DETERMINING THE RANGE OF PREDICTIONS FOR CALIBRATED AGENT-BASED SIMULATION MODELS

By

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DECLARATION

I declare that the work presented in this thesis is my own, except where stated otherwise.

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Abstract

Agent-based simulation is increasingly used to study systems in many areas of business and science nowadays. Agent-based simulation refers to simulations of systems that contain agent entities whose behaviour depends dynamically on the state of the system. This enables the agents to adapt their behaviour to changing conditions. For some applications, using agent-based simulation for prediction (rather than just for a better understanding) could be very powerful. For example, a company might wish to use a model of the population of their customers with word-of-mouth interactions to predict the sales of their product or the effect of an advertising campaign. However, the problem is that agent-based models typically have a very large number of parameters and many of these cannot be measured directly or estimated with sufficient precision.

The result is that a wide range of sets of parameter values may give an acceptable fit and are therefore feasible values. However, they may give quite different predictions. Therefore, simply choosing a single set of parameter values that produces a good fit may mean that the model results are incorrect and very misleading. The inverse problem has been studied in other areas of science including groundwater modelling (Brooks et al., 1994), but it appears that this issue has not yet been investigated for agent-based simulation.

In order to investigate the extent of this problem, in the research an agent-based consumer diffusion model was developed and treated as the real system. Selected output data from this model was used as measured values from the real world. In a pseudo-modelling exercise, this data was then used to calibrate agent-based models of the system, and a method similar to that of Brooks et al. (1994) was used to find the extent of the variations in predictions. The method had to be adapted since the model in this research is stochastic whereas the method had previously only been

applied to deterministic groundwater models.

In the model, a social network of individuals who interact with one another rather than a vast population of agents with many neutral contacts is represented. All agents are allocated to a diffusion social circle with a certain level of influence within the social network. These are constant attributes for that individual throughout the simulation. All agents initially have no knowledge or preference about the selected product. During the simulation, agents receive marketing communication messages (i.e. from company's advertisements, supermarkets, online search results etc.) and contact each other to exchange their knowledge and preferences about the product. There has been very little agent based modelling of this situation and the mechanisms developed represent a potential theoretical structure for this application. Sensitivity analysis was carried out and the model appears to produce realistic behaviour.

The adapted method was applied to four experiments of different amounts of observed data (initial periods of 70, 105, 140 and 175 days) to find the range of predictions of total sales in each case. The total sales for the real system model were 124 and the range of predictions for the four experiments were [58, 376], [79, 319], [91, 277], [109, 187]. As expected, the prediction range narrows as more data is available. However, the range of predictions is very wide for all four experiments and therefore the model would have limited usefulness for predicting sales in this type of situation. In particular, choosing a single set of parameter values is not appropriate and could produce very misleading results.

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I cherish the love that my parents have generously, continuously given to me at all times. Without them, this work would never have come into existence (literally).

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Some of the thesis published as papers presented in conferences.

Chapter 1

INTRODUCTION

1.1 INTRODUCTION

In recent years, agent-based (or individual-based) simulation has received a lot of attention. Agent-based simulation refers to simulations of systems that contain agent entities whose behaviour depends dynamically on the state of the system. This enables the agents to adapt their behaviour to changing conditions. In modelling such adaptive behaviour, agent-based simulation as a tool is commonly used in complexity science (Waldrop, 1993).

There is no standard definition of an agent. Some definitions list a set of properties, but a better approach is perhaps simply to say that an agent is an entity for which some cognitive process is modelled (Edmonds and Mohring, 2005). Usually, agents receive information from the environment (including other agents) and have internal rules that represent the cognitive decision process and determine how they respond. The rules can be a simple function of the inputs received or can be very complex incorporating various internal state parameters, which can include a model representing the agent's worldview of some part of the environment (such as predictions of other agents' behaviour). An example of a framework for complex cognitive processes is the PECS model, which has a hierarchical structure with states for physic (physical body), emotion, cognition and social status as well as sub-components for each of these (Schmidt and Schneider, 2004).

In some cases, the rules governing the agents' behaviour are fixed throughout the simulation, while in other cases, the rules can change to represent learning. The number of agents modelled can also vary from an individual agent through to a large population. Populations are usually heterogeneous with individual agents having different parameters or even quite different rules (e.g. different trading strategies in a stock-market simulation). Interactions between the agents are often a key part of the behaviour of the system. A very wide variety of applications have been studied using agent-based simulation, including stock markets, auctions, the spread of disease, ecosystems, military battles, crowd dynamics, sports games, transport, social behaviour, social networks, the development of technology, and consumer market behaviour (such as fads). For instance, the agents might represent stock brokers in stock markets, bidders in an auction, disease cells, autonomous characters in computer games, vehicles in traffic, chunks of code in software, people in crowds, economic regimes, or plants and animals in ecosystems.

For some applications, using agent-based simulation for prediction (rather than just better understanding) could be very powerful. For example, a company might wish to use a model of the population of their customers with word-of-mouth interactions to predict the sales of their product or the effect of an advertising campaign. However, the problem is that agent-based models typically have a very large number of parameters, and many of these cannot be measured directly or estimated with sufficient precision. The only other information available may be historical output data from the real system. Such data can be used to calibrate the model by finding parameter values that produce a good fit with the data. This is known as an inverse problem since it consists of using the outputs to determine the inputs. The problem is that there will usually be many solutions. There are two main reasons for this. The first is that there are often many parameters and few historical data values. The second is that any model that produces a good fit could be considered acceptable. A perfect fit is not expected because any simulation is a simplification of the real system and also there may be measurement errors in the historical data.

The result is that a wide range of sets of parameter values may give an acceptable fit and are therefore feasible values. However, they may give quite different predictions. Therefore, simply choosing a single set of parameter values that produces a good fit may mean that the model results are incorrect and very misleading. The inverse problem has been studied in other areas of science including groundwater modelling (Brooks et al., 1994), but it appears that this issue has not yet been investigated for agent-based simulation. An important difference between agent-based simulation and groundwater modelling is that agent-based simulation models are stochastic where groundwater models are deterministic.

The complex non-linear nature of most simulation models means that there is no simple equation for the feasible values of the parameters. Therefore, methods used for tackling the inverse problem have involved running the simulation and deriving alternative predictions in some way from those runs.

Formally I can define in this context:

- Calibration problem: The problem of obtaining the values of parameters that cannot be measured directly. This requires a calibration process using output data, which is an inverse problem.

- Inverse problem: Generally refers to problems where the answer is known but the question is unknown, and so knowledge of the answer is used to find the question. In this case, the process of using output data to determine model inputs, by finding parameter values that give model output that is close to the observed output values.

- Prediction problem: The problem of using a model to make predictions when there is uncertainty as to the parameter values. The different parameter values that give a good fit can produce a wide range of predictions and so the process of making predictions needs to take this into account.

This research sets out an approach, which is explained in the following chapters, to investigate these problems further for agent-based simulation by developing a consumer word-of-mouth model and searching for the range of predictions that arise from alternative acceptable calibrated parameter values.

1.2 RESEARCH OBJECTIVES

1.2.1 Main Objective

The main objective of the research is to investigate the effect of obtaining the parameter values of an agent-based model by calibration when using the model for prediction, by:

- developing and implementing a method based on previous research for obtaining an acceptable range of predictions from the alternative acceptable calibrations.
- comparing the range of predictions for different scenarios of the data available for calibration.

1.2.2 Secondary Objectives

The system studied in the research is consumer word-of-mouth (WOM) interactions. Few agent-based models have been built of this situation and there is no consensus as to the best way to model the agents or the interactions. Therefore, a secondary objective is to contribute towards modelling in this area by:

- developing a new agent-based WOM consumer model
- investigating the relationships between the parameters and the model output
- assessing whether the model produces realistic output.

1.3 BRIEF OVERVIEW OF APPROACH

The approach used was to develop an agent-based model and to treat this model as the real system. Output data from this model could then be taken as measured values from the real world and, in a pseudo-modelling exercise, used to calibrate an agent-based model of the system. The advantage of such a pseudo-modelling exercise is that the "real system" is completely known. Consequently, the model's predictions can be compared with the "true" future values, and the precise differences between the model and the real system are also known.

1.4 THESIS STRUCTURE

Figure 1.1 shows the structure of the thesis and how the chapters contribute to the problem being investigated. *Chapters 2, 3* and *4* set the work in context by reviewing previous literature, while *Chapter 5* set out the research methodology. *Chapter 6* explains how the model used in the research works. *Chapter 7* describes experiments, to understand the behaviour of the model, including sensitivity analysis. *Chapter 8* describes the implementation of the calibration method. Finally, *Chapter 9* summarises the contents of the thesis and describes possible future research.

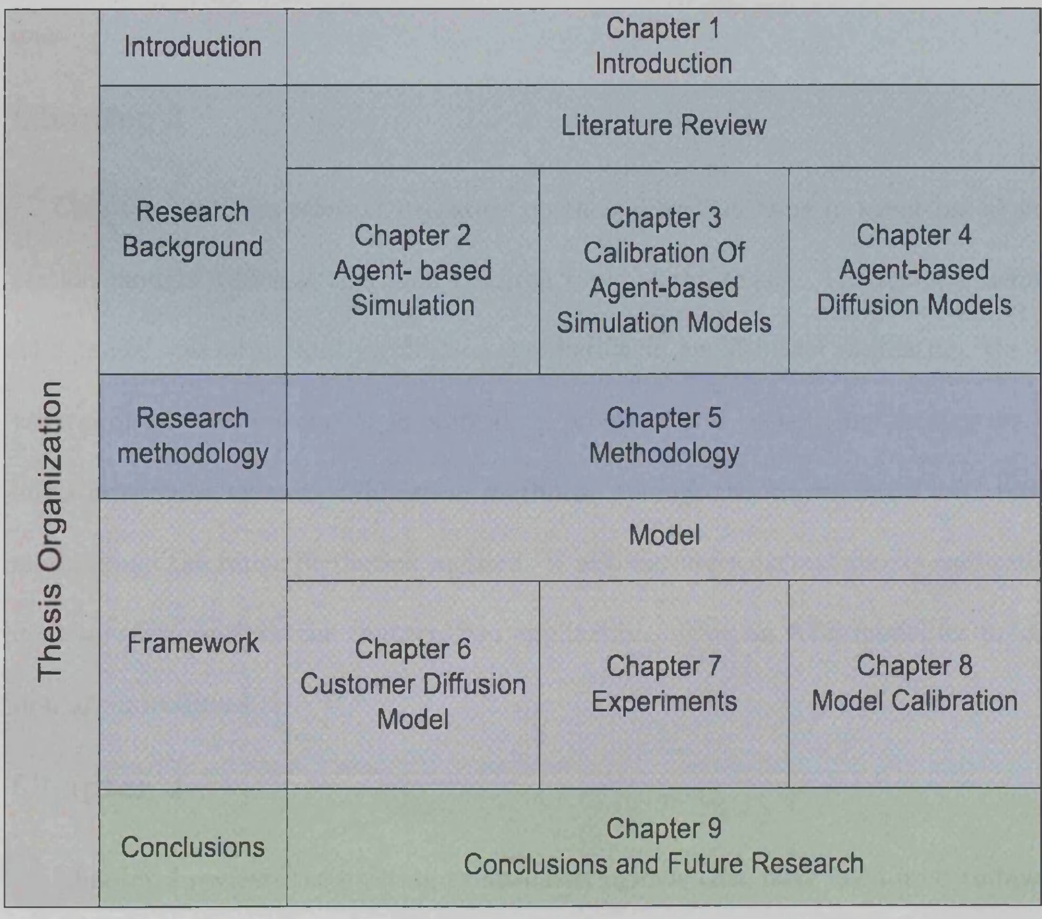

Figure 1.1: Thesis Structure

The following sections give an overview of each chapter.

Chapter 2

Chapter 2 provides a context for the rest of the thesis and discusses a range of ideas relevant to the validation of agent-based models. It defines complexity, complex adaptive systems and agent-based simulation (ABS). It identifies the main theoretical and methodological perspectives of ABS, and reviews recent work and key themes of discussion and debate in this field. In addition, this chapter reviews currently used agent-based simulation software packages and provides a brief introduction to each one.

Chapter 3

Chapter 3 reviews relevant literature on the calibration issue in agent-based simulation models, which is the main research topic of the thesis. The chapter defines ABS model validation and verification, prediction in agent-based simulation, the inverse problem, the parameter identification problem, and best-fitting parameters. It introduces some existing calibration methods, namely the Bayesian MCMC based method and the range prediction method. It also discusses current model-calibrating methods. To conclude the chapter, two applications using an ABS model for prediction are introduced.

Chapter 4

Chapter 4 reviews the existing agent-based models that have been used to investigate marketing phenomena, including the widely cited PECS model, the intelligent customer relationship management (iCRM) model from BT and the J-pop agent-based prediction model. It also briefly introduces the classic 1969 Bass diffusion model, which has now become the fundamental theoretic frame of most diffusion models. The chapter concludes with a discussion of the advantages and disadvantages of each of the above models.

Chapter 5

Chapter 5 illustrates the research methodology used in the research through a step-by-step simulation modelling plan for the research. In the end of the chapter, it gives a discussion of the advantages and disadvantages of the approach used in the research.

Chapter 6

Chapter 6 describes the model (an agent-based consumer word-of-mouth model) used to conduct the research. It introduces the model's structure (including agents' environments, agents' attributes and how agents' interactions will change one another's attributes), the model's parameters and the model's procedure. It also details the model's validation and verification, and describes a manual simulation and an Excel-based formulation used to verify the model.

Chapter 7

Chapter 7 describes some experiments conducted to understand the way the model behaves. It starts with the model output study to give a general idea of how the model behaves, followed by sensitivity analysis. Sensitivity analysis was conducted to investigate the impacts of various parameters, including the probability of losing knowledge at the end of each simulation day, the probability of an agent talking to agents from the same group, the probability of an agent receiving outside marketing information, the mean in the normal distribution of an agent's buying criterion and the mean in the normal distribution of an agent's unbiased true preference. An experiment on the knowledge and preferences of the outside marketing sources were also undertaken. Additionally, the number of agents in each group was varied.

Chapter 8

Chapter 8 demonstrates the step-by-step implementation of the calibration method used in the research on the agent-based WOM model. By adopting a similar method to that of Brooks et al. (1994) for searching for parameter sets that fit the model, the model used in the research shows a big prediction range in a variety of scenarios. It concludes that a calibrated model can still produce a big range of prediction and a careful calibration is needed to qualify the use of agent-based simulation models for prediction .

Chapter 9

Chapter 9 summarises the contents of the thesis and discusses the results of experiments, how these met the objectives of the research as set out in Chapter 1, the limitations of the model used and future areas of research.

Chapter 2

AGENT-BASED SIMULATION

Chapter Overview

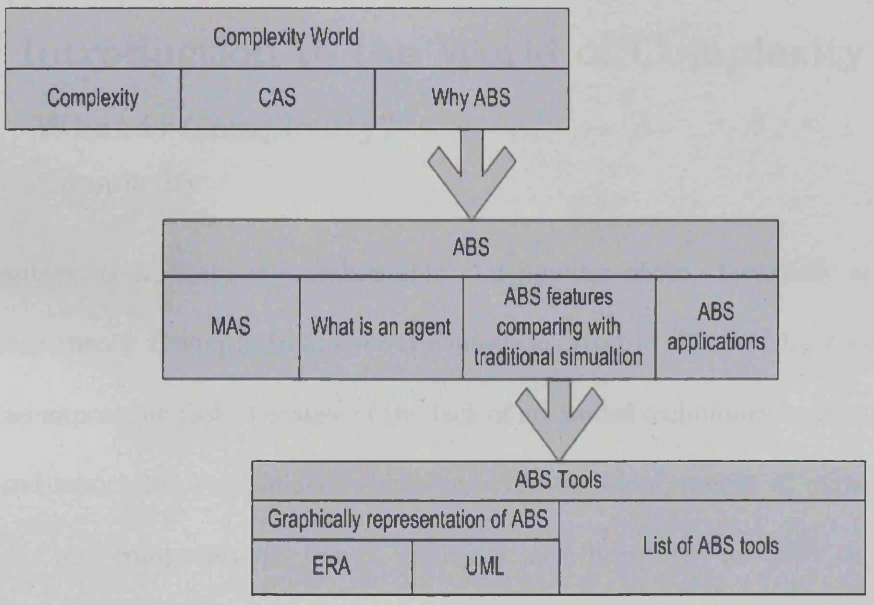

Figure 2.1: Contents of Chapter 2

This chapter provides a context for the rest of the thesis and discusses a range of ideas relevant to the validation of agent-based models. It defines complexity, the complex adaptive system and agent-based simulation (ABS). It identifies the main theoretical and methodological perspectives of ABS and reviews recent work and key themes of discussion and debate in this field.

Section 2.1 gives an introduction to the world of complexity and complex adaptive systems, which highlights the point that agent-based simulations are suitable for studying complex adaptive systems.

Section 2.2 describes the key features that an agent should have and explains the procedure of agent-based simulation as a modelling technique. It also compares agent-based simulation with traditional simulation. *Section 2.3* introduces the graphical representation methods for the ABS model (e.g. ERA and UML). It also summaries the main ABS modelling tool kits.

2.1 Introduction to the World of Complexity

2.1.1 What is Complexity?

2.1.1.1 Complexity

Nowadays, complexity is a fashionable and popular topic. Generally speaking, complexity theory attempts to answer the questions that in the past have been considered as impossible tasks because of the lack of advanced techniques, computational power and associated complexity. However, with the development of experimental technology and computational power, scientists have been able to study certain aspects of the complex world, and complexity theory has been applied to a variety of existing domains, such as stock markets, auctions, the spread of disease, ecosystems, military battles, crowd dynamics, sports games, transport, social behaviour, social networks, the development of technology, and consumer market behaviour (such as fads), though it is still not well-defined.

2.1.1.2 Fingerprints of The Complex

The following are a few of the most important "Fingerprints of the complex" recognized by Casti (1997):

- **Instability**: Complex systems tend to shift between many possible modes of behaviour, and the whole system can be affected dramatically by small changes (such as the "tipping point").

- **Adaptability**: Agents in the complex system are sometimes able to change their decision rules on the basis of partial information about the entire system.

- **Irreducibility/Emergence**: The complex system should be studied as a unified system. In other words, the behaviour of the system is determined by interactions among agents, so that it cannot be studied by looking at agents in isolation. Therefore, complex systems produce surprising outputs/behaviour. In other words, system behaviour patterns and properties cannot be predicted easily via individuals' rules of behaviour.

- **Memory**: Complex adaptive systems have memory, which is distributed throughout the whole system instead of being located at a specific place. The whole system behaviour is related to the system history.

- **Connectivity**: The complex system's elements are connected and interactive. What makes a system a system and not simply a collection of elements are the connections and interactions of the individual components of the system, as well as the effect of these linkages on the behaviour of the components.

Among the above fingerprints of the complex, Casti (1997) argues that the most distinguishing single feature of the complex system is "emergent behaviour". The appearance of the "emergent behaviour" is related to the whole system history behaviour and mainly due to the interaction between system parts. However, the system output is not usually predictable by analyzing separate system parts.

From the late 20th century, researchers began to explain this "emergent behaviour" as the result of non-linear world around us. In the nonlinear systems, it was found that capturing the exact rules/equations of their behaviour is sometimes of little help in predicting system outcomes. Real-world systems, especially those involving people, are generally too nonlinear to predict (Lucas, 1999). Researchers have found that the traditional theory was limited in terms of interpreting such Complex Adaptive Systems (CAS). They define the essence of CAS that they self-organise to improve/optimize the objective function and the system behaviour depends on the interactions of system parts (Lucas, 1999; Casti, 1997). Furthermore, Casti (1997) summarizes a number of characteristics he describes as the "Key Components" of CAS, namely:

- **Medium-sized number of agents**: The number of agents must be neither so small that all their interactions could be worked out very easily, nor so large that statistical aggregation methods could answer most kinds of questions about the system.

- **Intelligent and adaptive agents**: Agents are intelligent and autonomous;

they are capable of responding to external changes with the help of in-built behaviour rules and forming their self-maintaining systems with internal feedback paths.

- **Local information**: No agent has perfect information about the whole system. The agent only has "local" or "partial" information. In other words, there is no agent in the system who knows what every other agent is doing. Therefore, in the system, agents are making their decisions based on limited information.

We can take ecosystems as a typical example to examine the above key points of CAS. In an ecosystem, the system patterns emerge from "localized interactions and selection processes acting at lower levels. An essential aspect of such systems is nonlinearity, leading to historical dependency and multiple possible outcomes of dynamics" (Levin, 1998). In other words, in ecosystems, knowing a single species behaviour rule does not help with predicting the whole system emergent pattern. Such patterns arise from the interactions between species and are related to the system's previous status/pattern.

2.1.2 Tipping Points

Gladwell (2002) brought the term "tipping point" into CAS to describe the aforementioned "emergent behaviour" in a social context. The tipping point is a sociological term that refers to "the moment when something unique becomes common." For instance, a tipping point could refer to the moment of an epidemic outbreak (e.g. the dramatic moment in an epidemic when everything changes all at once), boiling point, critical mass etc. In order to define "tipping point" further, Gladwell (2002) identifies the following five key concepts of tipping point:

The Law of the Few Among the whole population, there are some people with much higher influence than others. Also, these people are willing to spread the information of social phenomena through a population. Without their aid, the "tipping point" is unlikely to occur.

The Stickiness Factor Messages about the new ideas or products must be found attractive or interesting by others (i.e. easy to remember, attractive for people to move to action.).

The Power of Context Gladwell claimed that human beings are more sensitive to their environment than they seem to be. The context changes can sometime tip an epidemic unexpectedly.

The Magic Number 150 Some researchers suggest 150 is the maximum number of people which an individual can have social relationships with (Dunbar's number[1]).

The New Product Cycle In an adoption innovation model (*Figure 4.2*), Rogers (1962) presented a bell curve of adaptation to a new phenomenon. When a new product was put into the market, the adopters were categorized into five groups based on their attitudes to the new product, namely: innovators, early adopters, early majority, late majority, and laggards. According to Roger's research, the

[1]Dr. Robin I. M. Dunbar: an evolutionary psychologist at the University of Liverpool School of Biological Sciences. The Dunbar's Number is still a conjecture, supported only by statistical and anecdotal evidence. But some researchers (sociologists, anthropologists, managers and, even some online game designers) have already used the number as proven fact.

majority adopters are "early majority and late majority" which accounted for 64 % of the population.

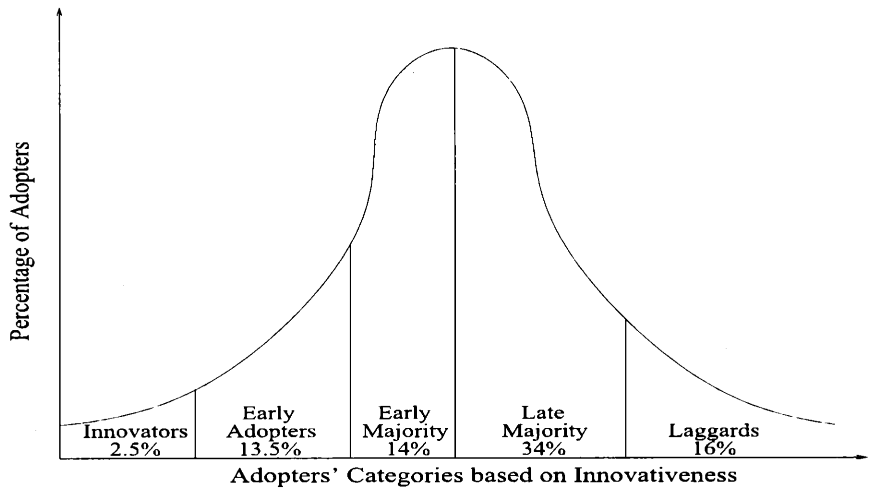

Figure 2.2: Rogers adoption innovation curve (Rogers, 1962)

2.1.3 Why is Agent-based Simulation Suitable to Study CAS?

The aim of simulation in general, is to gain insight into the systems that people do not completely understand. Agent-based simulations enable and aid the understanding of complex systems. Agent-based simulations are suitable for the study of CAS because the model is based on simple rules or algorithms by which the agents within a population behave, instead of the almost impossible task of building a mass detailed model where all interactions between agents and their effects are mapped out. Furthermore, three main reasons for adopting agent-based simulation to study CAS are:

- Medium number of agents: As mentioned above, with medium-sized numbers of agents, statistical analysis techniques do not work well.

- Complex interactions: Because of the complex and sometimes nonlinear, and discontinuous interactions between the heterogeneous agents, the behaviour of the system as whole is difficult to predict based on individual's behaviour. Traditional analytic techniques cannot cope with the complex interactions of CAS (Bonabeau, 2002a). That is also one reason why analytical tools have not been widely used in social science before.

- Intelligent and adaptive agents: When agents exhibit complex behaviour including learning and adaptation, they can be more easily represented as computer programs than with other traditional methods.

These features will be explained more with applications in *Section 2.2.4*.

2.2 Agent-based Simulation as a Young Field

Agent-based simulation is still a young and rapidly growing field. This section presents the findings of an extensive review of ABS literature. It also introduces some ABS applications to provide a general idea of how and in which fields ABS could be applied.

2.2.1 Multi-Agent Systems (MAS)

The agent concept was originally developed from MAS (multi-agent system). A multi-agent system is usually considered as a collection of "solving systems capable of autonomous reactive, pro-active, social behaviour" (Lomuscio, 1999). These solving systems work together to find the solution (Durfee et al., 1989). In this context, MAS are not models but problem-solving methods (O'Sullivan and Haklay, 2000).

Recently, MAS has been given a more general meaning. It refers to all systems which involve agents (as defined in the following section) (Jennings and Wooldridge, 1998). And the study of MAS focuses on systems in which many intelligent agents interact with each other.

2.2.2 What is an Agent?

Recently, agent-based simulation has become a commonly used term in the simulation literature. However, agreement on the precise definition of agent-based simulation has been difficult to achieve. Edmonds and Mohring (2005) gave the definition of an agent as an entity for which some cognitive process is modelled. Tunce (2001) defines agent-based simulation as the "use of agents for the generation of model behaviour in a simulation study". Reynolds (web page) says that "agent-based models are simulations based on the global consequences of local interactions of members of a population". Actually, none of these definitions contributes much towards a better understanding of agent-based simulation. However, Dickie (2002) tells us more about agent-based simulations; he states that "in agent-based simulation models, an entity's behaviour is generally modelled as a set of goals or actions. Agents control their own destiny, or in other words change their state based on their knowledge of the environment in which they are placed". From these different definitions of agent-based simulation, it is clear that the key to agent-based simulation is the agent notion. Such individuals (agents) might represent vehicles in traffic, plants and animals in ecosystems, chunks of code in software, autonomous characters in computer games, people in crowds or economic regimes.

In agent-based modelling, several authors have tried to identify the key aspects of an agent. Holland (1995) defines agents as "rule-based input-output elements whose rules can adapt to an environment". Dickie (2002) says that the character of an agent is that once an agent's knowledge has been built, the "behavioural mechanism acts on its degrees of freedom". An agent's degrees of freedom are the state variables the agent is able to affect. An agent's behaviour is a function that takes knowledge as an input, and outputs changes to the agent's degrees of freedom. A more comprehensive definition of an agent is given by Weiss (1999). He states that an agent is "a computational entity that can be viewed as perceiving and acting upon its environment and that it is autonomous in that its behaviour at least partially depends on its own experience". Rocha (1999) takes the view that the distinctive characteristic of agents is their autonomy, which means that an agent has the ability to act and make decisions without being controlled. Moreover, Schmidt (2000) and Holland (1995) review the features of agents, which will be discussed in the following parts:

Schmidt (2000) reviews the features of agents as the following aspects:

1. Autonomous behaviour: "Every agent is characterized by autonomous behaviour", e.g. an intelligent agent behaves autonomously without external control.

2. Individual world-view: Every agent perceives its surrounding external world according to its own model. This so-called conceptual model describes the intelligent agents' view of the outside world, and it is generally incomplete and frequently even incorrect.

3. Communicative and cooperative capacity: There is information to share and to

exchange between intelligent agents and their environment, which may consist of other intelligent systems and even other intelligent agents. Thus, intelligent agents exchange information with the external environment and with other intelligent agents in order to build up their own world view. In addition, the possibility of communication with other intelligent agents is the "precondition of common action in pursuit of a goal".

4. Intelligent behaviour: Agents are able to learn from the environment, as they have the capacity of "logical deduction". Therefore, intelligent agents can be used in unknown environments.

5. Spatial mobility: Intelligent agents are sometimes but not always required to display spatial mobility. (Spatial mobility is not included in the model in this research.)

On the other hand, Holland (1995) defines seven "basics" or characteristics of agents:

1. Aggregation.

    - Categorization (agent-level): In order to cope with their environments, agents group things with common characteristics and ignore differing characteristics.

    - Large-scale behaviour (multi-agent level): The collective behavioural patterns emerge from the aggregation of the individual agents' behaviour.

2. Tagging. Agents need to be individualized according to their identities.

3. Nonlinearity. In a multi-agent system, the integration or aggregation of agents is often non-linear. Thus, the resulting behaviour cannot be linearly predicted by decomposing the behaviour of individual agents.

4. Flows. The ABS relies on the connections (e.g. the flow and transfer of information, interactions etc.) between agents.

5. Diversity. Because MAS are designed to play different roles, multi-agent systems are typically heterogeneous. However, the agents' behaviours may be identified.

6. Internal Models. Every agent has its own internal model. The internal model organizes the individual agent's behaviour rule and can also enable agents to anticipate the expected inputs from their environment.

7. Building Blocks. Agents are built with simple components which make the coded construction easier. Therefore, model users can recombine the agents to produce a new agent with different behaviour and models.

Holland's basics give more emphasis to the emergence of large-scale behaviour or a multi-agents level (points 1,3,4,5), whereas Schmidt only mentions group behaviour in one point (point 3). Holland's basics are more comprehensive than Schmidt's and his paper also considers each part in great detail and has been widely cited. However, Schmidt's characteristics and Holland's basics share some common features; on one hand, they both agree that agents have their own internal models, i.e. they both have individual views of the world and rules of behaviour; on the other hand, they believe that agents are intelligent. Overall, agent-based simulation can be described as a simulation based on the global consequences of local interactions of autonomous intelligent members of a population (Reynolds, 1999a). In addition, an agent is an individual with a set of characteristics or attributes, a set of rules governing agent behaviours or decision-making capability, protocols for communicating responses to its environment and that interacts with other agents in the system.

2.2.3 ABS Vs Traditionally Simulation

Is agent-based simulation some new, fancy way of doing analysis? Is agent-based simulation just old wine in a new bottle? Although agent-based simulation is sometimes presented as if it were a new type of modelling, many "traditional" simulations feature some adaptive agent behaviour. For example, a simple queueing simulation may include a rule that customers (the agents) will not, with some probability, join the queue if its length exceeds a certain value, or that customers leave the queue if they have to wait too long.

In order to compare traditional simulation and ABS, the main typical characteristics (with some exceptions) are considered to be:

Typical characteristics of traditional simulation (examples: production line, call center, hospital, transport system):

1. The systems is one of queues and processes and so the main elements are the processors, the queues and the processed elements.

2. The range of behaviour of the processors is processing time and the next destination.

3. The item being processed (e.g. parts, customers, vehicles) is often passive although it may have heterogeneous characteristics.

4. The connections between the processors and queues (the flows through the system) are designed (top down) in the real system and so are focused in the simulation.

5. The stochasticity occurs in the arrivals of the element being processed, the processing times and sometimes the characteristics of the processed element, and the rules of the processed element.

6. Typical outputs of interest are throughout, queueing time and resource usage.

Typical characteristics of agent-based simulation (examples: stock market, auction, consumer market, military battle, crowds and epidemics):

1. The system is one of interacting agents and so the main elements are the agents and the environment.

2. There is a variety of behaviour of agents depending on current circumstances.

3. The agents have heterogeneous characteristics and rules of behaviour.

4. The agents' characteristics may change during the simulation (e.g. a change in strategy, an increase in knowledge, become ill or die).

5. The interactions between the agents are not designed but are unpredictable and are therefore modelled using random numbers.

6. The stochasticity occurs in the characteristics of the agents and their rules, the interactions between the agents and the interactions between the agents and the environment.

7. The item that passes between the agents (information, opinions, disease) is often intangible and its impact on the receiver depends upon the characteristic of the receiver.

8. Typical outputs of interests are aggregated agent behaviour (e.g. number of purchases, number of agents in different states) or the state of the environment (e.g. market price).

We can view both traditional and agent-based simulations as consisting of entities that remain in the system throughout the simulation and interactions between the entities through elements that between them. Viewed in this way, the corresponding entities remaining in the simulation are the processing entities and queues in traditional simulation (e.g. machines, servers), and the agents in ABS. Moreover, the elements passing through the connections between the entities are of a different nature as highlighted in the above list of characteristics. In traditional simulation, it is often a physical element that passes from one entity to the next without altering the entities (although sometimes information can be passed). The physical element usually follows a route through the system. In ABS, the agents exchange elements that alter their characteristics and the elements are just involved in separate transactions rather than following a route between several agents.

An interesting comparison is between a traditional call centre model and an agent based consumer model (where consumers exchange information and opinions about a product and ultimately purchase the product). Because in both cases, the elements remaining in the system are human beings and the systems centre around conversations. However, in the call centre, the conversations between the customer and the operator are modelled as simply a processing task which does not alter the characteristics of the customers or operators. The important data is the distributions for the call inter-arrival times and the processing times. In the consumer model, the precise time at which the conversations between the agents take place and the length of time of the conversations are not important. Instead it is the number of conversations and the way they affect the characteristics of the agents that matters.

Complexity and unpredictability in simulation behaviour usually arises mainly from the stochasticity. In traditional simulation, the most significant aspect is typically the time taken by the processed element as it passes through the system (i.e. arrival time and processing time). There can also be stochasticity in the characteristics and rule of the processed element. In agent-based simulation, the most significant aspect is typically the characteristics of the agent and the interaction between the agents. Therefore, the site of the stochasticity is different being related mainly to the processed element in traditional simulation (the characteristics and time) but being related mainly to the entities remaining in the system (i.e. the agent) in ABS.

In terms of applications, the systems studied by traditional simulation are often designed by an organisation to accomplish a particular task and they have a lot of control over the system. By contrast, the systems studied by ABS are often environments in which humans (or animals) can interact freely (within the rules of the environment). In such a situation, if there is an organisation with an objective (e.g. a company wants to maximise sales in the consumer market), then they have very limited control or influence over the system.

Overall, there are some differences between traditional simulation and ABS in the amount of control exerted on the real system, on the nature of the elements modelled and the way they interact, on the site of the stochasticity in the model and in the aspects of interest. It is arguable how fundamental these differences are although I would not consider them great enough for ABS to be considered as a new paradigm.

One of the causes of the greater use of agent-based simulation is that increasing computing power now makes such simulations feasible. There is also an appreciation that for some systems an agent-based approach may be necessary in order to capture the dynamics of the system.

2.2.4 Overview of ABS Application Areas

Quite a lot of research has recently been undertaken to apply agent-based simulations in various fields, including psychology and cognitive science, ecology and environment, economics and industry. Accordingly, there are a huge number of applications in the literature. For instance, the academic paper and book search-engine "*google scholar*" returned 5,780 results for the search phrase "agent-based simulation" (last access date 06/10/2007). Therefore, in this section, some examples will only be described at a level of detail intended to give an impression of the scope of the application areas. More details and examples can be found by consulting the literature cited. The literature that is most important to the research will be introduced in detail in the following chapters.

A comprehensive annotated list of ABS application areas is provided by Reynolds (1999a), as summarized in *Table 2.1*. Consulting Reynolds' (1999) list and other literature, we can take the following four general topic areas to introduce a few examples of the application of the ABS model (see *Table 2.2* and the following subsections).

Table 2.1: Summary of general ABS model area derived from Reynolds (1999a)

Animation and Interactive Multimedia	Animation and Interactive Multimedia
Traffic and Vehicle simulations	Traffic and vehicle simulations
Economics	Economics
Modelling Humans and Artificial Societies	Human crowds: motion and psychology
	Interpersonal communication
	Sociology
	Artificial societies
	Anthropology
	Emotion
Ecology and Biology	Non-species-specific models and other topics
	Bacteria
	Marine invertebrates
	Forests
	Insects
	Birds
	Mammals
	Fish
	Mixed ecosystems

2.2.4.1 Movement Patterns

Boids (a contraction of bird and android) simulation (Reynolds, 1987) is one of the earliest examples of applying an agent-based approach to a situation that was previously considered very difficult (O'Sullivan and Haklay, 2000). Reynolds tries to simulate the flocking behaviour of birds and other flocking animals, and to find the

Existing ABS application areas			
Movements ABS models	Economics ABS models	Sociological ABS models	Military ABS models

Table 2.2: The existing ABS application areas

rules for general agents (boids) to produce behaviour that appears realistic compared to the flocks, herds and schools of different animals in the real world. More agent-based models that are aiding researchers investigating biological phenomena can be found in Levy (1992), Resnick (1994) and Westervelt and Hopkins (1999).

Similar to the "Boids" model, a number of ABS models have been applied to the study of human movement (Bonabeau, 2002a), such as:

Pedestrian and crowd behaviour The "STREETS" model of people's shopping behaviour by Schelhorn et al. (1999) and the "SIMSTORE" (www.simworld.co.uk) model of customers behaviour in a real British supermarket (the Sainsbury's store at South Ruislip in west London) by Venables and Bilge (1998).

Evacuation Fire escape simulation by Still (1993) and Helbing et al. (2000). They tried to simulate the evacuation of a public space (stadium, station, city etc.) using agent-based simulations to capture interactions between people. Brailsford and Stubbins (2006) simulated the normal operation and emergency evacuation of a building in Southampton University. It was created by a combination of standard discrete event simulation (DES) using the package Simul8 and a social force simulation software package called "Pedestrian Escape Panic".

Flows These models are based on the work of Helbing (1992), Helbing and Molnar (1997), and Batty et al. (1999) to produce outputs that seem to match various observed human behaviours, like "lanes" on busy pavements. Examples include the "ResortScape" model of a theme park by Axtell and Epstein (1996) and "TRANSIMS" that simulates real time movements of every pedestrian and vehicle through a large metropolitan area transportation network by the Los Alamos National Laboratory (LANL). Other examples include vehicle routing models by Schreckenberg (2002) and Dia (2002).

2.2.4.2 Economic Agent-based Models

Dynamism is the main feature of financial markets. A large amount of interacting agents with individual behaviour rules (aimed at profit maximization) leads to the emergence of phenomena that make it difficult to make predictions in financial markets. Nearly forty years ago, the prevailing theory of the markets was presented by Fama (1970), who claimed that markets can be efficient based on the assumption of fully rational behaviour of all participating agents. However, such an efficient financial market theory has been questioned by complex market dynamics. From the observation of market behaviour, the market does not always reaches an equilibrium, as indicated in the traditional theories, and an agent's behaviour is not completely rational. Therefore, as instigated by the pioneering work of Anderson et al. (1988) and Arthur et al. (1997), more and more researchers have been applying ABS in economics in the last few years because ABS models allow for heterogenous and limited rational/irrational behaviour. Examples include a financial market model that exhibits realistic trading features by Raberto et al. (2001), and an agent-based economic 'laboratory' (N-ABLE) for analysing the economic factors by Schoenwald and Barton (2004).

2.2.4.3 Sociological Agent-based Models

A very prominent example in which agents are used to simulate artificial social systems is given by the Sugarscape experiments carried out by Epstein and Axtell (1996). In this well-known model, agents were given different rules of behaviour and the system was then run forward in time to see what macroscopic social structures emerged. It demonstrated well how simple rules of agents could produce complex whole population behaviours that were not predictable by individual agent's behaviour rules.

Healthcare model Brailsford et al. (2006) discussed some of the issues involved in simulation models which have human factors involved. Besides, they reviewed two health-care models for screening different diseases which attempted to incorporate human behaviour.

NASDAQ stock market model The NASDAQ[2] stock market model was built by the Bios Group in 1998. The model was built to investigate the impact of two proposed small regulatory changes. The simulation results indicated that a reduction in the market's tick size will reduce the market's ability to adjust the

[2]NASDAQ: the National Association of Security Dealers Automated Quotation

price.

On-line auction model Mizuta and Steiglitz (2001) built an on-line auction model. The model was developed based on a Vickrey auction (sealed-bid mechanism and the bidder with second-high-price wins, Vickery (1961)).

Two different types of bidders were identified as "early bidder" and "sniper bidder". The characteristic of the early bidder is "watch/modify/bid" and the sniper bidder is "wait/bid". Such dynamic auction behaviour cannot be easily described in the usual theoretical models; therefore, agent-based simulation was used. The model output shows that compared to sniper bidders, early bidders can win at a lower price but with a lower probability on average.

2.2.4.4 Military Agent-based Models

It is clear that there are always risk and cost factors involved in military simulation models. In addition, most military models involve the interactions of many submodules. Pew and Mavor (1998) gave an overview of the models applying ABS in the military. Ramaswamy et al. (2001) developed an java-based ABS model to detect and resolve interference in naval radar units. Each agent (a naval radar unit) in the model has its own strategy in identifying target ships. They also addressed the implementation and evaluation of interference diction and resolution problems. Schreckenberg et al. (2001) described how agent-based simulations could be used in the Air Force with the help of the AMBR (agent-based modelling and behavioural representation) program. The AMBR program developed new approaches to simulate intelligent behaviour and applied the knowledge derived to enhance the modelling and simulation capability of the Air Force.

2.3 Review of ABS Tools

2.3.1 Graphical Tools for ABS Model Building

A good graphical representation of the model structure can be very helpful in building the model. The diagram can represent the architecture of the model at a very high level of abstraction, while the logic of a single agent's behaviour will not normally be represented.

Unfortunately, ABS is not provided with a rich and well-defined set of diagrams to visually describe the internal logic of the model. There are a few attempts to graphically represent an agent-based model, the most widely cited one being the ERA (Environment Rules Agents) scheme introduced by Gilbert and Terna (2000). The ERA diagram highlights the type of agents involved in the simulation model and the way their "mind" is implemented, as shown in *Figure 2.3.* In particular, the ERA scheme is a methodology to separate the agent as a player from its mind, whose choices are determined by a rule master. The rule master can eventually have its rules changed by a rule maker. The rule master represents the adaptive proxy for the model, while the rule maker represents the evolutionary one.

The Unified Modelling Language (UML) is another promising graphical language to represent ABS models. UML is an open and extensible paradigm, and it is actually evolving towards software agent representation (AUML, 2003). It is a generally accepted, diffused and extensible modelling language that can be adapted for agent-based simulation models.

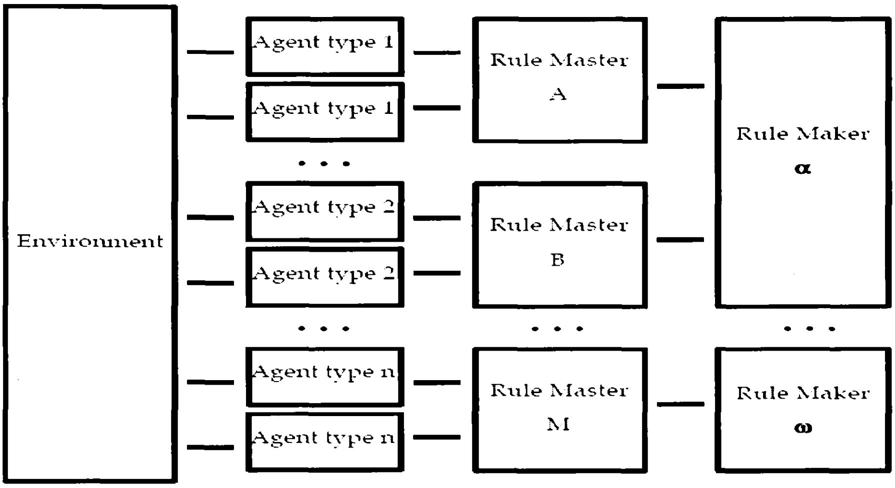

Figure 2.3: The Environment-Rules-Agents framework to build agent-based computational models (Gilbert and Terna, 2000)

2.3.2 Tools for ABS

The first question people usually ask about building an ABS model is "What language/software should I use?". Building a computer simulation model requires a high level of skill in computer programming. It is also important to test the code carefully, because the unexpected model outcomes might be caused either by code bugs or by the emerging properties of the model.

There are some open source software packages that have been developed to help with ABS model building. However, many of these are still under development and in the beta stage. The general impression of the ABS software scene presented is still one of infancy, though there are quite a few ABS communities (such as the Santa Fe Institute) and academics involved, the software packages still feel like early versions of the final products. Moreover, researchers and academics working in the field of ABS still often prefer to develop and code their own models (for instance, in this research, I experimented with some ABS software, but eventually decided to build the model from scratch) rather than rely on a third-party ABS software for the following reasons (Parker, 2001):

1. The conceptualisation of an ABS model is straightforward, which leads some to believe that it would be simple to construct from scratch.

2. It is time-consuming to learn third party software, especially when the software package is not well developed.

3. For validating and verifying the model, the researcher might find it is difficult to trace how the model works because some details of the working of the model are hidden from the developer by some software packages.

2.3.2.1 Three ABS Tools Groups

The ABS tools are divided into three main groups (Sonnessa, 2005):

- **Language-specific modelling environments** such as some commercial or freeware packages (i.e. Starlogo, Netlogo, Anylogic etc.). The Starlogo and Netlogo environments represent integrated applications, which provide the user with a language specifically designed to model spatial agent-based environments. This approach has two main advantages. First, they are easy to use, since the language provides a reduced set of instructions and hides the technicalities from the final user. Second, the modelling environment comes with a ready to use two-dimensional space, dramatically reducing the code-typing and the design time. The close dependence on the spatial representation is an advantage but, at the same time, also a great limitation on the tools' flexibility.

- **Open source libraries or frameworks**, based on standard programming languages such as Java, Object C and C++ (e.g. Swarm) and that require a high level of programming skills. The open source libraries are a set of functions offering tools in the middle between basic programming languages (C, C++, JAVA) and closed packages for simulation; they help programmers to develop their own software, with the help of a well-defined protocol and powerful tools to deal with agents' behaviours, interactions and time sequences. The users' simulation models are stand-alone programs that are written using the features provided by those libraries.

- **Computer network-based architectures**, particularly used to enable mobile autonomous agents (MAS). More and more ABS modelling packages tend to embed such functions. Within this category, the most important network-based frameworks are Cougaar and JADE. These frameworks are designed to create network services used by autonomous agents to perform some particular goals or to coordinate with other agents in the network, according to the Distributed Artificial Intelligence paradigm (DAI). Such kinds of architectures are not specifically designed for simulation, but by defining an appropriate time manager, it is possible to simulate virtual environments populated by intelligent agents. Obviously this kind of platform cannot represent a general purpose simulation environment, since they are particularly complex in the model's implementation and require a network of computers to perform a simulation run.

2.3.2.2 A Brief List of ABS Tools

Table 2.3 is a brief review of some of the most frequently used agent-based simulating software. More details of the ABS tools can be found in *Appendix A*.

Table 2.3: Summary of ABS tools

Category	**Example Packages**
Open Source	ABLE, Cougaar, Ecolab, JADE, JAS, MASON, Repast, Simpy, SWARM, ZEUS
Freeware	Ascape, NetLogo, Starlogo
Propietary	Agentsheets, Anylogic

Among the above ABS software packages, "SWARM" is the most well-known toolkit for ABS (Minar et al., 1996). The "SWARM" package was developed at the Santa Fe Institute (SFI). The advantages of this software have been shown in its use for various applications. The examples include game theory (Axelrod, 1984), epidemiology (Bagni et al., 2002), biology (Kauffman, 1993), and financial applications (Terna 2000 and LeBaron, 1996). The disadvantages of the package include:

1. It requires high programming skills in either Object C or Java,
2. It is Unix operation system-based, and therefore not very friendly to Microsoft Windows users, and
3. In the model validation phrase, it is difficult to trace how the model works due to the fact that some details of the workings of the model are hidden from the developer by Swarm libraries.

2.4 Summary

Agent-based simulation has attracted much interest lately, but an agreement on the definition of an agent has not yet been achieved. The simplest viewpoint is that an agent is an entity for which some cognitive process is modelled (Edmonds and Mohring, 2005). The cognitive processes here refer to obtaining and storing knowledge, and putting it to use. To some extent, ABS is a new simulation approach and with the benefit of much-increased computing power, it enables new types of simulations to be investigated. So far, it has been widely applied in many areas such as military, economics, sociology and movement patterns.

Chapter 3

CALIBRATION OF AGENT-BASED SIMULATION MODELS

CHAPTER OVERVIEW

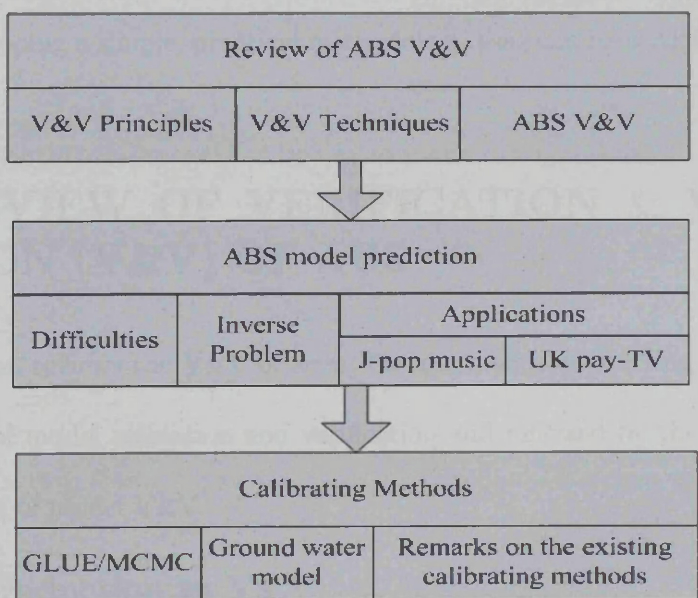

Figure 3.1: Contents of Chapter 3

This chapter provides the literature review for the calibration issue in agent-based simulation models, which is the main research topic. It defines ABS model validation & verification (V&V), prediction in agent-based simulation, calibration and the inverse problem. *Section 3.1* discusses the main theories and techniques of validation that are available to modellers when applying simulation using the agent-based approach. According to the purpose of the model (e.g. descriptive or predictive), there are different methods with which to validate the model. Different model types have different requirements for data, and there are difficulties in ABS model validation and prediction, which are explained in *Section 3.2*. *Section 3.3* introduces and remarks on the existing calibration procedures (i.e. MCMC based method and the Brooks et al. (1994) method). The research approach taken in this research is derived mainly from the second method. *Section 3.2.3* describes and reviews the existing ABS prediction models. It also explains how the thesis contributes to these discussions in terms of developing a simple, practical methodology that can help ABS users to make predictions.

3.1 REVIEW OF VERIFICATION & VALIDATION (V&V) OF ABS

This section reviews the V&V of agent-based simulation, starting from a general introduction of model validation and verification and followed by the principles and the techniques of model V&V.

3.1.1 Introduction to V&V

Verification and validation (V&V) are essential parts of the model development process if models are to be accepted and used to support decision-making. However, it is important to remember that validation does not imply verification, nor does verification imply validation. Basically, models must be matched against the phenomena being modelled and checked for errors at each stage of use. In addition, an easy and commonly used expression to clarify model verification and validation is: verification - building the ***model right***; validation - building the ***right model***.

Verification This is the process that makes sure the model does what it is intended to do from an operational perspective. Model verification is like debugging in programming. Models, especially simulation models, are often large computer programs. Therefore, all techniques that can help develop, debug, or maintain large computer programs are also useful for models.

Validation This ensures that the model meets its intended requirements in terms of the methods employed and the results obtained. In other words, validation tests whether the model could reproduce system behaviour with enough fidelity to satisfy the analysis objectives.

3.1.2 Principles of V&V

There has been a lot of work done in the research of simulation V&V (for example, Banks et al., 1988; Balci, 1995; Sargent, 1998; Brooks, 2001; Pidd, 2004). Some general accepted views on the V&V principles are as follows:

- All models are the simplification of the real system, therefore, it is impossible to claim that a model is 100% correct.

- The result of a model V&V is subject to the study/project objective. In other words, a V&V test may have completely different implications under different research objectives. For instance, comparing a model which is designed for a better understanding of the real system with a model which is designed for making accurate future prediction, a V&V test may focus on whether the model could reproduce a certain phenomenon in the first case while the latter case may focus on whether the test results could match the historical data.

- In general, V&V is the process of testing the model. It is the process for model builders/users to build up their confidence in using the model (i.e. better understanding of the real situation or to make predictions). Therefore, the V&V result should provide the model users with enough confidence in using the model for a specific project.

Particularly, one of the widely cited authors is Balci (1995), who has done a substantive work in simulation V&V research. Balci set out 15 principles for model VV&T (validation, verification and testing, Balci, 1995, 1998):

" **Principle 1:** The VV&T must be conducted throughout the entire life cycle of a simulation study;

Principle 2: The outcome of simulation model VV&T should not be considered as a binary variable where the model is absolutely correct or absolutely incorrect;

Principle 3: A simulation model is built with respect to the study objectives and its credibility is judged with respect to those objectives;

Principle 4: Simulation model VV&T requires independence to prevent developer's bias;

Principle 5: Simulation model VV&T is difficult and requires creativity and insight;

Principle 6: Simulation model credibility can be claimed only for the prescribed conditions for which the model is tested;

Principle 7: Complete simulation model testing is not possible;

Principle 8: Simulation model VV&T must be planned and documented;

Principle 9: Type I, II, and III errors must be prevented;

Principle 10: Errors should be detected as early as possible in the life cycle of a simulation study;

Principle 11: Multiple response problem must be recognized and resolved properly;

Principle 12: Successfully testing each submodel does not imply overall model credibility;

Principle 13: Double validation problem must be recognized and resolved properly;

Principle 14: Simulation model validity does not guarantee the credibility and acceptability of simulation results;

Principle 15: Formulated problem accuracy greatly affects the acceptability and credibility of simulation results."

3.1.3 V&V Techniques

In addition, Balci (1998) specified 75 different techniques, which were categorized into "informal, static, dynamic, symbolic, constraint and formal techniques" (details can be found in the cited paper). Similar to Balci's 75 techniques, but in more general terms, other authors commenting on V&V techniques include Sargent (1998), Banks et al. (1988), and Adrion et al. (1982).

Particularly, regarding the model validation, methods are commonly categorized as "black box" validation and "white box" validation (for example, Pidd, 2004).

Black box validation When using a black box method, the model output is compared with historical data to check whether it matches the historical data. In other words, only model input and output are examined and what happens inside the model is ignored.

White box validation In a white/open box validation, both the elements and the rules in the model are compared with the real system. In addition, the model assumptions and input distributions are also assessed. Compared with the black box method, the white box method looks into the inside of the model instead of only looking at the model input and output.

There are some commonly recognized issues about these two validation methods in the general simulation literature. Issues associated with "black box" validation include: first of all, the real world data may not be available (i.e. there is no such data in the real world as the model may simulate something not existing yet or there have been difficulties to collect such data). Secondly, there might be errors with the data collected. Thirdly, even if there are enough data, it is effectively only one sample from a distribution. We do not know the relative position of the sample collected compared with the whole population of alternative possible histories of the system. For instance, as shown in *Figure 3.2*, if the data is used for validation is effectively an extreme sample, the result could be very misleading. Finally, since all models are a simplification of the real world, which leads to a question of "how close is close enough to the real world". The answer is subjective and needs to be answered with reference to the objectives (rather than simply using the outcome of a statistical test). Issues associated with "white box" validation are that the result of white box validation depends on the model tester's knowledge of the real world. And therefore, different testers might have different validation results. In other words, it is subject to model tester's perspectives of the world.

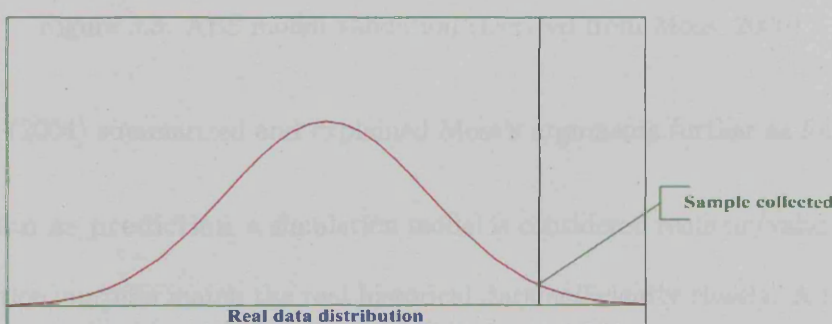

Figure 3.2: Issue with black box validation

3.1.4 V&V for ABS

A very important issue of ABS modelling is the validation of simulation models. In recent years, with more and more ABS applications being developed, researchers have become aware of the importance of validation issues in ABS. There are some discussions of ABS validation in the literature (for example, Moss, 2000; Hoog, 2004; Brown et al., 2005). However, very few papers among them have referred to the general simulation literature in the sense that they seem to be unaware of the aforementioned V&V methods and debate. Taking Moss (2000) as an example, he argued that the issue of validation depends on the point of view of the modeller, namely whether he or she has in mind a predictive or a descriptive model (Moss, 2000):

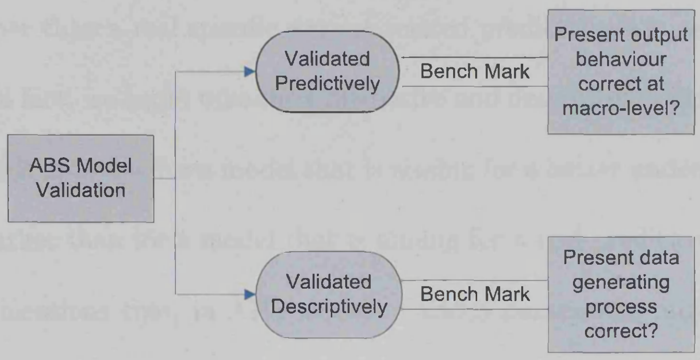

Figure 3.3: ABS model validation (Derived from Moss, 2000)

Hoog (2004) summarized and explained Moss's arguments further as follows:

Validation as prediction a simulation model is considered realistic/valid if its prediction/outputs match the real historical data sufficiently closely. A model has been validated predicatively if "the stylized facts at the macro-level match".

Validation as description a simulation model is considered realistic/valid if it describes phenomena and actual social processes associated with the individual agents, such as "beliefs, desires and trust". A model is validated descriptively if it provides a "correct representation of the data-generating process" . In other words, the model's individual behaviour is considered realistic at the micro-level. Moreover, this process is considered more subjective compared with the predictive validation process.

Based on the above two statement, Hoog (2004) suggested a joint criterion for ABS model validation: validated both descriptively at a micro-level and predictively at a macro-level. However, the prediction in Moss and Hoog's explanation differs to the general simulation literature. It emphasizes more whether the model could reproduce a pattern rather than a real specific project related prediction (e.g. aggregated sales prediction). In fact, we could take their predictive and descriptive validation methods as both methods applying for a model that is aiming for a better understanding of the real system rather than for a model that is aiming for a real prediction. In addition, Hoog (2004) mentions that, in ABS models, various parameters might produce the same output/phenomenon. However, he considered exactly specifying the individual agents as unimportant and therefore, he tended to ignore the difference between agents' parameters as long as the model output match the phenomenon pattern. In fact, when using an ABS model for prediction, this could be very problematic as a model that has passed black-box validation can produce seriously misleading results (Brooks et al., 1994). Such problems are often referred to as the parameter identification problem/inverse problem, which will be introduced in the following sections.

3.2 ABS MODEL PREDICTION

As discussed in *Chapter 2*, so far, ABS has been widely applied in many areas such as military, economics, sociology and movement patterns. In most of these applications, ABS was used as an understanding tool rather than a prediction tool. However, for some applications, using ABS for prediction (rather than just better understanding) could be very powerful. For example, a company might wish to use a model of the population of their customers with WOM interactions to predict the sales of the product or the effect of an advertising campaign. Or a broadcasting company may be interested in consumers' opinions on their newly launched TV programme. Moreover, there is a trend that the use of ABS is moving towards to prediction, especially in a business context. However, due to the difficulties of undertaking V&V on an ABS model, using an ABS model to predict the future is problematic. Some researchers have been aware of the issues. For instance, a recent special issue discussed the validation issues in ABS and debated the difficulties in using ABS models for prediction (Brown et al., 2005). But they did not reference the general simulation literature such as "white box" and "black box" validation. In general, they appear to have a relaxed view of model validation in the sense that they would recognize a model as a validated model if its output could match the general pattern in the real system/reference system. To some extent, a "white box" validation (model output matches a general pattern) might be enough for the descriptive use of models. However, regarding the predictive use of models, the validation criterion becomes more critical as the model needs to be able to predict a specific system rather than general pattern. This section will discuss this issue further.

3.2.1 Difficulties In Using ABS For Prediction

The difficulties in using ABS for prediction are mainly in the difficulties of knowing what the model parameters should be. Agent-based models typically have a very large number of parameters, and many of these cannot be measured directly or estimated with sufficient precision. The only other information available may be historical output data from the real system. Such data can be used to calibrate the model by finding parameter values that produce a good fit with the data. This is known as an inverse problem (see *Section 3.2.2*) since it consists of using the outputs to determine the inputs. The problem is that there will usually be many solutions. There are two main reasons for this. The first is that there are often many parameters and few historical data values. The second is that any model that produces a good fit should be considered acceptable. A perfect fit is not expected, because any simulation is a simplification of the real system and there may also be measurement errors in the historical data.

The result is that a wide range of sets of parameter values may give an acceptable fit and are therefore feasible values. However, they may give quite different predictions. As shown in *Figure 3.4*, Model 1, Model 2, Model 3 and Model 4 are all used to model real systems but with different parameters set. They produce outputs that will be used to compare with historical data. If this produces a good fit, then the model will be accepted by the modeller, as in *Figure 3.4*, Model 2, Model 3 and Model 4 are accepted, while Model 1 is refused due to a bad fit. In the next step, Model 2, Model 3 and Model 4 are used to make predictions. This results in different solutions (different size of solution circle in the graph represents different solutions).

3.2.2 Inverse Problem

The inverse problem can be described as a problem where the answer is known but the question is unknown. In other words, inverse problems are the determination of the present state of the system from the future observations or the identification of the parameters from observations of the evolution of the system.

The reason that inverse problem arises in agent-based modelling is that most

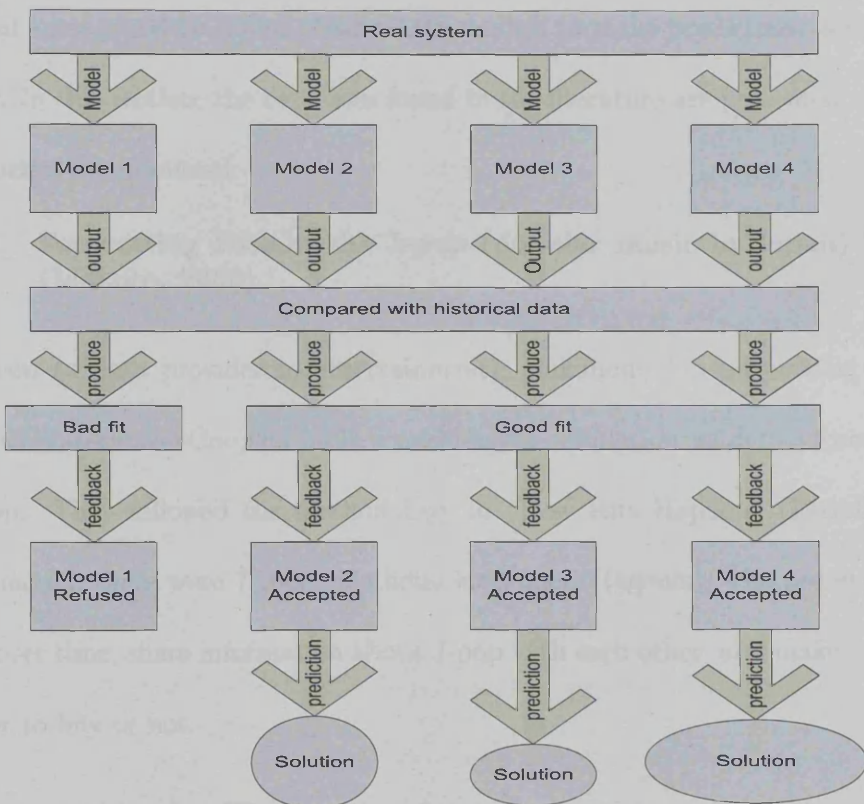

Figure 3.4: Difficulties using ABS to make predictions

agent-based models follow a comparison-validation approach that compares simulation outcomes with observed data. Therefore, it is unavoidable that, even if the model passes black-box validation tests and is able to produce outputs that are close to real historical data, the model is not necessarily validated.

3.2.3 Existing ABS Models Used For Predictions

Certainly, prediction is a possible and an important function of an ABS model. However, some complex adaptive systems (CAS) exhibit chaos, which make a great practical difficulty in any long-term predictions. Unfortunately, after an extensive search of the literature regarding using ABS models to make predictions, only two practical cases could be found. Using ABS models to make predictions is still in its infancy. In this section, the two cases found in the literature are presented, and each one's merits are discussed.

3.2.3.1 Forecasting Hits in the J-pop (popular music in Japan) Market (Makoto, 2000)

Oricon (a data provider in entertainment), Hakuhodo (an advertising agency) and PriceWaterhouse-Coopers built a multi-agent simulation model to forecast hits of J-pop. They followed the methodology in "How Hits Happen" (Farrell, 2000). In the model, there were 75,000 'synthetic consumers' (agents), who are exposed to media over time, share information about J-pop with each other, and make a decision whether to buy or not.

Prediction Objective The objective of their project is to predict and design a hit in the J-pop market.

Agents' Attributes There are 75,000 agents in the model, which represent consumers in the J-pop market. Agents receive information about a new CD through mass media, store, and word-of-mouth (WOM). In addition, their characteristics include their attitudes to the artist, their influences on their social network and their limited budgets.

Parameters In the model, there are five main parameters that influence agents' purchasing behaviours:

1. The artist's characteristic. This parameter is derived from Hakuhodo's (the advertising agency) data base. Hakuhodo conducted a consumer research

project named “the artist power survey”, which measures an artist’s characteristics by “quality of a song”, “whether the melody is easy to learn” and “appearance”, etc. For the new artists who were not part of the survey, judgements by experts were used in the model.

2. The released planned CD retail expectation degree, which can be obtained by Oricon from a national retail store panel. These data show the expectation of the store promotion, music trend prediction etc.

3. The number of airing times on radio. Music and media companies keep records on whether a radio station of a main city broadcasted the songs in a certain album. These data reflect the evaluation for a song according to DJs and were used in the model as a standard to express the degree of a prior campaign for an audience.

4. Television, commercial exposure [accumulation audience rating]. Similar to other new products, the campaign for the CD using TV commercials.The music may be used by a commercial for a product apart from a CD, which to widens the recognition of the song.

5. A tie-in with a TV programme, when the audience rating of a song in the J-pop market is used as a theme music of a TV programme as well. Cooperation with a popular programme (as measured by family viewing rate) may improve the recognition degree of a song very much.

Results The model could predict a hit phenomenon to some extent. The model validity was tested using the data of CDs released in the first quarter of 1999.

The model has a satisfactory prediction.

In addition, this project also provided diagnostic information and could serve as a 'flight simulator' in the entertainment industries. Overall, this paper is considered as the most relevant literature to the thesis. Unfortunately, Mr Makoto has left the company and is not able to disclose any more information about the project.

3.2.3.2 UK Pay-TV Subscriber Model (Twomey and Cadman, 2002)

In 2001, Beaufort[1] was sponsored by NTL [2] to build an ABS of the potential UK pay-TV subscriber market. There has been a dramatically change in the cable TV market over the last few years. Unlike the USA, the UK cable market is dominated by very few cable operators. This prevents any possibility of cross-sectional analysis across cable companies. Beaufort starts the project with a very simple subscriber behaviour model. The model can then be extended to consider more sophisticated behaviours.

Prediction Objective The objective is to build a scenario tool for investigating "what-if" questions to changes in regulatory and other market conditions. In addition, predicting the demand when price, package-content and package-size changes is also of interest.

Agents' Attributes These refer to an agent's sex, age, marital status, channel preferences and social grade. These data were obtained from monthly surveys of

[1] Beaufort International Ltd, London, UK. The group principal activity is providing management consultancy services.

[2] NTL is the UK's largest cable television operator

existing and potential subscribers. Besides, a bench mark of consumers' dissatisfaction when being required to purchase unwanted channels (as part of a package) was set based on the survey data. In addition, agents were given a simple decision-making rule, which included the "aggregation of utilities offered by the package of channels and comparing it with the price".

Parameters There are three main parameters that influence an agent's preference and corresponding subscriber behaviour: channel price, package-content and package-size.

Results As this model is part of a business consultancy project, no further project results were disclosed by its authors.

3.2.3.3 Discussion

Apparently, due to the complexity involved in agent-based modelling, applying agent-based simulation in prediction has not yet been well-developed. One of the main reasons could be that agent-based modelling tries to model a cognitive process which makes data collection extremely difficult when deciding upon parameter values for the model. Relatively easy data-collection compared with other systems in the cases of Makoto (2000) and Twomey and Cadman (2002) helped them to develop their models although it is unclear exactly how good that are for prediction. For instance, in the Makoto (2000) model, the artist's characteristics, the released planned CD retail expectation degree, television and commercial exposure and TV programme audience rating of the song in the J-pop market were data that was available.

3.3 CALIBRATING MODELS

There are some approaches found in the literature regarding model fitting. Two representative approaches (namely, the MCMC based method and Brooks et al. (1994) method) and the relevant literatures will be reviewed in this section.

3.3.1 Markov Chain Monte Carlo (MCMC) Based Calibrating Method

3.3.1.1 A Brief Introduction To MCMC

The basic idea behind MCMC is to draw a sample from the full posterior distribution, and then make inferences using the sample as a representative of the posterior distribution. For instance, we could calculate the sample mean and variance of the parameter from the sample.

It was developed as a stochastic simulation method by Metropolis et al. (1953) in the 1950s, and later refined and extended by Hastings (1970), Geman and Geman (1984), Gelman Rubin (1992), and Brooks (1998) (among others). More details about MCMC and related topics can be found both in the paper written by Liu (1999), which is considered to be a very comprehensive review paper of the MCMC method, as well as a MCMC sampling book written by Gilks et al. (1996). The MCMC method could be implemented in many ways. The most general one is the Metropolis-Hastings algorithm, developed by Metropolis et al. (1953) and further expanded by Hastings (1970). The core part of MCMC is how to construct a Markov chain by choosing a transitional probability so that it will finally converge to the equilibrium/stationary distribution $\pi(x)$.

The general idea is considering the ratio of the marginal distribution at time $t+1$ and t, and accept $x_{t+1} = x^{new}$ with a probability proportional to the ratio, p_{accept}; and $x_{t+1} = x_t$ with $1 - p_{accept}$. Here x^{new} is a random value proposed at time $t+1$. The process can be described as the following steps:

Step 1 Start with x^0, then iterate;

Step 2 Propose x^{new} from a proposal distribution $q(x^t, x^{new})$;

Step 3 Calculate ratio $p_{accept} = \frac{\pi(x^{new})q(x^{new},x^t)}{\pi(x^t)q(x^t,x^{new})}$; If the ratio$> 1$ then use $p_{accept} = 1$

Step 4 Determine the new value at time $t+1$ randomly (usually using a random number generator) where $x^{t+1} = x^{new}$ with probability p_{accept} and $x^{t+1} = x^t$ with probability $1 - p_{accept}$

After the above steps, run the chain until stationary and samples from the equilibrium/stationary distribution $\pi(x)$ can be used to find uncertainty around the outputs of interest.

3.3.1.2 Applying MCMC when Fitting Simulation Models

Generally, in terms of Bayesian statistics, the parameter values of a model are treated as random variables, and the aim is to find the posterior distribution $\pi(x)$ of them. The posterior distribution refers to a probability distribution associated with model parameter values. In addition, it depends on two types of information, namely, the prior distribution and the likelihood function. The prior distribution represents what we know about the parameters (our expectation of the parameter distribution), and the likelihood function represents how well the model fits the data (i.e. an objective function determining the distribution of the data being compared with the model output). Once these two factors have been defined, MCMC sampling plays its role. A Markov Chain which has the posterior probability distribution will be constructed. After a burn-in period, the samples generated from the Markov chain can be treated as coming from the posterior distribution. Based on such samples, the mean, variance and other statistical data can be calculated to find the uncertainty around outputs of interest.

3.3.1.3 Applications

The MCMC method has been used in fitting simulation models by several literatures (see Beven and Binley, 1992; Young et al., 1996; Chick, 1997; Nelson et al., 1997; Inoue and Chick, 1998; Andradttir and Bier, 2000; Currie, 2006, for examples). The representative examples are considered as follows:

- **An Agent-based Economics Model (Sallans et al., 2003):** Sallans et al. (2003) studied a discrete-time agent-based economic model. Sallans' model consists of three types of agents, namely, consumers, production firms, and financial traders, who operate in both a consumer market and a financial equities market. They introduced an innovative technique based on MCMC to validate the model, and used it to investigate the model parameter set, which would lead to a realistic model behaviours.

  The research was set out with the question "How good is a simulation?" and defined a good simulation as a simulation that can reproduce stylized features of real markets. These "good simulations" were transferred into an energy function to maximize the expected value of the profit and minimize the parameters' autocorrelations. The objective was to find parameter regimes where behaviour is good by sampling from the vector of simulation parameters θ subjected to the negative energy function $E(\theta)$:

$$P_E(\theta) = \exp\{-E(\theta)\}/Z$$

Again, as introduced in *Section 3.3.1.1*, Metropolis et al. (1953) algorithm is used to produce the stationary distribution $P_E(\theta)$.

This method enabled the large parameter spaces to be explored efficiently, so that the parameter set that leads to the reproduction of empirical phenomena can be found. To achieve this, Sallans et al. intersect parameter values histograms from the MCMC simulation runs to find common parameter settings.

- **GLUE Method**: The generalized likelihood uncertainty estimation (GLUE) techniques by Beven and Binley (Beven and Binley, 1992; Beven et al., 2000; Beven, 2000) are focused on prediction range. GLUE is an extension of the generalized sensitivity analysis (GSA, Hornberger and Spear, 1981; Spear et al., 1994). The GLUE technique is a more generalised method of choosing parameter sets from the whole range of possible parameters with a subjective likelihood of different parameter set. The approach obtained a range of predictions weighted by likelihood. In addition, the predictions can be compared with the observed behaviour. The likelihood measures how well the model and its associated parameter set fits the observed system behaviour.

- **Currie (2006) Method**: Currie (2006) applied the MCMC in fitting deterministic dynamic models and demonstrated the methodology with an example of a dynamic model of tuberculosis (TB) and human immunodeficiency virus (HIV). In her method, she considered all available sources of information: prior knowledge of the model parameter values (system experts or existing literature can usually provide information on the model parameter values) and data corresponding to the model output (data from a real system that the model output is meant to represent).

3.3.1.4 Merits Of The Method

The flexibility is a major advantage of the MCMC approach. It is straightforward to fit realistic models to complex data sets, which may have missing observations, measurement errors, multiple endpoints, or correlation structures (Dunson, 2001). Other advantages include:

Incorporation of prior information The MCMC approach is based on Bayes rule, which provides a rigorous way to incorporate data and prior information. In this method, the prior distribution and the likelihood of data are combined to gain a posterior distribution. This contains all the available information. Hence, it outperforms traditional methods in many cases.

Computation The ease in the computation of complex models. Briefly, MCMC algorithms generate a sequence of correlated samples through the time. The samples simulated at the same time are independent of each other. Once the distribution has converged to the target distribution , the samples generated by

MCMC could be remained, which can provide researchers the posterior distribution of interest (Dunson, 2001).

Capture dynamically information Due to the nature of the MCMC, it stores a lot of information dynamically during the run. Such information can be used for future analysis.

3.3.1.5 Drawbacks Of The Method

Expensive/Time-consuming A widely discussed drawback of the MCMC method is that running the full MCMC algorithm is very time-consuming. The Markov chain needs a very long burn-in and thinning period to make it converge to the equilibrium distribution. In addition, the variance of the estimator obtained from MCMC is usually high due to the correlated samples.

Data Validity The MCMC approach is very much dependent on the prior distribution, which is normally estimated by the modeller/field experts. Therefore, this will have a large influence on the sampling result.

3.3.2 Brooks et al. (1994) Method

Brooks et al. (1994) describe an approach to find the range of predictions of a groundwater model from alternative calibrations that were applied to an existing model of the Birmingham aquifer (Greswell et al., 1994). The aquifer was represented in the model by dividing it into rectangular cells. Each cell required geological parameters for the storage coefficient, S, and transmissivity, T. Recharge values, R, for the overall input of water to the cell from rainfall and other sources (such as water mains leakage) was also required. For cells containing part of a river, the streambed conductance, L, had to be specified. Abstraction of water from the wells was another important part of the model. Groundwater measurements were available at 12 sites for certain years although there were few readings before 1970 (m).

The fitness F of the model was measured using a weighted sum of the squared differences of the model values compared to the historical values (m), and a cut-off value was chosen as the criterion for a good fit. The objective of the original study was to predict groundwater levels for 2020, and in particular to identify areas of shallow groundwater where the water is close to the surface. Based on this objective an overall prediction measure, W, was devised of the extent of the model prediction of shallow groundwater for 2020. The simplex method (Nelder and Mead, 1965) was then used for searches for the local minima of the chosen functions ($f(W, F)$). The searches were to find the global maximum and minimum W values subject to a satisfactory F value across the parameter space (R,S,T,L).

The results were a considerable difference between the best and worst case prediction values W which all give a good fit. They therefore suggested using a single point prediction could be misleading in these circumstances. Instead, the appropriate approach is to take account of the alternative feasible calibrations and to evaluate, in some way, the different predictions they produce.

This method can be described in general terms as the following algorithm:

1. Define the parameter space of the range of values of the uncertain parameters.

2. Define a fitness measure, F, that measures how well the model matches the

observed data and set a cut-off value for an acceptable fit for the project.

3. Define a prediction measure, W, that measures the output of interest for the project.

4. Use an optimisation method on the parameter space to find the maximum and minimum W values for those points that give an acceptable fit. These maximum and minimum W values give the prediction range.

3.3.3 Remarks On The Existing Model Calibrating Methods

Comparing the GLUE/MCMC methods to the Brook's method, the main advantage of the Brook's method is that it focuses just on finding the range of predictions, which simplifies the search problem. This reduces the number of model runs required and enables it to be applied even when there is little information about the parameter values and when there are a large number of parameters. The main advantage of the GLUE/MCMC method approaches is that they provide more detailed information about the distribution of model outputs.

3.4 SUMMARY

Verification & Validation is very important for ABS modelling. The object of ABS model verification and validation is to balance these two impulses: the desire for the accuracy of prediction and the accuracy of the process (Brown et al., 2005). In general, ABS model V&V can be described as the process of answering questions such as "Can the model reproduce past behaviour?" and "Are the mechanisms and parameters of the model correct?" In some cases, even the first question is answered as yes, the model can reproduce past behaviour, the second question may still remain unsolved. For instance, even the model can reproduce the history data but with quite different structures (parameters). If that is the case, the impact of using such a model to make prediction remains a mystery. Moreover, agent-based models typically have a very large number of parameters, and many of these cannot be measured directly or estimated with sufficient precision. These inevitably make difficulties in using ABS models for prediction.

However, for some applications, using agent-based simulation for prediction (rather than just better understanding) could be very powerful. More and more researchers have been aware of such model identification problems in other areas and have tried to overcome them through different model-calibrating methods. For instance, some have tried to find the parameter region that can produce fit output and produce the possible prediction range with a certain confidence level (Brooks et al., 1994) and some take the MCMC based approaches which focus on giving the model prediction with an associated probability (Beven and Binley, 1992; Beven et al., 2000; Beven, 2000; Sallans et al., 2003; Currie, 2006).

Due to the complexity involved in ABS modelling, applying an ABS model in prediction has not yet been well-developed. Only two practical cases were found after a thorough search: Makoto (2000) successful Japanese J-pop CD sales market, and Twomey and Cadman (2002) UK pay-TV subscriber simulation. Hence, the use of an ABS model for prediction is still in its infancy. There are a great deal of questions left open for scholars.

Chapter 4

AGENT-BASED MARKETING DIFFUSION MODELS

CHAPTER OVERVIEW

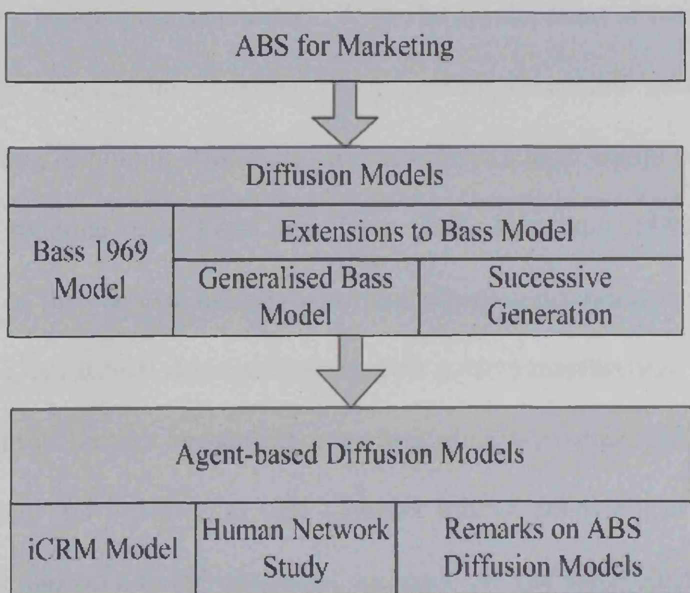

Figure 4.1: Contents of Chapter 4

This chapter starts with a general review of the existing agent-based models that are used to investigate marketing phenomena including the widely cited PECS model, the intelligent customer relationship management (iCRM) model from BT (British

Telecom), and the J-pop agent-based prediction model. *Section 4.2.1* then briefly introduces the classic 1969 Bass diffusion model (which has now become the fundamental theoretical frame of most diffusion models) and the main Bass model extensions. In addition, *Section 4.2.3* gives a discussion of the advantages and disadvantages of the Bass diffusion model. *Section 4.3* reviews two representative ABS diffusion models and debates the limitations in existing ABS diffusion models.

4.1 ABS FOR MARKETING

Agent-based modelling simulates complex marketing systems as swarms of agents (i.e. consumers, companies, economies). With the development of complex marketing system theories, scholars have realized the unique importance of autonomous agents for the modelling of human behaviour, and researchers have begun to adopt ABS in social science research (e.g. Bassu and Pryor, 1996; Bonabeau, 2002b; Janssen and Jager, 2000). In the classical human-behaviour simulation models, human beings are often modelled as rational decision makers with perfect information. However, these "classical" approaches are questioned more and more for being restricted to limited cognitive aspects (for instance, in such a model, human behaviour is reduced to cognitive abilities and cognitively controlled actions). At the same time, more complex theories about human behaviour come into the foreground in psychology. In contrast to the classical approaches, the latter also takes physical and emotional influences and social environments and social interactions into account (Dörner, 1999). Accordingly, there are increasing demands for more human-like agents in human behaviour simulation modelling. Fortunately, more and more researchers have been aware of the important role of agent-based models in modelling complex human behaviour. For instance, Urban and Schmidt (2000) introduced the PECS (Physical conditions, Emotion, Cognition, Social Status) reference model and from it developed some case studies. They claimed that the PECS reference model can provide a domain-independent model architecture to help scholars build human-like agents. However, this is a newly emerging subject, and there is still a great deal of work that needs to be conducted in applying agent-based simulation methods to marketing. In fact, most of the existing agent-based social science models are designed as a learning tool rather than as a predictive tool. The examples are described as follows:

- Container World project. This project delivered three agent-based models (the International Trade Model, the UK Competition Model and the Inland Distribution Model) designed to be used together. Container World was developed to provide strategic decisions in a continually changing world.

- iCRM model (intelligent Customer Relationship Management). iCRM tool uses an agent-based model to illustrate how iCRM investments can influence a targeted customer population. The model results provided the business decision makers with a clearer view of potential returns on such investments. This model takes into account the communication (customers exchanging their experiences) between members of a social network and also the powerful influence of WOM on the adoption of products and services. (Baxter et al., 2003)

- Bonabeau (2002b) built an agent-based model to simulate the decision-making process. The model simulates the "connections between consumer preferences, traits, constraints on purchase decisions, and apparel products."

- Bassu and Pryor (1996) conducted a project called "Aspen". They built an agent-based model to simulate the United States economy. In the model, they built more than 10000 agents representing various economic players (i.e. companies, banks, stock exchanges and households).

- Janssen and Jager (2000) built an agent-based model to analyze the effects of uncertainty and satisfaction on consumer behaviour. Their model included around 20 consumers (agents) and several other cognitive components.

- Brannon (1994) built InfoSumers which is a multi-agent simulator to simulate the diffusion of innovation in the clothing fashion market. In his model, the influence of interactions between suppliers and consumers in the textile market played an important factor in spreading the diffusion of innovation.

- Said et al. (2002) presented the CUBES (Customer Behavior Simulator) modelling approach based on interactions between virtual market actors and elementary behavioural attitudes in a competitive context .

4.2 DIFFUSION MODELS

"Diffusion" is the process by which a new idea or a new product becomes widely accepted by the market and the notion of diffusion is essentially a form of communication. In general, the diffusion of information can be divided into two types. One is Word-Of-Mouth (WOM) communication which relies on social networks constructed on human relationships. In WOM communication, information can be spread over a social community such as a college or a company. In addition, communication in this category can also be divided into two subcategories: face-to-face contacts and some sort of new type of communication brought about by technology (i.e. information on the internet, telecommunication, etc.). The other diffusion of information type is mass communication, where information can be spread to numerous people at the same time. For instance, TV and broadcasting are forms of mass communication (Tanimoto and Fujii, 2003).

In terms of diffusion modelling, the Bass diffusion model proposed by Frank Bass in 1969 was general recognized as the standard for analysing the growth of new consumer durable products in the marketing literature, which will be introduced briefly in the following section.

4.2.1 Bass (1969) Diffusion Model (BM)

The Bass diffusion model is concerned with the time of first-adoption of new consumer products. Rogers (1962) classifies adopters of innovations into various categories (as shown in *Figure 4.2*), based on the idea that certain individuals are inevitably more open to new products than others.

The Mathematical intuition behind the concept has been proposed by Bass (1969). One of the fundamental formulas is the Bass formula, which characterizes the spread of a new product in a market (Bass, 1969). Basically, the Bass diffusion model assumes a linear relationship between the probability of a particular individual adapting the product and the number of previous adopters, that is:

$$P(t) = p + (q/m)Y(t) \tag{4.1}$$

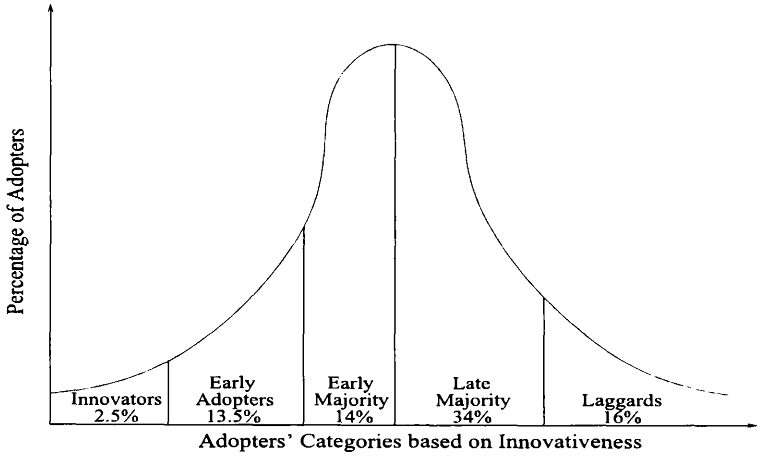

Figure 4.2: Rogers adoption innovation curve(Rogers, 1962)

where

$$P(t) = \frac{f(t)}{1 - F(t)} \text{ and } Y(t) = mF(t) \tag{4.2}$$

denote the probability of adoption conditional on those not yet adopting at time t and the number of buyers up to time t, respectively, where:

$f(t)$ is the likelihood function of adoption at time t;

$F(t) = \int_0^t f(s)dt$ is the cumulative function of the $f(t)$;

m is the market potential, which is the total number of consumers who eventually will adopt the product. In other words, it is the upper market limit.

Thus *Equation (4.1)* becomes

$$\frac{f(t)}{1 - F(t)} = p + qF(t). \tag{4.3}$$

Note that the parameter p and q can be interpreted as following:

p is the coefficient of innovation (external influence). It is used to measure the likelihood that a consumer who is currently not using the product will start using it due to the influence of mass media or other factors;

q is the coefficient of imitation (internal influence); It is used to measure the likelihood that a consumer who is currently not using the product will start using it due to the influence of WOM or the influence of people who have used the product (Bass, 1969).

Typical values of p and q (Bass et al., 1995) are the following:

- The average value of p has been found to be 0.03;
- The average value of q has been found to be 0.38.

Accordingly, the sales at time t is $S(t) = mf(t)$, which can be written as

$$S(t) = pm + (q-p)Y(t) - q/m[Y(t)]^2. \tag{4.4}$$

The $S(t)$ can be solved analytically in terms of p and q, and hence the time of peak sales t^*:

$$t^* = \frac{ln(\frac{q}{p})}{(p+q)}. \tag{4.5}$$

In practice, the Bass diffusion theory is easy to apply since parameters p and q are broadly studied in many markets and therefore, an indicative value can be obtained from literature (see Bass, 1969; Bass et al., 1994). Particularly, it is useful for a first assessment when no further details are available. Besides, there are two special cases of the Bass diffusion model by manipulating parameters p and q:

- when $q = 0$, the model reduces to the exponential distribution.
- when $p = 0$ the model reduces to the logistic distribution

However, attention must be paid, since the above standard model is only one of many diffusion models.

4.2.2 Extensions to Bass (1969) Diffusion Model

Since Bass published his diffusion model in 1969, it became broadly influential in marketing and management science. Many variations have been developed based on it (see Mahajan et al., 1990a, for a complete review). These extensions are either claiming further precision or being applied in specific circumstances. Two widely cited Bass diffusion model extensions will be presented in the following sections. One is to generalise the Bass model to include marketing and the other extends the Bass model to the study of repeat-purchasing products.

4.2.2.1 Generalized Bass Model (GBM)

The most basic diffusion fits very well on a range of new products and technology innovations. However, the model didn't consider decision variables such as pricing and advertising. By taking these variables into account, Bass et al. (1994) introduced a generalised Bass model, simply adding one more component "$x(t)$" into *Equation* (4.3):

$$\frac{f(t)}{1-F(t)} = (p + qF(t))x(t), \tag{4.6}$$

where $f(t)$, $F(t)$, p and q are defined in the same way as they are in *Equation* (4.3), and $x(t)$ is known as "current market effort" to reflect the current effect of dynamic marketing variables on the conditional probability of adoption at time t (Bass et al., 1994), e.g. a function of percentage change in price. Note that if $x(t)$ is constant, then the GBM is essentially equivalent to BM, this also explains why the BM works well without considering the decision variables. Similarly, the GBM has a closed-form solution which can be easily used.

4.2.2.2 Successive Generations

In the case that some products succeed one another in generations (i.e. technology products, computer games), Norton and Bass (1987) extended the model to sales of products with repeat-purchasing. Their model focuses on the substitution effect that a consumer who bought an old product is likely to buy one in the new generation to replace it.

The model can be fitted to a number of generations simultaneously, a multiple-generation example can be found in Norton and Bass (1987). They used this kind of model to capture a series of generations of innovation. The most innovative part of the model is to consider both the diffusion and substitution process at the same time while the Bass model can only deal with the diffusion process.

4.2.3 Remarks

The Bass model has been widely influential in marketing and management science. In 2004, the Bass model was selected as one of the ten most frequently cited papers in the 50-year history of Management Science.[1]

Up to now, there have been more than 850 papers published on the applications, refinements, and extensions to the Bass model. Applications (by Bass and his colleagues) include predictions of uptakes of satellite television, satellite telephone, new LCD projectors, wireless phone, satellite radio, wireless internet (2.5g and 3g) and

[1]Comments on "A New Product Growth Model for Consumer Durables: The Bass Model," Management Science, 50, No. 12, December 2004, pp. 1833-1840

many other technology forecasts.

4.2.3.1 Merits

The Bass diffusion model certainly has many merits as a heavily cited diffusion framework. Firstly, the model is a simple one, with only three parameters that can be easily estimated (m,p,q). Secondly, the model fits the trend in the sales growth of new products very well. Thirdly, the possibility of attaining a clear solution is very important, since the model gives a clear prediction that is proved to fit some phenomena (especially uptake peak) well, in contrast to the fact that a single solution to many marketing equations (i.e. the need to solve repeat integrals etc.) cannot be achieved. In particular, the model may be most appropriate for certain products with low prices (movies, books, music) or for products with very high benefits (agricultural and medical innovations)(Golder and Tellis, 1998). In the latter cases, the product adoptions depend primarily on diffusion of knowledge, social acceptability or popularity.

4.2.3.2 Limitations

However, the assumptions on which this model is based limit its applicability. There are three well-known drawbacks of the Bass model:

- It does not include marketing variables that could influence new product diffusion and sales (Golder and Tellis, 1998). For instance, price, customer afford ability and advertising, which are believed by many researchers to change the model's curve.

- The model's parameters are unstable. When new observations are added, these parameters can fluctuate substantially from year to year. For example, there are large fluctuations prior to the first peak in sales (Van den Bulte and Lilien, 1996).

- As a result of this instability, the model's forecasts are not accurate, unless the entire growth history is included. The model's forecasts are inaccurate before the sales peak and especially prior to the point of inflection (Mahajan et al., 1990b).

- It does not include uncertainty. In other words, due to the complexity theory, introduced in the previous chapter, the world is full of uncertainty. Randomness becomes an important factor in modelling, especially in modelling phenomena involved human behaviour. The area of diffusion certainly includes a large variety of human behaviours, and these behaviours are not necessarily logical.

Subsequent research has made progress, especially in extending the Bass model to include marketing variables and randomness. However, the extensions have come at the cost of simplicity: the new models are far more complex than the simple Bass model as shown in previous section. Therefore, the agent-based simulation can play its role by having new variables while keeping the model simple. The benefits of ABS modelling over other modelling techniques can be captured in the following statements:

- The ABS model can exhibit any particular behaviour as a result of interactions between its elements where the system behaviour may not be predicated by analysing individual behaviour (Hood, 1998).

- The ABS model captures emergent phenomena (Bonabeau, 2002b).

- Natural representations: the ABS model provides a natural description of a system. Because between the "target system" and the model representation, there is a simple, structural correspondence which makes the model easy to understand.

- ABS modelling is flexible: ABS models include the communication among agents. Agents are able to communicate with each other to share the product information. Sometimes, agents can imitate other agents in the population. Specific marketing company's can be included as a particular characteristic of the target population. Usually, traditional mathematical models are not able to cover such features because the complicated changing social networks make equation-based models too complex to be solved (Bonabeau, 2002b).

4.3 AGENT-BASED DIFFUSION MODELS

ABS has its merits (see *Chapter 2*) in the cases where people are influenced by their social context (what other people do in their social network). However, due to the nature of the variables (cognitive process) and the difficulty in parameter measurement, there are very few business applications. In fact, some academic attention has been given to applying agent-based simulations to diffusion situations that can be treated as a complex adaptive system. For example, Farrell (1998) and his colleagues developed a world with virtual agents to predict how and when hits happen. Working for Twentieth Century Fox, they modelled how a movie such as *Titanic* captivates the public and breaks box-office records worldwide. Two representative papers will be introduced in this section.

4.3.1 The iCRM Model (Baxter et al., 2003)

Baxter et al. (2003) built a generic model to allow companies to investigate the impacts of customer relationship management (CRM) strategies, with the aim of achieving better understanding through the comparison of scenarios rather than making specific predictions. The model has 500 agents, connected in a way that mimics a social network. The agents have perception values for the price and quality of a product, values that change based on interactions with other agents, their experience of the product and external factors (marketing, competition, CRM). The product is a repeat-purchase product (such as a subscription service), and each agent has a threshold value for the total of their price and quality perceptions, above which they purchase the product. Word-of-mouth interactions about the product between the agents become less frequent the longer they use the product, and there is also a loss of perception at each time-step to represent the effect of the competition. Customers may therefore be gained and lost as their perception values change.

Table 4.1: Key Components

Model Complexity	500 customers (heterogeneous agents with their own interpretations of the products attributes), single product (two parameters: price and quality)

Continued...

Validation	Not mentioned in the paper
Social Network Structure	Small-world model (Newman, 2003): define the degree of grouping within a network by the clustering coefficient C.
Theory Framework	Multiple-stage decision process (Rogers, 1962): **Acquisition** An agent receives information that will change their perception of the product through inter-agent communication and external factors. **Decision** If agents' perceptions are sufficiently high, agents make the choice of adopting the product. **Implementation** The explicit act of adoption or rejection (for repeat-purchase model use). **Confirmation** An agent's parameters are updated based on a study of the product or service.
Agents Decision Rule/ Utility Function	Since the model is a real business project, no detailed information regarding model equations can be found. In general, the agents in the model follow the rule that when an individual has a combined perception which exceeds their internal threshold it will adopt (or readopt) the product on offer.
Running of the Simulation	Start of the simulation: not mentioned in the paper; end of the simulation: 280 weeks.

Continued...

Conclusions	This is an illustrative model that can be used to compare the impact of different CRM strategies both in terms of market share and financial performance.

4.3.2 Diffusional Characteristics of Information on A Human Network Study (Kjima and Hirata, 2004)

Kjima and Hirata (2004) looked at the effect of different network structures, although the precise size and structure of the networks used is unclear. They used an SIR (susceptible / infected / removed) approach, based on disease transmission, for passing information between agents. The purchasing decision depended on the agent's enthusiasm for the product, which is a function of the utility of the product for the agent, the reliability of the information and the agent's attitude to risk.

Table 4.2: Key Components

Model Complexity	N/A[2]
Validation	N/A
Social Network Structure	Bipartite Network instead of traditional network. Each agent belongs to at least one group, which is his/her primal group (see *Figure4.3* for more information).

Continued...

[2]Not available from the paper published

Theory Framework	**SIR** model (Kermack and McKendrick, 1927): **I** infected agent: an agent who knows the information and is willing to let others know it. **S** susceptible agent: an agent who has not been infected but can be infected through interaction with an I-agent. **R** removed agent: an agent who stops diffusing information after a certain period.
Agents Decision Rule/ Utility Function	The utility function of an agent is decided by the product's attributes (k), the value of the product with respect to its attributes (z_k), the risk attitude (r^i) of agent i and the reliability of information (U^j). $$E^{ij} = \sum_k \alpha_k^i z_k - \frac{1}{2} r^i U^j$$
Running of the Simulation	Start of the simulation: seeding way; end of the simulation: when all the agents become S or R.
Conclusions	Some unique and interesting insights: 1. The cohesiveness of each group makes a large influence on the process. 2. It is easier for word-of-mouth to prevail in an oligopolistic society than in a mosaic society.

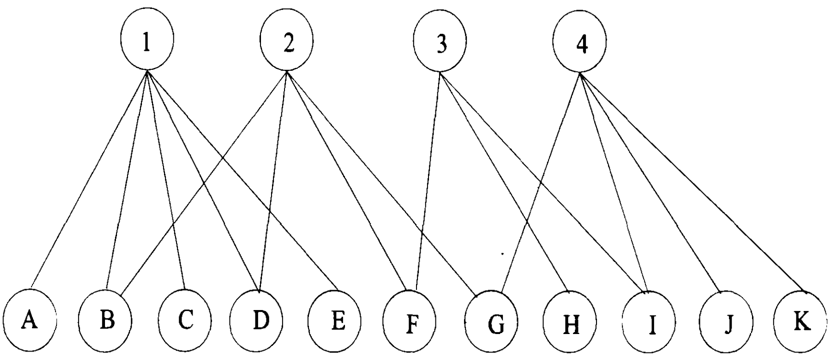

Figure 4.3: Bipartite network model

4.3.3 Remarks on the ABS Diffusion Models

In the context of the ABS model in marketing, agents are designed to reproduce their real-life counterparts. However, in different ABS models, the agents usually have different degrees of detail incorporated.

4.3.3.1 Elements of ABS diffusion models

In most cases, an agent in an ABS diffusion model usually consists of the following:

Attributes Generally speaking, agents' attributes include their age, sex and preferences.

Behaviour Rules Behaviours based on decision-making algorithms (e.g. utility maximization). In general, agents adopt the product or services on offer when their perception is greater than the set-up buying criterion (the buying criterion is decided by different agents' assumptions).

Social Network It is agreed that the social network is not completely random. People are segmented into social groups. Different people have different influences on the social network with which he/she is associated.

Figure 4.4 represents a random social network, where all individuals are connected randomly, while *Figure 4.5* represents a segmented social network, where individuals are clustered into different groups with connections with different groups.

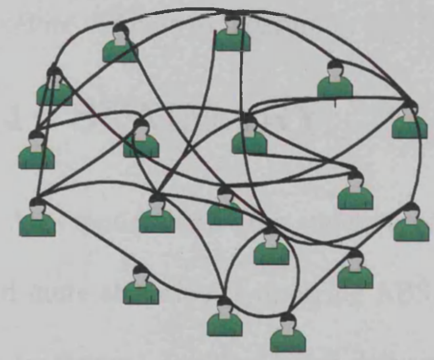

Figure 4.4: Random social network

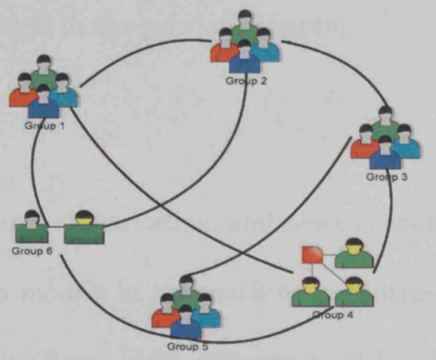

Figure 4.5: Small social network

In addition, uncertainty should also be included to represent the fact that consumers in the market do not necessarily behave rationally. In other words, they are not always aiming to optimize their benefits.

4.3.3.2 Limits of ABS diffusion models

An interesting observation found was that if the ABS diffusion model is used for predicting, it is not very successful, as it is used for achieving better understanding of the situation or the results are only treated as illustrative. For instance, the Farrell (1998) model was not very successful in contrast with the Baxter et al. (2003) iCRM model in the sense that the Farrell (1998) model could not predict when the hits happen. Predicting hits might be the most difficult thing to do, while understanding how hits happen is comparably easier (Farrell, 1998). And the Baxter et al. (2003) iCRM model enables decision-makers to understand the impact of their CRM strategies.

This is due to the fact that ABS diffusion models are normally based on numerous assumptions and simplifications. Since ABS diffusion models model the cognitive process, which is very difficult to validate and define, many of the parameters are therefore difficult to measure in real life, as stated in the previous chapter.

4.4 SUMMARY

ABS models have been widely used in the area of marketing, and researchers have paid more attention to applying ABS diffusion models in the market. In contrast to the traditional equation-based diffusion models (Bass, 1969, diffusion model and its extensions), an agent-based diffusion model has more advantages in terms of adding risk, randomness and heterogenous individual attributes into the model. It also gives a better understanding of the situation by providing the decision-makers with a platform on which different experiments can be tested.

However, ABS diffusion models do not perform well in making predictions (e.g. Farrell, 1998). This is mainly due to the difficulties in the measurement of numerous parameters. This is not be compared to Bass (1969) diffusion model and its extensions, in which only three or four parameters are required for a reasonably new product-adopting rate prediction.

Chapter 5

RESEARCH METHODOLOGY

CHAPTER OVERVIEW

This chapter introduces the methodology used in the research. It describes how the research has been conducted. A discussion of the merits and limits of the methodology is also provided at the end of the chapter.

5.1 METHODOLOGY

The aim of the research is to investigate the calibration problem for an agent-based simulation, which should give an indication of the limitations of using such models for prediction. An alternative viewpoint is that the study may indicate the amount of data required to produce a narrow range of predictions.

The approach devised is to develop an agent-based model (in this case, a consumer diffusion model) and to treat this model as the real system similar to the simulated reference map in Brown et al. (2005) as introduced in *Chapter 3*. Selected output data from this model will be used as measured values from the real world. In a pseudo-modelling exercise, this data will then be used to calibrate agent-based models of the system, and a method similar to those of Brooks et al. (1994) will be used to find the extent of the variations in predictions.

The research methodology can be summarized by the following three steps.

5.1.1 A Flow Chart of The Methodology

Figure 5.1 illustrates the methodology used in the research.

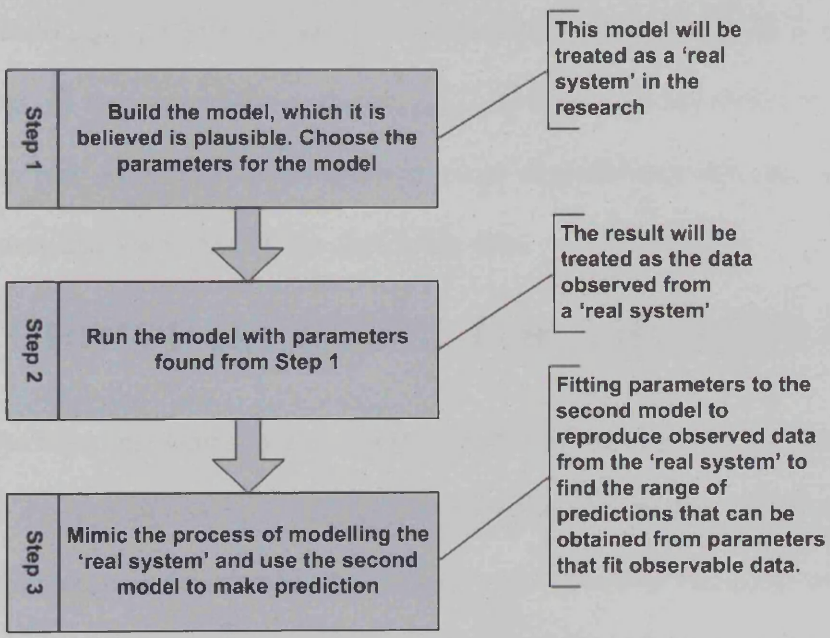

Figure 5.1: Flow chart of the proposed methodology

Step 1 Build the model that is believed to be plausible and choose parameters for it. The parameters chosen should make the model produce a reasonable result that can be accepted by field experts. Then, undertake multiple replications and see how much variation there is in the results. Presumably, the larger the population size, the smaller the effect of the randomness. Next, choose a population size so that the randomness effect is acceptably small. After all this, the model will be treated as the "real" system in the research. This model is referred to as $Model_{real\_system}$ in the later descriptions.

Step 2 Run $Model_{real\_system}$ with the parameters found from the previous step, and record the output from it. The result will be treated as data observed from the "real" system.

Step 3 Mimic the process of modelling the "real" system using a second model ($Model_{prediction}$) with the aim of the modelling project to make a prediction. Next, fit the parameters to $Model_{prediction}$ to reproduce the observed data from the "real" system in order to find the range of predictions that can be obtained from parameters that fit the observable data.

5.2 CHOICE OF MODEL FOR THE RESEARCH

For the following reasons, a model of the adoption of a new product is chosen for the research. First of all, due to the features of this situation (traditionally modelled by the Bass diffusion model introduced in *Chapter 4*), scholars are becoming aware of the advantages of applying an agent-based simulation model in such a situation. Second, during the research, a marketing expert was available for consultation, which was useful as the field expert's opinion helped to assess whether the model was plausible and realistic.

5.3 ADVANTAGES OF THE METHODOLOGY

The main advantage of such a pseudo-modelling exercise is that the real system is completely known. Consequently, the models' predictions can be compared with the "true" future values, and the precise differences between the models and the real system are also known, because the real system is designed by the modeller. The modeller has a full understanding of the model's structure. For instance, equations used in the model can be assumed to be correct.

This enables the research to isolate the effect of having to determine the parameter values by calibration. Since the structure of the "real system" and the model are identical the variance in predictions is entirely due to the calibration issue. If a real world system was modelled then the model structure would not match that of the real system (which, in any case, cannot be fully known) and it would not be possible to separate the effects of the differences in structure from the effects of calibration.

A further advantage of this approach is that it does not require the collection of real data, which can be difficult and time consuming. There are often also confidentiality problems in obtaining sales data.

5.4 DISADVANTAGES OF THE METHODOLOGY

The main potential disadvantage of the methodology is that the model used as the "real system" may not be realistic. If a real system was used then calibration tests could be applied to the model to assess its realism. However, our assessment can be made (in this case of $Model_{real\_system}$) by obtaining the opinions of a marketing expert.

5.5 DIFFERENCE IN THE METHODOLOGY TO THE PREVIOUS GROUNDWATER STUDY

Compared to the deterministic groundwater models, an additional problem for agent-based simulations is stochasticity. This is because heterogeneous populations are being modelled and therefore information for each individual in the real population will not usually be available. Instead, the model represents a typical population, and multiple replications are thus required to take account of the variations across possible populations. In particular, this means that a good fit with the historical data requires comparing the measured values against the range of values from the multiple replications, and predictions also need to be produced using multiple replications.

Chapter 6

AGENT-BASED CONSUMER MODEL

CHAPTER OVERVIEW

This chapter explains the agent-based consumer model used in this research. It describes the model's background, the conceptual model, the model's structure and the final parameters decided upon in the model. It also details the model validation and verification. It describes a manual simulation, and an Excel-based formulation test to verify and validate the model.

6.1 MODEL OVERVIEW

The application chosen for the research was a consumer word-of-mouth (WOM) model. The reasons for choosing this application were that:

- It is a situation in which agent behaviour is of importance.
- The ability to predict the future behaviour of the system would be very useful with significant potential commercial benefits.
- There has been very little agent based modelling of this situation and so the

model itself may contribute towards the development of theory and a better understanding.

- It is a common phenomenon which enabled me to use my own experience in developing the model. Input was also available from Mr Richard Meek in the marketing department.

The following assumptions have been made regarding the type of situation being modelled:

- The type of product is one with a short life-cycle, with a high likelihood of information and opinions being passed on between consumers by word-of-mouth.
- It is purchased as a one-off item (rather than a repeat purchase).
- Examples would include a computer game, a music album or a cinema ticket for a particular film.
- The population represented might be school or university students.

In the simulation model, a social network of individuals who interact with one another rather than a vast population of agents with many neutral contacts is represented. All agents are allocated to a diffusion social circle with a certain level of influence within the social network. All agents initially have no knowledge or preferences about the selected product. During the simulation, agents receive marketing communication messages (i.e. from company's advertisements, supermarkets, online search results etc.) and contact each other to exchange their knowledge and preferences about the product.

The model is run on a daily basis. At the start of the simulation, all the agents have no knowledge and no preferences about the product, since it is a new product. However, the company conducts an initial marketing campaign, and the agents may also see the product in the shops or read about it in the media. These interactions enable the agents to gain knowledge and change their preference in the initial stages of the simulation. The limited time of the campaign is modelled according to the probability of receiving outside information being reduced linearly down to 25% of the initial value over a period of 75 days.

The scenario investigated in the research was that the current time is several weeks after the product was launched (10 weeks for the first experiment, but this time was varied in subsequent experiments). Data for the total sales to date are available. The company wishes to predict the total sales of the product, the model should therefore match the sales to date and forecast the final sales. The key output variable is therefore sales.

The main output value of interest for the model is the number of purchases of the product in the population after the simulation period. Values up to a certain point (representing the time at which the modelling project is carried out) from the original model will be the historical data. The objective of the model will be to predict future sales.

6.2 MODEL DESCRIPTION

The following terms are used in this section:

- Characteristic: refers to aspects of the nature of the agent that are believed to exist in the real system such as the amount of influence the agent has within its social circle. They are assumed not to change during the simulation and are represented in the model using fixed attributes.

- Attribute: Variables attached to each agent. Fixed attributes are used to model characteristics. Variable attributes model changing views of the product.

- State: Refers to the values of an agent's attributes at a certain time point.

6.2.1 Agents' Attributes

The model contains a heterogeneous population of consumers (the agents). There is a lack of empirical data and no consensus in the literature on how consumers interact, on what happens when they interact or on what the important features and attributes are. Therefore, the attributes of the agents and the interactions between agents are based on our subjective views of the factors that are seen as important in the real world, whilst also trying to keep the model's structure as simple as possible.

Agent's views of the product In the model, each agent has two variable attributes whose values change as agents interact with each other and the environment or buy the product.

- **Knowledge** (K): *knowledge* here represents an agent's knowledge about the chosen product. It concerns the factors about the product. In other words, it is how much information the agent has about the product, defined as a figure ranging from 0 to 100. An agent with knowledge 0 means that the agent does not have any knowledge about the product at all, while an agent with knowledge 100 means that the agent has complete information about the product.

- **Preference** (P): *preference* is the agent's desire for the product. It implies how much the agent likes or dislikes the chosen product. It is shown as a figure ranging from -100 to 100. An agent with preference -100 means that the agent does not like the product at all and an agent with preference 100 indicates that the agent likes the product very much.

Agent's Characteristics The agents also have three fixed characteristics, assigned at the beginning of the simulation, which are all selected at random from probability distributions. These do not change during the simulation.

- **Influence** (I): this represents an agent's social standing within the population. Each agent is assigned an influence status, which cannot be changed for a particular product over the lifetime of the simulation. An agent with a high influence value means that their opinions have considerable weight in the conversations about the product. Influence can be any value between 0 and 20.

- **Unbiased true preference** (U): This parameter is introduced to represent the preference that an agent would have for the product with complete knowledge but with no peer pressure. It represents the underlying attractiveness of the product to the agent. In the model, unbiased true preference is a normally distributed value in the range $[-95, -35]$ and [35,95].

- **Buying criterion** (B): This is the preference value at which an agent buys the product. It represents varying attitudes regarding purchasing behaviour from cautious to free spending (some agents need less preference before they make a purchase than others). Agents buy when P $\geq P_{buy}$.

6.2.2 Environment Attributes

The environment communicates with agents in various ways and this is modelled in a similar way to inter-agent communications, with the environment having knowledge, preference and influence attributes, although these do not change during the simulation.

6.2.3 Interactions in the Model

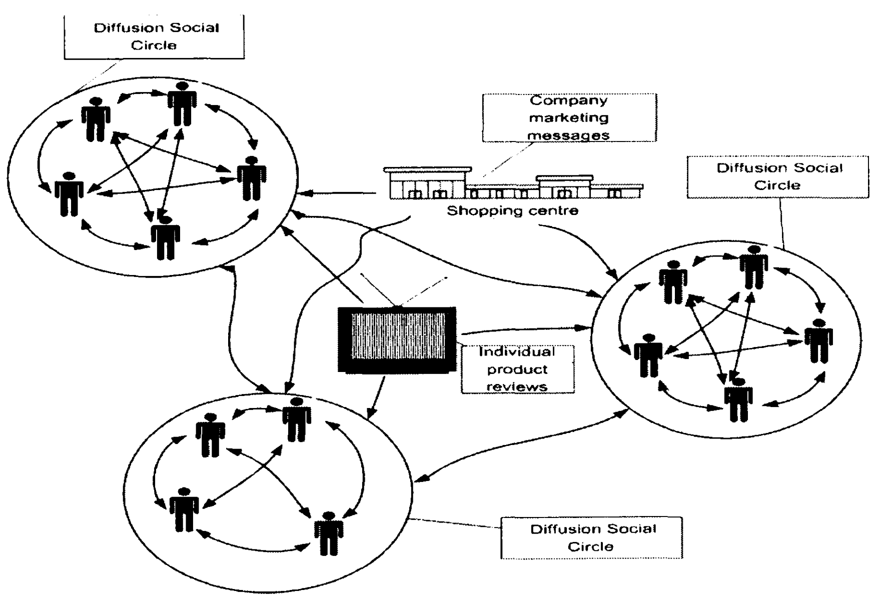

Figure 6.1: Interactions in the model

The interactions simulated in the model are conversations between agents about the product, and interactions between agents and the environment regarding the product (for instance, agents seeing adverts, reading product reviews or seeing the product in the shops). Another interaction in the model is agents buying the product and in this case, agents experiences of the product through using it. The strength of all these interactions will depend on the relative influence of the two communicating parties.

6.2.4 Rules on Contact

Notation:

In a single conversation where agent[a] contacts agent[b].

- K_a^{new} is agent[a]'s knowledge after the conversation.
- K_a^{old} is agent[a]'s knowledge before the conversation.
- K_b is agent[b]'s knowledge.
- P_a^{new} is agent[a]'s preference after the conversation.
- P_a^{old} is agent[a]'s preference before the conversation.
- P_b is agent[b]'s preference.
- I_b and I_a stand for agent[b] and agent[a]'s influence values respectively.
- U_a is agent[a]'s unbiased true preference.
- N_a is the number of purchases agent[a] has made (including the current purchase). N is introduced for repeated purchase behaviour.
- α is a random number between 0.01 and 0.15.

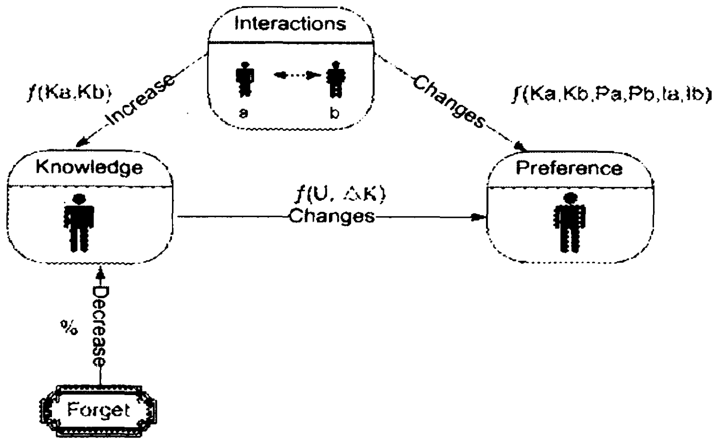

Figure 6.2: How agents change their knowledge and preferences

As shown in *Figure 6.2*, there are two ways for an agent to change its knowledge and preference. Agents gain knowledge from the interactions, which is a function of how much knowledge they know about the product and how much the other party knows about the product, using *Equation 6.1*. In addition, agent may forget some knowledge according to a random percentage. The agent's preference can be changed by the change of their knowledge; this is represented as a function of the change in knowledge and their unbiased true preference, using *Equation 6.3*. Additionally, the agent's preference can be changed by peer pressure, this is a function of the difference in knowledge, influence and preference between two parties, using *Equation 6.4*. Moreover, *Equation 6.3* applies every time there is a change in knowledge (i.e. after talking to another agent, after buying the product or if the agent loses knowledge at the end of each day).

Fixed equations are used for each of these interactions, and the forms of the equations for agent a interacting with agent / environment b are as follows:

Change in knowledge due to the interaction:

$$K_a^{new} = K_a^{old} + \alpha \times K_b \times \frac{(100 - K_a^{old})}{100} \tag{6.1}$$

It is assumed that in absence of peer pressure:

$$Preference = U \times \frac{K}{100} \tag{6.2}$$

Based on this underlying assumption, the effect on preference of the change in knowledge is:

$$P_a^{new} = P_a^{old} + U_a \times (\frac{K_a^{new} - K_a^{old}}{100}) \times (\frac{20}{20 + |P_a^{old} - \frac{U_a \times K_a^{old}}{100}|}) \tag{6.3}$$

The effect on preference of peer pressure is:

$$P_a^{new} = P_a^{old} + \alpha \times (P_b - P_a^{old}) \times \frac{(100 + K_b^{old} - K_a)}{200} \times \frac{(10 + I_b - I_a)}{20} \tag{6.4}$$

When an agent's P (preference) reaches P_{buy}, the agent buys the product, and *equations (6.5) and (6.3)* apply.

$$K_a^{new} = \frac{N_a \times K_a^{old} + 100}{N_a + 1} \tag{6.5}$$

(The situation modelled in this research is a one-off purchase and so $N_a = 1$ here.)

In each of these interactions, the agents may increase their knowledge, K, representing gaining information about the product (*Equation 6.1*). The change in knowledge depends on the existing knowledge of both parties. The gain in knowledge is the proportion of knowledge not known by the agent multiplied by the knowledge of the other party multiplied by a random proportion α. For example, if the agents knowledge is 70, then they will gain a random proportion of $0.3 \times \alpha$ of the other party's knowledge. This is based on the assumption that even if the other party knows less than the agent they will still probably have some different knowledge.

Whenever an agent's K value changes, this changes its P value as a function of the U value (*Equation 6.3*). The underlying assumption is *Equation 6.2* is absence of peer pressure (i.e. a linear relationship between P and K). Therefore, an increase in knowledge increases the preference by a proportion of the U value (with an adjustment to take account of existing peer pressure).

The preference, P, will also change, due to the influence of the preference of the other agent (peer pressure) or the environment (e.g. an opinion in a magazine review) (*Equation 6.4*). The strength of both of these interactions depends on the relative knowledge and influence of the two parties.

The agent buys the product when its preference, P, reaches its buying criterion, B. This increases the knowledge of the product by $\frac{1}{N_a+1}$ of the current lack of knowledge (*Equation 6.5*). As is the case with any change in knowledge, *Equation 6.3* is then used to change the agent's preference.

Interactions with the outside environment regarding the product are split into two types (*Figure 6.1*): information from the company and information from independent sources. These interactions use *Equations 6.1, 6.3* and *Equation 6.4* to change the agent's K and P in exactly the same way as interactions with other agents.

The population is divided into groups (representing social groupings), and each agent has a much higher probability of talking about the product to other agents within the group than to other agents outside the group. Agents also have a probability of losing some knowledge each day.

6.2.5 Remarks on Equations in the Model

In this subsection, the equations used in the model will be explained in more detail.

6.2.5.1 *Equation 6.1*

$$K_a^{new} = K_a^{old} + \alpha \times K_b \times \frac{(100 - K_a^{old})}{100}$$

This equation models that when an agent exchanges information with other agents or receives marketing information, it will gain knowledge. The agent's new knowledge equals to the agent's old knowledge K^{old} plus a random proportion of the other party's knowledge times the lack of knowledge this agent used to have.

6.2.5.2 *Equation 6.3*

$$P_a^{new} = P_a^{old} + U_a \times (\frac{K_a^{new} - K_a^{old}}{100}) \times (\frac{20}{20 + |P_a^{old} - \frac{U_a \times K_a^{old}}{100}|})$$

The above equation implements how an agent's preference changes when its knowledge changes, whether it goes up (gain knowledge) or down (lose knowledge).

$U_a \times (\frac{K_a^{new} - K_a^{old}}{100})$ represents the increase in preference the agent would have in the absence of peer pressure based on the absolute value of the current peer pressure effect being *equation 6.2* ($Preference = U \times \frac{K}{100}$).

$|P_a^{old} - \frac{U_a \times K_a^{old}}{100}|$ is the difference between what the agent's preference should be based on the agent's unbiased true preference and current knowledge and what it actually is.

$\frac{20}{20 + |P_a^{old} - \frac{U_a \times K_a^{old}}{100}|}$ was put into the equation to reduce the change in preference if there is peer pressure. This is based on the assumption that the impact of a change in knowledge will be lower if the agent is subject to peer pressure (i.e. the peer pressure will eliminate some of the effect of the change in knowledge).

6.2.5.3 *Equation 6.4*

$$P_a^{new} = P_a^{old} + \alpha \times (P_b - P_a^{old}) \times \frac{(100+K_b^{old}-K_a)}{200} \times \frac{(10+I_b-I_a)}{20}$$

This equation is used for an agent to change it's preference when making contact with other parties. The agent's new preference after a conversation with other parties equals the agent's old preference plus a random proportion of a measurement that takes into account the difference between two contacting parties' preferences, influences and knowledge. In general, this equation represents the fact that an agent's preference is changed by peer pressure.

$P_b - P_a^{old}$ is the difference between two parties' preferences.

$\frac{100+K_b^{old}-K_a}{200}$ is a ratio between 0 and 1 depending on the difference in knowledge (the greater the extra knowledge of the other party, the greater the effect that their preference has).

$\frac{10+I_b-I_a}{20}$ is used to reflect the impact of the relative difference of the two parties. Since I_b and I_a are numbers from 0 to 20, the maximum value for this part of the equation will be 1.5. Such a ratio implies that, if an agent has an extremely high influence, this agent tends to have a big effect on changing the opinion of an agent with an extremely low influence.

6.2.5.4 *Equation 6.5*

$$K_a^{new} = \frac{N_a \times K_a^{old}+100}{N_a+1}$$

This equation reflects the impact of the latest purchase on an agent's knowledge. In real life, it represents the fact that an agent gains more knowledge after they start using the product. Agents in the model were limited to one purchase only (in this case,

$N_a = 1$ and the equation transforms to $K_a^{new} = \frac{K_a^{old}+100}{2}$) but the general equation also shows how the model should be for repeat purchase (with each subsequent purchase having less of an effect).

6.2.6 Model Implementation

Algorithm in pseudo code:

```
SET UP AGENTS' ENVIRONMENT
            //environment:individual
            //source and company source
SET UP AGENTS' ALLOCATION MAP
           //assign agents' social group
INITIALIZE AGENTS' ATTRIBUTES
          //set up agent's initial unbiased preference,
          //influence,other probabilities etc.
FOR i=0, i<730, i++
          //730 simulation days
{
FOR EACH AGENT
{
AGENT RECEIVES INFORMATION FROM ENVIRONMENT
          //initially, agents obtain their knowledge and
          //preference from the environment
UPDATE AGENT'S STATUS
         // change agents' preference,
         //knowledge and purchase status
}
FOR EACH AGENT
{
AGENT CONTACTS OTHER AGENTS IN SAME GROUP
         //agents choose other agents from the same group with a given
         //probability to talk
UPDATE AGENT'S STATUS
         // change agents' preference,
         //knowledge and purchase status
}
FOR EACH AGENT
{
AGENT CONTACTS OTHER AGENTS FROM OTHER GROUPS
         //agents choose other agents from different groups with a given
         //probability to talk
```

```
UPDATE AGENT'S STATUS
         // change agents' preference,
         //knowledge and purchase status
}
FOR EACH AGENT
{
AGENT LOSES KNOWLEDGE
UPDATE AGENT'S STATUS
        // change agents' preference,
        // knowledge and purchase status
}
}

PRODUCE OUTPUT
          //record how many agents bought the product during the simulation
```

To implement the above pseudo code, C++ is applied. *Figure 6.3* shows how the objects (AGENT and SOURCE INFORMATION) work and provides a list of methods of the above objects:

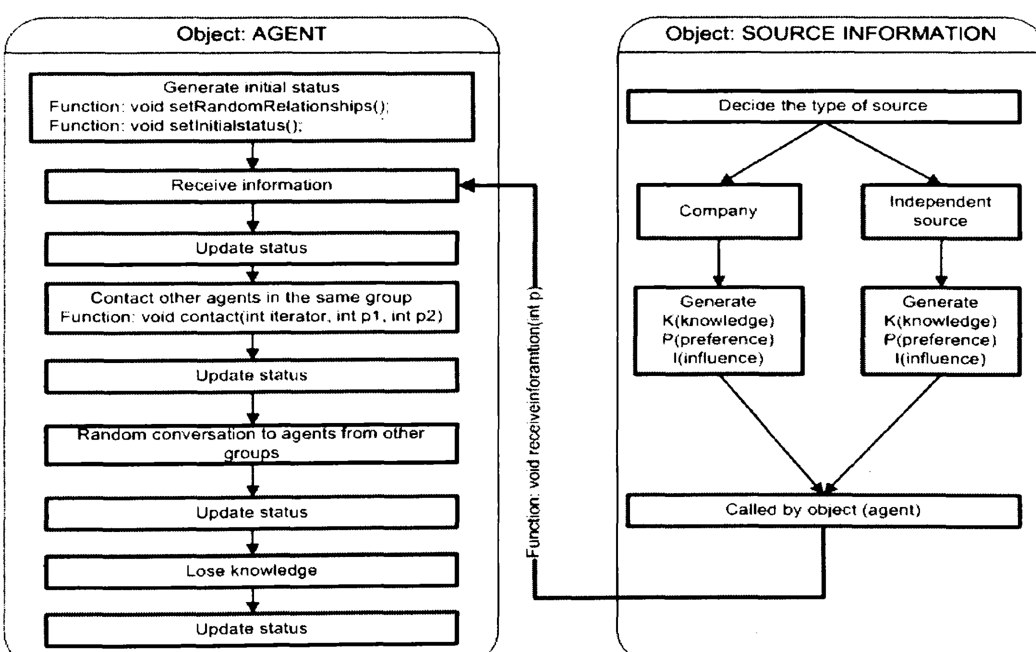

Figure 6.3: Objects: agent and source information

6.2.7 Default Parameters

Having constructed the model structure as described in the previous sections, the next stage in the modelling involved choosing the values for the parameters. The model with the default parameters is the "real system", and therefore the parameters were adjusted until the model showed plausible behaviour.

After a preliminary study of the model and discussion with a field expert (Mr. Richard Meek, who is an expert in the marketing field), the parameters were chosen, which result in 4.9% of the whole population buying the product in the first 70 simulation days (which was designed to reflect an advertisement campaign launched for the new product) and 24.7% of the population buying the product at the end of the whole simulation period, which are believed to be reasonable. The parameters are shown in *Table 6.1.*

Table 6.1: List of default parameters

Parameter	**Value**	**Parameter**	**Value**
$U_{mean\_positive}$	75	$U_{stdev\_positive}$	15
$U_{mean\_negative}$	-75	$U_{stdev\_positive}$	15
$P_{talk\_to\_same\_group}$	10%	$P_{positive\_Uvalue}$	90%
B_{mean}	65	B_{stdev}	10
$P_{receive\_information\_MIN}$	0%	$P_{receive\_information\_MAX}$	25%
$P_{lose\_knowledge}$	1%	$P_{\frac{company\_marketing\_infromation}{outside\_information}}$	80%
I_{mean_agent}	10	I_{stdev_agents}	3
$I_{company\_MIN}$	0	$I_{company\_MAX}$	5
$I_{mean\_independant}$	10	$I_{stdev\_independant}$	3
$Company\_knowledge_{mean}$	60	$Company\_knowledge_{stdev}$	15
$Company\_preference_{mean}$	60	$Company\_preference_{stdev}$	15

Continued...

$Independent\_knowledge_{mean}$	40	$Independent\_knowledge_{stdev}$	15
$Independent\_(+)preference_{mean}$	65	$Independent\_(+)preference_{stdev}$	10
$Independent\_\%_{positive\_preference}$	90%	$Random\_coefficient$	$0.1 \sim 0.15$
$Independent\_(-)preference_{mean}$	-65	$Independent\_(-)preference_{stdev}$	10
Population	500	*Simulation length(days)*	730
Random conversation	5	*Random Seed*	3
Agents group size from 2 to 8 (based on binomial distribution)			

Notes:

- The U (unbiased preference) value and B (buying criteria) value follow normal distribution with mean and standard deviation given in *Table 6.1.* The $U_{mean\_negative}$ parameter refers to the proportion of the population having negative U values (i.e. dislike), with the probability of an agent having a negative U value being 10%.

- The probability of the independent source having a negative preference is 20%.

- The number of random conversations each day refers to the total number of conversations outside the social groups in the population (i.e. five pairs of agents are picked at random from the population).

- The probability of agent receiving products' message from independent media or company's side follows uniform distribution between $P_{receive\_information\_MIN}$ and $P_{receive\_information\_MAX}$. The percentage of messages from the company in all outside information is $P_{\frac{company\_marketing\_infromation}{outside\_information}}$.

- The agent's influence value follows normal distribution $N(I_{mean\_agent}, I_{stdev\_agents})$ while the influence value of the company's messages follows uniform distribution between $I_{company\_MIN}$ and $I_{company\_MAX}$. The influence value of the information from independent media follows normal distribution.

- The company's knowledge and preference follow normal distribution.

- The population is divided into groups, sized between 2 and 8 as shown in *Figure 6.4*. A binomial distribution ($Binomial(7, 0.5)$) is used for the probabilities of the different group sizes, as it is considered to provide suitable values: for x between 2 and 8, the probability of group size $x = \frac{b(x-1,7,0.5)}{\sum_{i=1}^{7} b(i,7,0.5)}$, where $b(x, 7, 0.5)$ is the binomial probability of x successes from 7 trials with probability of success 0.5.

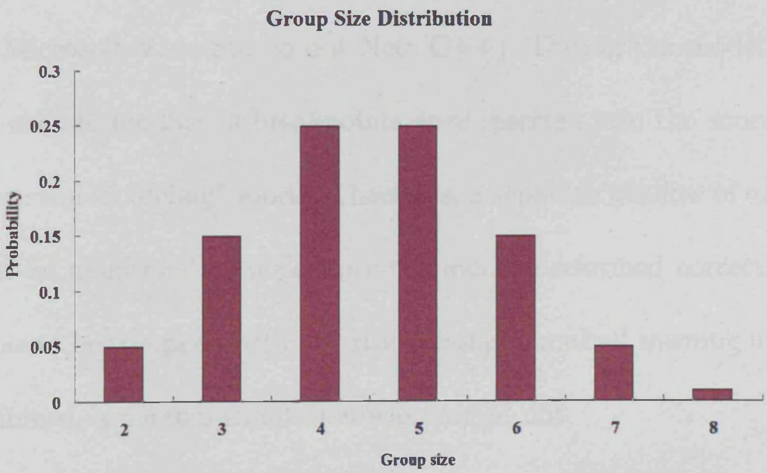

Figure 6.4: Group size distribution

6.3 MODEL VERIFICATION & VALIDATION

6.3.1 Model Verification

Verification is the process of making sure that the model does what it is intended to do from an operational perspective. The best strategy to identify bugs is to examine carefully behaviour rules and the internal data of the agents and then to verify agents' exact behaviour. Therefore, a manual simulation and an Excel-based simulation were carried out in order to verify the model.

6.3.1.1 Manual Simulation

First of all, I examined the internal behaviour and data of the agents and verified agents' exact behaviour. This process is easy to conduct with a high-level development library (e.g. Microsoft Visio Studio dot Net: C++). During the model's implementation phase, a large number of breakpoints were inserted into the source code. The model was then run in "debug" mode. Therefore, a separate window of each variable's changes could be monitored to make sure the model performed correctly. Once the model was able to be compiled with no error messages, and all warning messages were carefully examined, a manual simulation was carried out.

Due to the time difficulties in performing the manual simulation, the number of agents was fixed to 10. Additionally, only the first 10 days of the simulation were run. The knowledge and preference of each agent for each day were compared to the simulation results from the computer model. The initial parameters and random numbers used were from the computer model.

The results of the manual simulation matched the results gained from running the computer model, indicating that the computer model was running correctly for this scenario.

6.3.1.2 Excel Simulation

In order to test the model's algorithms (equations used in the model), a simulation in Excel was carried out after the manual simulation. Due to the complexity of the simulation, it was not possible to undertake a comprehensive simulation using Excel with 500 agents that included every single detail. Therefore, the simulation using Excel was limited to two agents with fixed knowledge, preference, unbiased true preference. And the computer model output matched the Excel results.

6.3.1.3 STRESS Test

A number of extreme values were used as model inputs to examine how the model behaved in extreme circumstances.

Table 6.2: List of extreme parameters and tests' results

Parameter	**Value tested**	**Pre-expectation**	**Result**
$U_{mean\_positive}$	-100	*Very few agents buy the product*	*Yes*
$U_{mean\_positive}$	100	*Very high percentage of the population buys the product*	*Yes*
$P_{lose\_knowledge}$	0	*Agents end up with very high knowledge*	*Yes*
$P_{lose\_knowledge}$	100	*No purchases were made*	*Yes*
$P_{talk\_to\_same\_group}$	100	*Simulation ends up with clearly clustered group preferences*	*Yes*

Continued...

$P_{talk\_to\_same\_group}$	0	*Simulation ends up with randomly distributed agents' preferences*	*Yes*
$P_{receive\_information}$	0	*No agents buy the product*	*Yes*
$P_{receive\_information}$	100	*Very high percentage of the population buys the product*	*Yes*
B_{mean}	100	*No agents buy the product*	*Yes*
B_{mean}	0	*Almost every agent buys the product*	*Yes*

As shown in *Table 6.2*, the model produced the results expected for extreme circumstances, thus adding to the confidence that the model does what it is intended to do.

6.3.2 Model Validation

The main objective of model validation is to test whether the model reproduces system behaviour with enough reliability to satisfy the project objectives. As far as this model is concerned, as described in the methodology chapter (*Chapter 5*), the main objective of the model is to build up a platform that can produce plausible marketing behaviour. Additionally, it is not a real case simulation, but a simulated marketing world simulation.

As mentioned in *Section 6.2.7*, a marketing expert[1], Mr Richard Meek, had some

[1]Mr Richard Meek, Lecturer (part time) in the Marketing Department, Management School, Lancaster University, UK

input in the model design phrase by explaining current marketing theory. He contributed to the validation of the model by confirming that the model produced reasonable behaviour from a marketing expert's point of view, which is the main aim of the model.

The model is therefore believed to be able to satisfy the original design requirement, due to the fact that it can produce plausible marketing behaviour.

6.4 OUTPUT VALUES

6.4.1 Main Output File Format

The output file in *Table 6.3* gives the main results from running the model. It records the total number of sales each day for the whole population (for each run if multiple replications are carried out).

Table 6.3: Example of main model output file

Sales	Run_1	Run_2	Run_3	...	Run_{1000}	Average
Day_1						
...						
Day_{730}						

6.4.2 Detailed Output File Format

This output file is for detailed model investigation as shown in *Table 6.4*. This output file records the individual values for the agents on each day of the simulation for a single run. The first three rows record selected fixed attribute values. The remainder of the file records the values for knowledge and preference at the end of each day, as well as whether the agent bought the product on that day.

Table 6.4: Example of model detailed output file

		$Agent_1$	$Agent_2$	$Agent_3$	...	$Agent_{500}$
Group number						
Unbiased T preference						
Buy criterion						
Day_1	Knowledge					
	Preference					
	Purchase					
...	...					
	...					
	...					
Day_{730}	Knowledge					
	Preference					
	Purchase					

6.5 SUMMARY

This chapter has introduced the model structure used in this research. It has given a general idea of how the basic model works. *Section 6.1* has provided an overview of the model. It has described the background and assumptions of the model. *Section 6.2* has introduced the agents' attributes, environment, agents' contact rules, etc. Moreover, in *Section 6.2.4*, the equations used in the model have been described in detail. *Section 6.2.7* has described how the default parameters were chosen and has given a list of the default parameters. The basic model with default parameters will be treated as the "real world", and the data collected from it will be treated as "real" data for future calibration use. *Section 6.3* has described the verification & validation of the model. Regarding model verification, a manual simulation along with a simulation in Excel were conducted to test whether the model did what it was designed to do. Additionally, the model was tested with various extreme values to check how it had performed in extreme circumstances. Regarding model validation, consultation of a marketing expert played an important role. His opinion contributed to the assessment of whether or not the model had produced a reasonable output. Both the verification and the validation results were positive. The confidence in the model was confirmed, thus allowing further experiments to be undertaken as outlined in the next chapter.

Chapter 7

MODEL BEHAVIOUR

CHAPTER OVERVIEW

This chapter describes the initial experiments that were conducted. It starts with the model output study to give a general idea of how the model behaves followed by sensitivity analysis of the different parameters. Sensitivity analysis has been conducted to investigate the impacts of changes in the following parameters: the probability of losing knowledge at the end of each simulation day, the probability for an agent to talk to agents from the same group, the probability for an agent to receive outside marketing information, the mean in the normal distribution of an agent's buying criterion and the mean in the normal distribution of an agent's unbiased true preference. In order to investigate the model dynamics, an experiment on the outside marketing sources' knowledge and preference has also been conducted. At the same time, the agents' group number has been tested.

7.1 MODEL BEHAVIOUR - BEHAVIOUR OF "REAL SYSTEM"

After the model was built, a few tests were conducted to examine if the model can produce plausible, realistic results.

The model was run with 500 agents, which represents a small community such as a school. The run length was 730 days (2 years). The model with the default parameters was run 1000 times and this was assumed to represent the total population (i.e. 1000 schools). The average of these 1000 replications therefore represents the true behaviour of the real system. The results gained were treated as real values observed from the real system in the later calibration phase.

7.1.1 Sales Distribution

Figure 7.1 shows the product life cycle of sales per day for 1000 replications and *Figure 7.2* shows the same data as cumulative sales. The highest sale appears around the 70th day when the average sales reach 1.318. There is quite a long tail with even a few sales taking place in the second year. This pattern accords with a typical new product launch pattern and it is therefore believed to be realistic.

The average total number of sales per replication is 123.591 (24.7% of the population of 500). There is considerable variability across the replications. *Figure7.3* shows the distribution of total sales for the 1000 replications and *Figure 7.4* shows the distribution of the first 10 weeks sales for the 1000 replications. Total sales appear to be approximately normally distributed.

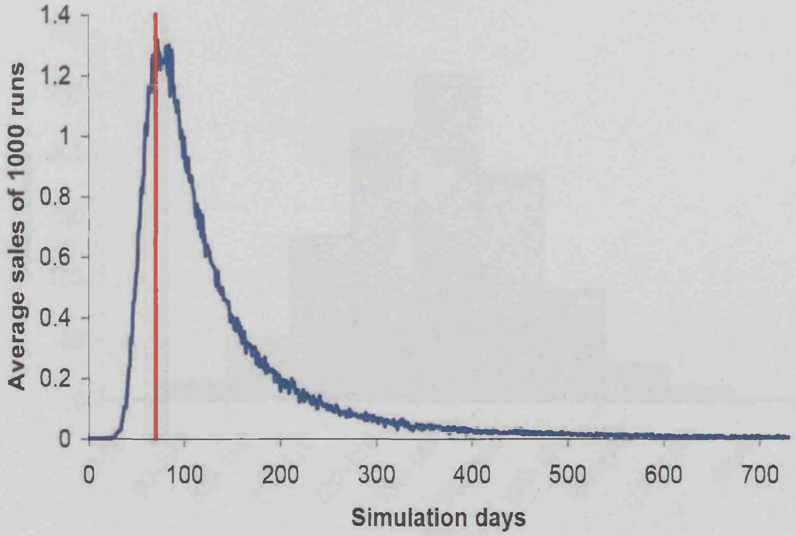

Figure 7.1: The product life cycle as measured by average number of buyers over 1000 simulation runs

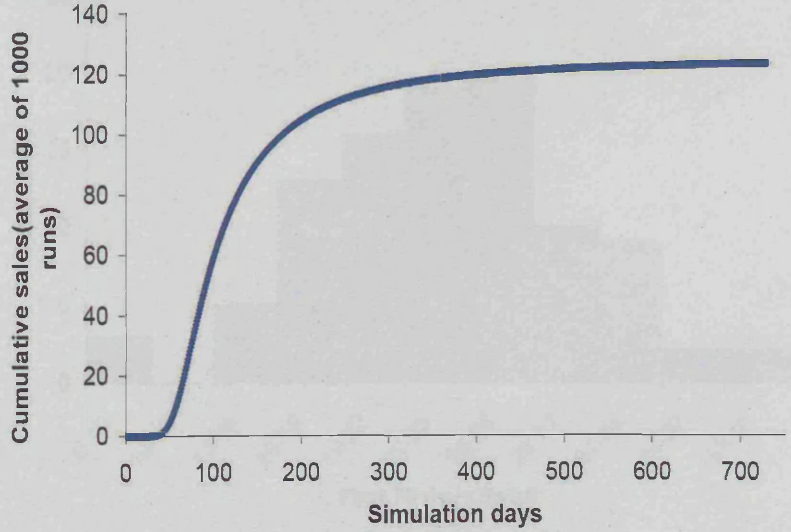

Figure 7.2: The cumulative sales(average of 1000 runs)

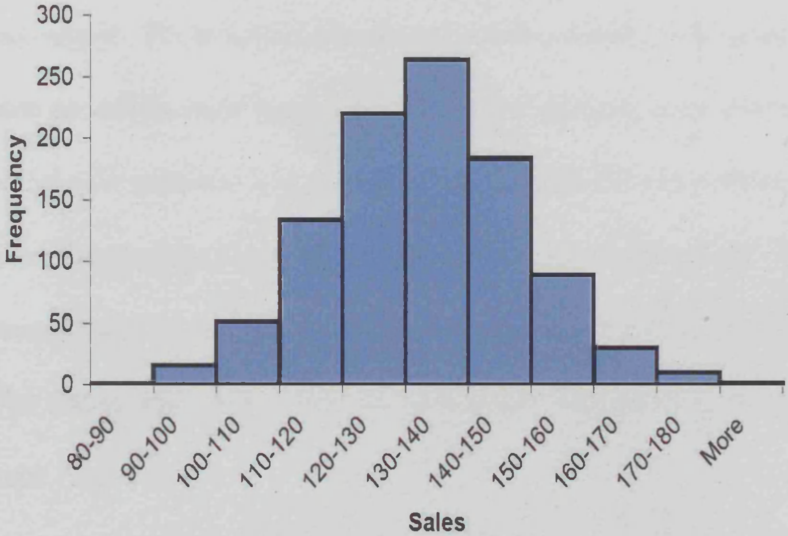

Figure 7.3: Histogram of sales

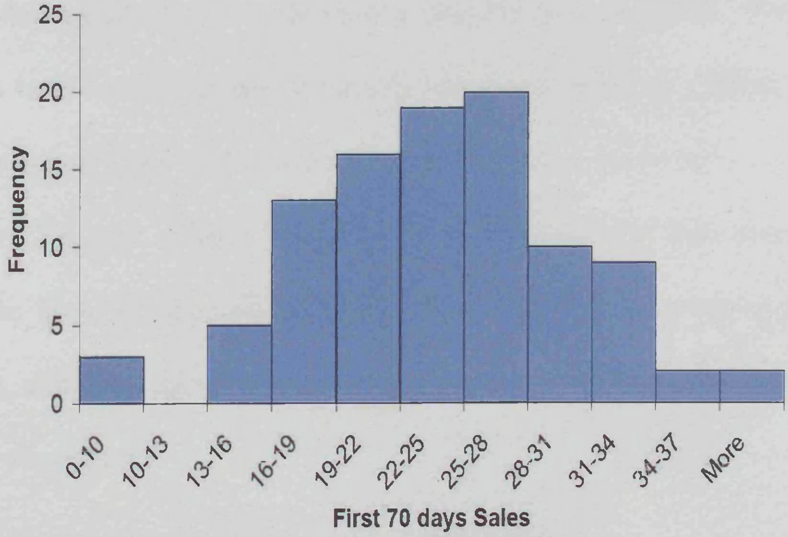

Figure 7.4: Histogram of the first 10 weeks sales

7.1.2 Peer Pressure

Various additional analysis were carried out to get a better understanding of the behaviour of the model. In this section, the model is run one time but with much more detailed output. For instance, the output result includes each agent's knowledge and preference as well as each agent's group number and purchase status.

The effect of peer pressure was examined by plotting the proportion of the group that purchased the product for all the groups in one run of the model. As mentioned in the previous Chapter, the group size varies between 2 and 8 and in this run, it happens to be 100 groups. The results for preference and purchase rate are shown in *Figure 7.5* and *Figure 7.6*.

Peer pressure is represented by the tight grouping of preference values in *Figure 7.5*, which shows each agents' preference in the end of a simulation. People from the same group tend to have similar preference. However, it is not realistic in the sense that people in real world would not have exact the same preference.

In *Figure 7.6*, peer pressure is evidenced by the relatively high proportion of extreme values. Many groups had 100% uptake meaning that all group members bought the product, and many groups had 0% uptake indicating that no purchases were made among the whole group.

7.2 SENSITIVITY ANALYSIS

"Sensitivity analysis studies the relationships between information flowing in and out of a model" (Saltelli and Scott, 2000). In general, sensitivity analysis is used to obtain a better understanding of model behaviour by looking at how much effect

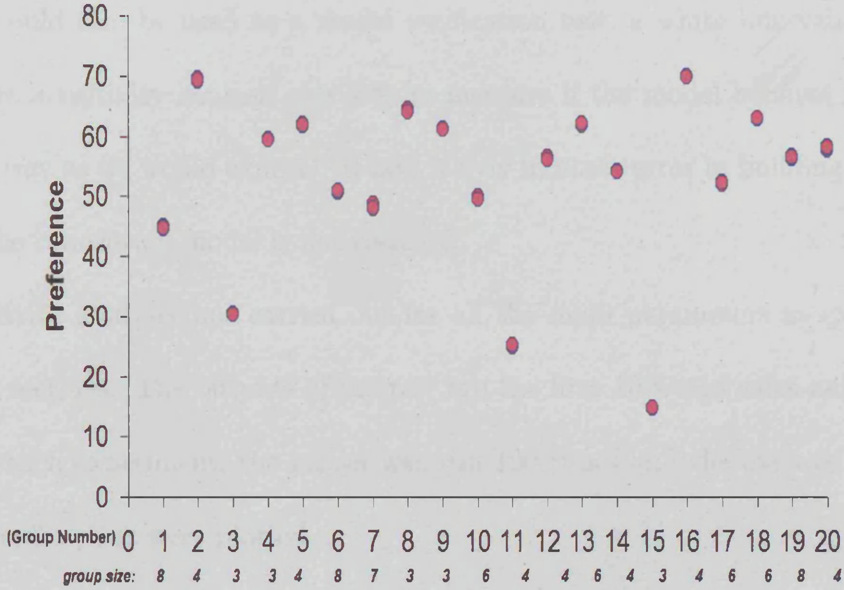

Figure 7.5: Grouped agent's preference

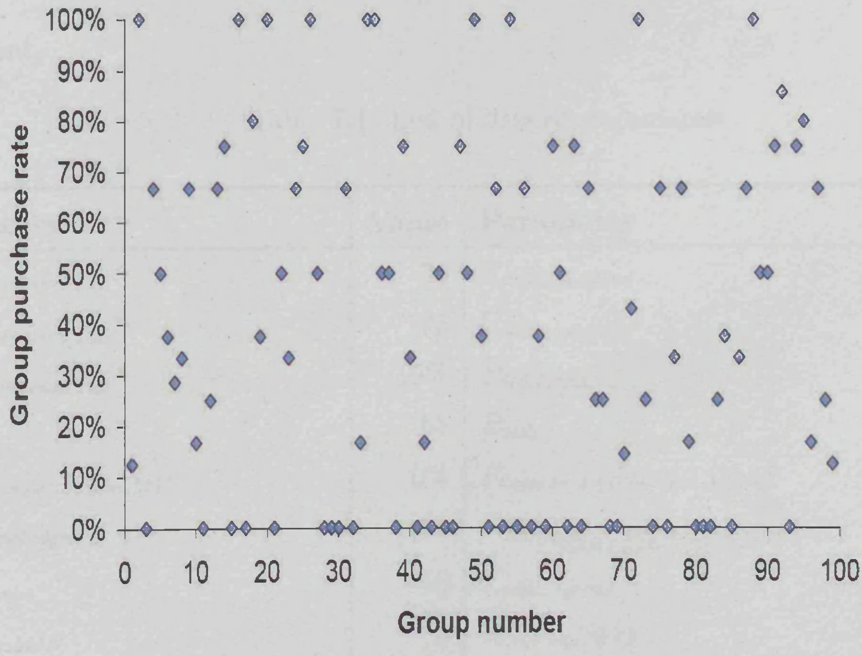

Figure 7.6: Purchase rate for each group

each input parameter has on the model output. Moreover, to some extent, sensitivity analysis could also be used as a model verification test, a white box validation. In such cases, sensitivity analysis can help to measure if the model behaves in realistic way (the way as we would expect). If not, it may indicate error in building the model or that the conceptual model is not realistic.

Sensitivity analysis was carried out for all the main parameters as explained in following sections. The outputs of interest are the first 10 weeks sales and the total sales. In each experiment, the model was run 100 times and the average results for the 100 replications were plotted.

7.2.1 Experiment Parameters

All experiments in the following sections will use the default parameters set (shown again in *Table 7.1*), though the parameter which needs to be tested will vary in each experiment.

Table 7.1: List of default parameters

Parameter	**Value**	**Parameter**	**Value**
$U_{mean\_positive}$	75	$U_{stdev\_positive}$	15
$U_{mean\_negative}$	-75	$U_{stdev\_positive}$	15
$P_{talk\_to\_same\_group}$	10%	$P_{positive\_Uvalue}$	90%
B_{mean}	65	B_{stdev}	10
$P_{receive\_information\_MIN}$	0%	$P_{receive\_information\_MAX}$	25%
$P_{lose\_knowledge}$	1%	$P_{\frac{company\_marketing\_infromation}{outside\_information}}$	80%
I_{mean_agent}	10	I_{stdev_agents}	3
$I_{company\_MIN}$	0	$I_{company\_MAX}$	5
$I_{mean\_independant}$	10	$I_{stdev\_independant}$	3

Continued...

$Company\_knowledge_{mean}$	60	$Company\_knowledge_{stdev}$	15
$Company\_preference_{mean}$	60	$Company\_preference_{stdev}$	15
$Independent\_knowledge_{mean}$	40	$Independent\_knowledge_{stdev}$	15
$Independent\_(+)preference_{mean}$	65	$Independent\_(+)preference_{stdev}$	10
$Independent\_\%_{positive\_preference}$	90%	*Random_coefficient*	$0.1 \sim 0.15$
$Independent\_(-)preference_{mean}$	-65	$Independent\_(-)preference_{stdev}$	10
Population	500	*Simulation length(days)*	730
Random conversation	5	*Random Seed*	3
Agents group size from 2 to 8 (based on binomial distribution)			

7.2.2 Experiment 1 - $P_{lose\_knowledge}$

Objective: Experiment 1 is the sensitivity analysis of the percentage of losing knowledge at the end of each simulation day. The expectation was that the more knowledge an agent loses each day, the slower the agent builds up its knowledge and consequently, the lower product uptake percentage in the whole population.

7.2.2.1 Parameters Used in The Experiment

In this experiment, the percentage of losing knowledge at the end of each simulation day($P_{lose\_knowledge}$) was changed from 0% to 5% with 0.50% increments.

7.2.2.2 Result

From *Figure 7.7*, a negative relationship between the sales and percentage of losing knowledge can be observed. In addition, the total sales as well as sales for first 10 weeks are sensitive to the percentage of losing knowledge in the range of [0% $\sim$ 2%]. The reason behind it is the fact that if an agent loses a high percentage of its knowledge at the end of the simulation day, it will be very difficult for the agent to build up its knowledge. Consequently, it will slow down the spread of the knowledge

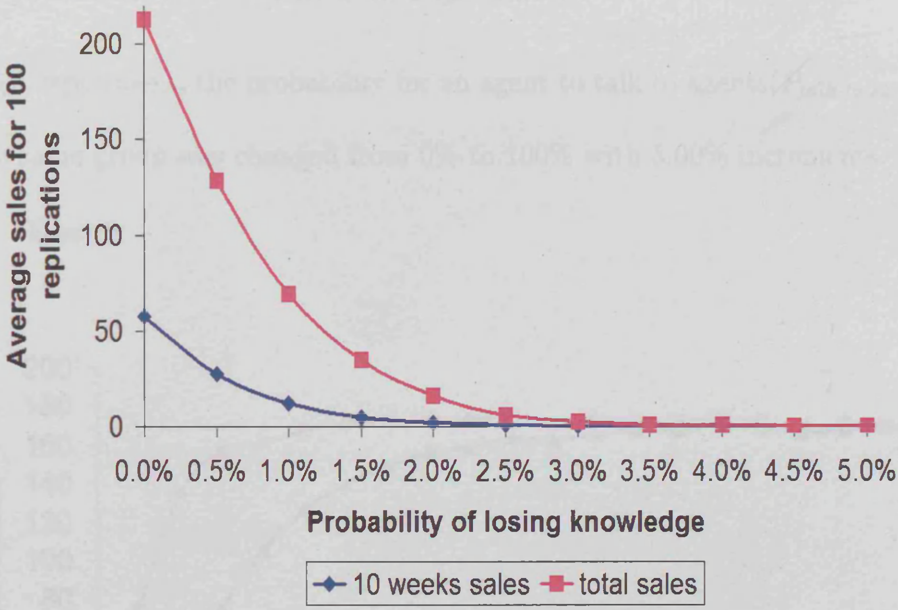

Figure 7.7: Sensitivity analysis of the percentage of losing knowledge

in the population. Accordingly, this leads to a very slow increase of agent's preference (because: $P_a^{new} = P_a^{old} + \alpha \times (P_b - P_a^{old}) \times \frac{(100+K_b^{old}-K_a)}{200} \times \frac{(10+I_b-I_a)}{20}$, a small ΔK will lead to a small P). The simulation, therefore, ends up with very few agents buying the product.

7.2.3 Experiment 2 - $P_{talk\_to\_same\_group}$

Objective: Experiment 2 is the sensitivity analysis of the probability for an agent to talk to agents from the same group. The likely effect here is that the more probability for an agent to talk to agents from the same group, the more similar the group behaviour.

7.2.3.1 Parameters Used in The Experiment

In this experiment, the probability for an agent to talk to agents($P_{talk\_to\_same\_group}$) from the same group was changed from 0% to 100% with 5.00% increments.

7.2.3.2 Result

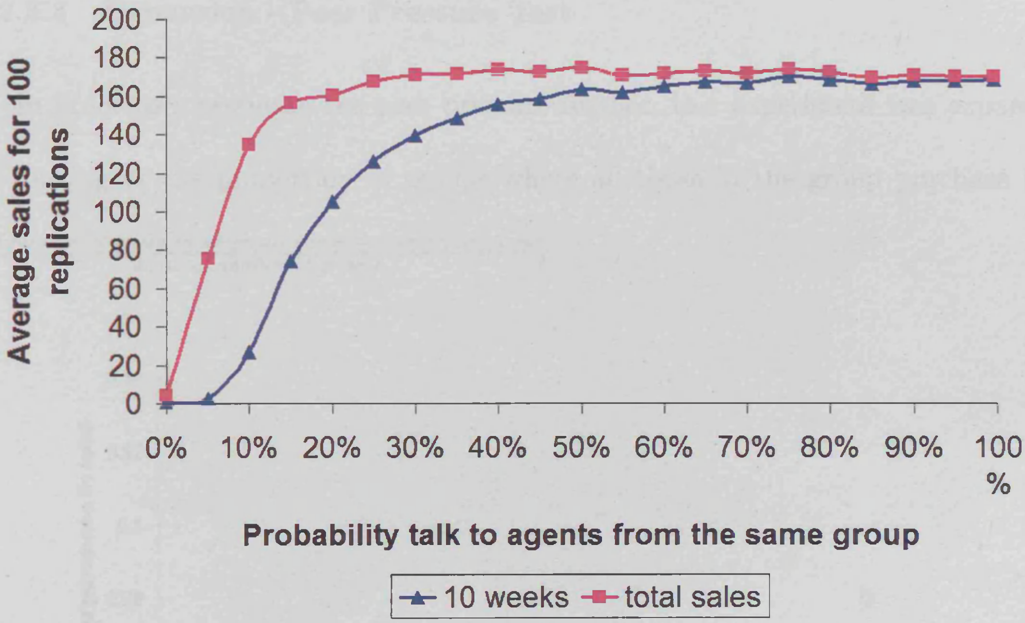

Figure 7.8: Probability for agent to talk to agents from the same group

From *Figure 7.8*, there is clearly a positive relationship between the sales and the probability for an agent to talk to agents from the same group for the lowest values of the probability. When $P_{talk\_to\_same\_group}$ is greater than 25%, the total sales tends to be constant but as the probability increase more agents make the purchase within the first 70 days. Another interesting observation is the upper limit of the total sales. In other words, while $P_{talk\_to\_same\_group}$ changes from 25% to 100%, the total sales stay in a stable range and do not go over 190. In practice, this indicates that if the social circle (social group) is fixed, increasing the probability for them to exchange information about the product only improves the sales up to a certain point. After this point, the probability for consumers to talk about the product will no longer be a crucial factor but will make the purchases occur more during the early stage of the new product.

7.2.3.3 Expansion - Peer Pressure Test

In order to investigate the peer pressure further, this experiment was expanded by looking at the proportion of groups where all those in the group purchase (i.e. $\frac{\textit{the number of group in which all group members purchased}}{\textit{the total number of groups}}$).

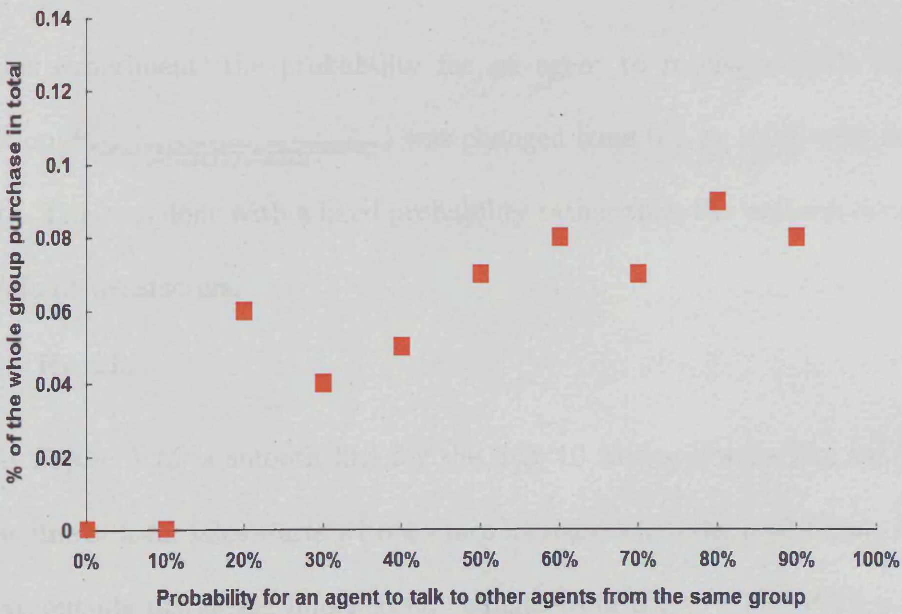

Figure 7.9: The group uptake percentage

In *Figure 7.9*, the x axis is $P_{talk\_to\_same\_group}$ which was changed from 0% to 100% with 10% increments. The y axis is the whole group uptake rate. There is clearly a positive correlation between them. This indicates that the more chance for the agents to talk to the other agents in the same group, the stronger peer pressure is in the group, which leads to the similar behaviour in the group.

7.2.4 Experiment 3 - $P_{\frac{company\_marketing\_infromation}{outside\_information}}$

Objective: Experiment 3 is the sensitivity analysis of the probability for an agent to receive outside marketing information. The expected effect here is that the more probability for an agent to receive outside marketing information, the more chance for the agent to gain a high knowledge of the product. Therefore, the agents' uptake behaviour will be decided more by the initial unbiased true preference value.

7.2.4.1 Parameters Used in The Experiment

In this experiment, the probability for an agent to receive outside marketing information($P_{\frac{company\_marketing\_infromation}{outside\_information}}$) was changed from 0% to 100% with 5.00% increments. This was done with a fixed probability rather than the uniform distribution of the default parameters.

7.2.4.2 Result

From *Figure 7.10*, a smooth line for the first 10 weeks of sales can be observed while the line of total sales starts with a sharp increase when the probability for agent to receive outside marketing information changes from 0% to 10%. With a value of 0% there is no outside information and so the agents are unable to gain knowledge or preference and so there are no sales. A probability of 5% is significant to generate about 100 sales although it takes a longer time for the agents to gain knowledge and so there are very few sales in the first 10 weeks. As the probability increases, total

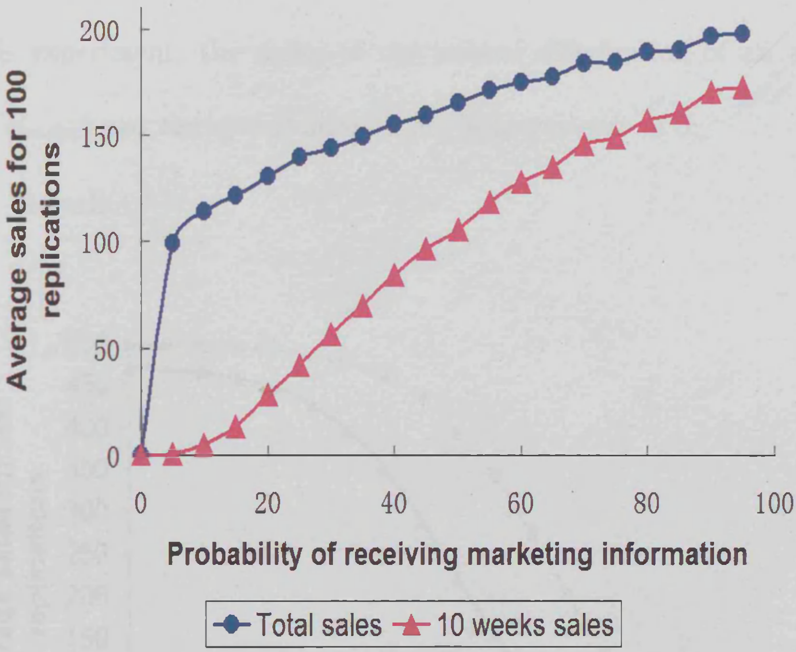

Figure 7.10: Probability for agent to receive outside marketing information

sales increases approximately linearly. However, even with a probability of 95% the total sales are only 200. This is because the probability reduces during the first 70 days to represent the initial advertising campaign and therefore there is a limit to the effect that increasing this parameter can have. At high values most of the sales are within the first 70 says.

7.2.5 Experiment 4 - B_{mean}

Objective: Experiment 4 is the sensitivity analysis of mean of the normal distribution of an agent's buying criterion. The expected effect here is the higher the mean, the lower the product uptake percentage in the whole population.

7.2.5.1 Parameters Used in The Experiment

In this experiment, the mean of the normal distribution of an agent's buying criterion(B_{mean}) was changed from 0 to 100 in increments of 5.

7.2.5.2 Result

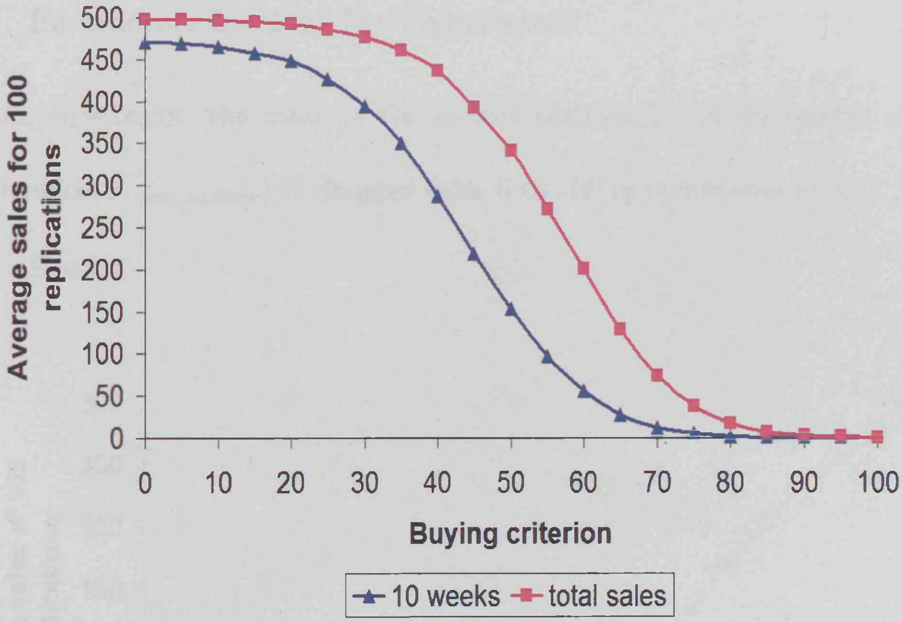

Figure 7.11: The mean of agent's buying criterion

From *Figure 7.11*, an obviously negative relationship between the mean in the normal distribution of an agent's buying criterion and the total sales can be observed. This is because when an agent's buying criterion increases, the agents are more picky in terms of making a purchase. For instance, when B_{mean} is 100, the total sales ends up with almost 0 since B_{mean} is too high for agent's preference to reach which leads to almost no purchases at the end of the simulation.

7.2.6 Experiment 5 - $U_{mean\_positive}$

Objective: Experiment 5 is the sensitivity analysis of the mean of the normal distribution of an agent's unbiased true preference. The expected effect here is that a higher mean will lead to higher preference values and therefore a higher uptake percentage.

7.2.6.1 Parameters Used in The Experiment

In this experiment, the mean of the normal distribution of an agent's unbiased true preference($U_{mean\_positive}$) is changed from 0 to 100 in increments of 5.

7.2.6.2 Result

Figure 7.12: The normal distribution mean of agent's unbiased true preference

In *Figure 7.12*, when $U_{mean\_positive}$ changes from 0 to 40, the total sales are close to 0. The total sales increases when $U_{mean\_positive}$ is more than 40 and shows a positive relationship with $U_{mean\_positive}$. However, even $U_{mean\_positive} = 100$ doesn't make all agents purchase the product. This is due to the fact that 10% of the agents are initially allocated with a negative U value and the existence of peer pressure within social group causes some agents with a high U value to end up with relatively low preference.

7.2.7 Experiment 6 - K_{out} and P_{out}

Objective: Experiment 6 was designed to investigate the effect of the company's knowledge and preference, which is passed on by the company's advertising campaign. The frequencies of receiving information from the company was investigated in Experiment 3. This experiment looks at the nature of information from company.

7.2.7.1 Parameters Used in The Experiment

In this experiment, the mean for the company's knowledge (K_{out}) and preference (P_{out}) were both changed from 0 to 100 in increments of 5. All the combinations of values were simulated giving a total number of parameter sets of $431 (= 21^2)$.

7.2.7.2 Result

Figure7.13 and *Figure7.14* shows that as K_{out} and P_{out} increase from 0 to 100, the total sales and the first 70 days sales increase accordingly. With the value of 0% there is no company knowledge and preference available in the market (no advertisement campaign launched), and so the only information source for the agent to gain knowledge will be the independent source, which is only weighted as 20% of the total market available information. The agents can only obtain knowledge and preference from a very limited source and so there are no sales in the first 70 days but considerable sales (80) by the end of simulation.

Preference P_{out} has more effect than knowledge K_{out} on the sales in the first 70 days. As shown in *Figure7.14*, with the K_{out} value of 0 and P_{out} value of 100, these reflect the fact that in the extreme condition that the company launched an advertisement campaign focusing only on publicising the brand instead of the specific product, the company can still generate some sales in the early stage (sales 40 after 70 days). This corresponds to real markets with some consumers making purchases based mainly on the brand.

7.2.8 Experiment 7 - Group Size

Objective: Experiment 7 was designed to explore the influence of social group size on agents' purchase behaviour.

7.2.8.1 Parameters Used in The Experiment

In this experiment, the social group size was changed from 0 to 160 in increments of 5. In order to make the experiment applicable, the group size was fixed to the testing parameters instead of using a binomial distribution. Because the population set in this model is 500, very large group sizes (bigger than 160, 32% of the whole population) were not considered. Small group sized (as used in the default parameters) were investigated in more detail by running the model for each size from 2 to 15.

7.2.8.2 Result

The results are shown in *Figure7.15* and *Figure 7.16*. As group size increases from 2 to 6, the total sales increases. This might relate to the way the program works as the

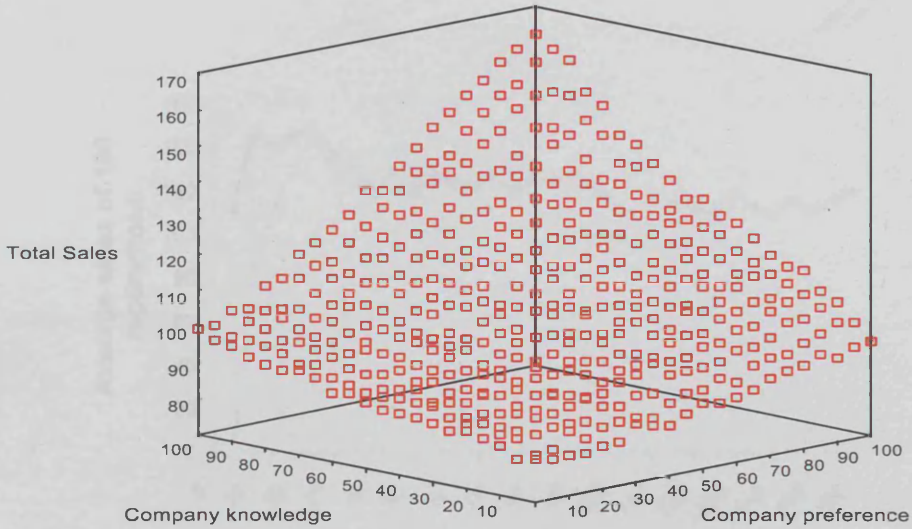

Figure 7.13: Scatter of sales, company knowledge and company preference

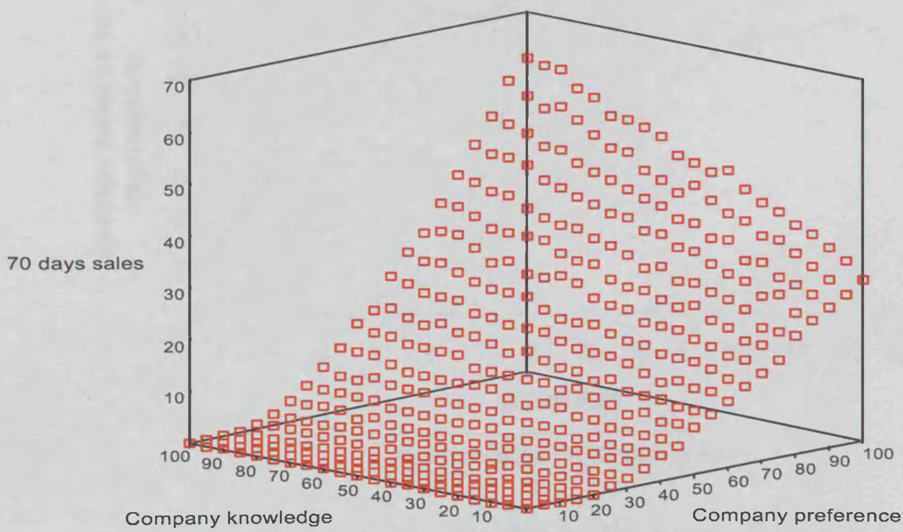

Figure 7.14: Scatter of sales, company knowledge and company preference

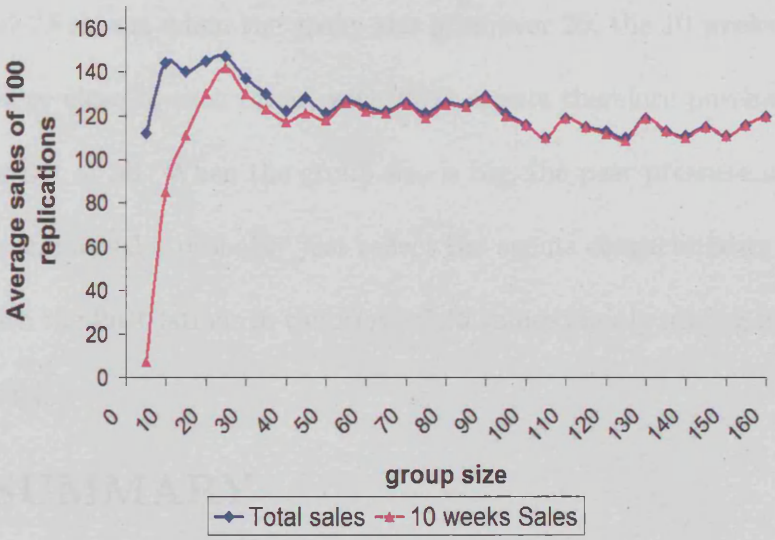

Figure 7.15: Sensitivity analysis on the social group size

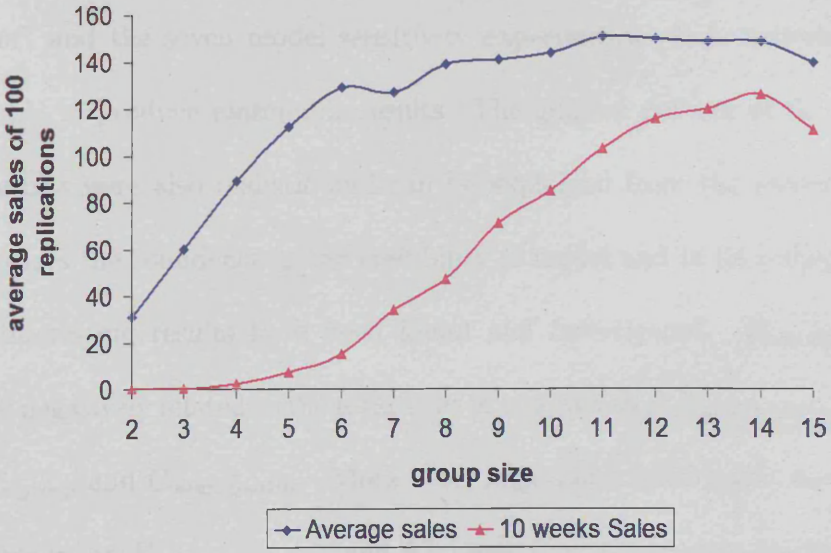

Figure 7.16: Sensitivity analysis on the social group size

probability for an agent to talk to each friend within the group is fixed, therefore, if the group is large, the agents will have more conversations each day which causes their knowledge to be built up quicker and consequently, the agent will purchase earlier. As *Figure7.15* shows, when the group size goes over 25, the 10 weeks sales and total sales are very close to each other, with most agents therefore purchasing in the first 70 days or not at all. When the group size is big, the peer pressure in the group will be weaker and so sales probably just reflect the agents characteristics (their U and B values) with the fluctuations in the *Figure7.15* values mainly arising from randomness in the model.

7.3 SUMMARY

This chapter described the behaviour observed from the model set up as the " real system" and the seven model sensitivity experiments. It is believed that the model is able to produce reasonable results. The general pattern of the sensitivity analysis results were also realistic and can be explained from the model structure, which increases the confidence in the credibility of model and in its coding.

Some interesting results have been found and investigated. $P_{lose\_knowledge}$ and B_{mean} were negatively related to the total sales in contrast to $P_{\frac{company\_marketing\_infromation}{outside\_information}}$, $P_{talk\_to\_same\_group}$ and $U_{mean\_positive}$. More over, high value sales limits were found in the experiments on $P_{talk\_to\_same\_group}$ and K_{out} and P_{out}. Consumers need a minimum amount of the company's information (i.e. advertising etc.) to build up their knowledge about the new product but the information's effect decreases when consumers get enough knowledge. In other words, the company's marketing information cannot increase consumers' interests in the product boundlessly. Regarding $U_{mean\_positive}$ and B_{mean}, the results are very straightforward, and can be explained by the model structure. The group size experiment implies a cognitive limit in an individuals' social group size. But due to the limit of population (500 in this case), further investigation could be done to reveal this limit.

Chapter 8

MODEL CALIBRATION

CHAPTER OVERVIEW

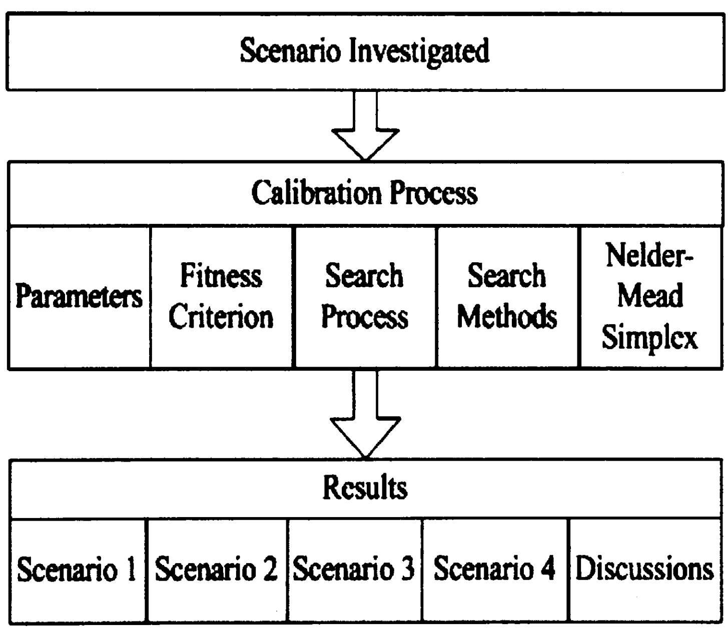

Figure 8.1: Contents of Chapter 8

In this chapter, details and results are set out from implementing the research methodology in *Chapter 5* to investigate the range of predictions from alternative calibrations. In the research, a similar method to Brooks et al. (1994) was chosen to search for parameter sets which produce a good fit with the calibration data. A detailed description of Brooks et al. (1994) paper can be found in *Chapter 3*.

8.1 SCENARIO INVESTIGATED

The artificial scenario devised is that a company launched a new product several weeks ago, and that actual sales to date are known (in this case obtained from the "real system" simulation). The product is a one-off product such as a popular film ticket, a DVD, a CD album or a new computer game, and there was an advertising campaign to promote the product. The company wishes to use the simulation model to predict the total sales that will be achieved over the two years life cycle of the product. The only data available for calibrating the model is the total sales to date. The aim of the calibration process is to find the highest and lowest total sales prediction for the parameters values that give a good fit with the total sales to date.

There are four "experiments" used in the research that differed only by the length of the initial period with the lengths being 70, 105, 140 and 175 days. The total sales to date used for calibration for these four initial periods were 24.4, 63.8, 86.5 and 99.0 respectively.

8.2 CALIBRATION PROCESS

8.2.1 Parameters Used for Calibration

Six parameters were chosen to be varied during the experiment as these were considered to be the most important parameters. All the other parameters were kept at the default values. The six parameters were:

- $U_{mean\_positive}$: The mean for positive unbiased true preference distribution.
- $U_{mean\_negative}$: The mean for negative unbiased true preference distribution.

- $P_{talk\_to\_same\_group}$: The probability for agents to contact other agents in the same social group.
- B_{mean} : The mean for the agent's buying criterion distribution.
- $P_{receive\_information\_MAX}$: The probability of agents receiving outside marketing information.
- $P_{lose\_knowledge}$: The probability for agents to lose knowledge about the product at the end of each simulation day.

In some of the tables in this Chapter, the parameters are abbreviated to U_p, U_n, P_{talk}, B_{mean}, P_{info}, and P_{lose} respectively.

8.2.2 Fitness Criterion

A criterion was set for the model to give a good fit, which required setting a fitness measure and a critical value that defines an acceptable fit. In the research, a single data value is available (total sales to date) and the measure needs to take account of the stochastic nature of the model. The choice of fitness measure is subjective and needs to reflect the desired accuracy of the model. The measure chosen here was the difference between the 95% confidence interval from 100 replications of the model for average sales to date per population and the actual value, with an acceptable fit being that the distance is 0 (i.e. the actual value lies within the interval). Using a large number of replications makes this quite a strict measure since the confidence interval is likely to be quite narrow.

A fitness function *Fitness*() (see *Equation 8.1*) was defined to implement this which takes the value of 0 if the true value is in the interval and 10 + the absolute difference between the true value and the interval if it is not in the interval. The 10 is an arbitrary value so that the function has a step between values inside and outside the confidence interval.

$$\boldsymbol{Fitness} = \begin{cases} 0 & if \quad \lambda \in 95\% Confidence\ Interval \\ \lambda - CI_{upperlimit} + 10 & if \quad \lambda > CI_{upperlimit} \\ CI_{lowerlimit} - \lambda + 10 & if \quad CI_{lowerlimit} > \lambda \end{cases} \tag{8.1}$$

Note: Confidence Interval: $[\overline{x} - t\ (\frac{\alpha}{2}\ ,\ \text{n}{-}1)\cdot\frac{S}{\sqrt{n}},\ \overline{x} + t\ (\frac{\alpha}{2},\ \text{n}{-}1)\cdot\frac{S}{\sqrt{n}}\ ]$ *where* α = 0.05 *(95 % confidence level),* $\overline{x}$ *is the sample mean, s is the sample standard deviation and n is the sample size. Sample here refers to the values collected from the model replications.*

8.2.3 Search Process

The search process consists of searching for parameter values that meet the fitness criterion and give the highest or lowest predictions for total sales. The same model structure as for the real system was used. In this respect the pseudo-modelling approach is removing an extra source of uncertainty compared to a real modelling situation in which the model is a simplification of the real system and may contain many assumptions and simplifications. This has the advantage that the range of predictions must be entirely due to the calibration process rather than due to differences in the structure of the real system and the model.

The search has to try and find the extreme values across the whole parameter space. However, there is no method that guarantees finding a global optimum for a complicated function.

In general, an extremum (maximum or minimum point) can be either local (the highest or lowest in a finite boundary) or global (the highest or lowest function value in the entire parameter space) as shown in *Figure* 8.13.

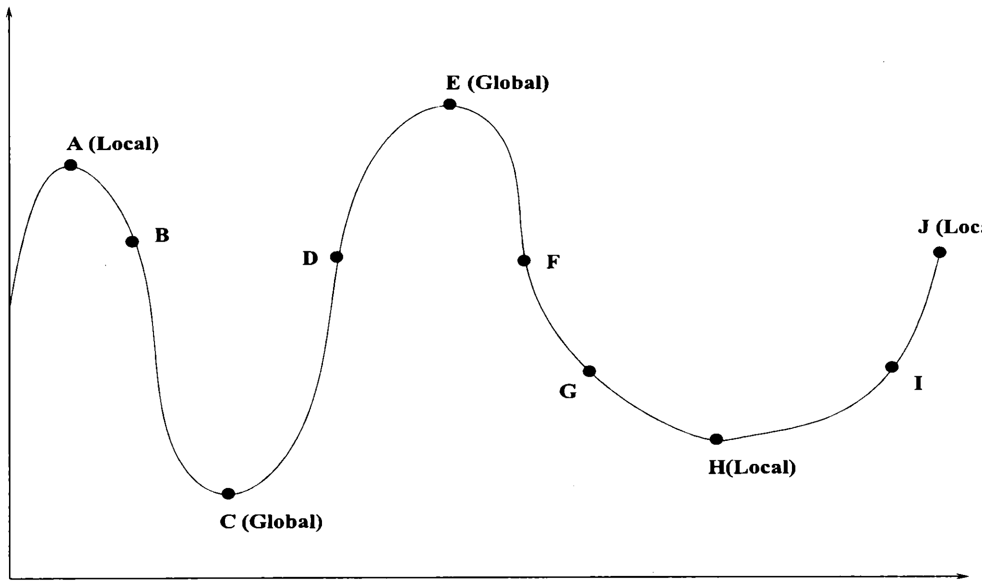

Figure 8.2: Extrema of a function in an interval (based on Press et al., 1992)

For instance, in this function interval, there are different extrema points: A, C, E, H, J. Points A and J are local maxima but not the global highest point, since E is the global highest point in this interval. In the same way, H is a local minimum but not the global lowest point in the interval. C is the global lowest point in this interval. Therefore, finding a global extremum is a very difficult task. Two standard heuristics are used widely (Press et al., 1992; Polak, 1971):

1. Search from different starting values (e.g. points B, D, F, G, I) and then choose the most extreme value of the searches.

2. Examine a local extremum by taking a finite amplitude step away from it, and check out if the objective function returns a better point.

The first of these heuristics was used in this research, with the starting values chosen from a grid of points and the Nelder-Mead simplex method used to search from selected starting values.

8.2.4 Search Method

The 5-step method shown in *Figure 8.3* was used to obtain a prediction range. These steps are explained in more detail in the following sections.

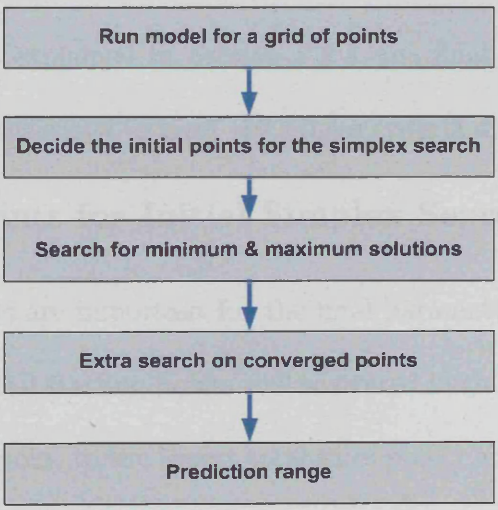

Figure 8.3: Model calibration process

8.2.5 Grid of Points

The purpose of initially running the model for a grid of points was to cover the whole parameter space so as to provide a wide range of possible starting points for the simplex searches. Three values (*Table 8.1*) were used for each of the six parameters, giving 729 ($= 3^6$) points in total. 10 replications were done for each point. The

Table 8.1: Grid parameter values

	Case 1	Case 2	Case 3
$U_{mean\_positive}$	60	75	90
$U_{mean\_negative}$	60	75	90
$P_{talk\_to\_same\_group}$	5	10	15
B_{mean}	50	65	80
$P_{receive\_information\_MAX}$	15%	25%	35%
$P_{lose\_knowledge}$	0.5%	1.0%	1.5%

average sales for the initial period and the average total sales were calculated for these 10 replications. The fitness of each point was also calculated using the fitness criterion function in *Section 8.2.2* (using the confidence interval calculated from the 10 replications). The grid points only used 10 replications to reduce the run time required. However, as explained in *Section 8.2.8*, the final points from the search procedure used 100 replications to meet the fitness criteria defined in *Section 8.2.2.*

8.2.6 Initial Points for Initial Simplex Search

As the initial points are important for the final parameter values, 9 points were chosen to make the initial starting points well scattered in the parameter space (from the highest total sales point to the lowest total sales point) and close to the potential fitted area (i.e. by choosing the three highest total sales with fitness 0 and the three lowest total sales with fitness 0). The following points from the 729 grid points were used as the starting points for the simplex searches:

- The highest total sales (H)
- The lowest total sales (L)
- The three highest total sales with fitness 0 ($H1, H2, H3$)

- The three lowest total sales with fitness 0 ($L1, L2, L3$)

- The default parameter values (D)

These nine sets of parameter values were then used to generate the initial simplexes for searching. The points for the initial simplex were constructed by adding 1 to each parameter value in turn and details can be found in *Appendix C*.

8.2.7 Nelder-Mead Downhill Simplex (Nelder and Mead, 1965)

Nelder-Mead algorithm is widely used to find the local minimum in a nonlinear problem. It was firstly proposed by Nelder and Mead (1965). The code used in the program to implement this algorithm was obtained from Numerical Recipes (Press et al., 1992). The basis of the algorithm is to search using a simplex in a multi-dimension space.

8.2.8 Nelder-Mead Searches

Two Nelder-Mead simplex searches were run for each of the nine initial points (see *Section 8.2.6*) to find the highest and lowest total sales with fitness value 0. The optimisation function used was:

$$\mathit{Function(F,\ S)} = \begin{cases} e^F + \mathrm{S} & \text{(to minimize sales)} \\ e^F - \mathrm{S} & \text{(to maximize sales)} \end{cases} \qquad (8.2)$$

where S is the total sales and F is the fitness measure.

The simplex searches only used 10 replications so as to reduce the run time required (which was still considerable even on a high performance cluster). Due to the stochasticity in the model, the Nelder-Mead searches did not meet the convergence criterion for stopping the program. In the program, 1000 was set as the maximum number of iterations (i.e. different points). Most of the time, the search ended up with a series of close parameter values which gave good fits and very similar total sales, and so the final parameters were treated as the search result. For each of the minimum sales case and the maximum sales case, the best value from the nine searches was identified. These points were then run using 100 replications to give the overall result and the final range of predictions. As already discussed, the search method cannot guarantee to find the global optimum and so there may be points that give higher and lower total sales with fitness 0. However, the range obtained is a lower bound for the true range.

In addition, parameters searching ranges were set as shown in *Table 8.2* based on the assumed feasible region for the model (according to the nature of the original model). If the search parameters go beyond the range, the model was programmed to return a fairly large number for the search function value to make the searching simplex move back to the feasible region.

Table 8.2: Summary of the parameter searching range

Parameters	**Range**
U_p	$0 - 100$
U_n	$0 - 100$
P_{talk}	$0 - 40$
B_{mean}	$0 - 100$
P_{info}	$0 - 60$
P_{lose}	$0 - 10\%$

8.3 'EXPERIMENT' 1 (INITIAL PERIOD: 70 DAYS

The total sales for the first 70 days (10 weeks) for the real system (i.e. the average of 1000 replications) was 24.35 and so this is the value used in the fitness function for

Experiment 1.

8.3.1 Grid of Points

The results of the grid of points was drawn in contour maps using the program ***Surfer 8.0*** which uses a "*Radial Basis Function*" as its interpolation method. Since it is impossible to draw a 7 dimensional graph, the 6 parameter values were transformed into 2 new values by *Equation 8.3*.

$$\begin{aligned} NewValue1 &= 90\% U_{mean\_positive} + 10\% U_{mean\_negative} - B_{mean} \\ NewValue2 &= 30\% P_{talk\_to\_same\_group} + 30\% P_{receive\_information\_MAX} - 40\% P_{lose\_knowledge} \end{aligned} \tag{8.3}$$

$NewValue1$ represents the difference between U value and B value and the weights $(90\%, 10\%)$ were set based on the percentage of agents having positive/negative U values in the whole population. $NewValue2$ represents the difference between the probability of gaining information and the probability of losing knowledge (in terms of the value, $P_{lose\_knowledge}$ is relatively smaller than the other two probabilities, it therefore was given more weight to emphasis the impact of losing knowledge on the total sales). With only three values for each parameter (and only 729 values in total), the grid points are quite well separated and as just explained, the 6 parameters are transformed into two new parameters. Therefore, the contour maps can only give an approximately indication of the response surfaces.

Figure 8.4 shows total sales plotted for the two new values and *Figure 8.5* shows the fitness values (using the first 70 days as the initial period) plotted for the two new values. The red lines in the map are the contour lines on which the fitness is 0. The contour map only gives an approximate indication of the pattern but I also have some additional confidence in the pattern from the examination of the grid data. The grid data contains dispersed points with zero fitness matching the pattern in the contour maps of scattered areas of good fit.

The same grid of points was used for all four experiments. The total sales are unchanged by the different experiments and therefore the sales contour map *Figure 8.4* applies to all. However, the fitness values are different because the comparison is with the total sales for the initial period.

As shown in histogram of the sales values with zero fitness in *Figure 8.6*, there were 40 points with zero fitness having sales ranging from 59 to 372. The sales between 60-80 appear more frequently than the sales in other ranges. There is only one sale in range 360-380. Some of these points had quite different parameter values, as represented by the widely scattered red lines in *Figure 8.5*. Comparing *Figure 8.5* with *Figure 8.4*, zero fitness contour lines correspond to sales of about 60, 120, 250 and 360. As we would expect, with a small amount of available data (70 days out of 720 days), the prediction range just from the grid points is wide. Furthermore, *Figure 8.4* also indicates more than one local optimum.

8.3.2 Initial Search Points

Table 8.3 shows the search starting points for the initial period of 70 days. The parameter values are generally as expected. Taking the H values as an example (the parameter values that gave the highest total sales), this point has the highest of values for $U_{mean\_positive}$ and $P_{receive\_information\_MAX}$, and the lowest values for the other four parameters. These are all as expected for high sales apart from $P_{talk\_to\_same\_group}$.

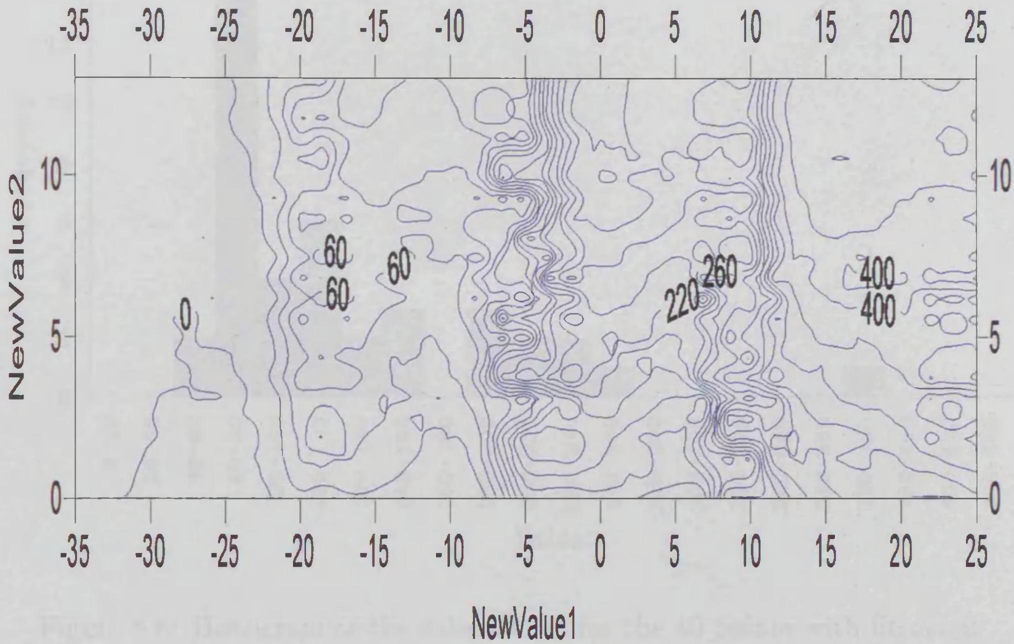

Figure 8.4: Contour map of the total sales

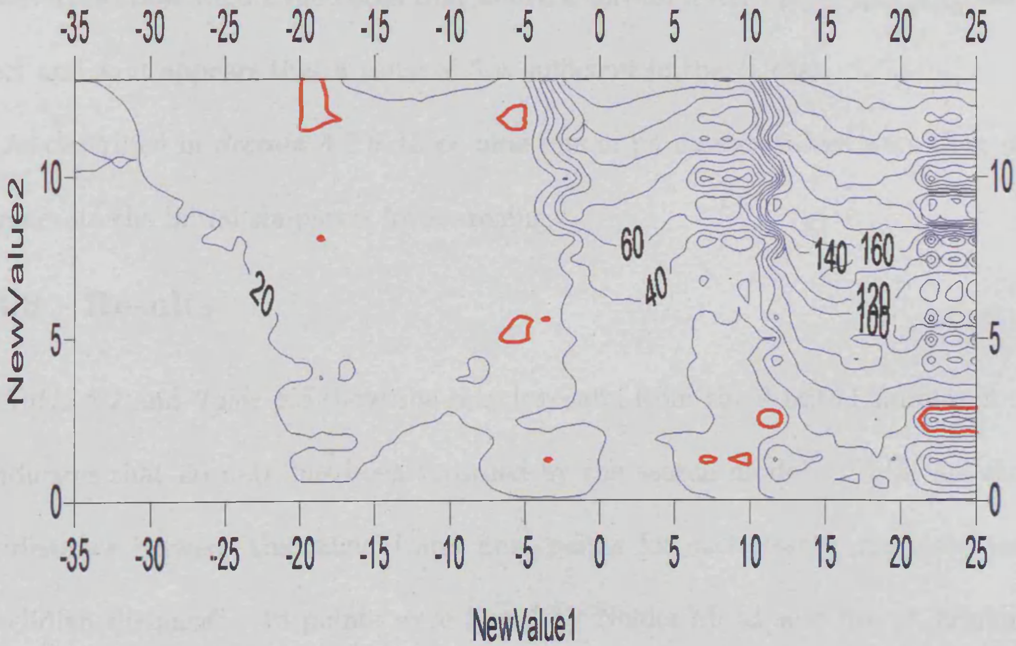

Figure 8.5: Contour map of the fitness (initial period: 70 days)

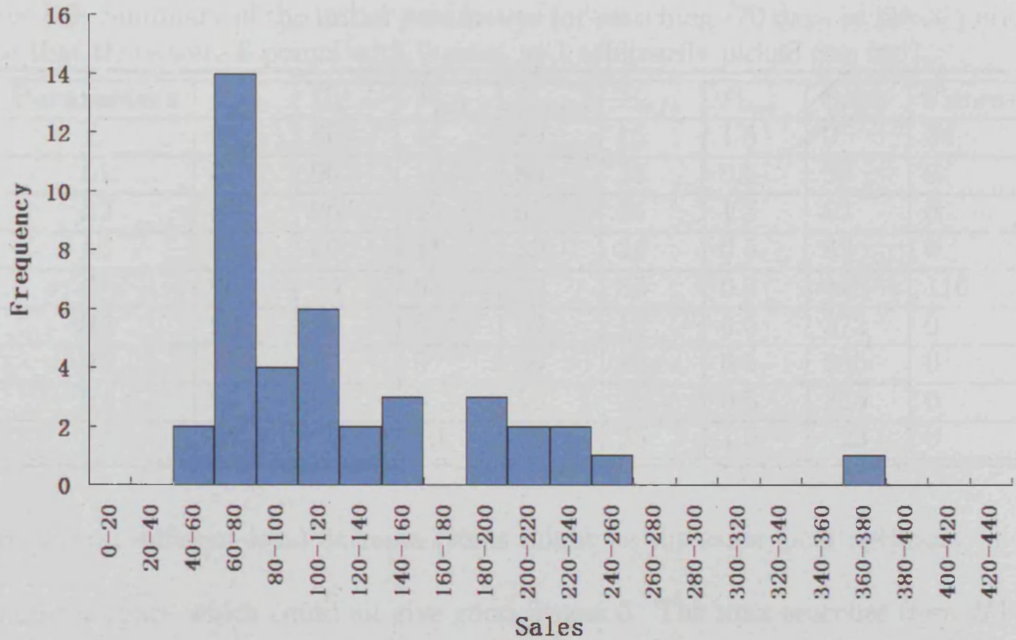

Figure 8.6: Histogram of the sales values for the 40 points with fitness 0

However, *Section 7.2.3.2* indicated that above a certain level, $P_{talk\_to\_same\_group}$ has no effect and so it appears that a value of 5 is sufficient in the H case.

As described in *Section 8.2.6*, these nine sets of parameter values were then used to generate the initial simplexes for searching.

8.3.3 Results

Table 8.4 and *Table 8.5* show the search results from the 9 initial simplexes. An * indicates that no data has been returned by the search model[1]. *Table 8.6* shows the distance between the original and final points for each search measured using "Euclidian distance". 16 points were found by Nelder-Mead searches (8 minimum and 8 maximum) as listed in *Tables 8.4 and Table 8.5.* This indicates that there

[1]Since the search model will stop after 1000 runs, if the search model can not manage to go into a zero fitness region after 1000 runs, the model stops and no data will be collected for that search. For instance, if the model constantly returned out-range parameters for the testing model.

Table 8.3: Summary of the initial parameters for searching (70 days as initial period), note that there were 6 points with 0 sales, so I arbitrarily picked one for L.

Parameters	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales	Fitness
L	60	60	5	80	15	1.5	0	24
$L1$	75	90	15	80	35	0.5	59	0
$L2$	60	90	15	65	25	1.5	62	0
$L3$	75	60	15	80	35	0.5	62	0
H	90	60	5	50	35	0.5	449	116
$H1$	90	90	5	50	15	0.5	372	0
$H2$	75	90	5	50	15	0.5	255	0
$H3$	75	90	5	50	25	0.5	235	0
D	75	75	10	65	25	1.0	123	0

were several different local extrema (some might be the same local optimum) in the parameter space which could all give good fitness 0. The max searches from $H1$-$H3$ and the min researches from $L1$-$L3$ tended not move far away from the original starting points. This could be because, as indicated by the contour map, the fitting regions are small and widely scattered. The searches for simplex H and L moved a relatively large distance. Since the initial fitness values for them (116 and 24 respectively) were high, it is to be expected that they have to move more distance from the original points in order to reach a fitting region.

In some cases the sales value at the end of the search in *Table 8.4* and *Table 8.5* is slightly worse than the starting point in *Table 8.3*. The reason is that different random numbers were used for the replications in the grid points and the search. Therefore, the sales value will be slightly different for the starting point of the search and in some cases the fitness value may not be 0.

In the minimum search, Simplex L returned the minimum sales of 59.3, although the sales returned by Simplex $L1$ (60.7) is very close to 59.3. The best point from the

Table 8.4: Search for Max Sales results with 70 days as initial period

Max	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	65.654	51.903	13.680	69.801	30.677	4.261	61.7
$Simplex_{L1}$	66.701	79.061	13.935	70.065	30.594	4.340	71.1
$Simplex_{L2}$	56.597	83.505	14.840	60.764	23.661	13.893	65.3
$Simplex_{L3}$	65.654	51.903	13.680	69.801	30.677	4.261	66.5
$Simplex_H$	66.456	55.443	3.080	37.519	29.932	9.213	236.3
$\mathbf{Simplex_{H1}}$	**91.785**	**90.898**	**5.994**	**50.453**	**15.151**	**5.024**	**371.7**
$Simplex_{H2}$	*	*	*	*	*	*	*
$Simplex_{H3}$	73.685	75.621	5.347	42.462	21.300	4.658	266.3
$Simplex_D$	82.501	81.851	10.991	68.348	28.294	11.453	133.2

Table 8.5: Search for Min Sales results with 70 days as initial period

Min	U_p	U_n	P_{talk}	$.B_{mean}$	P_{info}	P_{lose}	Sales
$\mathbf{Simplex_L}$	**55.777**	**82.407**	**14.525**	**60.131**	**23.439**	**14.240**	**59.3**
$Simplex_{L1}$	76.007	90.008	16.134	77.209	35.018	5.057	60.7
$Simplex_{L2}$	60.951	89.926	15.983	63.939	25.970	5.979	63.4
$Simplex_{L3}$	73.310	57.939	15.233	77.875	34.032	4.968	74.2
$Simplex_H$	*	*	*	*	*	*	*
$Simplex_{H1}$	75.677	89.658	5.912	50.788	25.836	15.883	211.4
$Simplex_{H2}$	78.213	93.757	6.435	52.254	16.662	5.244	261.2
$Simplex_{H3}$	75.092	95.207	6.202	53.457	15.690	5.216	259.4
$Simplex_D$	91.853	90.768	12.282	79.130	30.818	12.586	64.6

Table 8.6: The distance between the initial search points and search results

	L	$L1$	$L2$	$L3$	H	$H1$	$H2$	$H3$	D
Max Search	25.258	17.556	8.646	16.671	27.892	2.282	*	16.710	11.329
Min Search	32.821	3.177	9.237	3.551	*	21.038	5.865	11.280	27.894

maximum sales searches was Simplex $H1$.

The points for Simplex L and $H1$ were then run with 100 replications to give the final result. This is shown in *Table 8.7* which includes the 95% C.I. for the predictions of total sales. The extreme values of the confidence intervals are used for the prediction range. The results give a very wide prediction range for total sales of between 58.092 and 376.348. Examination of the parameter values may give an insight into the reason for these extreme values. The key aspect will be the relatively

Table 8.7: Prediction range with 70 days as initial period

	Min S	**Max** S
$U_{mean\_positive}$	55.777	91.785
$U_{mean\_negative}$	82.407	90.898
$P_{talk\_to\_same\_group}$	14.525%	5.994%
B_{mean}	60.131	50.453
$P_{receive\_information\_MAX}$	23.439%	15.151%
$P_{lose\_knowledge}$	1.424%	0.502%
First 70 days sales (95% confidence level)	[22.899, 24.840]	[22.727, 25.232]
Mean of the first 70 days sales	23.870	23.980
Prediction (95% confidence level)	[58.092, 60.987]	[366.091, 376.348]
Mean of prediction	59.540	371.220
Calibration benchmark	24.35	

different effects on the initial and total sales. For example, the probability of receiving outside information has more effect on sales during the initial period than on total sales based on the analysis in *Section 7.2.4*. Some of the other parameters will have a more similar effect on initial and total sales. The possible reason for the results could be that the highest sales case (Simplex $H1$) has a high mean U value (91.785) relative to B_{mean} (50.453) tending to produce high overall sales, whereas the low value for receiving information (i.e. P_{info} = 15.151) reduces sales in the initial period so that the model still fits. Low U (55.777) and a high probability of receiving outside information (i.e. P_{info} = 23.439) will then have the opposite effect in the lowest sales case (Simplex L).

Next, I repeated the above procedure on the other three experiments (with initial period of 105, 140, 175 days), and compared the results.

8.4 EXPERIMENT 2 (INITIAL PERIOD: 105 DAYS)

The total sales for the first 105 days (15 weeks) for the real system (i.e. the average of 1000 replications) was 63.79 and so this is the value used in the fitness function for Experiment 2.

8.4.1 Grid of Points

As mentioned in the previous section, the same grid of points was used here but compared to 105 days as the initial period. As shown in *Figure 8.7*, there are 35 points with zero fitness (decrease from 40 points in Experiment 1) with sales ranging from 79 to 323. Among these 35 points, there are 19 points that also had zero fitness for Experiment 1. Most new points have sales in the range of 80-100 and the range of 260-340. Comparing with *Figure 8.6*, the distribution of sales is concentrated on a narrower range. The most frequent sales category moved slightly to the right in the range of 80-100 and there are more sales evenly distributed in the range of 260-340. The histogram corresponds to the contour map of fitness for an initial period of 105 days in *Figure 8.8*. The zero fitness lines are approximately in the sales regions of 120, 260, and 100. As we would expect, when the available information increases, it is harder to find parameter values that fit.

8.4.2 Initial Search Points

Table 8.8 shows the initial starting points for an initial period of 105 days. Similar to Experiment 1, the parameter values are generally as expected, although $L1 - L3$ and $H1 - H3$ all differ to *Table 8.3*. Taking $L1 - L3$ as examples, comparing with

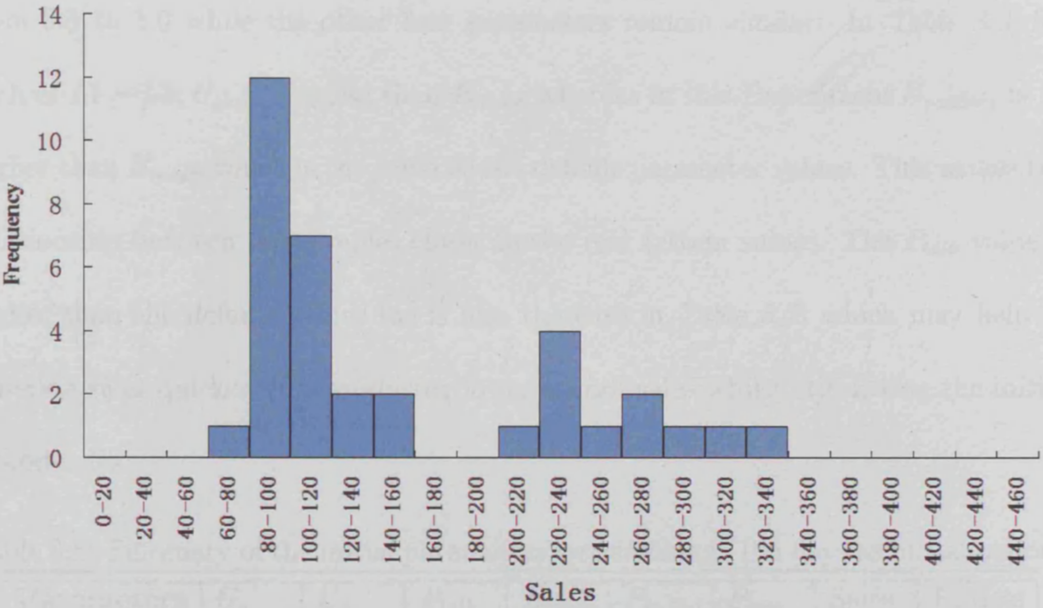

Figure 8.7: Histogram of the sales values for the 35 points with fitness 0

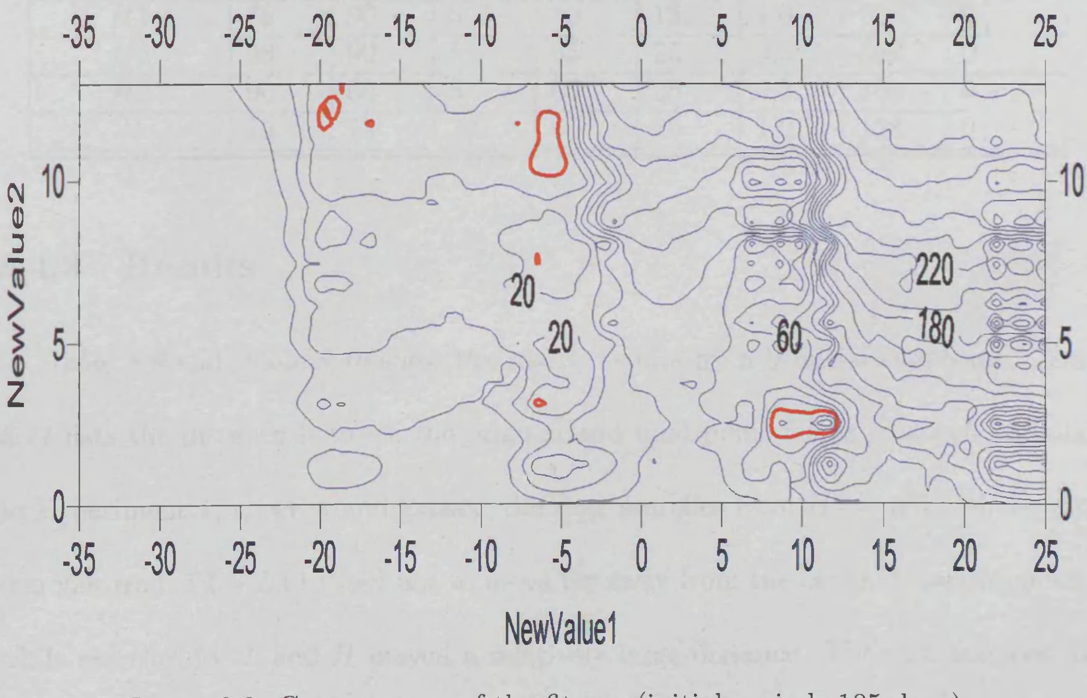

Figure 8.8: Contour map of the fitness (initial period: 105 days)

Table 8.3, all $U_{positive}$ values increased to the highest value (90) and all P_{lose} increased from 0.5 to 1.0 while the other four parameters remain similar. In *Table 8.3*, for each of $L1 - L3$, $U_{positive}$ is less than B_{mean} whereas in this Experiment $U_{positive}$ is 10 higher than B_{mean} which is the same as the default parameter values. This makes the relationship between these values closer to the real system values. The P_{talk} value is higher than the default values (as is also the case in *Table 8.3*) which may help to generate sales quicker thus producing lower overall sales whilst still fitting the initial period sales.

Table 8.8: Summary of the initial parameters for searching (105 days as initial period)

Parameters	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales	Fitness
L	60	60	5	80	15	1.5	0	64
$L1$	90	60	15	80	25	1.0	79	0
$L2$	90	75	15	80	25	1.0	81	0
$L3$	90	60	15	80	35	1.0	83	0
H	90	60	5	50	35	0.5	449	221
$H1$	75	90	5	50	15	1.0	323	0
$H2$	90	90	5	65	25	0.5	302	0
$H3$	90	60	5	65	25	0.5	300	0
D	75	75	10	65	25	1.0	123	0

8.4.3 Results

Table 8.9 and *Table 8.10* show the search results from 9 initial simplexes. *Table 8.11* lists the distance between the original and final point for each search. Similar to Experiment 1, as we would expect, the max searches from $H1 - H3$ and the min searches from $L1 - L3$ tended not to move far away from the original starting points while searches for L and H moved a relatively large distance. The min searches for $H2$ and $H3$, whilst not producing the best point, managed to find zero fitness points with much lower sales than initial points.

In the minimum search, Simplex $L1$ returned the minimum sales of 79.9, although the sales returned by Simplex $L2$ (80.2) and $L3$ (81.1) were both close to it. In Experiment 1, simplex L returned the best point and the Simplex $L1$ point here differs considerably in all the parameters apart from P_{talk} and P_{info}. In Experiment 1, the best point $U_{positive}$ is about 4 lower than B_{mean} whereas here it is about 11 higher. Simplex $H1$ returned the best point of 317.3 in the maximum sales searches. Compared to Simplex $H1$ in Experiment 1, which also returned the maximum sales, these two points are very similar except that the U has been decreased to 75.960 (91.785) and consequently the total sales has been decreased to 317.3 (371.7) due to the effect of the U value.

Table 8.12 shows the final results for the predictions of total sales based on Simplex $L1$ and $H1$ after running 100 replications. Similar to Experiment 1, the results give a very wide prediction range for total sales (using the outer values of the 95% C.I.) of between 78.764 and 318.623, although slightly narrower than Experiment 1.

8.5 EXPERIMENT 3 (INITIAL PERIOD: 140 DAYS)

The total sales for the first 140 days (20 weeks) for the real system (i.e. the average of 1000 replications) was 86.518 and so this is the value used in the fitness function for Experiment 3.

Table 8.9: Search for Max Sales results with 105 days as initial period

Max	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	68.792	81.174	6.231	46.398	18.703	9.517	249.7
$Simplex_{L1}$	*	*	*	*	*	*	*
$Simplex_{L2}$	82.457	54.700	13.993	73.160	32.042	9.158	103.4
$Simplex_{L3}$	83.654	89.577	12.675	74.5506	31.526	10.888	106.2
$Simplex_H$	*	*	*	*	*	*	*
$\mathbf{Simplex_{H1}}$	**75.960**	**89.955**	**5.996**	**50.442**	**15.024**	**5.028**	**317.3**
$Simplex_{H2}$	90.017	88.336	6.788	65.774	26.126	5.194	303.2
$Simplex_{H3}$	80.963	53.365	5.313	58.557	22.860	4.973	300.4
$Simplex_D$	78.026	77.167	10.955	67.582	26.404	11.364	116.9

Table 8.10: Search for Min Sales results with 105 days as initial period

Min	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	*	*	*	*	*	*	*
$\mathbf{Simplex_{L1}}$	**88.589**	**58.455**	**15.168**	**77.926**	**24.301**	**9.807**	**79.9**
$Simplex_{L2}$	89.078	58.747	15.624	78.495	34.415	9.916	80.2
$Simplex_{L3}$	87.583	57.681	15.387	77.652	24.650	10.054	81.1
$Simplex_H$	*	*	*	*	*	*	*
$Simplex_{H1}$	*	*	*	*	*	*	*
$Simplex_{H2}$	82.123	53.257	14.103	64.687	24.325	10.332	192.5
$Simplex_{H3}$	83.014	76.792	14.254	63.524	23.212	9.427	188.3
$Simplex_D$	82.277	54.583	13.958	73.756	31.972	9.138	99.9

Table 8.11: The distance between the initial search points and search results

	L	$L1$	$L2$	$L3$	H	$H1$	$H2$	$H3$	D
Max Search	41.231	*	23.814	30.380	*	7.798	1.463	1.730	5.026
Min Search	*	3.039	18.876	11.136	*	*	39.038	21.009	24.728

Table 8.12: Prediction range with 105 days as initial period

	Min S	**Max** S
$U_{mean\_positive}$	88.589	75.960
$U_{mean\_negative}$	58.455	89.955
$P_{talk\_to\_same\_group}$	15.168%	5.995%
B_{mean}	77.926	50.442
$P_{receive\_information\_MAX}$	24.301%	15.024%
$P_{lose\_knowledge}$	0.981%	0.503%
First 105 days sales (95% confidence level)	[58.821, 63.818]	[62.530, 68.051]
Mean of the first 105 days sales	61.320	65.290
Prediction (95% confidence level)	[78.764, 83.155]	[303.737, 318.623]
Mean of prediction	80.960	311.18
Calibration benchmark	63.79	

8.5.1 Grid of Points

The same grid of points as in previous section were used here again but compared to 140 days as the initial period. As shown in *Figure 8.9*, the number of zero fitness points reduced from 35 to 27 with the sales ranging from 89 to 267. Among these 27 sales values, there are 13 points with zero fitness remaining the same as in *Figure 8.6* and 15 points with zero fitness remaining the same as in *Figure 8.7*. Most new points have sales in the range of 100-120 and the range of 160-220. Comparing with *Figure 8.6* and *Figure 8.7*, the distribution of sales is concentrated on a narrower range with the most frequent sales moving slightly further to the right in the range of 100-120. The histogram corresponds to the contour map of fitness for an initial period of 140 days in *Figure 8.10*. Similar to *Figure 8.8*, the most zero fitness lines were approximately in the sales of 120, 260 and 100. There are still several distinct regions of zero fitness.

8.5.2 Initial Search Points

Table 8.13 shows the initial starting points for an initial period of 140 days. Similar to Experiment 1 and Experiment 2, the parameter values are generally as expected, although $L1 - L3$ and $H1 - H3$ all differ to those in *Table 8.3* and *Table 8.8*. Taking $L1 - L3$ as examples, whilst the $U_{positive}$ and B_{mean} values are lower than the previous two experiments, the difference here ($U_{positive}$ being 5 lower than B_{mean}) are the same as for Experiment 1 and quite different to Experiment 2.

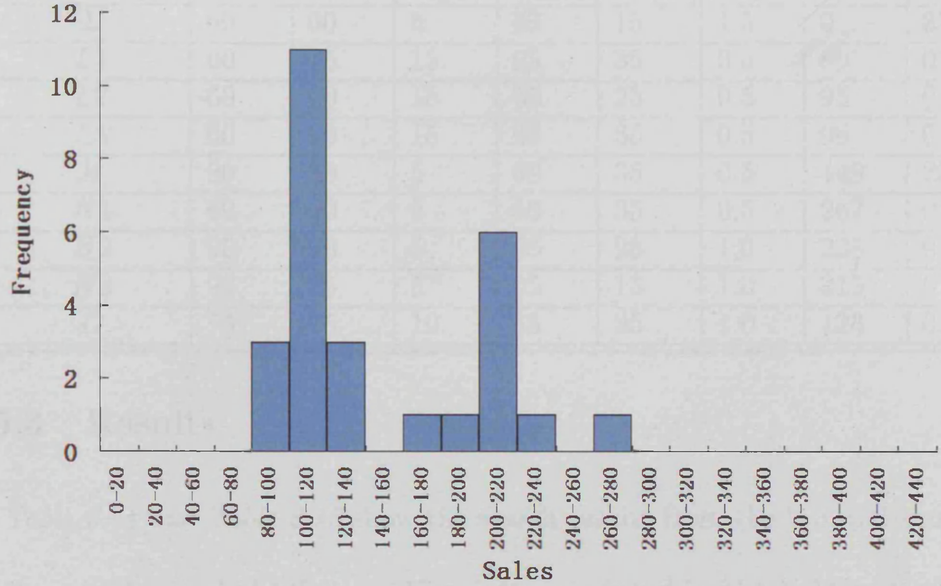

Figure 8.9: Histogram of the sales values for the 27 points with fitness 0

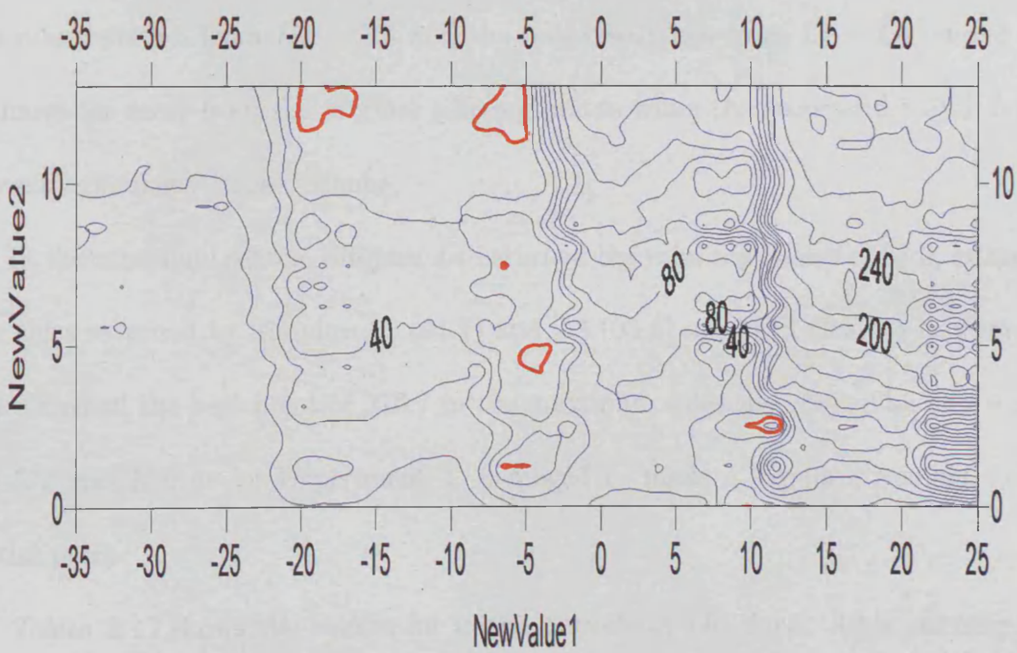

Figure 8.10: Contour map of the fitness (initial period: 140 days)

Table 8.13: Summary of the initial parameters for searching (140 days as initial period)

Parameters	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales	Fitness
L	60	60	5	80	15	1.5	0	87
$L1$	60	75	15	65	35	0.5	89	0
$L2$	60	60	15	65	25	0.5	95	0
$L3$	60	90	15	65	35	0.5	96	0
H	90	60	5	50	35	0.5	449	257
$H1$	60	90	5	50	35	0.5	267	0
$H2$	90	90	5	65	25	1.0	228	0
$H3$	90	75	5	65	15	1.0	215	0
D	75	75	10	65	25	1.0	123	0

8.5.3 Results

Table 8.14 and *Table 8.15* show the search results from the 9 initial simplexes as for the initial period of 140 days. 15 points were found by the Nelder-Mead searches while 3 searches did not return any results (L for both min and max searches and H for the max search). Similar to Experiment 1 and Experiment 2, as we would expect, the max searches from $H1 - H3$ and the min researches from $L1 - L3$ tended not to move far away from the original starting points while the successful search for H moved a relatively large distance.

In the minimum search, Simplex $L1$ returned the minimum sales of 88.6, although the sales returned by Simplex $L2$ (91.7) and $L3$ (93.6) are both close to it. Simplex $H1$ returned the best point of 276.7 in the maximum sales searches. The min search for $H2$ and $H3$, as for Experiment 2, managed to make a big improvement to the initial point.

Tables 8.17 shows the results for initial periods of 140 days. After running 100 replications for the best points, the width of prediction range reduced by 56.533

Table 8.14: Search for Max Sales results with 140 days as initial period

Max	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	*	*	*	*	*	*	*
$Simplex_{L1}$	60.9953	69.9324	14.8888	61.1272	33.4255	4.9181	108.6
$Simplex_{L2}$	57.2231	52.5186	13.3689	57.3234	22.1709	4.4118	118.1
$Simplex_{L3}$	64.3927	89.5965	15.1846	64.2369	34.9799	5.0395	105.7
$Simplex_H$	62.739	41.849	5.645	34.742	24.129	3.619	251.4
$\mathbf{Simplex_{H1}}$	**67.553**	**77.662**	**5.381**	**57.626**	**34.642**	**6.050**	**276.7**
$Simplex_{H2}$	88.096	97.915	5.442	70.685	29.664	10.720	231.5
$Simplex_{H3}$	87.013	80.464	5.453	69.722	18.896	10.836	226.7
$Simplex_D$	73.659	73.973	16.543	68.650	26.719	14.187	125.8

Table 8.15: Search for Min Sales results with 140 days as initial period

Min	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	*	*	*	*	*	*	*
$\mathbf{Simplex_{L1}}$	**65.0487**	**75.0385**	**16.003**	**66.0304**	**35.0414**	**5.0039**	**88.6**
$Simplex_{L2}$	61.7837	58.1012	14.399	62.8682	24.1672	4.8451	91.7
$Simplex_{L3}$	62.3013	87.739	14.574	63.2126	34.0592	4.8693	93.6
$Simplex_H$	*	*	*	*	*	*	*
$Simplex_{H1}$	42.641	63.496	3.583	35.077	31.306	4.020	240.7
$Simplex_{H2}$	88.239	98.037	5.480	70.808	14.735	12.762	121.4
$Simplex_{H3}$	86.499	81.692	5.545	70.779	18.849	13.012	124.6
$Simplex_D$	78.204	79.207	11.599	69.436	27.146	10.555	111.5

Table 8.16: The distance between the initial search points and search results

	L	$L1$	$L2$	$L3$	H	$H1$	$H2$	$H3$	D
Max Search	*	11.382	5.145	19.295	37.761	17.245	11.003	8.784	8.915
Min Search	*	3.891	11.970	6.845	*	30.802	14.651	10.706	7.424

Table 8.17: Prediction range with 140 days as initial period

	Min S	**Max** S
$U_{mean\_positive}$	65.049	67.553
$U_{mean\_negative}$	75.039	87.662
$P_{talk\_to\_same\_group}$	16.003%	5.381%
B_{mean}	66.030	57.626
$P_{receive\_information\_MAX}$	35.041%	34.642%
$P_{lose\_knowledge}$	0.500%	0.605%
First 140 days sales (95% confidence level)	[82.785, 86.835]	[79.364, 87.195]
Mean of the first 140 days sales	84.810	83.280
Prediction (95% confidence level)	[90.756, 95.084]	[260.877, 276.682]
Mean of prediction	92.920	268.780
Calibration benchmark	86.518	

to 183.325, which is smaller than the previous change between Experiment 1 and Experiment 2.

8.6 EXPERIMENT 4 (INITIAL PERIOD: 175 DAYS)

The total sales for the first 175 days (25 weeks) for the real system (i.e. the average of 1000 replications) was 99.0 and so this is the value used in the fitness function for Experiment 4.

8.6.1 Grid of Points

The same grid of points as in the previous Experiments was used here again but compared to 175 days as the initial period. As shown in *Figure 8.11*, the number of zero fitness points reduced from 27 to 23 with the sales ranging from 103 to 188. Among these 23 sales values, there are 15 points with zero fitness remaining the same as in *Figure 8.9*, 10 points with zero fitness remaining the same as in *Figure 8.7* and 10 points with zero fitness remaining the same as in *Figure 8.6*. Most new points emerge in the sales range of 120-140. Comparing with *Figure 8.6*, *Figure 8.7* and

Figure 8.9, the distribution of sales is concentrated on a narrower range with the most sales in the range of 100-140. The histogram corresponds to the contour map of fitness for an initial period of 175 days in *Figure 8.12*. Most zero fitness lines are approximately in the sales of 120 and 100. Whilst there are still several distinct zero fitness regions, the zero fitness regions have become smaller. As we would expect, when the available information increases (in other words, the calibration requirement becomes more strict), it is harder to find parameter values that fit.

8.6.2 Initial Search Points

Table 8.18 shows the initial starting points for an initial period of 175 days. Similar to the other three Experiments, the parameter values are generally as expected. $L1 - L3$ and $H1 - H3$ all differ to *Table 8.3*, *Table 8.8* and *Table 8.13*, although, $L1 - L3$ and $H1 - H3$ are close to *Table 8.8*. Taking $L1 - L3$ as examples, comparing with *Table 8.8*, all parameters are similar except that P_{lose} has decreased from 1.0 to 0.5. Based on the analysis in Chapter 7, P_{lose} (probability of losing knowledge) has a negative effect on the total sales. Therefore, as a result, a decreased P_{lose} will lead to the increased total sales, although it appears that here the interactions with the other parameters enables the model to fit the initial period whilst giving low overall sales.

8.6.3 Results

Table 8.19 and *Table 8.20* show the search results from the 9 initial simplexes for an initial period of 175 days. 14 points were found by the Nelder-Mead searches while 4 searches did not return any results (both max and min searches from L and H).

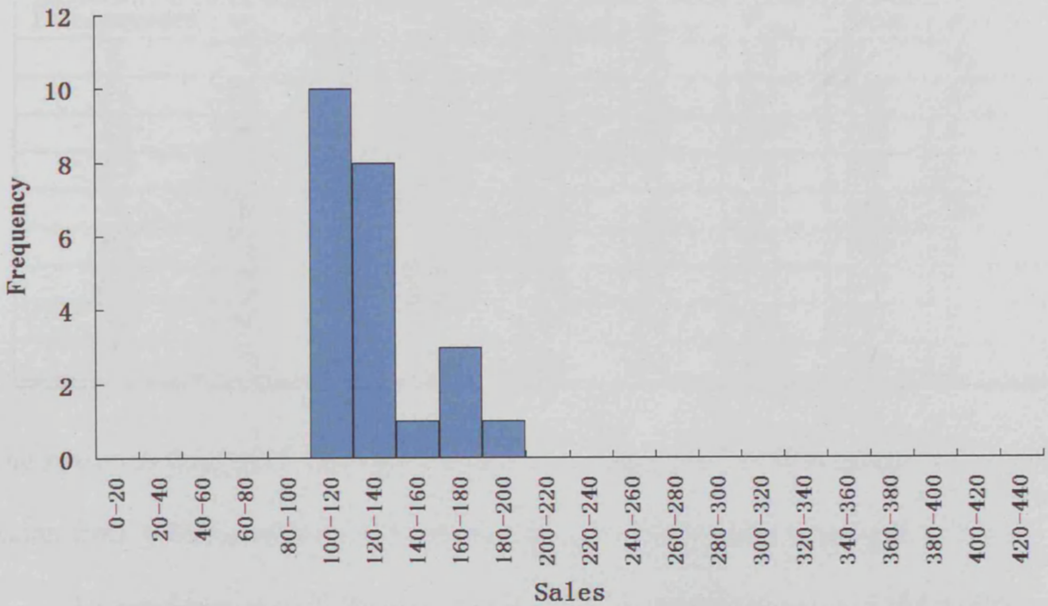

Figure 8.11: Histogram of the sales values for the 23 points with fitness 0

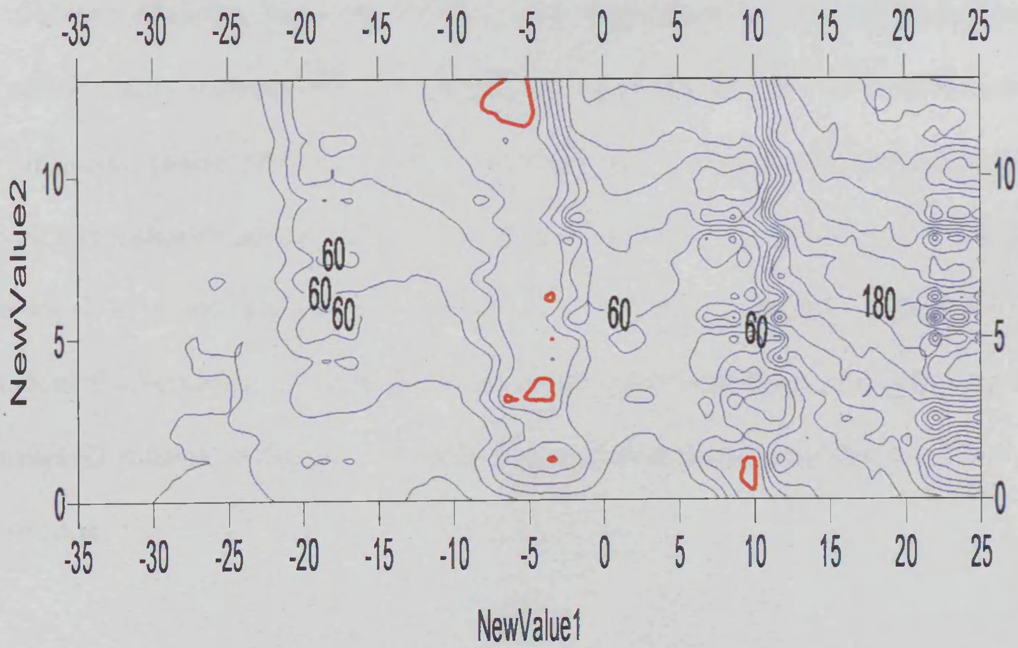

Figure 8.12: Contour map of the fitness (initial period: 175 days)

Table 8.18: Summary of the initial parameters for searching (175 days as initial period)

Parameters	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales	Fitness
L	60	60	5	80	15	1.5	0	99
$L1$	90	75	15	80	25	0.5	103	0
$L2$	90	60	15	80	35	0.5	106	0
$L3$	90	60	15	80	25	0.5	106	0
H	90	60	5	50	35	0.5	449	287
$H1$	75	75	5	65	35	0.5	188	0
$H2$	75	60	5	50	25	1.5	173	0
$H3$	60	75	5	50	25	1.0	171	0
D	75	75	10	65	25	1.0	123	0

The reason is that when the feasible region becomes smaller, it is harder to reach the region from a bad starting point (in this case, starting points with high fitness).

In the minimum search, Simplex D returned the minimum sales of 111.9, although the sales returned by Simplex $L1$ (109.5) was close to it. And Simplex $H1$ returned the best point of 189.5 in the maximum sales searches.

Tables 8.22 shows the results for the predictions of the total sales based on Simplex D and $H1$ after running 100 replications. As we would expect, the prediction range for an initial period of 175 days is considerably narrower than the prediction range which has a shorter initial period (70, 105 and 140 days). Comparing with 140 days, the width of prediction range was reduced by 105.048 to 78.277. Different to the previous Experiments, the minimum prediction range were found by searching from simplex D instead of Simplex $L1$, although the total sales from Simplex $L1$ is very close to it.

Table 8.19: Search for Max Sales results with 175 days as initial period

Max	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	*	*	*	*	*	*	*
$Simplex_{L1}$	83.434	67.540	13.980	76.439	29.434	4.829	136.9
$Simplex_{L2}$	85.134	54.870	14.462	73.821	32.284	4.762	133.2
$Simplex_{L3}$	84.463	56.545	14.222	74.865	28.067	4.359	128.6
$Simplex_H$	*	*	*	*	*	*	*
Simplex$_{\mathbf{H1}}$	**77.899**	**74.914**	**5.994**	**65.234**	**15.406**	**5.165**	**189.5**
$Simplex_{H2}$	85.340	58.708	6.203	67.587	15.368	5.052	181.2
$Simplex_{H3}$	86.419	75.942	4.276	50.486	15.476	15.189	178.4
$Simplex_D$	75.419	74.662	10.708	63.257	25.244	10.311	128.9

Table 8.20: Search for Min Sales results with 175 days as initial period

Min	U_p	U_n	P_{talk}	B_{mean}	P_{info}	P_{lose}	Sales
$Simplex_L$	*	*	*	*	*	*	*
$Simplex_{L1}$	87.608	72.161	14.485	74.156	31.468	6.769	109.5
$Simplex_{L2}$	81.984	54.106	14.272	72.806	33.853	7.699	118.6
$Simplex_{L3}$	84.898	73.914	15.465	73.674	25.006	7.786	114.4
$Simplex_H$	*	*	*	*	*	*	*
$Simplex_{H1}$	87.830	75.574	5.447	67.543	15.034	5.322	176.4
$Simplex_{H2}$	86.667	63.750	6.333	69.562	15.979	5.333	178.3
$Simplex_{H3}$	74.855	70.450	5.532	47.846	15.077	4.146	154.3
Simplex$_{\mathbf{D}}$	**71.809**	**70.937**	**10.318**	**61.339**	**23.591**	**9.401**	**111.9**

Table 8.21: The distance between the initial search points and search results

	L	$L1$	$L2$	$L3$	H	$H1$	$H2$	$H3$	D
Max Search	*	11.497	9.793	8.910	*	19.834	24.720	28.587	1.996
Min Search	*	9.652	12.644	16.359	*	23.882	26.636	19.469	6.522

Table 8.22: Prediction range with 175 days as initial period

	Min S	**Max** S
$U_{mean\_positive}$	71.80	77.899
$U_{mean\_negative}$	70.937	74.914
$P_{talk\_to\_same\_group}$	10.318%	5.994%
B_{mean}	61.3397	65.234
$P_{receive\_information\_MAX}$	23.591%	15.406%
$P_{lose\_knowledge}$	0.940%	0.516%
First 175 days sales (95% confidence level)	[95.409, 99.591]	[98.811,103.468]
Mean of the first 175 days sales	97.500	101.140
Prediction (95% confidence level)	[109.199,113.820]	[180.443,187.476]
Mean of prediction	111.510	183.960
Calibration benchmark	99.039	

8.7 DISCUSSIONS

Figure 8.13 is a graph of the range of predictions for the four different initial periods. The actual sales from the "real system" model was 123.5, and the actual sales for the four initial periods were 24.4, 63.8, 86.5 and 99.0 respectively. The prediction range is extremely wide for an initial period of 70 days and would provide very little useful information. As would be expected, the prediction range narrows as more information is obtained from a larger initial period. However, even for a period of 175 days, when 80% of the total actual sales have, in fact, been made, the range is still wide having a value of 78.

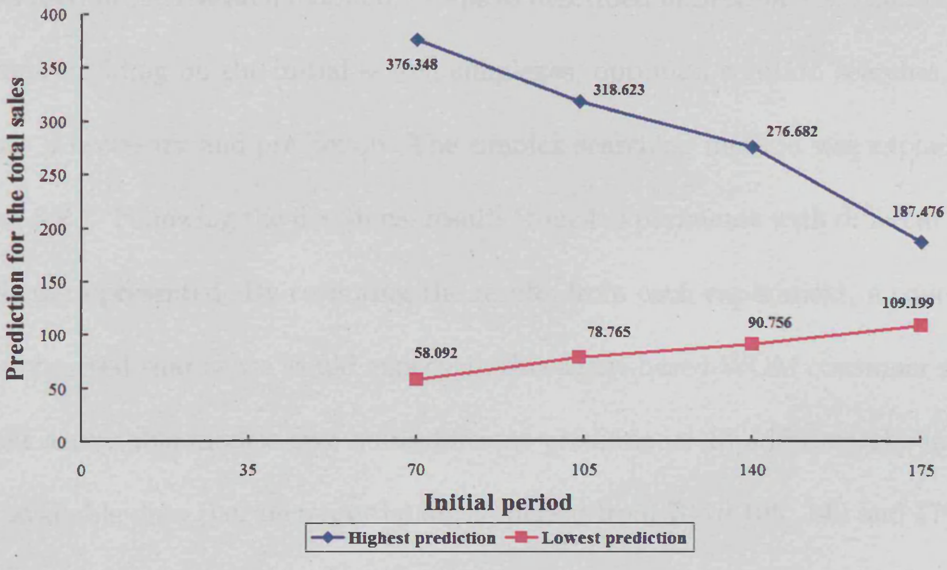

Figure 8.13: Prediction Range Vs Information available.

The circumstances of these experiments were that the model had a perfect structure but the parameter values were unknown and so could vary over a wide range. In this situation, it appears that it will often not be possible to make a precise prediction and therefore the usefulness of using such a model for prediction may be limited. Further work is needed to investigate other scenarios. In particular, the fitness function only used a single point. Fitting against several points (using sales at several points during the initial period) would be likely to produce a narrower prediction range, although it is harder to define what constitutes an acceptable fit in these circumstances.

8.8 SUMMARY

This chapter has described the calibration process used in the research. The chapter started with a brief introduction to the scenario investigated in *Section 8.1*. Then, *Section 8.2* gave an overview of the entire calibration process. The calibration process used in the research followed 5 steps as described in *Section 8.2.3*, namely grid of points, deciding on the initial search simplexes, optimum solution searches, extra searches if necessary and prediction. The simplex searching method was explained in *Section 8.2.7*. Following the five steps, results from 4 experiments with different initial periods were presented. By reviewing the results from each experiment, a conclusion can be observed that as we would expect, in this agent-based WOM consumer model, different acceptable models give quite different predictions. In addition, the increase of the available data (i.e. increase the initial period from 70 to 105, 140 and 175 days accordingly in this research) would narrow down the prediction region.

Chapter 9

CONCLUSIONS AND FUTURE RESEARCH

CHAPTER OVERVIEW

This chapter summaries and discusses the results of the experiments, how these meet the research objectives, limitations of the method used and the future research areas.

9.1 SUMMARY OF THE THESIS

This thesis reviewed the current literature on ABS, inverse problem, model calibration, and marketing simulation and described the development of an agent-based consumer word-of-mouth model and the implementation of a method for determining a range of prediction from alternative calibrations.

9.1.1 Main Arguments

Agent-based simulation has attracted much interest lately, but an agreement on the definition of an agent has not yet been achieved. The simplest viewpoint is that an agent is an entity for which some cognitive process is modelled (Edmonds and

Mohring, 2005). To some extent, ABS is a new simulation approach and with the benefit of much-increased computing power, it enables new types of simulations to be investigated. So far, it has been widely applied in many areas such as military, economics, sociology and movement patterns.

However, much of the ABS work has had the aim of increasing the understanding of the type of system rather than trying to reproduce a specific situation. Such an approach can be very valuable in producing important new insights and improving understanding. Simulation models in general are constructed by modelling local behaviour and then connecting the different parts together and allowing them to interact. Therefore any simulation model can provide useful information about the relationship between local structure and global behaviour, which can increase understanding. However, relating this to a particular real system implies that the model structure is a good representation of the important parts of the real system. If this is not the case then the implications drawn may be incorrect. It is therefore important to assess the validity of the model, although in the absence of a specific real system, validation can only consist of a subjective assessment of the plausibility of the model structure and of the responses (white box validation (Pidd, 2004)). For example, one of the early pieces of work was the boids simulation (Reynolds, 1999b), which tried to find rules for general "boid" agents to produce flocking behaviour that appeared realistic compared to the flocks, herds and schools of different animals in the real world. Some of the social science simulations are highly simplified models of virtual societies, such as the Sugarscape model (Epstein and Axtell, 1996). Criticisms of these sorts of models in some quarters have been that they are too divorced from reality to provide useful information about the real world and may reflect the prejudices of the model builder (see, for example, Lansing (2002) for a discussion of this debate).

9.1.1.1 Prediction, Model Calibration and the Inverse Problem

For some applications, using agent-based simulation for prediction (rather than just better understanding) could be very powerful. For example, a company might wish to use a model of the population of their customers with WOM interactions to predict the sales of the product or the effect of an advertising campaign. However, the problem is that agent-based models typically have a very large number of parameters and many of these cannot be measured directly or estimated with sufficient precision. The only other information available may be historical output data from the real system. Such data can be used to calibrate the model by finding parameter values that produce a good fit with the data. This is known as an inverse problem since it consists of using the outputs to determine the inputs. The problem is that there will usually be many solutions. There are two main reasons for this. The first is that there are often many parameters and few historical data values. The second is that any model that produces a good fit should be considered acceptable. A perfect fit is not expected because any simulation is a simplification of the real system and also there may be measurement errors in the historical data. The result is that a wide range of sets of parameter values may give an acceptable fit and are therefore feasible values. However, they may give quite different predictions.

9.1.1.2 Approach Used in This Research

This research investigated the calibration problem for an agent-based simulation, which gave an indication of the limitations of using such models for prediction.

The approach used was to develop an agent-based model and to treat this model as the real system. Output data from this model was then taken as measured values from the "real" world and, in a pseudo-modelling exercise, used to calibrate an agent-based model of the system. A method similar to that of Brooks et al. (1994) was then used to investigate the variations in predictions. The advantage of such a pseudo-modelling exercise is that the "real system" was completely known. Consequently, the models' predictions could be compared with the "true" future values, and the precise differences between the models and the real system was also known.

9.2 RESEARCH OBJECTIVES AND CONTRIBUTION TO THE FIELD

For each of the research objectives in Chapter 1, this section discusses the to which they were met and the contribution made.

9.2.1 Main Objectives

Objective: to develop and implement a method based on previous research for obtaining an acceptable range of predictions from the alternative acceptable calibrations.

This research has set out and implemented a method of finding a prediction range for ABS models. The research adapted the Brooks et al. (1994) method used in groundwater modelling and applied it to a word of mouth consumer model. Compared to the deterministic groundwater models, an additional problem for agent-based simulations is stochasticity. This is because heterogeneous populations are being modelled and information for each individual in the real population will not usually be available. Instead the model represents a typical population and multiple replications are required to take account of the variations across possible populations. Therefore the Brooks et al. (1994) method had to be adopted for a stochastic model, since a good fit with the historical data requires comparing the measured values against the range of values from multiple replications. The fitness measure used compared the real system value with the confidence interval from multiple replications, with a good fit defined as the real value lying within the interval. The predictions were also produced using multiple replications.

Objective: to compare the range of predictions for different scenarios of the data available for calibration.

A range of prediction was calculated for four scenarios of different initial periods of data collection. As would be expected, the larger the initial period the narrower the range of predictions. However the work quantified the range of predictions and found that the range was wide for all four scenarios.

9.2.2 Secondary Objectives

Objective: to develop a new agent-based WOM consumer model.

This research devised a simple structure for a WOM consumer model. Few models of this situation have been developed and the structure used was different to previous models. The structure is therefore a potential new modelling approach for this type of situation which could also provide an underlying theory or which further work could be based. The ability to model consumer word of mouth interacting effectively could have important benefits for business.

Objective: to investigate the relationships between the parameters and the model output.

The research investigated the effect of different parameters and how the structure works by a series of sensitivity analysis Chapter 7. It gave some interesting founding for marketing research.

Objective: to assess whether the model produces realistic output.

Based on the experiments conducted the model appears to produce realistic output and plausible behaviour. This provides some support for the structure of the model being a good approach.

9.3 FUTURE RESEARCH

In order to investigate the problem of using agent-based simulation for prediction further, the following work could be conducted:

1. **Different ways of measuring fitness:** In the current model, total sales of 10/15/20/25 weeks were used when measuring the fitness. However, other ways could be use. For instance, a portfolio of values (e.g. sales for each week) could be used to measure the fitness.

2. **Alternative model structure**: the model used in this research has the same

structure as the "real system". However, in future, different model structures other than the known one could be tried to check if a different result will be produced.

3. **Production type:** In the current model, the product type has been set out as a one-off purchase product. In future research, repeat purchase behaviour could be added into the model to investigate other product types.

4. **Alternative business decisions:** Different scenarios of business decisions could be tested apart from the scenario introduced in the thesis in order to help companies' decision making. For instance, a scenario that a company is considering putting more effort/money into the advertising campaign and wants to know the effects. This could be achieved by changing the influence level of outside source information in the model and changing the length of the advertising campaign.

Another area of future research could be to investigate the nature of word of mouth interaction as a further test of the model structure developed in this research. For example, it may be possible to conduct experiments investigating the transfer of knowledge and preference between subjects in conversations about a product.

9.4 FINAL CONCLUSIONS

Using agent-based simulation for prediction (rather than just better understanding) could be very powerful for some applications. However, the problem is that agent-based models typically have a very large number of parameters and many of these cannot be measured directly or estimated with sufficient precision. The only information available may be historical output data from the real system. Such data can be used to calibrate the model by finding parameter values that produce a good fit with the data. Moreover, any model that produces a good fit should be considered to be acceptable and different acceptable models may give quite different predictions as demonstrated in our research. A method which takes account of the different feasible parameter values (such as the approach described here) needs to be used in making predictions. The nature of agent-based models may therefore limit their usefulness for prediction except for situations in which the data for the parameters can be measured directly and accurately. The approach set out in this research could help to resolve such problem.

Appendix A

A Brief Introduction to ABS Packages

The description for each ABS package was directly from package's website.

A.1 List of agent-based simulation packages: Open Source

Name	Reference	Developer(s)
ABLE	http://www.alphaworks.ibm.com/tech/able	IBM
Description		
ABLE is a Java framework, component library, and productivity tool kit for building intelligent agents using machine learning and reasoning.		
Name	**Reference**	**Developer(s)**
Cougaar	http://www.cougaar.org/	DARPA
Description		
Cougaar is a Java-based architecture for the construction of large-scale distributed agent-based applications.		
Name	**Reference**	**Developer(s)**

Continued...

Ecolab	http://parallel.hpc.unsw.edu.au/ecolab	Russell Standish
Description		
Ecolab is a fairly complete agent-based simulation system. The model is implemented as a C++ object. Support for more advanced data structures and algorithms are available through the standard library in C++.		
Name	**Reference**	**Developer(s)**
JADE	http://sharon.cselt.it/projects/jade/	Telecom Italia Lab
Description		
JADE (Java Agent DEvelopment framework) is a software framework fully implemented in the Java language.		
Name	**Reference**	**Developer(s)**
JAS	http://jaslibrary.sourceforge.net/	Michele Sonnessa
Description		
JAS is a Java toolkit for creating agent-based simulations. It features a discrete-event time engine, APIs for network simulation design, and powerful yet easy-to-use implementations of Genetic Algorithms, Neural Networks and Classifier Systems.		
Name	**Reference**	**Developer(s)**
MASON	http://cs.gmu.edu/	George Mason University
Description		
MASON is a fast discrete-event multi-agent simulation library core in Java, designed to be the foundation for large custom-purpose Java simulations, and also to provide more than enough functionality for many lightweight simulation needs. MASON contains both a model library and an optional suite of visualization tools in 2D and 3D.		
Name	**Reference**	**Developer(s)**
Repast	http://repast.sourceforge.net/	Chicago University
Description		

Continued...

Repast is a software framework for creating agent-based simulations using the Java language. It provides a library of classes for creating, running, displaying and collecting data from an agent-based simulation. In addition, Repast can take snapshots of running simulations, and create QuickTime movies of simulations. Repast borrows much from the Swarm simulation toolkit and can properly be termed "Swarm-like".		
Name	**Reference**	**Developer(s)**
SimPy	http://simpy.sourceforge.net/	SimPy developer team
Description		
SimPy (Simulation in Python) is an object-oriented, process-based discrete-event simulation language based on standard Python and released under the GNU GPL. It provides the modeller with components of a simulation model including processes, for active components such as customers, messages, and vehicles, and resources for passive components that form limited-capacity congestion points such as servers, checkout counters, and tunnels. It also provides monitor variables to aid in gathering statistics. Random variants are provided by the standard Python random module.		
Swarm	http://wiki.Swarm.org	Swarm Development Group
Description		
Swarm is a software package for the multi-agent simulation of complex systems and was originally developed at the Santa Fe Institute. The basic architecture of Swarm is the simulation of collections of concurrently interacting agents: with this architecture, a large variety of agent-based models can be implemented.		
Name	**Reference**	**Developer(s)**
ZEUS	http://more.btexact.com/	BT. ISR agent research
Description		
The ZEUS toolkit provides a library of software components and tools that facilitate the rapid design, development, and deployment of agent systems.		

A.2 List of agent-based simulation packages: Freeware

Name	Reference	Developer(s)
Ascape	http://www.brook.edu/	The Brookings Institution
Description		
Ascape is a software framework for developing and analyzing agent-based models. In Ascape, agent objects exist within scapes; collections of agents such as arrays and lattices. These scapes are themselves agents, so that typical Ascape models are made up of "collections of collections" of agents.		
Name	**Reference**	**Developer(s)**
NetLogo	http://ccl.northwestern.edu/	Northwestern University
Description		
NetLogo is written in Java and it can therefore run on all major platforms (Mac, Windows, Linux, etc). NetLogo is a programmable modelling environment for simulating natural and social phenomena. It is particularly well-suited for modelling complex systems developing over time. Modellers can give instructions to hundreds or thousands of independent "agents", all operating in parallel.		
Name	**Reference**	**Developer(s)**
StarLogo	http://education.mit.edu/starlogo/	MIT
Description		
StarLogo is a programmable modelling environment for exploring the workings of decentralized systems -- systems that are organized without an organizer, and coordinated without a coordinator. With StarLogo, researchers can model (and gain insights into) many real-life phenomena, such as bird flocks, traffic jams, ant colonies, and market economies.		

A.3 List of agent-based simulation packages: Proprietary

Name	Reference	Developer(s)
AgentSheets	http://www.agentsheets.com/	AgentSheets, Inc.
Description		
AgentSheets features the unique Visual AgentTalk tactile and rule-based language to create, modify, and customize agent behaviour.		
Name	**Reference**	**Developer(s)**
AnyLogic	http://www.xjtek.com/	XJ Technologies
Description		
AnyLogic supports virtually all existing approaches to discrete event and continuous modelling, such as process flow diagrams, system dynamics, agent-based modelling, state charts and equation systems.		

Appendix B

Search Simplex

B.1 Initial Period $= 70$ days

Parameter simplex to give the highest sales

$$\mathbf{Simplex_H} = \begin{pmatrix} 90 & 60 & 5 & 50 & 35 & 5 \\ 91 & 60 & 5 & 50 & 35 & 5 \\ 90 & 61 & 5 & 50 & 35 & 5 \\ 90 & 60 & 6 & 50 & 35 & 5 \\ 90 & 60 & 5 & 51 & 35 & 5 \\ 90 & 60 & 5 & 50 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the highest sales but with fitness 0

$$\mathbf{Simplex_{H1}} = \begin{pmatrix} 90 & 90 & 5 & 50 & 15 & 5 \\ 91 & 90 & 5 & 50 & 15 & 5 \\ 90 & 91 & 5 & 50 & 15 & 5 \\ 90 & 90 & 6 & 50 & 15 & 5 \\ 90 & 90 & 5 & 51 & 15 & 5 \\ 90 & 90 & 5 & 50 & 16 & 6 \end{pmatrix}$$

Parameter simplex to give the second highest sales but with fitness 0

$$\mathbf{Simplex_{H2}} = \begin{pmatrix} 75 & 90 & 5 & 50 & 15 & 5 \\ 76 & 90 & 5 & 50 & 15 & 5 \\ 75 & 91 & 5 & 50 & 15 & 5 \\ 75 & 90 & 6 & 50 & 15 & 5 \\ 75 & 90 & 5 & 51 & 15 & 5 \\ 75 & 90 & 5 & 50 & 16 & 6 \end{pmatrix}$$

Parameter simplex to give the third highest sales but with fitness 0

$$\mathbf{Simplex_{H3}} = \begin{pmatrix} 75 & 90 & 5 & 50 & 25 & 15 \\ 76 & 90 & 5 & 50 & 25 & 15 \\ 75 & 91 & 5 & 50 & 25 & 15 \\ 75 & 90 & 6 & 50 & 25 & 15 \\ 75 & 90 & 5 & 51 & 25 & 15 \\ 75 & 90 & 5 & 50 & 26 & 16 \end{pmatrix}$$

Parameter simplex to give the lowest sales

$$\mathbf{Simplex_{L}} = \begin{pmatrix} 60 & 60 & 5 & 80 & 15 & 15 \\ 61 & 60 & 5 & 80 & 15 & 15 \\ 60 & 61 & 5 & 80 & 15 & 15 \\ 60 & 60 & 6 & 80 & 15 & 15 \\ 60 & 60 & 5 & 81 & 15 & 15 \\ 60 & 60 & 5 & 80 & 16 & 16 \end{pmatrix}$$

Parameter simplex to give the lowest sales but with fitness 0

$$\mathbf{Simplex_{L1}} = \begin{pmatrix} 75 & 90 & 15 & 80 & 35 & 5 \\ 76 & 90 & 15 & 80 & 35 & 5 \\ 75 & 91 & 15 & 80 & 35 & 5 \\ 75 & 90 & 16 & 80 & 35 & 5 \\ 75 & 90 & 15 & 81 & 35 & 5 \\ 75 & 90 & 15 & 80 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the second lowest sales but with fitness 0

$$\mathbf{Simplex_{L2}} = \begin{pmatrix} 60 & 90 & 15 & 65 & 25 & 15 \\ 61 & 90 & 15 & 65 & 25 & 15 \\ 60 & 91 & 15 & 65 & 25 & 15 \\ 60 & 90 & 16 & 65 & 25 & 15 \\ 60 & 90 & 15 & 66 & 25 & 15 \\ 60 & 90 & 15 & 65 & 26 & 16 \end{pmatrix}$$

Parameter simplex to give the third lowest sales but with fitness 0

$$\mathbf{Simplex_{L3}} = \begin{pmatrix} 75 & 60 & 15 & 80 & 35 & 5 \\ 76 & 60 & 15 & 80 & 35 & 5 \\ 75 & 61 & 15 & 80 & 35 & 5 \\ 75 & 60 & 16 & 80 & 35 & 5 \\ 75 & 60 & 15 & 81 & 35 & 5 \\ 75 & 60 & 15 & 80 & 36 & 6 \end{pmatrix}$$

Default parameter simplex

$$\mathbf{Simplex_{Default}} = \begin{pmatrix} 75 & 75 & 10 & 65 & 25 & 10 \\ 76 & 75 & 10 & 65 & 25 & 10 \\ 75 & 76 & 10 & 65 & 25 & 10 \\ 75 & 75 & 11 & 65 & 25 & 10 \\ 75 & 75 & 10 & 66 & 25 & 10 \\ 75 & 75 & 10 & 65 & 26 & 11 \end{pmatrix}$$

B.2 Initial Period = 105 days

Parameter simplex to give the highest sales

$$\mathbf{Simplex_{H}} = \begin{pmatrix} 90 & 60 & 5 & 50 & 35 & 5 \\ 91 & 60 & 5 & 50 & 35 & 5 \\ 90 & 61 & 5 & 50 & 35 & 5 \\ 90 & 60 & 6 & 50 & 35 & 5 \\ 90 & 60 & 5 & 51 & 35 & 5 \\ 90 & 60 & 5 & 50 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the highest sales but with fitness 0

$$\mathbf{Simplex_{H1}} = \begin{pmatrix} 75 & 90 & 5 & 50 & 15 & 10 \\ 76 & 90 & 5 & 50 & 15 & 10 \\ 75 & 91 & 5 & 50 & 15 & 10 \\ 75 & 90 & 6 & 50 & 15 & 10 \\ 75 & 90 & 5 & 51 & 15 & 10 \\ 75 & 90 & 5 & 50 & 16 & 11 \end{pmatrix}$$

Parameter simplex to give the second highest sales but with fitness 0

$$\mathbf{Simplex_{H2}} = \begin{pmatrix} 90 & 90 & 5 & 65 & 25 & 5 \\ 91 & 90 & 5 & 65 & 25 & 5 \\ 90 & 91 & 5 & 65 & 25 & 5 \\ 90 & 90 & 6 & 65 & 25 & 5 \\ 90 & 90 & 5 & 66 & 25 & 5 \\ 90 & 90 & 5 & 65 & 26 & 6 \end{pmatrix}$$

Parameter simplex to give the third highest sales but with fitness 0

$$\mathbf{Simplex_{H3}} = \begin{pmatrix} 90 & 60 & 5 & 65 & 25 & 5 \\ 91 & 60 & 5 & 65 & 25 & 5 \\ 90 & 61 & 5 & 65 & 25 & 5 \\ 90 & 60 & 6 & 65 & 25 & 5 \\ 90 & 60 & 5 & 66 & 25 & 5 \\ 90 & 60 & 5 & 65 & 26 & 6 \end{pmatrix}$$

Parameter simplex to give the lowest sales

$$\mathbf{Simplex_{L}} = \begin{pmatrix} 60 & 60 & 5 & 80 & 15 & 15 \\ 61 & 60 & 5 & 80 & 15 & 15 \\ 60 & 61 & 5 & 80 & 15 & 15 \\ 60 & 60 & 6 & 80 & 15 & 15 \\ 60 & 60 & 5 & 81 & 15 & 15 \\ 60 & 60 & 5 & 80 & 16 & 16 \end{pmatrix}$$

Parameter simplex to give the lowest sales but with fitness 0

$$\mathbf{Simplex_{L1}} = \begin{pmatrix} 90 & 60 & 15 & 80 & 25 & 10 \\ 91 & 60 & 15 & 80 & 25 & 10 \\ 90 & 61 & 15 & 80 & 25 & 10 \\ 90 & 60 & 16 & 80 & 25 & 10 \\ 90 & 60 & 15 & 81 & 25 & 10 \\ 90 & 60 & 15 & 80 & 26 & 11 \end{pmatrix}$$

Parameter simplex to give the second lowest sales but with fitness 0

$$\mathbf{Simplex_{L2}} = \begin{pmatrix} 90 & 75 & 15 & 80 & 25 & 10 \\ 91 & 75 & 15 & 80 & 25 & 10 \\ 90 & 76 & 15 & 80 & 25 & 10 \\ 90 & 75 & 16 & 80 & 25 & 10 \\ 90 & 75 & 15 & 81 & 25 & 10 \\ 90 & 75 & 15 & 80 & 26 & 11 \end{pmatrix}$$

Parameter simplex to give the third lowest sales but with fitness 0

$$\mathbf{Simplex_{L3}} = \begin{pmatrix} 90 & 60 & 15 & 80 & 35 & 10 \\ 91 & 60 & 15 & 80 & 35 & 10 \\ 90 & 61 & 15 & 80 & 35 & 10 \\ 90 & 60 & 16 & 80 & 35 & 10 \\ 90 & 60 & 15 & 81 & 35 & 10 \\ 90 & 60 & 15 & 80 & 36 & 11 \end{pmatrix}$$

Default parameter simplex

$$\mathbf{Simplex_{Default}} = \begin{pmatrix} 75 & 75 & 10 & 65 & 25 & 10 \\ 76 & 75 & 10 & 65 & 25 & 10 \\ 75 & 76 & 10 & 65 & 25 & 10 \\ 75 & 75 & 11 & 65 & 25 & 10 \\ 75 & 75 & 10 & 66 & 25 & 10 \\ 75 & 75 & 10 & 65 & 26 & 11 \end{pmatrix}$$

B.3 Initial Period = 140 days

Parameter simplex to give the highest sales

$$\mathbf{Simplex_H} = \begin{pmatrix} 90 & 60 & 5 & 50 & 35 & 5 \\ 91 & 60 & 5 & 50 & 35 & 5 \\ 90 & 61 & 5 & 50 & 35 & 5 \\ 90 & 60 & 6 & 50 & 35 & 5 \\ 90 & 60 & 5 & 51 & 35 & 5 \\ 90 & 60 & 5 & 50 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the highest sales but with fitness 0

$$\mathbf{Simplex_{H1}} = \begin{pmatrix} 60 & 90 & 5 & 50 & 35 & 5 \\ 61 & 90 & 5 & 50 & 35 & 5 \\ 60 & 91 & 5 & 50 & 35 & 5 \\ 60 & 90 & 6 & 50 & 35 & 5 \\ 60 & 90 & 5 & 51 & 35 & 5 \\ 60 & 90 & 5 & 50 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the second highest sales but with fitness 0

$$\mathbf{Simplex_{H2}} = \begin{pmatrix} 90 & 90 & 5 & 65 & 25 & 10 \\ 91 & 90 & 5 & 65 & 25 & 10 \\ 90 & 91 & 5 & 65 & 25 & 10 \\ 90 & 90 & 6 & 65 & 25 & 10 \\ 90 & 90 & 5 & 66 & 25 & 10 \\ 90 & 90 & 5 & 65 & 26 & 11 \end{pmatrix}$$

Parameter simplex to give the third highest sales but with fitness 0

$$\mathbf{Simplex_{H3}} = \begin{pmatrix} 90 & 75 & 5 & 65 & 15 & 10 \\ 91 & 75 & 5 & 65 & 15 & 10 \\ 90 & 76 & 5 & 65 & 15 & 10 \\ 90 & 75 & 6 & 65 & 15 & 10 \\ 90 & 75 & 5 & 66 & 15 & 10 \\ 90 & 75 & 5 & 65 & 16 & 11 \end{pmatrix}$$

Parameter simplex to give the lowest sales

$$\mathbf{Simplex_L} = \begin{pmatrix} 60 & 60 & 5 & 80 & 15 & 15 \\ 61 & 60 & 5 & 80 & 15 & 15 \\ 60 & 61 & 5 & 80 & 15 & 15 \\ 60 & 60 & 6 & 80 & 15 & 15 \\ 60 & 60 & 5 & 81 & 15 & 15 \\ 60 & 60 & 5 & 80 & 16 & 16 \end{pmatrix}$$

Parameter simplex to give the lowest sales but with fitness 0

$$\mathbf{Simplex_{L1}} = \begin{pmatrix} 60 & 75 & 15 & 65 & 35 & 5 \\ 61 & 75 & 15 & 65 & 35 & 5 \\ 60 & 76 & 15 & 65 & 35 & 5 \\ 60 & 75 & 16 & 65 & 35 & 5 \\ 60 & 75 & 15 & 66 & 35 & 5 \\ 60 & 75 & 15 & 65 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the second lowest sales but with fitness 0

$$\mathbf{Simplex_{L2}} = \begin{pmatrix} 60 & 60 & 15 & 65 & 25 & 5 \\ 61 & 60 & 15 & 65 & 25 & 5 \\ 60 & 61 & 15 & 65 & 25 & 5 \\ 60 & 60 & 16 & 65 & 25 & 5 \\ 60 & 60 & 15 & 66 & 25 & 5 \\ 60 & 60 & 15 & 65 & 26 & 6 \end{pmatrix}$$

Parameter simplex to give the third lowest sales but with fitness 0

$$\mathbf{Simplex_{L3}} = \begin{pmatrix} 60 & 90 & 15 & 65 & 35 & 5 \\ 61 & 90 & 15 & 65 & 35 & 5 \\ 60 & 91 & 15 & 65 & 35 & 5 \\ 60 & 90 & 16 & 65 & 35 & 5 \\ 60 & 90 & 15 & 66 & 35 & 5 \\ 60 & 90 & 15 & 65 & 36 & 6 \end{pmatrix}$$

Default parameter simplex

$$\textbf{Simplex}_{\textbf{Default}} = \begin{pmatrix} 75 & 75 & 10 & 65 & 25 & 10 \\ 76 & 75 & 10 & 65 & 25 & 10 \\ 75 & 76 & 10 & 65 & 25 & 10 \\ 75 & 75 & 11 & 65 & 25 & 10 \\ 75 & 75 & 10 & 66 & 25 & 10 \\ 75 & 75 & 10 & 65 & 26 & 11 \end{pmatrix}$$

B.4 Initial Period = 175 days

Parameter simplex to give the highest sales

$$\textbf{Simplex}_{\textbf{H}} = \begin{pmatrix} 90 & 60 & 5 & 50 & 35 & 5 \\ 91 & 60 & 5 & 50 & 35 & 5 \\ 90 & 61 & 5 & 50 & 35 & 5 \\ 90 & 60 & 6 & 50 & 35 & 5 \\ 90 & 60 & 5 & 51 & 35 & 5 \\ 90 & 60 & 5 & 50 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the highest sales but with fitness 0

$$\textbf{Simplex}_{\textbf{H1}} = \begin{pmatrix} 75 & 75 & 5 & 65 & 35 & 5 \\ 75 & 76 & 5 & 65 & 35 & 5 \\ 75 & 75 & 6 & 65 & 35 & 5 \\ 75 & 75 & 5 & 66 & 35 & 5 \\ 75 & 75 & 5 & 65 & 36 & 5 \\ 76 & 75 & 5 & 65 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the second highest sales but with fitness 0

$$\textbf{Simplex}_{\textbf{H2}} = \begin{pmatrix} 75 & 60 & 5 & 50 & 25 & 15 \\ 75 & 61 & 5 & 50 & 25 & 15 \\ 75 & 60 & 6 & 50 & 25 & 15 \\ 75 & 60 & 5 & 51 & 25 & 15 \\ 75 & 60 & 5 & 50 & 26 & 15 \\ 75 & 60 & 5 & 50 & 25 & 16 \end{pmatrix}$$

Parameter simplex to give the third highest sales but with fitness 0

$$\mathbf{Simplex_{H3}} = \begin{pmatrix} 61 & 75 & 5 & 65 & 25 & 10 \\ 60 & 76 & 5 & 65 & 25 & 10 \\ 60 & 75 & 6 & 65 & 25 & 10 \\ 60 & 75 & 5 & 66 & 25 & 10 \\ 60 & 75 & 5 & 65 & 26 & 10 \\ 60 & 75 & 5 & 65 & 25 & 11 \end{pmatrix}$$

Parameter simplex to give the lowest sales

$$\mathbf{Simplex_{L}} = \begin{pmatrix} 61 & 60 & 5 & 80 & 15 & 15 \\ 60 & 61 & 5 & 80 & 15 & 15 \\ 60 & 60 & 6 & 80 & 15 & 15 \\ 60 & 60 & 5 & 81 & 15 & 15 \\ 60 & 60 & 5 & 80 & 16 & 15 \\ 60 & 60 & 5 & 80 & 15 & 16 \end{pmatrix}$$

Parameter simplex to give the lowest sales but with fitness 0

$$\mathbf{Simplex_{L1}} = \begin{pmatrix} 90 & 75 & 15 & 80 & 25 & 5 \\ 91 & 75 & 15 & 80 & 25 & 5 \\ 91 & 75 & 16 & 80 & 25 & 5 \\ 91 & 75 & 16 & 81 & 25 & 5 \\ 91 & 75 & 16 & 81 & 26 & 5 \\ 91 & 75 & 16 & 81 & 26 & 6 \end{pmatrix}$$

Parameter simplex to give the second lowest sales but with fitness 0

$$\mathbf{Simplex_{L2}} = \begin{pmatrix} 90 & 60 & 15 & 80 & 35 & 5 \\ 91 & 60 & 15 & 80 & 35 & 5 \\ 91 & 60 & 16 & 80 & 35 & 5 \\ 91 & 60 & 16 & 81 & 35 & 5 \\ 91 & 60 & 16 & 81 & 36 & 5 \\ 91 & 60 & 16 & 81 & 36 & 6 \end{pmatrix}$$

Parameter simplex to give the third lowest sales but with fitness 0

$$\mathbf{Simplex_{L3}} = \begin{pmatrix} 90 & 60 & 15 & 80 & 25 & 5 \\ 91 & 60 & 15 & 80 & 25 & 5 \\ 91 & 60 & 16 & 80 & 25 & 5 \\ 91 & 60 & 16 & 81 & 25 & 5 \\ 91 & 60 & 16 & 81 & 26 & 5 \\ 91 & 60 & 16 & 81 & 26 & 6 \end{pmatrix}$$

Default parameter simplex

$$\mathbf{Simplex_{Default}} = \begin{pmatrix} 75 & 75 & 10 & 65 & 25 & 10 \\ 76 & 75 & 10 & 65 & 25 & 10 \\ 75 & 76 & 10 & 65 & 25 & 10 \\ 75 & 75 & 11 & 65 & 25 & 10 \\ 75 & 75 & 10 & 66 & 25 & 10 \\ 75 & 75 & 10 & 65 & 26 & 11 \end{pmatrix}$$

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