Three Essays in Executive Compensation

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Abstract

This thesis investigates three theoretical problems in executive compensation literature. They involve extension of a standard principal-agent model, incorporating taxation into the valuation of executive stock options, and the pricing of executive stock options in the presence of managerial effort.

Empirical literature has long addressed the endogeneity of capital structure and executive compensation. Yet few models, which optimally determine executive compensation, explicitly introduce capital structure choice. Chapter 2 proposes a principal-agent model in which the capital structure, compensation and managerial actions are simultaneously determined. Based on our numerical results leverage has two effects on managerial actions. One is to discipline the manager and the other is to replace the incentive effect of compensation. Two such effects exist because volatility is chosen by the manager.

The basic model is also extended to include debt-like compensation. Our results show that for a given leverage level, rewarding the manager with debt makes her work harder but take less risk. But debt compensation cannot limit risk neutral shareholders' risk appetite; we hence conclude that only a combination of capital and pay regulation, which restricts both risk-taking of shareholders and incentives of the manager, can significantly reduce the firm's risk.

Taxation is an important consideration in the design of executive (and employee) compensation. It directly affects the firm's revenue as well as the executive's after tax income. Once the compensation is granted, taxes also affect the early exercise strategy of the components of the compensation. Chapter 3 explores the executive (and employees) compensation with tax. Specifically, we build a tax-inclusive valuation model. The new feature of

the model is an addition of a tax decision, which allows the executive (and employees) to optimally sell stock to maximize after-tax terminal utility. The stock selling decision is very similar to an option exercise decision. The valuation model essentially has two embedded options: one option is when to exercise the stock option and the other option is when to sell the stock.

This new feature allows different exercise policies for executive stock options under different tax schemes. We apply the model to the US and the UK tax system. The findings suggest that restricted stock is the preferred form of compensation in the US. In the UK, restricted stock is only preferred when the executive has low wealth. We also investigate incentives of a special tax scheme – section 83b election – which gives employees a choice to pay income tax at grant date. This voluntary election allows the executive to accelerate tax on restricted stock. Our results suggest that 83b election is not optimal for the manager, who would get double-taxed. And it is not optimal for the issuing firm either, as restricted stock without the election can provide higher incentives at lower cost.

The value of executive stock options (ESOs) should depend on the manager's ability to influence firm value. ESOs are granted under the assumption that the executive could make the firm value increase. However, ESOs are always valued with no managerial influence. Chapter 4 examines valuation of ESOs, with the assumption that the manager can influence the firm value via her effort choice. The manager influences stock prices by exerting effort, which increases the firm's stock expected return. Effort leads to a disutility (which can be regarded as effort cost) to the risk-averse, utility-maximizing manager. In addition to the effort choice, the market asset is also introduced to the manager's investment set. Effort increases the manager's subjective valuation as well as the cost of ESO.

The standard principal-agent model is not strictly speaking consistent with general equilibrium models like CAPM. Managerial effort is generally not priced under these equilibrium models, because all managers are price-takers. For this reason, we assume that CAPM does not strictly hold when effort is introduced. Our results show that the manager's value and the cost increase with the correlation, because the manager delays a value destroying early exercise. We also show that the manager's subjective value of the ESO is higher than the cost only when the manager has low wealth, low risk-aversion, and the stock has a low volatility. Under these scenarios, the manager's marginal utility is high and effort has a large impact on the manager's valuation. As a result, the value is higher than the cost. These results suggest that managers of large public firms are less likely to value their ESOs higher than the cost; while managers of small non-public firms are likely to value their ESOs far higher than their cost. The result may explain why ESO is so popular in small startup firms, where ESO is most likely to be valued higher than the cost.

Declaration

I declare that this thesis consists of original work undertaken by myself at Lancaster University between 2007 and 2011. Chapter 2 is a co-authored paper with my supervisors Dr Grzegorz Pawlina and Dr Rafal Wojakowski. It was presented at 4th Doctoral Conference at Montpellier in 2011. Chapter 3 is a co-authored paper with Dr Martin Widdicks. Chapter 4 is a single authored paper by myself. Where work by other authors is referred to, it has been properly referenced.

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Chapter 1

Introduction

Executive compensation has been a multifaceted and controversial subject over the last three decades. The financial crisis of 2007 and the subsequent government bailout of major investment banks have raised a lot of controversy about current executive compensation practices. Banks receiving bailout funds have kept paying substantial bonuses to their executives.[1] Although public anger over top executives' pay resulted in some reforms of compensation practices and brought more restrictions on pay, it is not the first time the pay controversy has arisen.[2] In fact, executive compensation has been a heatedly researched area for the past decade. Existing work has uncovered many insights that significantly improve our understanding of executive compensation. Still, a consistent theory that captures observed practice is yet to be developed. A lot of work has to be done for better theories to emerge. This thesis is one of the works that attempts to move this process forward.

The main purpose of the thesis is to address three theoretical problems in executive compensation[3], which relate to three streams of literature: the principal-agent model, taxation and valuation of executive stock options (ESO). All of this literature has been extensively discussed and debated over the past three decades. Yet there are still questions left unanswered. In the following sections of this chapter, each stream of

[1] Conyon et al. [2011] document all these 'public anger' incidents in the US, France, Germany, Italy, Spain and the UK.

[2] Wells [2010] provides an excellent review of executive compensation in the 1930s. During the Depression-era, top CEOs' pay also sparked huge public outrage.

[3] Executive compensation is a topic that spans different disciplinary areas. For example, Devers et al. [2007] provide a comprehensive review of executive compensation across different subject areas – management, finance, accounting, economics and psychology.

literature is reviewed. As a preview of detailed analysis in the following three chapters, problems and unanswered questions in each stream are also discussed.

1.1 Principal-agent model

One major theoretical stream of executive compensation relates to the principal-agent problem (or agency theory), which stems from Ross [1973], Mirrlees [1976] and Holmstrom [1979]. The basic structure of these models is relatively simple, which usually involves two parties: a principal and an agent. The agent privately takes an action a that impacts on the payoff $x(a, \theta)$, which is a function of the action a and a random component θ; θ is normally distributed with a mean of 0 and a variance of 1. The agency problem is to determine how the payoff $x(a, \theta)$ should be shared optimally between the two parties. Assuming the principal is risk neutral and the agent has utility function $H(x, a)$, the principal's objective is to maximize expected payoff net payment to the agent,

$$\max_{s(x)} E(x - s(x)) \tag{1.1}$$

subject to,

$$E\big[H(s(x), a)\big] \geq \overline{H} \tag{1.2}$$

$$a \in \arg\max_{a} E\big[H(s(x), a)\big] \tag{1.3}$$

where $s(x)$ is the contract paid to the agent, and $\overline{H}$ is minimum expected utility for the agent to undertake the task. Condition (1.2) is the participation constraint, which guarantees the agent a minimum expected utility. Condition (1.3) is the incentive compatibility constraint, which ensures that the optimal contract leads to the best action. A convenient feature of the agency problem is that it can easily be applied to any principal-agent relationship. Jensen and Meckling [1976] integrate the agency problem to the theory of the firm, and show that an agency problem exists between shareholder and debtholder in the form of overinvestment.

A large body of literature has accumulated since then studying the agency problem associated with firms, especially the relationship between shareholder and manager. In fact, the contract design is the most prominent application of the principal-agent model. The model can be easily changed to accommodate equity-based compensation, which has been the most popular compensation instrument in the past three decades. A typical approach of modifying the standard model outlined above is to impose a functional form on the contract pay, so that

$$s(x) = bx + c \tag{1.4}$$

where b is the proportion of the payoff, x, the principal pays to the agent, which represents the stock of the firm. Constant c represents the agent's fixed wage. With some simplification, such a contract space can resemble a typical managerial compensation; in practice, managers also get bonuses, stock options, pension benefits and other incentive schemes. We only assume the stock and wage here to preserve linearity of the contract payoff, which allows for a tractable solution. The problem boils down to obtaining the optimal mix of b and c that maximizes the shareholder's value. To simplify the analysis, assume

$$\begin{aligned} x(a,\theta) &= a + \sigma\theta \\ H\big(s(x),a\big) &= -\frac{1}{R}\exp\Big[-R\Big(s(x) - \frac{1}{2}a^2\Big)\Big] \end{aligned} \tag{1.5}$$

The agent's utility function, $H\big(s(x),a\big)$ is exponential and effort averse. σ is the magnitude of the payoff risk. R is the agent's risk-aversion coefficient. Since the payoff is normally distributed and the agent's utility function $H\big(s(x),a\big)$ is exponential, mean-variance preference is used for expected utility. So

$$E\Big[H\big(s(x),a\big)\Big] = ab + c - \frac{1}{2}a^2 - \frac{1}{2}Rb^2\sigma^2 \tag{1.6}$$

Based on the first order condition of equation (1.3),

$$a = b \tag{1.7}$$

From the agent's participation constraint, the constant, c, is given by

$$c = \overline{H} + \frac{1}{2}Rb^2\sigma^2 - \frac{1}{2}b^2 \tag{1.8}$$

Now the principal's problem can be expressed in terms of b alone. His maximization follows,

$$\begin{aligned}\max_{s(x)} E(x - s(x)) =& E\big[(1-b)a + (1-b)\sigma\theta - c\big]\\ =&(1-b)a - c \\ =& b - \overline{H} - \frac{1}{2}Rb^2\sigma^2 - \frac{1}{2}b^2\end{aligned} \tag{1.9}$$

The first order condition with respect to b gives,

$$b = \frac{1}{1+R\sigma^2} \tag{1.10}$$

With some algebra arrangement, the complete solution is standard in the principal-agent literature,

$$a = b \quad , \quad b = \frac{1}{1+R\sigma^2} \quad , \quad c = \overline{H} + \frac{Rb^2\sigma^2 - 1}{2(1+R\sigma^2)} \quad , \tag{1.11}$$

The agent's performance based on expected payoff to the principal is,

$$P = E\big[x - s(x)\big] = \frac{1}{2(1+R\sigma^2)} - \overline{H} = \frac{1}{2}a - \overline{H} \tag{1.12}$$

Holmstrom and Milgrom [1987] have a continuous version of the above solution. Guo and Ou-Yang [2006] have a one-period solution similar to the one present here. There are two testable implications: the first one is that both effort a, and incentives (or pay-for-performance) b, are good predictors of the firm's performance; The second one is a negative relationship between incentives, b and exogenous risk, σ.

The simple structure and linear contracts of the standard model make solving the agency problem relatively easy.[4] Many studies build variations of the standard model to obtain testable implications. These studies mostly investigate relationship between variables in the model, e.g. between pay-for-performance and risk, pay-for-performance and firm performance, etc. Jin [2002] extends the basic model and allows the manager (agent) to trade the market asset. His model predicts that pay-performance sensitivity decreases with the firm-specific risk, and does not change with the market risk. These results are consistent with his empirical findings. Bitler et al. [2005] develop a principal-agent model that predicts the standard implications, which are pay-performance sen-

[4]Mean variance preferences, as shown in equation (1.6), are also needed to linearize expected utility.

sitivity decreasing with firm risk and increasing with firm performance. They find empirical support for these predictions using entrepreneurial data. Gao [2010] builds a principal-agent model with managerial hedging. His model shows that pay-performance sensitivity decreases with the agent's hedging cost. Empirically, he finds evidence supporting this prediction. Apart from the linear contracts used in many empirical studies, other works explore the convexity in executive compensation as equity-based pay is non-linear financial instruments. Innes [1990] solves a principal-agent model where agents have limited liability. Hemmer et al. [1999] investigate convexity in a principal-agent model. They show that convexity is introduced when agents have moderate levels of relative risk aversion.

Although the standard model has a tractable solution, its linearity makes application of the model to stock options difficult. Stock options are non-linear, convex instruments, which naturally make the model non-linear when included. Models, which include stock options, usually have less tractable solutions. However, inclusion of stock options in the principal-agent model can investigate a very important question: What is the optimal mix of compensation: stock or options? A problem that has puzzled the literature for decades and attracted widespread academic debate. Studies on this issue are many. Feltham and Wu [2001] build a principal-agent model to compare the incentive cost of stock and options. They show that options have less incentive cost when the agent can shift payoff risk, which suggests options are the optimal form of compensation. In an influential paper, Hall and Murphy [2002] reviewed the problem based on the certainty equivalent approach. Their investigation suggests that stock is the most efficient form of compensation. While empirical evidence provides little support for these results, they carefully interpret that a puzzle may exist, and suggest that more robust treatment would follow a principal-agent framework similar to Grossman and Hart [1983].

Their results (Hall and Murphy [2002]) lead to a series of calibration models that match principal-agent models to observed data. Armstrong et al. [2007] calibrate a principal-agent model to a dataset of Fortune 500 companies. They find that stock options are always part of the optimal contract. Dittmann and Maug [2007] calibrate a standard principal-agent model using US executive compensation data. However, they find stock is the optimal form of executive compensation and options should never be rewarded. Hence, they conclude that the standard principal-agent model typically used in the literature cannot explain observed compensation contracts.

Kadan and Swinkels [2008] introduce bankruptcy risk into the principal-agent model to investigate the optimal form of compensation. They empirically find that stock is only optimal when firms have a high probability of bankruptcy. Palmon et al. [2008] build a principal-agent model with stock, options and the agent having a limited liability. They show that options are the optimal form of compensation. They also investigate the optimal strike price of option and find that without taxation a slightly out-of-money option is the optimal compensation instrument. The studies mentioned above have mixed conclusions regarding the optimal form of compensation. It seems variations of the principal-agent model can lead to quite different results. This problem is still open to debate. In Chapter 3, we also investigate the optimal form of compensation using our valuation model.

Another problem that has attract recent attention is inside debt[5], such as pensions and deferred compensation, which constitutes a large form of executive compensation and has significant impact on executives' incentives. Prior literature solely considers equity (stock) based compensation mainly because of limited disclosure. Sundaram and Yermack [2007] show that inside debt has an impact on the firm's cost of debt and its capital structure. Edmans and Liu [2011] introduce inside debt into a principal-agent model, which involve changing the payoff function equation (1.4). They show inside debt is more effective than bonuses at curbing the overinvestment problem. Principal-agent models that incorporate inside-debt are rare. Debt compensation adds extra complexity to the 'stock or option' debate. The effect of inside debt is also considered in Chapter 2 of this thesis.

An interesting strand of studies calibrate the principal-agent model to observed data in order to examine efficiency of observed contracts. Dittmann et al. [2010] argue that managers are loss-averse and a principal-agent model with loss-averse agents can explain observed data remarkably well. Edmans et al. [2009] reformulate the original principal-agent model by introducing a multiplicative effort and calibrate the model to empirical contracts. Their model captures some features of observed data, so they conclude that the empirically observed compensation data actually reflects efficiency and

[5]This term was first coined in Jensen and Meckling [1976] to refer to compensation securities with payoffs similar to debt.

are consistent with their new model. More recently, Dittmann et al. [2011] calibrated a principal-agent model to demonstrate the unintended consequences of restriction on executive compensation. They show that rather than reducing risk-taking incentives, restriction on some components of compensation can lead to higher risk-taking incentives.

Finally, a small number of studies extend principal-agent models by endogenizing the risk choice, e.g. Cadenillas et al. [2004], Guo and Ou-Yang [2006] and Carlson and Lazrak [2010]. Since this is a major topic of Chapter 2, we reserve discussion of these papers for the next chapter.

1.2 Pricing and taxation of ESO

Another important strand of literature in executive compensation is the valuation of executive stock options (ESOs), which can be traced back to the option pricing theory of Black and Scholes [1973], Merton [1973] and Cox et al. [1979]. ESOs are American call options, which are typically granted to executives at the money as a form of compensation. However, once the options are granted, the executives cannot exercise it until a vesting period, which typically lasting 1-5 years, has passed. ESOs also have a long maturity, usually lasting about 10 years. ESOs differ from standard options in that they are not traded and holders can not sell the underlying stock to hedge their options exposure. Valuation of ESOs is essentially an incomplete market pricing problem, where a unique price is not available. Due to this reason, the standard Black-Scholes-Merton framework is not easily applicable to the problem. There is no generally accepted theory that can objectively price options under an incomplete market setting. In addition to the incompleteness, ESOs (in practice) are usually exercised differently than those of a standard option because of executives' personal wealth, risk aversion and ability to manage the firm. Firms also reset strike prices of granted options, a practice called repricing, when ESO goes too far out-of-the money. Due to these added complexities, ESO valuation attracts increasing academic attention.

The utility based (or so called certainty equivalent) approach is one of the most used valuation methods in the literature, as it provides a unique price for a given utility function. A certainty equivalent price is the riskless amount of cash that makes the manager's utility indifferent between accepting ESOs and taking the riskless cash.

In other words, utilities provided by the riskless cash and ESOs are equal. For this reason, this approach is also called utility indifference pricing. An advantage of this approach is that it incorporates the manager's subjective risk aversion parameter into the ESO price. Its major disadvantage is that explicit solutions are only available in some special cases, which may be already discussed in the literature (e.g. Henderson and Hobson [2009b]).

For a simple example, assume the manager's utility function without the ESO position is $U(w,t)$, where w is the manager's wealth and t represents time. When the manager has an ESO, her valuation function follows $J(w,s,t)$. The valuation function normally depends on the manager's wealth, risk aversion, and investment choice, where s is the stock price underlying the ESO. The certainty equivalent amount, C, is the cash amount that makes the utility from these two sources equal. It should satisfy the following relationship,

$$U(w+C,t) = J(w,s,t) \tag{1.13}$$

The left side of the equation has no risk, both w and C are certain at time t. So the certainty equivalent amount, C is

$$C = U^{-1}\big[J(w,s,t),t\big] - w \tag{1.14}$$

It is intuitively easy to understand the basics of this approach. The difficult part, however, is finding the value function, $J(w,s,t)$. Various numerical schemes are proposed in the literature to find the value function.

Lambert et al. [1991] are among the first to use the certainty equivalent approach to price ESOs. They compute the ESO value based on a single-period model, and find that the executive's subjective valuation is far lower than the market price (or option cost). Huddart [1994] and Kulatilaka and Marcus [1994] develop binomial tree models in a utility framework to compute certainty equivalents. With no market asset they assume that outside wealth is invested in the risk free asset. Detemple and Sundaresan [1999] solve the utility maximization in the presence of the market asset using a binomial model. In a continuous time framework, Henderson [2005] solves the portfolio choice problem with non-option assets optimally allocated between riskless and market assets. Carpenter et al. [2010] provide a comprehensive study of optimal exercise policy based on the utility maximizing, portfolio choice problem.

In the thesis, the certainty equivalent approach is used as the valuation method. Both Chapter 3 and 4 use a variation of certainty equivalent approach, assuming power law utility function, to value ESOs.[6]

Although pricing of ESOs in the utility framework has been extensively explored, there are very few studies that incorporate the certainty equivalent approach into the principal-agent model to price ESO. The problem lies in the fact that there is no unified general asset pricing model to accommodate features from both streams of literature. Ramakrishnan and Thakor [1984] attempt to incorporate moral hazard into a single-period asset pricing model. They argue that stock valuation should not be exogenous to the ownership structure of the firm, and moral hazard affects asset returns through unobservable managerial actions. They theorize that managerial effort is an unobservable pricing factor of the arbitrage pricing theory of Ross [1976]. Ou-Yang [2005] builds a continuous time asset pricing model with moral hazard. He shows under a principal-agent framework that asset returns still follow a modified CAPM relation. Managerial effort in his model is not a pricing factor, it affects asset price return through influence on systematic risk.

Taxation also plays a major role in the compensation literature. It directly determines which compensation instruments should be used, and when they should be used. This layer of complexity arises because ESOs differ by tax definition. For example, the most prominent ESOs in the US are incentive stock options and unqualified stock options, which have exactly the same option-styled feature. They only differ in tax terms which affect both income to the manager and cost to the firm. This difference in taxation actually has a pricing impact as it affects how the manager and the firm value ESOs. There are many studies that explore the taxation impact of ESO. For example, Babenko and Tserlukevich [2009] find that ESOs tend to be exercised when firms have high taxable income (exercise of ESOs leads to a tax deduction on corporate taxable income in the same tax year), so that granting options can save large US companies an

[6]Apart from the utility based approach, risk neutral valuation is also popular for its tractability. For example, Sircar and Xiong [2007] provide an analytical and flexible valuation framework, in a complete market setting, by assuming executives are risk neutral. Their model also considers resetting and reloading features of ESOs, which are quite difficult to implement in a utility based model. Cvitanic et al. [2008] explore the valuation problem considering ESO early exercise and job termination. In addition, the Statement of Accounting Standard 123(R), which is issued by the Financial Accounting Standard Board (FASB), also requires that firms value options according to "established principles of financial economic theory", which include two methods (lattice and modified Black-Scholes methods) for ESO valuations. Both methods are variations of risk neutral models.

average \$12.6 million every year. Graham et al. [2004] find that option deductions lead to large aggregate tax savings for US firms. Conventional option pricing theory does not consider taxation by assuming managers and firms are tax neutral (which means income and expense of different types have no impact on taxation). In Chapter 3, this assumption is relaxed and options are valued with a tax inclusive model.

1.3 Outline of the thesis

The thesis consists of the introduction followed by three chapters. In Chapters 2, 3, and 4, three different compensation problems are analyzed.

In Chapter 2 we develop a principal-agent model where compensation and capital structure decisions are made simultaneously. Specially, we consider a risk-averse manager whose compensation consists of stock and fixed wages. The manager optimally chooses the level of effort and volatility to maximize her terminal expected utility. Her action has a direct impact on the company value: effort choice influences mean of the firm's return, volatility choice impacts firm's risk. The manager incurs cost by exerting different actions; we assume that the cost of managerial action is convex in both effort and volatility. Different from the literature on risk shifting, we assume that it is both costly to increase and decrease volatility. Without managerial action, the firm's volatility stays at its normal level, therefore it is costly to change the firm's risk from its normal level, either increasing or decreasing it. The compensation contract, which consists of stock and fixed wage, is determined by the shareholders whose objective is to maximize the expected equity value net any payment to the manager. Agency cost arises because the manager is risk averse and may not choose actions that are in the best interests of shareholders. We estimate the agency costs between the manager and shareholders by comparing the first-best and the second-best solution. Under such a setting, we show that leverage has two effects on managerial action. One is to discipline for managerial effort and the other is to substitute for compensation incentives. Such a distinction exists because volatility is endogenous. We also extend the conventional principal-agent model to include debt-like compensation. Our results show that for a given leverage level, rewarding the manager with debt makes her choose higher effort and lower volatility. Since debt compensation does not reduce the risk appetite of shareholders, we hence conclude it is a combination of capital and pay regulation which limits both risk-taking of shareholders and incentives of the manager that can significantly reduce excessive risk-taking.

Chapter 3 contains an analysis of executive (and employees) compensation with tax. We introduce tax decisions, which involve the executive (and employees) optimally selling stock to maximize after-tax terminal utility, into the conventional option pricing model. Stock selling decisions are similar to option exercise decisions. The valuation model essentially has two embedded options, which allow different exercise behaviour for ESOs under different tax schemes. We then analyze whether the tax treatment could explain the widespread use of stock options for employees and executives across different countries. The findings support recent literature that restricted stock is a preferred form of compensation in the US. We also apply the model to a particular tax scheme – section 83b election. This election is only available to restricted stock to accelerate tax payment. Without this election, restricted stock is only taxed when vested (but not taxed when first granted) and at the income tax rate. Subsequent sale of stock is then taxed at a capital gain tax rate. If the manager elects 83b, then she is taxed at income tax rate when stocks are granted. Vesting does not incur any tax, but subsequent sale of stock is taxed at capital gain rate. Our results suggest that 83b election is neither optimal for the manager nor the issuing firm. Because it double-taxes the manager and results in low incentives, it is a very costly instrument to incentivize managers.

Chapter 4 considers the problem of the pricing ESOs with managerial effort. The model is an extension of Henderson [2005], Henderson [2007] and Carpenter et al. [2010]. Effort increases the firm's stock expected return, but is associated with a disutility to the risk-averse, utility-maximizing manager. The manager optimally exerts effort and exercises ESOs. In addition to effort choice, the manager can trade the market asset which is used as a partial hedge of the ESO position. Effort is included in the manager's valuation because she has private information about its value. Effort influences ESO cost only through exercise decisions but not directly through its impact on the stock price return. Because the cost is computed based on complete market dynamic hedging, all securities are priced at riskless rate. Conventional principal-agent models are mostly partial hence are not consistent with general equilibrium models like CAPM and managerial effort generally is not priced under these equilibrium models. For this reason, we adopt similar assumption to Ou-Yang [2005] that effort is not a priced factor, and CAPM does not strictly hold. Similar to a traded option, our results show that the manager's valuation increases with correlation. ESO cost also increases with correlation as the manager delays value destroying early exercise. We also show that the manager's subjective value of ESO is higher than cost when the manager has low wealth, low risk-aversion, and stock has a low volatility. Under these scenarios, the manager's marginal utility is high and effort has a significant impact on the manager's valuation, so that values are higher than cost. These results suggest that managers from large public firms are less likely to value their ESOs higher than cost; and managers of small non-public firms are likely to value their ESOs far higher than cost.

Chapter 5 concludes the thesis with a brief discussion of possible future extensions.

Chapter 2

Incentives, Managerial Risk-Taking and Capital Structure Choice

2.1 Introduction

Stock-based compensation has become an ever more important component of a manager's remuneration package. Hall and Murphy [2002] report that stock options accounted for over 40% of the total pay of the S&P 500 CEOs. As executive compensation raises some controversy, in particular during and in the aftermath of the recent credit crisis, a better understanding of managerial incentives induced by stock-based compensation seems crucial. While there is a large body of literature focusing on either how CEO compensation structure determines firm characteristics (e.g. Jin [2002]) or on the reverse problem, that is, how firm characteristics affect compensation structure (e.g. Coles et al. [2006], Brockman et al. [2010]), relatively little is known about the joint determination of the firm's operating strategy and its compensation structure. The objective of this chapter is to investigate the interaction between managerial incentives and firm characteristics. Specifically, we examine the effect of debt on the design of contracts and its impact on the manager's risk-taking behaviour.

Recent empirical literature suggests that compensation contracts and financing policies are jointly determined, yet to the best of our knowledge very few theoretical contributions have endogenized the capital structure choice in the evaluation of compensation contracts. Most studies treat financing and compensation decisions as independent, e.g. Dittmann and Maug [2007]. Notable exceptions are Cadenillas et al. [2004] and Carlson and Lazrak [2010]; both endogenize leverage and managerial actions. Still, they do not investigate the interaction between incentives and debt choice[7], which is exactly the focus of this chapter. We endogenize firm debt choice and compensation contracts, and determine both simultaneously as suggested in the empirical literature (e.g. Ortiz-Molina [2007]). Debt choice is important in determining the compensation contract because shareholders' return depends on debt which magnifies the manager's action choices. The manager's action, on the other hand, depends on shareholders' choice of compensation contract. So firm debt level, compensation and managerial risk-taking are simultaneously determined.

Specifically, we consider a risk-averse manager whose compensation consists of the firm's stock and a fixed wage. The manager chooses the level of effort and volatility to maximize the expected utility of her terminal wealth. Her action has a direct impact on the company value: the effort choice influences the mean of the firm's return, whereas the volatility choice impacts the firm's risk. The manager incurs a cost by exerting different actions and we assume that the cost of managerial action is convex in both effort and volatility. Unlike other contributions to the literature on risk shifting, we allow for a strictly positive cost of *both* an increase and a reduction of volatility. Without managerial action, the firm's volatility stays at its "normal" level (so it is costly to change the firm's risk from that level). Shareholders determine compensation contract; their objective is to maximize the expected equity value net of any payment to the manager. The agency cost arises because the manager is risk averse and may not choose actions that are in the best interests of shareholders.

Confirming prior studies, our results show that leverage has a similar impact to compensation incentives on inducing managerial effort and risk-taking. Therefore, leverage is a good substitute for incentives. Our results also show that the leverage-incentive relation is mixed, even in a very simple setup where debt is used to increase the size of the firm's assets. The leverage-incentive relationship is negative when volatility cost is low. This result has some empirical support e.g. Rajan and Zingales [1995] and Guay

[7]In Cadenillas et al. [2004] stock has no incentive effect; it is merely used to retain the manager. Carlson and Lazrak [2010] treat compensation as an exogenous choice.

[1999]. However, the relationship is positive when the volatility cost is high. This result is consistent with Lewellen [2006].

Another question we try to answer in this chapter is whether the design of the compensation package can mitigate managers' excessive risk-taking. This is a problem that is particularly important for the financial industry, where a debt-to-equity ratio of 20:1 is not uncommon among investment banks. Academic literature has offered a way of alleviating this problem – rewarding the manager with debt-like compensation, e.g. Sundaram and Yermack [2007] and Edmans and Liu [2011]. We extend the conventional principal-agent model to include debt-like compensation. Our results show that rewarding debt to the manager makes her work harder and take less risk. And leverage has a strictly negative relation with managerial risk-taking. Still, debt compensation results in an even higher leverage choice. Because rewarding debt-like compensation only mitigates managerial risk-taking, it by no means reduces shareholders' risk appetite. Unless there is a mechanism to mitigate risk-seeking behaviour of shareholders, designation of compensation contracts is unlikely to achieve systematic leverage reduction.

The paper closest to ours is Cadenillas et al. [2004] which implements a continuous time model for volatility and effort choice. Our one period model is complementary to theirs with managerial volatility choice. Our model is also similar to Palmon et al. [2008] who introduce limited liability to the shareholders' problem and solve the non-linear programme using simulation. Our model is also an extension of Guo and Ou-Yang [2006] with the leverage choice. Because of its rich setting, our model captures many existing features that are well established in the literature. First, our results show that the incentive-performance relationship is not always as straightforward as suggested by the simple principal-agent model. Second, the incentive-risk relation is not always negative as suggested in the literature, as low volatility cost and high leverage actually encourage risk-taking, which implies a positive incentive-risk relationship. Third, confirming the mixed incentive-leverage relationship observed in the literature (e.g. Florackis and Ozkan [2009] show that the incentive-leverage relationship is not monotonic), we demonstrate that whether incentives and leverage are positively or negatively related depends on the manager's risk aversion, skills and the company characteristics. Fourth, we show that debt is an alternative alignment device used to induce managerial action and manage the firm's overall risk.

Our findings come from considering in the model additional choice variables, namely endogenous volatility and leverage. To summarize, our contributions to the literature are as follows: Firstly, in contrast to Edmans and Liu [2011], where they argue that rewarding with debt can reduce risk-shifting, our results show that debt compensation does not reduce firm risk. This is because debt is endogenous in our model. Even though, for a given leverage level, rewarding with debt makes the manager work harder and take less risk (this is in line with Edmans and Liu [2011]). Shareholders can change leverage levels to offset the manager's risk-reducing choice. The net result is that the firm takes high risk by employing large leverage. In fact, observed leverage level could be even higher if not exogenously constrained (e.g. by bank capital requirement). Since debt-like compensation does not reduce the risk-taking appetite of shareholders, one could question the effectiveness of deferred compensation (which is essentially a debt-like compensation) at curbing excessive risk-taking.

Secondly, we clearly identify two effects of leverage on managerial actions. One is the disciplinary effect for managerial effort; the other is substitution for incentives. Such a distinction exists because volatility is endogenous. When volatility choice is not available to the manager, leverage strictly increases managerial effort. This is because leverage increases the firm's chance of bankruptcy, which can only be avoided by the manager's higher effort choice. Such disciplinary effect also presents when changing volatility is too costly (bottom two plots in Figure 2.5). When volatility is endogenous and can be implemented at low cost, leverage is a substitute for incentives (in our setup, incentives are company stocks), as firm bankruptcy can be prevented by an appropriate volatility choice. Therefore, increases in leverage are not likely to influence managerial actions (top two plots in Figure 2.5), but rather incentives rewarded to the manager.

These results show that controlling (by, e.g., regulation on pay) the manager's incentives are unlikely to reduce her risk-taking choices, as shareholders can maintain managerial risk-taking by increasing the firm's leverage. Alternatively, measures that combine capital requirement and pay regulation (which limits both risk-taking of the shareholders and the manager) can be more effective to reduce risk-taking attitudes.

The remainder of the chapter is organized as follows. In section 2.2, we introduce the model and in section 2.3 we outline the solution methodology. Section 2.4 contains numerical results whereas section 2.5 discusses empirical implications. Section 2.6 concludes.

2.2 The model

The structure of the basic model follows Holmstrom [1979]. There are two times, denoted by t_0 and t_1. The firm's asset value at t_1, X_1, is a function of managerial effort a. In addition to effort a, the manager can influence the company's volatility v (e.g., by choosing a certain operational policy). Both effort and volatility are costly to implement. Managerial effort increases the company's asset value but at the manager's personal cost. We assume volatility choice is also costly because it is costly for the manager to manage the firm's risk. This assumption is different to Cadenillas et al. [2004] where volatility is costless to change. Higher volatility also has a positive impact on the expected asset value. This can be interpreted as a risk premium. We assume that the manager optimally chooses effort, a, and volatility, v, to maximize her expected terminal utility. When the manager's actions are not observable, the compensation package must be chosen based on the rational expectations of shareholders, who anticipate the manager's choice.

Effort and volatility are endogenous variables of the model, which is essential to our understanding of the manager's action choices. We assume shareholders can issue debt and denote the face value of the debt, L. At t_1, the total value of company's equity is,

$$S(X_1) = (1 - C)(X_1 - L - w)^+, \tag{2.1}$$

where X_1 denotes levered firm asset value at maturity, t_1, w is manager's fixed wage, and $C \in [0, 1]$ is fraction of company stock awarded to the manager. The equity value at maturity, $S(X_1)$, is the firm's asset value net of debt and salary. Stock compensation is a convex instrument because the manager's loss is limited (as stock payoff cannot be negative) and her gain is uncapped.

The firm's levered asset value at t_1 equals

$$X_1 = (X_0 + B_0) \exp\left\{ \left[r + a + \alpha(\sigma + v)\right]T + (\sigma + v)\epsilon\sqrt{T} \right\}, \tag{2.2}$$

where $a \in (0, +\infty)$ is manager effort choice.[8] Effort increases the expected asset value, $E(X_1)$. $v \in (-\overline{\sigma}, \infty)$ is the manager's volatility choice; it can take negative values which is interpreted as reducing firm's total risk. σ is the firm's normal risk if there

[8] Although effort can go to infinity, it is never the optimal choice in the model.

is no action to change the firm's volatility. It is the normal level of risk when there is no managerial level intervention. $\overline{\sigma}$ represents maximum volatility level by which can be reduced; we impose the strict inequality $\overline{\sigma} < \sigma$, as there are physical limitations concerning how much risk the manager can reduce. X_0 is the firm's unlevered asset value at t_0, B_0 is the debt value at t_0. r is the risk-free interest rate. α is a positive constant that measures the benefits associated with higher risk. α is the reason the manager would want to increase firm volatility, otherwise the risk averse manager will only reduce firm volatility. Cadenillas et al. [2004] interpret α as the slope of the Capital Market Line, which depends on characteristics of the firm. We adopt the same interpretation here. $T = t_1 - t_0$ is the time interval between the two periods. ϵ is a random variable that takes value 1 and -1. Since X_1 is an exponential function, ϵ is chosen as

$$\epsilon = \begin{cases} 1 & \text{with probability} \quad p \\ -1 & \text{with probability} \quad 1-p \end{cases}, \tag{2.3}$$

where p is

$$p = \frac{1 - e^{-(\sigma+v)\sqrt{T}}}{e^{(\sigma+v)\sqrt{T}} - e^{-(\sigma+v)\sqrt{T}}} \tag{2.4}$$

Probability p is set so that $E[X_1]$ is always equal to $(X_0 + B_0)\exp\Big\{[r + a + \alpha(\sigma + v)]T\Big\}$.[9] In this setting, the manager's volatility choice affects the firm value as well as the probability of each value state, where prior literature assumes one of the two for tractability. For example, John and John [1993] assume that the managerial action only affects the probability of each state.[10] Edmans and Liu [2011] assume it only affects the value of each state. As shown, X_1 is a function of managerial effort and volatility. We assume $\sigma + v$ is the firm's total risk, which includes both systematic and idiosyncratic risk. In this sense, v is total risk choice.

[9] p is chosen to eliminate bias of the exponential function so that volatility only contributes to expected return though risk premium α.

[10] In John and John [1993], value of each state is constant but probability of each state is controlled by the manager.

2.2.1 Manager's problem

The manager is risk averse and has power law utility function $u(x)$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma} \quad , \quad \gamma > 0 \quad \text{and} \quad \gamma \neq 1 \quad , \tag{2.5}$$

where γ is the coefficient of risk-aversion. Utility is additively separable in income and action. This assumption is conventional in the literature. In addition to effort we explicitly introduce volatility as an action variable. $h(a, v)$ denotes the manager's cost function or disutility to effort,

$$h(a, v) = A_1 a^\theta + A_2 |v|^\beta, \quad \theta > 1, \quad \beta > 1, \tag{2.6}$$

where A_1, A_2, are both positive constants. θ and β are larger than 1 so that the cost function is always convex in both effort and volatility. Since it is costly to increase and decrease volatility, absolute value is used to make sure volatility cost is always positive.

The manager tries her best to avoid negative payoffs, as negative payoffs make her utility equal to negative infinity. This is because the manager's payoff will be 0 when the firm fails, as shown on equation (2.8) in the next page. Such a payoff leads to undesired results under the power law utility function. The cost function is similar to the linear case of Guo and Ou-Yang [2006]. It is increasing and convex in both effort and volatility. This cost function assumes that agent does not take any action to change risk (when $v = 0$), the risk of the firm value is simply σ.[11] The lowest value v could take is $-\overline{\sigma}$, and we assume the $-\overline{\sigma} < \sigma$ hold. This is a rather practical assumption as certain risks are not possible to reduce.

In our setup, it is costly to increase (positive value of v) and decrease (negative value of v) volatility. This volatility choice has a similar effect to hedging. For example, Bettis et al. [2001] assume that firms can change volatility by costly hedging. There is always cost involved when the manager varies the firm's basic risk level. This assumption is practically important, because most financial firms spend large sums to manage their risk. The manager incurs volatility cost because altering the firm's risk level requires

[11] We could also consider a family of Cobb-Douglas-type cost functions $h(a, v) = ka^\theta v^\beta$. However, interdependence between a and v results in the manager choosing either a or v of zero which makes the cost function always equal to zero. In that case, the manager can choose the other decision variable at infinity which makes the maximization problem unsolvable. Guo and Ou-Yang [2006] avoid this problem by assuming firm risk is infinite when managerial action is zero.

costly effort. Identifying the right risky project to invest in is a pressurized process that requires skills. Engaging in hedging strategies is also costly because the manager incurs a cost maintaining such a hedging programme. In this sense, the volatility cost is very similar to the effort cost.

Guo and Ou-Yang [2006] consider a similar setup, but they only allow for the case where it is costly to reduce risk. We argue that volatility cost is increasing on both sides, that is when either increasing or reducing risk. In addition to linear contracts we consider the convexity of stock compensation. Given the manager's utility and cost function, her objective function is given by,

$$\max_{a,v} E\Big\{u\big[M(X_1)\big] - h(a,v)\Big\}, \tag{2.7}$$

where $M(X_1)$ is the manager's wealth function at t_1, given by

$$M(X_1) = \begin{cases} 0 & \text{if} \quad X_1 \in [0, L] \\ X_1 - L & \text{if} \quad X_1 \in (L, L+w] \\ C(X_1 - L - w) + w & \text{if} \quad X_1 \in (L+w, +\infty) \end{cases} \tag{2.8}$$

X_1 is the firm's levered asset value defined in equation (2.2). The manager's action has direct impact on her compensation through effort and volatility choice. In the event of liquidation, we assume the following residual claim structure: debtholders have the highest seniority over residual claim, the manager or employees of the firm following next, shareholders following last. In the first case the firm asset value is lower than the face value of the debt. The firm liquidates and the manager receives nothing. Shareholders' value also goes to zero. When L is equal to zero, the firm is unlevered and value of equity is equal to the firm value.

The second case occurs when the firm asset value is higher than the debt value but this is insufficient to pay the manager's fixed wage w. Hence, the manager receives $X_1 - L$. Within the third range, the firm value is large enough both the manager's fixed wage w and stock grant C can be paid.

2.2.2 Shareholders' problem

We assume that shareholders decide on the managerial compensation policy. In addition, we also assume that shareholders are in control of the firm's leverage, L. Stulz [1990] and Cadenillas et al. [2004] also make similar assumptions about leverage choice, whereas Carlson and Lazrak [2010] assume that the manager decides leverage. The shareholders' objective is to maximize the firm's outside equity value

$$\max_{w,C,L} E\Big[S(X_1)\Big], \tag{2.9}$$

where $S(X_1)$ is given,

$$S(X_1) = \begin{cases} 0 & \text{if} \quad X_1 \in [0, L+w] \\ (1-C)(X_1 - L - w) & \text{if} \quad X_1 \in (L+w, +\infty) \end{cases} \tag{2.10}$$

$S(X_1)$ is firm equity value given the firm's asset value X_1. The first range represents the case where the firm asset value is lower than the combined face value of the debt and the managerial wage, w. The firm asset value is the residual claim to the debtholder, which is equal to X_1. The second range corresponds to the case where the firm asset value is large enough for both the manager's fixed wage w and debt repayment L.

The shareholders' problem is subject to the following condition,

$$\text{s.t.} \quad (a, v) \in \arg\max_{a,v} E\Big\{u\big[M(X_1)\big] - h(a, v)\Big\} \tag{2.11}$$

$$E\Big\{u\big[M(X_1)\big] - h(a, v)\Big\} \geq u(H_0) \tag{2.12}$$

These are participation and incentive compatibility constraints. Participation constraint (2.12) represents the minimum expected utility the manager is willing to work for the firm. Here H_0 is the certainty equivalent that represents the manager's external opportunities. In a general equilibrium model, H_0 would be determined by the competitive market. The second condition (2.11) is the incentive compatibility constraint which assumes that the manager works optimally to maximize her utility, given shareholders' choices. This constraint is essential to the second-best problem since shareholders cannot observe (hence enforce) the manager's action. The first-best problem is discussed in the next section.

The timeline of decisions is as follow: at the beginning of t_0, shareholders determine the optimal decision variables (w, C, L) to maximize firm's equity value. In determining these variables, shareholders anticipate the effect of these variables on the manager's action choices and the manager's expected utility. After observing firm leverage and the compensation package chosen by shareholders, the manager optimally chooses her effort and volatility to maximize expected terminal utility. If expected utility is higher than her reservation utility H_0, she accepts the offer. The game ends at t_1 when the firm liquidates, and shareholders pay off the manager and debtholders.

2.2.3 Equal seniority of managerial and debtholders' claims

The existing principal-agent literature commonly assumes that managers are compensated exclusively with cash and equity. Our analysis above makes similar assumptions. While empiricists find CEOs have a large amount of defined benefit pensions (e.g. Bebchuk and Jackson [2005], Sundaram and Yermack [2007]), which are unsecured obligations similar to a risky debt, these benefits would yield an equal claim with other creditors in the event of bankruptcy. This is a debt-like feature, thus pension benefits are debt-like compensation. Empirical studies (see Kabir et al. [2010] for another example) find a widespread use of debt-like compensation, but theories on this are almost non-existent; a notable first step is Edmans and Liu [2011] who argue that inside debt is a more effective solution to the risk-shifting problem than bonuses. In the previous setup, we limit the compensation package to equity and cash. A natural extension of the risk-shifting problem is directly introduce debt-related pay to the manager's compensation package, so that the manager does not only work to maximize probability of the firm's survival but also value of the firm at insolvency. The latter is very important to debtholders and reducing the risk-shifting problem, and it can easily be imposed in debt covenants. This approach also relaxes the riskless debt assumption where the manager does not work if the firm has a positive probability of bankruptcy. The managerial utility is no longer a negative infinity at bankruptcy, because her payoff is non-zero, as the debt compensation ensures the manager has a positive payoff in the event of firm failure.

Therefore, this alternative way of compensation results in the manager's payoff at t_1 being as follows,

$$M(X_1^D) = \begin{cases} eX_1^D & \text{if} \quad X_1^D \in [0, L] \\ C(X_1^D - L) + eL & \text{if} \quad X_1^D \in (L, +\infty) \end{cases} \tag{2.13}$$

where $M(X_1^D)$ represents the manager's payoff when she is rewarded with debt, in which case her total pay package consists of two components – equity and debt, slightly different from the equity-cash combination shown in equation (2.8). C denotes the fraction of the manager's equity reward and e represents the fraction of the manager's debt compensation. X_1^D denotes the firm's asset value when rewarding the manager with debt compensation,

$$X_1^D = (X_0 + B_0^D) \exp\left\{ \left[r + a + \alpha(\sigma + v)\right]T + (\sigma + v)\epsilon\sqrt{T} \right\}, \tag{2.14}$$

This value is very similar to X_1 in equation (2.2). The only difference is in the initial value of debt, B_0^D, which is now risky and needs to be valued with rational expectation. As the debt compensation package allows the firm's debt to be risky, so the firm may have a positive probability of failure. At maturity t_1 the debt value B_1^D is given by,

$$B_1^D = \begin{cases} (1-e)X_1^D & \text{if} \quad X_1^D \in [0, L] \\ (1-e)L & \text{if} \quad X_1^D \in (L, +\infty) \end{cases} \tag{2.15}$$

where L is face value of the risky debt. Since debtholders have the highest priority on the firm's residual claim, debtholders always receive $\min[(1-e)L, (1-e)X_1^D]$. It is worth noting that debtholders are fully aware of the manager's compensation contract. They know part of the manager's compensation comes from debt so that they correctly discount this value into the debt price. At t_0, the debt value is simply the discounted expected value of B_1^D,

$$B_0^D = e^{-rT} E[B_1^D(X_1^D, L)] \tag{2.16}$$

where $B_1^D(X_1^D, L)$ indicates dependence on both X_1^D and L. Since there is no interest payment, B_0^D is a zero-coupon bond. Solving this non-linear equation with respect to B_0^D identifies a unique B_0^D. We assume no tax advantage and no bankruptcy cost associates with debt financing.

The shareholders' objective under the debt compensation scheme is also very similar to equation (2.10) – they maximize the firm's equity value,

$$\max_{e,C,L} E\left[S(X_1^D)\right] \tag{2.17}$$

Where $S(X_1^D)$ is the firm's equity value when rewarding the manager with debt; at time t_1 it is given as follows,

$$S(X_1^D) = \begin{cases} 0 & \text{if} \quad X_1^D \in [0, L] \\ (1-C)(X_1^D - L) & \text{if} \quad X_1^D \in (L, +\infty) \end{cases} \tag{2.18}$$

Standard participation and incentive compatibility constraints apply here, which are the same as in equations (2.12) and (2.11). The complete payoff structure is reported in Table 2.1. As shown, the manager, shareholders and bondholders are actually playing a zero sum game with respect to the firm's asset value, e.g. X_1 or X_1^D.

Table 2.1: The table reports payoffs to the manager, shareholders and bondholders. The manager, shareholders and bondholders are playing a zero sum game, as their payoffs sum up to the firm's total asset value, equal to either X_1 or X_1^D.

Firm asset value, X_1 or X_1^D	$[0, L]$	$(L, L+w]$	$(L+w, +\infty)$
The manager is rewarded with equity, C, and fixed wage, w,			
The manager's payoff, $M(X_1)$	0	$X_1 - L$	$C(X_1 - L - w) + w$
Shareholders' payoff, $S(X_1)$	0	0	$(1-C)(X_1 - L - w)$
Bondholders' payoff, B_1	X_1	L	L
Total	X_1	X_1	X_1
The manager is rewarded with equity, C, and debt, e,			
The manager's payoff, $M(X_1^D)$	eX_1^D	$C(X_1^D - L) + eL$	$C(X_1^D - L) + eL$
Shareholders' payoff, $S(X_1^D)$	0	$(1-C)(X_1^D - L)$	$(1-C)(X_1^D - L)$
Bondholders' payoff, B_1^D	$(1-e)X_1^D$	$(1-e)L$	$(1-e)L$
Total	X_1^D	X_1^D	X_1^D

2.3 Solution methodology

In this section, we solve the manager's and shareholders' problems. Since the problem involves two parties, we solve the problem in two stages. In the second stage, we solve the problem for the manager. The solution of the manager problem is then used to solve the shareholders' problem. Both first-best and second-best are explored. Our problem does not admit a closed formed solution as mentioned previously this is largely due to shareholders' payoff function (which is exponential) and a power law utility.[12]

2.3.1 Manager's choices

First, we derive the solution to the manager's problem as described in the previous section. We ignore the shareholders' problem here, and treat all other shareholders' decision variables as given. In isolation of the shareholders' problem, the manager chooses her effort and volatility to maximize her terminal expected utility. From equation (2.7), the first-order conditions are:

$$\begin{aligned} &E\Big\{CX_1T\big[C(X_1-L-w)+w\big]^{-\gamma}\Big\}-\theta A_1a^{\theta-1}=0\\ &E\Big\{Y\frac{(X_1-L-w)^{1-\gamma}}{1-\gamma}+CX_1Z\big[C(X_1-L-w)+w\big]^{-\gamma}\Big\}-\beta A_2|v|^{\beta-1}=0 \end{aligned} \tag{2.19}$$

These are the first-order conditions in the region $X_1 \in (L+w, +\infty)$. When $X_1 \in (L, L+w)$, FOC is

$$\begin{aligned} &E\Big[X_1T(X_1-L)^{-\gamma}\Big]-\theta A_1a^{\theta-1}=0\\ &E\Big[Y\frac{(X_1-L)^{1-\gamma}}{1-\gamma}+X_1Z(X_1-L)^{-\gamma}\Big]-\beta A_2|v|^{\beta-1}=0 \end{aligned} \tag{2.20}$$

where X_1 is defined in equation (2.2), Z is a random variable that equals,

$$Z=\begin{cases} \alpha T+\sqrt{T} & \text{with probability} \quad p\\ \alpha T-\sqrt{T} & \text{with probability} \quad 1-p \end{cases} \tag{2.21}$$

[12]Cadenillas et al. [2004] use a simple payoff function which does not accommodate seniority structure of debt. They do not investigate incentives effect of leverage.

and Y is equal to

$$Y = \begin{cases} \frac{e^{(-v-\sigma)\sqrt{T}}}{e^{(v+\sigma)\sqrt{T}} - e^{(-v-\sigma)\sqrt{T}}} - \frac{\left(1-e^{(-v-\sigma)\sqrt{T}}\right)\left(e^{(v+\sigma)\sqrt{T}} + e^{(-v-\sigma)\sqrt{T}}\right)}{\left(e^{(v+\sigma)\sqrt{T}} - e^{(-v-\sigma)\sqrt{T}}\right)^2} & \text{with probability} \quad p \\ -\frac{e^{(-v-\sigma)\sqrt{T}}}{e^{(v+\sigma)\sqrt{T}} - e^{(-v-\sigma)\sqrt{T}}} + \frac{\left(1-e^{(-v-\sigma)\sqrt{T}}\right)\left(e^{(v+\sigma)\sqrt{T}} + e^{(-v-\sigma)\sqrt{T}}\right)}{\left(e^{(v+\sigma)\sqrt{T}} - e^{(-v-\sigma)\sqrt{T}}\right)^2} & \text{with probability} \quad 1-p \end{cases} \tag{2.22}$$

Equations (2.19) and (2.20) are non-linear, because of the exponential function in equation (2.2). The problem can be easily solved using iteration methods, the only difficulty is making a starting point that results in $X_1 \in (L + w, +\infty)$. To demonstrate the manager's optimal choice, Figure 2.1 plots an optimal solution for the manager problem with base case parameters, as a function of incentives, C.

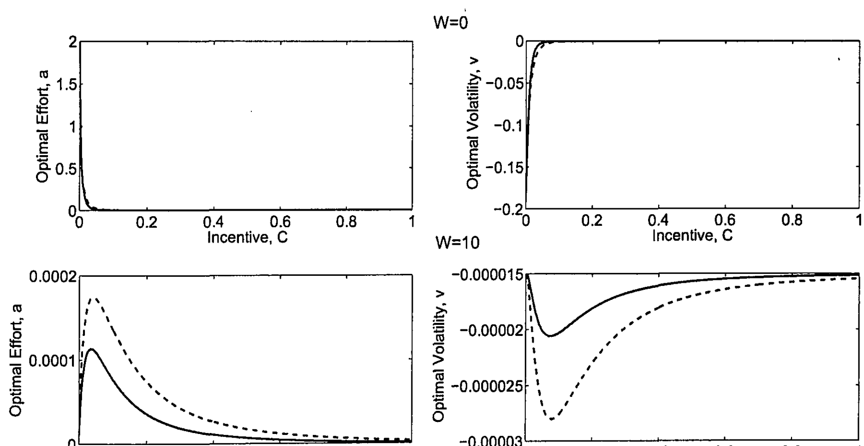

Figure 2.1: Optimal effort and volatility. Optimal effort and volatility with base case parameters. The figure plots managerial optimal effort and volatility with varying level of C, which denotes stock awarded to the manager. Parameter values are those of the base case: $X_0 = 100$, $A_1 = 0.5$, $A_2 = 2$, $T = 1$, $\alpha = 0.05$, $\sigma = 0.3$, $\theta = 2$, $\beta = 2$, $\gamma = 4$, $H_0 = 10$, $r = 0.03$, $T = 1$. The solid line represents leverage, $L = 0$, the dashed line represents leverage, $L = 50$. The upper two graphs plot optimal effort and volatility with low level of fixed wage, $w = 0$. The lower two graphs plot with high level of fixed wage, $w = 10$.

2.3.1.1 The manager's choices for base case

Managerial effort does not always increase with incentives. On the contrary, when the manager does not have any fixed wage, effort decreases dramatically with more incentives, as shown in the upper left graph of Figure 2.1. From first order condition (2.19), when $w = 0$ the optimal effort and volatility are a hyperbolic function of incentives C. Optimal effort strictly decreases with more incentives and volatility strictly increases with incentives. Maximum optimal effort occurs at $C = 0$ at which point effort $a = +\infty$, this property is special to the power law utility function, since the utility function equals negative infinity at 0. A similar result is obtained for the optimal volatility which reaches minimum at $C = 0$. It is worth noting that we constrain volatility choice with minimum of $-\overline{\sigma}$ as we do not allow the actual volatility, $v + \sigma$, to fall to zero. There is a limit as to how much volatility the manager is able to reduce. The combined effect of optimal effort and volatility is an extremely hard working manager that takes very little risk. This result contradicts the conventional thinking as it suggests the best incentives strategy is 'zero compensation' ($w = 0$ and $C = 0$), because the firm has no risk and has positive infinite drift. We point out this is not optimal when the shareholders' problem is presented. 'Zero compensation' is never optimal even though the manager works the hardest, because the extra condition (equation2.12) presents in the principal-agent model. 'Zero compensation' violates participation constraint and the manager simply does not work for the firm. The hyperbolic relation between incentives and effort does not hold when $w > 0$, in which case optimal effort and volatility is a concave function of incentives. Both effort and volatility admit a positive maximum with positive incentives. The results suggest that fixed wage is an important component of managerial compensation, since the manager's action changes radically when her fixed wage is 0.

Leverage plays a crucial role in determining the manager's action. Leverage magnifies the manager's action choice. Figure 2.1 shows a similar action pattern for both leverage levels, but action choices are in much larger magnitude when leverage is high. For example, when leverage is high, optimal effort is higher and optimal volatility is much lower suggesting the manager chooses higher effort and lower volatility to avoid default. When leverage is low, the manager chooses higher volatility and lower effort.

One obvious observation is that volatility is low when effort is high, and vice versa. Since optimal effort and volatility are simultaneously determined, depending on the relative cost of implementing each one, the manager may choose a high level of effort and a low level of volatility to maximize her expected utility. For example, when $C = 0.5$, the bottom two graphs in Figure 2.1 show relatively low levels of effort and a high level of volatility. From the shareholders' point of view, the best option to maximize expected equity value is to choose moderate levels of fixed wage and low levels of incentive so that the manager chooses a moderate level of effort and a positive level of volatility. The optimal levels of C and w are reported in the next section.

Figure 2.1 plots C up to 1 or 100%, in which case the manager owns the whole company. When $C = 1$, the manager is also the sole shareholder in the firm. The manager's action does not change because she is still risk-averse. The manager is not allowed to become a risk-neutral shareholder in our setup. It is worth noting that the optimal effort shown in Figure 2.1 is not the manager's actual observable performance, which is measured based on shareholders' return. It only shows, for given compensation packages, how the manager makes her effort choice. This feature is different to the standard literature which assumes the manager can only influence the mean of the firm asset value e.g. Dittmann and Maug [2007]. In the absence of volatility choice, high levels of effort always translate into high performance. In our model the firm's performance also depends on the manager's volatility choice, low effort choice does not necessarily translate to low observable performance. It may well be the case where high effort choice results in low performance (e.g. Guo and Ou-Yang [2006]).

2.3.2 First-best problem

As a benchmark for comparison, we also solve the first-best problem. In the first best case, the shareholders' can observe the manager's choice so that they can contract on the manager's actions; their objective function is

$$\max_{C,L,w,a,v} E\Big[S(X_1)\Big] \tag{2.23}$$

subject to

$$E\Big\{u\big[M(X_1)\big] - h(a,v)\Big\} \geq u(H_0) \tag{2.24}$$

where $S(X_1)$ and $M(X_1)$ are defined in equations (2.8) and (2.10). The incentive compatibility constraint is absent in this problem since the shareholder can issue a forcing contract to make sure the manager chooses a proper action. The problem is first-best as there is no moral hazard, and agency cost is equal to 0. Given the payoff structure of $M(X_1)$, $S(X_1)$ and utility function, a closed-form solution is very unlikely. It is worth mentioning that moral-hazard problem of this type are generally difficult to solve. The mathematical problem has received some academic attention (Prescott [2004], Su and Judd [2007] and Armstrong et al. [2007]).

2.3.3 Second-best problem

When managerial actions cannot be contracted upon, the solution is second-best. We outline here a numerical method that solves the optimal second-best problem. In the second stage of the problem the manager observes the compensation contract and firm leverage. If the contract offers expected utility higher than $u(H_0)$, the manager accepts the contract and optimally chooses the level of a and v.

In the first stage of the problem, shareholders choose the compensation contract (w, C) and the firm leverage L. We assume that shareholders can observe the value of X_1, and impose rational expectations. So shareholders correctly foresee the second stage managerial effort and volatility choice. It is not optimal for the shareholders to choose contract variables that provide the manager with less utility than $u(H_0)$, in which case the manager does not take the job. It is, however, optimal to choose a combination of C and w that corresponds to reservation utility, $u(H_0)$. In that case, the cost to the shareholders is minimized. The manager is indifferent between all these contract packages as all combinations provide her with at least the reservation utility. The solution seeking procedure is as follows: for a given leverage, L, vary combinations of C and w while maintaining the utility provided to the manager at $u(H_0)$. For each combination of C and w, the second stage problem in equation (2.19) and (2.20) is solved and shareholder value computed using equation (2.10). The optimal C and w are chosen to maximize shareholders' outside equity value. This procedure is then repeated for all leverage choices, L. The optimal leverage, L^*, is then chosen to maximize the outside equity value.

Essentially, the solution search procedure plots manager participation constraint in the shareholders' value space. The optimal solution is merely the point on the participation constraint that maximizes the shareholders' expected value. Figure 2.2 presents an illustration and plots manager participation constraint with two endogenous variables.

2.4 Numerical results

In choosing the appropriate parameters for the numerical results we try to approximately conform to Cadenillas et al. [2004] and Guo and Ou-Yang [2006]. We choose the firm's initial unlevered asset value, X_0, equal to 100 which is also used in Cadenillas et al. [2004]. We choose the coefficients of the effort function A_1 and A_2 to be 0.5 and 2 which is close to values used in Figure 11 of Guo and Ou-Yang [2006]. Effort and volatility cost parameters, θ and β, are both set to 2, which is used in Figure 12 of Guo and Ou-Yang [2006]. Time is chosen to be $T = 1$. The interest rate, r, is set to 0.03. The firm's initial volatility, σ, is set to 0.3 which is roughly the median volatility of executive stock option issues from Carpenter [1998]. The manager's coefficient of risk averse, γ, is assumed to be 4, following Kahl et al. [2003]. The manager's reservation wage, H_0, is 10.

As discussed in section 2.3, when more stock is awarded, the manager's fixed wage is reduced. The relevant amount of cash to be deducted from the fixed wage should provide same level of utility as that of the additional stock. In this scenario, the manager is indifferent between additional stock and the higher fixed pay. Optimally, shareholders choose a compensation package so that it just generates enough utility for the manager to participate. If the manager's reservation utility is $u(30)$ (which means $H_0 = 30$), then the compensation package must be such that stock awarded plus fixed wage provide a total utility of $u(30)$.

As a benchmark, we first present the solution to the first-best problem (see Table 2.2). For the parameters reported, it is optimal to reduce volatility which is equivalent to negative v. This is because shareholders trade off volatility for more leverage and effort. Since incentivizing managerial work is not important in the first-best case, shareholders choose to reduce volatility and rely on managerial effort to generate return.

The first-best problem admits multiple solutions (see left plot in Figure 2.3). This is because incentives are not important in the first-best problem as shareholders can enforce the manager's effort. The combination of C and w in Table 2.2 is just one set

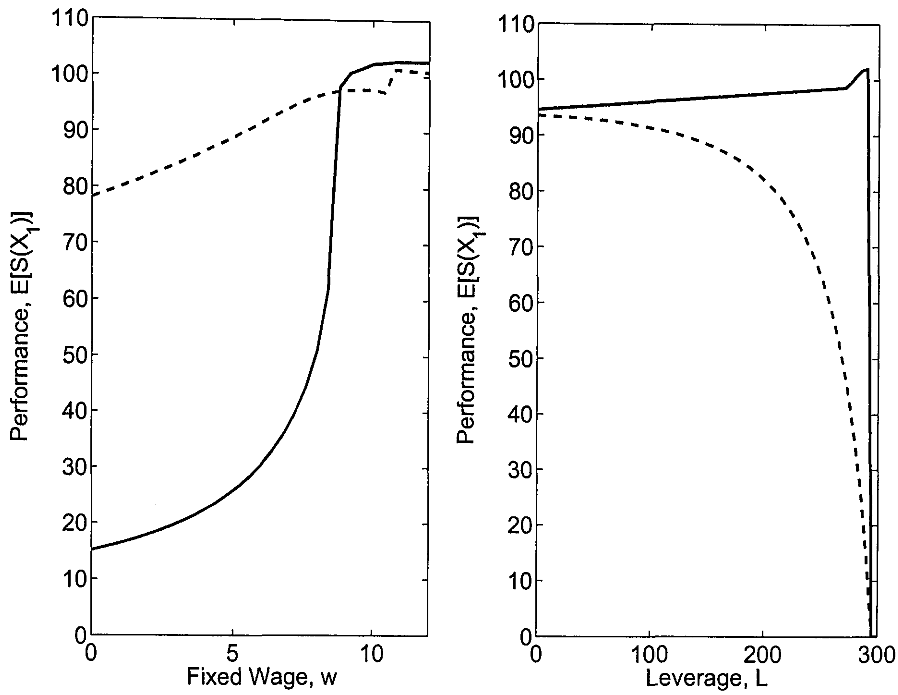

Figure 2.2: Second-best solution. The figure plots the manager's participation constraint (PC) with two pairs of endogenous variables. For each level of W. C is adjusted so that PC always binds. The optimal solution is obtained at $C^* = 0.71\%$, $w^* = 10.72$ and $L^* = 288.40$ with shareholders value $E[S(X_1)] = 102.67$. Figure on the left plots PC at $L = 288.40$ (the solid line) and $L = 200$ (the dashed line), the maximum is attained at $w = 10.72$. Figure on the right plots PC at $w = 10.72$ (the solid line) and $w = 2$ (the dashed line), the maximum is attended at $L = 288.40$.

of solutions on the shareholders' indifference curve. For an easy comparison, we fix the incentive, C, at zero so that the manager's reservation utility is entirely provided from the fixed wage. Results in Table 2.2 are expected. Interestingly, β and θ have a similar impact on shareholders' effort and volatility choices. As effort is increasing in both β and θ, whereas volatility is decreasing in both variables, it seems effort and volatility cost have similar impacts on shareholders' choices. When $\beta = 8$ and 10, the optimal volatility equals -0.2. This is merely the maximum amount of volatility shareholders can reduce.

Table 2.2: The table reports the first-best solution for various exogenous parameters. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $\overline{\sigma} = 0.2$, $A_1 = 0.5$, $A_1 = 2$. a and v is effort and volatility. C is optimal stock ownership awarded to the manager. w is fixed wage awarded to the manager. L is face value of debt. $E[S(X_1)]$ represents expected equity value. Shareholders maximize expected equity value with respect to five decision variables: managerial effort, a; firm volatility, v; stock ownership, C; fixed wage, w; and leverage, L.

Changing Parameter	a 10^{-4}	v 10^{-4}	C	w	L	$E[S(X_1)]$
Base Case	173.8	-1.92	0.00%	12.63	285.10	103.57
γ						
3	827.3	-52.5	0.00%	20.93	379.47	133.99
5	29.6	-0.01	0.00%	10.64	273.98	99.38
6	3.95	0.00	0.00%	10.17	272.59	98.79
7	0.37	0.00	0.00%	10.09	272.41	98.73
T						
0.25	121.1	-1.03	0.00%	11.00	573.94	94.47
0.50	148.2	-1.48	0.00%	11.66	402.99	97.66
3	208.6	-2.42	0.00%	15.28	174.16	127.31
5	221.9	-2.55	0.00%	17.37	147.41	154.24
α						
0.07	174.9	-1.85	0.00%	12.76	293.03	106.28
0.09	176.1	-1.78	0.00%	12.89	301.33	109.13
0.12	177.8	-1.69	0.00%	13.10	314.55	113.67
0.15	179.6	-1.63	0.00%	13.32	328.78	118.56
β						
4	177.5	-505.8	0.00%	13.16	357.56	104.45
6	179.4	-1857	0.00%	16.41	916.02	111.90
8	218.6	-2000	0.00%	16.79	1150.52	121.51
10	220.6	-2000	0.00%	16.78	1154.32	121.90
θ						
3	779.8	-5.22	0.00%	16.79	372.66	134.04
4	1530	-8.50	0.00%	22.26	603.37	213.45
5	2269	-12.2	0.00%	34.12	1407.67	490.64
5.5	2618	-14.3	0.00%	54.32	3552.99	1231.6

2.4.1 Agency costs

Now we are ready to present the solution to the general second-best problem. Table 2.3 presents the optimal solution for the first-best and second-best problem with various

Table 2.3: The table reports the second-best solution for various exogenous parameters. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $\overline{\sigma} = 0.2$, $A_1 = 0.5$, $A_1 = 2$. a and v is manager's optimal effort and volatility choice. C is optimal stock ownership awarded to the manager. w is manager's fixed wage. L is face value of debt. $E[S(X_1)]$ represents expected equity value. Shareholders maximize expected equity value with respect to stock, C, fixed wage, w, and leverage, L.

Changing Parameter	a 10^{-4}	v 10^{-4}	C	w	L	$E[S(X_1)]$
Base Case	130.7	-30.7	0.71%	10.72	288.40	102.67
γ						
3	723.2	-169.1	1.87%	14.79	391.44	130.44
5	16.8	-3.97	0.23%	10.01	274.70	99.19
6	1.86	-0.44	0.19%	9.94	273.29	98.76
7	0.17	-0.04	0.06%	9.94	272.42	98.72
T						
0.25	118.8	-6.76	0.13%	10.38	577.83	93.46
0.50	120.3	-16.5	0.43%	10.43	406.72	96.95
3	135.9	-32.3	0.97%	11.90	176.73	125.96
5	158.7	-50.1	1.54%	12.79	150.52	152.29
α						
0.07	133.1	-30.7	0.74%	10.74	296.32	105.38
0.09	135.5	-30.5	0.75%	10.77	304.60	108.21
0.12	139.3	-30.4	0.79%	10.80	317.84	112.73
0.15	143.2	-30.3	0.83%	10.84	332.12	117.61
β						
4	0.34	31.3	0.09%	9.91	269.20	98.74
6	0.22	275.6	0.06%	9.94	245.21	98.80
8	0.20	784.4	0.06%	9.94	205.28	98.86
10	0.14	1370	0.05%	9.95	170.14	99.08
θ						
3	743.1	-19.53	1.25%	12.84	375.63	133.19
4	1509	-16.26	1.40%	15.11	605.56	212.75
5	2263	-15.57	1.32%	18.82	1409.36	490.12
5.5	2616	-15.63	1.05%	23.64	3554.20	1231.13

ranges of exogenous parameters; the first rows are results for the base case discussed previously. Comparing results between Tables 2.2 and 2.3, shareholders' value $E[S(X_1)]$ is slightly higher in the first-best case, around 2% higher for most parameters. Optimal incentives, C, reported for the second-best are around 1%, depending on exogenous

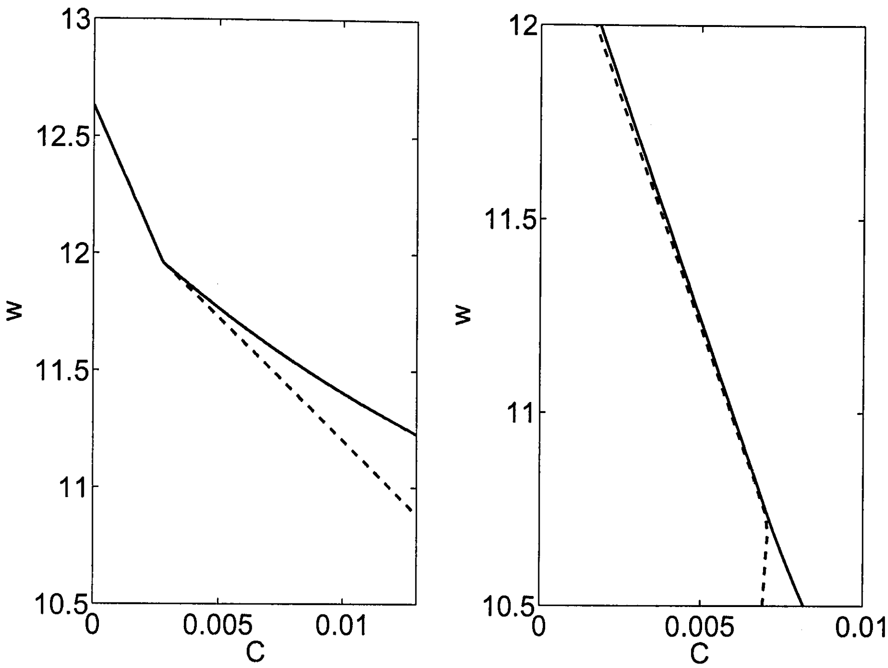

Figure 2.3: Indifference curve. This figure plots the indifference curves of the manager and shareholders. The figure plots manager participation constraint (PC) and shareholder indifference curve in C and w space. The left figure is the first-best problem; the right figure is the second-best. The solid lines are the manager's PC and the dashed lines are the shareholders' indifference curves. Exogenous parameters are those of the base case.

parameters. These results are quantitatively similar to Suntheim [2010], who reports bank managers' average ownership is 0.23%. For some parameter values, incentives are very small, less than 0.1%. This is because shareholders only reward incentives to managers who are not very risk averse ($\gamma < 5$). Unlike many studies that allow for negative stock ownership, e.g. Feltham and Wu [2001] and Dittmann and Maug [2007], we restrict both incentive and fixed wages to be positive.

2.4.2 Incentive-performance relation

Since our model does not have a closed formed solution, we draw inference from our numerical results. Earlier literature proposes a positive incentive-performance relation (performance is measured by firm's total asset value after deducting compensation to the manager and debt to bondholders), because incentives always induce positive effort.

In the absence of volatility choice, it implies a positive incentive-performance relation. Our results also show the positive incentive-effort relation as higher incentives always lead to a higher effort choice (e.g. first section of Table 2.3 show effort and incentives decreases with more risk aversion, γ). However, the incentive-performance relationship is not always positive. For example a higher value of β results in higher performance but lower incentives. It suggests a negative incentive-performance relation. Since managerial performance also depends on volatility choice, which shows no persistent relation with performance, it is hard to infer any quantitative relation between incentive and performance.

Such a nonlinear relation is not present in the classical model where both incentives and performance can go to infinity.[13] This problem only exists when volatility is endogenous. Therefore, we also report results when volatility choice is not available (see Table 2.4, 2.5). These results additionally support this argument, as the incentive-performance relation is positive for all variations of exogenous variables except for θ. Since θ can also measure the manager's skills (as high θ means the manager can exert more effort with less cost), the manager with high value of θ would have higher reservation wage if this is a full equilibrium model. To show this point, we isolate the impact of endogenous leverage on the incentive-performance relation and report results when leverage is fixed at zero (see Table 2.7). The incentive-performance relation is positive for the three reported exogenous variables γ, T, and α. But it is not monotonic in θ. When the manager is very skilled, optimal incentives are simply bounded by the participation constraint.[14] In this sense, optimal incentives do not increase even when the manager is more skilled. So the incentive-performance relation is not positive. Fully endogenous θ and H_0 would solve the problem, and we expect the same positive incentive-performance relation when volatility is not endogenous.

Table 2.4: First-best solution with no volatility choice. The table reports the first-best solution for various exogenous parameters. Other exogenous parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $A_1 = 0.5$, $A_1 = 2$. a is optimal effort choice. C is optimal stock ownership awarded to the manager. w is fixed wage awarded to the manager. L is face value of debt. $E[S(X_1)]$ represents the expected outside equity value. Shareholders maximize the expected outside equity value with respect to four decision variables: managerial effort, a; stock ownership, C; fixed wage, w; and leverage, L.

Changing Parameter	a 10^{-4}	C	w	L	$E[S(X_1)]$
Base Case	173.8	0.00%	12.63	284.88	103.57
γ					
3	830.7	0.00%	20.73	370.25	133.51
5	29.6	0.00%	10.64	273.98	99.38
6	3.61	0.00%	10.17	272.59	98.79
7	0.37	0.00%	10.09	272.42	98.73
T					
0.25	121.1	0.00%	11.00	573.72	94.47
0.50	148.1	0.00%	11.65	402.76	97.65
3	208.6	0.00%	15.28	173.96	127.30
5	221.9	0.00%	17.36	147.21	154.22
α					
0.07	174.9	0.00%	12.76	292.81	106.28
0.09	176.1	0.00%	12.89	301.11	109.13
0.12	177.8	0.00%	13.10	314.33	113.67
0.15	179.6	0.00%	13.31	328.56	118.56
θ					
3	780.1	0.00%	16.77	371.76	133.99
4	1531	0.00%	22.21	600.32	213.09
5	2272	0.00%	33.88	1389.44	486.65
5.5	2623	0.00%	53.32	3433.70	1196.97

2.4.3 Effect of leverage

Previous theoretical literature investigating manager's risk attitude largely ignores the impact of firm leverage, or treats leverage as exogenously given. We explicitly endogenize leverage in our model. When leverage is endogenous, the relationship between leverage and other endogenous variables is driven by a common exogenous variable. Table 2.3 reports optimal solutions for various exogenous variables. Leverage has a mixed relationship with volatility across different variations of exogenous variables (column 6 of Table 2.3). When volatility cost is low, optimal volatility shows a negative relationship with leverage. Because low volatility cost implies that the manager chooses high volatility, shareholders anticipate the manager's high volatility choice and reduce leverage to avoid potential bankruptcy. The optimal leverage almost halves between $\beta = 4$ and 10. The optimal volatility is around 0.14 when $\beta = 10$, and is quite significant compared to the base case result of -0.004. The optimal leverage is only chosen so that the firm has a zero probability of bankruptcy. Shareholders always take just enough leverage so that the firm has no probability of failure. This mechanism also drives the mixed incentive-performance relation as we showed in the previous subsection. On the other hand, effort is positively related to both leverage and incentives, a relation that is well understood in the principal-agent literature, because both incentives and leverage have same effects on the manager's effort choice.

2.4.4 Effect of endogenous volatility

To investigate effect of endogenous volatility, we also report (Tables 2.4, 2.5 and 2.7) results for the conventional principal-agent model where the manager can only exert effort. The results support the argument that the manager's tradeoff utility between effort and volatility. With endogenous volatility choice, the optimal leverage is slightly higher when the effort cost is high. This is because negative volatility choice affords the firm more leverage.

[13]See equations 10 and 11 of Guo and Ou-Yang [2006], where an infinite skilled manager can have incentive infinitely close to 1 and the principal's (not strictly speaking shareholder) payoff always increases with incentive. So that the incentive-performance relation is always positive.

[14]The reason is as follows: the optimal solution is obtain at small value of, w, because θ is so large that awarding the manager with fixed wage is simply not optimal. And optimal incentives are hyperbolic function of optimal effort when w is zero, hence incentives should be as small as possible to extract higher managerial effort. But contract is bounded below by the participation constraint, so the optimal incentives, C, are simply implied by the participation constraint.

Table 2.5: Second-best solution with no volatility choice. The table reports second-best solution for various exogenous parameters. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $\overline{\sigma} = 0.2$, $A_1 = 0.5$, $A_1 = 2$. a and v is manager's optimal effort and volatility choice. C is optimal stock ownership awarded to the manager. w is manager's fixed wage. L is face value of debt. $E[S(X_1)]$ represents expected equity value. Shareholders maximize expected equity value with respect to stock, C, fixed wage, w, and leverage, L.

Changing Parameter	a 10^{-4}	C	w	L	$E[S(X_1)]$
Base Case	140.1	0.85%	10.53	286.74	103.11
γ					
3	785.1	2.07%	14.20	373.20	132.72
5	17.3	0.29%	9.90	274.58	99.23
6	1.71	0.10%	9.92	272.68	98.77
7	0.17	0.06%	9.95	272.42	98.72
T					
0.25	81.0	0.47%	9.96	575.44	94.27
0.50	109.9	0.67%	10.14	404.64	97.34
3	184.6	1.02%	11.79	175.72	126.45
5	202.9	1.01%	12.77	148.96	153.02
α					
0.07	142.5	0.87%	10.56	294.62	105.83
0.09	144.8	0.88%	10.60	302.88	108.69
0.12	148.4	0.91%	10.64	316.04	113.23
0.15	151.9	0.94%	10.68	330.20	118.13
θ					
3	750.2	1.27%	12.76	407.20	133.47
4	1516	1.41%	15.04	601.64	212.69
5	2268	1.31%	18.71	1390.14	486.43
5.5	2622	0.89%	28.12	3434.04	1196.86

However, when effort cost is low, endogenous volatility leads to lower leverage (see column 6 of Table 2.3 and column 5 of Table 2.5). This is because effort impacts more on the firm's value than volatility does. Overall, when effort and volatility cost are in the same magnitude, endogenous volatility is likely to mildly increase the optimal leverage.

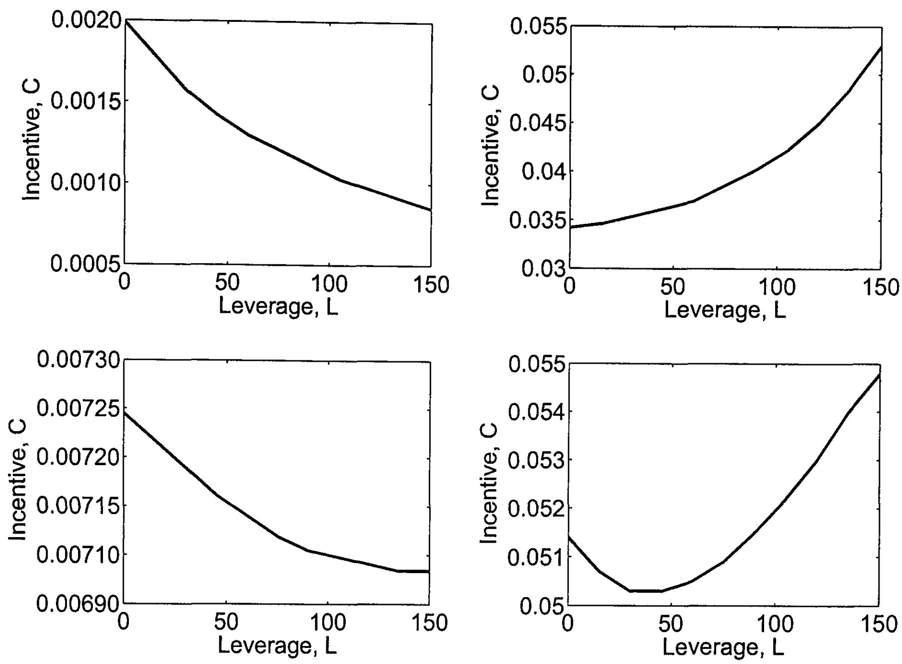

Figure 2.4: Mixed incentive-leverage relation. This figure plots optimal incentives with varying level of leverage. L is firm leverage, optimal incentives are solved by keeping leverage exogenous. All plots use base case parameters unless otherwise specified. Plot on the top left uses $\beta = 4$. Plot on the top right uses $\gamma = 3$. Bottom left plot uses $T = 3$, bottom right plot uses $\theta = 3$.

2.4.5 Incentive-leverage relationship

Optimal incentives show mixed results with firm leverage. To demonstrate this, we fix leverage at various exogenous levels and solve the optimal problem. As shown in Figure 3.2, the incentive-leverage relation is not monotonic and it depends on the manager's risk aversion, effort and volatility cost, and time. For example, the relationship is positive for $\gamma = 3$, negative for $\beta = 4$ and positive again for $T = 3$. For $\theta = 3$, it is not monotonic. It seems that the incentive-leverage relationship is very sensitive to changes in exogenous variables, as optimal compensation is made to balance risk-inducing effects of incentives and leverage. This result also holds even though volatility is not endogenous; we reports results with no volatility choice (Table 2.7), where incentives again show mixed relation with leverage.

Table 2.6: This table reports second-best optimal solution with different level of fixed leverage. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $\overline{\sigma} = 0.2$,$T = 1$, $A_1 = 0.5$, $A_2 = 2$. a and v is the manager's optimal effort and volatility choice. C is optimal stock ownership awarded to the manager. w is manager's fixed wage. L is face value of debt. $E[S(X_1)]$ represents expected equity value.

Leverage	$L=0$					$L=80$					$L=150$				
Changing Parameter	a 10^{-4}	v 10^{-4}	C	w	$E[S(X_1)]$	a 10^{-4}	v 10^{-4}	C	w	$E[S(X_1)]$	a 10^{-4}	v 10^{-4}	C	w	$E[S(X_1)]$
Base Case	0.28	0.00	0.27%	9.74	94.60	0.48	-0.00	0.26%	9.75	95.82	0.67	-0.01	0.27%	9.75	96.88
γ															
3	33.4	-2.24	3.45%	6.79	94.78	64.5	-8.41	3.89%	6.73	96.37	119.4	-23.1	5.31%	6.37	98.06
5	0.00	0.00	0.01%	9.99	94.60	0.00	0.00	0.02%	9.98	95.81	0.01	0.00	0.02%	9.98	96.87
6	0.00	0.00	0.00%	10.00	94.60	0.00	0.00	0.01%	9.99	95.81	0.00	0.00	0.01%	9.99	96.87
7	0.00	0.00	0.00%	10.00	94.60	0.00	0.00	0.00%	10.00	95.81	0.00	0.00	0.00%	10.00	96.87
T															
0.25	0.06	0.00	0.06%	9.94	91.13	0.12	0.00	0.06%	9.94	91.43	0.16	0.00	0.07%	9.94	91.70
0.50	0.14	0.00	0.14%	9.87	92.28	0.26	0.00	0.14%	9.87	92.88	0.35	0.00	0.14%	9.87	93.41
3	0.77	-0.03	0.73%	9.27	104.50	1.23	0.00	0.71%	9.33	108.25	1.58	-0.14	0.69%	9.40	111.57
5	5.36	-0.16	1.06%	8.91	115.42	8.21	-0.45	1.07%	9.07	121.94	200.6	-19.8	1.12%	12.86	152.48
α															
0.07	0.29	0.00	0.27%	9.74	95.23	0.50	0.00	0.27%	9.74	96.93	0.66	-0.00	0.26%	9.75	98.42
0.09	0.29	0.00	0.27%	9.74	95.87	0.50	0.00	0.27%	9.74	98.06	0.68	0.00	0.27%	9.74	99.98
0.12	0.28	0.01	0.27%	9.74	96.82	0.49	0.01	0.26%	9.74	99.76	0.67	0.00	0.26%	9.74	102.33
0.15	0.30	0.01	0.28%	9.73	97.79	0.49	0.01	0.26%	9.74	101.48	0.69	0.01	0.27%	9.73	104.70
β															
4	0.21	39.7	0.20%	9.81	94.62	0.21	39.7	0.12%	9.89	95.85	0.21	39.7	0.08%	9.92	96.93
6	0.19	328.1	0.18%	9.83	94.78	0.19	328.1	0.10%	9.90	96.12	0.19	328.1	0.07%	9.93	97.30
8	0.15	818.9	0.15%	9.86	95.03	0.15	818.9	0.08%	9.92	96.58	0.16	818.9	0.06%	9.94	97.93
10	0.13	1371	0.13%	9.88	95.32	0.13	1371	0.07%	9.93	97.09	0.13	1371	0.05%	9.95	98.64
θ															
3	178.2	-0.63	5.14%	5.29	95.99	232.9	-1.57	5.11%	5.98	98.89	281.5	-2.65	5.48%	6.50	101.79
4	763.5	-1.82	9.86%	0.64	101.20	905.7	-3.51	9.74%	2.08	109.52	987.2	-4.66	9.57%	3.42	117.65
5	1389	-1.93	9.97%	0.00	108.18	1602	-3.95	10.93%	0.00	122.99	1732	-5.49	11.97%	0.00	136.81
5.5	1687	-1.92	9.70%	0.00	111.82	1906	-3.80	10.33%	0.00	129.86	2033	-5.17	10.99%	0.00	146.67

Table 2.7: Optimal compensation with no volatility choice. This table reports second-best optimal solution with different level of fixed leverage. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $T = 1$, $A_1 = 0.5$, $A_2 = 2$. a is the manager's optimal effort choice. C is optimal stock ownership awarded to the manager. w is manager's fixed wage. L is face value of debt. $E[S(X_1)]$ represents expected equity value.

Leverage	$L = 0$				$L = 80$				$L = 150$			
Changing Parameter	a 10^{-4}	C	w	$E[S(X_1)]$	a 10^{-4}	C	w	$E[S(X_1)]$	a 10^{-4}	C	w	$E[S(X_1)]$
Base Case	0.29	0.28%	9.74	94.60	0.48	0.26%	9.75	95.82	0.67	0.27%	9.75	96.88
γ												
3	33.5	3.46%	6.78	94.78	65.2	3.93%	6.70	96.38	124.5	5.48%	6.29	98.08
5	0.00	0.02%	9.98	94.60	0.00	0.02%	9.98	95.81	0.00	0.02%	9.98	96.87
6	0.00	0.01%	9.99	94.60	0.00	0.01%	9.99	95.81	0.00	0.01%	9.99	96.87
7	0.00	0.00%	10.00	94.60	0.00	0.00%	9.99	95.81	0.00	0.00%	9.99	96.87
T												
0.25	0.02	0.07%	9.94	91.13	0.03	0.07%	9.94	91.43	0.04	0.07%	9.94	91.70
0.50	0.07	0.14%	9.87	92.28	0.13	0.14%	9.87	92.88	0.18	0.14%	9.87	93.41
3	2.29	0.73%	9.27	104.50	3.66	0.71%	9.33	108.25	4.81	0.70%	9.39	111.57
5	5.36	1.06%	8.91	115.42	8.21	1.07%	9.07	121.94	207.2	1.93%	12.72	152.53
α												
0.07	0.29	0.27%	9.74	95.23	0.50	0.27%	9.74	96.93	0.69	0.27%	9.74	98.42
0.09	0.29	0.27%	9.74	95.87	0.50	0.27%	9.74	98.06	0.68	0.27%	9.74	99.98
0.12	0.28	0.27%	9.74	96.82	0.49	0.26%	9.74	99.76	0.70	0.27%	9.73	102.33
0.15	0.28	0.27%	9.74	97.79	0.51	0.27%	9.73	101.48	0.69	0.27%	9.73	104.70
θ												
3	178.2	5.14%	5.29	95.99	232.9	5.12%	5.97	98.89	282.9	5.51%	6.42	103.39
4	763.3	9.85%	0.65	101.20	906.7	9.77%	2.06	109.52	988.3	9.59%	3.41	117.66
5	1386	9.97%	0.00	108.18	1602	10.94%	0.00	122.99	1702	10.79%	0.86	138.73
5.5	1687	9.70%	0.00	111.82	1907	10.34%	0.00	129.86	2034	11.00%	0.00	146.68

2.4.6 Exogenous leverage

When leverage is endogenous, companies choose the highest possible leverage level to extract returns from the manager's action. All optimal solutions are obtained at the highest possible leverage, where slight increases in leverage beyond the optimal level would result in firm bankruptcy. This result is commonly known in the banking industry where all banks take as much leverage as they possibly can. However, leverage of, for instance, banks are not fully endogenous because of capital requirements[15], which impose an upper bound on leverage. We report optimal solutions under three exogenously specified leverage levels. Since regulated maximum (or externally constrained) leverage ratios are lower than the otherwise optimal one, our reported (see Table 2.6) leverage is relatively low compared to the optimally chosen.

Interestingly, there are some cases in which the optimal effort and volatility do not change across the three levels of leverage. For example, when $\beta = 4$ optimal effort and volatility is 0.21 and 39.7 for all three levels of leverage, even though optimal incentives change from 0.20% to 0.08%. In this case optimal effort and volatility is insensitive to the leverage changes, which only affect the optimal incentives. Under this scenario, leverage and incentives are a similar way of making the manager behave optimally. Previous literature has similar findings that leverage is another incentive tool (which may act as a substitute to incentive compensation) used to encourage managerial hard work and risk-taking. Our results certainly demonstrate such arguments.

However, such indifference effects are only observed when volatility is a choice variable (see Table 2.7) and the volatility cost is low. When the volatility cost is high, leverage acts as a disciplinary tool for effort as it forces the manager to exert more effort. When the volatility cost is low, leverage acts as a substitute for incentives. Since leverage merely impacts on optimal incentives but has no effect on the manager's action choice. Figure 2.5 clearly demonstrates this effect. When volatility cost is high, optimal volatility is indifferent from zero across all leverage levels. But effort shows a positive relation with leverage, even though incentives stay constant. On the other hand, both effort and volatility show little variation across leverage levels when volatility cost is low. Leverage does not change the manager's action choice but does change the optimal incentives. These are two effects of leverage: discipline for effort and substitute

[15]For example, Suntheim [2010] reports banks' Tier 1 capital ratio have little cross sectional variation.

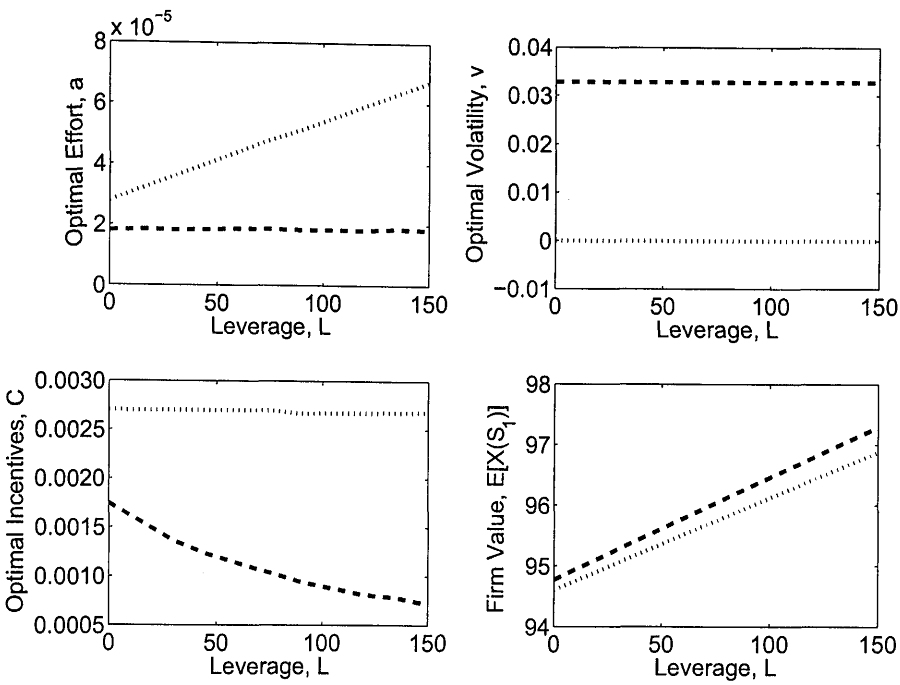

Figure 2.5: Two effects of leverage. This figure demonstrates disciplinary and substitution effects of leverage. The optimal solution is solved at each exogenous leverage level. The dotted line is based on high volatility cost ($\beta = 2$); the dashed line is based on low volatility cost ($\beta = 6$). All other parameter values are base cases as in Figure 2.2

for incentives. Such a distinction exists because of endogenous volatility choice, which balances shareholders' leverage and incentives choices.

Finally, it needs to be emphasized that even though the firm is allowed to go bankrupt, it is never optimal for the shareholders and the manager to do so. Shareholders anticipate the manager's skills and choose a debt level that is manageable so that the firm never goes bankrupt. If there is default at any state of the economy, the manager's utility is not maximized. Because debt default upon the firm's bankruptcy results in the manager receiving nothing, due to power law utility function when wealth is zero the manager's expected utility is negative infinity. This immediately violates participation constraint. The manager simply does not work if the firm has positive probability of bankruptcy. Shareholders anticipate the manager's action and choose a safe level of debt that is robust against negative shocks, so that the firm never fails. Bondholders expect shareholders debt choice and correctly price their lendings. The net result is a riskless debt that always pays back its face value. This result is similar to riskless debt in Cadenillas et al. [2004] where the value of the firm is always larger than the nominal value of debt. The riskless assumption is relaxed when the manager is awarded with debt, which we discuss next.

2.4.7 Effects of the equal seniority of managerial and debtholders' claims

Table 2.8 reports optimal second-best solution for the alternative compensation package, where the manager is awarded with equity and debt. In this case, debt is risky and the firm may have positive probability of failure. This, however, results in infinite leverage choice, as shareholders only care about the expected return of the firm. The manager, on the other hand, is also protected through debt compensation, which pays out the same compensation even if the firm fails. Under such a compensation structure, infinite leverage is always optimal no matter how risk averse the manager is. This is because derived utility function is always convex when leverage is large enough.

Although this model implies infinite leverage, such a solution is not entirely implausible as banks could have much higher leverage than at present if there were no minimum capital requirement. We report the results for three exogenous levels of leverage similar to the previous section (when leverage is zero, the problem collapses to no debt compensation so we focus at positive leverage levels). In addition to normal range of leverage, we also report results for $L = 2000$, which is equivalent to a capital to debt ratio of 1:20.

Without doubt rewarding the manager with debt makes her very conservative in terms of exerting effort and choosing volatility. Comparing results between Tables 2.3 and 2.8 shows that (for the same level of leverage) optimal volatility is always lower when the manager is rewarded with inside debt. Obviously, such optimal choices are achieved at the expense of the shareholders. All shareholders' values are lower when the firm rewards with debt. This result is expected, as debtholders can earn returns higher than risk free rate (see Table 2.9). Although both debtholders' and shareholders' expected returns increase with leverage, shareholders' return would be much higher if rewarding the manager with equity and fixed wage. Debt compensation in this regard balances payoffs for both parties.

Table 2.8: Managerial compensation with inside debt. The table reports second-best solution for three leverage levels, with various exogenous parameters. Other non-varying parameters are as follows: $X_0 = 100$, $\sigma = 0.3$, $\overline{\sigma} = 0.2$, $A_1 = 0.5$, $A_1 = 2$. a and v is manager's optimal effort and volatility choice. C is optimal stock ownership awarded to the manager. e is manager's debt compensation, denotes as percentage of the firm's leverage, e.g. $e = 1\%$ indicates the manager receives 1% of L as compensation. L is face value of debt. $E[S(X_1^D)]$ represents expected equity value. Shareholders maximize expected equity value with respect to stock, C, fixed wage, e, and leverage, L.

Leverage	$L = 80$					$L = 150$					$L = 2000$				
Changing Parameter	a 10^{-4}	v 10^{-4}	C	e	$E[S(X_1^D)]$	a 10^{-4}	v 10^{-4}	C	e	$E[S(X_1^D)]$	a 10^{-4}	v 10^{-4}	C	e	$E[S(X_1^D)]$
Base Case	0.72	-0.01	0.42%	12.01%	95.67	0.85	-0.01	0.35%	6.45%	96.73	15.54	-4.18	0.15%	0.70%	119.60
γ															
3	67.13	-9.04	4.20%	8.07%	96.23	120.8	-23.49	5.51%	4.13%	97.89	196.6	-42.29	1.95%	0.63%	149.33
5	0.03	0.00	0.15%	12.32%	95.66	0.02	0.00	0.10%	6.60%	96.72	1.57	-0.44	0.00%	0.72%	117.17
6	0.00	0.00	0.11%	12.37%	95.66	0.00	0.00	0.07%	6.62%	96.72	0.16	-0.05	0.00%	0.73%	116.76
7	0.00	0.00	0.10%	12.38%	95.66	0.00	0.00	0.06%	6.63%	96.72	0.02	-0.00	0.00%	0.74%	116.59
T															
0.25	0.09	0.00	0.21%	12.26%	91.39	0.08	0.00	0.14%	6.58%	91.66	3.15	-1.67	0.00%	0.56%	97.83
0.5	0.23	-0.00	0.27%	12.19%	92.80	0.25	-0.00	0.21%	6.54%	93.33	6.87	-2.64	0.00%	0.62%	105.15
3	3.99	-0.14	0.82%	11.54%	107.82	4.93	-0.23	0.75%	6.23%	111.12	45.79	-7.67	0.12%	1.00%	180.47
5	8.49	-0.49	1.17%	11.23%	121.20	66.41	-6.41	1.07%	6.48%	134.11	71.57	-9.46	0.00%	1.26%	248.21
α															
0.07	0.80	-0.01	0.47%	11.95%	96.73	0.91	-0.01	0.38%	6.43%	98.22	15.57	-4.10	0.15%	0.69%	129.99
0.09	0.89	-0.00	0.52%	11.89%	97.80	0.97	-0.01	0.40%	6.41%	99.71	15.62	-4.01	0.15%	0.69%	140.53
0.12	1.02	-0.00	0.59%	11.79%	99.41	1.07	-0.00	0.44%	6.38%	101.98	15.72	-3.88	0.15%	0.68%	156.65
0.15	1.16	0.00	0.66%	11.69%	101.04	1.17	0.00	0.48%	6.35%	104.26	15.85	-3.76	0.16%	0.67%	173.14
β															
4	0.27	38.06	0.15%	12.32%	95.70	0.28	37.32	0.12%	6.59%	96.77	15.47	-571.4	0.35%	0.65%	116.06
6	0.23	323.9	0.13%	12.34%	95.95	0.24	320.3	0.10%	6.60%	97.12	18.70	-1606	0.86%	0.55%	109.20
8	0.20	810.0	0.12%	12.36%	96.38	0.22	801.9	0.09%	6.61%	97.72	24.35	-2000	1.33%	0.49%	107.25
10	0.18	1356	0.10%	12.38%	96.86	0.19	1342	0.08%	6.62%	98.38	24.95	-2000	1.39%	0.48%	107.37
θ															
3	238.3	-1.68	5.53%	7.04%	98.66	284.2	-2.72	5.74%	4.20%	101.51	342.4	-4.30	1.12%	0.66%	176.57
4	949.4	-4.11	11.52%	0.79%	109.37	1011	-5.03	10.57%	1.81%	117.28	949.7	-4.16	0.93%	0.60%	298.85
5	1602	-3.95	10.93%	0.00%	122.99	1732	-5.49	11.97%	0.00%	136.81	1618	-4.09	0.75%	0.53%	460.97
5.5	1906	-3.80	10.34%	0.00%	129.86	2033	-5.17	10.99%	0.00%	146.66	2567	-14.43	33.90%	0.00%	502.03

Table 2.9: Debtholder and shareholder return. This table reports the price of the risky debt (B_0^D), debtholder return (μ_S) and shareholder return (μ_S) under the alternative compensation scheme where the manager is rewarded with equity and debt. Parameter values are those of Table 2.6. Debtholder return is computed, $\mu_D = \frac{\log(L)-\log(B_0^D)}{T}$, where L is face value of debt as reported below. B_0^D is simultaneously computed with the optimal solution. Similarly shareholder return is computed, $\mu_S = \frac{\log\{E[S(X_1^D)]\}-\log(X_0)}{T}$, where values of $E[S(X_1^D)]$ are those reported in Table 2.8.

Leverage	$L=80$			$L=150$			$L=2000$		
Changing parameters	B_0^D	μ_D (%)	μ_S (%)	B_0^D	μ_D (%)	μ_S (%)	B_0^D	μ_D (%)	μ_S (%)
Base Case	68.31	15.80	-4.43	136.18	9.66	-3.32	1513.94	27.84	17.90
γ									
3	71.37	11.42	-3.84	139.55	7.22	-2.13	1544.08	25.87	40.10
5	68.07	16.15	-4.44	135.95	9.83	-3.34	1511.25	28.02	15.84
6	68.03	16.20	-4.44	135.93	9.85	-3.34	1510.73	28.06	15.49
7	68.02	16.22	-4.44	135.92	9.86	-3.34	1510.47	28.07	15.35
T									
0.25	69.66	55.34	-35.99	139.08	30.24	-34.84	1782.15	46.13	-8.79
0.50	69.20	29.00	-14.94	138.10	16.53	-13.80	1667.41	36.38	10.05
3	64.68	7.09	2.51	128.55	5.15	3.52	1175.06	17.73	19.68
5	61.12	5.38	3.85	117.69	4.85	5.87	978.98	14.29	18.18
α									
0.07	68.36	15.72	-3.33	136.21	9.64	-1.80	1521.45	27.35	26.23
0.09	68.41	15.65	-2.23	136.24	9.62	-0.29	1529.07	26.85	34.02
0.12	68.48	15.54	-0.59	136.28	9.59	1.96	1540.75	26.09	44.88
0.15	68.56	15.43	1.03	136.32	9.56	4.17	1552.70	25.32	54.90
β									
4	68.07	16.14	-4.40	135.97	9.82	-3.28	1600.64	22.27	14.90
6	68.05	16.18	-4.13	135.96	9.83	-2.92	1771.52	12.13	8.80
8	68.04	16.20	-3.69	135.95	9.84	-2.31	1931.44	3.49	2.00
10	68.03	16.21	-3.19	135.94	9.84	-1.63	1931.52	3.48	-0.37
θ									
3	72.17	10.30	-1.35	139.45	7.29	1.50	1556.61	25.06	56.85
4	77.02	3.80	8.95	142.94	4.82	15.94	1645.49	19.51	109.48
5	77.64	3.00	20.69	145.57	3.00	31.35	1763.02	12.61	152.82
5.5	77.64	3.00	31.20	145.57	3.00	44.64	1940.89	3.00	196.86

Debt compensation also leads to a higher effort choice, as optimal efforts are always higher. Generally, the relationship between leverage and other endogenous variables is the same as in the original model, and volatility and leverage is now negatively related as higher leverage always results in low volatility choice. This is in contrast to the substitution effect found in the original setup. One implication of a negative leverage-volatility relationship is that high leverage can lead the manager to reduce volatility substantially, e.g. for each value of β, volatility reduction is quite significant. When β (measure of volatility cost) is larger than 8, optimal volatility actually hits the lower bound, $\overline{\sigma} = 0.2$. Such an action is intuitive, as volatility cost is so low that the manager relies solely on reducing volatility to maximize her expected utility. Overall, debt compensation makes the manager work harder and take less risk, and significantly reduces the manager's risk-taking for a given leverage level.

2.5 Empirical implications

Our results show that the incentive-leverage relationship is not monotonic, depending on the firm's investment opportunities and the manager's skills. When volatility cost is high, shareholders choose high incentives exposing the manager to more risk which makes her choose lower volatility. This results in a positive relationship between incentives and leverage. When the volatility cost is low, shareholders choose low incentives and high leverage to encourage risk-taking, and the incentive-leverage relationship is negative. Results in Table 2.6 demonstrate that incentives are negatively related to leverage when the volatility cost is low, and positively related when managerial risk-aversion is low. These results explain mixed findings in the empirical literature; for example Coles et al. [2006], Ortiz-Molina [2007] and Gao [2010] find negative incentive-leverage whereas Lewellen [2006] finds a positive relationship.

It would be very interesting to test the relationship between leverage and incentives in a structural model, where compensation contracts and firm volatility are simultaneously determined. Coles et al. [2006] test this and find a positive relationship. This result is different from predictions of our model, where incentives have a mixed relationship with leverage, as we assume the manager can influence the firm's return and volatility by exerting optimal action, which is not explicitly considered in their paper. To the best of our knowledge, previous empirical studies do not explicitly take this point into account. It would be appealing to revisit this problem with two more endogenous variables – firm return and volatility – to account for the effect of the manager's action.

Our results also show that there exists a flat region of action (for firms with low volatility cost) where exogenous leverage only affects the optimal incentives choice.

Since capital requirements (an exogenous leverage level) for banks have increased after the credit crunch (see Slovik and Cournede [2011] for recent capital requirements changes), it would be interesting to test such implications based on this natural experiment. As expected, the same level of risk-taking should be observed for large, international banks which have relatively low volatility cost, with a significant reduction in their CEOs' incentives.

Debt-like compensation helps to reduce managerial risk-taking as well as making the manager work hard. This interesting phenomenon is demonstrated in our numerical results, and empirical work could potentially explore this area, in which management's debt-like compensation is ignored. It is an ongoing debate that the bank manager's incentive is highly skewed and in part contributed to the financial downfall in 2008. Since then new legislation has been passed to defer most bankers' pay, which is a feature similar to our debt-like compensation. It would be interesting to revisit these banks executives' incentives position taking into account their inside debt positions.

2.6 Conclusions

We develop a model explicitly accounting for the manager's ability to influence risk. Different from previous literature in risk shifting (Cadenillas et al. [2004], Guo and Ou-Yang [2006], Carlson and Lazrak [2010]) we assume that the manager can both increase or decrease the firm's risk at a cost. Our results demonstrate that the relationship between incentives and performance is not always positive; it depends on managerial skills and firm characteristics.

In addition to the manager's ability to influence risk, we explicitly consider the connection between leverage and compensation. Leverage is important to the shareholders because it magnifies the firm return. It is also important to the manager because the decision of the optimal compensation contract largely depends on leverage. Our results demonstrate that leverage can be regarded as a substitute for incentives only when volatility is endogenous, since leverage encourages similar managerial actions as those of compensation incentives. When volatility is not endogenous, leverage serves as a disciplinary tool as higher leverage always leads to higher effort choice.

We also extend the conventional principal-agent model to include debt-like compensation. For a given leverage level, our results show that debt-like compensation not only reduces managerial risk-taking but also increases the manager's effort choice. Although the resulting model implies that infinite leverage is optimal, we argue this is hardly implausible as most banks would have extremely high leverage (possibly even higher than current level) were there no capital requirements.

In conclusion, the chapter introduces endogenous volatility and leverage to a one-period principal-agent model. Our results show that leverage is negatively related to incentives when volatility cost is low, which has some empirical support, e.g Guay [1999]. Also, leverage has a positive relationship with incentives when the manager is not very risk-averse, consistent with Coles et al. [2006]. We argue that this is due to managers' action choice not being explicitly considered in most empirical studies. Besides the complex relation mentioned in the literature, we show that even when bondholder-shareholder agency cost is absent, the incentive-leverage relationship is still ambiguous. Unless managerial actions are explicitly considered, empirical studies are likely to find mixed results.

A useful extension of the original model would be to explicitly incorporate firm bankruptcy. Two features can be added to the model in order to accommodate this feature. Firstly, as we previously discussed, the manager's utility becomes infinitely negative when firm has positive probability of failure. One way to avoid infinity is to allow the manager to have outside wealth, so that, even if the firm fails, the manager's wealth is strictly positive. The second problem with original model is that the firm chooses infinite leverage when the manager has debt compensation. This problem can also be avoided by introducing a fractional bankruptcy cost, similar to Leland [1994] and Leland and Toft [1996], so that there is penalty cost to additional leverage, which bounds shareholders' leverage choice. Cadenillas et al. [2004] claim that the numerical solution is not available for the bankruptcy problem mentioned above. It would be very interesting to revisit their claim with these added features.

It would be very interesting to investigate a version of the problem where bondholder-shareholder conflict is present. Our setup assumes that leverage has a simple impact on the firm asset value, that is scaling up of the firm's size. It would be interesting to account for agency conflict involved between bondholders and shareholders. In such a setting, shareholders would choose incentives to balance agency cost between the manager and bondholders. Another interesting extension would be to introduce outside investment options for the managers. If the manager has other investment options, she would choose effort and volatility differently. That may lead to a very different risk-taking behaviour.

Chapter 3

Executive Compensation with Tax

3.1 Introduction

Employee stock options and restricted stock grants are subject to a variety of tax laws. These tax considerations could potentially have a substantial effect on the optimal exercise (or sale of these grants) and the value of the grants to the employee. Subject to qualifying constraints, Incentive Stock Options (ISO) in the US, and Enterprise Management Incentives (EMI) and Company Share Option Plans (CSOP) in the UK offer the employee the chance to pay long-term capital gains tax on their profits from option exercise. Naturally, these plans may be more valuable to the employee than other forms of compensation, keeping the cost of compensation to the firm fixed. We develop a continuous time utility model where, by carefully considering the optimal exercise and sale for the employee, we are able to derive the employee's valuation of the option and the cost to the firm of issuing the option.

In the UK, for most reasonable levels of wealth and risk aversion, EMIs and CSOPs provide the largest value to an employee given a fixed cost to the firm. This is largely because in the UK, the firm is able to deduct the employee's option payoff from their corporate taxable income. However, the result is not true for all employees, as employees with high levels of risk aversion or low levels of wealth would prefer to exercise the options before meeting the tax qualifying conditions and thus forgo the potential tax savings. In the US, payouts from ISOs are not corporate tax deductible, thus, for reasonable levels of effective corporate tax, ISOs are the least effective form of compensation considered as they provide the lowest value to the employee for a given cost to the firm.

The other compensation contracts considered are stock options that do not qualify for the potentially lower tax rate; these are called nonqualified stock options (NQSO) in the US and unapproved option plans (UOP) in the UK. Payoffs from these options are taxed as regular income when the option is exercised, and are eligible for corporate tax deduction. We also consider restricted stock plans. These are taxed as regular income at the date of issuance even though they cannot be sold until the vesting period. It is often possible for employees to make a Section 83b election, allowing them to defer the tax payment until the stock vests. We value the restricted stock plans by considering the possibility of a Section 83b election.

Our findings are in agreement with McDonald [2004] in that making the 83b election reduces the value of the restricted stock grant to the employee. Blouin and Carter [2007] argue that an 83b election increases employee incentives. But their analysis is based on a particular scenario, that is stock price goes up in the future. With no assumption on future stock price, our results show that 83b election also lowers employee incentives, in particular the pay-for-performance measure.[16] Given non-optimality of this election, there appears to be little rational justification of the 83b election for either the employee or the firm.

Stock options have become an important instrument for rewarding employees in recent years, but no agreement has been reached in the research community as to why granting options is such a popular practice. Economic theories propose some possible explanations: providing incentive to employees (Hall and Murphy [2002]), providing non-debt tax shield (Graham [2003]), employee optimism (Oyer and Schaefer [2005]) and employee retention (Oyer [2004]); none of these explanations could single-handedly explain the wide popularity of the board-based option plan. There is also an ongoing debate regarding the best type of compensation instruments – stocks or options. Prior literature, such as Hall and Murphy [2002] and Dittmann and Maug [2007], argue that it is not optimal to reward options as compensation because they are an inefficient way of rewarding employees. Despite these conclusions, firms continue to use options in their compensation plans.

[16] Pay-for-performance, incentives and Δ are all interchangeable terms.

Thus, as well as determining the value and cost of issuance, we also consider the incentives provided by the plans, in particular the pay-for-performance measure. We find that in the UK cost effective incentives are best provided by EMIs and CSOPs in line with the intention of creating such plans. In the US, ISOs provide the least cost-effective incentives, unless the corporate tax rate is very low. This is a surprising result, given that the tax breaks for ISOs were seemingly designed to provide incentives to employees. This result may imply that only loss making firms choose to issue ISOs or that employees are extracting benefits from the firm.

Finally, we look at the expected lifetimes of the options. Any incentives provided by option plans only last as long as the grant is held by the employee and so this is an important consideration. Also, empirical corporate finance research (for example, Malmendier and Tate [2005]) is interested in when options are exercised as it may be possible to infer certain behavioral biases or inside information from employee behaviour. It is well known in the literature that employees exercise compensation options earlier than they would with otherwise traded options, e.g. Carpenter et al. [2010]. This is because employees are risk averse and suffer trading restriction. However, pricing and exercising also depends upon tax planning. We find that the incentive feature typically causes employees to exercise their options earlier because they expect to hold stocks (obtained from exercising ISOs) one more year for tax purposes, hence they exercise the option earlier to reduce total holding time.

The implications of these findings are that tax considerations are very important in determining the value and optimal exercise strategy for the employees. In the UK, due to the favorable taxation of EMIs and CSOPs, the optimal compensation contract typically consists of these incentive option plans, even when not considering pay-for-performance incentives. In the US, due to unfavorable tax rules, ISOs are not a cost-effective way of paying employees or providing incentives unless companies are not paying corporate tax, or the effective rate is very low. We find that, depending upon the type of option contract, the optimal exercise decision can be substantially different, with ISOs typically being exercised earlier than NQSOs; when attempting to calibrate models to observed option data this is an important consideration. Finally, we find little justification for the offering of a Section 83b election for restricted stock plans.

The rest of the chapter proceeds as follows. Section 3.2 outlines tax and accounting rules in the US and UK. Section 3.3 introduces the valuation model for various compensation instruments and discusses the incentives provided. Section 3.4 reports our results, and Section 3.5 concludes.

3.2 US and UK tax rules

3.2.1 Stock option

There are primarily two ways that firms in the US can grant employees stock options: ISOs and NQSOs. ISOs are not commonly used in practice. According to Hall and Liebman [2000], ISOs only account for 5 percent of total option grants. They are not popular because of two major reasons.[17] First, there is a $100,000 limit on ISOs that an employee can acquire during each calendar year. But few firms grant up to this limit potentially because of the second reason: the cost of ISO is not deductible against corporate tax. It is simply a costly form of compensation. For an employee, the ISO is not taxed at grant and exercise date. It is only taxed when the employee sells the stock, and at a lower capital gains tax rate. The ISO is tax advantageous to the employee but costly to the issuing firm.

NQSOs are widely used in the US. While NQSOs are similar to ISOs by design, they have some flexible features that make them very attractive to firms. Firstly NQSOs can be awarded to anyone – employees, executives, consultants, etc. Second, although NQSOs holders are taxed at an income tax rate at exercise date, issuing firms receive a parallel deduction against corporate income at the same time. NQSO is tax advantageous to the issuing firm but costly to its recipient. Table 3.1 shows detailed tax rules for each of these option granting schemes.

In the UK, there are similar schemes that provide favorable tax treatment to employees: EMI and CSOP[18] which are taxed similarly to ISO in the US. The tax rules on those option schemes are quite detailed and have different qualifying condition.[19]

[17] Hall and Liebman [2000] report ISO account for 5% of total option grant in public firms. According to a survey conducted by Bureau of Labor Statistics, in 1999, ISO accounts for 31.2% of total stock option grant in private industry. The survey report is available online at: http://www.bls.gov/opub/cwc/archive/spring2001art1.pdf

[18] HM Revenue & Customs provides detailed (and quite technical) tax rules on each of these option schemes. They are online at: http://www.hmrc.gov.uk/manuals/essum/ESSUM40000.htm

[19] EMIs are aiming at small firms that have gross assets under £30 million and subject to £120,000 per employee limits and £3 million limit for all employees. CSOPs are taxed the same way as EMIs but have more restrictions. Under CSOP scheme, any employee can hold qualifying options for no more than £30,000. CSOPs is only vested 3 years after grant, exercising CSOP options less than 3 year of grant disqualify the options' tax advantage. So their use is more limited.

Table 3.1: Taxation on ISO and NQSO. This table reports stock option tax rules in the US and UK. US tax rules are based on Hall and Liebman [2000], Graham [2003] and McDonald [2004]. UK tax rules are based on 'Employee Share Schemes User Guide' produced by HM Revenue & Customs, which is available at: http://www.hmrc.gov.uk/manuals/essum/index.htm.

		US		UK	
		Employees	Corporation	Employees	Corporation
NQSO, also known as unapproved option plan in the UK	Grant Date T_0	No tax liability incur	No taxable event	No tax liability incur	No taxable event
	Exercise Date τ_e	Income tax on the difference between stock price at exercise and strike price $P_{\tau_e} - X$	Tax deductible expense equal to difference of stock price at exercise and strike price $P_{\tau_e} - X$	Income tax on the difference between stock price at exercise and strike price $P_{\tau_e} - X$	Tax deductible expense equal to difference of stock price at exercise and strike price $P_{\tau_e} - X$
	Sale of Stock τ_s	Long-term capital gains tax on the difference between stock price at sale and stock price at exercise $P_{\tau_s} - P_{\tau_e}$ if $\tau_s - \tau_e > 1$. Or short-term capital gains tax on the difference if $\tau_s - \tau_e < 1$.	No taxable event	Capital gains tax on the difference between stock price at sale and stock price at exercise $P_{\tau_s} - P_{\tau_e}$	No taxable event
ISO, also known as EMI and CSOP in the UK	Grant Date T_0	No tax liability incur	No taxable event	No tax liability incur	No taxable event
	Exercise Date τ_e	No tax liability incur	No taxable event	No tax liability incur. For CSOP condition, $\tau_e - T_0 \geq 3$ must satisfy, otherwise the option is disqualified and taxed as unapproved options	Tax deductible expense equal to difference of stock price at exercise and strike price $P_{\tau_e} - X$
	Sale of Stock τ_s	Long-term capital gains tax on the difference between stock price at sale and strike price $P_{\tau_s} - X$ under the condition that $\tau_e - T_0 \geq 2$ and $\tau_s - \tau_e \geq 1$, otherwise income tax on the difference	No taxable event	Capital gains tax on the difference between stock price at sale and strike price $P_{\tau_s} - X$	No taxable event

Similar to ISOs in the US, employees are not taxed at the grant or exercise date but at the long-term capital gains rate when employees sell the shares. The crucial difference between ISOs and EMIs however is that firms get a corporate tax deduction when employees exercise an EMI or CSOP option. In addition to the EMI/CSOP, the UK has its equivalent of NQSO, which is called UOP. It is taxed in the same way as an NQSO.

Apart from ISOs and NQSOs, and EMIs and CSOPs, there are also the employee stock purchase plan (ESPP) in the US and save as you earn (SAYE) options scheme in the UK, which offer option grants to employees. Both schemes require employees to make savings toward future exercise of the granted options. ESPPs are taxed similar to ISOs where firms cannot take a tax deduction. SAYE is taxed similar to EMIs and CSOPs, where firms also get corporate tax deductions for share benefit provided under this scheme.

3.2.2 Restricted stock

For employees, restricted stock plans are subject to the income tax rate when the stock vests. It is not taxed at date of grant unless the employee chooses to make an 83b election. Firms receive corporate tax deduction for granting restricted stocks. Under an 83b election, the employee pays income tax upon grant and capital gains tax upon the sale of stock. Restricted stock plans are taxed the same in the UK and the US; there are also equivalent rules in the UK that provide similar 83b election.

3.2.3 Relative tax advantage

Typically from a valuation point of view, stock options are inefficient because they provide less value to employees for the same dollar outlay (see Hall and Murphy [2002]). For US companies paying corporate tax, ISOs are less efficient than NQSOs since they do not attract any corporate tax deduction, leading to a larger cost of issuance. To illustrate the point, consider the employee's valuation of the stock is $\overline{P}$ and the market valuation of the stock is P. For a risk-averse employee with restricted stock, $\overline{P} < P$,

$$\frac{\overline{P}(1-\tau_I)}{P(1-\tau_C)} < \frac{1-\tau_I}{1-\tau_C} \tag{3.1}$$

where τ_C and τ_I are the corporate and income tax rates, respectively. The right hand side is the value to cost ratio of cash compensation. The left hand side represents value to cost ratio for restricted stock. From this simple inequality, restricted stock provides less value to the employee (for the same amount of dollar outlay) than cash compensation.[20] Since cost of restricted stock is not affected by sales behaviour (option cost is affected by exercise behaviour), cash will always dominate restricted stock in terms of value to the employee per dollar spent by the firm.

A slightly different argument can be applied to ISOs and NQSOs at exercise time;

$$\frac{\overline{P}-X}{P-X} < \frac{(\overline{P}-X)(1-\tau_I)}{(P-X)(1-\tau_C)} \qquad \text{if} \quad \tau_I < \tau_C \tag{3.2}$$

X is the strike price of stock options. The fraction, $\frac{\overline{P}-X}{P-X}$, is the payoff ratio of ISOs, where the numerator is the payoff the employee receives when exercising the option, since the risk averse employee values stock at $\overline{P}$. The denumerator is simply the company's cash outlay at exercised date for granting ISOs. The right hand side of the equation is the payoff ratio of NQSOs. This initutive result leads to a well known result (e.g. Graham [2003]) that when the corporate tax rate is higher than the income tax rate, NQSOs are preferred to ISOs.

It is also straightforward to show that restricted stock is preferred to NQSOs,

$$\frac{\max(\overline{P}-X,0)(1-\tau_I)}{\max(P-X,0)(1-\tau_C)} < \frac{\overline{P}(1-\tau_I)}{P(1-\tau_C)} \tag{3.3}$$

This relation holds for any tax rate. From the above simple analysis, we show that cash has the largest payoff ratio, followed by restricted stock, then NQSO, at last ISO. The same argument also applies to UK option plans, where EMI/CSOP has payoff ratio,

$$\frac{\overline{P}-X}{(P-X)(1-\tau_C)} \tag{3.4}$$

It is unclear if this ratio is larger than that of cash (and also that of restricted stock), as it depends on income tax rate as well as the employee's risk aversion. Our more elaborate model in section 3.4 shows that these relations (shown above) hold for current US and UK tax rates.

Although these inequalities are a good approximation of each instruments' relative tax advantage, they are not value to cost ratio we show in later section, which is

[20] Note we use a simplifying assumption that income tax for the employee is $\overline{P}\tau_I$; actually her income tax is equal to $P\tau_I$, though it does not affect the direction of the inequality.

more accurate at describing the employee's problem. Section 3.4 shows that exercise behaviour, capital gains tax, risk aversion and wealth are also important in determining relative tax advantage. In fact, section 3.4.5 demonstrate that NQSOs can be preferred even if the income tax rate is substantially higher than the corporate tax rate. It is also worth noting that the above analysis does not consider the incentives provided by the compensation instruments either, which as we will show are important reasons of their popularity.

3.3 The model

In this section, we develop the tax inclusive pricing model. There are two major additions to the standard partial differential equation (PDE) pricing technique (see Carpenter [1998], Carpenter et al. [2010] and Pollet et al. [2011]). Firstly, the model incorporates tax payments into the executive's and firm's option valuation, by adjusting expiry and exercise conditions of the appropriate PDE. Secondly, upon exercising the option, the employee receives a stock that may incur additional tax payments once it has been sold. The stock sale decision could incur short-term capital gains, long-term capital gains or income tax depending upon the option plan. This stock sale decision is equivalent to the exercise of an option with a zero strike price and so is modeled as an embedded option. The value and cost of a compensation option (ISO[21] or NQSO) depends upon the decision to exercise the option, the decision to exercise the stock and also the tax payable at the point of exercise and sale. As usual, exercise and sale decisions depend upon employee characteristics such as liquid wealth and risk aversion.

Consistent with the standard executive stock option literature (e.g., Kulatilaka and Marcus [1994]; Carpenter [1998]; Hall and Murphy [2002]) we use a terminal wealth power utility model (the model in Chapter 2 also uses the same utility function). As shown by Lambert et al. [1991] the employee's value is different from the firm's valuation as the employee faces a large non-diversifiable risk. Conversely, the firm's shareholders are fully diversified. We first determine the employee's optimal exercise strategy and her valuation, then separately calculate the firm's value of the option under the risk-neutral measure using the employee's exercise strategy.

[21] In the following analysis we will use ISO to also mean CSOP or EMI, unless otherwise specified.

The employee has constant relative risk aversion and is granted an option[22] at time 0. Exercise decisions are taken to maximize the utility of wealth, W, at the maturity of the options T. The utility of wealth at this time is given by:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma} \tag{3.5}$$

where γ is the coefficient of risk aversion. The employee invests all of the non-option wealth in the riskless asset. So, that

$$dW_t = rW_t dt \tag{3.6}$$

where r is the risk-free rate. The stock price, P, follows a geometric Brownian motion,

$$dP_t = \mu P_t dt + \sigma P_t dZ_t \tag{3.7}$$

where μ and σ are the mean and volatility of the stock price return respectively and dZ is a Brownian motion. To make the executive rationally select the riskless portfolio, the expected return of the stock price is chosen to be equal to the risk-free rate, $\mu = r$.

Note that although unrealistic, this constraint retains the key features of the full portfolio allocation and option exercise problem, without the increased complexity of including a market asset. If the CAPM holds and the executive is allowed to hold a market asset, then it would not be optimal to allocate any wealth to the stock. Instead, wealth would be optimally allocated to the market asset as it dominates the stock. Without the market asset, imposing the restriction that $\mu = r$ ensures that there is no desire for the executive to hold stock and that the executive's desire to hold the option is only due to the possibility that the option could produce a large payoff.

Additionally, as we are also considering the sale of stock after exercise, choosing $\mu > r$ without a market asset leads to the employee having additional reasons (other than tax savings) to hold the stock. In the presence of a market asset, if the stock is priced according to CAPM then the market asset would be a preferable asset to hold unless the tax savings of holding the stock are substantial. As above, choosing $\mu = r$

[22]We consider restricted stock as an option with zero exercise price.

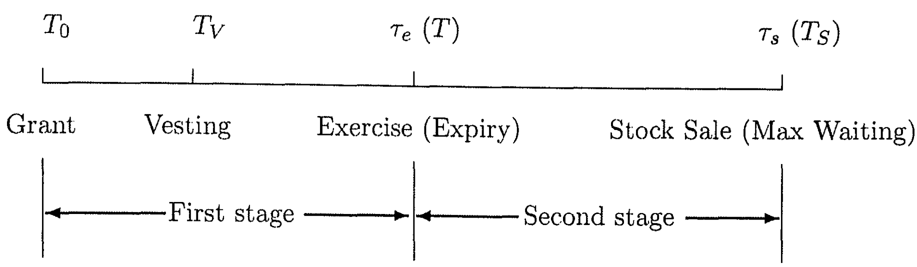

Figure 3.1: Timeline of the valuation model. This figure demonstrates the timeline of the valuation model. Two choice variables are in the model; they are optimal exercise time, τ_e and optimal sell time, τ_s. The employee optimally chooses both times to maximise expected terminal utility. The first stage is the conventional utility-based pricing model where the employee optimally exercises the option to maximize expected terminal utility. The maximum exercise time is option's expiry, T. The first stage ends once the option is exercised. The second stage is where the employee optimally decides to sell the stock. T_S is the maximum waiting time after the option exercise, as the tax benefit of waiting ends on that date. Stock is always sold at T_S if not already sold early. The second stage ends once the stock is sold. The model then works backward in time from τ_s or T_S and solve stock value at exercise date, τ_e. Then backward to solve option value at grant date, T_0.

retains this feature, as without tax considerations the employee would prefer to invest in the risk-free asset.

We model the employee exercise decision as a two-stage problem, in the first stage the employee chooses when to exercise the option and buy the stock at strike price; at the second stage the employee chooses when to sell the stock at the market price. As usual with PDE methods, we work backwards in time to price the employee's stock in the second stage, and then use this value to determine the employee's option price in the first stage. Naturally, both of these decisions depend on the appropriate tax rate as well as employee and stock characteristics. Figure 3.1 shows the timeline of the model.

3.3.1 Second stage

Since the model allows exercise in stages, wealth cannot be eliminated as a state variable. Note that once the option holding has been exercised, the terminal wealth can only be determined by knowing the sale time. We consider the second stage problem first where the employee has already exercised the option and holds a share of the firm stock. The employee chooses to sell the stock to maximize expected terminal utility. We use $S(W_t, P_t, t)$ to denote the employee's value function, then her goal is to maximize the expected utility of time $T_S(=\tau_e+T_W)$ wealth,

$$S(P_t,W_t,t)=\max_{\tau_s\in(\tau_e,T_S]} E_t\Big[S^*\big(W_{\tau_s}+K_{\tau_s},\tau_s\big)\Big] \tag{3.8}$$

Where τ_s is the optimal selling time for the stock. S^* is the indirect utility once stock has been sold, its functional form is derived below. K_{τ_s} is the payoff from selling the stock which depends on value of τ_s as we will see below. The final possible date for sale is T_S which is the option exercise date τ_e plus the waiting time (T_W) before the tax benefit can be taken. Typically, after 1 year has elapsed there is no tax benefit to hold the share and it will automatically be sold, so $T_W=1$ for this stage.

In the absence of the sale, the value function S (at the risk of ambiguity, we suppress its arguments and subscripts for concision) satisfies the following Bellman equation,

$$S_t dt+rWS_W dt+\mu PS_P dt+\frac{1}{2}\sigma^2P^2S_{PP}dt=0 \tag{3.9}$$

where the subscripts denote partial differentiation with respect to each argument. This leads to the no-sale partial differential equation,

$$S_t+rWS_W+\mu PS_P+\frac{1}{2}\sigma^2P^2S_{PP}=0 \tag{3.10}$$

After the stock has been sold, the executive's problem is reduced to investing in liquid wealth. The employee's Bellman equation in this case is

$$E_t\left[S_t^*dt+S_W^*dW+\frac{1}{2}S_{WW}^*(dW)^2\right]=0 \tag{3.11}$$

Since W follows a non-stochastic process, the employee's PDE is reduced to,

$$S_t^*+rWS_W^*=0 \tag{3.12}$$

it can be easily verified that the problem has the solution,

$$S^*(W_t,t)=e^{(1-\gamma)r(T_S-t)}\frac{W_t^{1-\gamma}}{1-\gamma} \tag{3.13}$$

Typically the tax benefits of the ISO are not realized if the stock is sold within one year of exercise and then all profits from the option are taxed at income tax rates. After one year of exercise, then for ISOs option profits are taxed at the long-term capital gains rate. The sale of the stock means that the employee receives X and pays capital gains tax (τ_{CG}) on the (positive difference) between P_t and X. At T_S, the terminal condition is

$$S(P_{T_S}, W_{T_S}, T_S) = \frac{(W_{T_S} + K_{T_S})^{1-\gamma}}{1-\gamma} \tag{3.14}$$

where K_{T_S} is payoff from selling the stock,

$$K_{T_S} = \begin{cases} P_{T_S} & \text{if} \quad P_{T_S} < X \\ (1-\tau_{GC})(P_{T_S} - X) + X & \text{if} \quad P_{T_S} > X \end{cases} \tag{3.15}$$

The employee sell the stock at T_S because there is no longer any tax benefit associated with waiting. Since the riskless asset provides same return, holding risky stock is simply not optimal.

Prior to T_S it is still possible to sell the stock early but here the employee has to pay income tax, τ_I, on any options profits. So at time $t < T_S$, it is optimal for the employee to sell the stock early if

$$S(P_t, W_t, t) < S^*(W_t + K_t, t) \tag{3.16}$$

where now K_t is payoff from selling the stock before T_S,

$$K_t = \begin{cases} P_t & \text{if} \quad P_t < X \\ (1-\tau_I)(P_t - X) + X & \text{if} \quad P_t > X \end{cases} \tag{3.17}$$

Thus, on early sale $S(P_t, W_t, t) = S^*(W_t + K_t, t)$. One thing to note is that as the payoff is always P_t if the stock price falls below X, it is always optimal to sell early as there is no tax benefit from holding the stock. Following Lambert et al. [1991] the employee's stock value can be calculated by considering the certainty equivalent. From equation (3.10) we can find the executive's value function $S(P_t, W_t, t)$ and from equation (3.13) we can determine the amount of money the executive would be willing to exchange for the option,

$$P_{\text{tax}}(P_t, W_t, t) = \left((1-\gamma)e^{-(1-\gamma)r(T_S-t)}S(P_t, W_t, t)\right)^{\frac{1}{1-\gamma}} - W \tag{3.18}$$

The value $P_{\text{tax}}(P_t, W_t, t)$ is the employee's valuation of the stock at the point of exercising when the stock price is P_t and the employee has liquid wealth W_t. As this expression will also be used in the first stage of the problem, this is a crucial input into the option exercise decision in the next subsection.

It is tempting to think that we have to work out this second stage for all options and restricted stock plans considered. In the case of NQSOs, the holder of the stock can choose to sell the stock immediately, incurring no extra tax payments, or to hold the stock potentially for up to one year where any profits from holding the stock incur long-term capital gains. However, in this scenario the employee has already paid income tax from the payoff of the option and so there is no additional tax saving to be made from holding the stock any further. For this reason, it is always sold in place of the risk-free rate as soon as the option is exercised. Thus we do not need to consider the second stage for these options.

Similarly, with restricted stock, once the stock vests, the employee pays income tax on the value P_{T_V} at the vesting date. At this point they could hold the stock for another year, paying only long-term capital gains on the profits, but they could also sell the stock and invest in the risk-free asset, which is a superior investment. Thus, for restricted stock, we also do not need to consider the second stage.

3.3.2 First stage

Now we consider the first stage of our option pricing formula – where the employee still holds an unexercised option. As before, the employee maximizes expected terminal utility. Similar to the second stage problem, we now assume that the employee's valuation function is $V(W_t, P_t, t)$, which is a function of the employee's wealth, the stock price, and time. Formally,

$$V(P_t, W_t, t) = \max_{\tau_e \in [T_V, T]} E_t \left[V^* \left(W_{\tau_e} + P_{\text{tax}}(P_{\tau_e}, W_{\tau_e}.\tau_e) - X, \tau_e\right)\right] \tag{3.19}$$

Where τ_e is option's optimal exercise time, T_V is the vesting period of the option, T is maturity of the option. $P_{\text{tax}}(P_{\tau_e}, W_{\tau_e}, \tau_e)$ is value of the stock obtained from second stage, it represents value of the stock to the risk-averse employee given an optimal sale strategy. V^* is the employee's valuation function defined in equation (3.22).

In this case, the employee's portfolio consists of W invested in the risk free asset and the stock option. This is different from the second stage where the employee has one stock. Similar to the second stage problem, in the absence of exercise the valuation function V satisfies the Bellman equation,

$$V_t dt + rWV_W dt + \mu PV_P dt + \frac{1}{2}\sigma^2 P^2 V_{PP} dt = 0 \tag{3.20}$$

so the no-exercise partial differential equation is

$$V_t + rWV_W + \mu PV_P + \frac{1}{2}\sigma^2 P^2 V_{PP} = 0 \tag{3.21}$$

Analogously to the second stage problem, once the option has been exercised the value function is,

$$V^*(W_t, t) = e^{(1-\gamma)r(T-t)}\frac{W_t^{1-\gamma}}{1-\gamma} \tag{3.22}$$

Upon exercise the employee pays the strike price X and receives a stock which can be sold at any future time. For an ISO we have determined the value of the stock to the employee in the previous subsection as $P_{\text{tax}}(P_t, W_t, t)$. So it is optimal to exercise the option if

$$V(P_{\tau_e}, W_{\tau_e}, \tau_e) < V^*\left(W_{\tau_e} + P_{\text{tax}}(P_{\tau_e}, W_{\tau_e}, \tau_e) - X, \tau_e\right) \tag{3.23}$$

At maturity (time T), the terminal condition for the value function is,

$$V(P_T, W_T, T) = V^*\left(W_T + \max(P_{\text{tax}}(P_T, W_T, T) - X, 0), T\right). \tag{3.24}$$

as here the option will only be exercised if $P_{\text{tax}}(P_T, W_T, T) - X > 0$.

The value of the option to the employee can again be determined by considering the certainty equivalent. The certainty equivalent of a option is the riskless amount the employee is willing to give up at time $t = 0$, so

$$V^*(W_0 + C, 0) = V(P_0, W_0, 0) \tag{3.25}$$

where C is the value of option. Based on equation (3.22) and (3.25), the ISO price can be obtained,

$$C_{\mathrm{ISO}}(P, W) = \left((1-\gamma)e^{-(1-\gamma)rT} V(P_0, W_0, 0)\right)^{\frac{1}{1-\gamma}} - W_0. \tag{3.26}$$

3.3.2.1 NQSOs and restricted stock

NQSOs have slightly different early exercise and terminal conditions as tax is paid at the point of exercise rather than when the stock is sold. As such, the early exercise is optimal (where τ_e is an exercise time) if

$$V(P_{\tau_e}, W_{\tau_e}, \tau_e) < V^*\left(W_{\tau_e} + (1-\tau_I)(P_{\tau_e} - X)^+, \tau_e\right) \tag{3.27}$$

where τ_I is the income tax rate. For the same reasons before, NQSOs are already taxed at option exercise date; it is simply not optimal to hold stock any longer. The stock is sold immediately after the option exercise. Hence the stock is valued at P_t rather than $P_{\mathrm{tax}}(P_t, W_t, t)$, as it is already settled in cash. At expiry, a similar condition applies,

$$V(P_T, W_T, T) = V^*\left(W_T + (1-\tau_I)(P_T - X)^+, T\right). \tag{3.28}$$

and

$$C_{\mathrm{NQSO}}(P, W) = \left((1-\gamma)e^{-(1-\gamma)rT} V(P_0, W_0, 0)\right)^{\frac{1}{1-\gamma}} - W_0. \tag{3.29}$$

For restricted stock it depends upon whether the 83b election has been selected or not. In the absence of an 83b election, as soon as the stock vests, the employee pays income tax. Sale is not possible before the vesting date. The restricted stock price then satisfies equation (3.10) with terminal condition

$$S(P_{T_V}, W_{T_V}, T_V) = \frac{(W_{T_V} + P_{T_V}(1-\tau_I))^{1-\gamma}}{1-\gamma} \tag{3.30}$$

where T_V is the vesting period for the stock. Thus the current value of this restricted stock is given by

$$P_{\mathrm{No83b}}(P, W) = \left((1-\gamma)e^{-(1-\gamma)rT_V}S(P_0, W_0, 0)\right)^{\frac{1}{1-\gamma}} - W_0 \tag{3.31}$$

The problem is slightly more complex when the employee makes a 83b election. Assume the stock is awarded at the initial price of P_0. The employee's valuation function, $S(P_t, W_t, t)$, is the same as that of a standard restricted stock. Similar to equation (3.30), the terminal condition of a 83b elected restricted stock is,

$$S(P_{T_V}, W_{T_V}, T_V) = S^*(W_{T_V} + P_{T_V} - \tau_{CG}(P_{T_V} - P_0)^+, T_V) \tag{3.32}$$

where S^* is valuation function defined in equation (3.13), T_V is the vesting date, at which the stock is sold. At issuance the value of the restricted stock is

$$P_{\mathrm{83b}}(P, W) = ((1-\gamma)e^{-(1-\gamma)rT_V}S(P_0, W_0, 0))^{\frac{1}{1-\gamma}} - W_0 - \tau_I P_0 \tag{3.33}$$

This result is slightly different from that of equation (3.31), as the last term $\tau_I P_0$ is income tax paid on the initial stock price.

3.3.3 Firm's valuation

Shareholders of the firm's are well diversified investors and so it is appropriate to value the option from their perspective by using a risk-neutral approach. The firm's valuation determines the stock option cost; it is also the amount of the grant that is expensed in the firm's income statement. Denote the risk neutral value by F. In the absence of exercise, the proportional drift of F under the risk-neutral measure is equal to the risk free rate,

$$E^Q[dF] = rFdt \tag{3.34}$$

As exercise is dependent upon the employee's wealth, W, the firm's cost also depends on W. This results in the following partial differential equation for F (note subscripts denote partial differentiation),

$$F_t + rWF_W + PF_P + \frac{1}{2}\sigma^2 P^2 F_{PP} - rF = 0 \tag{3.35}$$

The exercise strategy is determined by the employee's optimization problem outlined in the previous sections. At an exercise time t for particular values of W and P then

$$F(P_t, W_t, t) = (P_t - X)^+ - Y\tau_C(P_t - X)^+ \tag{3.36}$$

Where τ_C is corporate tax rate. Y is the indicator function defined in equation (3.37) below.

$$Y = \begin{cases} 1 & \text{if the option is NQSO} \\ 0 & \text{if the option is ISO} \end{cases} \tag{3.37}$$

The firm receives a tax deduction when an NQSO is exercised, but no deduction is available for ISO exercises. Smooth pasting conditions do not apply here because exercise is chosen to maximize the employee's value function V and not to minimize F. In other words, the exercise boundary is exogenous to the valuation model. For this reason, F is not necessarily continuously differentiable in W or P, although it must be continuous. The terminal condition for the PDE is

$$F(P_T, W_T, T) = (P_T - X)^+ - Y\tau_C(P_T - X)^+ \tag{3.38}$$

3.3.4 Expected time to exercise

We also calculate the expected lifetime of the entire option grant, θ. The expected lifetime represents the time until the compensation has expired or been exercised. In the absence of expiry, exercise, θ, follows a martingale. This is because without exercise, θ always equals option maturity, T.

$$E[d\theta] = 0 \tag{3.39}$$

The corresponding PDE is

$$\theta_t + rW\theta_W + \mu P\theta_P + \frac{1}{2}\sigma^2 P^2 \theta_{PP} = 0 \tag{3.40}$$

If (P_t, W_t, t) is a point on the exercise boundary, then the lifetime of the option is known with certainty and so

$$\theta(P_t, W_t, t) = t \tag{3.41}$$

At expiry the option lifetime is also known with certainty and so the terminal condition for the PDE is known,

$$\theta(P_T, W_T, T) = T \tag{3.42}$$

For restricted stock, expected time to exercise is equal to vesting period because holding stock does not provide higher return. Pollet et al. [2011] use the same measure to compute option expected lifetime.

3.3.5 Tax deduction

Option exercises offer firms an important non-debt tax shield. As firms can carry tax losses two years backwards as well as 20 years forward, deduction of option exercise can be realized in any year of the 22-year period or not realized at all if firms have a huge amount of other deductions. Actual ex post tax effects also depend on other firm characteristics. It is extremely difficult to know this 'deduction profit' beforehand, for example Graham et al. [2004] develop a simulation model to forecast tax deduction. For simplicity, we assume that the deductions of option exercise are recognized in the same year the option is exercised. Partially recognizing deduction throughout the 22-year period would have limited effect on our results, unless marginal corporate tax rate is a function of tax deduction.

3.3.6 Measure metrics

To evaluate the effectiveness of different compensation instruments, we use three measurement metrics to compare the incentives provided by different compensation packages. The first metric measures how much the executive's valuation differs from the shareholders issuance cost. Due to trading restriction and taxation, the employee's valuation is different from cost incurred by the company. We divide the subjective valuation of the employee to the company's cost. This gives a ratio,

$$\text{Efficiency Metric} = \frac{V(X, W_0, 0)}{F(X, W_0, 0)} \tag{3.43}$$

The higher this ratio is, the more effective the compensation instrument is. Since both value and cost prices are after tax, it is natural to compare the efficiency ratio of options with that of a cash salary; we define the cash ratio as follows,

$$\text{Cash ratio} = \frac{1 - \tau_I}{1 - \tau_C} \tag{3.44}$$

The higher the cash ratio, the higher income the employee gets for each dollar the company paid. As both company and employees are not tax neutral, different types of compensation instruments have different efficiency.

In addition to the efficiency measure, the second metric measures the incentive effects of the compensation package. We compute delta option value which is the rate of change of the employee's grant value with respect to changes in the stock price. Scaling delta by the company cost, then we know how much incentive each compensation instrument is providing per dollar/pound. This gives

$$\text{Delta metric} = \frac{\Delta(X, W_0, 0)}{F(X, W_0, 0)} \tag{3.45}$$

where

$$\Delta(X, W_0, 0) = \frac{V(X + dP, W_0, 0) - V(X - dP, W_0, 0)}{2dP} \tag{3.46}$$

and dP is a small change in the stock price. The higher the delta metric, the more incentives are provided for each dollar.

While the first two metrics measure the incentive and risk-taking effects of the compensation package, the final metric considers the expected lifetime of the compensation package. The analysis of retention should not be independent of the cost incurred, so we again scale expected time to exercise with company issuance cost.

$$\text{Retention metric} = \frac{\theta(X, W_0, 0)}{F(X, W_0, 0)} \tag{3.47}$$

This is considered as a proxy for how many years of retention the firm buys for each dollar that it spends. For restricted stock, this metric is very easy to compute as stocks have expected time to exercise equal to vesting period.

For options, the metric directly relates to wealth, as both θ and F are functions of W. In addition to direct dependence on wealth, θ will be high when the employee has high wealth, because wealth reduces risk aversion and delays value destroying early exercise. The employee's exercise choice then feeds back into the firm's valuation, increases the option's cost. Therefore, there is a positive correlation between θ and F.

The positive correlation biases the retention metric downward. This is due to risk aversion effect on both value of θ and F. When the employee is risk neutral, correlation between θ and F reduces to zero. In this case θ equals maturity of the option and F equals Black-Scholes value. As risk aversion increases, correlation between θ and F increases, and both values of θ and F go down. However, risk aversion has larger impact on θ, which resulted its value to drop faster than F, similar effects are also observed for efficiency and delta metrics, see Figures 3.2, 3.4. So the retention metric reduces as correlation between the two increases. Although the positive correlation biases the 'retention' effect, it still provides a general indication as to how much the firm is paying for the options' expected life.

3.4 Results and discussion

3.4.1 Parameter choice

We set firm volatility equal to 30% which is roughly the median volatility of executive stock option issuers from Carpenter [1998]. The risk free rate, r, is assumed to be equal to 4% and T, the time to expiration, equal to 10 years following Carpenter [1998] and Hall and Murphy [2002]. The vesting period, T_V is set to 3 years following a 1999 survey by the US Bureau of Labor Statistics which reports that '3 years was the average period needed for full grant vesting'.[23] In addition to the basic parameters of the firm stock, we assume personal income tax rate, τ_I, is equal to 0.33 which is currently the highest marginal tax rate in the US. Corporate tax rate, τ_c, is assumed to be 0.35; it is also the highest tax rate for corporations. Long-term capital gains tax τ_{CG} is assumed to be

[23] Report available at: http://www.bls.gov/opub/cwc/archive/spring2001art1.pdf

0.15. For simplicity we assume short-term capital gains rate is equal to the employee's income tax rate. For UK options, we use a similar assumption where all tax rates are from the highest tranche; according to HM Revenue & Customs, these rates are $\tau_I = 0.5$, $\tau_c = 0.28$ and $\tau_{CG} = 0.28$. All options in the chapter are at-the-money with an initial stock price $P_0 = 1$. We consider a range of starting liquid wealth from 0.25 to 5, which is a reasonable approximate of the employee's outside wealth (e.g. Hall and Murphy [2000, 2002] and Carpenter et al. [2010]).

3.4.2 Value to cost ratio

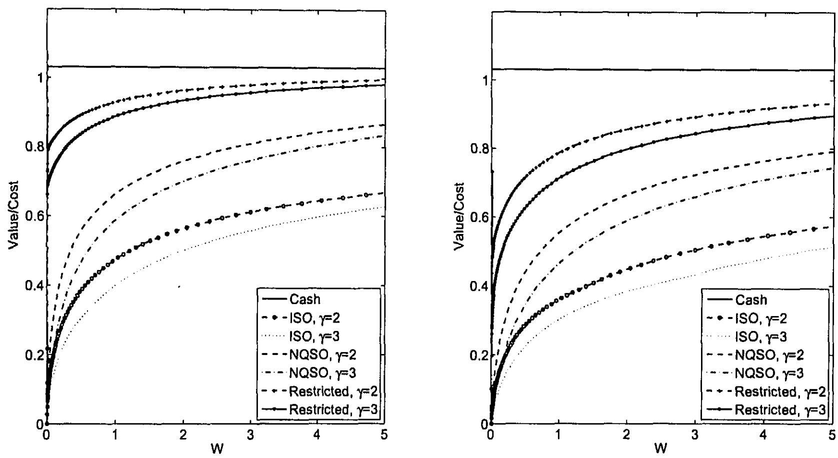

Figure 3.2: Efficiency metric. This figure plots the efficiency metric for at-the-money ISOs, NQSOs and restricted stock. The plot on the left has low volatility, $\sigma = 0.3$ and the plot on the right has high volatility, $\sigma = 0.5$. Other parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, stock price volatility, $\sigma = 0.3$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$

It is without doubt that ISOs are valued higher than NQSOs; see Table 3.2, for example, where ISO values are consistently higher than their respective NQSOs. Even though the employee values ISO at a high level, it is more costly for the firm to issue. All ISOs costs shown in Table 3.2 are higher than NQSOs' costs. Although the employee is heavily taxed for receiving an NQSO, the amount that is taxable to the employees is also available for corporate tax deduction, which substantially reduces the issuing cost.

Table 3.2: Value and cost of ISO and NQSO. This table reports value and cost of at-the-money ISO and NQSO. Parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$.

			$\sigma = 0.3$					$\sigma = 0.5$				
			Wealth					Wealth				
			0.25	0.5	1	2	5	0.25	0.5	1	2	5
		γ										
ISO	Value	2	0.139	0.183	0.229	0.275	0.329	0.123	0.169	0.219	0.279	0.364
	Cost		0.467	0.476	0.484	0.489	0.492	0.575	0.591	0.608	0.623	0.636
	Value	3	0.103	0.143	0.191	0.243	0.308	0.087	0.129	0.180	0.235	0.323
	Cost		0.455	0.468	0.479	0.485	0.491	0.559	0.576	0.595	0.612	0.630
	Value	4	0.083	0.121	0.164	0.219	0.296	0.067	0.105	0.154	0.211	0.297
	Cost		0.447	0.461	0.474	0.482	0.490	0.550	0.568	0.585	0.604	0.626
	Value	5	0.070	0.106	0.150	0.203	0.290	0.055	0.090	0.137	0.195	0.284
	Cost		0.447	0.455	0.468	0.480	0.489	0.547	0.560	0.578	0.598	0.621
	Value	6	0.061	0.096	0.140	0.192	0.291	0.047	0.079	0.125	0.185	0.284
	Cost		0.439	0.454	0.464	0.478	0.488	0.541	0.559	0.575	0.593	0.618
NQSO	Value	2	0.135	0.172	0.208	0.241	0.278	0.123	0.169	0.219	0.269	0.328
	Cost		0.303	0.309	0.315	0.318	0.320	0.374	0.384	0.395	0.405	0.414
	Value	3	0.103	0.142	0.182	0.221	0.267	0.087	0.129	0.180	0.234	0.305
	Cost		0.296	0.304	0.310	0.316	0.320	0.364	0.375	0.387	0.398	0.410
	Value	4	0.083	0.121	0.163	0.207	0.262	0.067	0.105	0.154	0.211	0.291
	Cost		0.291	0.300	0.307	0.313	0.319	0.357	0.369	0.380	0.392	0.406
	Value	5	0.070	0.106	0.150	0.197	0.263	0.055	0.090	0.137	0.195	0.284
	Cost		0.290	0.296	0.304	0.312	0.318	0.355	0.364	0.376	0.388	0.404
	Value	6	0.061	0.096	0.140	0.191	0.269	0.047	0.079	0.125	0.185	0.284
	Cost		0.285	0.295	0.302	0.310	0.317	0.352	0.363	0.373	0.385	0.402
Restricted Stock	Value	2	0.557	0.580	0.604	0.626	0.648	0.422	0.466	0.512	0.557	0.606
	Cost		0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650
	Value	3	0.513	0.544	0.577	0.608	0.639	0.356	0.408	0.464	0.519	0.583
	Cost		0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650
	Value	4	0.476	0.513	0.554	0.591	0.631	0.308	0.365	0.427	0.489	0.563
	Cost		0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650
	Value	5	0.443	0.487	0.533	0.577	0.625	0.272	0.332	0.397	0.464	0.548
	Cost		0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650
	Value	6	0.414	0.463	0.514	0.564	0.621	0.244	0.305	0.372	0.443	0.535
	Cost		0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650

That is why NQSOs have far higher efficiency ratios than those of ISOs (see Figure 3.2).

Restricted stock has the highest value and the cost out of the three forms of compensation instruments. Furthermore, it is equivalent to options with zero strike price. From a value and cost perspective, restricted stock has the highest efficiency ratio (Figure 3.2). This result is consistent with the view that restricted stock is the optimal form of compensation (Hall and Murphy [2002], Dittmann and Maug [2007]) and also with our prior analysis in section 3.2.3. However, if firms give compensation based on efficiency, they may do so by giving the employee cash instead, because per dollar cost, cash provides the highest efficiency ratio. But cash provides no incentives to the employee. Judging the best form of compensation depends on other measurement metrics.

Since employee valuation of stock options is utility based, it heavily depends on the employee's risk aversion. For example, in Figure 3.3 ISOs efficiency ratio changes considerably across different risk aversion. It is also worth noting that since ISOs issuing cost is not tax deductible, its efficiency ratio does not depend on corporate tax rate (section 3.4.5 has more detail on this). So cash may not have the highest efficiency ratio. Figure 3.3 also plots value and cost ratio when the employee is risk neutral at $\gamma = 0$. As expected, the ratio does not change with wealth as risk neutral employee's preference does not depend on wealth level. The value to cost ratio stays at 0.85, because both the employee and the firm are risk neutral. They all treat the ISO position as cash, since the risk neutral employee always sold the stock late to enjoy a favourable capital gains tax rate. The value to cost ratio is merely $\frac{1-\tau_{CG}}{1-\tau_C}$, which equals 0.85 for tax rates reported in Figure 3.3.

3.4.3 Delta metric

Even though options have the lowest efficiency ratio (ISO being the worst), they outperform cash and restricted stock in the delta metric. At high wealth, NQSOs provide the highest incentives per dollar cost, followed by ISOs, then restricted stock. At low wealth, restricted stock has the highest delta metric. At high volatility (Figure 3.4, right plot), the delta metric for all instruments drops. This is because a risk averse employee discounts risky instruments, so that value decreases with volatility. Firm cost,

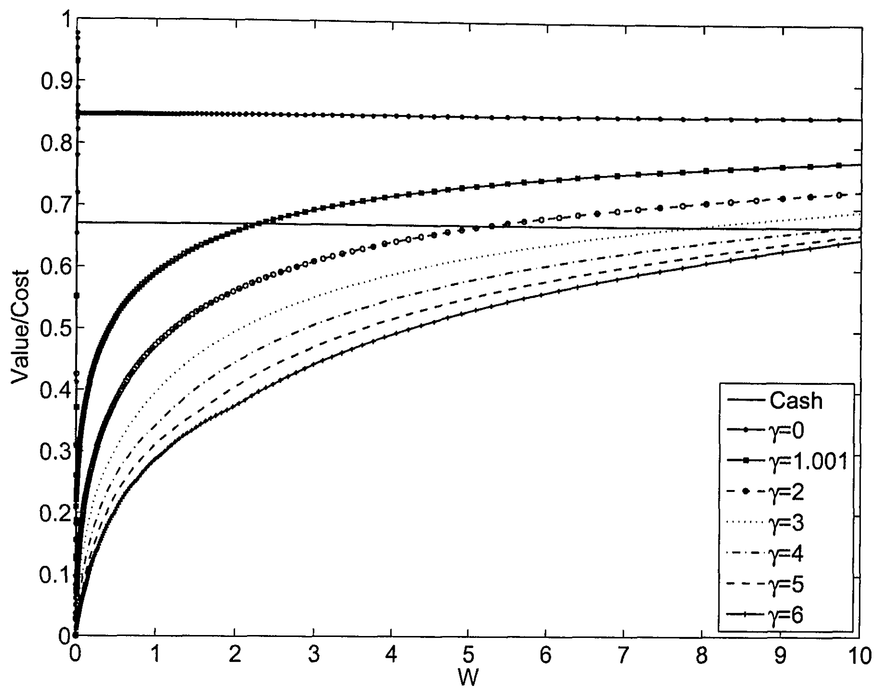

Figure 3.3: ISO with no corporate tax . This figure plots efficiency metrics for at-the-money ISOs with different risk aversion. The corporate tax rate is set to zero ($\tau_C = 0$) in the figure. Other parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, stock price volatility, $\sigma = 0.3$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$.

on the other hand, is computed based on risk-neutrality, and increases with volatility. The combined effect is a low delta metric at high volatility. Among all the instruments, options have the largest delta/value change because of its inherent risk.

Although NQSO outperforms ISO in both efficiency and delta metrics because of favorable tax deduction rules in the US, NQSO lost its incentives advantage under the UK tax rule. Figure 3.5 reports same metrics for UK tax system. EMI/CSOP is the UK equivalent of ISO that gives the employee favorable tax, similar to ISO in the US. UOP has its US equivalent NQSO. ER stands for entrepreneurs relief which is a special relief only available in the UK; under this tax scheme, employees only pay capital gains tax at 10%. Though the qualifying conditions for this scheme are quite strict, we still report results based on this scheme. Results in Figure 3.5 demonstrate

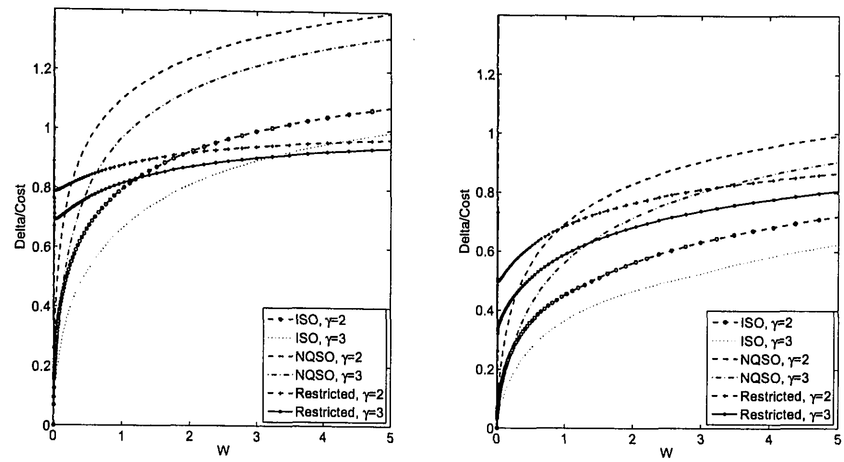

Figure 3.4: Delta metric . This figure plots delta metric for at-the-money ISOs, NQSOs and restricted stock for varying level of wealth. Plot on the left has low volatility, $\sigma = 0.3$ and plot on the right has high volatility, $\sigma = 0.5$. Other parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$

how taxation can affect the effectiveness of stock options. Under the UK's tax system, cash no longer provides the best value/cost ratio; restricted stock only dominates stock options at low wealth. When the employee has personal wealth larger than 2.5, ISO (or EMI/CSOP) is the most effective compensation instrument. For every pound of firm cost, ISO (or EMI/CSOP) also provides far greater delta than that of restricted stock. These results provide one explanation that most UK firms use ISO (or EMI/CSOP) up to their respective statutory limit.

3.4.4 Early exercise boundary

While restricted stock is sold once vested to avoid capital gains tax (because we assume risky stock has return equal to riskless asset), stock options are held much longer after the vesting date. The flexibility of the model allows us to easily investigate the exercise boundary for different compensation options. Figure 3.6 plots the exercise boundary of NQSOs and ISOs with different wealth levels and different time to expiration. When liquid wealth is high, at $W = 3$, the ISO is exercised earlier than that of NQSO because

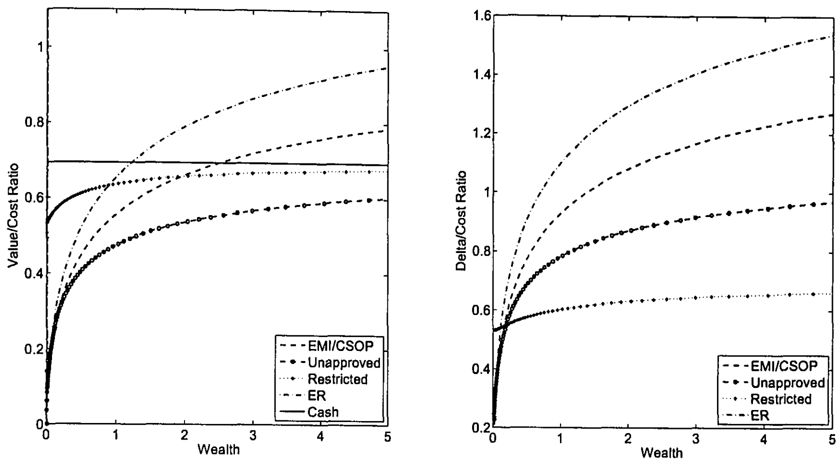

Figure 3.5: Efficiency and delta metric, the UK case. This figure plots efficiency and delta metric for at-the-money ISOs (or EMI/CSOP in the UK), NQSOs (or unapproved option scheme) and restricted stock. ER (entrepreneurs relief) is special tax scheme that provides qualifying employees with capital tax rate of 10%. UK tax rate are: income tax rate, $\tau_I = 0.5$, capital gains tax, $\tau_{CG} = 0.28$, corporate tax rate, $\tau_c = 0.28$. Other parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, $\sigma = 0.3$.

the employee expects to hold stock from exercising ISO for another year, as shown in Figure 3.7. Stocks (from high wealth state, $W > 0.75$) are held until one year has elapsed.

When liquid wealth is low, at $W = 0.15$, ISO and NQSO are exercised in almost the same way. Their exercise boundaries are almost identical. This is because at low wealth, the employee becomes very risk averse and impatient. She simply forgoes the tax benefits from ISO and exercises as if it was a NQSO. Results in Table 3.2 and 3.3 also suggest that ISOs are treated as NQSOs. For example, when $\gamma = 4$, $W = 0.25$ and $\sigma = 0.3$, both ISO and NQSO have the same value, 0.083. Their expected exercise time is the same too, at 6.032. This raises the question – why grant risk averse employees ISOs which cost the issuing firm a large sum but are treated by the employee as NQSOs? The firm can simply grant NQSOs instead which achieves the same level of retention as of ISOs.

When liquid wealth is moderately low, for example at $W = 0.5$, the employee is not very risk averse. ISO values are higher than that of NQSO, so that holding the ISO

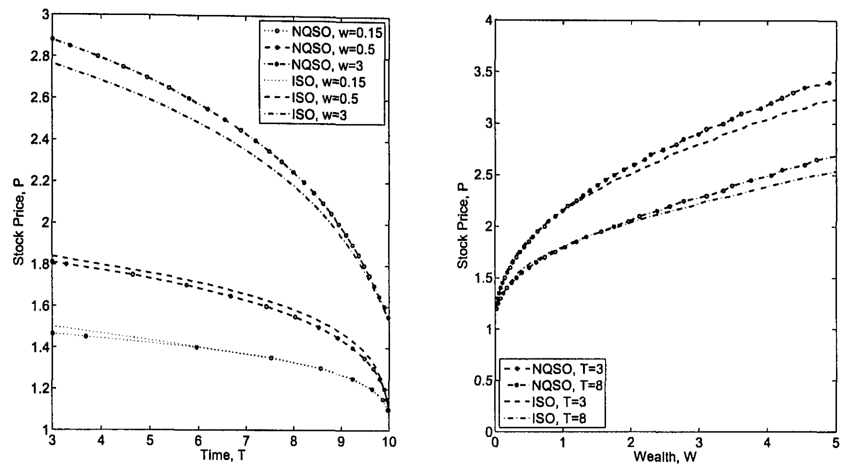

Figure 3.6: Option exercise boundary. The figure plots option exercise boundary at both time and wealth dimension. Options are always exercised above the exercise boundary. Parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, stock price volatility, $\sigma = 0.3$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$.

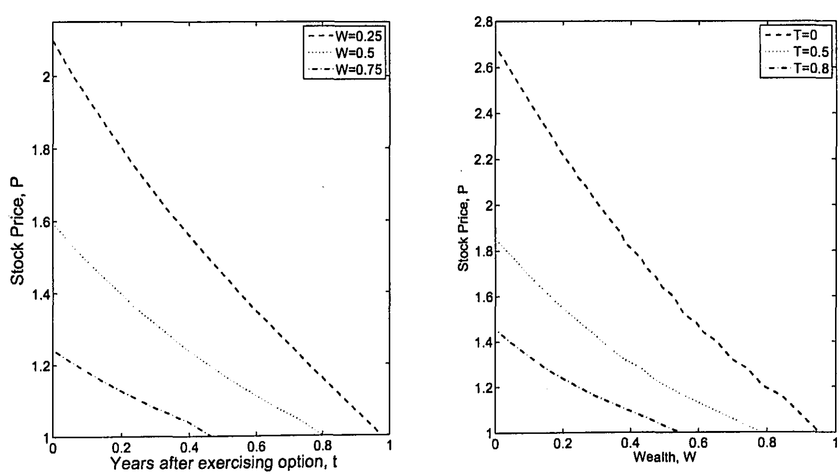

Figure 3.7: Stock selling boundary. The figure plots stock sell boundary at both time and wealth dimension. Parameters are those of Figure 3.6. Stocks are always sold below the selling boundary.

longer leads to a higher terminal utility. ISO is held longer even though the employee is fully aware that she is holding stocks from exercising ISO one more year. These results suggest that the company should grant ISOs to a relatively low risk averse employee and NQSOs to high risk averse employees.

Table 3.3: Expected exercise time of options. Expected exercise time are computed based on equation (3.40). Parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, income tax rate, $\tau_I = 0.33$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$.

		$\sigma = 0.3$					$\sigma = 0.5$				
		Wealth					Wealth				
		0.25	0.5	1	2	5	0.25	0.5	1	2	5
	γ										
ISO	2	6.888	7.391	7.924	8.446	9.106	7.026	7.383	7.788	8.227	8.656
	3	6.329	6.950	7.553	8.079	8.783	6.691	7.046	7.483	7.896	8.435
	4	6.032	6.622	7.261	7.825	8.561	6.498	6.859	7.237	7.683	8.303
	5	6.014	6.333	6.926	7.657	8.410	6.437	6.697	7.075	7.530	8.169
	6	5.712	6.287	6.756	7.506	8.303	6.338	6.681	7.011	7.413	8.072
NQSO	2	6.852	7.365	7.941	8.525	9.239	7.026	7.383	7.787	8.204	8.718
	3	6.329	6.899	7.465	8.104	8.913	6.691	7.046	7.483	7.896	8.465
	4	6.032	6.622	7.168	7.806	8.675	6.498	6.859	7.237	7.683	8.289
	5	6.014	6.333	6.926	7.599	8.509	6.437	6.697	7.075	7.530	8.166
	6	5.712	6.287	6.756	7.415	8.391	6.338	6.681	7.011	7.413	8.072

Figure 3.7 plots the stock sale boundary of ISOs. Stock obtained from exercising NQSOs is always sold immediately. As explained previously, NQSO is taxed at exercise (and at income tax rate), which also moves the tax base to the current stock price. It is optimal to sell once exercised to avoid capital gains tax. On the other hand, stock obtained from exercising ISOs is only sold early when the stock price is sufficiently high and the employee has low wealth. For example, when wealth is 0.25, stock from early exercised ISOs is held slightly longer. As wealth increases, stock is held longer. When wealth is high enough, stock from exercising ISOs is always sold after the one year period to avoid higher income tax rate. This explains why ISOs are exercised earlier than NQSOs at high wealth. As stocks from ISOs are held for another year, the employee exercises the option early to offset the additional waiting before selling the stock.

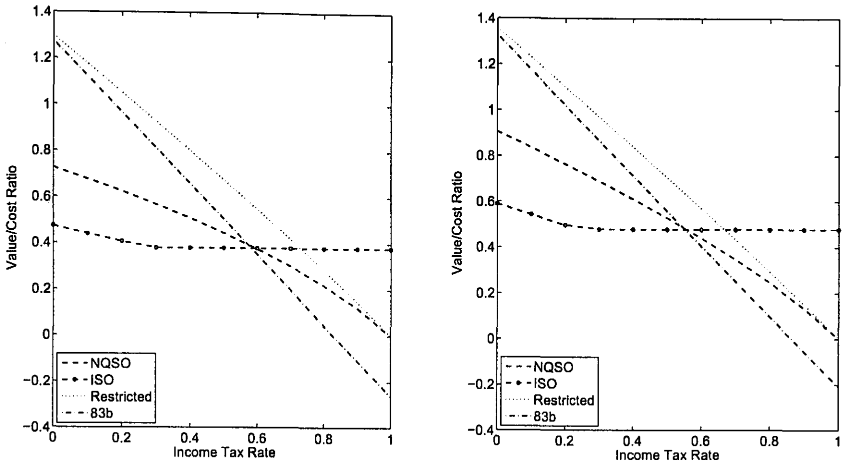

Figure 3.8: Income tax effect. The figure plots value to cost ratio with varying level of income tax rate for all four compensation instruments – Non-qualified stock option (NQSO), incentive stock option (ISO), restricted stock and restricted stock with 83b election. Restricted stock is based on initial stock price of $P_0 = 1$. The left plot uses wealth, $W = 0.5$ and the plot on the right uses wealth, $W = 1$. Parameter values are: initial stock price, $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 3$, stock price return, $\mu = 0.04$, stock price volatility, $\sigma = 0.3$, capital gains tax, $\tau_{CG} = 0.15$, corporate tax rate, $\tau_c = 0.35$.

3.4.5 Tax effect

Using assumptions outlined in section 3.2.3, we compare the tax effect on efficiency of compensation instruments. Figures 3.8, 3.9, 3.10 plot efficiency ratios with a varying level of employee's income tax, corporate tax and capital gains tax rate. While varying one tax rate, we keep the other two tax rates fixed at the benchmark rates which are $\tau_I = 0.33$, $\tau_c = 0.35$, $\tau_{CG} = 0.15$.

3.4.5.1 Income tax

Holding the corporate tax and capital tax constant, we plot the efficiency ratio with varying levels of income tax. Similar to Graham [2003], who argues that NQSO is preferred over ISO when the corporate tax rate is higher than the income tax rate, NQSO is preferred when income tax is higher than corporate tax rate. However, our result suggests that for NQSO to be preferred, income tax rate has to be substantially higher (almost twice) than the corporate tax rate. The magnitude is different from

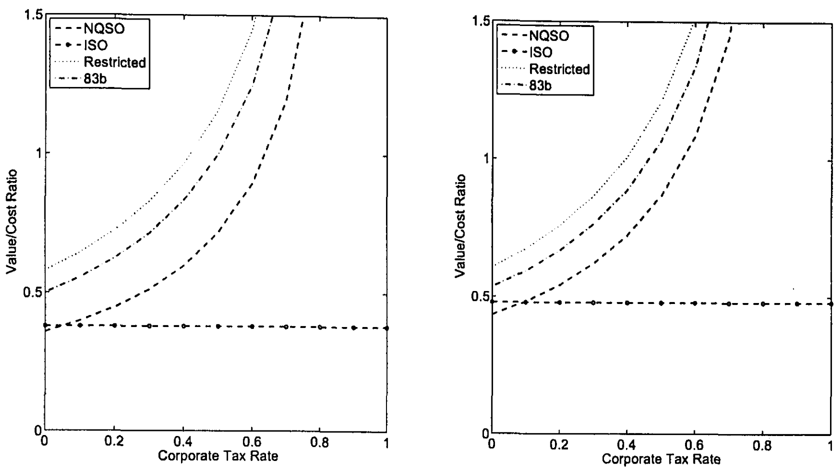

Figure 3.9: Corporate tax effect. The figure plots value to cost ratio with varying level of corporate tax rate for all four compensation instruments – Non-qualified stock option (NQSO), incentive stock option (ISO), restricted stock and restricted stock with 83b election. Restricted stock is based on initial stock price of $P_0 = 1$. Parameter values are those outlined in Figure 3.8. The left plot uses wealth, $W_0 = 0.5$ and the plot on the right use wealth, $W_0 = 1$.

our intuitive result outlined in section 2, because early exercise delays the tax liability. When income tax is 100%, NQSO is simply worth nothing since all value of the stock is taxed away. The ISO, however, is not affected by change of income tax. When income tax is low, it is optimal to sell the stock (which is obtained by exercising the option) early since income tax is lower than capital gains tax. As income tax increases, this early selling advantage disappears. It is then always taxed at the capital gains rate, so that the efficiency of an ISO stays constant and eventually becomes advantageous to NQSO.

Restricted stock with an 83b election has efficiency lower than 0, because it is double taxed at both income and capital gains tax rate. For example, when income tax is 100%, the employee with restricted stock and 83b election has to pay 100% of the stock price at grant date, so she values the stock at zero. Subsequent sale of the stock is taxed again at a capital gains rate which means the value of the stock is lower than zero. With the effect of risk aversion and vesting, its value should be worth even less.

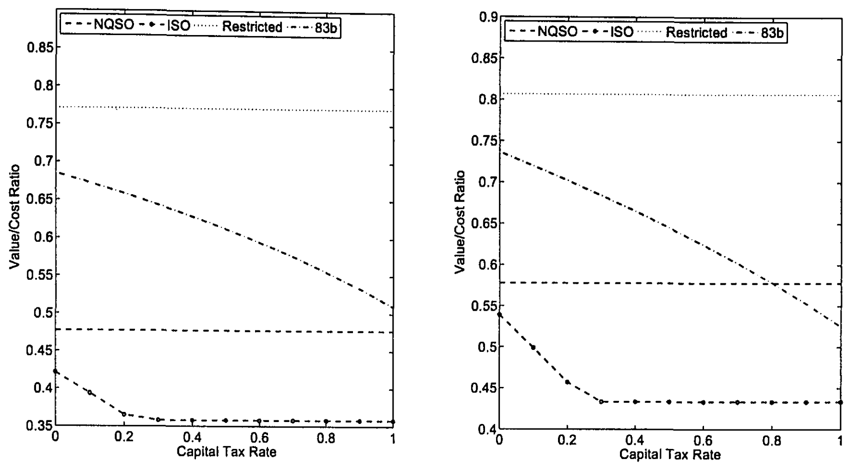

Figure 3.10: Capital gains tax effect. The figure plots value to cost ratio with varying level of capital gains tax rate for all four compensation instruments – Non-qualified stock option (NQSO), incentive stock option (ISO), restricted stock and restricted stock with 83b election. Restricted stock is based on initial stock price of $P_0 = 1$. Parameter values are those outlined in Figure 3.8. The left plot uses wealth, $W_0 = 0.5$ and the plot on the right use wealth, $W_0 = 1$.

3.4.5.2 Corporate tax

Since firms receive deductions for granting NQSOs and restricted stock, efficiency goes to infinity when the corporate tax rate equals 1. At this point the issuance cost of compensation is zero. ISOs are not eligible for tax deduction; their efficiency ratio stays constant with every level of corporate tax.

3.4.5.3 Capital gains tax

Restricted stock and NQSO are not affected by the level of capital gains tax. This is because holding stock is not optimal (stock offers same rate of return as that of the riskless asset) so that stocks are always sold once vested (or obtained by exercising). Efficiency of restricted stock with 83b election drops as capital gains tax increases, since capital gains tax is charged at difference of grant price and sale price. This makes the value of 83b elected stock drop with the capital gains tax rate. ISOs' efficiency drops when the capital gains tax rate is low, and stays constant when capital gains tax is high. This is because prematurely selling stock (which is obtained from exercising ISO)

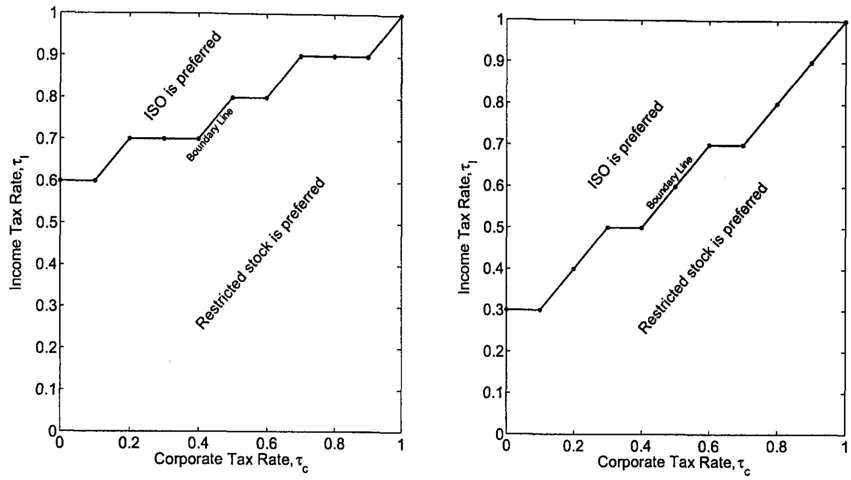

Figure 3.11: Tax preference. The figure compares value to cost ratios of four compensation instruments – ISO, NQSO, restricted stock, restricted stock with 83b election. Based on value to cost ratio, above the boundary line, ISO is always preferred. Below it, restricted stock is always preferred. NQSO and 83b are not shown in the graph because they are dominated by either ISO or restricted stock. Figure 3.12 has more details on this issue. The left plot uses initial wealth $W_0 = 0.25$ and the right plot uses initial wealth $W_0 = 5$. Other parameter values are those outlined in Figure 3.8, capital gains tax, τ_{CG} is 0.15.

attracts income tax; when capital gains tax is low selling stock early is not optimal, stock is always held after one year to get a lower tax rate. As the capital gains increases, this late sale premium drops and eventually disappears. So stock is always sold early to be taxed at a lower income tax rate.

3.4.5.4 Tax preference

Figure 3.11 compare value to cost ratio of all compensation instruments in a income/corporate tax space. On the upper left corner of both plots, ISO is preferred with higher value to cost ratio. This is because income taxes on ISOs can be avoided by selling stock early, in which case it is taxed at capital gains tax. That is why ISO has high value to cost ratio even though income tax rate is 100%. Restricted stock is preferred in the lower right corner as its value to cost ratio dominates that of both NQSO and 83b elected stock. This result is expected as our previous discussion in section 3.2.3 shows that restricted stock is always preferred to NQSO. Stock with 83b

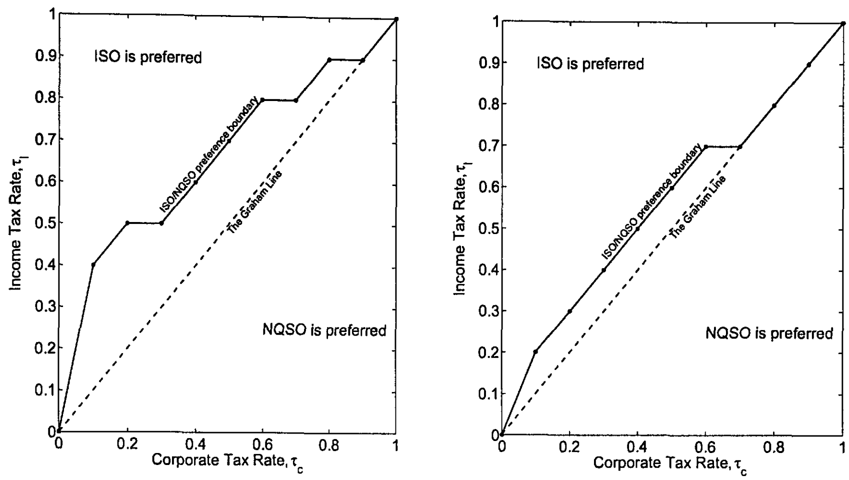

Figure 3.12: ISO and NQSO preference. The figure compares value to cost ratios of ISO and NQSO with varying level of corporate tax and income tax. Based on value to cost ratio, above the preference boundary, ISO is always preferred. Below it, NQSO is always preferred. The Graham Line is the well known result from Graham et al. [2004], which argues that when corporate tax rate is higher than income tax rate, NQSO is preferred to ISO. This argument is demonstrated in equation (3.2). The left plot uses initial wealth $W_0 = 0.25$ and the right plot uses initial wealth $W_0 = 5$. Other parameter values are those outlined in Figure 3.8, capital gains tax, τ_{CG} is 0.15.

election is not preferred either as our discussion in the later section shows that 83b election is never optimal.

The right plot in Figure 3.11 shows the same results but with high level of wealth. The boundary line that delineates the two regions shifts down, because of the wealth effect on the employee's valuation which increase options' value to cost ratio. A similar wealth effect is also observed in Figure 3.12, which compares ISO and NQSO in the same tax space. This figure demonstrates our earlier claim that NQSOs can be preferred even if the income tax rate is substantially higher than the corporate tax rate. As the left plot in Figure 3.12 shows NQSO is preferred even though income tax is twice corporate tax. There is a large derivation from the conventional argument suggested in Graham et al. [2004], which argue that NQSO is preferred to ISO when corporate tax rate is higher than the income tax rate. Such a derivation exists due to the employee's risk aversion. As the right plot in Figure 3.12 shows when wealth level is high, the preference boundary converges back to The Graham Line.

From a simple analysis of this section, it seems ISOs' efficiency is less sensitive to the changes of tax rate, but more sensitive to change of wealth (or risk aversion). In fact, both ISO's value and cost are insensitive to tax changes. Section 83b election can result in negative values to the employee because of double taxation.

3.4.6 Stock or options: a principal-agent argument

The on-going debate about the optimal mix of compensation package has tilted toward stock in recent years as documented in Conyon et al. [2011]. One explanation that the empirical literature offers is mandatory expensing of options, which disarms the options' tax and accounting advantage. Dittmann and Maug [2007] argue that stock is the optimal form of compensation even though options offer a higher incentive per dollar of cost. The reason for this is that the employee values options at far lower price than the market does; in order to provide the same level of compensation to the manager, options actually cost more to the firm. This result holds well in a one-period principal-agent model, where participation and incentive constraints have to be satisfied. Their results hold well under the US tax rules. But under the UK tax rules, as we show next, ISO is the best form of compensation,

To demonstrate the problem, we use a numerical example similar to Dittmann and Maug [2007]. In a standard principal-agent setting, the employee only works if the compensation offered exceeds her reservation utility, which is known as the participation constraint. In addition, the contract has to incentivize her action, which evokes the incentive compatibility constraint. Since we make no assumption about the employee's production function, we assume that the compensation contract is designed to achieve a certain level of incentives, I. We also assume that the reservation wage is R. This means compensation awarded to the employee has at least a value of R for her to accept the job. So,

$$C(m, n, P, W) + m(1 - \tau_I) = R \tag{3.48}$$

$$\Delta\big(m(1 - \tau_I), n, P, W\big) = I \tag{3.49}$$

where n denotes number of stock or options awarded to the employee, and m denote the amount of cash salary awarded to the employee. And $C(m, n, P, W)$ is the value

Table 3.4: Incentive effect of compensation contract. This table reports different compensation contracts that provide the employee the same level of value and delta. The compensation contract consists of number of stocks or options (n) and amount of fixed wage (m) paid. n and m are obtained by keeping Δ and value (of the employee) at fixed levels; firm cost then computes accordingly. For convenience of comparison, the fixed levels Δ and value are chosen to be that of 5 ISOs ($n = 5$) plus fixed wage of 5 ($m = 5$), then total value of the compensation is computed based on, Value $= C_{\text{ISO}}(m(1-\tau_I), n, X, W) + (1-\tau_I)m$. Incentives are merely that of, $\Delta(m(1-\tau_I), n, X, W)$. Other combinations of compensation contracts are obtained by varying n and m so that all compensation contracts have the same incentives and value. m amount of fixed wage has two effects on the employee's valuation: firstly, it increases the employee's value by $(1-\tau_I)m$, this is the riskless part of her compensation. Secondly, it increases the employee's valuation of ESO or restricted stock because the employee's wealth is increased by $(1-\tau_I)m$ which affects her valuation of all risky instruments. Firm cost is computed based on, $F_{\text{Cost}} = F(m, n, X, W) + (1-\tau_C)m$. Retention is defined: $\frac{\theta\left(m(1-\tau_I),n,X,W,0\right)}{F_{\text{Cost}}}$. Parameter values are those of Figure 3.6.

Wealth	0.25	0.5	1	2	5
Fixed Δ	1.758	1.791	1.851	1.953	2.170
Fixed value	4.380	4.403	4.444	4.514	4.663
Panel A: Restricted stock					
n	3.110	3.153	3.232	3.362	3.628
m	3.749	3.735	3.707	3.653	3.571
Firm Cost	4.458	4.477	4.510	4.560	4.679
Retention	0.673	0.670	0.665	0.658	0.641
Panel B: Incentive stock option					
n	5	5	5	5	5
m	5	5	5	5	5
Firm Cost	5.654	5.657	5.663	5.673	5.690
Retention	1.361	1.366	1.361	1.368	1.416
Panel C: Non qualified stock option					
n	5.605	5.625	5.660	5.719	5.838
m	4.988	4.989	4.990	4.991	4.994
Firm Cost	4.988	4.994	5.006	5.041	5.096
Retention	1.575	1.580	1.567	1.565	1.603
Panel D: Restricted stock with 83b election					
n	3.731	3.777	3.858	3.986	4.228
m	3.659	3.640	3.605	3.549	3.449
Firm Cost	4.804	4.821	4.851	4.898	4.990
Retention	0.625	0.622	0.618	0.612	0.601

Table 3.5: Incentive effect of compensation contract, the UK case. Tax rates for the UK are: income tax rate, $\tau_I = 0.5$, capital gains tax, $\tau_{CG} = 0.28$, corporate tax rate, $\tau_c = 0.28$. Other parameter values are the same as those of 3.6. EMI/CSOP is the UK version of ISO. Unapproved option scheme is the UK version of NQSO.

Wealth	0.25	0.5	1	2	5
Fixed Δ	1.445	1.481	1.545	1.649	1.857
Fixed value	3.342	3.366	3.410	3.482	3.624
			Panel A: Restricted stock		
n	3.450	3.515	3.627	3.805	4.145
m	3.614	3.589	3.546	3.481	3.354
Firm Cost	5.086	5.115	5.165	5.246	5.399
Retention	0.590	0.587	0.581	0.572	0.556
			Panel B: EMI/CSOP		
n	5	5	5	5	5
m	5	5	5	5	5
Firm Cost	4.228	4.231	4.237	4.245	4.259
Retention	1.802	1.815	1.793	1.187	1.908
			Panel C: Unapproved option scheme		
n	6.325	6.351	6.400	6.472	6.523
m	4.983	4.985	4.985	4.991	4.994
Firm Cost	5.328	5.331	5.337	5.345	5.359
Retention	1.430	1.441	1.424	1.451	1.516

of the compensation instruments given m, n, P and W. The employee's valuation is a function of cash and the number of stocks or options awarded, as both m and n change the employee's liquid wealth. Following the same logic, $\Delta\big(m(1-\tau_I), n, P, W\big)$ is also a function of m and n. This requires making a slight adjustment to the original model to include m and n. The firm cost is computed once m and n are determined.

Equation (3.48) is the participation constraint and equation (3.49) is the incentive compatibility constraint. It is worth noting that this is not strictly a principal-agent model, as the value and incentives of restricted stock and options changes dynamically with time and stock price. A continuous-time principal agent model, where incentives and participation constraints are continuously adjusted, is more formal in this regard. Our point is very simple: the two conditions are binding at the time the compensation package is accepted, but not every time afterwards.

For a given level of wealth, the above two equations define a unique contract mix. Since all stock and options values are computed as certainty equivalent dollar amounts, it is straightforward to compare these results. Table 3.4 shows the compensation mix with different levels of wealth. The results for the US are consistent with Dittmann and Maug [2007], who argue that use of restricted stock is the preferred form of compensation, as the firm incurs less cost for providing the same level of compensation and incentives.

This result, however, does not always hold in the UK. When the wealth level is high, options dominate restricted stock with a lower cost to the firm. This is due to a favorable tax treatment, higher income and lower corporate tax rates. The conclusion in Dittmann and Maug [2007] is based on the US system where tax favourable options do not induce corporate tax deduction. Since options are always valued lower than restricted stock when there is no tax, unless taxes are designed to favour options, it is always cost efficient to reward stock. The UK results, as in Table 3.5, show that CSOP/EMI (or ISO) options have lower firm cost. This may partly explain why EMI/CSOPs are always used up to their statutory limits.

Compensation awarded loses its incentive effect once sold. It is also typical for firms to reward new instruments as old ones expire. As documented in the previous section, the expected lifetime of restricted stock is merely equal to its vesting period, which provides a very limited period of retention. Stock options have an expected lifetime far exceeding their vesting period. Once a compensation instrument is sold, its incentive effects on the employee are lost. So a new one has to be awarded to retain and incentivize the employee. We also report retention metrics for all compensation instruments.

Retention is far higher for options than it is for stocks, because options have an expected lifetime that is far longer than stock.[24] From a static one-period principal agent model, retention is not a problem because the contract lasts only one period. Taking into account the retention cost, options are the preferred form of compensation. Setting the contract period at 10 years, options cost as little as half that of stock to retain the employee (because options have retention metrics almost twice that of restricted stock). Under the UK system, the retention of an option is almost three times that of stock, suggesting stock is even more costly when retaining employees. This may again partly explain the popularity of stock options in startup technology firms where employee retention is crucial.

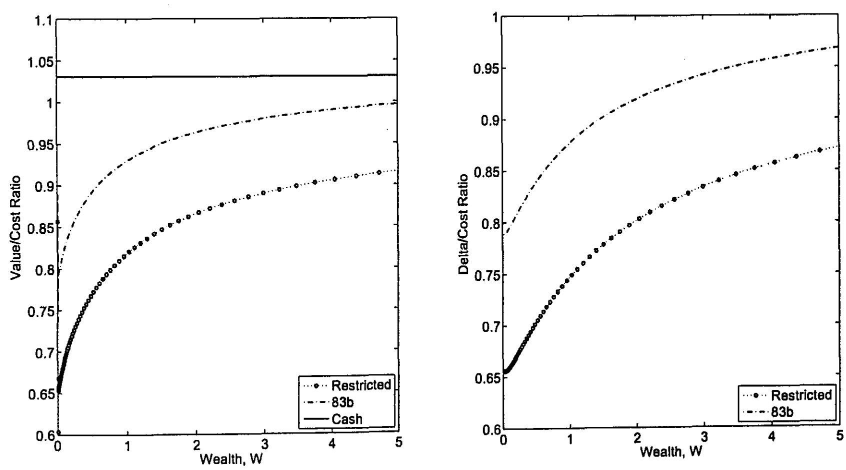

Figure 3.13: Section 83b election. The figure plot efficiency and delta ratio of section 83b election. Parameter values are those outlined in Figure 3.6.

[24]In arriving at the retention measure, we make an simplifying assumption that new stock and option grants have the same expected lifetime as that previously granted. While restricted stock in our setting has the expected lifetime equal to the vesting period (there is no variation with respect to stock price either; in other words, it is not random), options' expected lifetime varies with stock, strike price, vesting period and maturity. Actual exercise behaviour (and eventual sale of stock) is path dependent, so is option lifetime.

3.4.7 Section 83b election

Section 83b election has a value far lower than a standard restricted stock. This is due to the fact that 83b election only provides the tax saving in a particular scenario – the stock price goes up in the future. This result is also consistent with McDonald [2003] who shows that it is generally not optimal to make 83b elections. While from a valuation view it is not optimal to make 83b elections, Blouin and Carter [2007] argue that the 83b election increases employee incentives. Our results show that 83b election actually decreases incentives. As Figure 3.13 shows, incentives from the 83b election are much lower than standard restricted stock. This is because the the 83b election only increases incentives when stock price goes up. When price falls, incentives go down for the 83b elected stock.

To show this point, we also report the results for the principal-agent argument shown in the previous section. The results shown in panel D of Table 3.4 support the fact that section 83b is not an optimal form of compensation. As for the same level of value and incentives, 83b election costs the firm more than the standard restricted stock.

3.5 Conclusions

This chapter advances our understanding of the employee stock option valuation with an in-depth study of options under different tax schemes. None of the previous literature on ESO valuation has incorporated taxation, which is a deciding factor of the employee's exercise decision. We introduce a simple model that incorporates tax rules on different compensation instruments. Based on the three metrics introduced, we evaluate the effectiveness of different compensation instruments; our result show that, in the US, nonqualifying stock option (NQSO) is the most efficient compensation instruments in terms of value, incentives and retention. We also show that ISOs are generally exercised earlier than NQSOs, because employees expect to hold stock (from exercising ISO) one more year.

Our results also show the 83b election, which accelerate income tax on restricted stock, is not optimal and can lead to double taxes to its recipient. Contrary to Blouin and Carter [2007], who argue that 83b election increases incentives, our results show that 83b reduces incentives. Under the election, incentives only increase in a particular scenario, that is when stock price goes up. Since it is difficult to know future stock price ex-ante, the only plausible explanation for making such election is employee over-confidence.

Based on a similar argument in Dittmann and Maug [2007], we calibrate the after tax price of compensation instruments to a simple principal-agent model. Our results show that incentive effects alone do not justify the wide use of options in the US because stocks can provide the same level of incentives at lower compensation cost. However, options can lead to far lower retention costs to the firm; in our numerical example stocks cost almost twice that of options to induce employee incentives. We also calibrate the model based on UK tax rules. Unlike ISOs in the UK, tax favourable options schemes (EMI/CSOPs) provide better value, incentives and retention. Possibly this is one reason why those options are always used up to their respective statutory limit. Although tax minimization is not a main motive for an option grant (Babenko and Tserlukevich [2009], Graham [2003]), tax rules certainly explain the wide popularity of NQSOs in the US and EMI/CSOPs in the UK.

3.6 Appendix

3.6.1 Numerical scheme

PDEs in this chapter are solved using the implicit finite difference method. We take equation 3.10 as an example to illustrate the numerical procedure. For convenience we restate the equation below,

$$S_t + rWS_W + \mu PS_P + \frac{1}{2}\sigma^2 P^2 S_{PP} = 0 \tag{3.50}$$

The equation can be log transformed to make coefficient on differential term constant, we do this to the wealth term. So that the equation becomes,

$$S_t + rS_w + \mu PS_P + \frac{1}{2}\sigma^2 P^2 S_{PP} = 0 \tag{3.51}$$

where $w = \log(W)$. Only wealth is transformed, as similar transformation on stock space needs interpolation when calculating the final price. The wealth space is discretized as follow,

$$w(k) = w_0 \exp(k\Delta w) \qquad for \quad k = -N, ...0..., N \tag{3.52}$$

and w_0 and Δw is defined,

$$\begin{aligned} w_0 &= w_{\max} \exp(-N\Delta w) \\ \Delta w &= \frac{\log\left(\frac{w_{\max}}{w_{\min}}\right)}{2N} \end{aligned} \tag{3.53}$$

$w_{\max}$ and $w_{\min}$ are maximum and minimum of the wealth space. Since utility is not defined for negative wealth, $w_{\min}$ is chosen to be a small value. We choose 0.01 for computation used in the chapter.

The stock space is not transformed, so we follow conventional scheme

$$P(j) = P_{\min} + j\Delta P \qquad \text{for} \quad j = 0, ..., M \tag{3.54}$$

and ΔP is

$$\Delta P = \frac{P_{\max} - P_{\min}}{M} \tag{3.55}$$

Time space is similarly defined,

$$\Delta t = \frac{T}{L} \tag{3.56}$$

where M, N, and L define grid size of the problem space. Use $S_{j,k,i}$ to denote each node on the three-dimensional space. The PDE is discretized to the following *implicit* difference equation,

$$\begin{aligned} &\frac{S_{j,k,i+1} - S_{j,k,i}}{\Delta t} + r\frac{S_{j,k+1,i} - S_{j,k-1,i}}{2\Delta w} + \mu P(j)\frac{S_{j+1,k,i} - S_{j-1,k,i}}{2\Delta P} \\ &\frac{1}{2}\sigma^2 P(j)^2 \frac{S_{j+1,k,i} - 2S_{j,k,i} + S_{j-1,k,i}}{\Delta P^2} = 0 \end{aligned} \tag{3.57}$$

At each node $S_{j,k,i}$, the equation has the following solution.

$$S_{j,k,i} = \left(\frac{S_{j,k,i+1}}{\Delta t} + r\frac{S_{j,k+1,i} - S_{j,k-1,i}}{2\Delta w} + \mu P(j)\frac{S_{j+1,k,i} - S_{j-1,k,i}}{2\Delta P} \right. \\ \left. \frac{1}{2}\sigma^2 P(j)^2 \frac{S_{j+1,k,i} + S_{j-1,k,i}}{\Delta P^2} \right) \Bigg/ \left(\frac{\sigma^2 P(j)^2}{\Delta P^2} + \frac{1}{\Delta t} \right) \tag{3.58}$$

Although this is not the full solution to the equation, the above solution can easily be implemented using successive-over-relaxation (SOR) for iteration. Since this is a linear problem with constant coefficient on wealth, SOR converge nicely. Though different variations of SOR can be used to increase the convergence rate and boost computation performance.

3.6.2 Results with no taxation

When there is no taxation, our model collapses down to a simple utility-based framework with no portfolio choice. Although embedding portfolio choice in the utility framework is a topical area for ESO valuation, for example, Leung and Sircar [2009], Grasselli and Henderson [2009] and Carpenter et al. [2010], it is not very relevant to the taxation problem introduced in this chapter. In fact, our non-tax model becomes so simple that the only comparables come from standard Black-Scholes. This is because wealth in our model is invested in the riskless asset and there is no optimal portfolio choice which (if included) will significantly complicate the model. The main difference between ISO and NQSO is in their respective tax rules; if all tax rates are set to 0 both ISO and NQSO will be valued the same. Stock obtained from exercising ISO (and NQSO) is also immediately sold for there is no advantage of waiting.

Table 3.6 reports ISO and NQSO value and cost; all tax rates are set to 0; vesting is also set to 0 so that it can be compared to Black-Scholes. Table 3.6 shows that the firm cost almost converges to Black-Scholes when risk aversion is low and wealth is high. As expected, ISO and NQSO have identical value and cost because they are essentially the same option when there is no tax.

Table 3.6: This table reports value and cost of ISO and NQSO with no taxation and no vesting. Since ISO and NQSO are only different in tax terms, their values are identical (as well as costs) when there are no taxes. Parameters are: $P_0 = 1$, risk free rate, $r = 0.04$, time to maturity, $T = 10$, vesting period, $T_V = 0$, stock price return, $\mu = 0.04$, income tax rate, $\tau_I = 0$, capital gains tax, $\tau_{CG} = 0$, corporate tax rate, $\tau_c = 0$. Without taxation and vesting, the model collapses down to a simple utility-pricing framework, which can easily compared against the Black-Scholes.

	$\sigma = 0.3$					$\sigma = 0.5$				
	Wealth					Wealth				
	0.25	0.5	1	2	5	0.25	0.5	1	2	5
					ISO value					
γ										
1.001	0.265	0.306	0.348	0.386	0.429	0.294	0.346	0.402	0.458	0.523
2	0.187	0.235	0.285	0.335	0.392	0.198	0.254	0.317	0.382	0.464
3	0.146	0.194	0.246	0.300	0.366	0.152	0.205	0.267	0.335	0.425
4	0.121	0.167	0.219	0.275	0.347	0.125	0.174	0.234	0.302	0.396
5	0.104	0.147	0.199	0.255	0.332	0.106	0.152	0.210	0.277	0.374
6	0.092	0.133	0.183	0.240	0.321	0.093	0.136	0.192	0.258	0.358
					ISO cost					
γ										
1.001	0.474	0.482	0.489	0.492	0.493	0.564	0.588	0.608	0.625	0.641
2	0.443	0.461	0.475	0.485	0.492	0.498	0.534	0.567	0.596	0.625
3	0.422	0.443	0.463	0.478	0.489	0.464	0.499	0.537	0.573	0.611
4	0.404	0.431	0.453	0.471	0.486	0.435	0.476	0.518	0.555	0.599
5	0.393	0.422	0.444	0.465	0.483	0.418	0.461	0.499	0.541	0.588
6	0.393	0.414	0.437	0.459	0.480	0.419	0.452	0.489	0.527	0.579
					NQSO value					
γ										
1.001	0.265	0.306	0.348	0.386	0.429	0.294	0.346	0.402	0.458	0.523
2	0.187	0.235	0.285	0.335	0.392	0.198	0.254	0.317	0.382	0.464
3	0.146	0.194	0.246	0.300	0.366	0.152	0.205	0.267	0.335	0.425
4	0.121	0.167	0.219	0.275	0.347	0.125	0.174	0.234	0.302	0.396
5	0.104	0.147	0.199	0.255	0.332	0.106	0.152	0.210	0.277	0.374
6	0.092	0.133	0.183	0.240	0.321	0.093	0.136	0.192	0.258	0.358
					NQSO cost					
γ										
1.001	0.474	0.482	0.489	0.492	0.493	0.564	0.588	0.608	0.625	0.641
2	0.443	0.461	0.475	0.485	0.492	0.498	0.534	0.567	0.596	0.625
3	0.422	0.443	0.463	0.478	0.489	0.464	0.499	0.537	0.573	0.611
4	0.404	0.431	0.453	0.471	0.486	0.435	0.476	0.518	0.555	0.599
5	0.393	0.422	0.444	0.465	0.483	0.418	0.461	0.499	0.541	0.588
6	0.393	0.414	0.437	0.459	0.480	0.419	0.452	0.489	0.527	0.579
Black-Scholes	0.494	0.494	0.494	0.494	0.494	0.654	0.654	0.654	0.654	0.654

Chapter 4

Managerial Effort and the Valuation of Executive Stock Options

4.1 Introduction

Valuation of executive stock options has been a widely researched area of finance, yet few studies explicitly consider the feedback effect of incentives, in which the manager's action directly influences the company stock price. Managers clearly have the ability to influence the company stock price as, essentially, it is the purpose of granting them stock options. The objective of this chapter is to investigate the value and cost of executive stock options by explicitly introducing managerial effort into a valuation framework.

We consider a risk-averse manager whose welfare depends on the market and the company's stock price. The manager is awarded with an ESO whose value depends on the company stock price. The manager is also restricted from trading in the company stock. This is essentially an incomplete market problem, since the manager cannot dynamically hedge her restricted position. Following the literature, the manager's restricted position is priced using a certainty equivalent approach (e.g. Lambert et al. [1991], Carpenter [1998], Henderson and Hobson [2009b]). Although the manager cannot trade the company stock, she can influence stock price by exerting costly effort.

4. MANAGERIAL EFFORT AND THE VALUATION OF EXECUTIVE STOCK OPTIONS

We introduce effort as an explicit choice variable for the manager. In particular, the manager can increase the drift rate of the stock price process by exerting effort. In addition to the ESO, the manager can invest her personal wealth in the market portfolio and the riskless asset. The market portfolio is the manager's hedging vehicle, which she uses to partially hedge the exposure to the ESO.

It is widely accepted that executives value stock options lower than the market (e.g. Huddart [1994], Kulatilaka and Marcus [1994], Detemple and Sundaresan [1999], Meulbroek [2001]). Managers are subject to trading restrictions, which means that they are only rewarded for the systematic part of the total risk (because they can only hedge exposures by trading the market asset). Yet they are exposed to the total risk of the firm. Managers thus subjectively discount the option value. However, such a view is not entirely correct when the manager has the ability to influence the stock price. We show that, under certain circumstances, the effort choice can lead the manager to value stock options higher than the market does.

This is because managerial effort is directly priced in the company stock price which then feeds back into the manager's terminal utility. These higher than market valuations can occur under any (or any combination) of the following circumstances: the manager has low wealth, the manager has low risk aversion, and the manager is highly skilled (from a modelling perspective, this means her effort has large impact on the firm's expected return). Under these circumstances, the manager's marginal utility is high and effort plays a major role in the manager's valuation. Following these results, we conclude that managers in large public firms are less likely to value their ESO positions higher than the market does, as they are wealthy individuals and well compensated for their skills. On the other hand, managers in small and non-public firms are more likely to value their ESOs higher than the market, because they have low wealth and may not be appropriately compensated for their talent.

The models closest to ours are Henderson [2005] and Carpenter et al. [2010]. Henderson [2005] examines ESO value and incentives in a utility maximization framework. She solves the problem using an exponential utility function and the solution obtained does not depend on personal wealth. This desirable feature is included in our model, as executives' wealth is bound to impact both the ESO value and managerial effort. Carpenter et al. [2010] solve the problem for power law utility and implement many variations to the original portfolio-decision model. Our model complements theirs, with a new feature – managerial effort is a choice variable.

4.1.1 Related literature

Utility maximization in continuous time was first introduced in Merton [1969] and Merton [1971], based on the Hamilton-Jacobi-Bellman equation and requires the underlying process to be Markovian. Pliska [1986], Cox and Huang [1989], Cox and Huang [1991], and Karatzas et al. [1987] extend the utility maximization problem using the martingale method. Karatzas et al. [1991] and He and Pearson [1991] extend Merton's original problem to an incomplete market. Adopting the martingale method, Henderson [2002] values a contingent claim with a non-traded underlying asset. When there is no ESO, Merton [1969] shows that the manager maintains a constant proportion in the market portfolio. However when the manager has an ESO, which restricts her from trading the underlying company stock, she exhibits different portfolio selection behavior. In a similar setup, Kahl et al. [2003] show that liquidity restrictions have a major effect on the executive's optimal investment. Similar continuous-time problems which involve non-traded stochastic income are also explored in Duffie et al. [1997], who consider an infinite-horizon problem with a trading constraint. Tepla [2000] employs the martingale approach to study the non-traded option problem with exponential utility. Henderson [2002] solves non-traded options with general payoff function for both exponential and power law utility. Leung and Sircar [2009] extend Henderson's result to a more general framework with early exercise, vesting and exogenous job termination. Grasselli and Henderson [2009] extends the incomplete market problem with a multiple exercise feature.

In the principal-agent framework, Feltham and Wu [2001] explicitly model managerial effort in a one-period model. They find that an option grant is optimal when the manager can influence both the drift and the risk of company stock price. As reviewed in the introduction of the thesis, Jin [2002] develops a single-period principal-agent model to examine empirical relationships between risk and incentives. Bitler et al. [2005] develop a principal-agent model with entrepreneurial effort and find that effort increases with ownership. Cadenillas et al. [2004] consider managerial effort and risk-taking behavior with unlevered and levered restricted stock. They find that levered stock is optimal for a skilled manager. Agliardi and Andergassen [2003] explicitly model managerial effort in Merton's set up.

The remainder of the chapter is organized as follows. Section 4.2 sets up the model with non-traded assets. Section 4.3 examines the value and cost of ESO using a single asset model. Section 4.4 analyses the general two asset model. Section 4.5 concludes with possible further research.

4.2 The model

In this section, we model the portfolio choice of a manager who has a non-traded American call option. For easier exposition, we discuss the European case first where the option cannot be exercised before maturity. The non-traded option is awarded to the manager at time $t = 0$. It has a payoff function $C_T = (S_T - K)^+$ at maturity $t = T$. S_T and K are company stock price at $t = T$ and option strike price, respectively. The manager is prohibited from hedging the option position; more specifically the manager is not allowed to trade in the company stock S. This is a traditional option with trading restriction. The investment set includes three assets: a riskless bond, a market portfolio and the company stock.

The riskless bond or money market fund B follows

$$dB_t = rB_t dt \tag{4.1}$$

where r is the constant interest rate. The market portfolio M follows

$$\frac{dM_t}{M_t} = \alpha dt + \nu dZ_t^M \tag{4.2}$$

where α is the drift of the market return and ν is the volatility of the market return. Both α and ν are constant. The market risk premium is $\alpha - r$ and market price of risk is $\lambda = \frac{\alpha - r}{\nu}$. The manager can trade in the riskless bond and the market portfolio. Although the manager is not allowed to trade in the company stock, other outside investors without any restriction can still trade it. Market price of the company stock has the following dynamics,

$$\frac{dS_t}{S_t} = \mu dt + \delta a_t dt + \sigma dZ_t \tag{4.3}$$

where constant μ is the mean of stock price return without any managerial influence; constant σ is the standard deviation of stock price return. a_t is manager's instantaneous effort choice, observable only to the manager. $\delta \in (0, \infty)$ is an exogenous constant and determines the impact of managerial effort on stock price. It depends upon characteristics of the manager; in a similar one-period setup, Palmon et al. [2008] assume δ of $\frac{1}{2}$. δ can also be interpreted as managerial skill or talent, since the higher the δ the larger impact the effort has. The manager can improve the firm's performance by exerting effort a. Effort benefits the manager with an increased stock price return, but it is costly for her to implement. As the manager exerts effort to increase her portfolio value, she also incurs the cost of such hard work. There is a trade-off between the benefits and cost. Assume that the cost of effort a is a function $G(a)$. Following the related literature in the principal-agent model (e.g.Feltham and Wu [2001], Jin [2002] and Cadenillas et al. [2004]), a good candidate with increasing and convex cost for $G(a)$ is quadratic. So the stock price process consists of three components – company stock return, managerial effort and stock volatility. This setup is similar to the static model of Jin [2002]. Both Z_t^M and Z_t are correlated Brownian motions and

$$Z_t^M = \rho Z_t + \sqrt{1-\rho^2} X_t \tag{4.4}$$

where X_t is a Brownian motion correlated with neither Z_t^M nor Z_t. The manager can use market portfolio to hedge her exposure to the restricted ESO. Following Merton [1969, 1971], the manager's personal wealth follows the process

$$dW_t = r\phi_t W_t dt + (1-\phi_t) W_t \frac{dM_t}{M_t} \tag{4.5}$$

where ϕ_t is the instantaneous proportion of W_t invested in the riskless asset and $1-\phi_t$ is proportion invested in the market portfolio.

In a conventional equilibrium model, managers are price-taker so that effortful actions of managers have no impact on expected asset return, CAPM is a perfect example.

Much of the previous work has been done to reconcile the principal-agent model with the general equilibrium asset pricing model. For example, Ramakrishnan and Thakor [1984] derived an equilibrium asset price model with effort choice in a one-period model. They assumed that effort increases stock expected return, and effort is a pricing factor of Arbitrage Pricing Theory. Ou-Yang [2005] extended the model to a continuous-time equilibrium model. His main results are that managers' action impact firm expected return through their impact on systematic risk. We adopt a similar assumption in this chapter.

CAPM does not strictly hold in our model. We assume that

$$\frac{\mu - r}{\rho\sigma} < \frac{\alpha - r}{\nu} \tag{4.6}$$

where the fraction on the left hand side is the stock's no effort market price of risk. The fraction on the right hand side is the market price of risk. This inequality makes sure that, without the manager's effort, the stock's price of systematic risk is strictly below that of the market. However, we do *not* impose the following condition to enforce CAPM,

$$\frac{\mu + \delta a - r}{\rho\sigma} = \frac{\alpha - r}{\nu} \tag{4.7}$$

where the left hand side is the stock's with effort market price of risk. Instead, we assume the market price of the firm follows,

$$\frac{\mu + \delta A - r}{\rho\sigma} = \frac{\alpha - r}{\nu} \tag{4.8}$$

where A is expected level of effort implied by CAPM. If the manager chooses her optimal effort, $a = A$, then the firm's stock is correctly priced by CAPM. Equation (4.8) implies that there exists an equilibrium effort level, which is aggregate of effort levels from all firms in the market (as assumed in Ou-Yang [2005]). Our argument is that the firm's price of risk depends on the manager's effort choice, which depends on the manager's skills and risk aversion. The CAPM relation does not always hold between the company stock and the market portfolio because of this.[25]

[25]This is after all a partial equilibrium model; the stock price should also be influenced by the manager's compensation which then determines the manager's effort choice.

We introduce effort in the model because the manager has perfect information about it. She uses after effort stock price to value her ESO portfolio. Even though shareholders cannot observe effort, their valuation does not depend on it. This is an elegant feature of Black-Scholes pricing formula, which completely removes expected return from option valuation. Since shareholders are risk neutral and can trade both the firm's stock and the market, they can ignore the impact of managerial effort when valuing ESO position. Or in other words, shareholders' valuation does not directly depend on managerial effort.[26]

The manager is assumed to be risk averse and have power utility function,

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \tag{4.9}$$

without exercise the manager's portfolio choice is the original Merton problem with an option position. Her indirect utility function is,

$$J(W_t, S_t, t) = \max_{a_t, \phi_t} E\Big[U\big((S_T - K)^+ + W_T\big) - \frac{1}{2}\int_0^T a_t^2 dt\Big] \tag{4.10}$$

W and S form a joint Markov process. The integration term is accumulative cost (disutility) of effort. The dynamic decision problem of the manager is to optimally choose her effort a and proportion ϕ to maximize her expected terminal utility subject to the dynamic budget constraint in equation (4.5). Assuming small time Δt and $t + \Delta t \leq T$, then the value function can be rewritten as

$$J(W_t, S_t, t) = \max_{a_t, \phi_t} E_t[J(W_t + \Delta W, S_t + \Delta S, t + \Delta t)] - \frac{1}{2}\int_t^{t+\Delta t} a_t^2 ds \tag{4.11}$$

Using Ito's formula and Taylor's theorem, dropping arguments in $J(W_t, S_t, t)$ and using subscripts to represent partial differentiation value, J satisfies the HJB equation

$$\begin{aligned}\max_{\phi_t, a_t} \Big[&-\frac{1}{2}a_t^2 + J_t + S(\mu + \delta a_t)J_S + \frac{1}{2}S^2\sigma^2 J_{SS} + W[r\phi_t + \alpha(1-\phi_t)]J_W \\ &+ \frac{1}{2}(1-\phi_t)^2\nu^2 W^2 J_{WW} + (1-\phi_t)WS\nu\sigma\rho J_{WS}\Big] = 0\end{aligned} \tag{4.12}$$

[26] Functionally, there is no relationship between shareholders' valuation and effort. But effort affects the manager's exercise choice, which indirectly affects shareholders' valuation.

Differentiating the equation with respect to a_t and ϕ_t gives the following first-order condition,

$$a_t^* = \delta S J_S$$
$$\phi_t^* = 1 - \frac{(r-\alpha)J_W}{\nu^2 W J_{WW}} + \frac{S\sigma\rho J_{SW}}{2\nu W J_{WW}} \tag{4.13}$$

Solving the above equation numerically is quite involved and it is left for the Appendix at the end of the chapter. Without the option position (or after the option position is exercised), there is no non-traded asset and the manager's new objective function is,

$$J^*(W_t, t) = \max_{\phi_t} E\big[U(W_T)\big] \tag{4.14}$$

The manager optimally chooses her investment portfolio to maximize terminal payoff. The objective function $J^*(W, t)$ satisfies the HJB equation,

$$\max_{\phi}\left[J_t^* + J_W^*[r\phi_t + (1-\phi_t)\alpha]W + \frac{1}{2}J_{WW}^*(1-\phi_t)^2\nu^2W^2\right] = 0 \tag{4.15}$$

Once the options are exercised, effort becomes redundant as the manager will not exercise the option if exerting effort leads to higher expected terminal utility. Essentially, the manager is choosing between her own effort and the market effort (which is implied in α). So effort drops out from the valuation PDE. Optimizing over ϕ and bringing the optimal ϕ^* into the equation, we have the PDE

$$J_t^* + J_W^*\big[r\phi_t^* + (1-\phi_t^*)\alpha\big]W + \frac{1}{2}J_{WW}^*(1-\phi_t^*)^2\nu^2W^2 = 0 \tag{4.16}$$

From equation 22 of Merton [1969], solution of J should take form of

$$J^*(W_t, t) = \frac{W_t^{1-\gamma}}{1-\gamma}b(t) \tag{4.17}$$

where $b(t)$ is a function of time t. Bringing the solution back into the fundamental

PDE, $b(t)$ should satisfy the following ordinary differential equation,

$$\frac{b'(t)}{1-\gamma} + b(t)g + b(t)k = 0 \tag{4.18}$$

where $g = r + \frac{(r-\alpha)^2}{\nu^2\gamma}$ and $k = -\frac{(r-\alpha)^2}{2\nu^2\gamma}$, given terminal condition $J^*(W_T, T) = U(W_T)$. The solution of the objective function is

$$J^*(W,t) = \frac{W^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\nu^2\gamma}\Big)(T-t)\right] \tag{4.19}$$

where the optimal proportion $\phi_t^* = 1 - \frac{\alpha-r}{\nu^2\gamma}$. The manager's problem outlined above does not involve exercising the option position early, but the problem is already quite complicated to solve. In the Appendix at the end of the chapter, the same European problem is discretized to a one-period binomial model.

With the introduction of early exercise, the manager's problem becomes a utility maximization with optimal stopping. Her indirect utility function is,

$$I(W_t, S_t, t) = \max_{T_V \leq \tau < T, 0 \leq a < +\infty, \phi} E\Big[J^*(W_\tau + (S_\tau - K)^+, \tau) - \frac{1}{2}\int_0^\tau a_t^2 dt\Big] \tag{4.20}$$

where T_V is the vesting period of the ESO, τ is the optimal stopping time of the option. $J^*(W_t, t)$, as defined in equation (4.19), is the manager's indirect utility after her option position is exercised. The second term in the square bracket is accumulative cost of effort up to the exercise time. Apart from its cost, effort is also beneficial to the manager. It is because S_t, as defined in equation 4.3, is a function of effort. The manager can take all the gains from effort at exercise time, as reflected in the exercise value term $(S_\tau - K)^+$. As explained in equation (4.15), the manager can hold the option to receive return from her own effort (which also incurs a cost), or she can exercise the option to get the market effort (which is implied in the market return, α). Note the manager will not exercise the option if holding the option and exert effort can lead to higher terminal utility.

It is obvious that $I(W_t, S_t, t)$ also satisfy PDE (4.12). Once it is determined, based on equation (4.19) the manager's valuation is then,

$$V(W_0, S_0) = \left\{(1-\gamma)\exp\left[-(1-\gamma)\left(r + \frac{(r-\alpha)^2}{2\nu^2\gamma}\right)T\right]I(W_0, S_0, 0)\right\}^{\frac{1}{1-\gamma}} - W_0 \quad (4.21)$$

where $V(W_0, S_0)$ is the manager's subjective valuation of the option, given initial wealth, W_0 and initial stock price, S_0. Since $I(W_t, S_t, t)$ and $J(W_t, S_t, t)$ both satisfy the same PDE, appendix provides details on how to numerically solve for $J(W_t, S_t, t)$.

4.2.1 Value and cost

One interesting question of managerial influence is how managerial effort affects the firm's cost, as effort affects the manager's exercise behavior which also impacts on the ESO cost. Following well known results (e.g. Lambert et al. [1991], Carpenter et al. [2010]), the cost of option can be represented as the risk-neutral expectation of the riskless discounted option payoff:

$$C = E^Q\left[e^{-r\tau}(S_\tau - K)^+\right] \quad (4.22)$$

where E^Q is expectation taken with respect to the risk neutral probability measure; τ is the manager's optimal exercise policy. Using Ito's lemma, cost of option C should satisfy the following PDE;

$$\begin{aligned} &C_t + rSC_S + \frac{1}{2}S^2\sigma^2 C_{SS} + rWC_W + \frac{1}{2}W^2(1-\phi)^2\nu^2 C_{WW} \\ &+ \rho\sigma\nu SW(1-\phi)C_{SW} - rC = 0 \end{aligned} \quad (4.23)$$

where managerial effort does not directly enter into this PDE. But equation (4.12) and (4.23) are solved simultaneously where optimal exercise policy from equation (4.12) are fed into equation (4.23). The key feature of this setup is that managerial effort actually affects firm cost of ESO.

Table 4.1: Value and cost of ESO for the single asset case. All ESOs are at-the-money with stock price $S = 1$, and strike price $K = 1$. γ is the manager's risk aversion, σ is stock volatility, δ is measure of managerial skill. Other parameter values are: interest rate $r = 0.03$, stock return $\mu = 0.03$, wealth $W = 2$. Vesting period, $T_V = 3$. The results are obtained by numerically solving equation 4.24. Details of the numerical scheme are in the Appendix. Maximum firm cost of ESOs are: 0.369 for $\sigma = 0.2$ and 0.702 for $\sigma = 0.6$. Maximum cost is computed using equation 4.24 but with cost-maximizing exercise strategy.

	$\sigma = 0.2$					$\sigma = 0.6$				
γ	2	3	4	5	6	2	3	4	5	6
					Panel A: ESO value					
δ										
0	0.223	0.202	0.184	0.170	0.158	0.292	0.232	0.193	0.165	0.145
0.5	0.316	0.225	0.193	0.173	0.160	0.313	0.237	0.195	0.166	0.145
1	0.926	0.332	0.221	0.183	0.163	0.388	0.254	0.200	0.168	0.146
1.5	1.929	0.577	0.280	0.201	0.170	0.581	0.284	0.208	0.170	0.147
2	2.953	0.886	0.380	0.229	0.179	1.019	0.335	0.220	0.174	0.148
					Panel B: ESO cost					
δ										
0	0.315	0.300	0.290	0.281	0.273	0.577	0.556	0.540	0.530	0.518
0.5	0.360	0.314	0.293	0.283	0.274	0.584	0.556	0.540	0.530	0.521
1	0.369	0.352	0.308	0.288	0.275	0.609	0.563	0.541	0.531	0.522
1.5	0.369	0.366	0.332	0.297	0.280	0.653	0.572	0.547	0.532	0.522
2	0.369	0.368	0.350	0.309	0.284	0.685	0.586	0.549	0.532	0.523
					Panel C: Initial effort					
δ										
0	0	0	0	0	0	0	0	0	0	0
0.5	0.077	0.028	0.012	0.005	0.002	0.042	0.014	0.005	0.002	0.001
1	0.150	0.059	0.024	0.010	0.004	0.095	0.030	0.011	0.004	0.002
1.5	0.141	0.074	0.035	0.015	0.006	0.162	0.047	0.017	0.007	0.003
2	0.120	0.070	0.042	0.020	0.009	0.204	0.065	0.023	0.009	0.004

4.3 A single asset case

The problem is generally complicated since the manager is allowed to trade market portfolio. We first consider the simple case where the manager cannot make the portfolio-choice. In this simple case, the manager is rewarded with the stock option and her

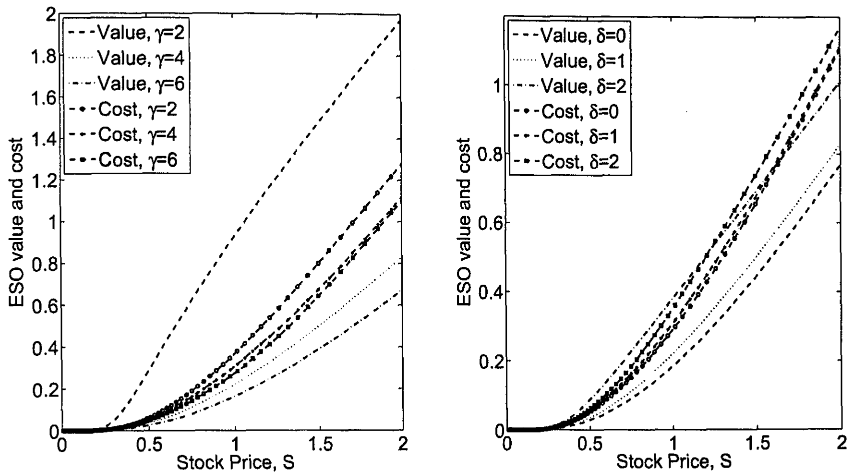

Figure 4.1: Value and cost with single asset model. This figure plots subjective value and firm cost of ESO with different stock prices. In the left plot, δ is set equal to 1. In the right plot, γ is set equal to 4. Volatility, σ is set equal to 0.2 for both plots. Other parameter values are the same as Table 4.1

outside wealth is invested in the riskless asset. For this simple one-asset case, we also assume that $\mu = r$, so that the risk premium of the stock is purely contributed by managerial effort, δa. This is to make sure that the manager wants to hold the option longer only because of her positive effort choice, but not because of higher value of μ so that the natural advantage of the option over the riskless asset is removed, otherwise the manager would choose to hold the option longer (to extract positive drift $\mu - r$) even though she is not exerting effort.[27] Using a similar argument based on HJB equation, the manager's valuation PDE is then,

$$J_t + \mu S J_S + \frac{1}{2}\delta^2 S^2 (J_S)^2 + \frac{1}{2}\sigma^2 S^2 J_{SS} = 0 \tag{4.24}$$

where δ is a measure of the manager's skill, and the third term comes from the manager's optimal effort choice. We report results for the single asset case in Table 4.1.

Obviously, there are a few cases where the manager's subjective value is higher than the firm cost. These cases occur when the manager is very skilled (high value of δ) and not very risk averse (low value of γ) and the stock has a relatively low volatility. When

[27] Such an argument is also valid under the conventional setup where the manager has no effort choice.

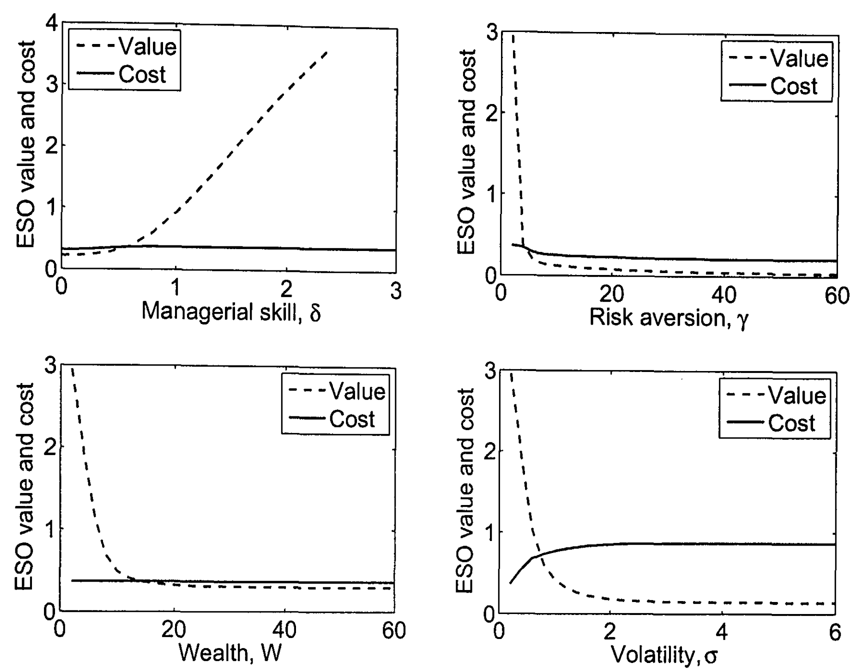

Figure 4.2: Value and cost with other parameters. This figure plots subjective value and firm cost of ESO with different risk aversion, γ, managerial skills, δ, wealth, W and volatility, σ. Other parameter values are the same as Table 4.1

these conditions are not satisfied, ESO values are still lower than the firm's cost. For example, when volatility is 0.2, the value is larger than cost only when δ is larger than 1. This is a very large value of δ, as initial effort under this scenario is around 12% which is almost three times that of the interest rate. The stock drift with managerial effort is almost 25%. Although talented managers can significantly improve the value of a company, such a high return for a publicly listed company is very rare, certainly not for every manager. Cadenillas et al. [2004] make a reasonable assumption that δ is 0.05. Under this assumption, our model shows that reported ESO subjective values are strictly lower than their respective firm cost.

Figures 4.1 and 4.2 demonstrate the point that the value is only higher than the cost when managerial skill is extremely high. The value is increasing and unbounded in δ, and it is infinity when δ goes to infinity. As this is a partial equilibrium model where there is no endogenous compensation, value can go to infinity while holding other exogenous variables fixed. For example, wealth is held fixed in both Figure 4.1 and 4.2. Ideally, a skilled manager is compensated more, which increases the manager's wealth and brings down her subjective value, so that the infinite value is never realized.

4.4 The general model

With the market asset, the problem becomes more involved. In order to have tractable results for the portfolio optimization problem, many previous contributions assume exponential utility function (e.g. Henderson [2005]). Carpenter et al. [2010] assume constant relative risk aversion utility and solve the problem using the finite difference method. We use a similar approach to solve the optimization problem.

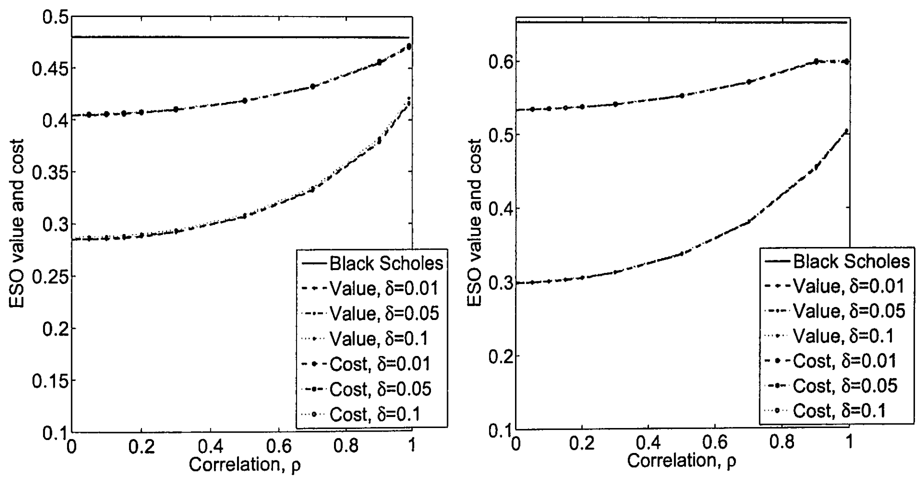

Figure 4.3: ESO value and cost with varying correlation. This figure plots value and cost for various correlation, ρ and managerial talent, δ. The plot on the left has low volatility at $\sigma = 0.3$, the plot on the right has high volatility at $\sigma = 0.5$. Wealth, $W = 0.6$. Parameter values are those of Table 4.2.

One major addition that the two asset model offers is correlation between the stock and the market. This section focuses on the effect of correlation. Similar to the previous model with the market asset, the correlation has a significant impact on the executive's valuation (see Figure 4.3). As the correlation increases, the manager can increasingly hedge her ESO position using the market asset. The ESO position becomes increasingly similar to a traded option, so both the value and the cost of ESO increase with the

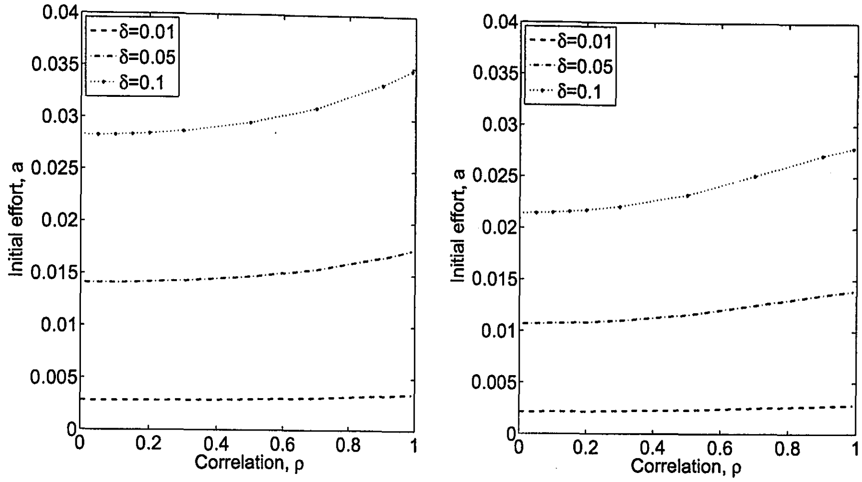

Figure 4.4: Initial effort and correlation. This figure plots initial effort varying level of correlation. The plot on the left has low volatility at $\sigma = 0.3$, the plot on the right has high volatility at $\sigma = 0.5$. Wealth, $W = 0.6$. Other parameter values are those of Table 4.2.

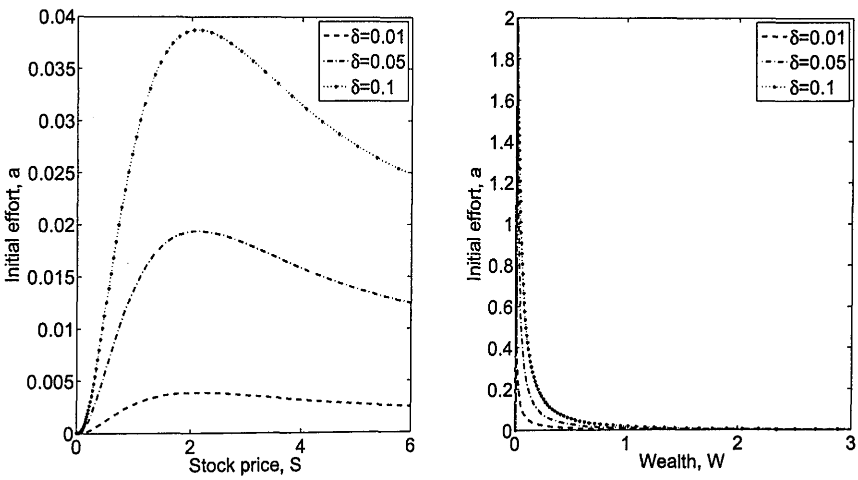

Figure 4.5: Initial effort and wealth. This figure plots initial effort with varying level of stock price and wealth. Correlation, $\rho = 0.5$. The left plot has wealth, $W = 0.6$. The right plot has stock price, $S = 1$. Other parameter values are those of Table 4.2.

correlation. In addition to the value and cost reported, Figure 4.4 shows the initial managerial effort[28] for different levels of correlation, ρ. Effort also increases with ρ; this

[28]Effort is path dependent, Figure 4.4 only reports effort level when ESOs are granted.

Table 4.2: Value and cost of ESO for the two asset case. All ESOs are at-the-money with stock price $S = 1$, and the strike price $K = 1$. σ is stock volatility, δ is measure of managerial skill. The manager has risk aversion, $\gamma = 2$. Other parameter values are: interest rate $r = 0.03$, market asset return, $\alpha = 0.08$, market asset volatility, $\nu = 0.2$. Option has maturity, $T = 10$ years and vesting period , $T_V = 3$. Stock return is assumed to be $\mu = r + k\left(\frac{\alpha - r}{\nu}\rho\sigma\right)$, where $0 < k < 1$, to make sure no effort stock return is lower than that implied by CAPM. k is set to 0.9 for results in the table.

	$\sigma = 0.3$					$\sigma = 0.5$				
ρ	0	0.2	0.5	0.7	0.9	0	0.2	0.5	0.7	0.9
					ESO value, $W = 0.6$					
δ										
0.01	0.232	0.236	0.260	0.293	0.351	0.226	0.234	0.268	0.312	0.373
0.05	0.233	0.238	0.261	0.294	0.353	0.226	0.234	0.268	0.313	0.374
0.1	0.237	0.241	0.265	0.299	0.360	0.228	0.236	0.270	0.316	0.377
					ESO cost, $W = 0.6$					
δ										
0.01	0.380	0.383	0.397	0.415	0.441	0.506	0.511	0.529	0.550	0.555
0.05	0.380	0.384	0.398	0.416	0.443	0.506	0.511	0.529	0.550	0.555
0.1	0.382	0.386	0.400	0.418	0.444	0.507	0.512	0.530	0.550	0.555
					ESO value, $W = 1.2$					
δ										
0.01	0.285	0.288	0.306	0.332	0.378	0.298	0.305	0.336	0.380	0.453
0.05	0.285	0.288	0.307	0.332	0.379	0.298	0.305	0.337	0.381	0.455
0.1	0.287	0.290	0.309	0.335	0.382	0.299	0.306	0.338	0.382	0.457
					ESO cost, $W = 1.2$					
δ										
0.01	0.404	0.407	0.418	0.432	0.455	0.533	0.537	0.553	0.572	0.599
0.05	0.404	0.407	0.418	0.432	0.456	0.534	0.537	0.553	0.572	0.600
0.1	0.405	0.408	0.419	0.433	0.457	0.534	0.538	0.553	0.572	0.601

is because with high value of ρ the manager can increasingly hedge her ESO position. So she delays the value destroying early exercise and replies on effort to increase her terminal expected utility. As a result, value also increases with correlation.

The ESO cost does not converge to maximum value (which is equal to the Black-Scholes value) when ρ goes to 1. This is because the manager has to keep her wealth

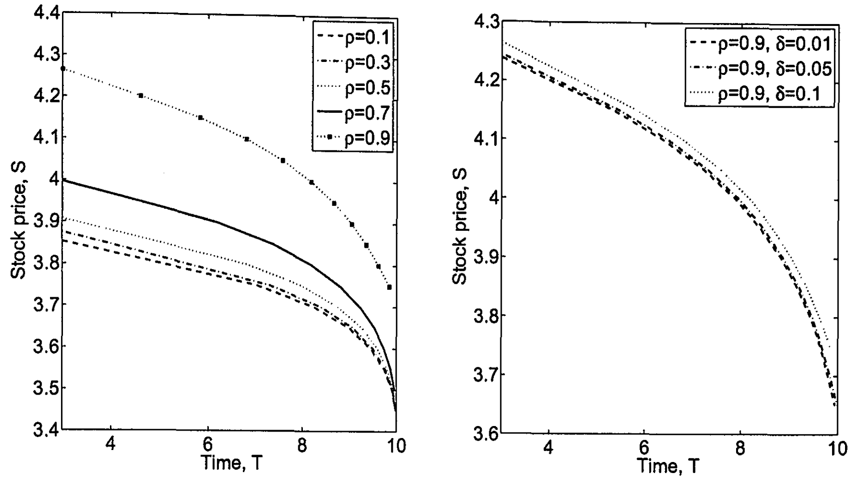

Figure 4.6: Early exercise boundary, low volatility case. This figure plots exercise boundary for various correlations, ρ and managerial talent, δ, volatility is $\sigma = 0.3$. Wealth is $W = 0.6$. Other parameter values are those of Table 4.2.

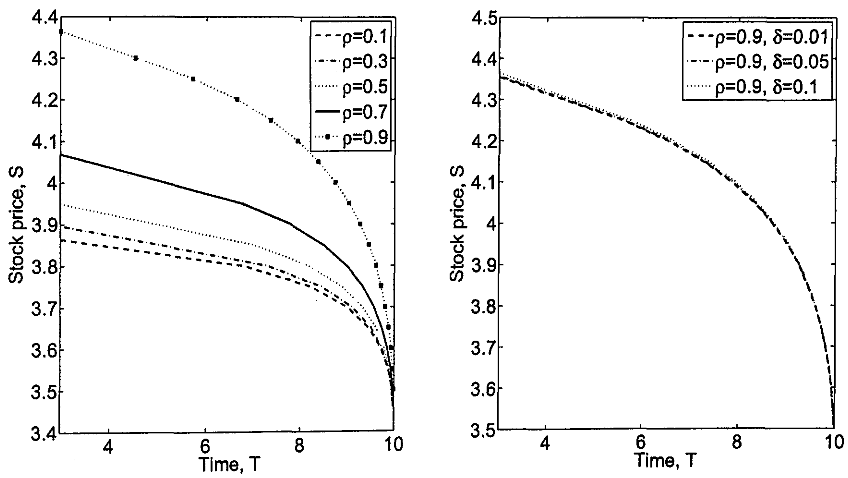

Figure 4.7: Early exercise boundary, high volatility case. This figure plots exercise boundary for various correlations, ρ and managerial talent, δ, volatility is $\sigma = 0.5$. Wealth is $W = 0.6$. Other parameter values are those of Table 4.2.

positive to prevent expected utility from going to the negative infinity.[29] As Carpenter et al. [2010] points out, the manager is essentially solving the problem with a non-

[29]This is a similar problem to that of Chapter 2, where power law utility function gives the negative infinity when wealth is negative.

negativity constraint on wealth. For cost to converge to the Black-Scholes, an additional condition is needed. That is, the manager has to be risk neutral, which removes the non-negativity constraint from the problem.

The effect of effort on both the value and the cost is very marginal (see Figure 4.3). Numerical results are reported in Table 4.2. First, it is obvious to see the effect of effort. Both value and cost increases with managerial talent, δ. The problem, however, is the magnitude of the difference. For example, when $\rho = 0.5$, $\sigma = 0.5$ and $W = 0.6$, a 10 times increase in managerial skill (from $\delta = 0.01$ to 0.1) only results in a less than 1% increases in value (from 0.268 to 0.270) and an even smaller increase in the cost (from 0.529 to 0.530).

This result is due to the manager's wealth, which affects the manager's marginal utility and effort level. Figure 4.5 shows the effort level with varying levels of stock price and wealth. Effort is largest when wealth is lowest. In other words, effort has the largest impact on value when wealth is relatively low, although values at low wealth (as reported in Table 4.2) are lower than values at high wealth. This is because the wealth effect dominates the effort effect, so that the value increases with wealth. Figure 4.5 also reports the initial effort with varying levels of stock price. Effort reaches its maximum with stock price equal approximately twice the strike price, then gradually diminishes. Effort also has a large impact on value when the stock price is higher than the strike price; still, it does not guarantee that the value would be higher than the cost as the wealth effect dominates in the manager's valuation.

In addition to the value and the cost, Figures 4.6 and 4.7 report the optimal exercise boundary for various levels of correlation and managerial skill.[30] Higher correlation indicates that the manager can increasingly hedge her ESO positions so that she would hold options longer. The differences between $\rho = 0$ and $\rho = 0.9$ are quite large, but the exercise boundary does not go to infinity (which implies the manager never exercises ESOs early) as ρ goes to 1. The reason is already mentioned above: it is a constraint that is imposed by the utility function. For the exercise boundary to go to infinity, the manager has to be risk neutral. Managerial skills, δ, on the other hand, have only a marginal impact on the exercise boundary. This is also confirmed by numerical results

[30] We report only results for positive correlations for one reason. Our setup allows only positive effort, and effort increases stock price of risk, $\frac{\mu+a-r}{\sigma}$. A negative correlation means the stock has negative risk premium and effort has to be negative to increase stock price of risk.

in Table 4.2. δ has a similar impact on the value and exercise boundary as that of the stock price return, as high δ results in high effort choice which increases return of the stock.[31]

4.5 Discussion and conclusion

In this chapter, we build a continuous time utility model to value ESOs under the explicit assumption that the manager has the ability to influence the stock price. The traditional view that ESOs are valued lower than the Black-Scholes valuation does not always hold. By explicitly considering the manager's ability (effort), we show that a risk averse manager can value ESOs more highly than the market. These higher than market valuations occur under certain cases: when the manager is not very risk averse, or when she has low wealth, when she is highly skilled, or when the stock has low volatility. Under these scenarios, the manager's marginal utility is high and effort has a large impact on the manager's valuation.

However, in a general equilibrium model (or in an efficient competitive market) where skilled talents are appropriately rewarded, highly skilled managers are likely to be paid with large sums of fixed wage, which raises their wealth and reduces their risk aversion. In this case, effort has an impact on valuation but not large enough to significantly change exercise behavior, nor would effort make the manager's valuation higher than that of the market.

The practical implications of these results are as follows. Managers from large public firms are less likely to value their ESOs higher than the cost. Because of competition they are rewarded with large amounts of fixed wages which substantially raises their wealth level. Skilled managers are also likely to be paid more, which also reduces their valuation. On the other hand, managers of small non-public firms are likely to value their ESOs far higher than their cost, as they are not paid with large fixed wage. They may not have large outside wealth either. Competition for these positions is less intense, which implies that skilled managers may not be rewarded more than their peers. All these factors increase the manager's valuation. These results may partly

[31] We report results for a small range of δ, because the single asset example already demonstrates that value (cost is bounded by Black-Scholes) is unbound in δ. Such results also hold in the two assets case.

explain why ESOs are so popular in startup firms, where managers are most likely to value ESOs higher than the market. It also offers an explanation of why empirical research (for publicly traded firms) finds mixed relationship between ESO incentives and firm performance. This may be due to managerial effort having a limited impact on firms' stock price.

Finally, a natural extension of this chapter would be to incorporate the shareholders' problem in the model. One possible formulation of such problem would be treating the shareholders' objective as a static maximization, similar to that outlined in Chapter 3. Shareholders optimally choose number of ESOs rewarded to the manager, in order to maximize firm value (net cost of options position) at option expiry date. The maximization takes into account reservation and incentive constraints, but it would be difficult to make both constraints binding throughout the ESOs' lifespan. A simplified assumption would be to make both constraints only binding at the grant date (as shown in Chapter 3). It would be very interesting to see how ESOs are valued under such a model, since shareholders have an impact on the manager's effort choice and her ESO valuation.

4.6 Appendix

4.6.1 A binomial tree example

Based on the European framework outlined in the model, we can find the cash amount (or certainty equivalent) that the manager is willing to receive in exchange for the position in stock option. Following Lambert et al. [1991], Carpenter [1998] and Hall and Murphy [2002], the CE amount is simply the amount which makes the manager indifferent between receiving riskless cash and accepting the risky ESO. Solving for the CE in a continuous framework is quite involved and is demonstrated in the next section. A simpler one-period binomial tree is used in this section to show the computation of the CE.

Consider two correlated risky assets in the market. Suppose the two assets S_t and M_t have initial value of S_0 and M_0, at the end of period 1 their respective values are

$S\varepsilon$ and $M\eta$. There are four states at the end of period 1:

$$(S_1, M_1) = \begin{cases} S\varepsilon^1 & M\eta^1 \\ S\varepsilon^2 & M\eta^2 \\ S\varepsilon^3 & M\eta^1 \\ S\varepsilon^4 & M\eta^2 \end{cases}$$

with each state has equal probability $\frac{1}{4}$. The tree is calibrated using the parametrization of He [1990] with each parameter computed as follows:

$$\begin{aligned}
\varepsilon^1 &= \exp\left[(\mu+\delta a)h + \left(\sigma\rho + \sigma\sqrt{1-\rho^2}\right)\sqrt{h}\right] \\
\varepsilon^2 &= \exp\left[(\mu+\delta a)h - \left(\sigma\rho + \sigma\sqrt{1-\rho^2}\right)\sqrt{h}\right] \\
\varepsilon^3 &= \exp\left[(\mu+\delta a)h + \left(\sigma\rho - \sigma\sqrt{1-\rho^2}\right)\sqrt{h}\right] \\
\varepsilon^4 &= \exp\left[(\mu+\delta a)h - \left(\sigma\rho - \sigma\sqrt{1-\rho^2}\right)\sqrt{h}\right] \\
\eta^1 &= \exp\left[\alpha h + \nu\sqrt{h}\right] \\
\eta^2 &= \exp\left[\alpha h - \nu\sqrt{h}\right]
\end{aligned}$$

All parameters are discretized directly using their continuous counterpart, e.g. $\frac{dM_t}{M_t} = \alpha dt + \nu dZ_t^M$, where $h = T$ since this is a 1-period tree $h = T/1 = T$. The correlation between two diffusion terms $dZ_t^M = \rho dZ_t + \sqrt{1-\rho^2}dX_t$ where X_t is Brownian motion independent of both Z_t^M and Z_t.

The initial wealth $w = W_0 = \phi B + (1-\phi)M_0$. At the end of period 1, wealth is $W_1 = \phi W_0(1+r) + (1-\phi)W_0\eta$. Using the same notation as in the continuous time case, the option that underlying the non-traded asset S_t has payoff function C_T. For a call option $C_T = (S_T - K)^+$. Assuming the manager's disutility function $K(a) = \frac{1}{2}a^2$. The manager's objective under this one period model is

$$J(w,1) = \sup_{\phi,a} E\big[U(W_T + C_T)\big] - \frac{1}{2}a^2$$

Where $J(w,1)$ is objective function with non-traded call option. Similarly, $J(w,0)$ is objective function without non-traded option. The number 0 and 1 in the argument indicate either there is 1 option or there is 0 option. The manager's decision problem is to optimally choose ϕ and a to maximize her expected terminal utility. The price P of the non-traded option can be solved using

$$J(w-P,1) = J(w,0)$$

As stated above $J(w,0)$ is simply the objective function without the non-traded call option. P is the price that leaves the manager indifferent (her utility being unchanged) between paying the price P to receive the option, and leaving the position unhedged. With this simple structure, the computation of the non-traded option price is relatively easy. However, extending the two-periods tree to multi-periods is a daunting task. Musiela and Zariphopoulou [2004] investigate the incomplete binomial model; they propose a probabilistic iterative algorithm for a multi-periods framework.

A numerical example For the below numerical example power law utility $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$ is used and parameter values are: $S_0 = 1$, $K = 1$, $W_0 = 1$, $r = 0.03$, $\alpha = 0.08$, $\nu = 0.2$, $\mu = 0.025$, $\sigma = 0.2$, $\rho = 0.1$, $T = 1$, $\gamma = 2$, $\delta = 0.1$. The expected utility is then

$$J(W_0, C_0) = \max_{\phi,a}\left[E\Big(U(C_1 + W_1)\Big)\right] - \frac{1}{2}a^2 \tag{4.25}$$

To maximize expected utility, the optimal hedge $1-\phi$ and optimal effort is 0.928 and 0.044. Using $J(W_0 - P, 1) = J(W_0, 0)$, the price of the non-traded option is $P = 0.105$. To make comparison, the market value (or firm cost) of an equivalent traded option is computed using 1-period Cox, Ross, Rubinstein binomial tree.[32] $P^{CRR} = 0.116$ is the price of an equivalent traded option. The non-traded stock option is worth slightly less than a traded option. This is because of the risk aversion and non-tradeability. Effort also impacts on price, but in this example $\delta = 0.1$ so its effect is relatively small to make any significant difference.

[32]Since the option is always exercised at the end of time 1, results from CRR model equals firm cost of this 1-period ESO.

4.6.2 Numerical solution

To solve equation (4.12) numerically, we first take a log transformation. Similar to Carpenter et al. [2010], defining $s = \log(S)$, $w = \log(W)$ and $f(w, s, t) = J(W, S, t)$, so that

$$J_t = f_t \tag{4.26}$$

$$J_W = \frac{f_w}{W} \tag{4.27}$$

$$J_{WW} = \frac{f_{ww} - f_w}{W^2} \tag{4.28}$$

$$J_S = \frac{f_s}{S} \tag{4.29}$$

$$J_{SS} = \frac{f_{ss} - f_s}{S^2} \tag{4.30}$$

$$J_{WS} = \frac{f_{ws}}{WS} \tag{4.31}$$

After this transformation, it is not difficult to see that the coefficients on linear terms are constant. The new PDE is

$$\begin{aligned}\max_{\phi_t, a_t} \Big[& -\frac{1}{2}a_t^2 + f_t + (\mu + \delta a_t - \frac{1}{2}\sigma^2) f_s + \frac{1}{2}\sigma^2 f_{ss} + [r\phi_t + \alpha(1 - \phi_t) \\ & - \frac{1}{2}(1 - \phi_t)^2 \nu^2] f_w + \frac{1}{2}(1 - \phi_t)^2 \nu^2 f_{ww} + (1 - \phi_t)\nu\sigma\rho f_{ws}\Big] = 0\end{aligned} \tag{4.32}$$

Differentiating equation (4.12) with respect ϕ_t and a_t gives the following first-order conditions

$$\begin{aligned} a_t^* &= \delta f_s \\ \phi_t^* &= \frac{\nu^2 f_{ww} + (\alpha - r - \nu^2) f_w + \nu\sigma\rho f_{ws}}{\nu^2 (f_{ww} - f_w)} \end{aligned} \tag{4.33}$$

Bring a_t^* and ϕ_t^* back into equation (4.12),

$$\begin{aligned} &f_t + (\mu - \frac{1}{2}\sigma^2)f_s + \frac{1}{2}\delta^2(f_s)^2 + \frac{1}{2}\sigma^2 f_{ss} \\ &+ [r\phi_t^* + \alpha(1-\phi_t^*) - \frac{1}{2}(1-\phi_t^*)^2\nu^2]f_w \\ &+ \frac{1}{2}(1-\phi_t^*)^2\nu^2 f_{ww} + (1-\phi_t^*)\nu\sigma\rho f_{ws} = 0 \end{aligned} \tag{4.34}$$

This is a nonlinear PDE with three variables. There is no known closed-form solution. Some attempts have been made in the theoretical literature to solve the problem in closed-form. For example, Henderson and Hobson [2002] solve the problem for exponential utility with a simple payoff function $C_T = kS_T$, where k is a known constant. Henderson [2002] solve the problem for general payoff function $C_T = h(S_t)$ using series-based approximation, where $h(S_t)$ is a general function of S_t. Henderson [2005] explored a similar problem using exponential utility.

Following a similar argument, company cost has PDE,

$$\begin{aligned} &C_t + (r - \frac{1}{2}\sigma^2)C_s + \frac{1}{2}\sigma^2 C_{ss} + \left[r - \frac{1}{2}(1-\phi_t^*)^2\nu^2\right]C_w \\ &+ \frac{1}{2}(1-\phi_t^*)^2\nu^2 C_{ww} + (1-\phi_t^*)\nu\sigma\rho C_{ws} - rC = 0 \end{aligned} \tag{4.35}$$

where ϕ^* is given in equation 4.33. Note this equation is also log-transformed. For the single asset model, very similar PDEs can be derived. The manager's valuation PDE follows

$$f_t + (\mu - \frac{1}{2}\sigma^2)f_s + \frac{1}{2}\delta^2(f_s)^2 + \frac{1}{2}\sigma^2 f_{ss} = 0 \tag{4.36}$$

The equation is after log-transformation, where equation (4.24) is the original equation. Under the single asset model, firm cost PDE is merely that of Black-Scholes, hence is omitted here.

4.6.3 Solution methods: single asset model

For computational purposes, all PDEs outlined in the Appendix are log transformed. All PDEs are solved using the finite difference method (FDM), but detailed schemes are slightly different for different PDEs. We use a stable and explicit scheme called the alternating direction explicit (ADE) method, which is introduced in Duffy [2009].

The prime advantages of ADE are that it is explicit (no need to solve the system of equations at each time step, hence great computational speed) and it is unconditionally stable (which most explicit methods lack).

For FDM we need firstly to define a numerical grid that represents the PDE. The grid values for single asset case are defined as follow,

$$\begin{aligned} f_{j,i} &= f(j\Delta s, i\Delta t) \\ \text{for} \quad j &= 0, ..., M \quad \text{and} \quad i = 0, ..., N, \end{aligned} \tag{4.37}$$

where both M and N are positive integers that determine the size of stock and time discretization. Δs and Δt are grid spacing on stock and time direction, so that $f_{j,i}$ is the grid value at grid node (j, i).

Derivatives of function at each grid node (j, i) are then approximated as

$$f_t = \frac{f_{j,i+1} - f_{j,i}}{\Delta t} \tag{4.38}$$

for the time derivative, and

$$f_s = \frac{f_{j+1,i} - f_{j-1,i}}{2\Delta s} \tag{4.39}$$

for the first stock derivative. Second stock derivative is,

$$f_{ss} = \frac{f_{j+1,i} - 2f_{j,i} + f_{j-1,i}}{(\Delta s)^2} \tag{4.40}$$

The single asset PDE by finite difference is then,

$$\begin{aligned} \frac{f_{j,i+1} - f_{j,i}}{\Delta t} = &- (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,i} - f_{j-1,i}}{2\Delta s}\right) \\ &- \frac{1}{2}\delta^2\left(\frac{f_{j+1,i} - f_{j-1,i}}{2\Delta s}\right)^2 \\ &- \frac{1}{2}\sigma^2\left(\frac{f_{j+1,i} - 2f_{j,i} + f_{j-1,i}}{(\Delta s)^2}\right) \end{aligned} \tag{4.41}$$

This is a typical implicit approximation, so that only $f_{j,i+1}$ is known in the above equation. Because of the nonlinear term on the first stock derivative, it is not obvious how to solve the system of equations analytically. SOR is good choice for this mildly nonlinear problem, but its convergence (including its different variants) is quite slow. However, the problem can easily be transformed into two *explicit* equations, the first one is

$$\begin{aligned}\frac{f_{j,i+1} - f_{j,i}}{\Delta t} = &- (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,i} - f_{j-1,i+1}}{2\Delta s}\right) \\ &- \frac{1}{2}\delta^2\left(\frac{f_{j+1,i} - f_{j-1,i+1}}{2\Delta s}\right)^2 \\ &- \frac{1}{2}\sigma^2\left(\frac{f_{j+1,i} - f_{j,i} - f_{j,i+1} + f_{j-1,i+1}}{(\Delta s)^2}\right)\end{aligned} \tag{4.42}$$

At time step i, all grid values at $i+1$ is known. So the explicit equation gives the solution,

$$\begin{aligned}f_{j,i}^D = &\left[\frac{f_{j,i+1}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,i} - f_{j-1,i+1}}{2\Delta s}\right)\right. \\ &+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,i} - f_{j-1,i+1}}{2\Delta s}\right)^2 \\ &\left.+ \frac{1}{2}\sigma^2\left(\frac{f_{j+1,i} - f_{j,i+1} + f_{j-1,i+1}}{(\Delta s)^2}\right)\right] \Big/ \left(\frac{1}{\Delta t} + \frac{1}{2}\frac{\sigma^2}{(\Delta s)^2}\right)\end{aligned} \tag{4.43}$$

This equation is solved from $j = M - 1$ all the way 'down' to $j = 1$. At $j = M$, grid value is simply given by the upper boundary condition. The second explicit equation is,

$$\begin{aligned}\frac{f_{j,i+1} - f_{j,i}}{\Delta t} = &- (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,i+1} - f_{j-1,i}}{2\Delta s}\right) \\ &- \frac{1}{2}\delta^2\left(\frac{f_{j+1,i+1} - f_{j-1,i}}{2\Delta s}\right)^2 \\ &- \frac{1}{2}\sigma^2\left(\frac{f_{j+1,i+1} - f_{j,i} - f_{j,i+1} + f_{j-1,i}}{(\Delta s)^2}\right)\end{aligned} \tag{4.44}$$

which can be solved similarly as,

$$\begin{aligned} f_{j,i}^{U} = & \left[\frac{f_{j,i+1}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2) \left(\frac{f_{j+1,i+1} - f_{j-1,i}}{2\Delta s} \right) \right. \\ & + \frac{1}{2}\delta^2 \left(\frac{f_{j+1,i+1} - f_{j-1,i}}{2\Delta s} \right)^2 \\ & \left. + \frac{1}{2}\sigma^2 \left(\frac{f_{j+1,i+1} - f_{j,i+1} + f_{j-1,i}}{(\Delta s)^2} \right) \right] \Big/ \left(\frac{1}{\Delta t} + \frac{1}{2} \frac{\sigma^2}{(\Delta s)^2} \right) \end{aligned} \tag{4.45}$$

The equation is solved from $j = 1$ all the way 'up' to $j = M - 1$. At $j = 0$, grid value is given by the lower boundary condition. The solution at node (i, j) is then obtained by averaging the two terms

$$f_{j,i} = \frac{f_{j,i}^{U} + f_{j,i}^{D}}{2} \tag{4.46}$$

Given the manager's utility function, the boundary conditions mentioned above and terminal condition are given,

$$\begin{aligned} f_{0,i} &= \frac{W^{1-\gamma}}{1-\gamma} \\ f_{M,i} &= \frac{\left[n(M\Delta s - K)^{+} \exp\left(r(N-i)\Delta t \right) + W \right]^{1-\gamma}}{1-\gamma} \\ f_{j,N} &= \frac{\left[n(j\Delta s - K)^{+} + W \right]^{1-\gamma}}{1-\gamma} \end{aligned} \tag{4.47}$$

where K is the exercise price of the option and W is the manager's wealth which is exogenous to the model. In addition to boundary and terminal conditions, the value of exercising the option is,

$$f_{j,i}^{E} = \frac{\left[n(j\Delta s - K)^{+} \exp\left(r(N-i)\Delta t \right) + W \right]^{1-\gamma}}{1-\gamma} \tag{4.48}$$

At every node, solution $f_{j,i}$ is replaced with $f_{j,i}^{E}$ if $f_{j,i}^{E} > f_{j,i}$.

4.6.4 Solution methods: two asset model

The valuation PDE as outlined in previous section is highly nonlinear. Brennan et al. [1997] solve a similar PDE using successive-over-relaxation, but this method converges slowly when the solution gird becomes large. Carpenter et al. [2010] solve the problem using splitting methods which subdivides the PDE into two ODEs and greatly increases computational performance. However, splitting methods have its problem: firstly it creates fictional time step where boundary conditions are not known. Secondly most splitting methods are still implicit which mean a system of equations has to be solved at each time step; it is very costly to solve such systems especially when grid size becomes large. ADE, on the other hand, is explicit and easy to generalize to a higher dimension. The ADE method outlined below is conditionally stable due to explicit approximation of the nonlinear terms. On the other hand, the splitting method is more stable and yield better results. Still, ADE is faster than the splitting method. Since the splitting method is widely used, we only talk about the ADE method. For the two asset model, grid values are defined,

$$f_{j,k,i} = f(j\Delta s, w_0 + k\Delta w, i\Delta t)$$
$$\text{for} \quad j = 0, ..., M, \quad k = 0, ..., L, \quad \text{and} \quad i = 0, ..., N, \tag{4.49}$$

where $w_0 > 0$ is the lowest value of wealth on the grid; it is kept positive to make sure boundary conditions are well defined. For all computations in this chapter, $w_0 = 0.01$ is used. Derivative approximations are as follows,

$$f_t = \frac{f_{j,k,i+1} - f_{j,k,i}}{\Delta t} \tag{4.50}$$

$$f_s = \frac{f_{j+1,k,i} - f_{j-1,k,i}}{2\Delta s} \tag{4.51}$$

$$f_{ss} = \frac{f_{j+1,k,i} - 2f_{j,k,i} + f_{j-1,k,i}}{(\Delta s)^2} \tag{4.52}$$

$$f_w = \frac{f_{j,k+1,i} - f_{j,k-1,i}}{2\Delta w} \tag{4.53}$$

$$f_{ww} = \frac{f_{j,k+1,i} - 2f_{j,k,i} + f_{j,k-1,i}}{(\Delta w)^2} \tag{4.54}$$

The above approximations are implicit. With explicit approximation on wealth direction and also the mixed derivative,

$$\begin{aligned}
f_w &= \frac{f_{j,k+1,i+1} - f_{j,k-1,i+1}}{2\Delta w} \\
f_{ww} &= \frac{f_{j,k+1,i+1} - 2f_{j,k,i+1} + f_{j,k-1,i+1}}{(\Delta w)^2} \\
f_{ws} &= \frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w \Delta s}
\end{aligned} \tag{4.55}$$

These explicit terms are used to linearize the PDE. Based on PDE (4.34), and difference approximation of derivatives, the valuation function (4.34) has the following implicit form,

$$\begin{aligned}
&\frac{f_{j,k,i+1} - f_{j,k,i}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,k,i} - f_{j-1,k,i}}{2\Delta s}\right) \\
&+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,k,i} - f_{j-1,k,i}}{2\Delta s}\right)^2 + \frac{1}{2}\sigma^2\left(\frac{f_{j+1,k,i} - 2f_{j,k,i} + f_{j-1,k,i}}{(\Delta s)^2}\right) \\
&+ [r\phi^* + \alpha(1-\phi^*) - \frac{1}{2}(1-\phi^*)^2\nu^2]\left(\frac{f_{j,k+1,i} - f_{j,k-1,i}}{2\Delta w}\right) \\
&+ \frac{1}{2}(1-\phi^*)^2\nu^2\left(\frac{f_{j,k+1,i} - 2f_{j,k,i} + f_{j,k-1,i}}{(\Delta w)^2}\right) \\
&+ (1-\phi^*)\nu\sigma\rho\left(\frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w\Delta s}\right) = 0
\end{aligned} \tag{4.56}$$

Note that ϕ^* is approximated explicitly based on equation (4.55) where formula of ϕ^* is given in equation (4.33). Mixed derivative is also tackled explicitly. After treating all nonlinear terms explicitly, the valuation PDE is linear. Brennan et al. [1997], Kahl et al. [2003] and Carpenter et al. [2010] use the same linearization scheme. Similar to the single asset case, the difference equation is transformed to two *explicit* equations,

$$\begin{aligned}
&\frac{f_{j,k,i+1} - f_{j,k,i}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,k,i} - f_{j-1,k,i+1}}{2\Delta s}\right) \\
&+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,k,i} - f_{j-1,k,i+1}}{2\Delta s}\right)^2 + \frac{1}{2}\sigma^2\left(\frac{f_{j+1,k,i} - f_{j,k,i} - f_{j,k,i+1} + f_{j-1,k,i+1}}{(\Delta s)^2}\right) \\
&+ [r\phi^* + \alpha(1-\phi^*) - \frac{1}{2}(1-\phi^*)^2\nu^2]\left(\frac{f_{j,k+1,i} - f_{j,k-1,i+1}}{2\Delta w}\right) \\
&+ \frac{1}{2}(1-\phi^*)^2\nu^2\left(\frac{f_{j,k+1,i} - f_{j,k,i} - f_{j,k,i+1} + f_{j,k-1,i+1}}{(\Delta w)^2}\right) \\
&+ (1-\phi^*)\nu\sigma\rho\left(\frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w\Delta s}\right) = 0
\end{aligned} \tag{4.57}$$

and

$$\begin{aligned}
&\frac{f_{j,k,i+1} - f_{j,k,i}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,k,i+1} - f_{j-1,k,i}}{2\Delta s}\right) \\
&+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,k,i+1} - f_{j-1,k,i}}{2\Delta s}\right)^2 + \frac{1}{2}\sigma^2\left(\frac{f_{j+1,k,i+1} - f_{j,k,i} - f_{j,k,i+1} + f_{j-1,k,i}}{(\Delta s)^2}\right) \\
&+ [r\phi^* + \alpha(1-\phi^*) - \frac{1}{2}(1-\phi^*)^2\nu^2]\left(\frac{f_{j,k+1,i+1} - f_{j,k-1,i}}{2\Delta w}\right) \\
&+ \frac{1}{2}(1-\phi^*)^2\nu^2\left(\frac{f_{j,k+1,i+1} - f_{j,k,i} - f_{j,k,i+1} + f_{j,k-1,i}}{(\Delta w)^2}\right) \\
&+ (1-\phi^*)\nu\sigma\rho\left(\frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w\Delta s}\right) = 0
\end{aligned} \tag{4.58}$$

with solutions

$$
\begin{aligned}
f^D_{j,k,i} = &\left[\frac{f_{j,k,i+1}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,k,i} - f_{j-1,k,i+1}}{2\Delta s}\right)\right. \\
&+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,k,i} - f_{j-1,k,i+1}}{2\Delta s}\right)^2 + \frac{1}{2}\sigma^2\left(\frac{f_{j+1,k,i} - f_{j,k,i+1} + f_{j-1,k,i+1}}{(\Delta s)^2}\right) \\
&+ [r\phi^* + \alpha(1-\phi^*) - \frac{1}{2}(1-\phi^*)^2\nu^2]\left(\frac{f_{j,k+1,i} - f_{j,k-1,i+1}}{2\Delta w}\right) \\
&+ \frac{1}{2}(1-\phi^*)^2\nu^2\left(\frac{f_{j,k+1,i} - f_{j,k,i+1} + f_{j,k-1,i+1}}{(\Delta w)^2}\right) \\
&\left.+ (1-\phi^*)\nu\sigma\rho\left(\frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w\Delta s}\right)\right] \Big/ F(j,k,i)
\end{aligned}
\tag{4.59}
$$

and

$$
\begin{aligned}
f^U_{j,k,i} = &\left[\frac{f_{j,k,i+1}}{\Delta t} + (\mu - \frac{1}{2}\sigma^2)\left(\frac{f_{j+1,k,i+1} - f_{j-1,k,i}}{2\Delta s}\right)\right. \\
&+ \frac{1}{2}\delta^2\left(\frac{f_{j+1,k,i+1} - f_{j-1,k,i}}{2\Delta s}\right)^2 + \frac{1}{2}\sigma^2\left(\frac{f_{j+1,k,i+1} - f_{j,k,i+1} + f_{j-1,k,i}}{(\Delta s)^2}\right) \\
&+ [r\phi^* + \alpha(1-\phi^*) - \frac{1}{2}(1-\phi^*)^2\nu^2]\left(\frac{f_{j,k+1,i+1} - f_{j,k-1,i}}{2\Delta w}\right) \\
&+ \frac{1}{2}(1-\phi^*)^2\nu^2\left(\frac{f_{j,k+1,i+1} - f_{j,k,i+1} + f_{j,k-1,i}}{(\Delta w)^2}\right) \\
&\left.+ (1-\phi^*)\nu\sigma\rho\left(\frac{f_{j+1,k+1,i+1} - f_{j+1,k-1,i+1} - f_{j-1,k+1,i+1} + f_{j-1,k-1,i+1}}{4\Delta w\Delta s}\right)\right] \Big/ F(j,k,i)
\end{aligned}
\tag{4.60}
$$

where

$$
F(j,k,i) = \frac{1}{\Delta t} + \frac{\sigma^2}{2(\Delta s)^2} + \frac{(1-\phi^*)^2\nu^2}{2(\Delta w)^2}
\tag{4.61}
$$

Table 4.3: This table reports value and cost of ESO with no effort. The stock return is set to be consistent with CAPM. In order to ensure accuracy of our results, parameter values are chosen to be those of Carpenter, Stanton and Wallance [2010], they are: $P_0 = 1$, risk free rate, $r = 0.05$, time to maturity, $T = 10$, vesting period, $T_V = 2$, market return, $\mu = 0.13$, market volatility, $\nu = 0.2$, initial wealth, $W_0 = 0.6$, risk aversion coefficient, $\gamma = 4$. CSW value and cost are results from Table 3 of Carpenter, Stanton and Wallance [2010].

	Volatility=0.50, Beta=1.2			
Parameter	Value	Cost	CSW value	CSW cost
Panel A: Beta effects				
0.0	0.158	0.424	0.158	0.425
0.5	0.166	0.429	0.166	0.430
0.9	0.180	0.439	0.180	0.440
1.2	0.196	0.450	0.197	0.451
1.4	0.211	0.460	0.210	0.460
Panel B: Volatility effects holding beta constant				
0.25	0.409	0.467	0.411	0.467
0.30	0.297	0.427	0.298	0.426
0.40	0.230	0.421	0.229	0.421
0.50	0.196	0.450	0.195	0.450
0.60	0.172	0.485	0.170	0.486
Panel C: Volatility effects holding correlation constant				
0.25	0.215	0.340	0.214	0.339
0.30	0.213	0.359	0.212	0.359
0.40	0.206	0.404	0.205	0.404
0.50	0.196	0.450	0.195	0.450
0.60	0.184	0.493	0.182	0.494

So the solution at node (j, k, i) is,

$$f_{j,k,i} = \frac{f^U_{j,k,i} + f^D_{j,k,i}}{2} \tag{4.62}$$

Given the manager's utility function, the boundary conditions are,

$$
\begin{aligned}
f_{0,k,i} &= \frac{(w_0 + k\Delta w)^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\beta^2\gamma}(T - i\Delta t)\Big)\right] \\
f_{M,k,i} &= \frac{\big(M\Delta s - K + w_0 + k\Delta w\big)^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\beta^2\gamma}(T - i\Delta t)\Big)\right] \\
f_{j,0,i} &= \frac{\big((j\Delta s - K)^+ + w_0\big)^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\beta^2\gamma}(T - i\Delta t)\Big)\right] \\
f_{j,N,i} &= \frac{\big((j\Delta s - K)^+ + w_0 + N\Delta w\big)^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\beta^2\gamma}(T - i\Delta t)\Big)\right]
\end{aligned}
\tag{4.63}
$$

with terminal condition,

$$
f_{j,k,L} = \frac{\big((j\Delta s - K)^+ + w_0 + k\Delta w\big)^{1-\gamma}}{1-\gamma} \tag{4.64}
$$

Finally, the exercise value is

$$
f^E_{j,k,i} = \frac{\big((j\Delta s - K)^+ + w_0 + k\Delta w\big)^{1-\gamma}}{1-\gamma} \exp\left[(1-\gamma)\Big(r + \frac{(r-\alpha)^2}{2\beta^2\gamma}(T - i\Delta t)\Big)\right] \tag{4.65}
$$

At every node, solution $f_{j,k,i}$ is replaced with $f^E_{j,k,i}$ if $f^E_{j,k,i} > f_{j,k,i}$.

For robustness check, Table 4.3 compares our results with those of Carpenter, Stanton and Wallance [2010]. All results in Table 4.3 are computed based on 200 steps in stock price space (which range from 0 to 4), 200 steps in wealth space (which range from 0 to 5), and 10,000 steps in time space. The differences in our results may due to different treatments of boundary conditions and inter-time step boundary in the splitting method, which Carpenter is not explicit about (the small difference still presents even we increase grid size to $400 \times 400 \times 10,000$). As Duffy [2009] suggest that inter-time step boundary conditions of splitting methods are still poorly understood. We expect to have exact solutions of Carpenter's if we impose their boundary conditions.

Our boundary conditions are shown in equation (4.63). We also impose these boundary conditions on inter-time steps.

Henderson and Hobson [2009a] found that the cost valuation function C, which satisfies equation (4.35), is convex in wealth at some parameter values. We did not find any convexity in the parameters reported in this chapter. This *may* be due to the fact the numercial scheme performs relatively poorly at low wealth level, as the manager's utility is negative infinity when wealth approaches zero. We fix the wealth at a finitely low level (it is 0.01 for all our numerical results) to cope with this problem. Carpenter et al. [2010] also noted that convexity in wealth does not affect their numerical results.

Chapter 5

Future research

Although chapters 2, 3, and 4 are self-contained essays that answer specific questions, they by no means complete our understanding of executive compensation. There are multiple avenues to extend findings of these chapters.

Firstly, debt in chapter 2 only has a scaling effect, which increases the firm asset base and magnifies managerial actions. This is a reasonable assumption for a financial institution, which uses debt to lever up investments. It may not be true for a non-financial firm as debt can be used for other purposes, e.g. saving corporate tax, researching new technology, etc. Also, the firm's operation may not be easily scalable. A new model can explore this by introducing alternative interactions between debt and other firm characteristics. Another possible extension is to incorporate shareholder-debtholder agency cost to the model, so that the model has two agency problems: shareholder-manager and shareholder-debtholder. Such a model can investigate interaction between the two agency relationships.

Secondly, in chapter 3 we assume that the effective tax rate is always fixed at the highest level. It is a rather simplified assumption made for exposition purposes. As the effective tax rate is a function of corporate income and deduction, relaxing this assumption requires full knowledge of firms' financial information and a forecast of possible future income, so that more accurate estimate of effective tax rate can be used in the valuation model. It is essentially a model that combines Graham et al. [2004] and our tax-inclusive valuation model.

Thirdly, the model in chapter 4, as most studies in the literature, is only partial. ESO positions are exogenously given. In an equilibrium model, both ESO position and managerial effort should be choice variables. It would be very interesting to make the model a full equilibrium one where compensation contracts, effort choice and ESO pricing are simultaneously determined. However, it is not obvious how such a complex (principal-agent) model can be easily embedded in a equilibrium one. Ou-Yang [2005] solves the problem for linear contracts; it is far from obvious how this can be applied to non-linear instruments such as options.

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