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# The effect of a movable mass on the aeroelastic stability of composite hingeless rotor blades in hover

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## Abstract

In this paper, the aeroelastic stability of a composite hingeless rotor blade with a chordwise movable mass is investigated. The point mass is located near the tip of the blade and its chordwise location is variable with respect to the elastic axis and can be moved during the flight. This movable mass is added to the blade to actuate the blade twist during flight. By actuating the mass in the chord direction of the blade during the flight, a bending moment which is the result of the centrifugal force of the mass and its offset is induced on the blade. This bending moment induces twist in the blade, due to bend-twist coupling in the composite lamination. The blade is modelled by using the geometrically exact fully intrinsic beam equations along with the variational asymptotic beam sectional analysis. The aerodynamic loads are simulated by using the two-dimensional strip theory combined with a uniform inflow. The nonlinear partial differential aeroelastic equations are discretized by a time-space scheme, and the converged results are compared with those reported in the literature and a very good match is observed. The results show that by positioning the mass near the tip of the blade, and also by using the ply angle of about 30 degree in this configuration, the highest possible twist change is achieved when the mass moves from the leading edge to the trailing edge of the blade. Moreover, the spanwise location of the mass slightly changes the stability boundaries, while the chordwise movement significantly affects the aeroelastic instability.

**Keywords:** Morphing blade, Aeroelastic stability, composite material, bend-twist coupling, fully intrinsic equations, concentrated mass.

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## 25 **Introduction**

26 During the past decades of development in rotorcraft industry, different concepts have been suggested  
27 to enhance the performance of the vehicle and at the same time to decrease the pollution, noise and  
28 vibration, by changing the shape of the blade. Blade twist morphing is a concept which could modify  
29 the shape of the blade in flight to achieve the best performance in each flight condition. For helicopter  
30 rotors, the twist distribution that minimizes the power requirement is different in each flight condition  
31 [1]. Therefore, the predefined blade twist variation normally is chosen as a compromise between  
32 different flight conditions. Blade twist morphing changes the blade twist during flight to allow the  
33 rotorcraft to fly in an optimum condition in terms of twist variation. Han et al. [2] showed how the  
34 performance of a helicopter during flight could be improved by dynamic blade twist. They  
35 demonstrated that the dynamic blade twist improves the performance and reduces the rotor power  
36 requirement. Chen and Chopra [3] studied the effect of piezoelectric actuators on the twist change of  
37 blades. The piezoelectric patches were positioned on the top and bottom of the blade and about  $0.4^\circ$  of  
38 twist change was achieved. Then this concept was tested in hover condition, and it was proved that a  
39 linear twist change of about  $0.6^\circ$  can modify the rotor lift by 10% [4]. Reduction and control of the  
40 vibratory loads of a composite box beam blade with using the smart materials was considered by  
41 Chattopadhyay et al. [5]. It was found that the number of actuators and their location have significant  
42 effect on the reduction of dynamic loads. Cesnik et al. [6] developed an analytical model for  
43 modelling an active twist rotor blade with distributed anisotropic strain actuators. This active twist  
44 rotor aimed to reduce the vibration and noise of the blade, and good correlation with experiments was  
45 observed. This study developed further to cover the forward flight condition by Shin and Cesnik [7].  
46 Prashant and Sung [8] analysed how the active twist concept affect the active vibration reduction of  
47 composite blades with imperfections. They showed that the rotor vibratory loads and also the energy  
48 input may be influenced by introducing imperfections to the blade. Twist distribution modification of  
49 a tiltrotor blade based on shape memory alloy torque tube was studied by Prahlad and Chopra [9]. In  
50 this study, the actuation behaviour was tuned by the heat treatment of SMAs. Mistry et al. [1]  
51 developed a warp-induced twist variation concept for rotary-wing applications. In this method, the

52 twist of the blade changes by rotation of a threaded rod. More recently, Amoozgar et al. [10, 11]  
53 developed a novel concept for twist morphing of composite blades. In this study, the twist of the blade  
54 was the result of mass movement and stiffness tailoring of the composite blade.

55 Aeroelastic analysis of helicopter rotor blades is a key design requirement. Hingeless rotor blades are  
56 normally considered as cantilevered beams, and the common type of aeroelastic instability is the one  
57 characterized by the coupling between lead-lag bending, flap bending, and torsion deflections of  
58 blades. The frequency of this instability is usually near to the lead-lag natural frequency [12]. There  
59 are some review papers dedicated to surveying different models used for composite rotor blade  
60 analysis [13]. One of the first studies concerning with the aeroelastic stability of composite rotor  
61 blades, was considered by Hong and Chopra [14]. It was found that depending on the laminate design  
62 of the box beam, different stability characteristics may be obtained. Panda and Chopra [15]  
63 determined the aeroelastic stability and response of composite hingeless rotor blades based on  
64 moderate deflection beam theory in forward flight. The effect of ply orientation and elastic coupling  
65 on the vibration and stability was shown. The effect of transverse shear deformation on the modelling  
66 of the rotor blade for aeroelastic analysis and response of composite rotors has been presented by  
67 Smith and Chopra [16] in forward flight. They also showed that the unsteady aerodynamic increases  
68 the vibratory load up to 30%. Kim and Dugundji [17] investigated the large amplitude, nonlinear  
69 aeroelastic behaviour of composite hingeless rotor blades in hover condition. Numerical results  
70 showed that in moderate amplitude, the nonlinear aerodynamics is dominant, and nonlinear static-  
71 dynamic structural couplings can affect the aeroelastic behaviour at large amplitudes. The aeroelastic  
72 response and vibratory loads of an elastically tailored composite rotor blade has been determined by  
73 Smith [18]. It was highlighted that the positive or negative elastic couplings have stabilizing or  
74 destabilizing effect on the lag mode damping. Tracy and Chopra [19] studied the aeroelastic stability  
75 of a composite hingeless rotor blade in hover flight. In the positive collective pitch angles, the lag  
76 damping mode stabilizes with negative chordwise bending-torsion coupling. The influence of fibre  
77 orientation and stacking sequence on the aeroelastic stability of composite rotor blades has been

78 investigated by Jeon et al. [20]. The lag mode instability is influenced by the bending-twist coupling  
79 in the symmetric lamination, and the extension-twist coupling in the antisymmetric configuration.

80 A new formulation based on exact beam formulation and unsteady dynamic wake aerodynamic model  
81 was considered by Shang et al. [21] for aeroelastic stability analysis of composite hingeless rotor  
82 blades. The initial twist and curvature of the composite blade can improve the aeroelastic stability and  
83 reduce the static loads. Jeon and Lee [22] considered the aeroelasticity of a composite rotor blade  
84 using a finite element method based on large deflection beam theory in forward flight. It was  
85 proposed that when the deflection is large, the full finite element should be used instead of modal  
86 approach to predict the stability behaviour accurately. An analytical model for investigating the  
87 aeroelasticity of composite blades with swept tips was proposed by Friedmann et al. [23]. The tip  
88 sweep can have destabilizing effect on the blade, while this instability can be removed in some certain  
89 ply angles of the composite blade. Bao et al. [24] designed and tested several Mach scaled composite  
90 blades to reduce the vibratory loads of the blade and good correlation in hover condition was  
91 observed. Friedmann et al. [25] examined the compatibility between the composite cross-sectional  
92 analysis based on variational asymptotic approach and a moderate deflection beam model, and the  
93 results were validated with experimental data. The aeroelastic stability behaviour of an isolated  
94 composite hingeless rotor blade has been determined by Fulton and Hodges [26]. The blade was  
95 modelled by a geometrically exact beam formulation without any restrictions on the rotations and  
96 displacements magnitudes. The analysis showed that the non-classical couplings affect the aeroelastic  
97 stabilities, and therefore must be considered in general purpose analysis. Lim and Lee [27] studied the  
98 aeroelastic analysis of bearingless rotor blades considering a composite flexbeam by using a large  
99 deflection beam theory. They showed that the bending-torsional coupling of the composite layup  
100 could change the stability of the lag mode. The aeroelastic stability of composite hingeless rotors by  
101 using the free-wake aerodynamic model has been also considered by Xiao et al. [28].

102 Byers and Gandhi [29-31] explored the influence of a moving mass in the spanwise direction on the  
103 aeroelastic stability to produce a vibration absorber. They showed the Coriolis forces couple the  
104 flapping and the lead-lag motion together and hence affect the rotor stability. The effect of embedded

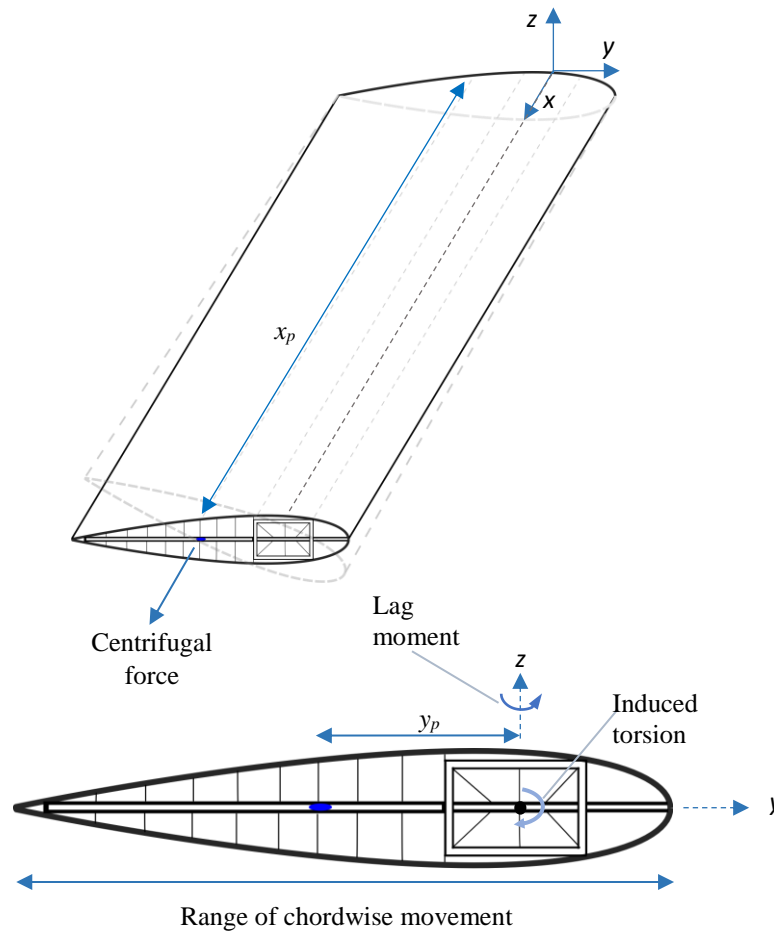
105 chordwise absorbers on the stability of the rotor system was studied experimentally and analytically  
106 by Kang et al. [32]. They showed that using the chordwise absorbers improves the stability of rotors.  
107 To add to the aforementioned literature, in this study a new twist morphing concept based on the mass  
108 movement is introduced, and the effect of this morphing concept on the aeroelastic stability  
109 boundaries of the composite hingeless rotor blade in hovering condition is determined. The blade  
110 structural model is based on the geometrically exact fully intrinsic beam equations [33] and the  
111 aerodynamic loading on the blade is a combination of the quasi-steady strip aerodynamic theory and  
112 the uniform inflow [34]. The added mass is modelled as a concentrated mass attached to the blade  
113 which can move in different directions [10], and the cross-sectional characteristics are determined by  
114 using the variational asymptotic approach [35].

### 115 **Problem Statement**

116 A composite hingeless rotor blade is modelled here as a cantilevered beam attached to the blade hub.  
117 The blade has a composite rectangular closed box section as a spar. The blade is equipped with a track  
118 attached to the spar to allow for point mass chordwise movement in flight. This mass movement  
119 produces a variable in-plane bending moment due to the centrifugal force acting on the mass. This  
120 bending moment then turns to an equivalent torsional moment through the spar lag-torsion coupling.  
121 Therefore, the twist of the blade depends on the point mass chordwise movement. The point mass  
122 location along the blade coordinate system is denoted here as  $x_p$  and  $y_p$ , respectively. Figure 1 shows a  
123 schematic of the morphing system described above. The chordwise position of the mass ( $y_p$ ), shown in  
124 Figure 1, is able to change in flight and assumes a suitable mechanism may be designed. Thus, when  
125 the flight condition changes, the required blade twist, in terms of the optimum performance, changes.  
126 The spanwise location of the added mass is a fixed design variable. It is assumed that the added mass  
127 doesn't have any offset in the  $z$  direction ( $z_p=0$ ). The origin of the coordinate system is located at the  
128 root of the blade and on the quarter chord of the section. The  $x$  axis is along the blade span, and the  $y$   
129 axis is along the chord of the blade toward the leading edge of the blade.

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Figure 1: Schematic of the morphing twist change concept

### 138 Aeroelastic Modelling

139 The aeroelastic modelling of the blade is composed of two modules, the structural model and the  
 140 aerodynamic loading model. The three-dimensional structural model of the blade can be divided into  
 141 two parts. In the first part, a two-dimensional cross-sectional analysis is carried out by using the  
 142 variational asymptotic approach [35], and the cross-sectional properties are obtained. Then the global  
 143 behaviour of the blade is modelled by the one-dimensional nonlinear geometrically exact fully  
 144 intrinsic beam equations [33]. This formulation has been successfully used for structural analysis of  
 145 stationary and rotating beams [36-38].

146 The geometrically exact fully intrinsic beam equations express the nonlinear behaviour of generally  
 147 anisotropic, initially twisted and curved beam as

$$\begin{aligned}
148 \quad & \partial F_1 / \partial x_1 + K_2 F_3 - K_3 F_2 + f_{aero_1} = \partial P_1 / \partial t + \Omega_2 P_3 - \Omega_3 P_2 \\
149 \quad & \partial F_2 / \partial x_1 + K_3 F_1 - K_1 F_3 + f_{aero_2} = \partial P_2 / \partial t + \Omega_3 P_1 - \Omega_1 P_3 \\
150 \quad & \partial F_3 / \partial x_1 + K_1 F_2 - K_2 F_1 + f_{aero_3} = \partial P_3 / \partial t + \Omega_1 P_2 - \Omega_2 P_1 \\
151 \quad & \partial M_1 / \partial x_1 + K_2 M_3 - K_3 M_2 + 2\gamma_{12} F_3 - 2\gamma_{13} F_2 + m_{aero_1} = \partial H_1 / \partial t + \Omega_2 H_3 - \Omega_3 H_2 + V_2 P_3 - V_3 P_2 \\
152 \quad & \partial M_2 / \partial x_1 + K_3 M_1 - K_1 M_3 + 2\gamma_{13} F_1 - (1 + \gamma_{11}) F_3 + m_{aero_2} = \partial H_2 / \partial t + \Omega_3 H_1 - \Omega_1 H_3 + V_3 P_1 - V_1 P_3 \\
153 \quad & \partial M_3 / \partial x_1 + K_1 M_2 - K_2 M_1 + (1 + \gamma_{11}) F_2 - 2\gamma_{12} F_1 + m_{aero_3} = \partial H_3 / \partial t + \Omega_1 H_2 - \Omega_2 H_1 + V_1 P_2 - V_2 P_1 \\
154 \quad & \partial V_1 / \partial x_1 + K_2 V_3 - K_3 V_2 + 2\gamma_{12} \Omega_3 - 2\gamma_{13} \Omega_2 = \partial \gamma_{11} / \partial t \tag{1} \\
155 \quad & \partial V_2 / \partial x_1 + K_3 V_1 - K_1 V_3 - (1 + \gamma_{11}) \Omega_3 + 2\gamma_{13} \Omega_1 = 2\partial \gamma_{12} / \partial t \\
156 \quad & \partial V_3 / \partial x_1 + K_1 V_2 - K_2 V_1 + (1 + \gamma_{11}) \Omega_2 - 2\gamma_{12} \Omega_1 = 2\partial \gamma_{13} / \partial t \\
157 \quad & \partial \Omega_1 / \partial x_1 + K_2 \Omega_3 - K_3 \Omega_2 = \partial \kappa_1 / \partial t \\
158 \quad & \partial \Omega_2 / \partial x_1 + K_3 \Omega_1 - K_1 \Omega_3 = \partial \kappa_2 / \partial t \\
159 \quad & \partial \Omega_3 / \partial x_1 + K_1 \Omega_2 - K_2 \Omega_1 = \partial \kappa_3 / \partial t
\end{aligned}$$

160 where,  $x_l$  is the spanwise coordinate of the beam reference line,  $F_i$  and  $M_i$  for  $i=1,2,3$ , are the internal  
161 forces and moments,  $V_i$  and  $\Omega_i$  are the linear and angular velocities,  $P_i$  and  $H_i$  are the sectional linear  
162 and angular momenta, respectively.  $K_i$  is the final curvature of the deformed beam, and  $\gamma_{1i}$  and  $\kappa_{1i}$   
163 are the strain measures. The aerodynamic force and moments on the blade are defined here by  $f_{aero_i}$   
164 and  $m_{aero_i}$  for  $i=1,2,3$ . All these parameters are defined in the deformed coordinate system except the  
165 initial curvature. The details of the formulation can be found in [33]. The cross-sectional properties of  
166 the composite spar is determined by VABS [35], which are then introduced in the beam formulation  
167 through the stiffness matrix as

$$168 \quad \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{12} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{13} & A_{23} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{13} & D_{23} & D_{33} \end{bmatrix} \begin{bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix} \tag{2}$$



169 where  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{D}$  are the composite spar cross-section stiffness components. It is noted that these  
 170 stiffness matrices are different from the stiffness matrices obtained based on lamination theory. As the  
 171 beam is clamped to the root, the fixed boundary conditions are applied to close the formulation.

172 The aerodynamic loads in the hover condition based on the intrinsic expression of the Greenberg's  
 173 theory [34] is defined as

$$\mathbf{f}_{aero} = \mathbf{C}^{Ba} \mathbf{f}_a \quad (3)$$

$$\mathbf{m}_{aero} = \mathbf{C}^{Ba} \mathbf{m}_a + \mathbf{C}^{Ba} \mathbf{x}_a \mathbf{f}_a$$

174 where  $\mathbf{x}_a$  is the offset between the beam reference line, and the aerodynamic centre, and  $\mathbf{C}^{Ba}$  is the  
 175 direction cosine matrix of deformed frame with respect to aerodynamic frame. In this study, it is  
 176 assumed that the offset of the aerodynamic centre from the elastic axis is zero. The aerodynamic force  
 177 and moment equations in the aerodynamic reference frame are [34]

$$\mathbf{f}_a = \rho_\infty b \begin{bmatrix} 0 \\ c_{l_a} V_{a_3}^2 - c_{d_0} V_T V_{a_2}^2 + c_{d_a} V_{a_3} V_{a_2} \\ -c_{l_a} V_{a_2} \left( V_{a_3} - \frac{\Omega_a b}{2} \right) - \frac{c_{l_a} \dot{V}_{a_3} b}{2} - c_{d_0} V_T V_{a_3} + c_{d_a} V_{a_3}^2 \end{bmatrix} \quad (4)$$

$$\mathbf{m}_a = 2\rho_\infty b^2 \begin{bmatrix} -bc_{l_a} V_{a_2} \Omega_a / 8 - c_{l_a} (b^2 \dot{\Omega}_a / 32 - b \dot{V}_{a_3} / 8) \\ 0 \\ 0 \end{bmatrix}$$

178 where  $c_{l_a}$ ,  $c_{d_0}$ , and  $c_{d_a}$  are the airfoil lift and drag coefficients, respectively. The variables with  
 179 subscript (  $_a$  ) are expressed in the aerodynamic reference frame. The induced inflow velocity  
 180 corrects the vertical component of the velocity as follow

$$V_{a_{3T}} = V_{a_3} + \lambda \quad (5)$$

181 The uniform induced inflow velocity determined by the blade element momentum theory at  $3/4$  span,  $\lambda$ ,  
 182 is given as [39]

$$\lambda = \text{sgn}[\theta + \phi(0.75R)] \frac{\pi\sigma}{8} \Omega R \left( \sqrt{1 + \frac{12}{\pi\sigma} |\theta + \phi(0.75R)|} - 1 \right) \quad (6)$$

183 where  $\sigma$  is the blade solidity, and  $\theta$  and  $\phi$  are the blade pitch angle and elastic twist angle,  
 184 respectively.

185 Finally, by combining the structural and aerodynamic models together, the full aeroelastic equations  
 186 can be obtained. To solve the nonlinear aeroelastic equations, a time-space discretization scheme is  
 187 used [33]. In this method, every unknown variable is defined on the right and left hand sides of each  
 188 node. By applying this scheme to the governing equations, the discretized equations of motion for the  
 189  $n^{\text{th}}$  element in the vector format will be:

$$\begin{aligned} \frac{\hat{\mathbf{f}}_l^{n+1} - \hat{\mathbf{f}}_r^n}{dt} + (\tilde{\mathbf{k}}^n + \check{\mathbf{k}}^n) \bar{\mathbf{F}}^n + \bar{\mathbf{f}}_{aero}^n &= \dot{\bar{\mathbf{P}}}^n + \tilde{\boldsymbol{\Omega}}^n \bar{\mathbf{P}}^n \\ \frac{\hat{\mathbf{M}}_l^{n+1} - \hat{\mathbf{M}}_r^n}{dt} + (\tilde{\mathbf{k}}^n + \check{\mathbf{k}}^n) \bar{\mathbf{M}}^n + (\tilde{\mathbf{e}}_1 + \tilde{\mathbf{V}}^n) \bar{\mathbf{F}}^n + \bar{\mathbf{m}}_{aero}^n &= \dot{\bar{\mathbf{H}}}^n + \tilde{\boldsymbol{\Omega}}^n \bar{\mathbf{H}}^n + \tilde{\mathbf{V}}^n \bar{\mathbf{P}}^n \\ \frac{\hat{\mathbf{V}}_l^{n+1} - \hat{\mathbf{V}}_r^n}{dt} + (\tilde{\mathbf{k}}^n + \check{\mathbf{k}}^n) \bar{\mathbf{V}}^n + (\tilde{\mathbf{e}}_1 + \tilde{\mathbf{V}}^n) \bar{\boldsymbol{\Omega}}^n &= \dot{\bar{\mathbf{V}}}^n \\ \frac{\hat{\boldsymbol{\Omega}}_l^{n+1} - \hat{\boldsymbol{\Omega}}_r^n}{dt} + (\tilde{\mathbf{k}}^n + \check{\mathbf{k}}^n) \bar{\boldsymbol{\Omega}}^n &= \dot{\bar{\mathbf{k}}}^n \end{aligned} \quad (7)$$

190 where, subscripts  $r$  and  $l$  refer to the right and left hand sides of each node,  $(\hat{\quad})$  represents the nodal  
 191 value of each variable, and  $(\tilde{\quad})$ , the tilde operator, converts any vector to its corresponding matrix.  
 192 The element variable,  $(\bar{\quad})$ , defines the average of each variable such as  $\mathbf{F}$ , as follows:

$$\bar{\mathbf{F}}^n = \frac{\hat{\mathbf{f}}_l^{n+1} + \hat{\mathbf{f}}_r^n}{2} \quad (8)$$

193 In this way, any discontinuity, such as the point mass, can be taken into account. Hence, the following  
 194 nodal equations are used to consider the nodal mass effect on the equations of motion:

$$\hat{\mathbf{F}}_r^n - \hat{\mathbf{C}}_{lr}^{nT} \hat{\mathbf{F}}_l^n + \hat{\mathbf{f}}_{aero}^n = \dot{\hat{\mathbf{P}}}_r^n + \tilde{\boldsymbol{\Omega}}_r^n \hat{\mathbf{P}}_r^n \quad (9)$$

$$\hat{\mathbf{M}}_r^n - \hat{\mathbf{C}}_{lr}^{nT} \hat{\mathbf{M}}_l^n + \hat{\mathbf{m}}_{aero}^n = \dot{\hat{\mathbf{H}}}_r^n + \tilde{\boldsymbol{\Omega}}_r^n \hat{\mathbf{H}}_r^n + \tilde{\mathbf{V}}_r^n \hat{\mathbf{P}}_r^n \quad (10)$$

195 where,  $\hat{\mathbf{C}}_{lr}^n$  is the slope discontinuity, and in this case it is simply the identity matrix. The added mass  
 196 is introduced to the formulation through the generalized momentum-velocity relation as follows:

$$\begin{Bmatrix} \hat{\mathbf{P}}_r \\ \hat{\mathbf{H}}_r \end{Bmatrix} = \begin{bmatrix} \hat{m} \Delta & -\hat{m} \tilde{\chi} \\ \hat{m} \tilde{\chi} & \hat{I} \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{V}}_r \\ \hat{\boldsymbol{\Omega}}_r \end{Bmatrix} \quad (11)$$

197 where,  $\hat{m}$ ,  $\hat{I}$ , and  $\hat{X}$  are the added mass value, moment of inertia, and location from the beam reference  
198 line. In this study, the moment of inertia of the added mass ( $\hat{I}$ ) about its centroid is assumed to be  
199 zero. The added mass value and its offset from the reference line is zero everywhere except the  
200 location at which the mass is added to the blade:

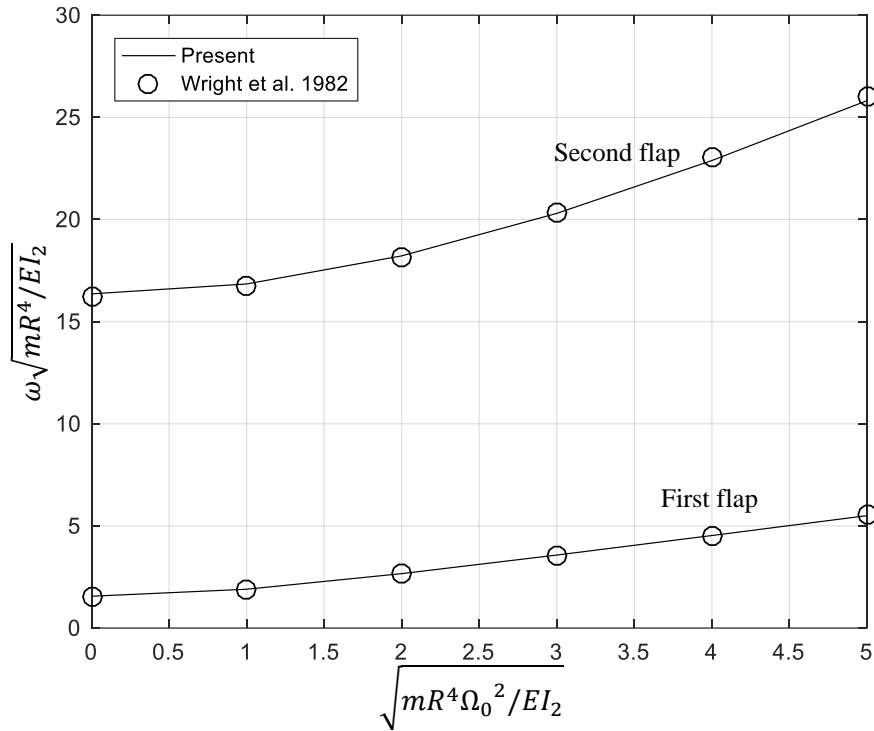
$$\hat{I} = 0, \quad \hat{m}_M = m_p, \quad \hat{X}_M = [0 \quad y_p \quad z_p]^T \quad (12)$$

201 where  $M$  is the node at which the mass is added.

202 First by removing all the time derivatives terms, the steady-state condition of the system is obtained.  
203 Then the nonlinear equations are linearized about the steady-state solution, and the aeroelastic  
204 frequency and damping are determined from the eigenvalue analysis. The aeroelastic stability of the  
205 hingeless rotor blade here is investigated here by checking the lead-lag mode damping as this mode is  
206 more prone to suffer from aeroelastic instability due to low values of drag force in this direction [12].

## 207 **Numerical Results**

208 To check the validity of the developed aeroelastic code, two cases are considered and compared with  
209 those reported in the literature. First the effect of adding a tip point mass with a weight equal to the  
210 blade weight, on the nondimensional first and second flap modes of an isotropic cantilevered beam is  
211 considered. The obtained results are compared with those reported by Wright et al. [40] in Figure 2,  
212 and shown to be a good match. Here  $m$  is the mass per unit length,  $R$  is the blade length,  $\Omega_0$  is the  
213 rotating velocity, and  $EI_2$  is the flap bending stiffness of the blade. It is noted that here the added tip  
214 mass value is equal to the blade overall mass.



215

216

217 **Figure 2: Comparison of the change in frequency parameter with respect to the nondimensional rotating**  
 218 **speed for an isotropic beam with a point mass**

219 To check the accuracy of the aeroelastic analysis results, a composite blade identical to the one used  
 220 in [26] is considered next. The blade spar is a rectangular box section, with outer dimensions of  
 221 12.804 mm and 8.944 mm with a wall thickness of 0.804 mm. The spar is made of AS4/3501-6  
 222 graphite/epoxy with material properties described in Table 1. Each wall of the spar box has 6 layers of  
 223  $[0_2, \zeta_4]$  and the layups are antisymmetric with respect to the mid-plane of the cross-section as shown  
 224 in Figure 3.

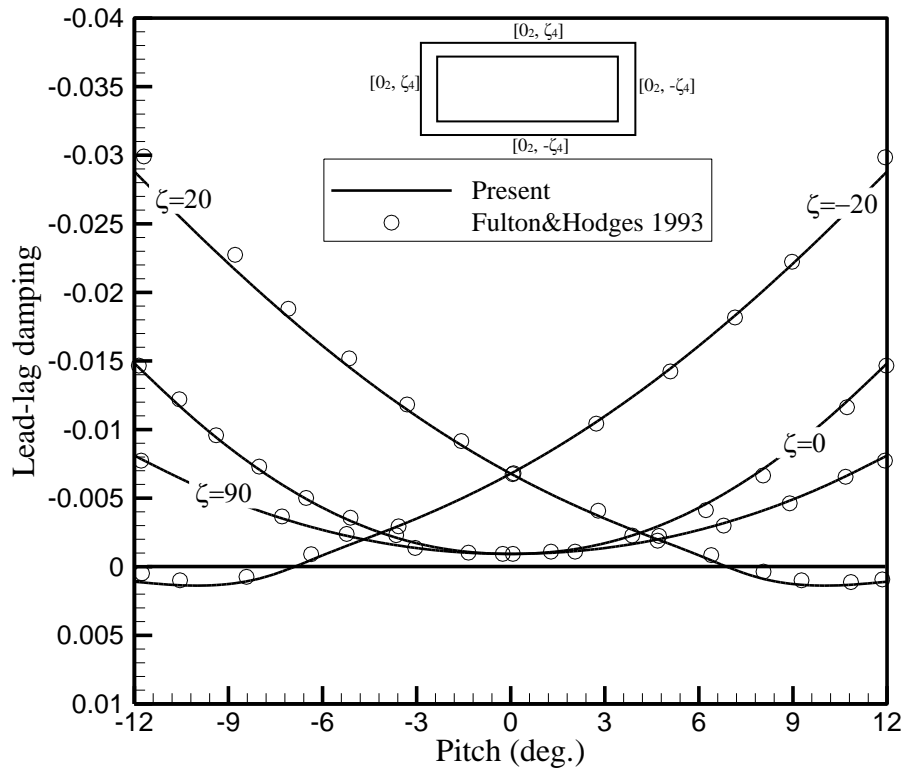
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Table 1: AS4/3501-6 graphite/epoxy material properties [26]

Material property	Value
$E_{11}$ (Gpa)	142
$E_{22} = E_{33}$ (Gpa)	9.81
$G_{12} = G_{13}$ (Gpa)	6
$G_{23}$ (Gpa)	3.77

$v_{12} = v_{13}$	0.3
$v_{23}$	0.34

226 The rotor blade characteristics are listed in Table 2. The nondimensional aeroelastic lead-lag damping  
 227 of this box-beam case is determined for various blade pitch angles, and compared with those obtained  
 228 by Fulton and Hodges [26] in Figure 3. The Timoshenko sectional stiffness matrices determined by  
 229 VABS for this case are presented in Table 3.



230

231

**Figure 3: Comparison of the lead-lag damping of the composite blade**

232

233

Table 2: Hingeless rotor blade characteristics

Parameter	Definition	Value
$\gamma = 3\rho_{\infty}c_{l_a}cR/m$	Lock number	5.593
$\sigma = N_b c / \pi R$	Solidity	0.0572
$c/R$	Chord/blade length	0.08986

$N_b$	Number of blades	2
$c_{d_0}/c_{l_a}$	Drag coefficient to lift coefficient ratio	0.0079/6.283

234

235

Table 3: Timoshenko stiffness matrix

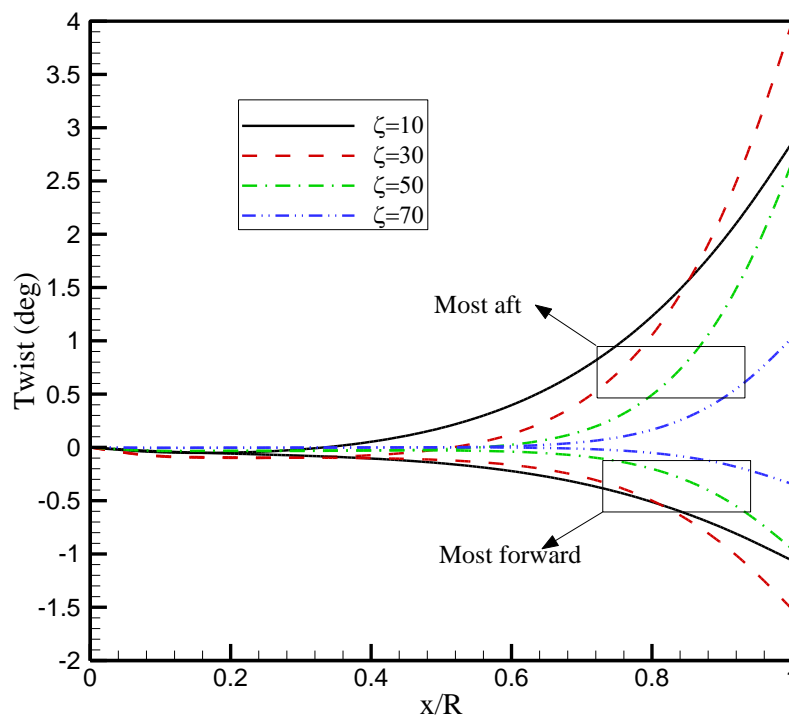
Stiffness	$\zeta=0^\circ$	$\zeta=20^\circ$	$\zeta=90^\circ$
$A_{11}$ (N)	$4.6 \times 10^6$	$3.63 \times 10^6$	$1.68 \times 10^6$
$A_{22}$ (N)	$1.07 \times 10^5$	$2.19 \times 10^5$	$9.95 \times 10^4$
$A_{33}$ (N)	$6.3 \times 10^4$	$1.26 \times 10^5$	$5.54 \times 10^4$
$B_{11}$ (N.m)	0	$3.37 \times 10^3$	0
$B_{22}$ (N.m)	0	$1.64 \times 10^3$	0
$B_{33}$ (N.m)	0	$1.66 \times 10^3$	0
$D_{11}$ (N.m <sup>2</sup> )	4.6	$1.07 \times 10^1$	4.6
$D_{22}$ (N.m <sup>2</sup> )	$5.6 \times 10^1$	$4.3 \times 10^1$	$2.0 \times 10^1$
$D_{33}$ (N.m <sup>2</sup> )	$1.0 \times 10^2$	$7.79 \times 10^1$	$3.7 \times 10^1$

236

237 By evaluating the previous two test cases, it is clear that the developed aeroelastic code is capable of  
 238 predicting the effect of added mass on the aeroelastic stability of composite hingeless rotor blades in  
 239 hover. In what follows, the effect of added mass on the twist morphing and the aeroelastic stability of  
 240 the composite blades is analysed. The blade characteristics are same as the one used in the previous  
 241 section, except that the layup arrangement here is no longer antisymmetric. This is because here the  
 242 bend-twist coupling is the main source of the twist morphing, and therefore, a unidirectional laminate  
 243 consisting of 6 plies with fibre angle  $\zeta$  in each wall is considered ( $[\zeta]_6$ ). This composite configuration  
 244 means that the  $\mathbf{B}$  components of the stiffness matrix (Eq. (2)) become zero, while other stiffness  
 245 values ( $\mathbf{A}$ ,  $\mathbf{D}$ ) are non-zero. This is because the lag-torsion coupling is the source of the twist change  
 246 in this paper. Note that in all cases from here on, the rotor angular velocity is  $\Omega_R=1000$  rpm, and it is

247 assumed that the centroid of the spar box is coincident with centre of mass of the blade at the quarter  
248 chord of the NACA 0012 airfoil.

249 The mass magnitude is considered here as a fraction of the blade mass itself, and denoted as  $\mu=m_p/m$ .  
250 First the effect of added mass on the twist change of the composite blade is examined. In this case, the  
251 aerodynamic loading isn't considered. Figure 4 shows the effect of different ply angles on the  
252 spanwise twist distribution of the blade for two locations of the mass. The upper domain is for the  
253 most aft position of the mass, while the lower domain is for the most forward location of the mass. By  
254 introducing the mass, the twist distribution of the blade changes, and the rate of change depends on  
255 the ply orientation. By moving the mass from the trailing edge to the leading edge of the cross-  
256 section, about  $\delta\phi_{tip} = 5.5^\circ$  tip twist change is induced when the ply angle is  $\zeta=30^\circ$ .



257

258 **Figure 4: The twist distribution of the blade for different layup angles and two chordwise positions**

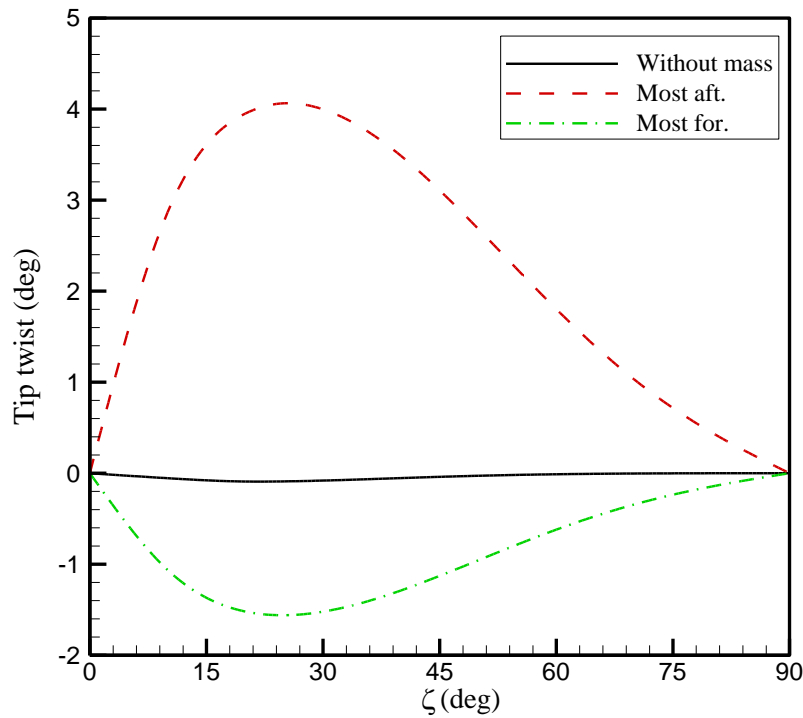
259

$$(\mu=0.05, x_p/R=1)$$

260 The effect of different ply angles on the tip twist value of the blade is determined and shown in Figure  
261 5. Here the spanwise location of the mass is at the tip of the blade. By increasing the ply angle, the tip  
262 twist first increases until a ply angle of about  $\zeta=25^\circ$ , and then decreases. This is true for both

263 chordwise locations of the mass. This ply angle is representative of the highest bending-twist coupling  
 264 in the composite blade in this configuration. It should be noted that the ply angle not only changes the  
 265 lag-torsion coupling but also it influences the rotating frequencies of the blade to some extent.

266



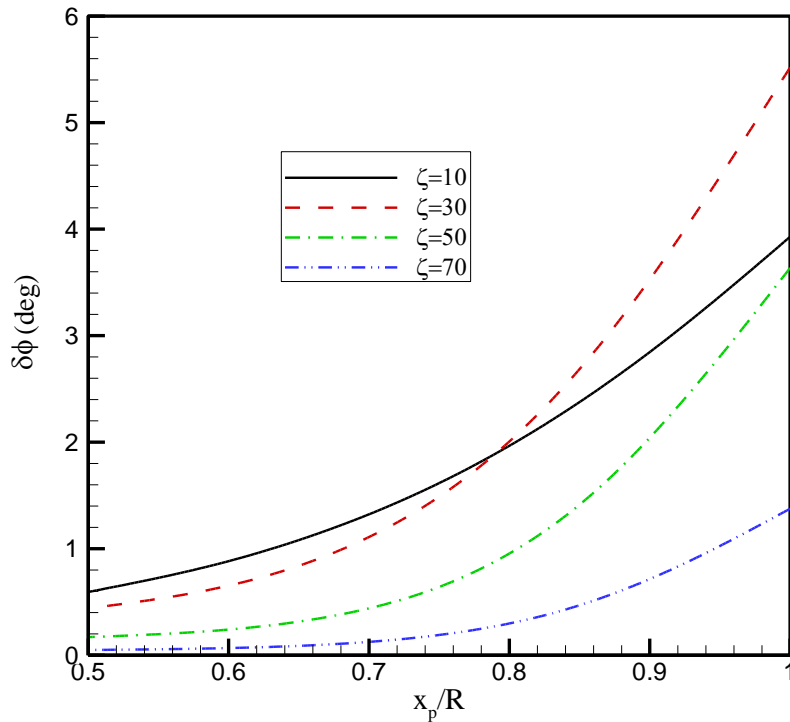
267

268 **Figure 5: The blade tip twist versus layup angle for ( $\mu=0.05$ ,  $x_p/R=1$ )**

269 Figure 6 demonstrates the effect of spanwise location of the point mass on the actuation range of  
 270 blade tip twist for different layup angles. The actuation range of blade tip twist is the difference of the  
 271 tip twist between the aft and forward locations of the point mass; and therefore indicates the potential  
 272 degree to which tip twist may be morphed in flight. The highest tip twist change occurs when the  
 273 mass is located at the tip of the blade for all ply angles. Moreover, the maximum tip twist change for  
 274 the spanwise location of the mass between  $0.5 < x/R < 0.8$  is for  $\zeta=10^\circ$ , while from here on to the tip of  
 275 the blade is for  $\zeta=30^\circ$ .

276



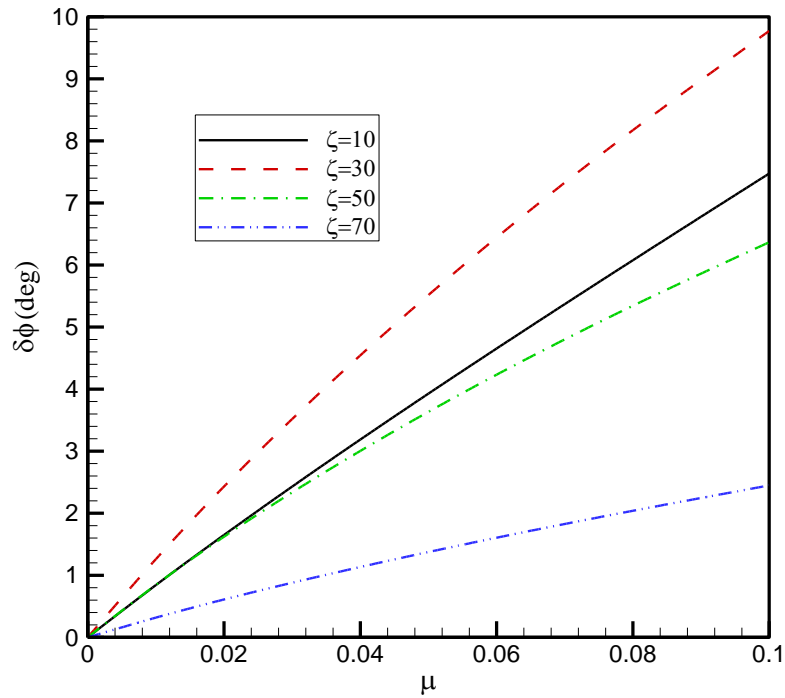


277

278 **Figure 6: The blade tip twist actuation range versus spanwise location for different ply angles ( $\mu=0.05$ )**

279 Finally, the effect of non-dimensional mass magnitude on the tip twist actuation range of the  
 280 composite blade is analysed and shown in Figure 7. By increasing the mass magnitude, the amount of  
 281 twist change increases for all ply angles. By considering all the results presented above, it is  
 282 highlighted here that the mass magnitude and location affects the twist change of the blade  
 283 dramatically. Depending on the spar configuration, there is one layup orientation that results in the  
 284 highest twist change in the blade. Therefore, in terms of the blade twist morphing, moving a mass  
 285 near the tip of blade has positive effects. Now in the what follows, the effect of the mass on the  
 286 aeroelastic stability of composite blade is discussed.

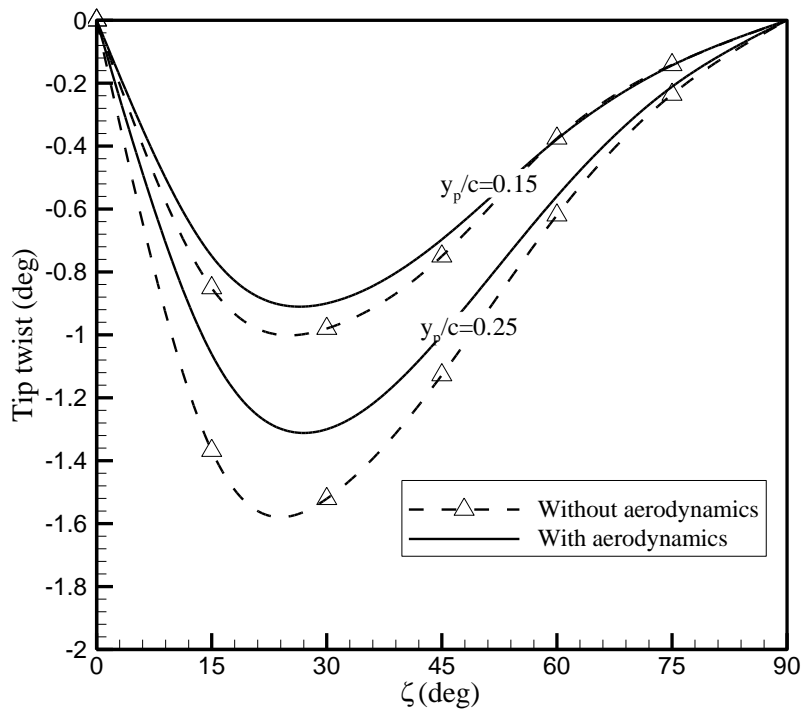
287 Figure 8 shows the effect of aerodynamic loads on the tip twist change of the blade when the mass is  
 288 located at the tip. In this case, the blade pitch angle is zero. It is clear that by adding the aerodynamic  
 289 loads and moments to the blade, the tip twist decreases. This is because the blade flap angle tends to  
 290 decrease the lag bending moment applied on the point mass. This highlights the importance of the  
 291 aerodynamic loads on the effectiveness of this morphing concept.



292

293

Figure 7: The blade tip twist actuation range versus mass magnitude ( $x_p/R=1$ )



294

295 Figure 8: The blade tip twist value with and without aerodynamic loads for different ply angles ( $\mu=0.05$ ,

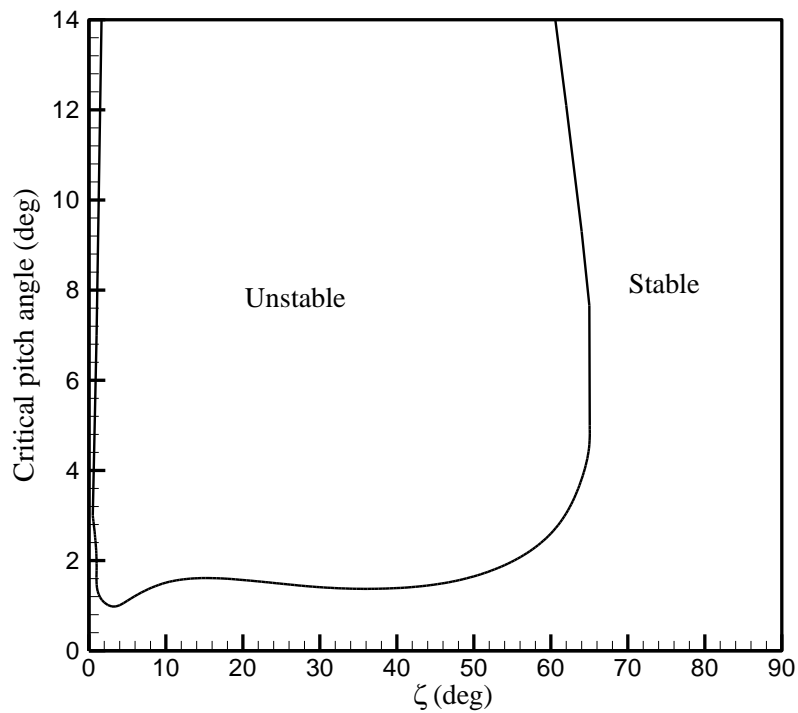
296

$x_p/R=1$ )

297 Figure 9 illustrates the aeroelastic stability of the composite blade with respect to different layup

298 angles. In this case the blade does not include any added mass. The blade is stable for ply angles

299 higher than about  $\zeta=65^\circ$  and smaller than  $\zeta=1^\circ$ . The instability domain is almost the same for all layup  
 300 angles between these two boundaries, but the domain tends to get larger for ply angles between  
 301  $1^\circ < \zeta < 10^\circ$ . It is noted that as this blade is unstable in the region between  $1^\circ < \zeta < 65^\circ$ , and the ply angle  
 302 that needs to be selected for designing the morphing mechanism must be outside this range. However,  
 303 it could be possible to design a cross-section to achieve required level of twist change subject to the  
 304 aeroelastic instability constraints.

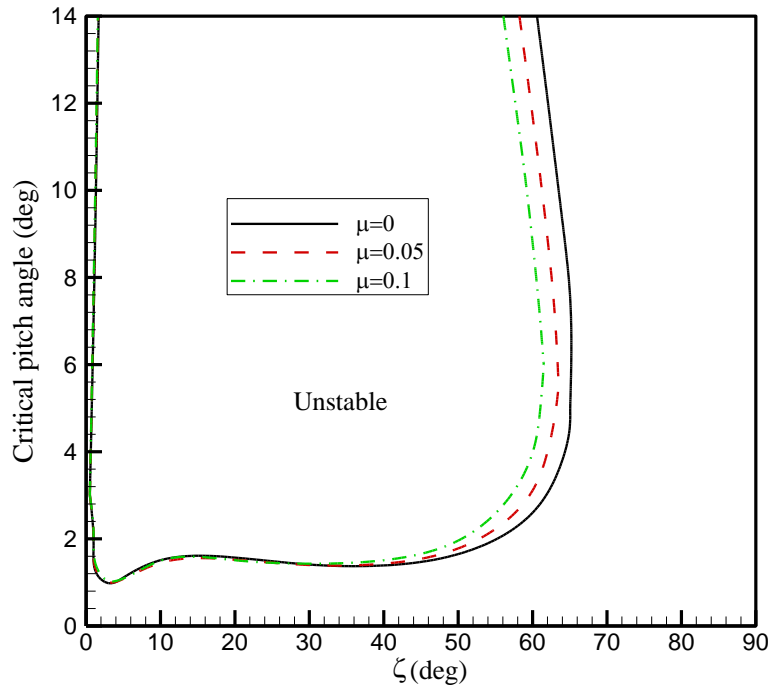


305

306 **Figure 9: The stability boundary of the composite blade without added mass**

307 The effect of nondimensional mass magnitude on the lead-lag aeroelastic stability boundaries is shown  
 308 in Figure 10. In this case, the mass is located at the tip of the blade on the shear centre of the section.  
 309 By increasing the mass magnitude, the unstable region decreases. Therefore, the point mass located at  
 310 the shear centre of the blade, has a stabilizing effect. The left boundary of the unstable region stays  
 311 unchanged by the additional mass until  $\zeta=40^\circ$ . Moreover, by introducing the mass to the blade, the  
 312 layup angle that the blade enters into the stable region decreases. Therefore, by adding a mass to the  
 313 blade and locating it exactly on the elastic axis of the blade, the aeroelastic stability of the blade  
 314 increases.

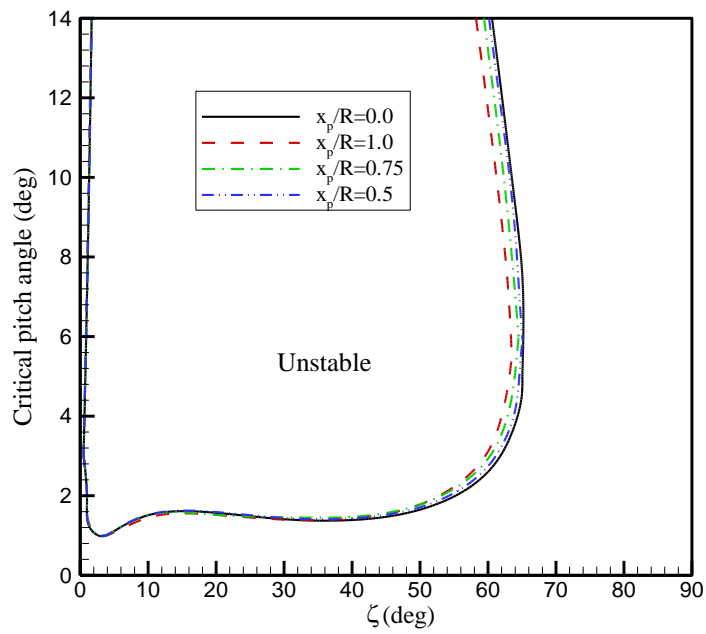
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316

317

Figure 10: Stability boundaries of the lag mode for different point mass magnitudes ( $y_p/c=0, x_p/R=1$ )



318

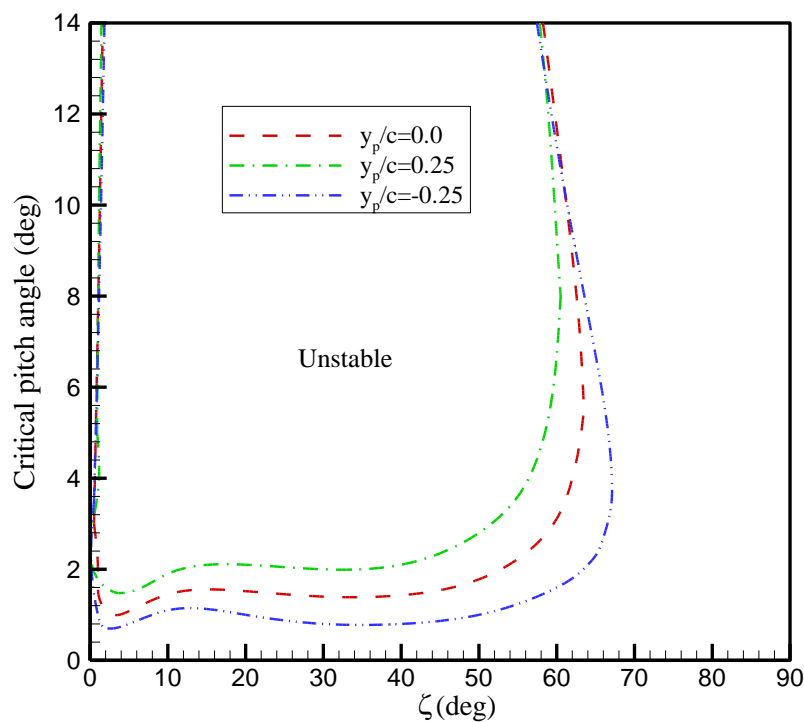
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320

Figure 11: Stability boundaries of the lag mode for different spanwise locations of the point mass ( $\mu=0.05, y_p/c=0$ )

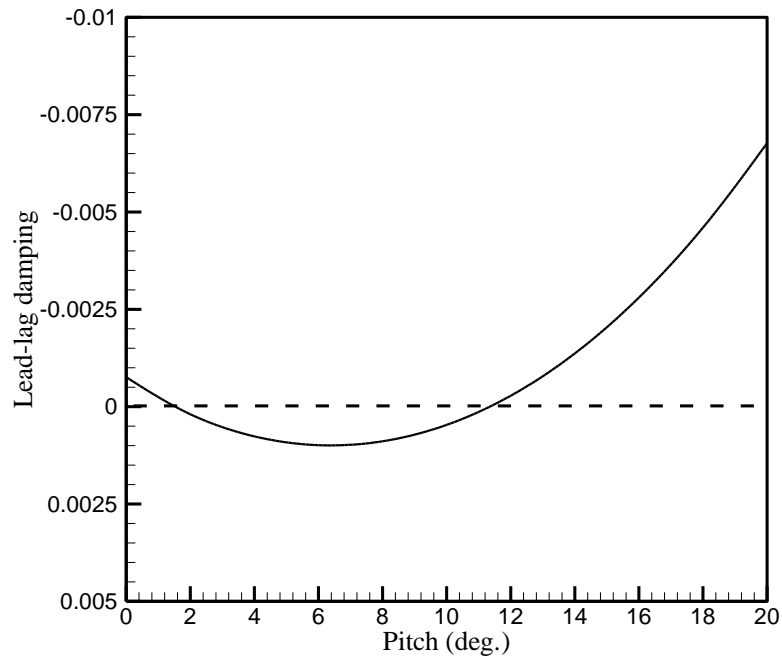
321 Figure 11 shows how the spanwise location of the added mass changes the lead-lag aeroelastic  
 322 stability boundaries. Here, the mass value is 5 percent of the blade weight ( $\mu=0.05$ ) and located at the  
 323 shear centre of the blade. By moving the mass from the mid-span to the tip of the blade, the unstable  
 324 region shrinks. It is noted that the point mass spanwise location has a minor effect on the lead-lag  
 325 aeroelastic instability. This is because the mass doesn't produce any lag moment and therefore, the  
 326 bend-twist coupling doesn't produce any torsional moment.

327 The effect of chordwise movement of the point mass on the lead-lag aeroelastic stability of the  
 328 composite blade is shown in Figure 12. By moving the mass from the leading edge to the trailing  
 329 edge, the size of the unstable region increases. This indicates that the chordwise location of the mass  
 330 critically changes the stability characteristics of the blade. Moreover, the ply angle at which the blade  
 331 enters the stable region increases by moving the mass toward the trailing edge. By moving the mass  
 332 toward the trailing edge, the lag moment due to the centrifugal force of the added mass, produces a  
 333 nose up pitching moment which in turn increases the aerodynamic loads. Therefore, moving the mass  
 334 toward the trailing edge has a destabilizing effect on the blade in this configuration.



335  
 336 **Figure 12: Stability boundaries of the lag mode for different chordwise locations of the point mass ( $\mu=0.05$ ,**  
 337  **$x_p/R=1$ )**

338 Finally, the aeroelastic lead-lag damping variation with respect to the blade pitch angle for the above  
 339 case ( $y_p/c = -0.25$ ) when the ply angle is  $\zeta = 60^\circ$  is shown in Figure 13. By increasing the blade pitch,  
 340 the damping first decreases and then increases. Therefore, the blade pitch angle in this case first has a  
 341 destabilizing effect and then has a stabilizing effect. This is the reason that in the above cases for a  
 342 constant ply angle, by increasing the pitch angle, the instability region decreases.



343

344 **Figure 13: Normalized lead-lag damping for different pitch angles ( $\mu=0.05$ ,  $x_p/R=1$ ,  $y_p/c=-0.25$ ,  $\zeta=60^\circ$ )**

345

346 **Conclusion**

347 The aeroelastic stability of the a composite hingeless rotor blade with an added mass is studied. The  
 348 added mass is used as an actuation method to change the twist of the blade. By moving the mass in the  
 349 chordwise direction, the bending-twist coupling of the composite layup of spar induces a torsional  
 350 moment on the blade. This torsional moment then changes the twist of the blade. As the added mass  
 351 may change the aeroelastic stability of the blade, the effect of its spanwise and chordwise location,  
 352 and also its magnitude, on the blade lead-lag stability boundaries is evaluated. Introducing the mass to  
 353 the blade, can change the twist distribution of the blade. The amount of twist induced in the blade  
 354 depends on the layup angle, mass magnitude, mass location, and angular velocity of the rotor. The  
 355 added mass influences the lead-lag aeroelastic instability of the blade. By moving the mass towards

356 the tip of the blade, the instability region decreases, while by moving the mass from the leading edge  
357 to the trailing edge of the blade, the unstable domain increases. Moreover, the mass magnitude also  
358 affects the stability boundaries of the blade. Therefore, this morphing concept may be subjected to  
359 aeroelastic instabilities, and in designing or modifying a blade to work with this morphing concept, it  
360 is essential to consider the aeroelastic stability as a design constraint.

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