-Based Distributed Fixed-time Economic Dispatch under Uncertain Information in Microgrids**

### 7.1 Problem Formulation

### 7.1.1 Objective Function in the Microgrid

The objectives of an islanded microgrid are to fulfil the demand required by loads to maintain the supply-demand balance and to minimise the operating cost of contained DERs. To achieve the economical operation of the microgrid, as discussed below, cost functions are designed for DERs according to their different features.

### **Controllable Distributed Generators**

In the proposed model, the microgrid consists of CGs as the controllable generators. Denoting  $\mathcal{N}_{CG}$  as the set of CGs in the microgrid, the generation cost of the *j*th CG, is expressed as a quadratic function of active power generation [141],

$$C_j(P_j) := a_j P_j^2 + b_j P_j + c_j, \quad \forall j \in \mathcal{N}_{CG},$$
(7.1)

where  $a_j$ ,  $b_j$  and  $c_j > 0$  are cost coefficients of the *j*th CG.

#### **Battery Energy Storage Systems**

For the optimal plan of BESS operation, a key factor is the degradation of battery cells subjected to repeated charging/discharging cycles. Experiments in [142] have shown that the increase in the cycle depth would suffer a constant marginal cost if the battery is limited within a certain operational range. Also, [143] introduces a fixed per-kWh degradation model to represent battery degradation. In this work, the fixed per-kWh degradation model is modified by replacing the DoD of the battery under nominal conditions within a certain DoD range, i.e.,  $\forall b \in \mathcal{N}_b$  with  $\mathcal{N}_b$  is the set of BESSs,

$$B_b(P_b) := \frac{|P_b|}{CN_b(D_b^{\max} - D_b^{\min})/100\% \cdot 2C_b^{cap.}},$$
(7.2)

where  $P_b$  is the power output of the *b*th BESS;  $CN_b$  is the cycle number that the battery could be operated within  $[D_b^{\min}, D_b^{\max}]$ .  $C_b^{cap}$  is the capacity of the *b*th BESS. Therefore, the operating cost of a BESS is formulated as

$$C_b(P_b) := C_{\text{ini}} B_b(P_b) + a_b P_b^2 + c_b, \forall b \in \mathcal{N}_b,$$

$$(7.3)$$

where  $C_{\text{ini.}}$  is the initial investment cost of BESSs; the last two terms penalize the fast charging/discharging that could be harmful to battery life [144], and  $a_b$  and  $c_b$  are positive constants. Therefore,  $C_b(P_b) := a_b P_b^2 + b_b |P_b| + c_b$  with  $b_b = \frac{C_{\text{ini.}}}{CN_b(D_b^{\text{max}} - D_b^{\text{min}})/100\% \cdot 2C_b^{cap.}}$ .

#### **Renewable Generators**

Since current Feed-in Tariffs (FITs) are designed to encourage the uptake of renewable and low-carbon generation [145], the generation cost of RGs could be neglected. In this work, the objective of RGs is to track the short-term forecast of the maximum renewable generation capacity. To this end, a pseudo-cost function is designed to minimize the mismatch between the predicated generation and actual generation, i.e.,

$$C_r(P_r) := c_r (P_r - P_r^{\text{s.t.}})^2$$
  
=  $c_r P_r^2 - 2c_r P_r P_r^{\text{s.t.}} + c_r (P_r^{\text{s.t.}})^2, \forall r \in \mathcal{N}_{\text{RG}},$  (7.4)

where  $P_r$  and  $P_r^{\text{s.t.}}$  are the power generation and the short-term predicated maximum generation capacity of the *r*th RG.  $c_r$  is defined as  $\frac{1}{P_r^{\text{s.t.}}}$  such that  $a_r = \frac{1}{P_r^{\text{s.t.}}}$ ,  $b_r = -2$  and  $c_r = P_r^{\text{s.t.}}$ . Therefore, the equal power sharing of RGs respecting the short-term forecast of the maximum capacity is achieved when the marginal cost of RGs is equal [146].

To conclude, the objective function of local controllers in the microgrid is formulated as

min 
$$\left\{\sum_{j}^{N_{\text{CG}}} C_j(P_j) + \sum_{b}^{N_b} C_b(P_b) + \sum_{r}^{N_{\text{RG}}} C_r(P_r)\right\}$$
 (7.5a)

s.t. 
$$\sum_{j}^{N_{\text{CG}}} P_j + \sum_{b}^{N_b} P_b + \sum_{r}^{N_{\text{RG}}} P_r = P_d,$$
 (7.5b)

$$P_j^{\min} \le P_j \le P_j^{\min}, \ \forall j \in \mathcal{N}_{CG},$$
 (7.5c)

$$P_b^{\min} \le P_b \le P_b^{\min}, \ \forall b \in \mathcal{N}_b,$$
 (7.5d)

$$P_r^{\min} \le P_r \le P_r^{\min}, \quad \forall r \in \mathcal{N}_{\mathrm{RG}}.$$
(7.5e)

where  $P_d$  is the load demand, and (7.5b) is adopted to ensure the supply-demand balance in the microgrid.  $P_{(\cdot)}^{\min}$  and  $P_{(\cdot)}^{\max}$  are the minimal power capacity and the maximum power capacity of the (·), respectively. For convenience, we denote  $\mathcal{N} := \mathcal{N}_{CG} \cup \mathcal{N}_b \cup \mathcal{N}_{RG}$ . We write  $C_P = \sum_{i}^{N} C_i(P_i), \forall i \in \mathcal{N}$ , where  $P_i$  is used to represent the power of the *i*th participant. Therefore, the Lagrangian function of (7.5) is expressed by

$$L(P,\lambda) = \sum_{i \in \mathcal{N}} C_i(P_i) + \lambda(P_d - \sum_{i \in \mathcal{N}} (P_i))$$
(7.6)

where  $P = [P_1, ..., P_N]^T$  and N is the total number of the participants, and  $\lambda$  is a Lagrangian multiplier. Using saddle point dynamics [147], the centralised solution is given by

$$P_i^* = \frac{\lambda^* - b_i}{2a_i}, \quad \lambda^* = \frac{P_d - \sum_{i \in \mathcal{N}} \frac{b_i}{2a_i}}{\sum_{i \in \mathcal{N}} \frac{1}{2a_i}}, \tag{7.7}$$

where  $P_i^*$  is the optimal solution of (7.5). When considering the capacity constraints, the marginal cost is the partial derivative of  $C_i$  w.r.t.  $P_i$ , such as  $\lambda_i = a_i P_i + b_i$ . In [49], a well-known solution is introduced as the equal incremental cost criterion, i.e.,

$$\begin{cases} 2a_iP_i + b_i = \lambda^*, & \text{if } P_i^{\min} < P_i < P_i^{\max} \\ 2a_iP_i + b_i > \lambda^*, & \text{if } P_i = P_i^{\min}, \\ 2a_iP_i + b_i < \lambda^*, & \text{if } P_i = P_i^{\max}, \end{cases}$$
(7.8)

where  $\lambda^*$  is the optimal incremental cost.

The above optimization problem can be solved by either centralized methods or distributed methods. However, with increasing penetration level of the intermittent and uncertain renewable generation, these methods may lose their effectiveness since the operating condition of a microgrid could change frequently and unexpectedly. Furthermore, uncertain information exists in the computation processes of gradients and communication channels [148]. Furthermore, the predicated information of renewable generation could be inaccurate, and capacity loss could be caused by repeated charging/discharging cycle results in a varying battery capacity. The uncertainties could significantly undermine the accuracy and effectiveness of traditional EMS. Therefore, a novel distributed EMS with faster convergence and robustness against uncertain information is required to meet the new challenges of microgrid development.

# 7.2 Distributed Fixed-time Energy Management System under Uncertainty

In this section, a distributed fixed-time EMS under uncertain information is proposed for each microgrid. The uncertainties mentioned in Section 7.1.1 are modelled by a time varying signal  $\omega_i(t)$  in the update dynamics of the *i*th agent, i.e.,  $\dot{P}_i = u_i + \omega_i(t)$ , i = [1, ..., N], where *N* is the numbers of participants in the microgrid.

### 7.2.1 Algorithm Design

A distributed EMS with a fast convergence speed is given by

$$\dot{P}_i = u_i^o + u_i^r + \omega_i(t), \tag{7.9a}$$

$$u_i^o = -\alpha_1 \sum_{j=1}^N a_{ij} \operatorname{sig}^{p_1}(\lambda_i - \lambda_j) - \beta_1 \sum_{j=1}^N a_{ij} \operatorname{sig}^{q_1}(\lambda_i - \lambda_j),$$
(7.9b)

$$\dot{\lambda}_i = 2a_i \left( u_i^o + \alpha_2 \operatorname{sig}^{p_2}(P_i - \frac{\lambda_i - b_i}{2a_i}) \right)$$
(7.9c)

$$+\beta_2 \operatorname{sig}^{q_2}(P_i - \frac{\lambda_i - b_i}{2a_i}) \bigg), \qquad (7.9d)$$

$$u_i^r = -k_1 \left( \alpha_0 \operatorname{sig}^{\frac{1}{2}}(s_i) + \beta_0 \operatorname{sig}^{q_0}(s_i) \right) + \phi_i,$$
(7.9e)

$$\dot{\phi}_{i} = -k_{2} \left( \alpha_{0}^{2} \frac{1}{2} \operatorname{sign}(s_{i}) + \alpha_{0} \beta_{0} (\frac{1}{2} + q_{0}) \operatorname{sig}^{\frac{1}{2} + q_{0} - 1}(s_{i}) + \beta_{0}^{2} q_{0} \operatorname{sig}^{2q_{0} - 1}(s_{i}) \right),$$
(7.9f)

$$s_i = P_i - P_i(0) - \int_0^t u_i^o(\tau) d\tau,$$
 (7.9g)

where  $\operatorname{sig}^{p}(\cdot) = \operatorname{sign}(\cdot)|\cdot|^{p}$ .  $k_{1}$  and  $k_{2}$  are positive constants to be designed.  $p_{(\cdot)}$  and  $q_{(\cdot)}$  are real numbers satisfying  $0 < p_{(\cdot)} < 1 < q_{(\cdot)}$ .  $\alpha_{(\cdot)}$  and  $\beta_{(\cdot)}$  are positive constants.

**Theorem 7.2.1.** The proposed distributed optimisation algorithm (7.9) solves the formulated problem (7.5) in fixed-time despite the existence of uncertain information.

*Proof.* The proof includes the following steps: 1) Prove that  $\lim_{t\to T_0} s_i = \dot{s}_i = 0$ ; 2) Prove that  $\lim_{t\to T_1} P_i = \frac{\lambda_i - b_i}{2a_i}$ ; 3) Prove that  $\lim_{t\to T_2} \lambda_i = \lambda_j = \bar{\lambda}$ ; 4) By 3),  $\lambda_i = \lambda^*$ ,  $P_i = P_i^*$ .

1) Taking the time derivative of (7.9g), for  $i \in \mathcal{N}$ , one has

$$\dot{s}_{i} = -k_{1} \left( \alpha_{0} \operatorname{sig}^{\frac{1}{2}}(s_{i}) + \beta_{0} \operatorname{sig}^{q_{0}}(s_{i}) \right) + \psi_{i}, \quad \psi_{i} = \phi_{i} + \omega_{i}(t)$$
(7.10a)  
$$\dot{\psi}_{i} = -k_{2} \left( \alpha_{0}^{2} \frac{1}{2} \operatorname{sign}(s_{i}) + \alpha_{0} \beta_{0}(\frac{1}{2} + q_{0}) \operatorname{sig}^{\frac{1}{2} + q_{0} - 1}(s_{i}) + \beta_{0}^{2} q_{0} \operatorname{sig}^{2q - 1}(s_{i}) \right) + \dot{\omega}_{i}$$
(7.10b)

Define a vector  $\zeta_i^T = [\xi_i, \psi_i]$  with  $\xi_i = \alpha_0 \operatorname{sig}^{\frac{1}{2}}(s_i) + \beta_0 \operatorname{sig}^{q_0}(s_i)$ . Note that  $\psi_i = \xi_i' \xi_i$ , where

 $\xi'_{i} = \frac{1}{2}\alpha_{0}|s_{i}|^{-\frac{1}{2}} + q_{0}\beta_{0}|s_{i}|^{q_{0}-1}$ . Therefore, we can write

$$\dot{\zeta}_{i} = \xi_{i}' \begin{bmatrix} -k_{1}\xi_{i} + \psi_{i} \\ -k_{2}\phi_{i} + \frac{\dot{\omega}_{i}}{\xi_{i}'} \end{bmatrix} = \xi_{i}'(A\xi_{i} + B\frac{\dot{\omega}_{i}}{\xi_{i}'}), \qquad (7.11)$$

where  $A = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ . To facilitate the forthcoming robust stability analysis,

assuming that the perturbation term satisfies the following condition, i.e.,

$$\vartheta_{i} = \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}^{T} \begin{bmatrix} R & S \\ S^{T} & -1 \end{bmatrix} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix} \ge 0.$$
(7.12)

where  $R = -\frac{1}{2}(\kappa_1 \kappa_2^T + \kappa_2 \kappa_1^T)$  and  $S = \frac{1}{2}(\kappa_1^T + \kappa_2^T)$ . In particular, consider that  $\kappa_2^T = -\kappa_1^T = g \begin{bmatrix} 1 & 0 \end{bmatrix}$  with g > 0 such that  $R = \kappa_1 \kappa_2^T$  and S = 0. Therefore, assuming that  $|\dot{\omega}_i|$  is bounded such as  $|\dot{\omega}_1| \le \left|\frac{1}{2}g\alpha_0^2\right|$ ,  $\vartheta_i \ge 0$  is guaranteed by  $g(\xi_i'\xi_i) \ge |\dot{\omega}_i|$ . In the sequel, a convergence analysis of (7.10) is provided. A Lyapunov function candidate is chosen as  $V = \sum_{i=1}^N V_i = V_1 + V_2$ , where  $V_1 = \sum_{i=1}^N V_{1,i} = \sum_{i=1}^N \zeta_i^T \mathcal{P}_i \zeta_i$  with a positive symmetric definite matrix  $\mathcal{P}_i \in \mathbb{R}^{2\times 2}$ , and  $V_2 = \sum_{i=1}^N V_{2,i} = \sum_{i=1}^N \mu_1 |\xi_i|^2 - \mu_2 \operatorname{sig}^{\frac{1}{q_0}}(\xi_i) \operatorname{sig}^{\frac{2q_0-1}{q_0}}(\psi_i) + \mu_3 \phi_i^2$ . First, the time derivative of  $V_1$  along (7.9) is given by

$$\dot{V}_{1} = \sum_{i=1}^{N} \xi_{i}^{\prime} \left( \zeta_{i}^{T} (A^{T} \mathcal{P}_{i} + \mathcal{P}_{i} A) \zeta_{i} + \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} B^{T} \mathcal{P}_{i} \zeta_{i} + \zeta_{i}^{T} \mathcal{P}_{i} B \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \right)$$

$$= \sum_{i=1}^{N} \xi_{i}^{\prime} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}^{T} \begin{bmatrix} A^{T} \mathcal{P}_{i} + \mathcal{P}_{i} A & \mathcal{P}_{i} B \\ B^{T} \mathcal{P}_{i} & 0 \end{bmatrix} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}$$

$$\leq \sum_{i=1}^{N} \xi_{i}^{\prime} \left( \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}^{T} \begin{bmatrix} A^{T} \mathcal{P}_{i} + \mathcal{P}_{i} A & \mathcal{P}_{i} B \\ B^{T} \mathcal{P}_{i} & 0 \end{bmatrix} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix} + \vartheta_{i} \right).$$
(7.13)

Let  $\varepsilon > 0$  such that the following Algebraic Riccati Inequality,

$$A^{T}\mathcal{P}_{i} + \mathcal{P}_{i}A + \varepsilon\mathcal{P}_{i} + R + \mathcal{P}_{i}BB^{T}\mathcal{P}_{i} \le 0, \qquad (7.14)$$

are satisfied, and we further have

$$\dot{V}_{1} \leq \sum_{i=1}^{N} \xi_{i}^{\prime} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}^{T} \begin{bmatrix} A^{T} \mathcal{P}_{i} + \mathcal{P}_{i}A + R & \mathcal{P}_{i}B \\ B^{T} \mathcal{P}_{i} & -1 \end{bmatrix} \begin{bmatrix} \zeta_{i} \\ \frac{\dot{\omega}_{i}}{\xi_{i}^{\prime}} \end{bmatrix}$$
$$\leq \sum_{i=1}^{N} -\varepsilon \xi_{i}^{\prime} V_{1,i}$$
$$= \sum_{i=1}^{N} -\varepsilon \left( \frac{1}{2} \alpha_{0} |s_{i}|^{-\frac{1}{2}} + q_{0} \beta_{0} |s_{i}|^{q_{0}-1} \right) V_{1,i}.$$
(7.15)

Recall  $\lambda_{\min}(\mathcal{P}_i) \|\zeta_i\|_2^2 \leq \zeta_i^T \mathcal{P}_i \zeta_i \leq \lambda_{\max}(\mathcal{P}_i) \|\zeta_i\|_2^2$ , and  $|s_i|^2 \leq \left\|\frac{1}{\alpha_0}\zeta_i\right\|_2 \leq \left(\frac{V_1^{\frac{1}{2}}}{\alpha_0\lambda_{\min}^{\frac{1}{2}}(\mathcal{P}_i)}\right)$ . Therefore

$$\dot{V}_{1} \leq -\varepsilon \frac{1}{2} \alpha_{0}^{2} \sum_{i=1}^{N} \lambda_{\min}^{\frac{1}{2}}(\mathcal{P}_{i}) V_{1,i}^{\frac{1}{2}} - \varepsilon q_{0} \beta_{0} \sum_{i=1}^{N} |s_{i}|^{q_{0}-1} V_{1,i}$$

$$\leq -\varepsilon \frac{1}{2} \alpha_{0}^{2} r_{0} \left(\sum_{i=1}^{N} V_{1,i}\right)^{\frac{1}{2}} = -\varepsilon \frac{1}{2} \alpha_{0}^{2} r_{0} V_{1}^{\frac{1}{2}}, \qquad (7.16)$$

where  $r_0 = \min_{i=1,...,N} \lambda_{\min}^{\frac{1}{2}}(\mathcal{P}_i) \ge 0$ , and  $\lambda_{\min}(\mathcal{P}_i)$  is the minimal eigenvalue of  $\mathcal{P}_i$ .

Then, the time derivative of  $V_2$  is given as

$$\dot{V}_{2} = \sum_{i=1}^{N} \left( -\left|\xi_{i}\right|^{\frac{1-q_{0}}{q_{0}}} \xi_{i}' \left(2\mu_{1}k_{1}\left|\xi_{i}\right|^{\frac{3q_{0}-1}{q_{0}}} + \frac{1}{q_{0}}\mu_{2}|\psi_{i}|^{\frac{3q_{0}-1}{q_{0}}} - \frac{1}{q_{0}}\mu_{2}k_{1}\xi_{i}\operatorname{sig}^{\frac{2q_{0}-1}{q_{0}}}(\psi_{i}) - \tilde{k}_{2}\frac{2q_{0}-1}{q_{0}}\mu_{2}\left|\xi_{i}\right|^{2}|\psi_{i}|^{\frac{q_{0}-1}{q_{0}}} - 2(\mu_{1} - \tilde{k}_{2}\mu_{3})\operatorname{sig}^{\frac{2q_{0}-1}{q_{0}}}(\xi_{i})\psi_{i}\right)\right),$$

$$(7.17)$$

where due to  $|\dot{\omega}_i| \leq g |\xi_i'\xi_i|$ , there exists  $\dot{\omega}_i = \tilde{g}\xi_i'\xi_i$  with  $\tilde{g} \leq g$ , and then  $\tilde{k}_2 = k_2 - \tilde{g}$ . Using Young's inequality such as  $\Theta \Xi \leq c^{\sigma_1} \frac{\Theta^{\sigma_1}}{\sigma_1} + c^{-\sigma_2} \frac{\Xi^{\sigma_2}}{\sigma_2}$  with  $\frac{1}{\sigma_1} + \frac{1}{\sigma_2} = 1$  for sig  $\frac{2q_0-1}{q_0}(\xi_i)\psi_i$ ,  $\xi_i \operatorname{sig}^{\frac{2q_0-1}{q_0}}(\psi_i)$  and  $|\xi_i|^2 |\psi_i|^{\frac{q_0-1}{q_0}}$ , respectively, one can obtain that

$$\dot{V}_{2} \leq \sum_{i=1}^{N} \left( -\left|\xi_{i}\right|^{\frac{1-q_{0}}{q_{0}}} \xi_{i}' \left(\Gamma_{1}\left|\xi_{i}\right|^{\frac{3q_{0}-1}{q_{0}}} + \Gamma_{2}\left|\psi_{i}\right|^{\frac{3q_{0}-1}{q_{0}}}\right) \right),$$
(7.18)

where choosing  $\mu_1 = k_2\mu_3$ ,  $\Gamma_1 = \mu_2\Upsilon_1 - 2g\mu_3 \frac{2q_0-1}{3q_0-1} \delta_1^{\frac{3q_0-1}{2q_0-1}}$  with  $\Upsilon_1 = 2k_2k_1 - k_1 \frac{1}{3q_0-1} \gamma_2^{\frac{3q_0-1}{q_0}} - 2(k_2 - g) \frac{2q_0-1}{3q_0-1} \gamma_3^{\frac{3q_0-1}{2q_0}}$  and  $\Gamma_2 = \frac{1}{q_0}\mu_2\Upsilon_2 - 2g\mu_3 \frac{q_0}{3q_0-1} \delta_1^{-\frac{3q_0-1}{q_0}}$  with  $\Upsilon_2 = 1 - k_1 \frac{2q_0-1}{3q_0-1} \gamma_2^{-\frac{3q_0-1}{2q_0-1}} - (k_2 - g) \frac{(2q_0-1)(q_0-1)}{(3q_0-1)q_0} \gamma_3^{-\frac{3q_0-1}{q_0-1}}$ . Following the similar analysis of the single system in [149], there always exists  $\left(\frac{q_0^2}{3q_0-1} \frac{2g\mu_3}{\mu_2\Upsilon_2}\right)^{\frac{q_0}{3q_0-1}} < \Upsilon_1 < \left(\frac{3q_0-1}{2q_0-1} \frac{\mu_2\Upsilon_1}{2g\mu_3}\right)^{\frac{2q_0}{3q_0-1}}$  to guarantee  $\Gamma_1 > 0$  and

 $\Gamma_2 > 0$  by selecting  $k_2$  large enough. Then, denoting  $\Gamma_{\min} = \min\{\Gamma_1, \Gamma_2\}$  and  $k_{\min} =$  $\min_{i=1,...,N} \min_{s_i} \frac{p\alpha_0 |s_i|^{p-q_0} + q_0\beta_0}{(\alpha_0 |s_i|^{p-q_0} + \beta_0)^{\frac{q_0-1}{q_0}}}, \text{ we obtains that}$ 

$$\dot{V}_{2} \leq -\Gamma_{\min}k_{\min}\sum_{i=1}^{N} \left( \left|\xi_{i}\right|^{\frac{3q_{0}-1}{q_{0}}} + \left|\psi_{i}\right|^{\frac{3q_{0}-1}{q_{0}}} \right) \leq 0.$$
(7.19)

Notably, using Young's inequality for  $\sin^{\frac{1}{q}}(\xi_i)\sin^{\frac{2q-1}{q}}(\psi_i)$ , it can be obtained that  $\exists \delta_4 > 0$ ,  $\left(\mu_1 - \mu_2 \frac{\delta_4}{2q_0}\right) |\xi_i|^2 + \left(\mu_3 - \frac{2q_0-1}{2q_0}\mu_2 \delta_4^{-\frac{2q_0}{2q_0-1}}\right) \psi_i^2 \le V_{2,i} \le \left(\mu_1 + \mu_2 \frac{\delta_4}{2q_0}\right) |\xi_i|^2 + \left(\mu_3 + \frac{2q_0-1}{2q_0}\mu_2 \delta_4^{-\frac{2q_0-1}{2q_0}}\right) |\xi_i|^2 + \left(\mu_3 + \frac{2q_0-1}{2q_0}\mu_2 \delta_4^{-\frac{2q_0-1}{2q_0}}\right$  $\frac{2q_0-1}{2q_0}\mu_2\delta_4^{-\frac{2q_0}{2q_0-1}}\bigg)\psi_i^2, \text{ and } V_{2,i} \text{ is positive definite if } \frac{(2q_0-1)^{\frac{2q_0-1}{2q_0}}}{2q_0k_2^{\frac{1}{2q_0}}} \le \frac{\mu_3}{\mu_2} \text{ which is guaranteed}$ by selecting  $k_2$  large enough. Then we have  $V_{2,i} \leq 2\mu_3 \max\{1,k_2\}(|\xi_i|^2 + \psi_i^2)$ . It finally obtains that

$$\dot{V}_{2} \leq -2\Gamma_{\min}k_{\min}\left(\frac{1}{4\mu_{3}\max\{1,k_{2}\}}\right)^{\frac{3q_{0}-1}{2q_{0}}}\sum_{i=1}^{N}V_{2,i}^{\frac{3q_{0}-1}{2q_{0}}}$$
$$\leq -2\Gamma_{\min}k_{\min}\left(\frac{1}{4\mu_{3}\max\{1,k_{2}\}}\right)^{\frac{3q_{0}-1}{2q_{0}}}\left(\sum_{i=1}^{N}V_{2,i}\right)^{\frac{3q_{0}-1}{2q_{0}}}.$$
(7.20)

Notably, recall that

$$\lambda_{\min}(\mathcal{P}_i) \|\zeta_i\|_2^2 \le \zeta_i^T \mathcal{P}_i \zeta_i \le \lambda_{\max}(\mathcal{P}_i) \|\zeta_i\|_2^2$$
(7.21)

and

$$\theta_{\min} \|\zeta_i\|_2^2 \le V_{2,i} \le 2\mu_3 \max\{1,k_2\} \|\zeta_i\|_2^2$$
(7.22)

where  $\theta_{\min} = \min\{\mu_1 + \mu_2 \frac{\delta_4}{2q_0}, \mu_3 + \frac{2q_0 - 1}{2q_0} \mu_2 \delta_4^{-\frac{2q_0}{2q_0 - 1}}\}$  so V

$$V_i \le (\lambda_{\max}(\mathcal{P}_i) + 2\mu_3 \max\{1, k_2\}) \|\zeta_i\|_2^2.$$
 (7.23)

Thus

$$\begin{split} \dot{V} &\leq -\sum_{i=1}^{N} \varepsilon \frac{1}{2} \alpha_{0}^{2} \lambda_{\min}(\mathcal{P}_{i}) \left\| \zeta_{i} \right\|_{2} - \frac{1}{q_{0}} \Gamma_{\min} k_{\min} \left( \frac{1}{4\mu_{3} \max\{1, k_{2}\}} \right)^{\frac{3q_{0}-1}{2q_{0}}} \theta_{\min}^{\frac{3q_{0}-1}{2q_{0}}} \left\| \zeta_{i} \right\|_{2}^{\frac{3q_{0}-1}{2q_{0}}} \\ &\leq -\mathbf{v}_{1} V^{\frac{1}{2}} - \mathbf{v}_{2} V^{\frac{3q_{0}-1}{2q_{0}}}, \end{split}$$
(7.24)

where  $\mathbf{v}_1 = \frac{\varepsilon_2^1 \alpha_0^2 \min_{i=1,\dots,N} \lambda_{\min}(\mathcal{P}_i)}{(\max_{i=1,\dots,N} \lambda_{\max}(\mathcal{P}_i) + 2\mu_3 \max\{1,k_2\})}$  and

$$\mathbf{v}_{2} = \frac{1}{q_{0}} k_{\min} \left( \frac{\theta_{\min}}{4\mu_{3} \max\{1, k_{2}\} (\max_{i=1, \dots, N} \lambda_{\max}(\mathcal{P}_{i}) + 2\mu_{3} \max\{1, k_{2}\})} \right).$$
(7.25)

Using Thereom 5 in [150] with k = 1, it can be concluded that V = 0 with a setting time  $T_0$ . Therefore,  $s_i$  converges to zero in a fixed-time with  $T_0 = \frac{1}{v_2(1-\frac{3q_0-1}{2q_0})} + \frac{2}{v_1}$ , i.e.,  $\lim_{t \to T_0} s_i = \dot{s}_i = 0$ .

2) For  $t \ge T_0$ , it has  $\dot{P}_i = u_i^o$ . Define  $\bar{P}_i = \frac{\lambda_i - b_i}{2a_i}$ , and a Lyapunov function candidate as  $V_3 = \frac{1}{2} \sum_{i=1}^{N} (P_i - \bar{P}_i)^2$ . The time derivative of  $V_3$  along (7.9) is given by

$$\begin{split} \dot{V}_{3} &= -\sum_{i=1}^{N} (P_{i} - \bar{P}_{i}) \left( \alpha_{2} \operatorname{sig}^{p_{2}} (P_{i} - \frac{\lambda_{i} - b_{i}}{2a_{i}}) \right) \\ &+ \beta_{2} \operatorname{sig}^{q_{2}} (P_{i} - \frac{\lambda_{i} - b_{i}}{2a_{i}}) \right) \\ &= -\alpha_{2} \sum_{i=1}^{N} \left( (P_{i} - \bar{P}_{i})^{2} \right)^{\frac{1 + p_{2}}{2}} - \beta_{2} \sum_{i=1}^{N} \left( (P_{i} - \bar{P}_{i})^{2} \right)^{\frac{1 + q_{2}}{2}} \\ &\leq -\alpha_{2} \left( \sum_{i=1}^{N} (P_{i} - \bar{P}_{i})^{2} \right)^{\frac{1 + p_{2}}{2}} - \beta_{2} N^{\frac{1 - q_{2}}{2}} \left( \sum_{i=1}^{N} (P_{i} - \bar{P}_{i})^{2} \right)^{\frac{1 + q_{2}}{2}} \\ &= -2^{\frac{1 + p_{2}}{2}} \alpha_{2} \left( V_{2} \right)^{\frac{1 + p_{2}}{2}} - 2^{\frac{1 + q_{2}}{2}} \beta_{2} N^{\frac{1 - q_{2}}{2}} \left( V_{2} \right)^{\frac{1 + q_{2}}{2}}. \end{split}$$
(7.26)

where the inequality comes from Lemma 1 in [151]. Finally, using Thereom 5 in [150], it can be concluded that  $\lim_{t\to T_1} P_i = \frac{\lambda_i - b_i}{2a_i}$  with a setting time  $T_1 = \frac{1}{2^{\frac{1+p_2}{2}}\alpha_2} + \frac{1}{2^{\frac{1+q_2}{2}}\beta_2 N^{\frac{1-q_2}{2}}} + T_0$ .

3) In this step, we aim to show that  $\lim_{t\to T_2} \lambda_i = \lambda_j$ . From 2), as  $t \ge T_1$ , it has  $P_i = \frac{\lambda_i - b_i}{2a_i}$  and therefore,

$$\dot{\lambda}_{i} = -2a_{i} \bigg( \alpha_{1} \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{p_{1}}(\lambda_{i} - \lambda_{j}) - \beta_{1} \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{q_{1}}(\lambda_{i} - \lambda_{j}) \bigg).$$
(7.27)

A Lyapunov function candidate is defined as  $V_4 = \sum_{i=1}^{N} \frac{1}{4a_i} e_i^2$  with  $e_i = \lambda_i - \frac{1}{N} \sum_{j=1}^{N} \lambda_j$ . The time derivative of  $V_4$  along (7.9) is given by

$$\dot{V}_{4} = -\alpha_{1} \sum_{i=1}^{N} e_{i} \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{p_{1}}(\lambda_{i} - \lambda_{j}) - \beta_{1} \sum_{i=1}^{N} e_{i} \sum_{j=1}^{N} a_{ij} \operatorname{sig}^{q_{1}}(\lambda_{i} - \lambda_{j})$$

$$= -\frac{1}{2} \alpha_{1} \sum_{i,j=1}^{N} a_{ij} |e_{i} - e_{j}|^{p_{1}+1} - \frac{1}{2} \beta_{1} \sum_{i,j=1}^{N} a_{ij} |e_{i} - e_{j}|^{q_{1}+1}$$

$$\leq -\nu_{3} V_{4}^{\frac{p_{1}+1}{2}} - \nu_{4} V_{4}^{\frac{q_{1}+1}{2}}, \qquad (7.28)$$

where  $v_3 = \frac{1}{2} \alpha_1 \left( \frac{8\lambda_2(\pounds)}{\min_{i=1,\dots,N} a_i} \right)^{\frac{p_1+1}{2}}$  and  $v_4 = \frac{1}{2} \beta_1 N^{\frac{1-q_1}{1}} \left( \frac{8\lambda_2(\pounds)}{\min_{i=1,\dots,N} a_i} \right)^{\frac{q_1+1}{2}}$  Therefore, we can conclude  $\lim_{t \to T_2} \lambda_i = \lambda_j$  with the setting time  $T_2 = T_1 + \frac{1}{v_3(1-p_1)} + \frac{1}{v_4(q_1-1)}$  by Thereom 5 in [150].

4) From (7.27), we have  $\sum_{i=1}^{N} \frac{\dot{\lambda}_i}{2a_i} = 0$  which implies  $\sum_{i=1}^{N} \frac{\lambda_i(t)}{2a_i}$  is invariant for  $t \ge T_1$ , i.e.,  $\sum_{i=1}^{N} \frac{\lambda_i(t)}{2a_i} = \sum_{i=1}^{N} \frac{\lambda_i(T_1)}{2a_i} = \sum_{i=1}^{N} (P_i(T_1) + \frac{b_i}{2a_i})$ , since  $\lim_{t\to T_1} P_i = \frac{\lambda_i(T_1) - b_i}{a_i}$ . Invoking 3), as  $t \ge T_2$ , one obtains  $\lambda_i = \lambda_j = \bar{\lambda} = \frac{\sum_{i=1}^{N} (P_i(T_1) + \frac{b_i}{2a_i})}{\sum_{i=1}^{N} \frac{1}{2a_i}}$ ,  $\forall i, j \in N$ . In addition, from (7.9g), and assuming  $\sum_{i=1}^{N} P_i(0) = P_D$ , it has

$$\sum_{i=1}^{N} P_{i} = \alpha_{1} \sum_{i,j=1}^{N} a_{ij} \int_{0}^{t} (\lambda_{j}(s) - \lambda_{i}(s))^{1 - \frac{p_{1}}{q_{1}}} ds$$
$$+ \beta_{1} \sum_{j=1}^{N} a_{ij} \int_{0}^{t} (\lambda_{j}(s) - \lambda_{i}(s))^{1 + \frac{p_{1}}{q_{1}}} + \sum_{i=1}^{N} P_{i}(0) = P_{D}.$$
(7.29)

Thus, based on (7.2.1) and (7.29), we have  $\bar{\lambda} = \frac{P_D + \sum_{i=1}^{N} \frac{b_i}{2a_i}}{\sum_{i=1}^{N} \frac{1}{2a_i}} = \lambda^*$ , which further implies  $P_i = P_i^* = \frac{\lambda^* - b_i}{2a_i}$  as  $t \ge T_2$ . The proof is completed.

### 7.3 Simulation Study

In this section, we demonstrate the performance of the proposed algorithm through several case studies under different operational conditions. Two configurations of the islanded microgrid are considered in this study, which are simulated using a modified IEEE 13-bus system in Matlab/Matpower. In *Case* 7.1, the effectiveness of this algorithm is firstly verified under constant load demands, and a scalability analysis is provided. *Case* 7.2 tests the performance of the proposed algorithm under plug-and-play operation. Lastly, in *Case* 7.3, the focus is the capability of the proposed algorithm to handle time-varying operational conditions.

In above case studies, we assumes that the uncertainties of the *j*th CG and the *b*th BESS are  $\omega_b = j\sin(0.5it)$  and  $\omega_b = 1.5b\sin(0.6bt)$ . For the *r*th RG, the uncertainty is  $\omega_r = r\sin(0.8rt) + d_r$ , where  $d_r$  is the constant uncertainty to emulate the short-term prediction error.

### 7.3.1 Parameter Setup

The minimum/maximum power rates of all BESSs are [-10 10] MW. The total cycle number of battery cells of BESSs is 10000 when BESSs operate within the state of charge [20,80]

%.

### **Configuration 1**

In this configuration, one CG, four BESSs and one PV are located at the microgrid. The parameters of CGs are given as  $a_{1,1} = 0.047$  and  $b_{1,1} = 0.58$ . The capacities of BESSs are [12 7 10 8] MWh. The initial investment costs of battery cells of three BESSs are 0.58 \$/Wh, 0.52 \$/Wh, 0.5 \$/Wh and 0.6 \$/Wh, respectively. Therefore, the parameters of BESSs are  $a_{b,1} = [0.185, 0.215, 0.198, 0.172], \forall b \in \mathcal{N}_b^3$ . The short-term prediction of the maximum PV generation is 2.5 MW.

#### **Configuration 2**

In this configuration, the microgrid is operated under increased penetration of renewable generation, i.e., two CGs, two BESSs, one WT and one PV. The parameters of CGs are given as  $a_{j,1} = [0.052, 0.059]$  and  $b_{j,1} = [0.68, 0.55], \forall j \in \mathcal{N}_{CG}^2$ . The capacities of BESSs are [9, 7] MWh. The initial investment costs of battery cells of three BESSs are 0.52 \$/Wh and 0.5 \$/Wh, respectively. The parameters of two BESSs are  $a_{b,1} = [0.185, 0.208], \forall b \in \mathcal{N}_b^2$ . The short-term predictions of the maximum WT/PV generation are [3, 2.5] MW

### 7.3.2 Algorithm Implementation with Local Constraints

The proposed method in (7.9) does not include the local inequality constraints. In this section, to avoid violations of the local constraints of capacities, an algorithm in Algorithm 2 is further refined for the *i*th participant.

### 7.3.3 Case 7.1

In this case, the effectiveness of the proposed algorithm is verified by the microgrid under the Configuration 1. The local load demand is 70 MW. The results are given in Figs. 7.1 - 7.3. As shown in Figs. 7.1 - 7.2, the incremental costs of all participants converge to the

#### Algorithm 2 Distributed Fixed-time EMS in the Microgrid

**Initialization:** for all  $i \in \mathcal{N}$ 

Local EMS:  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ ;  $\beta_0$ ,  $\beta_1$  and  $\beta_2$ ;  $k_1$  and  $k_2$ ;  $p_1$ , and  $p_2$ ;  $q_0$ ,  $q_1$  and  $q_2$ ; Local device:  $\sum_{i \in \mathcal{N}} P_i(0) = P_d(0)$ 

### **Consensus Algorithm: :**

1: Update  $(P_i, \lambda_i)$  for  $i \in \mathcal{N}$  using (7.9), and check the local capacity by

$$P_{i} = \begin{cases} P_{i}^{\max}, & \text{if } P_{i} > P_{i}^{\max} \\ P_{i}^{\min}, & \text{if } P_{i} < P_{i}^{\min} \end{cases}$$
(7.30)

Define two auxiliary variables, i.e.,  $\rho_i$ ,  $\varsigma_i$ ,

$$\rho_i = \begin{cases} \frac{\lambda_i - b_i}{2a_i} - P_i, \text{ if } i \in \Omega\\ 0, \text{ Otherwise} \end{cases} \quad \varsigma_i = \begin{cases} 0, \text{ if } i \in \Omega\\ \frac{1}{2a_i}, \text{ Otherwise} \end{cases}$$

where  $\Omega$  is the set of participants at their minimum/maximum capacity;

2: Update  $(P_i, \lambda_i)$  with local constraints by

$$x_{i} = \begin{cases} P_{i}^{\min}/P_{i}^{\max} \text{ if } i \in \Omega\\ \frac{\lambda^{*}-b_{i}}{2a_{i}}, \text{ Otherwise} \end{cases}$$
(7.31)

where  $\lambda^* = \lambda_i + \rho_i / \zeta_i$  and  $\dot{Z}_i = -\alpha_3 \sum_{j=1}^N a_{ij} \operatorname{sig}^{p_3}(Z_i - Z_j) - \beta_3 \sum_{j=1}^N a_{ij} \operatorname{sig}^{q_3}(Z_i - Z_j)$ 3: **return**  $P_i^*$  if  $P_i^{\min} \leq P_i^* \leq P_i^{\max}$ ;

Table 7.1: The computation time of different numbers of participants

Num. of participants	6	18	48	96	102
Computation time	0.025s	0.037s	0.057s	0.084s	0.149s

optimal value in the fixed time, and in the meantime, the active power outputs converge to the corresponding optimal values. The total active power satisfies the supply-demand equality constraint as shown in Fig. 7.3. Results have shown that the optimisation objective is fulfilled in the fixed-time.

To further investigate the scalability, we adopt the proposed algorithm in different numbers of participants in a modified IEEE 123-bus system on a PC with Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 4GB RAM. The computation time is given in Table 7.3.3. The result shows that even for 102 participants (34 CGs, 51 BESSs and 17 RGs), the proposed algorithm can still solve the problem within 1s that proves the capability of practical implementation in a large-scale system.



Figure 7.1: The incremental cost of participants



Figure 7.2: The active power updates of participants



Figure 7.3: The total active power of participants



Figure 7.4: The incremental cost of participants under plug-and-play operation



Figure 7.5: The active power updates of participants under plug-and-play operation

### 7.3.4 Case 7.2

This case study tests the plug-and-play adaptability of the proposed algorithm. We suppose that BESS4 in the microgrid fails to operate at 10 s and recovers at 15 s. During the failure, BESS4 loses its communication with its neighbours, and the remaining participants remain connected in order to meet the planning requirement and the load demand. The results are given in Figs. 7.4 - 7.6, which show that the active power outputs of the remaining participants will converge to the new optimal values, and that the mismatch caused by the BESS4 failure is eliminated. Furthermore, these values will converge to those of the previous ones after BESS4 is reconnected. The results clearly show the plug-and-play capability of the proposed algorithm.



Figure 7.6: The total active power of participants under plug-and-play operation

### 7.3.5 Case 7.3

In this case, the microgrid under Configuration 2 is considered, which contains an in- creased level of renewable generation. To investigate the effect of the curtailment of renewable generation, it is considered that the microgrid suffers the curtailments of WT and PV generation given by [2, 1.5] MW at 5 s. The results are shown in Figs. 7.7 - 7.9, which are the incremental cost, active power outputs, and total active power of participants. The results show that when the curtailment occurs, the active power outputs of participants will converge to new optimal values with a new common point of incremental costs. Also, in Fig. 7.9, the system balance is guaranteed by the proposed algorithm.

To further investigate the performance of the proposed algorithm under time-varying conditions, a varying demand is considered. Therefore, the proposed algorithm must converge to optimal results before the next change in the operation. The load demand is changed during the simulation, such as 62 MW at the beginning, 65 MW at 6 s, and 60 MW at 12 s. The results are shown in Figs 7.10 - 7.12, which are incremental costs, active power outputs, and total active power of participants. The results show that the proposed algorithm will automatically respond to each change in load demands and can fast converge to optimal solutions before the next change successfully.



Figure 7.7: The incremental cost under curtailments of renewable generation



Figure 7.8: The active power update under curtailments of renewable generation



Figure 7.9: The total active power under curtailments of renewable generation



Figure 7.10: The incremental cost under varying demand



Figure 7.11: The active power update under varying demand



Figure 7.12: The total active power under varying demand

### 7.4 Conclusion

This section proposed a novel distributed EMS to address the power management problem in islanded microgrids based on the MAS framework. The proposed algorithm can be adopted in a fully distributed manner to reduce the complexity of communication and computation. In addition, it guarantees a fast convergence speed and robustness against uncertain information simultaneously. Several studies have proved that the proposed algorithm can converge towards the optimal solution with the requirements of fixed-time and robust performances.

# **Chapter 8**

# **Cooperative Optimal Control of Battery Energy Storage System under Wind Uncertainties in a Microgrid**

### 8.1 Introduction

Since high penetration renewable sources will be integrated into future power systems, energy storage systems are often installed to maintain frequency stability in a microgrid. The operation condition of a microgrid may change frequently due to the intermittency of renewable sources, and BESSs will be charged/discharged accordingly to smooth and balance the generation of renewable sources. Thus, ESSs should be coordinated in a proper way to ensure the supply-demand balance while increasing their profits and energy efficiency.

In this chapter, a distributed optimal solution for multiple BESSs is to maintain supply- demand balance while maximizing their welfare and energy efficiency is proposed to BESSs by enhancing the coordination through a local communication. Each BESS is designated as an agent, and it only utilizes local information to interact with the neighbouring agents. Additionally, since the participants in microgrid may not be willing to release their information about cost functions, or even the local gradient with other neighbouring agents, the proposed solution could be implemented without private information to the individual agents.



Figure 8.1: Multiple BESSs System

# 8.2 Network Model of Multiple Battery Energy Storage Systems

### 8.2.1 Overview of Multiple Battery Energy Storage Systems

Fig. 8.1 shows a MAS framework for an architecture of multiple BESSs and wind power generation, which is connected to the main grid through the PCC. The PCC of the main grid is used to observe the power delivered/withdrawn and decide the operation mode of the microgrid. Each BESS consists of several lithium-ion batteries interfaced with a DC-AC inverter. In the proposed framework, a BESS is designated as an agent, and controlled by an EMS. A communication network is embedded in the EMS to transfer the information to and from its neighbouring agents in order to achieve a determined objective. In addition, following the control strategy in [73], a storage system controller (SSC) is applied to adjust the power output of the BESS to the signal generated by the communication network.

# 8.3 Distributed Energy Management of Battery Energy Storage Systems under Wind Power uncertainties

In a microgrid, the frequency may change rapidly and frequently due to the uncertainties of wind power generation, which is mainly influenced by the supply-demand mismatch of the active power. The active power balance at  $t_0$  in a microgrid can be represented as

$$\sum_{k \in S_W} P_{W,k}^0 + \sum_{i \in S_B} P_{B,i}^0 = \sum_{j \in S_l} P_{l,j}^0,$$
(8.1)

where  $S_W$ ,  $S_B$  and  $S_l$  are the index set of WTSs, BESSs and load demands, respectively;  $P_{B,i}^0$  is the charging/discharging power of *i*th BESS that can be positive/negative;  $P_{W,k}^0$  and  $P_{l,j}^0$  are the wind power generation and load demand of *k*th WT and *j*th load, respectively.

Although wind power generation can relieve the burden of frequency regulation, it is deficient in terms of accuracy due to its intermittent nature. Furthermore, when wind power generation is controlled in the MPPT mode, it cannot be treated as dispatchable. In contrast, BESSs have fast response properties and exhibit high performance. As a result, installing BESSs in a power grid is a promising solution for absorbing excessive power and compensating for insufficient power.

Then the net power required from the BESS for frequency regulation in the short-term can be calculated as:

$$\sum_{i \in S_B} P_{B,i} = \Delta P_D = \sum_{k \in S_W} \Delta P_{W,k} - \sum_{j \in S_l} \Delta P_{l,j},$$
(8.2)

where  $P_{B,i}$  is the power output of *i*th BESS;  $\Delta P_{W,k}$  and  $\Delta P_{l,j}$  are active power change of *k*th WT and *j*th load, respectively.

However, as indicated in [97], the output power level is one of the factors affecting the energy efficiency of a BESS. Thus, a factor called inner energy rate is defined as (8.3) to express the rate of inner charging or discharging energy of a BESS for output power.

$$\varepsilon_i = \frac{\Delta E_{B,i}}{P_{B,i}}.$$
(8.3)

where  $\Delta E_{B,i}$  is the change of battery's inner energy.

Note that the price of electricity is an important factor that can encourage the BESS to participate in frequency regulation. As indicated in [29, 152, 153], the price of electricity is

an effective method of energy management for the participants in a microgrid to adjust their electricity consumption in a cost-effective way. ToU pricing is adopted,  $\rho(t)$ , as a variable to coordinate BESSs to discover the maximum efficient point of the operation. Thus, the objective is defined as maximizing the total welfare of BESSs by adjusting the price while considering the efficiency the BESSs and maintaining the active power balance, i.e.,

Max 
$$\sum_{i \in S_B} \left( \rho(t) P_{B,i} - \varepsilon_i P_{B,i} \right)$$
(8.4)

where  $\varepsilon_i$  is the inner rate energy. The experiment in [97] shows that the function of the inner rate energy can be written in a piecewise linear function as:

$$\varepsilon_i = \alpha_i P_{B,i} + \beta_i. \tag{8.5}$$

By substituting (8.5) into (8.4), the objective function can be rewritten as

Max 
$$\sum_{i \in S_B} \left( \rho(t) P_{B,i} - \left( \alpha_i P_{B,i}^2 + \beta_i P_{B,i} \right) \right)$$
  
s.t 
$$\sum_{i \in S_B} P_{B,i} = \Delta P_D$$
$$P_{B,i}^m \le P_{B,i} \le P_{B,i}^M$$
(8.6)

where  $P_{B,i}^m$  and  $P_{B,i}^M$  are the lower and upper bound of *i*th BESS, respectively. The equality constraint describes the balance between the net power demand and the total output power of BESSs, and the inequality constraints are the local boundary of the output power for the *i*th BESS.

It should be noted that the transmission losses are inevitable, and these are about 5% - 7% of the total load according to the energy information administration (EIA) [154]. Thus, it can be modelled by multiplying the load by 5% - 7%.

For multiple BESSs, they may be operated on different SoCs due to the difference in efficiency. Since the power is predominately shared among BESSs with higher SoCs, some BESSs will be overloaded even when the required power is lower than the total power capacities of BESSs. As a result, a proper power sharing method is required to coordinate BESSs based on their energy level. To this end, the objective function is rewritten as,

Min 
$$\sum_{i \in S_B} f_i(P_{B,i})$$
(8.7)

s.t 
$$\sum_{i \in S_B} \eta_i P_{B,i} = \Delta P_D \tag{8.8}$$

$$h_i(P_{B,i}) = (P_{B,i}^m - P_{B,i})(P_{B,i}^M - P_{B,i}) \le 0,$$
(8.9)

where  $f_i(P_{B,i}) = (\alpha_i P_{B,i}^2 + \beta_i P_{B,i}) - \rho(t) P_{B,i}$ .  $\eta_i$  is the weight on the contribution of *i*th BESS. It can be defined as a ratio of the energy level in order to prevent BESSs from running out prematurely, i.e.,

$$\eta_i = \frac{E_i}{\sum_{i \in S_B} E_i}$$

where  $E_i$  is the energy level of *i*th BESS.

A central coordinator could be deployed to solve the above problem. The coordinator communicates with each BESS in the network by a bi-directional communication line to collect the data required to solve the problem, such as the objective functions, the operational constraints and the actual power output. With the received data, it solves the problem and broadcasts the reference to all connected BESSs. However, due to the high penetration of wind power generation, the centralized strategy may lose its control efficiency if operating conditions change frequently and unpredictably. In the following section, a distributed algorithm is proposed to coordinate the BESSs to maintain the active power balance, and in the meantime, a coordination scheme of BESSs and WTs is presented to overcome the problem of wind power generation errors.

### 8.4 Consensus-based Cooperative Algorithm Design

The formulated optimization problem of BESSs in (8.7) is a convex problem with both equality and inequality constraints. In this section, the solution set is firstly characterized by the so-called refined Slater condition and theKKT optimality conditions. A consensus-based distributed cooperative algorithm is presented for discovering the optimal solution of the energy management problem. Then the implementation of the proposed algorithm is presented.

### 8.4.1 Solution Set of Distributed Energy Management

Let *L* be the Lagrangian equation with the Lagrangian multiplier  $\lambda$ , one has

$$L = \sum_{i \in S_B} \left( \left( \alpha_i P_{B,i}^2 + \beta_i P_{B,i} \right) - \rho(t) P_{B,i} \right) + \left( \sum_{i \in S_B} \lambda \eta_i P_{B,i} - \Delta P_D \right), \quad (8.10)$$

It should be noted that the inequality constraints are unnecessary to be added in the augmented function due to they are local and can be treated as boundaries of the problem domain [155]. These inequality constraints can be taken into account by applying additional projection operations, which do not affect the convergence analysis as shown in [127].

Remark 8.4.1. By using the dual decomposition [156], one has

$$L = \sum_{i \in S_B} \left( \left( \alpha_i P_{B,i}^2 + \beta_i P_{B,i} \right) - (\rho(t) - \lambda_i \eta_i) P_{B,i} \right) + \Delta P_D.$$
(8.11)

It can be found that the Lagrange multiplier  $\lambda_i$  acts as a virtual price to adjust the real price and therefore BESSs are coordinated to find the maximum efficient point.

Since the formulated problem is a convex optimization problem with affine constraints, the global optimality can be ensured by using KKT optimality conditions [157]. Following the KKT conditions, a point  $P_B^* = \{P_{B,1}^*, \dots, P_{B,n}^*\} \in \mathcal{R}^n$  is a solution of the energy management problem if and only if there exists a point  $\lambda^*$  such that

$$\nabla f_i(P_{B,i}^*) + \lambda^* \eta_i = 0, \quad i \in \{1, \dots, n\}$$
  
$$\eta_1 P_{B,1}^* + \dots + \eta_i P_{B,i}^* = \Delta P_D.$$
(8.12)

### 8.4.2 Distributed Cooperative Algorithm Design

In this section, a distributed algorithm is presented for the energy management of BESSs. Following [158], the active power dynamics of *i*th BESS can be written as

$$P_{B,i} = u_{B,i}, (8.13)$$

where  $u_{B,i}$  is the control input for *i*th BESS. In a microgrid, the distributed energy management of BESSs aims to maximize the efficiency of BEESs under a properly designed

strategy, which can be achieved by controlling the references of BESSs. Then the design objective is defined such that the control input only requires the information from neighbouring BESSs, and then  $P_{B,i}$  converges to the solution set of the proposed problem (8.7).

To this end, the solution for the problem (8.7) is the following continuous-time distributed algorithm,

$$\dot{P}_{B,i} = -\nabla f_i(P_{B,i}) - \eta_i \lambda_i$$
  
$$\dot{\lambda}_i = -\gamma_1 \sum_{i \in n} a_{ij} (\lambda_i - \lambda_j) - \gamma_2 \sum_{i \in n} a_{ij} (z_i - z_j) + \gamma_3 (\eta_i P_{B,i} - r_i \Delta P_D)$$
  
$$\dot{z}_i = \sum_{i \in n} a_{ij} (\lambda_i - \lambda_j)$$
(8.14)

where  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3 > 0$  are the positive constants, respectively. We assume  $r_i$  is the ability that *i*th BESS is able to detect the total required load, and denote  $r = [r_1, \dots, r_n]^T$  with  $\mathbf{1}_n^T r = 1$ , for  $i \in n$ , where  $\mathbf{1}_n = [1, 1, \dots]^T \in \mathbb{R}^n$ .

To solve the optimization problem in a distributed way, inspired by the centralized saddlepoint dynamics in [159], a mismatch estimator is introduced based on of the idea of distributed average estimation; the purpose is to observe global information. With the estimator to observe the supply-demand mismatch, the algorithm (8.14) then can be implemented in a distributed way since the only information required is the states of *i*th BESS, i.e.  $\lambda_i$  and  $z_i$ . As a result, each BESS only needs to send/receive the value of  $\lambda_i$  and  $z_i$  to its neighbour BESSs. While considering the privacy concerns of BESSs, the proposed algorithm does not require BESSs to share their gradients of the cost function with neighbouring BESSs. Furthermore, in view of Remark. 8.4.1, the ToU signal  $\rho(t)$  will be adjusted by  $\lambda_i$  and therefore multiple BESSs are coordinated to discover the most efficient point.

According to (8.14), the control input for *i*th BESS is designed as

$$u_{B,i} = -\nabla f_i(P_{B,i}) - \eta_i \lambda_i. \tag{8.15}$$

**Remark 8.4.2.** To keep the BESS operating in a feasible mode, various local constraints can be integrated into the proposed algorithm, such as the SoC constraint and the constraints of minimum/maximum charging/discharging duration. These local constraints can be dealt with by the proposed algorithm by including corresponding projection operation.

### 8.4.3 Convergence Analysis

To facilitate convergence analysis, the proposed algorithm (8.14) is written in a compacted form, such as

$$\dot{P}_{B} = -\nabla f(P_{B}) - \eta \lambda$$
  
$$\dot{\lambda} = -\gamma_{1} \mathcal{L} \lambda - \gamma_{2} \mathcal{L} z + \gamma_{3} (\eta P_{B} - r \Delta P_{D})$$
  
$$\dot{z} = \mathcal{L} \lambda$$
(8.16)

where  $P_B = [P_{B,1}, \ldots, P_{B,n}]^T$ ,  $\lambda = [\lambda_1, \ldots, \lambda_n]^T$ ,  $\nabla f(P_B) = \sum_{i \in S_B} f_i(P_{B,i})$  and  $z = [z_1, \ldots, z_n]^T$ , respectively, and let  $\eta = \text{Diag}(\eta_1, \ldots, \eta_n)$ . The inspiration of our algorithm is based on multiple time-scale operations, which applies a distributed estimator to distribute the centralized saddle-point algorithm. To analyze the convergence, we first consider the equilibrium point  $(\overline{P}_B, \overline{\lambda}, \overline{z})$  of the proposed algorithm. When executing the algorithm over a connected and weight-balanced graph, it results in

$$\mathbf{1}_{n}^{T}\dot{z} = \mathbf{1}_{n}^{T}\mathcal{L}\overline{\lambda} = 0, \qquad (8.17)$$

where the fact that  $\mathbf{1}_{n}^{T} \mathcal{L} = 0$  is used to deduce (8.17). Then, the equilibrium point can be obtained by

$$0 = -\nabla f(\overline{P}_B) - \eta \overline{\lambda} \tag{8.18a}$$

$$0 = -\gamma_1 \mathcal{L}\overline{\lambda} - \gamma_2 \mathcal{L}\overline{z} + \gamma_3 (\eta \overline{P}_B - r\Delta P_D)$$
(8.18b)

$$0 = \mathcal{L}\overline{\lambda}.$$
 (8.18c)

Left multiplying (8.18c) by  $\mathbf{1}_n^T$  gives

$$\gamma_3 \mathbf{1}_n^T \left( \eta \overline{P}_B - r P_D \right) = 0 \tag{8.19a}$$

$$\nabla f(\overline{P}_B) + \eta \overline{\lambda} \mathbf{1}_n = 0. \tag{8.19b}$$

Thus, it is shown that the equilibrium point  $(\overline{P}_B, \overline{\lambda}, \overline{z})$  satisfies

$$\overline{P}_{B,i} = P_B^*, \ \overline{\lambda}_i = \lambda^*, \ \overline{z}_i = z^*, \ \text{for} \ i \in n,$$
(8.20)

where  $(P_B^*, \lambda^*, z^*)$  is the solution set of the optimization problem given in (8.12). For the convenience of the convergence analysis, the states in (8.16) are translated to the equilibrium point as

$$\hat{P}_B = \left(P_B - \overline{P}_B\right), \ \hat{\lambda} = m^T \left(\lambda - \overline{\lambda}\right), \ \hat{z} = m^T \left(z - \overline{z}\right), \tag{8.21}$$

where  $(\overline{P}_B, \overline{\lambda}, \overline{z})$  is any equilibrium point of (8.16) and *m* can be defined as in [160]. Thus, one has

$$\dot{P}_B = -\phi - \eta m \hat{\lambda} \tag{8.22a}$$

$$\hat{\lambda} = -\gamma_1 m^T \mathcal{L} m \hat{\lambda} - \gamma_2 m^T \mathcal{L} m \hat{z} - \gamma_3 \eta m \hat{P}_B$$
(8.22b)

$$\dot{\hat{z}} = m^T \mathcal{L} m \hat{\lambda} \tag{8.22c}$$

where  $\phi = \left( \nabla f(\hat{P}_B + \overline{P}_B) - f(\overline{P}_B) \right)$ . To study the stability of the proposed algorithm, we consider a candidate Lyapunov function

$$V = \frac{1}{2}\gamma_{3}\hat{P}_{B}^{T}\hat{P}_{B} + \frac{1}{2}\hat{\lambda}^{T}\hat{\lambda} + \frac{1}{2}\gamma_{2}\hat{z}^{T}\hat{z}.$$
(8.23)

The Lie derivative of V along with (8.22) is given as

$$\begin{split} \dot{V} &= \gamma_3 \hat{P}_B^T (-\phi - \eta m \hat{\lambda}) + \gamma_2 \hat{z}^T (m^T \mathcal{L} m \hat{\lambda}) + \hat{\lambda}^T (-\gamma_1 m^T \mathcal{L} m \hat{\lambda} - \gamma_2 m^T \mathcal{L} m \hat{z} - \gamma_3 \eta m \hat{P}_B) \\ &= -\gamma_3 \hat{P}_B^T \phi - \gamma_1 \hat{\lambda}^T m^T \mathcal{L} m \hat{\lambda} \\ &= -\gamma_3 (\hat{P}_B + P_B^* - P_B^*) \phi - \gamma_1 \hat{\lambda}^T m^T \mathcal{L} m \hat{\lambda}, \\ &\leq 0 \end{split}$$
(8.24)

where the convexity of the cost function, i.e.,  $\gamma_3(\hat{P}_B + P_B^* - P_B^*)\phi \ge 0$  is invoked for obtaining the last inequality. So far, it is shown that the trajectories of (8.22) and also (8.14) are bounded as  $\dot{V} \le 0$ . With the invariant set Theorem 1 in [160], it can be concluded that the points on  $\dot{V} = 0$  are the equilibrium points of the algorithm.

## 8.4.4 The Coordination of BESSs under Wind Power Generation Error

The intermittent nature of wind power generation means that it may lose its accuracy for maintaining supply-demand balance, whereas BESSs have the characteristic of fast response and high performance for balancing short-term variations in network power. For this reason, wind power generation is supplemented by BESSs in real applications.

As depicted in Fig. 8.2, a brief scheme is provided to coordinate the BESSs and wind power generation for maintaining system stability. The power management system (PMS) can collect the operation points of all participants in the grid to observe the supply-demand balance.

As shown in the power regulation part, if there is an imbalance in net power, wind power will be employed to compensate for the mismatch as much as possible firstly. However, the wind power may be insufficient and inaccurate, and there are also control errors connected with wind power generation. Therefore, a regulation signal will be generated based on both the mismatched power and the control error of wind power generation. Following the regulation signal ( $\Delta P_D$ ), BESSs are then implemented to absorb excessive power or compensate for insufficient power according to the proposed algorithm.

### 8.4.5 Algorithm Implementation

By referring to each BESS as an agent, the proposed algorithm is implemented in a distributed manner under a MAS framework. Each agent consists of two control levels. The top control level consists of three function modules, i.e., the measurement module, the communication module and the optimal solution discovery module. The measurement module obtains and updates the local information, and the communication module exchanges the information with the neighbouring agents. With the information provided by the measurement and communication module, the optimal solution discovery module updates the information and generates the output power reference for the bottom control level. Once the power reference is generated, the bottom-level control is implemented to control the agent to track this reference, which can follow the control scheme introduced in Fig. 8.3.

### **8.5** Simulation Results and analysis

In this section, four case studies are provided to verify the effectiveness of the proposed optimization algorithm. In *Case* 8.1, the proposed algorithm is compared with another resource management approach to show the advantages of our algorithm. In *Case* 8.2, a modified IEEE 14-bus system with five BESSs and two WTs is built in the MATLAB/Simulink that to demonstrate the convergence of the proposed algorithm under constant renewable generation and load demand. *Case* 8.3 is carried out with uncertain output power from the renewable generation. Finally, the scalability of the proposed algorithm is investigated in *Case* 8.4 where a 30-BESSs system is built in the MATLAB/Simulink.



Figure 8.2: Coordination scheme of wind generation and BESSs



Figure 8.3: The control diagram for a BESS

Table 8.1: Parameters of simulation studies								
	$\alpha_i$	$\beta_i$	$P_{B,i}^{\min}$ (kW)	$P_{B,i}^{\max}(\mathbf{kW})$	$\eta_i$			
BESS1	0.2037	1.0460	0	60	0.20			
BESS2	0.2815	1.2309	0	65	0.16			
BESS3	0.1987	1.0292	0	68	0.24			
BESS4	0.2092	1.1245	0	62	0.22			
BESS5	0.2247	1.0996	0	70	0.18			

During the simulation studies, the ToU signal is adopted as Fig. 8.4 to imitate the tariff in a real application, which simplifies the signal in [161] but keeping the same property. The simulation parameters are summarised in Table. 8.1 that are adopted from the experimental results in [97], and the topology of the communication network is implemented to be identical to the physical network.

### 8.5.1 Case 8.1

To reveal the effectiveness of the proposed algorithm, the algorithm in this study is first compared with another continuous algorithm in [3]. Without loss of generality, the ToU pricing and the supply-demand mismatch are assumed to be a constant value and the operation condition is assumed to be the same for this comparison study. As shown in Figs. 8.5 - 8.6, the marginal cost of each BESS can converge to its optimal value under both of the algorithms. However, by replacing the global mismatch estimator by a distributed estimator, the convergence speed of the proposed algorithm is faster than the algorithm in [3]. Additionally,



Figure 8.4: The time of use tariff

the proposed algorithm guarantees optimality without sharing the information about its own cost function.

### 8.5.2 Case 8.2

In this case study, the performance of the proposed distributed strategy is investigated in the IEEE 14-bus system. The microgrid is operated in the islanded mode and the supply-demand mismatch is set to be a constant value as 180kW. As shown in Fig. 8.7, the marginal cost of each BESS converges to the optimal value, and with the change in the ToU tariff during the simulation, the marginal cost will converge to a new optimal value according to the price signal. Figs. 8.8 - 8.10 depict the output power references, the supply-demand mismatch estimation, and the total output power of BESSs, respectively. The results show that the output power references converge to the optimal value according to the marginal cost update and at the same time, the supply-demand mismatch is eliminated by the proposed algorithm, which indicates its promising application in real-time control.



Figure 8.5: The marginal cost update under the proposed algorithm



Figure 8.6: The marginal cost update under the algorithm in [3]



Figure 8.7: The marginal cost update under the proposed algorithm



Figure 8.8: The output power update under the proposed algorithm



Figure 8.9: The power balance estimation



Figure 8.10: The total output power of BESSs



Figure 8.11: The output power profile of the wind power generation

### 8.5.3 Case 8.3

Since the output power from the wind power generation will be uncertain and time-varying, the required power from BESSs will be time-varying when high penetration renewable sources are integrated into the microgrid. In this case, a simulation study is carried out under a time-varying output power from the wind power generation to verify the effectiveness of the proposed algorithm. The microgrid is supposed to be islanded in 0s intentionally. The WTs in Fig. 5.2 are controlled in the reactive power regulation mode, and the output power from each WT is given in Fig. 8.11, respectively. During the simulation period, each BESS implements the proposed algorithm and is controlled by the strategy in Fig.8.3. With the coordination scheme in Fig. 8.2, Figs. 8.12 - 8.13 present the results for the output power and the power balance estimation, respectively. It can be observed that the power outputs from BESSs converge to the optimal value, and at the same time, the supply-demand mismatch

### 8.5.4 Case 8.4

To extend the proposed algorithm to a large-scale system, the algorithm should be designed to converge to the optimal value in a timely manner. Thus, the scalability of the proposed



Figure 8.12: The output power update under the proposed algorithm



Figure 8.13: The power balance estimation


Figure 8.14: Single line diagram of the 30-BESS system

algorithm is investigated in this case. To this end, a 30-BESS system is built in Matlab/Simulink, and the single line diagram of the 30-BESS system is shown in Fig. 8.14. The communication network is designed to be weight-balanced and strongly connected, and the supply-demand mismatch is assumed as 1000 kW. The scalability is demonstrated by observing the results in Figs. 8.15 - 8.16, which show that the output power of each BESS converges to their optimal values while the deviations between demand and supply power converge to zero.



Figure 8.15: The output power update under the proposed algorithm



Figure 8.16: The power balance estimation

#### 8.6 Conclusion

The coordination problem of BESSs in a microgrid under the high penetration of renewable sources is investigated in this chapter. A distributed cooperative control strategy is proposed for BESSs to maintain the supply-demand balance in a microgrid while increasing their profits and energy efficiency. Based on the proposed MAS framework, the proposed solution can be implemented in a distributed manner without a central coordinator. In addition, the results indicate that the optimal solution is achieved without releasing the information about the cost function. The effectiveness and the scalability of the proposed distributed strategy are further demonstrated by the simulation using the IEEE test systems and 30-BESS system.

### **Chapter 9**

#### **Future Works**

#### 9.1 Future Research

In further research, there are many open problems to the application of distributed optimization on energy management of smart grids.

- 1. For coordinating the charging process of PEVs, the characteristic of driving behaviours and load profiles at different specified time and areas should be further considered in the optimization problem formulation.
- 2. In absence of power flow constraints, the current solutions can not solve the energy management problem in the type of optimal power flow with some system operation constraints, such as voltage, current and line capacity limits. This flaw should be further investigated.
- **3.** It is possible to incorporate big data analysis for energy management problems. In a smart grid, there are thousands of smart meters that generate a huge scale of data in a given time-window. The amounts of data require to be analysed properly and used for improving energy-use efficiency. However, this part is still not investigated deeply.
- 4. Most existing results for energy management problems in smart grids are designed in an open-loop feature, which may face challenges such as model and forecasting inaccuracies. To capture time-varying operation and economic objectives and constraints,

the time-varying optimization formalism can be leveraged, which is designed by using the feedback-based information to cope with these inaccuracies.

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# Appendix A

## **Data for Test Systems**

#### A.1 Line Data for IEEE 14-bus System

Line	From	То	Line impedance (p.u.)		Susceptance (half charging)					
numbers			Resistance	Reactance	(p.u.)					
1	1	2	0.01938	0.05917	0.02640					
2	1	5	0.05403	0.22304	0.02190					
3	2	3	0.04699	0.19797	0.01870					
4	2	4	0.05811	0.17632	0.2460					
5	2	5	0.05695	0.17388	0.01700					
6	3	4	0.06701	0.17103	0.01730					
7	4	5	0.01335	0.04211	0.00640					
8	4	7	0	0.20912	0					
9	4	9	0	0.55618	0					
10	5	6	0	0.25202	0					
11	6	11	0.09498	0.19890	0					
12	6	12	0.12291	0.25581	0					
13	6	13	0.06615	0.13027	0					
14	7	8	0	0.17615	0					
15	7	9	0	0.11001	0					
16	9	10	0.03181	0.08450	0					
17	9	14	0.12711	0.27038	0					
18	10	11	0.08205	0.19988	0					
19	12	13	0.22092	0.19988	0					
20	13	14	0.17093	0.34802	0					

Table A.1: Line Data for IEEE 14-bus System

### A.2 Line Data for IEEE 30-bus System

Lino								
Line numbers	From	То	impedence (n.v.)		Susceptance (half charging) (p.u.)			
			Impedance (p.u.)					
	1	•	Resistance	Reactance	0.02			
l	1	2	0.02	0.06	0.03			
2	1	3	0.05	0.20	0.02			
3	2	4	0.06	0.18	0.02			
4	2	5	0.05	0.02	0			
5	2	6	0.06	0.18	0.02			
6	3	4	0.01	0.04	0			
7	4	6	0.01	0.04	0			
8	4	12	0	0.23	0			
9	5	7	0.05	0.12	0.01			
10	6	7	0.03	0.08	0			
11	6	8	0.01	0.09	0			
12	6	9	0	0.21	0			
13	6	10	0	0.56	0			
14	6	28	0.07	0.06	0.01			
15	8	28	0.06	0.20	0.02			
16	9	11	0	0.21	0			
17	9	10	0	0.11	0			
18	10	20	0.09	0.21	0			
19	10	17	0.03	0.09	0			
20	10	21	0.03	0.08	0			
21	10	22	0.07	0.15	0			
22	12	13	0	0.14	0			
23	12	14	0.12	0.26	0			
24	12	15	0.07	0.13	0			
25	12	16	0.01	0.12	0			
26	14	15	0.22	0.12	0			
27	15	18	0.11	0.22	0			

Table A.2: Line Data for IEEE 30-bus System

Line Line Susceptance (half charging) То impedance (p.u.) From numbers (p.u.) Resistance Reactance 28 15 23 0.10 0.21 0 0 29 16 17 0.08 0.19 30 18 19 0.06 0.13 0 31 19 20 0.03 0.07 0 22 0.22 0 32 21 0.01 33 22 24 0.11 0.18 0 0.27 34 23 24 0.13 0 24 25 0 35 0.19 0.33 36 25 0.25 0.38 0 26 37 25 27 0.11 0.21 0 38 27 29 0.22 0.4 0 39 27 0.22 0.60 0 30 0 40 28 27 0 0.4 0 41 29 30 0.24 0.45

Table A.3: Continued Line Data for IEEE 30-bus System