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# Impact of User Number on Massive MIMO with a Practical Number of Antennas

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*Abstract*—**This paper evaluates the impact of spatially multiplexing more users within a massive multiple-input, multiple-output (MIMO) system. Recent works on massive MIMO show that there is a peak value for sum spectrum efficiency (SE) achieved by serving a certain number of users. It was shown that until the sum SE reached its peak value, the maximum sum SE is achieved by serving all users simultaneously. These results were based on perfect channel state information (CSI), Shannon capacity calculations or using a very large number of antennas at the base station (BS). As opposed to the aforementioned results, we show that the maximum sum SE with practical number of antennas could be achieved by decreasing the number of users before the sum SE reached its peak value. This is shown by calculating the sum SE based on the Error Vector Magnitude (EVM) performance. Sum SE with zero-forcing (ZF) and matched filtering (MF) is presented for uplink (UL) data transmission in a single-cell. Channel matrices formed from both independent and identically distributed (IID) Rayleigh samples and measured data from a real massive MIMO system were used for performance evaluation. The impact of adding more users is also demonstrated by real constellations captured from an indoor massive MIMO trial.**

*Index Terms*—**Massive MIMO, 5G, Channel Estimation, EVM**

## I. INTRODUCTION

Massive multiple-input, multiple-output (MIMO) is a multi-user (MU) MIMO system with a large number of antennas at the base station (BS) serving several users within the same time and frequency resource [1]. Field trials such as those recently conducted by ZTE [2] and Facebook [3] support the trend towards using this technology in future 5G wireless systems. The unprecedented per-cell bandwidth efficiency of 146.5 bit/s/Hz achieved in [4] has encouraged industries to consider massive MIMO as key 5G technology that could drastically enhance the capacity of sub-6 GHz communications in future wireless networks. Furthermore, recent results in [5] and [6] provide some of the first indications for how a real massive MIMO system could perform under mobile conditions.

The theoretical results in [1] show that significant capacity improvements are possible in MU MIMO by increasing the number of antenna elements at the BS. The "channel hardening" synonymous with massive MIMO [7] was observed in the field trials documented in [8], [4] and [9]. In theory, the user channel vectors become pairwise orthogonal as the number of BS antennas is increased, facilitating the effective use of matched filtering (MF) [10]. The user-side channel Gram matrix from the field trials in [4] and [9] indicates the level of spatial orthogonality achieved when using a practical number of antennas may not be ideal. When the user channels become more correlated, it is likely that zero-forcing (ZF) or Minimum Mean Square Error (MMSE) will be required for reliable data transmission.

Accurate channel state information (CSI) is crucial for correct MU MIMO operation [1]. Several pieces of research have investigated the effect of inaccurate CSI in massive MIMO. The work in [11] illustrates the impact of hardware impairments for the downlink (DL) of a single cell with different channel conditions. It shows how the number of BS antenna elements and their spacings can affect the Error Vector Magnitude (EVM) at the user equipment (UE) side. The hardware impairment impact for a single-cell scenario was covered in [12]. The paper shows how the number of antennas effect the average sum rate with different channel models. The effect of imperfect channel reciprocity and CSI error was also covered in [13] for DL transmissions. The paper introduced a model that considers the impact of RF mismatches on the linear precoding for time division duplex (TDD) massive MIMO system. The relationship between the output signal to interference plus noise ratio (SINR) and the amplitude error variance were illustrated with ZF and MF. The impact of inaccurate CSI in a single-cell for both, uplink (UL) and DL was covered in [14]. The impact of non-ideal hardware on the capacity limits was covered in [15] for both, UL and DL in a multi-cell scenario. It shows how the pilot length and the number of antennas at the BS are affected by the relative estimation error per antenna. The multi-cell scenario was then covered in [16] for both UL and DL. It shows how the number of antennas, pilot allocation and hardware impairments affect the spectral efficiency.

This paper evaluates the performance degradation in massive MIMO that can occur due to CSI error accumulation as more users are spatially multiplexed. The work focuses on the uplink transmission for single-cell scenario with practical number of antennas at the BS operating with real hardware. An indoor channel, captured from the trial in [4], and independent and identically distributed (IID) Rayleigh channel were used for the simulations in this paper. Two linear decoding techniques are covered: ZF and MF. CSI errors can occur for many reasons such as local oscillator

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(LO) phase noise, reciprocity calibration inaccuracies and quantization errors in analog to digital converters. Cumulative error amplification is derived to illustrate the impact of channel hardening and the number of users on amplifying the effect of the errors caused by hardware impairments. Theoretical and realistic spectrum efficiency (SE) calculation then is used to evaluate the performance difference between scenarios.

## II. SYSTEM MODEL

A single-cell Massive-MIMO architecture is considered in this work. The base station is equipped with a large number of antennas (M) and serves a number of active single-antenna users (K) where (M $\gg$ K). The system operates in TDD mode and uses the same time-frequency resources for all users. Each UE transmits a frequency-orthogonal UL pilot for the channel estimation. The estimated channel matrix between the UEs and the BS is denoted by $\hat{\boldsymbol{H}} \in \mathbb{C}^{\mathrm{M}\times\mathrm{K}}$. While the actual channel matrix during the uplink data transmission is denoted by $\boldsymbol{H} \in \mathbb{C}^{\mathrm{M}\times\mathrm{K}}$, which is given by

$$\boldsymbol{H} = \hat{\boldsymbol{H}} + \boldsymbol{E} \tag{1}$$

where $\boldsymbol{E} \in \mathbb{C}^{\mathrm{M}\times\mathrm{K}}$ is the difference between the estimated channel and the actual channel during the uplink data transmission. This error could be caused by hardware impairments, interpolation across frequency, large-scale channel attenuation and any other potential sources. In this paper the error in the channel estimation between the UE and the BS is modeled as a complex Gaussian distribution $\sim \mathcal{CN}\left(\mathbf{0}, \sigma_e^2 \boldsymbol{I}_M\right)$, where $\boldsymbol{I}_M$ is the M$\times$M identity matrix and $\sigma_e^2$ is the error variance. The equalized UL signal $\hat{\boldsymbol{x}} \in \mathbb{C}^{\mathbb{K}}$ can be expressed as

$$\hat{\boldsymbol{x}} = \boldsymbol{W}(\sqrt{\rho_{ul}}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}) \tag{2}$$

where $x$ represents the transmitted symbol vector from all users in the same cell, normalized as $\mathbb{E}\{|x_k|\} = 1$. The corresponding UL transmit power is denoted by $\rho_{ul}$. Simple UL power control is assumed in this paper. The UL transmit power is adjusted so the received signal to noise ratio (SNR) from all users is the same. The additive noise vector is denoted by $\boldsymbol{n}$. The noise variance from the antennas at the BS is modeled as $\sim \mathcal{CN}\left(\mathbf{0}, \sigma_n^2 \boldsymbol{I}_M\right)$, where $\sigma_n^2$ is the noise variance . $\boldsymbol{W} \in \mathbb{C}^{\mathrm{K}\times\mathrm{M}}$is the linear decoder matrix, formed using MMSE, ZF or MF.

## III. ACCUMULATIVE ERROR AMPLIFICATION

With imperfect channel estimation the error in equation 1 will always be added to the estimated channel. In sections A and B below, the impact of adding users with inaccurate CSI is explained for MF and ZF in the UL. From (2), the equalized signal vector can be written as follows:

$$\begin{aligned}\hat{\boldsymbol{x}} &= \sqrt{\rho_{ul}}\boldsymbol{W}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{W}\boldsymbol{n}\\ &= \sqrt{\rho_{ul}}\left(\boldsymbol{W}\boldsymbol{E} + \boldsymbol{W}\hat{\boldsymbol{H}}\right)\boldsymbol{x} + \boldsymbol{W}\boldsymbol{n}\\ &= \sqrt{\rho_{ul}}\boldsymbol{W}\boldsymbol{E}\boldsymbol{x} + \sqrt{\rho_{ul}}\boldsymbol{W}\hat{\boldsymbol{H}}\boldsymbol{x} + \boldsymbol{W}\boldsymbol{n}\end{aligned} \tag{3}$$

The number of users affect the actual equalized signal for the target user k. This can be shown by rearranging (3) as follows

$$\hat{x}_k = \underbrace{\sqrt{\rho_{ul_k}}\sum_{i=1}^{K}(\boldsymbol{W}\boldsymbol{E})_{k,i}\,\boldsymbol{x}_i}_{\text{Cumulative Error}} + \overbrace{\sqrt{\rho_{ul_k}}\sum_{i=1}^{K}\left(\boldsymbol{W}\hat{\boldsymbol{H}}\right)_{k,i}\boldsymbol{x}_i}^{\text{Desired Signal+Interference}} + z_k \tag{4}$$

where $z_k$ is the amplified noise for user k caused by the decoder. The "Cumulative Error" part represents the interference introduced by CSI estimation inaccuracies. The "Desired Signal + Interference" part consists of the UL transmitted symbol from user k and the interference caused by the inter-user spatial correlation.

### A. Match Filter Receiver

MF is a low complexity operation where the decoder used in the equalization process can be written as $\boldsymbol{W} = \hat{\boldsymbol{H}}^H$.The equalized signal vector can be written as follows:

$$\hat{x}_k = \underbrace{\sqrt{\rho_{ul}}\sum_{i=1}^{K}\left(\hat{\boldsymbol{H}}^H\boldsymbol{E}\right)_{k,i}\boldsymbol{x}_i}_{\text{Cumulative Error}} + \overbrace{\sqrt{\rho_{ul}}\sum_{i=1}^{K}\left(\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)_{k,i}\boldsymbol{x}_i}^{\text{Desired Signal + Interference}} + z_k \tag{5}$$

By increasing the number of antennas at the BS, the ratio between diagonal elements and non-diagonal elements of the Gram matrix $\boldsymbol{G} = \hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}$ will also increase. This is known as the channel hardening effect as previously mentioned [7]. The "Desired Signal + Interference" part is extremely sensitive upon the number of antennas at the BS. The "Cumulative Error" part is less impacted by the number of antennas since the matrix elements resulting from the matrix multiplication of $\left(\hat{\boldsymbol{H}}^H\boldsymbol{E}\right)$ are far smaller than the channel Gram matrix when UL power control is applied. The interference from both parts is increased by increasing the number of users.

### B. Zero Forcing Receiver

ZF tends to improve the performance by suppressing the interference between users and enhancing the number of simultaneous users. The decoder used in the equalization can be written as $\boldsymbol{W} = \left(\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)^{-1}\hat{\boldsymbol{H}}^H$. The equalized signal vector can be written as follows:

$$\hat{x}_k = \underbrace{\sqrt{\rho_{ul_k}}\sum_{i=1}^{K}\left(\left(\left(\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)^{-1}\hat{\boldsymbol{H}}^H\boldsymbol{E}\right)_{k,i}\boldsymbol{x}_i\right)}_{\text{Cumulative Error}} + \sqrt{\rho_{ul_k}}\boldsymbol{x}_i + z_k \tag{6}$$

Unlike MF, the interference is only caused from the "Cumulative Error" part which is introduced by CSI estimation inaccuracies. This interference is increased by increasing number of users. Similar to MF, the channel Gram matrix impacts the interference value. Although ZF suppresses the interference from the estimated channel between users, it amplifies the interference introduced by the inaccurate CSI. This interference is caused by the inverse of the channel Gram matrix in the "Cumulative Error" part. Although the value of this part in ZF is larger than the one in MF, the overall interference in MF is still higher than that of ZF.

## IV. SPECTRAL EFFICIENCY EVALUATION

Two different scenarios were considered to evaluate the massive MIMO performance for differing numbers of users when CSI errors are present. An IID Rayleigh channel is used in the first scenario, where M=128 and K is an even number $\in [2, 22]$. In the second scenario, the real channel captured from the trial in [4] was used. The trial took place at the University of Bristol. A patch panel antenna array was setup in a 4×32 to serve 22 user clients in LOS and placed 24.8m away. These scenarios were run through an uplink massive MIMO simulator developed at the University of Bristol. In addition to randomly generated channels, new vectors can be transmitted through channels previously captured by a real system for more extensive analysis. For both scenarios, an error variance of 0.01 (1%) was used for the simulations shown. Whilst this value may be higher or lower in reality, it serves to illustrate the potential effects of CSI inaccuracies in massive MIMO. The users were introduced in an order where the least overall correlation are considered first to minimise the inter-user interference (IUI) damage with each step . More detailed estimations for real system error variances will be provided in future work.

### A. Theoretical Evaluation

Fig. 1 shows SE comparison between the 1st and the 2nd scenario as the number of active users increases across an SNR range of 0−50 dB. The sub-plots in the first row show the median achievable SE per user by using ZF and MF for both scenarios. Their equivalent sum SE are the sub-plots in the second row. The median per user SE is always 16.59 bits/s/Hz at 50 dB SNR in case of one active user since there is no interference source. For ZF it can be seen in (a) and (b) that a median per user spectral efficiency of greater than 4 bits/s/Hz can always be maintained at 30 dB SNR. Increasing the SNR beyond 30 dB improves lower numbers of active users, but plateaus at 4.89 bits/s/Hz in (a) and 4.219 bits/s/Hz in (b) for 22 users at 40 dB SNR. Sub-plot (e) and (f) in Fig. 1 show the sum SE for 1st and 2nd scenario respectively. With ZF, the median per user SE difference between 1st and 2nd scenario is small since the interference is only caused by the cumulative error part in (6) where the off-diagonal values of the Gram matrix for 1st scenario is less than the ones from 2nd scenario. Despite the small difference in per user SE, the maximum sum SE in 1st scenario outperform the one from 2nd scenario by 14.89 bits/s/Hz since 22 users are served simultaneously. With MF, the median per user SE is greatly affected by adding more users as it is shown in Fig. 1, sub-plots (c) and (d). The maximum sum SE is achieved when only two users are selected in both scenarios. For 22 users, the sum SE in 1st scenario is greater by 6.98 bits/s/Hz at 50 dB SNR.

### B. Practical Evaluation

A common measurement of signal quality used in 3GPP long-term evolution (LTE) standards is the EVM. It is a comprehensive metric because it takes all the elements the transmitted symbol was affected by into account. Higher modulation and coding schemes (MCS) are supported when the EVM is smaller [17]. From [18] and [17], the EVM can by given as

$$\mathrm{EVM}_k = \sqrt{\frac{\frac{1}{N}\sum_{n=1}^{N} |S_r(n) - S_t(n)|^2}{\frac{1}{N}\sum_{n=1}^{N} |S_t(n)|^2}} \times 100 \tag{7}$$

where $N$ is the number of symbols the EVM was measured over. $S_r(n)$ is the nth normalized received symbol and $S_t(n)$ is the ideal value of the nth symbol. For comprehensive analysis, the EVM was plotted for the 1st and the 2nd scenarios using ZF and MF. Based on the 3GPP LTE standards, the required EVM for 64 quadrature amplitude modulation (QAM) is 9% [17]. The required EVM to achieve 256 QAM is currently being considered and simulation campaigns shown it might be in the range of 1.5% – 4% [19].

Fig. 2 and Fig. 3 show the EVM results with ZF for the 1st and the 2nd scenario respectively. For the 1st scenario, the EVM range was between 1.5% and 4.5% which corresponds to K from 2 till 22 with 40 dB SNR. In the 2nd scenario, the EVM performance was slightly degraded and its range becomes between 1.9% and 5.8%. The increment in the EVM range is caused by the high spatial correlation in the 2nd scenario which amplifies the impact of inaccurate CSI. Table I and Table II show the sum SE results at 40 dB SNR for the 1st and 2nd scenarios respectively. In the 1st scenario, the maximum sum SE is achieved by serving 19 users simultaneously with 256-QAM. By adding more users, 64-QAM is used which won't increase the sum SE unless 26 users are served simultaneously. In the 2nd scenario, the maximum sum SE is achieved by serving 16 users simultaneously with 256-QAM. This value can only be increased by adding more than five users. As it is shown in both scenarios, by adding more users the EVM performance becomes worse and the sum SE could be decreased. The degradation in the EVM performance can be seen in Fig. 5, which shows a realistic 64-QAM constellations captured from the trial in [20]. The constellations in the left was captured with 24 active users. The constellations in the right was captured after removing two users randomly where the observed EVM was enhanced just by removing two users since the "Cumulative Error" part in (6) was reduced by two users. The EVM performance with perfect CSI is shown in Fig. 4 where the left plot is for the 1st

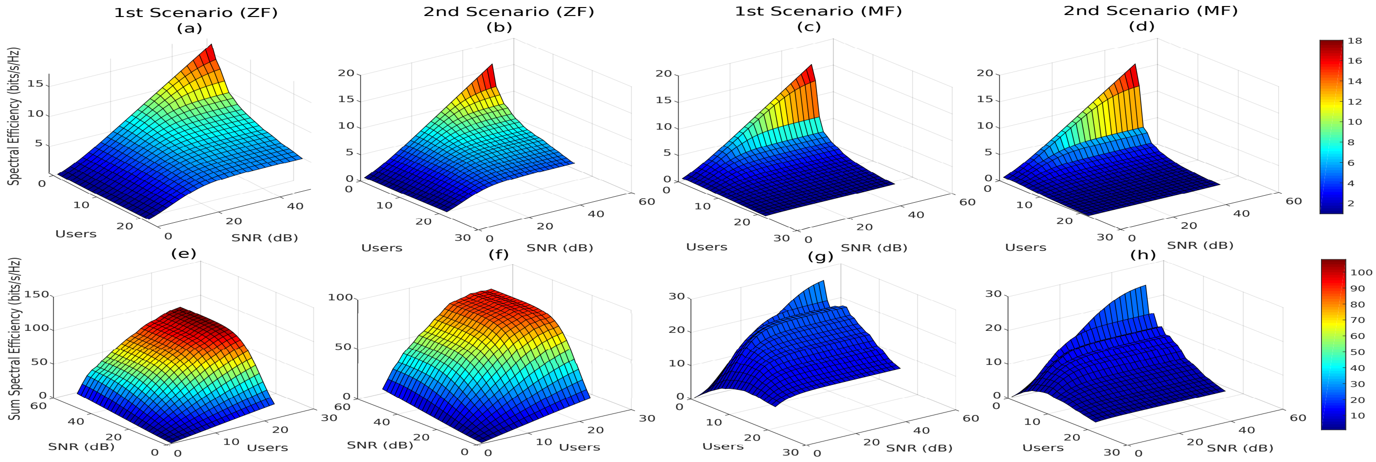

Fig. 1: SE (1st row ) and sum SE (2nd row) comparison between IID Rayleigh channel and measured indoor channel with ZF and MF.

scenario and the right plot is for the 2nd scenario. The EVM performance was enhanced and its range becomes between 0.85% and 0.97% in 1st scenario and between 0.89% and 1.18% in 2nd scenario which corresponds to K from 2 till 22 with 40 dB SNR. With perfect CSI, the EVM performance is slightly affected by the number of users since it only amplifies the noise. Fig. 6 shows the EVM results with MF for 1st and 2nd scenarios with 40 dB SNR. The sub-plot a and b are for the 1st scenario with perfect and inaccurate CSI respectively. The sub-plot c and d are for the 2nd scenario with perfect and inaccurate CSI respectively. The impact of inaccurate CSI can be ignored in both scenarios. The EVM performance is highly affected by the number of users. Compared to ZF, the EVM value corresponds to 22 users is increased by 30.5% in the 1st scenario and 64% in the 2nd scenario. In the 1st scenario, the maximum number of users can be served is 8 using quadrature phase-shift keying (QPSK). In the second scenario, the maximum number of users is reduced by two using the same MCS. Maximum sum SE of 8.832 bits/s/Hz is achieved by serving two users with 256-QAM in both scenarios.

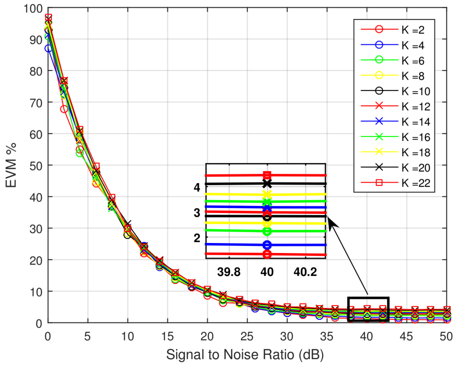

Fig. 2: EVM performance with ZF in IID Rayleigh channel.

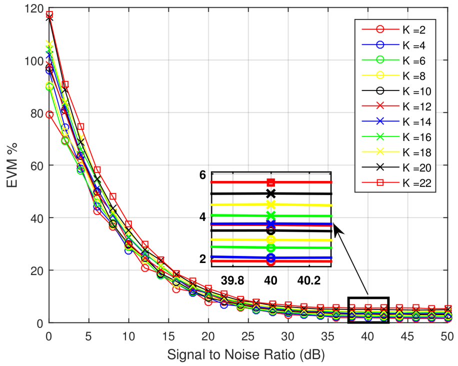

Fig. 3: EVM performance with ZF in measured indoor channel.

TABLE I:
Realistic Performance (IID Rayleigh Scenario with ZF).

| | Serving all Users | | Maximizing Sum SE |
|---|---|---|---|
| K | 22 | 25 | 19 |
| MCS | 64-QAM | 64-QAM | 256-QAM |
| Sum SE | 72.864 b/s/Hz | 82.8 b/s/Hz | 83.9 b/s/Hz |

TABLE II:
Realistic Performance (Measured Indoor Scenario with ZF).

| | Serving all Users | | Maximizing Sum SE |
|---|---|---|---|
| K | 17 | 21 | 16 |
| MCS | 64-QAM | 64-QAM | 256-QAM |
| Sum SE | 56.3 b/s/Hz | 69.55 b/s/Hz | 70.656 b/s/Hz |

## V. CONCLUSIONS

In this paper, the impact of the user number upon maximizing sum SE in a single-cell massive MIMO has been illustrated. MF and ZF decoders were covered in the UL data transmission case. All the different sources for CSI error were considered as one combined error in the resulting channel estimate. A practical method based on EVM performance for realistic SE calculation was compared with the theoretical method based on Shannon capacity. By using both IID Rayleigh and measured massive MIMO channels with 1% CSI estimation