Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.
Digital Object Identifier 10.1109/ACCESS.2017.DOI

Performance Analysis of User Ordering Schemes in Cooperative Power-domain Non-Orthogonal Multiple Access Network

JIAN ZHANG[1], JIANHUA GE[1], (Member, IEEE), QIANG NI[2], (Senior Member, IEEE) MIAOWEN WEN[1,3], (Member, IEEE), AND YANGYANG ZHANG[1]
[1]The State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an 710071, China
[2]School of Computing and Communications, Lancaster University, Lancaster LA1 4WA, United Kingdom
[3]School of Electronic and Information Engineering, South China University of Technology, Guangzhou, China

Corresponding author: Jian Zhang (e-mail: zhj_xidian@126.com).

This work was supported in part by the National Basic Research Program of China (973 Program) under Grant 2012CB316100, in part by the 111 Project under Grant B08038, in part by the National Natural Science Foundation of China under Grant 61501347, in part by the the Royal Society project under Grant IEC170324, in part by the EPSRC IAA project under Grant CSA7114, in part by the EPSRC project under Grant EP/K011693/1, and in part by the EU FP7 CROWN project under Grant PIRSES-GA-2013-610524.

ABSTRACT Non-orthogonal multiple access (NOMA) has recently received much attention as a candidate technique for the fifth generation (5G) networks. In this paper, considering both the direct and relay-aid paths, we investigate the performance of a downlink NOMA-based cooperative system, and further analyze two different user ordering schemes. The outage probability, diveristy gain and ergodic rate are studied as three benchmarks to evaluate the system performance. For different user ordering schemes, the exact outage probabilities of users are first solved in closed form. Then, the outage behavior in the high signal-to-noise ratio (SNR) region is discussed to obtain the diversity gain. In addition, closed-form expression of ergodic rate for the strongest user, and upper bounds for the rest users at high SNR are provided. Finally, numerical results verify the accuracy of our analysis and demonstrate that, sorting users based on relay-aided path can provide larger ergodic sum rate in some cases. By contrast, sorting users based on direct path can provide better diversity gain, and the corresponding performance is less sensitive to relay's location.

INDEX TERMS NOMA, user ordering, cooperative networks, performance analysis.

I. INTRODUCTION

NON -orthogonal multiple access (NOMA) is one of the promising techniques to improve spectrum efficiency in 5G cellular communications [1], [2]. The key idea of power-domain NOMA[1] is to serve multiple users simultaneously at the same frequency, same spreading codes but different power levels. By applying the superposition coding at the transmitter and successive interference cancellation (SIC) at the receiver, NOMA is expected to achieve larger connectivity, greater cell-edge throughput, higher spectral efficiency and better fairness than conventional orthogonal multiple access (OMA) [6], [7].

[1]NOMA can be realized in power domain and the other domains [3]-[5]. In this paper, we mainly focus on the power-domain NOMA, and we use "NOMA" to represent "power-domain NOMA" hereafter.

To further enhance the performance, many researchers have been attempting to integrate various mature techniques into NOMA system. In particular, as an effective method to enhance the performance in terms of service coverage and transmission reliability, a few different cooperative scenarios which combine NOMA with relay have been proposed [8]-[12]. The first cooperative scenario mainly considered the cooperation among users. In such scenarios, the stronger user also acts as a role of decode-and-forward (DF) relay to assist the weak user, thus resulting in better fairness and higher diversity gain. Based on this mechanism, Wei *et al.* [9] have proposed a hybrid downlink-and-uplink cooperative NOMA scheme and achieved a better tradeoff between spectral efficiency and signal reception reliability. As a further variant, in [10] and [11], this cooperative mechanism has been extended to cognitive radio systems, where secondary users also serve as relays to improve the performance of both primary and secondary networks.

On the other hand, the downlink cooperative scenario that contains a dedicated relay has been extensively studied [13]-[19]. In this scenario, a dedicated relay is configured to bridge the transmission between source and multiple cell-edge users. In this context, references [13] and [14] studied the performance in amplify-and-forward (AF) relay-aided NOMA systems. By presenting closed-form expressions and simulations, the authors have demonstrated that NOMA can achieve better outage performance and larger ergodic rate than conventional OMA. As a further advance, in [15] and [16], multiple-antenna technique has been applied to this system to obtain larger diversity gain. By contrast, considering the independent but not necessarily identically distributed (i.n.i.d.) fading, the study of [17] has analyzed and compared DF system with AF one. Wan $et\ al.$ have also pointed out that DF protocol significantly outperforms AF one in terms of ergodic sum rate and exhibits better outage performance at low signal-to-noise ratio (SNR). In [18], this cooperative mechanism has been extended to cognitive networks, and the corresponding resource allocation algorithm has been well studied.

However, although many studies have been contributed to analyze cooperative NOMA systems, some key issues may still remain, and one of which is the user ordering. At the transmitter, users are sorted according to their channel conditions, and more power is allocated to the users with worse channel conditions, thus resulting in a better trade-off between the system throughput and user fairness [20], [21]. Also, at the receiver, the benefit of NOMA depends critically on SIC strategy which requires appropriate user ordering [22]. In non-cooperative scenarios, the user ordering scheme is straightforward since there only contains direct path. However, the cases are complicated in cooperative NOMA. In such scenarios, the signal sent by the source arrives at the destination through diverse paths: one directly from source node and the other through relay node. Performance will, thus, highly depend on which path determines the user ordering. If properly designed, the superiority of both key NOMA components (superposition coding and SIC), can be guaranteed. However, to the best of our knowledge, very few works have concentrated on the analysis of user ordering issue in cooperative NOMA. And this is the gap which this paper aspires to fill.

In this paper, the performance of two major user ordering schemes in cooperative NOMA is analyzed. Moreover, compared with our previous works [13]-[17], to reflect a more realistic scenario, we first consider that both the direct and relay-aided paths are available. Then, to evaluate the system performance more comprehensively, we further adopt maximum-ratio-combining (MRC) criterion instead of selection combining (SC) at user side. The main contributions of this paper are summarised as follows:

1) A downlink cooperative NOMA network is considered in this paper. Considering the impacts of both direct and relay-aided paths, the performance of two major user ordering schemes is analyzed and compared.
2) Outage performance is first analyzed as a criterion to evaluate the two different user ordering schemes. Closed-form expressions for users' outage probability are derived. Then, by investigating the asymptotic behavior under high-SNR assumption, we further obtain the corresponding diversity gain.
3) The ergodic rate is analyzed as an alternative benchmark to evaluate the two user ordering schemes. Since the sum rate of such systems highly depends on the strongest user[2] at high SNR [13]-[17], we thus obtain the exact closed-form expression of ergodic rate for the strongest user. For the rest users, we also obtain the corresponding upper bounds at high SNR.
4) Comprehensive simulations are provided to evaluate the two user ordering schemes and our analyses. In addition, the impacts of several important coefficients, including power allocation coefficients, and relay position, are also discussed via simulations.

The rest of this paper is organized as follows. We introduce the system model and some basic assumptions in Section II. Section III and IV analyze the outage performance and diversity gain, respectively. The ergodic rate is discussed in Section V. Numerical results are provided in Section VI. Finally, Section VII concludes this work.

$Notations$: Throughout this paper, $P(\cdot)$ symbolizes probability. Λ^c symbolizes the complementary set of event Λ. $F_X(\cdot)$ and $f_X(\cdot)$ symbolize the cumulative distribution function (CDF) and the probability density function (PDF) of a random variable X, respectively.

II. SYSTEM MODEL

In this section, we introduce a cooperative NOMA system model, and some basic assumptions are listed.

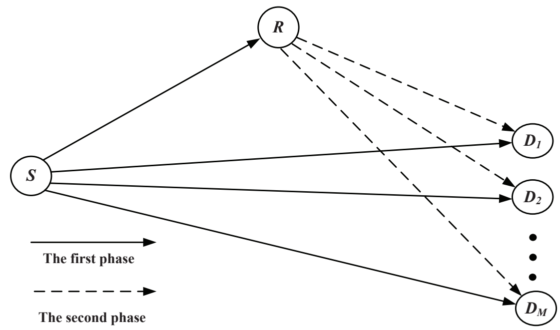

FIGURE 1: Downlink cooperative NOMA system.

As depicted in Fig. 1, we consider a common downlink cooperative network, where a source S communicates with M users $\mathcal{D} = \{D_1, D_2, \ldots, D_M\}$ via a dedicated DF relay,

[2]According to the principle of power-domain NOMA, the users should be sorted based on their channel conditions. That is to say, the strongest user is the one who has the best channel condition.

and the users are clustered to form homogeneous network topology [23]. All nodes in this system are equipped with a single antenna and all nodes know the exact channel state information (CSI). Unlike the assumption of absent direct links in previous works, we consider the direct links between S and $\mathcal{D}$ are also present to model a more realistic situation. All the wireless links are assumed to experience independent Rayleigh fading and additive white Gaussian noise (AWGN). The channel vector between S and $\mathcal{D}$ is denoted by $h_{SD} = [h_{SD_1}, h_{SD_2}, \cdots, h_{SD_M}]$, where $h_{SD_m} \sim \mathcal{CN}(0, \Omega_X)$, $m = 1, 2, \cdots, M$. The channel gain between S and R is denoted by h_{SR}, where $h_{SR} \sim \mathcal{CN}(0, \Omega_Y)$. Similarly between R and $\mathcal{D}$, the channel vector is denoted by $h_{RD} = [h_{RD_1}, h_{RD_2}, \cdots, h_{RD_M}]$, where $h_{RD_m} \sim \mathcal{CN}(0, \Omega_Z)$, $m = 1, 2, \cdots, M$. For notational simplicity, let $\lambda_X = |h_X|^2$, where $X \in \{SR, SD_i, RD_i\}$ with $i \in \{1, 2, \cdots M\}$.

As described in [13]-[17], the whole transmission of this half-duplex relay system is also completed in two consecutive phases.

During the first phase, the source S broadcasts the superimposed signal x_S to R and $\mathcal{D}$ simultaneously, where x_S is given by $x_S = \sum_{i=1}^{M} \sqrt{a_i P_S} x_i$, P_S denotes the transmit power at S, x_i denotes the signal of D_i, and a_i denotes the corresponding power coefficient. Therefore, the received signals at R and D_i can be written as $y_R = h_{SR} \sum_{i=1}^{M} \sqrt{a_i P_S} x_i + n_R$ and $y_{SD_m} = h_{SD_m} \sum_{i=1}^{M} \sqrt{a_i P_S} x_i + n_{D_m}^1$, where $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ and $n_{D_m}^1 \sim \mathcal{CN}(0, \sigma_{D_m^1}^2)$, denote the AWGNs at R and D_m, respectively.

Based on the principle of NOMA, the DF relay decodes the signals of $\mathcal{D}$ using SIC, Thus, the corresponding signal-to-interference-and-noise ratio (SINR) at relay is given by:

$$\gamma_{SR}^m = \begin{cases} \frac{a_m \lambda_{SR}}{\tilde{a}_m \lambda_{SR} + \frac{1}{\rho}}, & m < M \\ \rho a_M \lambda_{SR}, & m = M, \end{cases} \tag{1}$$

where $\rho \triangleq \frac{P_S}{\sigma_R^2}$ represents the average signal-to-noise ratio (SNR), and $\tilde{a}_m = \sum_{i=m+1}^{M} a_i$.

Meanwhile, SIC will also be carried out at $\mathcal{D}$. User D_n should decode the signal of user D_m first before decoding its own signal ($n > m > 1$), and the signal of D_n will be treated as noise at D_m. As a result, the SINR for D_m to decode its own signal can be calculated as:

$$\gamma_{SD}^m = \frac{a_m \lambda_{SD_m}}{\tilde{a}_m \lambda_{SD_m} + \frac{1}{\rho}}. \tag{2}$$

The SINRs for D_M to decode the signal of D_m and its own signal can be calculated respectively as:

$$\gamma_{SD}^{M \to m} = \frac{a_m \lambda_{SD_M}}{\tilde{a}_m \lambda_{SD_M} + \frac{1}{\rho}}, \tag{3}$$

$$\gamma_{SD}^{M} = \rho a_M \lambda_{SD_M}. \tag{4}$$

During the second phase, the relay rebuilds the superposition code and retransmits it to all users with power P_R [17]. Therefore, the received signal at D_m can be written as $y_{RD_m} = h_{RD}^m \sum_{i=1}^{M} \sqrt{a_i P_R} x_i + n_{D_m}^2$, where $n_{D_m^2} \sim \mathcal{CN}(0, \sigma_{D_m^2}^2)$ denotes the AWGN at D_m in the second phase. As in [13]-[17], we also assume that $P_S = P_R = P$, $\sigma_R^2 = \sigma_{D_m^1}^2 = \sigma_{D_m^2}^2 = \sigma^2$. Similar to (2)-(4), the corresponding SINRs of link $R \to D$ can be expressed as:

$$\gamma_{RD}^m = \frac{a_m \lambda_{RD_m}}{\tilde{a}_m \lambda_{RD_m} + \frac{1}{\rho}}, \tag{5}$$

$$\gamma_{RD}^{M \to m} = \frac{a_m \lambda_{RD_M}}{\tilde{a}_m \lambda_{RD_M} + \frac{1}{\rho}}, \tag{6}$$

$$\gamma_{RD}^{M} = \rho a_M \lambda_{RD_M}. \tag{7}$$

Finally, by using the MRC criterion, all users combine the received signals of the two-phase transmission. Taking into account the impact of both direct and relay-aided links, in the following sections, we analyze and compare two different user ordering schemes, i.e., the users are sorted according the channels gains of relay-aided links as $\lambda_{RD_1} \le \lambda_{RD_2} \le \cdots \le \lambda_{RD_M}$, or according the channels gains of direct links as $\lambda_{SD_1} \le \lambda_{SD_2} \le \cdots \le \lambda_{SD_M}$.

III. OUTAGE PERFORMANCE

To evaluate the two different user ordering schemes, in this section, outage probability is characterized as a benchmark criterion of system performance.

A. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF RELAY-AIDED ($R \to D$) LINKS.

From the mechanism described in the last section, the link $S \to R$ has a great impact on the SINR at users. If relay could decode the signals correctly, the user can combine the signals from both links $S \to \mathcal{D}$ and $R \to \mathcal{D}$. Otherwise, only the signal from link $S \to \mathcal{D}$ is available. Therefore, the outage probability of the m-th user[3] can be given by

$$\begin{aligned} P_{out}^m = & \underbrace{P\left(\gamma_{SR}^m < \gamma_{tar}^m, \gamma_{SD}^m < \gamma_{tar}^m\right)}_{\Psi_1} \\ & + \underbrace{P\left(\gamma_{SR}^m \ge \gamma_{tar}^m, \gamma_{SD}^m + \gamma_{RD}^m < \gamma_{tar}^m\right)}_{\Psi_2}, \end{aligned} \tag{8}$$

where γ_{tar}^m denotes the target SINR for D_m. The target rate for D_m is given by $R_m = \frac{1}{2}\log_2(1 + \gamma_{tar}^m)$.

The first part in (8) can be obtained as

$$\begin{aligned} \Psi_1 = & P\left(\frac{a_m \lambda_{SR}}{\tilde{a}_m \lambda_{SR} + \frac{1}{\rho}} < \gamma_{tar}^m\right) P\left(\frac{a_m \lambda_{SD_m}}{\tilde{a}_1 \lambda_{SD_m} + \frac{1}{\rho}} < \gamma_{tar}^m\right) \\ = & F_{\lambda_{SR}}(\tau_m) F_{\lambda_{SD_m}}(\tau_m) = \left(1 - e^{-\frac{\tau_m}{\Omega_Y}}\right)\left(1 - e^{-\frac{\tau_m}{\Omega_X}}\right), \end{aligned} \tag{9}$$

where $\tau_m = \frac{\gamma_{tar}^m}{\rho(a_m - \gamma_{tar}^m a_n)}$. It is assumed that $\gamma_{tar}^m < \frac{a_m}{a_n}$; otherwise the outage probability of D_m is always one.

[3]For mathematical tractability, in Section III, we mainly consider the two-user scenario, i.e., $M = 2$, or only two among M ($M > 2$) users are paired to perform NOMA. The scenario where all the M ($M > 2$) users participate NOMA are discussed in the Sections IV, V and VI.

Then, the second part of (8) can be calculated as

$$\begin{aligned}\Psi_2 &= P\big(\gamma_{SR}^m \geq \gamma_{tar}^m\big)P\big(\gamma_{SD}^m + \gamma_{RD}^m < \gamma_{tar}^m\big)\\ &= e^{-\frac{\tau_m}{\Omega_Y}}\underbrace{P\big(\gamma_{SD}^m + \gamma_{RD}^m < \gamma_{tar}^m\big)}_{\Psi_3}.\end{aligned} \tag{10}$$

And Ψ_3 can be further rewritten as follows:

$$\begin{aligned}\Psi_3 =& 1 - P((2\rho^2 a_m a_n - \gamma_{tar}^m \rho^2 {a_n}^2)\lambda_{RD_m}\lambda_{SD_m}\\ &+(\rho a_m - \gamma_{tar}^m \rho a_n)(\lambda_{RD_m} + \lambda_{SD_m}) \geq \gamma_{tar}^m).\end{aligned} \tag{11}$$

Recall the condition $\gamma_{tar}^m < \frac{a_m}{a_n}$, we have $2\rho^2 a_m a_n - \gamma_{tar}^m \rho^2 {a_n}^2 > 0$ and $\rho a_m - \gamma_{tar}^m \rho a_n > 0$. After some algebraic manipulations, (11) can be revised as

$$\begin{aligned}\Psi_3 &= 1 - P(\lambda_{RD_m} \geq \max\{0, \tfrac{\gamma_{tar}^m - b\lambda_{SD_m}}{c\lambda_{SD_m}+b}\})\\ &= \begin{cases} 1 - P(\lambda_{RD_m} > \frac{\gamma_{tar}^m - b\lambda_{SD_m}}{c\lambda_{SD_m}+b}), & 0 < \lambda_{SD_m} < \frac{\gamma_{tar}^m}{b};\\ 1 - P(\lambda_{RD_m} > 0), & \lambda_{SD_m} \geq \frac{\gamma_{tar}^m}{b},\end{cases}\end{aligned} \tag{12}$$

where $b = (\rho a_m - \gamma_{tar}^m \rho a_n)$ and $c = 2\rho^2 a_m a_n - \gamma_{tar}^m \rho^2 {a_n}^2$.

With the aid of [26], the CDF of λ_{RD_m} is given by

$$F_{\lambda_{RD_m}}(x) = \sum_{i=m}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}e^{-\frac{(j+M-i)x}{\Omega_Z}}. \tag{13}$$

Then, Ψ_3 can be expressed as

$$\begin{aligned}\Psi_3 &= 1 - \Big(\int_0^{\frac{\gamma_{tar}^m}{b}} f_{\lambda_{SD_m}}(x)dx \int_{\frac{\gamma_{tar}^m - bx}{cx+b}}^{\infty} f_{\lambda_{RD_m}}(y)dy\\ &\quad + \int_{\frac{\gamma_{tar}^m}{b}}^{\infty} f_{\lambda_{SD_m}}(x)dx \int_0^{\infty} f_{\lambda_{RD_m}}(y)dy\Big)\\ &= \Delta e^{\frac{(j+M-i)b}{\Omega_Z c}}\underbrace{\int_0^{\frac{\gamma_{tar}^m}{b}} e^{-[\frac{x}{\Omega_X}+\frac{(j+M-i)\gamma_{tar}^m+\frac{(j+M-i)b^2}{c}}{\Omega_Z cx+\Omega_Z b}]}dx}_{\Psi_4},\end{aligned} \tag{14}$$

where $\Delta = \frac{1}{\Omega_X}\sum\limits_{i=m}^{M}\sum\limits_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}$. Let r denote $\Omega_Z cx+\Omega_Z b$, and d denote $(j+M-i)\gamma_{tar}^m + \frac{(j+M-i)b^2}{c}$, after applying the series expansion of the exponential functions in (14), we have:

$$\begin{aligned}\Psi_4 &= \int_{\Omega_Z b}^{\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b} e^{-\frac{r-\Omega_Z b}{\Omega_X\Omega_Z c}}e^{-\frac{d}{r}}dr\\ &= e^{\frac{b}{\Omega_X c}}\Bigg[\sum_{t=2}^{\infty}\frac{(-1)^t}{t!}d^t\int_{\Omega_Z b}^{\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b} e^{-\frac{r}{\Omega_X\Omega_Z c}}r^{-t}dr\\ &\quad + \int_{\Omega_Z b}^{\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b} e^{-\frac{r}{\Omega_X\Omega_Z c}}dr\\ &\quad - d\int_{\Omega_Z b}^{\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b} e^{-\frac{r}{\Omega_X\Omega_Z c}}r^{-1}dr\Bigg],\end{aligned} \tag{15}$$

The second part of Ψ_4 can be directly given by

$$\Psi_4^2 = \Omega_X\Omega_Z c\Big(e^{-\frac{b}{\Omega_X c}} - e^{-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}}\Big). \tag{16}$$

With the help of [27, eq.(3.351.4)] , the first part of Ψ_4 can be calculated as

$$\begin{aligned}\Psi_4^1 =& (-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left(-b\frac{1}{\Omega_X c}\right)}{(t-1)!} + \frac{e^{-\frac{b}{\Omega_X c}}}{(\Omega_Z b)^{t-1}}\\ &\times\sum_{k=0}^{t-2}\frac{(-1)^k\left(\frac{1}{\Omega_X\Omega_Z c}\right)^k(\Omega_Z b)^k}{(t-1)(t-2)\cdots(t-1-k)} - \Bigg[(-1)^t\\ &\times\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left[-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}\right]}{(t-1)!} + \frac{e^{-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}}}{\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^{t-1}}\\ &\times\sum_{l=0}^{t-2}\frac{(-1)^l\left(\frac{1}{\Omega_X\Omega_Z c}\right)^l\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^l}{(t-1)(t-2)\cdots(t-1-l)}\Bigg],\end{aligned} \tag{17}$$

where $Ei(x) = \int_{-\infty}^{x}\frac{e^t}{t}dt$, $x < 0$. With the aid of [27, eq.(3.352.2)], the third part of Ψ_4 can be calculated as

$$\Psi_4^3 = d\Big[Ei\Big(-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}\Big) - Ei\Big(-\frac{b}{\Omega_X c}\Big)\Big]. \tag{18}$$

Finally, by combining (8)-(10) and (14)-(18), the closed-form expression of the outage probability for the weak user D_m can be written as

$$\begin{aligned}P_{out}^m =& \big(1 - e^{-\frac{\tau_m}{\Omega_Y}}\big)\big(1 - e^{-\frac{\tau_m}{\Omega_X}}\big)\\ &+\Delta e^{-\frac{\tau_m}{\Omega_Y}+\frac{(j+M-i)b}{\Omega_Z c}+\frac{b}{\Omega_X c}}\frac{1}{\Omega_Z c}\Big(\sum_{t=2}^{\infty}\frac{(-1)^t}{t!}d^t\Psi_4^1 + \Psi_4^2 - \Psi_4^3\Big).\end{aligned} \tag{19}$$

On the other hand, the n-th user D_n should decode the signal of D_m first before decoding its own signal, thus the outage probability of D_n can be given by

$$\begin{aligned}P_{out}^n =& \Big[1 - P\big(\gamma_{SR}^n \geq \gamma_{tar}^n, \gamma_{SR}^m \geq \gamma_{tar}^m\big)\Big]\\ &\times\Big[1 - P\big(\gamma_{SD}^n \geq \gamma_{tar}^n, \gamma_{SD}^{n\to m} \geq \gamma_{tar}^m\big)\Big]\\ &+P\big(\gamma_{SR}^n \geq \gamma_{tar}^n, \gamma_{SR}^m \geq \gamma_{tar}^m\big)\Big[1\\ &-P(\gamma_{SD}^n + \gamma_{RD}^n \geq \gamma_{tar}^n, \gamma_{SD}^{n\to m} + \gamma_{RD}^{n\to m} \geq \gamma_{tar}^m)\Big].\end{aligned} \tag{20}$$

Define the first part of (20) as Ψ_5, and the second part as Ψ_6. Subsequently, Ψ_5 can be further calculated as

$$\begin{aligned}\Psi_5 =& \Big[1 - P\Big(\rho a_n\lambda_{SR} \geq \gamma_{tar}^n, \frac{a_m\lambda_{SR}}{\tilde{a}_m\lambda_{SR}+\frac{1}{\rho}} \geq \gamma_{tar}^m\Big)\Big]\\ &\times\Big[1 - P\Big(\rho a_n\lambda_{SD_n} \geq \gamma_{tar}^n, \frac{a_m\lambda_{SD_n}}{a_n\lambda_{SD_n}+\frac{1}{\rho}} \geq \gamma_{tar}^m\Big)\Big]\\ =& \Big[1 - P\big(\lambda_{SR} \geq \max\{\tau^n, \tau^m\} \triangleq \theta\big)\Big]\\ &\times\Big[1 - P\big(\lambda_{SD_n} \geq \theta\big)\Big]\\ =& \Big(1 - e^{-\frac{\theta}{\Omega_Y}}\Big)\Big(1 - e^{-\frac{\theta}{\Omega_X}}\Big),\end{aligned} \tag{21}$$

where γ_{tar}^n represents the target SINR of D_n and $\tau^n = \frac{\gamma_{tar}^n}{\rho a_n}$. The target rate for D_n is given by $R_n = \frac{1}{2}\log_2(1 + \gamma_{tar}^n)$.

Then, Ψ_6 can be attained as

$$\begin{aligned}\Psi_6 =& P\left(|h_{SR}|^2 \geq \theta\right)\big[1 \\ & -\underbrace{P(\lambda_{RD_n}+\lambda_{SD_n} \geq \tau^n, \gamma_{SD}^{n\to m}+\gamma_{RD}^{n\to m} \geq \gamma_{tar}^m)}_{\Psi_7}\big] \\ =& e^{-\frac{\theta}{\Omega_Y}}(1-\Psi_7). \end{aligned} \tag{22}$$

By substituting (3) and (6) into (22) and following similar steps to (12), Ψ_7 can be written as

$$\Psi_7 = P\left(\lambda_{RD_n} \geq \max\left\{0, \frac{\gamma_{tar}^m - b\lambda_{SD_n}}{c\lambda_{SD_n}+b}, \tau^n - \lambda_{SD_n}\right\}\right). \tag{23}$$

Ψ_7 can be further calculated based on the relationship between the function $\frac{\gamma_{tar}^m - b\lambda_{SD_n}}{c\lambda_{SD_n}+b}$ and function $\tau^n - \lambda_{SD_n}$. For the case of $\tau^n \geq \frac{\gamma_{tar}^m}{b}$, the outage probability of the n-th user D_n is given by:

$$\begin{aligned}P_{out}^n =& \left(1-e^{-\frac{\theta}{\Omega_Y}}\right)\left(1-e^{-\frac{\theta}{\Omega_X}}\right) \\ &+\frac{1}{\Omega_X}\sum_{i=n}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}e^{-\frac{\theta}{\Omega_Y}-\frac{(j+M-i)\tau^n}{\Omega_Z}} \\ &\times\frac{\Omega_X\Omega_Z}{[\Omega_Z-\Omega_X(j+M-i)]}\left(1-e^{-\frac{[\Omega_Z-\Omega_X(j+M-i)]\tau^n}{\Omega_X\Omega_Z}}\right).\end{aligned} \tag{24}$$

Proof: See Appendix A.

On the other hand, for the case of $\tau^n < \frac{\gamma_{tar}^m}{b}$ and $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) < 0$, P_{out}^n is given by (25). For the case of $\tau^n < \frac{\gamma_{tar}^m}{b}$ and $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) \geq 0$, the outage probability of D_n can be derived as (26), which are shown at the top of the next page.

B. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF DIRECT ($S \to D$) LINKS.

In this user ordering scheme, $\{\lambda_{SD_i}\}$ are ordered instead of $\{\lambda_{RD_i}\}, i \in \{1, 2, \cdots M\}$. Therefore, the CDF of $\{\lambda_{SD_i}\}$ is given by

$$F_{\lambda_{SD_m}}(x) = \sum_{i=m}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}e^{-\frac{(j+M-i)x}{\Omega_X}}. \tag{27}$$

Then, the corresponding outage probability can be calculated following the similar steps as in subsection III.A. In this situation, the closed-form expression of D_m can be given by:

$$\begin{aligned}P_{out}^m =& (1-e^{-\frac{\tau_m}{\Omega_Y}})\sum_{i=m}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}e^{-\frac{(j+M-i)\tau_m}{\Omega_X}} \\ &+\frac{M!}{\Omega_X\Omega_Z c(m-1)!(M-m)!}\sum_{l=0}^{m-1}(-1)^l\binom{m-1}{l} \\ &\times e^{\frac{b}{\Omega_Z c}+\frac{b}{\Omega_X c}-\frac{\tau_m}{\Omega_Y}}\left[\sum_{t=1}^{\infty}\frac{(-1)^t}{t!}d^t\Psi_{12}^1+\Psi_{12}^2-\Psi_{12}^3\right],\end{aligned} \tag{28}$$

where

$$\begin{aligned}\Psi_{12}^1 =& (-1)^t\frac{(\frac{M-m+l+1}{\Omega_X\Omega_Z c})^{(t-1)}Ei(-\frac{M-m+l+1}{\Omega_X c}b)}{(t-1)!} \\ &+\frac{e^{-\frac{M-m+l+1}{\Omega_X c}b}}{(\Omega_Z b)^{t-1}}\sum_{k=0}^{t-2}\frac{(-1)^k(\frac{M-m+l+1}{\Omega_X c})^k(b)^k}{(t-1)(t-2)\cdots(t-1-k)} \\ &-\Bigg[(-1)^t\frac{(\frac{M-m+l+1}{\Omega_X\Omega_Z c})^{(t-1)}Ei[-\frac{M-m+l+1}{\Omega_X c}(c\frac{\gamma_{tar}^m}{b}+b)]}{(t-1)!} \\ &+\frac{e^{-\frac{M-m+l+1}{\Omega_X c}(c\frac{\gamma_{tar}^m}{b}+b)}}{(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b)^{t-1}}\sum_{k=0}^{t-2}\frac{(-1)^k(\frac{M-m+l+1}{\Omega_X c})^k(\frac{c\gamma_{tar}^m}{b}+b)^k}{(t-1)(t-2)\cdots(t-1-k)}\Bigg],\end{aligned}$$

$$\Psi_{12}^2 = \frac{\Omega_X\Omega_Z c}{M-m+l+1}e^{-\frac{(M-m+l+1)\Omega_Z b}{\Omega_X\Omega_Z c}}\left(1-e^{-\frac{(M-m+l+1)\Omega_Z c\gamma_{tar}^m}{b\Omega_X\Omega_Z c}}\right),$$

and
$$\begin{aligned}\Psi_{12}^3 =& (\gamma_{tar}^m+\frac{b^2}{c})\{Ei[-\frac{(M-m+l+1)}{\Omega_X c}(\frac{c\gamma_{tar}^m}{b}+b)] \\ &-Ei[-\frac{(M-m+l+1)b}{\Omega_X c}]\}.\end{aligned}$$

As for the strong user D_n, if $\tau^n \geq \frac{\gamma_{tar}^m}{b}$, the closed-form expression of strong user can be given by:

$$\begin{aligned}P_{out}^n =& \left(1-e^{-\frac{\theta}{\Omega_Y}}\right)\sum_{i=n}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}e^{-\frac{(j+M-i)\theta}{\Omega_X}} \\ &+\frac{M!\Omega_Z}{(n-1)!(M-n)![\Omega_Z(M-n+k+1)-\Omega_X]}e^{-\frac{\tau^n}{\Omega_Z}-\frac{\theta}{\Omega_Y}} \\ &\times\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\left[1-e^{-\frac{[\Omega_Z(M-n+k+1)-\Omega_X]\tau^n}{\Omega_X\Omega_Z}}\right].\end{aligned} \tag{29}$$

Finally, For the case of $\tau^n < \frac{\gamma_{tar}^m}{b}$, P_{out}^n can be also calculated following the similar steps to Appendix A, B, and (24)-(26).

IV. DIVERSITY GAIN

Since the closed-form expression of the outage probability in Section III is complex, it is also essential to study the diversity gain to provide more insights. To proceed, we define $\Lambda_{R_m,out}$ as the event that R cannot decode the signals of $\{D_1, D_2, ..., D_m\}$ successfully. Then the probability of $\Lambda_{R_m,out}$ is given by:

$$P(\Lambda_{R_m,out}) = 1 - P(\lambda_{SR} > \tau_m^*) = 1 - e^{-\frac{\tau_m^*}{\Omega_Y}}, \tag{30}$$

where $\tau_m^* = \max\{\tau_1, \tau_2, \cdots, \tau_m\}$ with $m < M$, and $\tau_m^* = \max\{\tau_1, \tau_2, \cdots, \frac{\gamma_{tar}^M}{\rho a_M}\}$ with $m = M$.

Next, we define Φ_m as the outage event of D_m, $\Phi_{m,j}$ as the event that D_m fails to decode the signal of D_j $(1 \leq j \leq m)$ after MRC, and $\Phi_{m,j}^c$ as the complementary set of $\Phi_{m,j}$. The outage probability of Φ_m can be formulated as

$$P(\Phi_m) = 1 - P(\Phi_{m,1}^c \cap \Phi_{m,1}^c \cap \cdots \cap \Phi_{m,m}^c), \tag{31}$$

in which, the probability of $\Phi_{m,j}$ can be written as

$$\begin{aligned}P(\Phi_{m,j}) =& P(\Lambda_{R_m,out})P(\gamma_{SD}^{m\to j} < \gamma_{tar}^j) \\ &+P(\Lambda_{R_m,out}^c)P(\gamma_{SD}^{m\to j}+\gamma_{RD}^{m\to j} < \gamma_{tar}^j).\end{aligned} \tag{32}$$

It can be observed that the first part of (32) equals to the outage probability that only the direct links are available and D_m fails to decode D_j. The second part equals to the outage probability that R successfully decodes the signals of $\mathcal{D}$ but outage event happens as well. Therefore, the probability of

$$
\begin{aligned}
P_{out}^{n} = & \left(1-e^{-\frac{\theta}{\Omega_Y}}\right)\left(1-e^{-\frac{\theta}{\Omega_X}}\right)+\frac{1}{\Omega_Z c}\Delta' e^{-\frac{\theta}{\Omega_Y}+\frac{(j+M-i)b}{\Omega_Z c}+\frac{b}{\Omega_X c}}\left\{\sum_{t=2}^{\infty}\frac{(-1)^t}{t!}d^t(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left(-b\frac{1}{\Omega_X c}\right)}{(t-1)!}\right.\\
& +\frac{e^{-\frac{b}{\Omega_X c}}}{(\Omega_Z b)^{t-1}}\sum_{k=0}^{t-2}\frac{(-1)^k\left(\frac{1}{\Omega_X\Omega_Z c}\right)^k(\Omega_Z b)^k}{(t-1)(t-2)\cdots(t-1-k)}-[(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left[-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}\right]}{(t-1)!}\\
& +\frac{e^{-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}}}{\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^{t-1}}\sum_{l=0}^{t-2}\frac{(-1)^l\left(\frac{1}{\Omega_X\Omega_Z c}\right)^l\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^l}{(t-1)(t-2)\cdots(t-1-l)}]+\Omega_X\Omega_Z c\left(e^{-\frac{b}{\Omega_X c}}-e^{-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}}\right)\\
& \left.-d\left[Ei\left(-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}\right)-Ei\left(-\frac{b}{\Omega_X c}\right)\right]\right\}.
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
P_{out}^{n}= & \left(1-e^{-\frac{\theta}{\Omega_Y}}\right)\left(1-e^{-\frac{\theta}{\Omega_X}}\right)+\Delta' e^{-\frac{\theta}{\Omega_Y}+\frac{(j+M-i)b}{\Omega_Z c}+\frac{b}{\Omega_X c}}\frac{1}{\Omega_Z c}\left\{\mu_{x_1}+\Omega_X\Omega_Z c\left(e^{-\frac{b}{\Omega_X c}}-e^{-\frac{cx_1+b}{\Omega_X c}}\right)-d\left[Ei\left(-\frac{cx_1+b}{\Omega_X c}\right)\right.\right.\\
& \left.\left.-Ei\left(-\frac{b}{\Omega_X c}\right)\right]\right\}+\Delta' e^{-\frac{\theta}{\Omega_X}-\frac{(j+M-i)\tau^n}{\Omega_Z}}\frac{\Omega_X\Omega_Z}{[\Omega_Z-\Omega_X(j+M-i)]}\left\{e^{-\frac{[\Omega_Z-\Omega_X(j+M-i)]x_1}{\Omega_X\Omega_Z}}-e^{-\frac{[\Omega_Z-\Omega_X(j+M-i)]x_2}{\Omega_X\Omega_Z}}\right\}\\
& +\Delta' e^{-\frac{\theta}{\Omega_X}+\frac{(j+M-i)b}{\Omega_Z c}+\frac{b}{\Omega_X c}}\frac{1}{\Omega_Z c}\left\{\mu_{x_2}+\Omega_X\Omega_Z c\left(e^{-\frac{cx_2+b}{\Omega_X c}}-e^{-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}}\right)-d\left[Ei\left(-\frac{\frac{c\gamma_{tar}^m}{b}+b}{\Omega_X c}\right)-Ei\left(-\frac{cx_2+b}{\Omega_X c}\right)\right]\right\},
\end{aligned}
\tag{26}
$$

where $\Delta'=\frac{1}{\Omega_X}\sum_{i=n}^{M}\sum_{j=0}^{i}(-1)^j\binom{i}{j}\binom{M}{i}$, $x_1=\frac{\tau^n c-\sqrt{(\tau^n c)^2-4c(\gamma_{tar}^m-b\tau^n)}}{2c}$ and $x_2=\frac{\tau^n c+\sqrt{(\tau^n c)^2-4c(\gamma_{tar}^m-b\tau^n)}}{2c}$,

$$
\begin{aligned}
\mu_{x_1} = & \sum_{t=2}^{\infty}\frac{(-1)^t}{t!}d^t\left\{(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left(-b\frac{1}{\Omega_X c}\right)}{(t-1)!}+\frac{e^{-\frac{b}{\Omega_X c}}}{(\Omega_Z b)^{t-1}}\sum_{k=0}^{t-2}\frac{(-1)^k\left(\frac{1}{\Omega_X\Omega_Z c}\right)^k(\Omega_Z b)^k}{(t-1)(t-2)\cdots(t-1-k)}\right.\\
& \left.-(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left[-(cx_1+b)\frac{1}{\Omega_X c}\right]}{(t-1)!}-\frac{e^{-(cx_1+b)\frac{1}{\Omega_X c}}}{(\Omega_Z cx_1+\Omega_Z b)^{t-1}}\sum_{l=0}^{t-2}\frac{(-1)^l\left(\frac{1}{\Omega_X\Omega_Z c}\right)^l(\Omega_Z cx_1+\Omega_Z b)^l}{(t-1)(t-2)\cdots(t-1-l)}\right\},\\
\mu_{x_2} = & \sum_{t=2}^{\infty}\frac{(-1)^t}{t!}d^t\left\{(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left(-\frac{cx_2+b}{\Omega_X c}\right)}{(t-1)!}+\frac{e^{-\frac{cx_2+b}{\Omega_X c}}}{(\Omega_Z cx_2+\Omega_Z b)^{t-1}}\sum_{k=0}^{t-2}\frac{(-1)^k\left(\frac{1}{\Omega_X\Omega_Z c}\right)^k(\Omega_Z cx_2+\Omega_Z b)^k}{(t-1)(t-2)\cdots(t-1-k)}\right.\\
& \left.-(-1)^t\frac{\left(\frac{1}{\Omega_X\Omega_Z c}\right)^{(t-1)}Ei\left[-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}\right]}{(t-1)!}-\frac{e^{-\left(\frac{c\gamma_{tar}^m}{b}+b\right)\frac{1}{\Omega_X c}}}{\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^{t-1}}\sum_{l=0}^{t-2}\frac{(-1)^l\left(\frac{1}{\Omega_X\Omega_Z c}\right)^l\left(\frac{\Omega_Z c\gamma_{tar}^m}{b}+\Omega_Z b\right)^l}{(t-1)(t-2)\cdots(t-1-l)}\right\}.
\end{aligned}
$$

$Proof$: See Appendix B.

Φ_m can be bounded as

$$
\begin{aligned}
P(\Phi_m) \leq & \underbrace{P(\lambda_{SR} < \tau_m^*)P(\lambda_{SD_m} < \tau_m^*)}_{J_1}\\
& \underbrace{+P(\lambda_{SR}\geq \tau_m^*)P(\lambda_{RD_m} < \tau_m^*)P(\lambda_{SD_m}< \tau_m^*)}_{J_2}.
\end{aligned}
\tag{33}
$$

A. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF RELAY-AIDED ($R \to D$) LINKS.

In this user ordering scheme, λ_{SD_m} is an unordered variable, then J_1 can be given by

$$
\begin{aligned}
J_1 &= P(\lambda_{SR} < \tau_m^*)P(\lambda_{SD_m} < \tau_m^*)\\
&= (1-e^{-\frac{\tau_m^*}{\Omega_Y}})(1-e^{-\frac{\tau_m^*}{\Omega_X}}) \propto \frac{1}{\rho^2},
\end{aligned}
\tag{34}
$$

where $\propto$ denotes the approximation at high SNR. Also, $P(\lambda_{SR}\geq\tau_m^*)P(\lambda_{SD_m}<\tau_m^*)$ in J_2 is given by $e^{-\frac{\tau_m^*}{\Omega_Y}}(1-e^{-\frac{\tau_m^*}{\Omega_X}})$. Then, the PDF of λ_{RD_m} is given by

$$
\begin{aligned}
f_{\lambda_{RD_m}}(x) = & \frac{M!}{\Omega_Z(m-1)!(M-m)!}\\
& \times\sum_{i=0}^{m-1}(-1)^i\binom{m-1}{i}e^{-\frac{(M-m+i+1)x}{\Omega_Z}}.
\end{aligned}
\tag{35}
$$

The high-SNR approximations of $P(\lambda_{RD_m} < \tau_m^*)$ can be obtained following the steps to (36)-(39), which can be approximated as $\frac{1}{\rho^m}$. By substituting J_1, and J_2 into (33), we can conclude that all the users can experience a same diversity order as two.

B. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF DIRECT ($S \to D$) LINKS.

In this scheme, the PDF of λ_{SD_m} is given by

$$
\begin{aligned}
f_{\lambda_{SD_m}}(x) = & \frac{M!}{(m-1)!(M-m)!}\frac{1}{\Omega_X}\\
& \times\sum_{i=0}^{m-1}(-1)^i\binom{m-1}{i}e^{-\frac{(M-m+i+1)x}{\Omega_X}}.
\end{aligned}
\tag{36}
$$

Then, $P(\lambda_{SD_m} < \tau_m^*)$ can be obtained as:

$$\begin{aligned} P(\lambda_{SD_m}<\tau_m^*) &= \int_0^{\tau_m^*} f_{\lambda_{SD_m}}(x)dx \\ &= \frac{M!}{(m-1)!(M-m)!}\frac{1}{M-m+i+1} \\ &\times \sum_{i=0}^{m-1}(-1)^i \binom{m-1}{i}\left(1-e^{-\frac{(M-m+i+1)\tau_m^*}{\Omega_X}}\right) \\ &\triangleq \sum_{l=1}^{\infty}\frac{(-1)^{l+1}M!\tau_m^{*\,l}}{(m-1)!(M-m)!l!\Omega_X{}^l}\sum_{j=0}^{l-1}\binom{l-1}{j} \\ &\times (M-m+1)^{l-1-j}\sum_{i=0}^{m-1}\binom{m-1}{i}(-1)^i i^j, \end{aligned} \tag{37}$$

where $\triangleq$ denotes the series expansion of exponential functions.

Recall Equations (24)-(25) in [13], and sums of the binomial coefficients in [27, (Eq.(0.154.3)] and [27, (Eq.(0.154.4)]. It is interesting to observe that, in the high SNR region, all the components containing i^j in (37), $j < (m-1)$, can be removed. Also, the components containing i^j, $j > (m-1)$, can also be ignored. Therefore, J_1 can be approximated as

$$J_1 \simeq \frac{M!\tau_m^{*\,m+1}}{(M-m)!m!\Omega_X{}^m\Omega_Y} \propto \frac{1}{\rho^{m+1}}. \tag{38}$$

Meanwhile, since λ_{RD_m} is an unordered variable, $P(\lambda_{SR} \geq \tau_m^*)P(\lambda_{RD_m} < \tau_m^*)$ in J_2 can be directly obtained as:

$$P(\lambda_{SR}\geq \tau_m^*)P(\lambda_{RD_m} < \tau_m^*) = e^{-\frac{\tau_m^*}{\Omega_Y}}(1-e^{-\frac{\tau_m^*}{\Omega_Z}}) \propto \frac{1}{\rho}. \tag{39}$$

Therefore, at high SNR regions , J_2 can be also approximated as $\frac{1}{\rho^{m+1}}$. By combining J_1 and J_2 in (33), we can conclude that, in this user ordering scheme, the m-th user can achieve a diversity order of $m+1$.

V. ERGODIC RATE

In Sections III and IV, we mainly investigate the outage performance and diversity gain under the assumption that each user has a preset quality of service (QoS) requirement, and it is shown that the second user ordering scheme can achieve better diversity order. However, the diversity order by itself does not tell the entire story. Due to the significant benefits of NOMA on better user fairness and larger capacity, it is also interesting to discuss the users' ergodic rate as an alternative criterion.

As described in [24], in this situation, the user's rate is determined opportunistically by the user's channel condition instead of a preset value.

A. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF RELAY-AIDED ($R \to D$) LINKS.

We first focus on the ergodic rate of the m-th ($1 \leq m \leq M-1$) user. According to the mechanism described in Section II, γ_m is given by:

$$\begin{aligned} \gamma_m =& \max\{min\{\gamma_{SD}^m, \gamma_{SD}^{m\to m+1}, \cdots, \gamma_{SD}^{m\to M}\}, min\{\gamma_{SR}^m, \\ & \gamma_{RD}^m+\gamma_{SD}^m, \gamma_{RD}^{m\to m+1}+\gamma_{SD}^{m\to m+1}, \cdots, \gamma_{RD}^{m\to M}+\gamma_{SD}^{m\to M}\}\}. \end{aligned} \tag{40}$$

In light of (2), (3), (5), and (6), in the high-SNR region ($\rho \to \infty$), γ_m can be approximated as $a_m/\widetilde{a_m}$, which means when $\rho \to \infty$, the ergodic sum rate of this system is mainly determined by the M-th user rather than the rest $M-1$ users.

As for the M-th user, the corresponding SINR can be expressed as:

$$\begin{aligned} \gamma_M =& \max\{\gamma_{SD}^M, min(\gamma_{SR}^M, \gamma_{RD}^M+\gamma_{SD}^M)\} \\ =& a_M\rho\max\left\{\lambda_{SD_M}, min\{\lambda_{SR}, \lambda_{RD_M}+\lambda_{SD_M}\}\right\}. \end{aligned} \tag{41}$$

To address γ_M, we let ω=max$[\lambda_{SD_M}, min(\lambda_{SR}, \lambda_{RD_M}+\lambda_{SD_M})]$, and the CDF of ω can be expressed as:

$$\begin{aligned} F_\omega(x) =& 1-e^{-\frac{x}{\Omega_Y}}-e^{-\frac{x}{\Omega_X}}+e^{-\frac{x}{\Omega_Y}-\frac{x}{\Omega_X}}+\sum_{k=0}^{M}\binom{M}{k}(-1)^{k+1} \\ &\times\frac{k}{\Omega_X k-\Omega_Z}\Big[\frac{\Omega_Z}{k}(e^{-\frac{x}{\Omega_Y}-\frac{kx}{\Omega_Z}}-e^{-\frac{x}{\Omega_Y}})-\Omega_X(e^{-\frac{x}{\Omega_X}-\frac{x}{\Omega_Y}}-e^{-\frac{x}{\Omega_Y}}) \\ &-\frac{\Omega_Z}{k}(e^{-\frac{kx}{\Omega_Z}-\frac{x}{\Omega_Y}-\frac{x}{\Omega_X}}-e^{-\frac{x}{\Omega_Y}-\frac{x}{\Omega_X}})+\Omega_X(e^{-\frac{2x}{\Omega_X}-\frac{x}{\Omega_Y}}-e^{-\frac{x}{\Omega_Y}-\frac{x}{\Omega_X}})\Big]. \end{aligned} \tag{42}$$

Proof: See Appendix C.

Then, the ergodic rate of M-th user is given by

$$R^M = E\Big[\frac{1}{2}\log(1+\gamma_M)\Big] = \frac{a_M\rho}{2\ln 2}\int_0^\infty \frac{1-F_\omega(x)}{1+a_M\rho x}dx. \tag{43}$$

By substituting (42) into (43) and with the aid of [27, (Eq.(3.352.4)], we can calculate the ergodic rate of M-th user as (44).

B. ALL USERS ARE SORTED ACCORDING TO THE CHANNELS GAINS OF DIRECT ($S \to D$) LINKS.

On the other hand, if only the direct links are ordered, the m-th user's SINR γ_m is given by:

$$\begin{aligned} \gamma_m = \max\big[&\gamma_{SD}^m, min(\gamma_{SR}^m, \gamma_{RD}^m+\gamma_{SD}^m, \\ &\gamma_{RD}^{m\to m+1}+\gamma_{SD}^{m\to m+1}, \cdots, \gamma_{RD}^{m\to M}+\gamma_{SD}^{m\to M})\big]. \end{aligned} \tag{45}$$

Similarly, again applying the high SNR approximation, (45) can be approximated as $a_m/\widetilde{a_m}$.

Also, the M-th user's SINR is still given by (41). Note that in this situation, the ordered variables are $\{\lambda_{SD_m}\}$ instead of $\{\lambda_{RD_m}\}$, so the CDF of ω is different. Following the similar steps as subsection V.A, R^M can be given by (46), as shown at the top of pape 8.

VI. NUMERICAL RESULTS

In this section, computer simulations are provided to evaluate the two different user ordering schemes (scheme A: sort users according to links $R \to D$, scheme B: sort users according to links $S \to D$), and the corresponding analytical results. We consider that the base station, the relay node, and users are located on a straight line. $\Omega_Y = d_1{}^{-\alpha}$, $\Omega_Z = (1-d_1)^{-\alpha}$, $\Omega_X = 1$ and $\alpha = 4$, where d_1 and

$$R^M = \frac{1}{2\ln 2}\Big[\Gamma_1 - \sum_{k=0}^{M}\left(\begin{array}{c} M \\ k \end{array}\right)(-1)^{k+1}\frac{1}{\Omega_X k - \Omega_Z}(\Gamma_2 - \Gamma_3)\Big], \tag{44}$$

where $\Gamma_1 = -e^{\frac{1}{a_M\rho\Omega_Y}} Ei\Big(-\frac{1}{a_M\rho\Omega_Y}\Big) - e^{\frac{1}{a_M\rho\Omega_X}} Ei\Big(-\frac{1}{a_M\rho\Omega_X}\Big) + e^{\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_X\Omega_Y}} Ei\Big(-\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_X\Omega_Y}\Big),$

$$\Gamma_2=\Omega_Z\Big[-e^{\frac{\Omega_Z+k\Omega_Y}{a_M\rho\Omega_Y\Omega_Z}}Ei\Big(-\frac{\Omega_Z+k\Omega_Y}{a_M\rho\Omega_Y\Omega_Z}\Big)+e^{\frac{1}{a_M\rho\Omega_Y}}Ei\Big(-\frac{1}{a_M\rho\Omega_Y}\Big)\Big]-k\Omega_X\Big[-e^{\frac{\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}}Ei\Big(-\frac{\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}\Big)+e^{\frac{1}{a_M\rho\Omega_Y}}Ei\Big(-\frac{1}{a_M\rho\Omega_Y}\Big)\Big],$$

$$\begin{aligned}\Gamma_3 = \Omega_Z\Big[&-e^{\frac{k\Omega_Y\Omega_X+\Omega_Z\Omega_X+\Omega_Z\Omega_Y}{a_M\rho\Omega_Z\Omega_Y\Omega_X}} Ei\Big(-\frac{k\Omega_Y\Omega_X+\Omega_Z\Omega_X+\Omega_Z\Omega_Y}{a_M\rho\Omega_Z\Omega_Y\Omega_X}\Big) + e^{\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_Y\Omega_X}} Ei\Big(-\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_Y\Omega_X}\Big)\Big]\\ -k\Omega_X\Big[&-e^{\frac{2\Omega_Y+\Omega_X}{a_M\rho\Omega_Y\Omega_X}} Ei\Big(-\frac{2\Omega_Y+\Omega_X}{a_M\rho\Omega_Y\Omega_X}\Big) + e^{\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_Y\Omega_X}} Ei\Big(-\frac{\Omega_X+\Omega_Y}{a_M\rho\Omega_Y\Omega_X}\Big)\Big].\end{aligned}$$

$$R^M = \frac{1}{2\ln 2}\Big[\Gamma_4 - \sum_{j=0}^{M}\left(\begin{array}{c} M \\ j \end{array}\right)\sum_{k=0}^{M}\left(\begin{array}{c} M \\ k \end{array}\right)(-1)^{k+1+j}(\Gamma_5+\Gamma_6)\Big], \tag{46}$$

where $\Gamma_4 = \sum_{j_1=1}^{M}\left(\begin{array}{c} M \\ j_1 \end{array}\right)(-1)^{j_1} e^{\frac{j_1}{a_M\rho\Omega_X}} Ei\Big(-\frac{j_1}{a_M\rho\Omega_X}\Big) - \sum_{j_2=0}^{M}\left(\begin{array}{c} M \\ j_2 \end{array}\right)(-1)^{j_2} e^{\frac{j_2\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}} Ei\Big(-\frac{j_2\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}\Big),$

$$\Gamma_5 = \frac{\Omega_X}{\Omega_Z k - \Omega_X}\Big[-e^{\frac{j\Omega_Y+\Omega_X+k\Omega_Y}{a_M\rho\Omega_X\Omega_Y}} Ei\Big(-\frac{j\Omega_Y+\Omega_X+k\Omega_Y}{a_M\rho\Omega_X\Omega_Y}\Big) + e^{\frac{j\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}} Ei\Big(-\frac{j\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}\Big)\Big],$$

$$\Gamma_6 = \frac{k\Omega_Z}{\Omega_Z k - \Omega_X}\Big[-e^{\frac{j\Omega_Y\Omega_Z+\Omega_X\Omega_Z+\Omega_X\Omega_Y}{a_M\rho\Omega_X\Omega_Y\Omega_Z}} Ei\Big(-\frac{j\Omega_Y\Omega_Z+\Omega_X\Omega_Z+\Omega_X\Omega_Y}{a_M\rho\Omega_X\Omega_Y\Omega_Z}\Big) + e^{\frac{j\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}} Ei\Big(-\frac{j\Omega_Y+\Omega_X}{a_M\rho\Omega_X\Omega_Y}\Big)\Big].$$

$1-d_1$ denote the distance of $S \to R$ and $R \to D$ after normalization[4].

A. OUTAGE PERFORMANCE & DIVERSITY GAIN

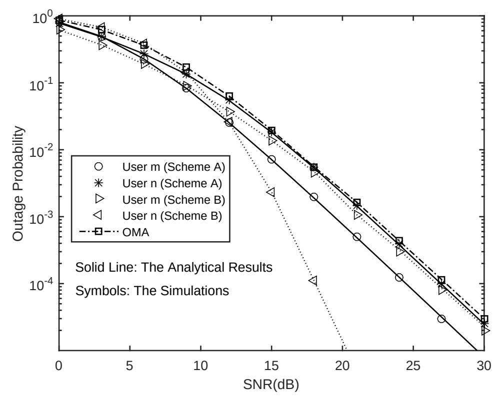

FIGURE 2: Two users scenario with $m = 1$, $n = M = 4$, $a_m = 0.8$, $a_n = 0.2$, $d_1 = 0.5$, $\gamma_{tar}^m = 2.5dB$, and $\gamma_{tar}^n = 4dB$.

By using the Monte Carlo method, in Figs. 2 and 3, we first provide the outage performance comparisons among different user ordering schemes by varying the average SNR. In Fig. 2, we also set the OMA system as a comparison candidate, where the target SINR γ_{OMA} of user in OMA system satisfies $\frac{1}{2}\log_2(1+\gamma_{OMA}) = R_m + R_n$. As can

[4]This path loss model has been well explained in [25], and it has also been widely used in numerous works [13]-[17]

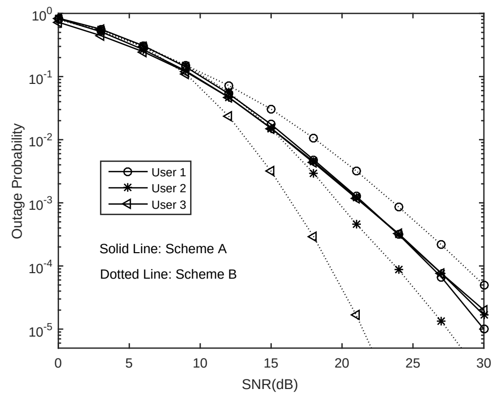

FIGURE 3: Three users scenario with $M = 3$ $a_1 = 1/2$, $a_2 = 1/3$, $a_3 = 1/6$, $\gamma_{tar}^1 = 0.9dB$, $\gamma_{tar}^2 = 1.5dB$, and $\gamma_{tar}^3 = 2dB$.

be observed, both two NOMA schemes outperform conventional OMA scheme. The reason is that NOMA can serve the two users simultaneously in two time phases, whereas four time phases are needed to complete the transmission of two users for conventional OMA scheme. Then, we can see that the simulation results match very well with the derived analytical results in (19), (24)-(26), and (28)-(29), which exactly verifies the accuracy of our derivations. In Fig. 3, we further present the outage performance of three users scenario as an extension. It can be observed that, for scheme A, the outage probabilities of all users show similar trends, and all users can experience the same diversity gain of two. By contrast, for scheme B, the outage probabilities of different users show different trends, and the m-th user can achieve a diversity gain of $m+1$, which is also consistent with our analysis provided in Section IV.

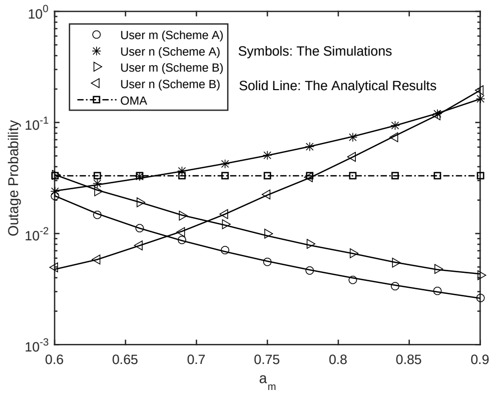

FIGURE 4: Outage probability vs. a_m with $d_1 = 0.5$, $m=1$, $n=M=4$, $\rho = 10dB$, $\gamma_{tar}^{m} = 1dB$, and $\gamma_{tar}^{n} = 3dB$.

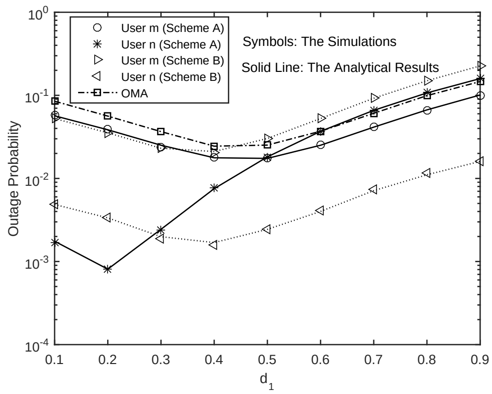

FIGURE 5: Outage probability vs. d_1 with $m=1$, $n=M=4$, $\rho = 15dB$, $d_1 = 0.5$, $a_m = 0.8$, $a_n = 0.2$, $\gamma_{tar}^{m} = 1dB$, and $\gamma_{tar}^{n} = 3dB$.

In Figs. 4 and 5, we illustrate outage performance by varying a_m and d_1, respectively. Firstly, it can be seen that the exact analytical results match very well with the simulations in both Figs. 4 and 5. Then, from Fig. 4, we can observe that a_m has a considerable impact on the outage probability of users. For fixed preset target SINR, the outage probability of user m decreases and that of user n increases with a_m. Moreover, the position of relay also has a significant impact on outage performance (see Fig. 5), especially for scheme A. Since the relay is more likely to decode and retransmit the signals successfully when it is near to the source ($d_1 \leq 0.4$), sorting user according to the links $R \to D$ is more efficient in this case. However, when relay is far away from source, the decoding is more difficult for relay, and the outage performance gap between scheme A and OMA narrows with d_1.

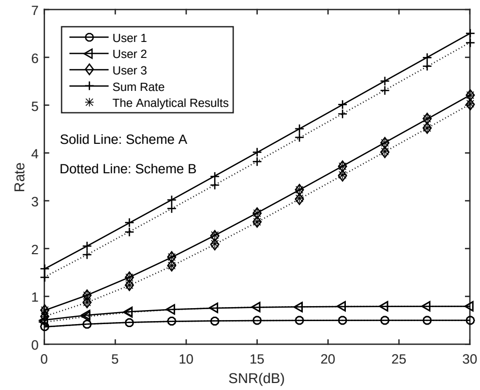

FIGURE 6: Ergodic rate vs. ρ with $d_1 = 0.5$, $a_1 = 1/2$, $a_2 = 1/3$, $a_3 = 1/6$.

B. ERGODIC RATE

To obtain more insightful results, we next analyze the ergodic rate of users under different user ordering schemes. In Fig. 6, the ergodic rate is shown as a function of average SNR. Firstly, we can see the exact analytical results in (44) and (46) match very well with the simulations, which confirms the correctness of our derivations. Then, it can be seen that, for user m ($m < M$), the ergodic rate nearly remains unchanged at high SNR. By contrast, the ergodic rate of the M-th user is highly related to average SNR. That is to say, in the high-SNR region, the ergodic sum rate mainly depends on the M-th user, which is also consistent with the description in Section V. Another observation from Fig. 6 is the difference of two user ordering schemes. When the relay's position is in the middle between source and users, user D_M's ergodic rate of scheme A is larger than that of scheme B, thus also resulting in a larger ergodic sum rate.

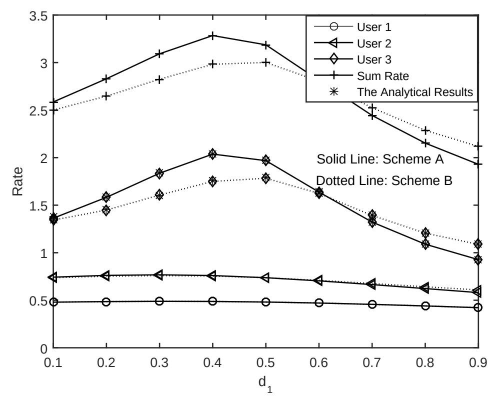

FIGURE 7: Ergodic rate vs. d_1 with $\rho = 10dB$, $a_1 = 1/2$, $a_2 = 1/3$, $a_3 = 1/6$.

Fig. 7 depicts the performance of different user ordering schemes by varying d_1. As can be observed, the position of relay has a significant impact on the ergodic rate. When the position of relay is near to source, scheme A can achieve larger ergodic sum rate than scheme B. On the contrary, when

$d_1 > 0.6$, scheme B is the better one. This is because the relay's decoding task is more difficult when it is far from the source. Comparing scheme A with scheme B, we can find that, scheme A can achieve larger ergodic rate in some cases, while scheme B is less sensitive to relay's location. This observation also justifies the analysis in Fig. 5.

VII. CONCLUSION

In this paper, two major user ordering schemes in cooperative NOMA have been analyzed and compared from two aspects: the outage performance and ergodic rate. As for outage performance, we have derived the closed-form expressions of users, and then obtained the corresponding diversity gain. As for ergodic rate, exact closed-form expression of the strongest user, and upper bounds of the rest users in the high-SNR region have been derived. Monte Carlo simulations have demonstrated that, on one hand, sorting users based on relay-aid path can provide larger ergodic sum rate in some cases, and on the other hand, sorting users based on direct path is less sensitive to the relay's location, and can provide larger diversity gain.

.

APPENDIX A PROOF OF (24)

To address (23), we first assume $\tau^n \geq \frac{\gamma_{tar}^m}{b}$. Note that $b > 0$ and $c > 0$, it can be observed that $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) > 0$. Consequently, if $\lambda_{SD_n} \in \left(\frac{\tau^n c - \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c}, \frac{\tau^n c + \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c}\right)$, we have $\frac{\gamma_{tar}^m - b\lambda_{SD_n}}{c\lambda_{SD_n} + b} < \tau^n - \lambda_{SD_n}$. Furthermore, it can be also observed that $\frac{\tau^n c - \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c} < 0$ as well as $\frac{\tau^n c + \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c} > \tau^n$. Accordingly, Ψ_7 can be calculated as (47):

$$\begin{aligned}\Psi_7 = & \int_0^{\tau^n} f_{\lambda_{SD_n}}(x)dx \int_{\tau^n - x}^{\infty} f_{\lambda_{RD_n}}(y)dy \\ & + \int_{\tau^n}^{\infty} f_{\lambda_{SD_n}}(x)dx \int_0^{\infty} f_{\lambda_{RD_n}}(y)dy \\ & = 1 + \frac{1}{\Omega_X} \sum_{i=n}^{M} \sum_{j=0}^{i} (-1)^{j+1} \binom{i}{j} \binom{M}{i} e^{-\frac{(j+M-i)\tau^n}{\Omega_Z}} \\ & \times \frac{\Omega_X \Omega_Z}{[\Omega_Z - \Omega_X(j+M-i)]} \left(1 - e^{-\frac{[\Omega_Z - \Omega_X(j+M-i)]\tau^n}{\Omega_X \Omega_Z}}\right).\end{aligned} \tag{47}$$

Therefore, by combining (20)-(22), and (47), the outage probability of the n-th user D_n can be attained as (24) for the case of $\tau^n \geq \frac{\gamma_{tar}^m}{b}$.

APPENDIX B PROOF OF (25) AND (26)

If $\tau^n < \frac{\gamma_{tar}^m}{b}$, $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) \geq 0$ may not always hold. Then, for the case of $\tau^n < \frac{\gamma_{tar}^m}{b}$ and $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) < 0$, we have $\frac{\gamma_{tar}^m - b\lambda_{SD_n}}{c\lambda_{SD_n} + b} > \tau^n - \lambda_{SD_n}$, and Ψ_7 can be calculated as:

$$\begin{aligned}\Psi_7 = & \int_0^{\frac{\gamma_{tar}^m}{b}} f_{\lambda_{SD_n}}(x)dx \int_{\frac{\gamma_{tar}^m - bx}{cx+b}}^{\infty} f_{\lambda_{RD_n}}(y)dy \\ & + \int_{\frac{\gamma_{tar}^m}{b}}^{\infty} f_{\lambda_{SD_n}}(x)dx \int_0^{\infty} f_{\lambda_{RD_n}}(y)dy.\end{aligned} \tag{48}$$

Consequently, by following the similar steps to (14)-(18), the outage probability of D_n in this case can be derived as (25).

For the case of $\tau^n < \frac{\gamma_{tar}^m}{b}$ and $(\tau^n)^2c - 4(\gamma_{tar}^m - b\tau^n) \geq 0$, it can be verified that both the two intersection points of the two functions $\frac{\gamma_{tar}^m - b\lambda_{SD_n}}{c\lambda_{SD_n} + b}$ and $\tau^n - \lambda_{SD_n}$ exist. Further define $x_1 = \frac{\tau^n c - \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c}$ and $x_2 = \frac{\tau^n c + \sqrt{(\tau^n c)^2 - 4c(\gamma_{tar}^m - b\tau^n)}}{2c}$. Note that $0 < x_1 \leq x_2 < \frac{\gamma_{tar}^m}{b}$, and Ψ_7 can be given by

$$\begin{aligned}\Psi_7 = & \underbrace{\int_0^{x_1} f_{\lambda_{SD_n}}(x)dx \int_{\frac{\gamma_{tar}^m - bx}{cx+b}}^{\infty} f_{\lambda_{RD_n}}(y)dy}_{\Psi_8} \\ & + \underbrace{\int_{x_1}^{x_2} f_{\lambda_{SD_n}}(x)dx \int_{\tau^n - x}^{\infty} f_{\lambda_{RD_n}}(y)dy}_{\Psi_9} \\ & + \underbrace{\int_{x_2}^{\frac{\gamma_{tar}^m}{b}} f_{\lambda_{SD_n}}(x)dx \int_{\frac{\gamma_{tar}^m - bx}{cx+b}}^{\infty} f_{\lambda_{RD_n}}(y)dy}_{\Psi_{10}} \\ & + \underbrace{\int_{\frac{\gamma_{tar}^m}{b}}^{\infty} f_{\lambda_{SD_n}}(x)dx \int_0^{\infty} f_{\lambda_{RD_n}}(y)dy}_{\Psi_{11}}.\end{aligned} \tag{49}$$

Therefore, following the similar steps to (14)-(18), Ψ_8 can be calculated as

$$\begin{aligned}\Psi_8 = & 1 - e^{-\frac{x_1}{\Omega_X}} + e^{\frac{(j+M-i)b}{\Omega_Z c} + \frac{b}{\Omega_X c}} \frac{1}{\Omega_X \Omega_Z c} \sum_{i=n}^{M} \sum_{j=0}^{i} (-1)^{j+1} \\ & \times \binom{i}{j} \binom{M}{i} \Big\{ \mu_{x_1} + \Omega_X \Omega_Z c \Big(e^{-\frac{b}{\Omega_X c}} - e^{-\frac{cx_1 + b}{\Omega_X c}} \Big) \\ & - d\Big[Ei\big(-\tfrac{cx_1 + b}{\Omega_X c}\big) - Ei\Big(-\tfrac{b}{\Omega_X c}\Big)\Big]\Big\}.\end{aligned} \tag{50}$$

Then, following the similar steps in (47), Ψ_9 can be calculated as

$$\begin{aligned}\Psi_9 = & \int_{x_1}^{x_2} \frac{1}{\Omega_X} e^{-\frac{x}{\Omega_X}} dx \int_{\tau^n - x}^{\infty} f_{\lambda_{RD_n}}(y)dy \\ & = e^{-\frac{x_1}{\Omega_X}} - e^{-\frac{x_2}{\Omega_X}} + \frac{1}{\Omega_X} \sum_{i=n}^{M} \sum_{j=0}^{i} (-1)^{j+1} \binom{i}{j} \binom{M}{i} \\ & \times e^{-\frac{(j+M-i)\tau^n}{\Omega_Z}} \frac{\Omega_X \Omega_Z}{[\Omega_Z - \Omega_X(j+M-i)]} \\ & \times \Big(e^{-\frac{[\Omega_Z - \Omega_X(j+M-i)]x_1}{\Omega_X \Omega_Z}} - e^{-\frac{[\Omega_Z - \Omega_X(j+M-i)]x_2}{\Omega_X \Omega_Z}} \Big).\end{aligned} \tag{51}$$

Similar to Ψ_8, Ψ_{10} can be calculated as

$$\begin{aligned}\Psi_{10} = & e^{-\frac{x_2}{\Omega_X}} - e^{-\frac{\frac{\gamma_{tar}^m}{b}}{\Omega_X}} + e^{\frac{(j+M-i)b}{\Omega_Z c} + \frac{b}{\Omega_X c}} \frac{1}{\Omega_X \Omega_Z c} \\ & \times \sum_{i=n}^{M} \sum_{j=0}^{i} (-1)^{j+1} \binom{i}{j} \binom{M}{i} \Big\{ \mu_{x_2} + \Omega_X \Omega_Z c \\ & \times \Big(e^{-\frac{cx_2 + b}{\Omega_X c}} - e^{-\frac{c\frac{\gamma_{tar}^m}{b} + b}{\Omega_X c}} \Big) - d\Big[Ei\Big(-\frac{c\frac{\gamma_{tar}^m}{b} + b}{\Omega_X c}\Big) \\ & - Ei\Big(-\frac{cx_2 + b}{\Omega_X c}\Big)\Big]\Big\}.\end{aligned} \tag{52}$$

Ψ_{11} can be obtained directly as

$$\Psi_{11} = \int_{\frac{\gamma_{tar}^m}{b}}^{\infty} \frac{1}{\Omega_X} e^{-\frac{x}{\Omega_X}} dx \int_0^{\infty} f_{\lambda_{RD_n}}(y) dy = e^{-\frac{\gamma_{tar}^m}{b\Omega_X}}. \tag{53}$$

Finally, by substituting (49)-(53) into (20), we can derive the outage probability of the n-th user, thus completing the proof of (26).

APPENDIX C PROOF OF (42)

In this scheme, the PDF of λ_{RD_M} is given by

$$f_{\lambda_{RD_M}}(x) = \frac{k}{\Omega_Z} \sum_{k=0}^{M} \binom{M}{k} (-1)^{k+1} e^{-\frac{xk}{\Omega_Z}}. \tag{54}$$

Subsequently, the PDF of $\lambda_{RD_M} + \lambda_{SD_M}$ can be expressed by

$$\begin{aligned} & f_{\lambda_{RD_M}+\lambda_{SD_M}}(z) = \textstyle\int_{-\infty}^{+\infty} f_{\lambda_{RD_M}}(x) f_{\lambda_{SD_M}}(z-x) dx \\ & = \sum_{k=0}^{M} \binom{M}{k} (-1)^{k+1} \tfrac{k}{\Omega_Z} \tfrac{1}{\Omega_X} e^{-\frac{z}{\Omega_X}} \textstyle\int_0^z e^{-\frac{(\Omega_X k - \Omega_Z)x}{\Omega_X \Omega_Z}} dx \\ & = \sum_{k=0}^{M} \binom{M}{k} (-1)^{k+1} \tfrac{k}{\Omega_X k - \Omega_Z} \left(e^{-\frac{z}{\Omega_X}} - e^{-\frac{kz}{\Omega_Z}}\right). \end{aligned} \tag{55}$$

Accordingly, the CDF of $\lambda_{RD_M} + \lambda_{SD_M}$ can be calculated based on (55). Finally, the CDF of ω is given by

$$\begin{aligned} & F_\omega(x) = F_{\lambda_{SD_M}}(x) F_{min(\lambda_{SR}, \lambda_{RD_M}+\lambda_{SD_M})}(x) \\ & = F_{\lambda_{SD_M}}(x)\{1 - [1 - F_{\lambda_{SR}}(x)][1 - F_{\lambda_{RD_M}+\lambda_{SD_M}}(x)]\} \\ & = \left(1 - e^{-\frac{x}{\Omega_X}}\right)\Big\{1 - e^{-\frac{x}{\Omega_Y}} \Big\{1 - \sum_{k=0}^{M} \binom{M}{k} (-1)^{k+1} \\ & \times \tfrac{k}{\Omega_X k - \Omega_Z} \left[\tfrac{\Omega_Z}{k}(e^{-\frac{kx}{\Omega_Z}} - 1) - \Omega_X(e^{-\frac{x}{\Omega_X}} - 1)\right]\Big\}\Big\}. \end{aligned} \tag{56}$$

REFERENCES

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, "What will 5G be?" in IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065-1082, June 2014.

[2] Q. Li, H. Niu, A. Papathanassiou, and G. Wu, "5G network capacity:Key elements and technologies," in IEEE Veh. Technol. Mag., vol. 9, no. 1, pp. 71-78, Mar. 2014.

[3] L. Yang, J. Chen, Q. Ni, J. Shi, and X. Xue, "NOMA-enabled Cooperative Unicast-Multicast: Design and Outage Analysis," in IEEE Trans. on Wireless Commun., vol. 16, no. 12, pp. 7870-7889, Dec. 2017.

[4] Y. Sun, D. W. K. Ng, Z. Ding, and R. Schober, "Optimal Joint Power and Subcarrier Allocation for Full-Duplex Multicarrier Non-Orthogonal Multiple Access Systems," in IEEE Trans. on Commun., vol. 65, no. 3, pp. 1077-1091, Jan. 2017.

[5] Y. Sun, D. W. K. Ng, Z. Ding and R. Schober, "Optimal Joint Power and Subcarrier Allocation for MC-NOMA Systems," in 2016 IEEE Global Communications Conference (GLOBECOM), Washington, DC, USA, Dec. 2016, pp. 1-6.

[6] V. W. S. Wong, et al., Key Technologies for 5G Wireless Systems. Cambridge, UK, 2017.

[7] A. Benjebbovu et al., "Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access," in Proc. IEEE ISPACS, 2013, pp. 770-774.

[8] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," in IEEE Commun. Lett., vol. 19, no. 8, pp. 1462-1465, Aug. 2015.

[9] Z. Wei, L. Dai, D. W. K. Ng, and J. Yuan,"Performance Analysis of a Hybrid Downlink-Uplink Cooperative NOMA Scheme", in 2017 Vehicular Technology Conference (VTC Spring), Sydney, NSW, Australia, Jun. 2017, doi: 10.1109/VTCSpring.2017.8108407.

[10] L. Lv, J. Chen, and Q. Ni, "Cooperative Non-Orthogonal Multiple Access in Cognitive Radio," in IEEE Commun. Letters, vol. 20, no. 10, pp. 2059 - 2062, Aug. 2016.

[11] L. Lv, J. Chen, Q. Ni, and Z. Ding. "Design of Cooperative Non-Orthogonal Multicast Cognitive Multiple Access for 5G Systems: User Scheduling and Performance Analysis", in IEEE Trans on Commun., vol. 65, no. 6, pp. 2641-2656, Jun. 2017.

[12] G. Liu, X. Chen, Z. Ding, Z. Ma, and F. R. Yu, "Hybrid Half-Duplex/Full-Duplex Cooperative Non-Orthogonal Multiple Access With Transmit Power Adaptation," in IEEE Trans. on Wireless Commun., vol. 17, no.1, pp. 506-519, Nov. 2017.

[13] J. Men and J. Ge, "Performance analysis of non-orthogonal multiple access in downlink cooperative network," in IET Commun., vol. 9, no. 18, pp. 2267-2273, Dec. 2015.

[14] J. Men, and J. Ge, C. Zhang, "Performance analysis of Non-Orthogonal Multiple Access for relaying networks over Nakagami-m fading channels," in IEEE Trans. on Vehicular Technology, vol. 66, no. 2, pp. 1200-1208, Feb. 2017.

[15] J. Men, and J. Ge, "Non-orthogonal multiple access for multiple-antenna relaying networks," in IEEE Commun. Lett., vol. 19, no. 10, pp. 1686-1689, Oct. 2015.

[16] Y. Zhang, J. Ge, and E. Serpedin, "Performance analysis of non-orthogonal multiple access for downlink networks with antenna selection over Nakagami-m fading channels," in IEEE Trans. on Vehicular Technology, vol. 66, no. 11, pp. 10590-10594, Sep. 2017.

[17] D. Wan, M. Wen, F. Ji, Y. Liu, and Y. Huang, "Cooperative NOMA Systems With Partial Channel State Information Over Nakagami-m Fading Channels", in IEEE Trans. on Commun., vol. 66, no. 3, pp. 947-958, Nov. 2017.

[18] Y. Sun, D. W. K. Ng, and R. Schober, "Resource Allocation for MC-NOMA Systems with Cognitive Relaying," in 2017 IEEE Globecom Workshops (GC Wkshps), Singapore, Singapore, Jan. 2018, doi: 10.1109/GLOCOMW.2017.8269087.

[19] Y. Xiao, L. Hao, Z. Ma, Z. Ding, Z. Zhang, and P. Fan, "Forwarding Strategy Selection in Dual-Hop NOMA Relaying Systems", in IEEE Commun. Lett.,doi: 10.1109/LCOMM.2018.2803809, to be published in 2018.

[20] Z. Song, Q. Ni, and X. Sun,"Distributed Power Allocation for Nonorthogonal Multiple Access Heterogeneous Networks", in IEEE Commun. Letters, vol. 22, no. 3, pp. 622-625, Jan. 2018.

[21] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K.Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in Proc. IEEE Vehicular Technology Conference (VTC), Dresden, Germany, Jun. 2013.

[22] Z. Ding, X. Lei, G. Karagiannidis, R. Schober, J. Yuan, and V. Bhargava, "A Survey on Non-Orthogonal Multiple Access for 5G Networks: Research Challenges and Future Trends," in IEEE J. Sel. Areas Commun., vol. 35, no. 10, pp. 2181-2195, Jul. 2017.

[23] Y. Xu, H. Sun, R. Q. Hu, and Y. Qian, "Cooperative non-orthogonal multiple access in heterogeneous networks," in Proc. IEEE Global Commun. Conf. (GlobeCom), San Diego, CA, USA, Dec. 2015, pp. 1-6.

[24] Z. Ding, Z. Yang, P. Z. Fan, and H. V. Poor, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," in IEEE Signal Process. Lett., vol. 21, no. 12, pp. 1501-1505, Dec. 2014.

[25] J. Li, L. J. Cimini, J. Ge, C. Zhang, and H. Feng, "Optimal and Suboptimal Joint Relay and Antenna Selection for Two-Way Amplify-and-Forward Relaying," in IEEE Trans. on Wireless Commun., vol. 15, no. 2, pp. 980-993, Feb. 2016.

[26] H. A. David and H. N. Nagaraja, 'Order Statistics', 3rd edn, New York, USA, 2003.

[27] I. S. Gradshteyn, and I. M. Ryzhik. 'Table of Integrals, Series and Products', Academic Press, New York, 6th edn, 2000.

JIAN ZHANG received the B.Sc. and M.Sc. degrees in School of Telecommunications Engineering from Xidian University, Xi'an, China, in 2010, and 2014, respectively. He is currently working towards the Ph.D. degree in communication and information system at the same university. In particular, He is very interested in performance analysis of wireless communication systems. His current research interests are in the fields of cooperative networks, non-orthogonal multiple access (NOMA), and energy harvesting.

JIANHUA GE received his Ph.D. degree in Schoolof Telecommunications Engineering from Xidian University, Xi'an, China, in 1990. He is currently a Professor and the Deputy Director of the State Key Laboratory of Integrated Services Networks, School of Telecommunications Engineering, Xidian University. He has worked on digital television (DTV) standardization as a DTV technical expert. His research interests include digital communications, digital video broadcasting, performance enhancement techniques for 4G/5G cellular communication systems.

QIANG NI (M'04-SM'08) received the B.Sc., M.Sc., and Ph.D. degrees in engineering from the Huazhong University of Science and Technology, China, in 1993, 1996, and 1999, respectively. He is currently a Professor and the Head of the Communication Systems Group, School of Computing and Communications, Lancaster University, Lancaster, U.K. His research interests include the area of future generation communications and networking, including green communications and networking, cognitive radio network systems, heterogeneous networks, 5G, SDN, cloud networks, energy harvesting, wireless information and power transfer, IoTs, cyber physical systems, machine learning, big data analytics, and vehicular networks. He has authored or co-authored over 200 papers in these areas. He was an IEEE 802.11 Wireless Standard Working Group Voting Member and a Contributor to the IEEE Wireless Standards.

MIAOWEN WEN (M'14-SM'18) received the B.S. degree from Beijing Jiaotong University, Beijing, China, in 2009, and the Ph.D. degree from Peking University, Beijing, China, in 2014. From 2012 to 2013, he was a Visiting Student Research Collaborator with Princeton University, Princeton, NJ, USA. He is currently an Associate Professor with the South China University of Technology, Guangzhou, China. He has published a Springer book entitled Index Modulation for 5G Wireless Communications and over 90 research papers, which include over 50 journal papers and over 30 conference papers. His research interests include index modulation, non-orthogonal multiple access, physical layer security, and molecular communications.

Dr. Wen was the recipient of the Excellent Doctoral Dissertation Award from Peking University and the Best Paper Awards from the IEEE International Conference on Intelligent Transportation Systems Telecommunications (ITST) in 2012, the IEEE International Conference on Intelligent Transportation Systems (ITSC) in 2014, and the IEEE International Conference on Computing, Networking and Communications (ICNC) in 2016. He was recognized as an Exemplary Reviewer for the IEEE Communications Letters in 2017. He is currently serving as a symposium co-chair for the IEEE ICNC2019, a workshop co-chair for the IEEE/CIC International Conference on Communications in China (ICCC'2018), and a Guest Editor for IEEE Journal on Selected Areas in Communications (Special Issue on Spatial Modulation for Emerging Wireless Systems) and for IEEE Journal of Selected Topics in Signal Processing (Special Issue on Index Modulation for Future Wireless Networks: A Signal Processing Perspective). He has served on the Editorial Boards of several international journals, including the IEEE Access, the EURASIP Journal on Wireless Communications and Networking, the ETRI Journal, and the Physical Communication (Elsevier).

YANGYANG ZHANG received the B.S. degree from the School of Telecommunications Engineering, Xi'an Polytechnic University, Xi'an, China, in 2014. She is currently pursuing the Ph.D. degree in communication and information system with Xidian University. She is very interested in performance analysis of wireless communication systems. Her current research interests are in the fields of cooperative networks, non-orthogonal multiple access, and energy harvesting.